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SPECTRAL MODELS BASED ON BOUSSINESQ EQUATIONS

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Abstract: For a stationary wave field, spectral models represent the surface wave motion sufficiently accurately. There are however two drawbacks in using nonlinear spectral models: a) The computational time involved when simulating the random wave field; the number of operations needed is $O(N^2)$ (N is the number of frequency components). b) Spectral models are usually one-equation reduction of the two-equation time domain model, which does not then have the same characteristics of the original model. Here we explore two approaches to solve these problems. First, we use the hybrid FFT technique (Bredmose et al. 2004) both to speed up the calculations and to incorporate higher order nonlinear terms. Second, we look at frequency domain transformation without reducing the model to one equation. Significant improvement in computational speed was obtained with the hybrid FFT approach. The models also show good agreement to the data reported by Mase and Kirby (1992).

INTRODUCTION

The nonlinear aspects of nearshore wave transformation and breaking has long been realized for accurate characterization of the nearshore wave field and its attendant effects on the nearshore environment, in particular that of sediment transport and morphology. While widely-used phase-averaged models such as SWAN (Booij et al. 1999) can give reasonably accurate predictions of waveheights in the nearshore, they cannot offer any additional information concerning the near bottom velocity skewnesses required for moment-based sediment transport calculations (e.g. Bailard 1981). Often, recourse is made toward coupling the linear waveheights with empirical relationships between waveheights and velocity skewnesses.

Phase-resolving nonlinear models, in contrast, are able to replicate the required velocity moments for incorporation into sediment transport formulations. These models are usually divided into *time-domain* models (in which the free surface is evolved in space and time) and *frequency-domain* or *spectral* models (in which the free surface is assumed to be time-periodic and the amplitudes of the free surface are evolved in space). Many of the most important recent developments in time-domain modeling have been in the area of *extended Boussinesq equations*, in which the weakly-dispersive, weakly-nonlinear Boussinesq equations (Peregrine 1967) have been extended to increase accuracy in regions where high nonlinearity and increased dispersion is necessary (Madsen et al. 1991; Nwogu 1993; Wei et al. 1995; Madsen and Schaffer 1998). In the frequency domain, models fall primarily into two classes: extended Boussinesq frequency-domain models (Kaihatu and Kirby 1994; Kofoed-Hansen and Rasmussen 1998; Kaihatu and Kirby 1998; Bredmose et al. 2004) and nonlinear mild-slope equation models (Kaihatu and Kirby 1992; Agnon et al. 1993; Kaihatu and Kirby 1995; Agnon and Sheremet 1997; Kaihatu 2001). Further developments have included improvements to treatments of the nonlinearity and dissipation mechanisms; a summary of the state of the art appears in Kaihatu (2003).

The recently-completed NOPP Nearshore Community Model (NearCoM; http://chinacat.coastal.udel.edu/~kirby/programs/nearcom/index.html) is a modular approach to nearshore and coastal modeling. The simulation of nearshore processes is divided up into wave, circulation and sediment transport modules, each with their own spatial and temporal scales, with coupling between the models provided by a computational backbone. This implies that the frequency-domain approach would be a sufficient paradigm for representation of the wave motion in the model system, particularly if the wavefield can be considered steady-state over the time scale of the circulation and sediment transport processes. One drawback of the nonlinear frequency domain models, however, is the computational time involved when simulating multi-frequency spectral wave propagation; the number of operations for N frequency components go up as $O(N^2)$ due to the nonlinear terms. A recently-developed alternative used for calculating the nonlinear terms in a frequency-domain model has been the hybrid time/frequency domain approach of Bredmose (2002). Building on the pseudo-spectral modeling techniques of Canuto et al. (1987), Bredmose (2002) implemented an approach in which the nonlinear terms were retained in the time domain. The individual components of the nonlinear terms were transformed into the frequency domain, along with all associated derivatives with respect to time and space. The time series of each component was then calculated via Fast Fourier Transform (FFT), and the nonlinear products assembled. The combined products in time series form were then re-transformed into the frequency domain using FFT and applied to the rest of the frequency domain equation. All told, this reduced the number of computations to $O(N\log N)$, a significant savings. The implication is, however, that a strong connection between the frequency domain and the time domain exists, otherwise there would be no analogue for the frequency domain nonlinear terms in the time domain. By default, this causes a change in focus from nonlinear mild-slope equations to extended Boussinesq equations, since the development of nonlinear mild-slope equations assume periodic solutions *apriori*.

This, however, also forces the question of which extended Boussinesq model to use. Various one-equation reductions of the models of Madsen et al. (1991) and Nwogu (1993) seem like obvious (and equivalent) choices, since the differences between the various models were most apparent in their time-domain formulations. However, Kaihatu (2003) showed the potential consequences of the incorrect choice of time domain model from which to begin, as well as the potential difficulties of developing frequency domain models from one-equation reductions of the two-equation time domain models. (These difficulties were briefly mentioned in Chen and Liu 1995 and Kaihatu and Kirby 1998, but not further explored).

In this study we implement the hybrid FFT technique into a version of the Boussinesq model derived by Madsen and Schaffer (1998) and Veeramony (1999). We make use of the relative ease of calculating nonlinear terms to retain full nonlinearity, using only the dispersion parameter μ (=kh, where k is the wavenumber and h the water depth) as a truncation measure. We then also look at the frequency domain transformation of the model of Nwogu (1993), done without reduction to a single equation. We then compare the two types of models to data.

EXTENDED BOUSSINESQ MODEL OF MADSEN AND SCHAFFER (1998) AND VEERAMONY (1999) – TRANSFORMATION INTO FREQUENCY DOMAIN:

The extended Boussinesq model of Madsen et al. (1991) was enhanced by the addition of terms which fortify the model's nonlinear characteristics and dispersion comparisons to that of fully-dispersive linear theory. This enhancement was independently derived by Madsen and Schaffer (1998) and Veeramony (1999) (written for one horizontal dimension):

$$\eta_t + [(h+\eta)\bar{u}]_x = 0 \qquad (1)$$

$$\bar{u}_t + \bar{u}\bar{u}_x + g\eta_x + Bgh^2\eta_{xxx} + \left(B - \frac{1}{3}\right)h^2\bar{u}_{xxt} - hh_x\bar{u}_{xt} + \Lambda = O\left(\mu^4\right) \qquad (2)$$

where $\bar{u}$ is the depth-averaged horizontal velocity, η is the free-surface elevation, g is gravitational acceleration, B is a free parameter to tune the dispersion agreement with linear theory (B=-1/15 leads to acceptable agreement) and the subscripts denote differentiation with respect to x or t. The term denoted Λ contains terms of $O(\delta\mu^2, \delta^2\mu^2, \delta^3\mu^2)$, thus retaining all terms up to $O(\mu^4)$. The model is thus "fully nonlinear" for $O(\mu^2)$.

Our first step is to reduce (1) and (2) to a single wave equation for η. We then derive a frequency domain equation from the result in two forms. The *weakly-nonlinear* frequency domain equation retains only quadratic nonlinearity, while the *fully-nonlinear* frequency domain equation retains all nonlinearity for the order of dispersion. We then check the properties of the wave equation against that of the appropriate Stokes theory. Figure 1 shows the second harmonic amplitude predicted by the weakly and fully nonlinear wave equation models against that predicted by the original two equation model (Equations 1 and 2); these amplitudes are normalized against the second harmonic amplitude predicted by second order Stokes theory, and serves as a check on the changes enacted on the properties of the system by the wave equation reduction. It is apparent from the figure that the prediction of the second harmonic amplitude is unchanged between the two-equation model and the fully-nonlinear wave equation model, while the weakly-nonlinear wave equation model shows a very strong decay in the accuracy of the second amplitude prediction relative to the fully-nonlinear wave equation. Other properties of the full two-equation model were also compared to the wave equation models; it can be shown that the third harmonic amplitude, as well as the nonlinear dispersion, is overpredicted by the fully nonlinear wave equation model relative to the two-equation model. These same properties are strongly underpredicted by the weakly nonlinear wave equation model.

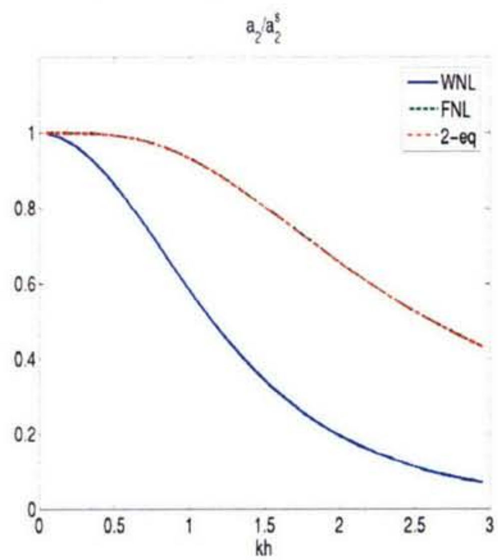

Figure 1. Amplitude of second harmonic normalized by Stokes second order amplitude, as a function of relative depth *kh*.

We now transform the model into the frequency domain. Assumption of time periodicity and slow spatial variation of the free surface amplitudes yields the following:

$$ib_1 A_{nx} + b_2 A_n + ib_3 h_x A_n + ib_4 k_{nx} A_n = \Gamma \tag{3}$$

where A_n is the complex amplitude of the (time-periodic) free surface, b_1 through b_4 are linear transformation coefficients, the subscript n is an index for the frequency components, and the term Γ is:

$$\Gamma = (h\overline{u}\overline{u}_x)_x - (\overline{u}\eta)_{xt} - \left(B - \frac{1}{3}\right)h^2(\overline{u}\eta)_{xxxt} + hh_x(\overline{u}\eta)_{xxt} + (h\Lambda)_x \tag{4}$$

Quadratic nonlinearity (shown explicitly in Eq. 4) is straightforwardly treated using the usual convolution methods with associated resonant triad reduction (e.g. Freilich and Guza 1984). The cubic and quartic nonlinear terms in Λ, however, are not. Even the reduction to weak nonlinearity, however, still require significant computational time. To overcome this, then, we use the hybrid FFT approach of Bredmose (2002) for the nonlinear terms.

We first take the derivatives of all the terms inside the nonlinear products in Λ in the frequency domain. Thus:

$$\eta_t = -i\omega_n\hat{\eta}_n \qquad \eta_x = ik_n\hat{\eta}_n \tag{5}$$

and we use linear transformations to go between velocity and free surface elevations. Then we use an inverse FFT to obtain the time series for each term in the nonlinear products. We then multiply the time series together to assemble the nonlinear terms in the time domain, then FFT the completed nonlinear terms into the frequency domain. Thus we completely describe the nonlinearity in the frequency domain, and at a fraction of the computational cost. Additionally, the retention of cubic and quartic nonlinearity offers no significant additional computational difficulties, since in the time domain this would amount to an additional multiplication of a time series.

FREQUENCY DOMAIN TRANSFORMATION OF EXTENDED BOUSSINESQ EQUATIONS OF NWOGU (1993)

One of our motivations for this work initially was to investigate a suitable treatment of the extended Boussinesq model of Nwogu (1993). We chose this model because it forms the core of the University of Delaware FUNWAVE model (Wei et al. 1995) and we had thought that a suitable frequency domain analogue of this would be useful. However, as noted by Kaihatu (2003), the transformation of the original equations into a nonlinear wave equation for the free surface, and subsequently into the frequency domain, is far from straightforward. We circumvent these difficulties by transforming the original equations directly into the frequency domain.

Beginning with the original equations of Nwogu (1993):

$$\eta_t + [(h+\eta)u_\alpha]_x + \left[\left(\frac{z_\alpha^2}{2} - \frac{h^2}{6}\right)hu_{\alpha_{xx}} + \left(z_\alpha + \frac{h}{2}\right)h(hu_\alpha)_{xx}\right]_x = O(\delta\mu^2, \delta^2) \tag{6}$$

$$u_{\alpha_t} + g\eta_x + u_\alpha u_{\alpha_x} + z_\alpha\left(\frac{z_\alpha}{2}u_{\alpha_{xxt}} + (hu_{\alpha_t})_{xx}\right) = O(\delta\mu^2, \delta^2) \tag{7}$$

where u_α is the horizontal velocity at a particular location z_α in the water column. This position is chosen such that the resulting linear dispersion relation is a good match to linear theory. Nwogu (1993) determined that z_α=-0.552h leads to an acceptable match. Instead of combining the equations, we assume that both the free surface η and the velocity u_α are periodic with time, and that the usual triad resonance conditions can be applied. This leads to two frequency domain equations (one continuity and one momentum equation):

$$
\begin{aligned}
&-i\omega_n A_n + \left\{\left(1-\left[3\alpha+2\sqrt{1+2\alpha}\right]k_n^2h^2\right)h_x - 3k_n\left(\alpha+\frac{1}{3}\right)h^3k_{nx} - ik_n^3h^3\left(\alpha+\frac{1}{3}\right)+ik_nh\right\}B_n \\
&+\left\{\left(1-3k_n^2h^2\left(\alpha+\frac{1}{3}\right)\right)h + 2i\left[3\alpha+2\sqrt{1+2\alpha}\right]k_nh^2h_x + 3i\left(\alpha+\frac{1}{3}\right)h^3k_{nx}\right\}B_{nx} = \\
&-\frac{i}{4}\left(\sum_{l=1}^{n-1}(k_l+k_{n-l})(A_lB_{n-l}+B_lA_{n-l})e^{i\theta_{l,n-l}} + 2\sum_{l=1}^{N-n}(k_{n+l}-k_l)(A_l^*B_{n+l}+B_l^*A_{n+l})e^{i\theta_{n+l,-l}}\right)
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&gA_{nx} + igk_nA_n + \left\{i\omega_n\left[\alpha k_n^2h^2-1\right] + \omega_n\alpha h^2k_{nx} + \omega_nh\left[2\sqrt{1+2\alpha}-2\right]h_xk_n\right\}B_n \\
&+\left[2\omega_n\alpha k_nh^2 - i\omega_nh\left[2\sqrt{1+2\alpha}-2\right]h_x\right]B_{nx} = \\
&-\frac{i}{4}\left\{\sum_{l=1}^{n-1}(k_l+k_{n-l})B_lB_{n-l}e^{i\theta_{l,n-l}} + 2\sum_{l=1}^{N-n}(k_{n+l}-k_l)B_l^*B_{n+l}e^{i\theta_{n+l,-l}}\right\}
\end{aligned}
\tag{9}
$$

where:

$$
\alpha = \left(\frac{z_\alpha^2}{2h^2}+\frac{z_\alpha}{h}\right) \tag{10}
$$

and B_n is the complex amplitude of u_α. (We note here that these equations were derived independently by Bredmose 2002). In contrast to Kaihatu and Kirby (1998), who optimized the shoaling behavior of their wave equation model with a shoaling parameter, the shoaling behavior of the present model should closely mimic that of the original time domain equation with no further optimization. Figure 2 shows a test of linear shoaling behavior from the model for several wave periods compared to that of linear theory. Even when starting at $kh \sim 2$, the model shows good fidelity to the shoaling behavior of linear theory.

TEST OF MODELS: EXPERIMENT OF MASE AND KIRBY (1992):

Mase and Kirby (1992) conducted a laboratory experiment in which random waves were propagated up a slope and the free surface measured at various locations on the slope. The unique aspect of their Case 2 experiment, to which we compare, is that the Pierson-Moskowitz spectrum at the wavemaker had a peak frequency of 1 Hz; this leads to kh=1 at the spectral peak and over the constant-depth part of the basin. This is a demanding test for nearshore nonlinear wave models because of the relatively strong dispersion effects. The reader is referred to Mase and Kirby (1992) for the salient details of the experiment.

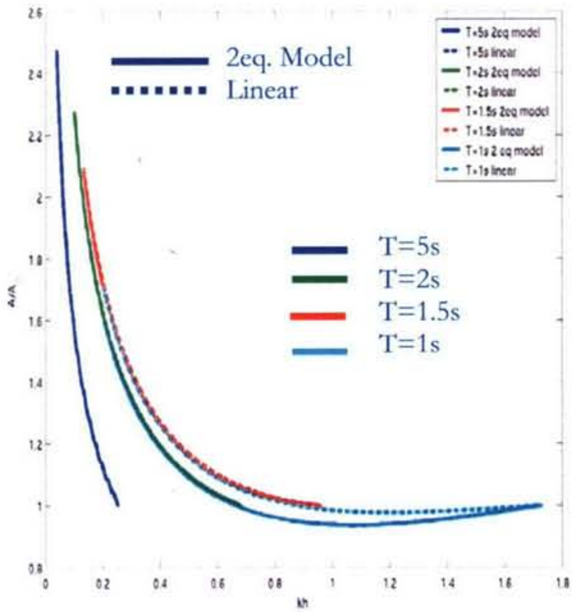

Figure 2. Test of linear shoaling behavior of frequency domain transformation of model of Nwogu (1993).

Since wave breaking and dissipation occurs in the experiment, we require a dissipation mechanism in the models. We use the dissipation mechanism of Thornton and Guza (1983) with the frequency distribution mechanism of Mase and Kirby (1992); this mechanism splits the distribution of dissipation between one that is independent of frequency and another that weights the dissipation by the square of the frequency. Kirby and Kaihatu (1996) discuss the implications of the split on the resulting physics of wave breaking and give physical reasoning for the frequency-squared distribution. The dissipation term was added to the wave equation model, and to the momentum equation of the two-equation model.

We show comparisons of the weakly and fully nonlinear wave equation models (Eq. 4 and truncations thereof) and the two-equation model (Eq. 8 and 9) to the data in Figure 3. The models based on Eq. 4 and 5 were truncated at N=400 frequency components, while the two-equation model simulation was truncated at N=300 frequency components; this was simply the result of miscommunication between the co-authors. The comparisons are made at four of the twelve gage locations in the experiment, but are spaced to show the various stages of evolution. Offshore (h= 0.35m) all models compare relatively well to the data. As shoaling begins to affect the wavefield (h=0.25m) the models lose accuracy in the high frequency range, quite probably due to the underprediction of linear shoaling of the models compared to linear theory. Nonlinear effects in shoaling begin to become apparent at h=0.125 m, and the second harmonic of the frequency peak begins to show some amplification above the background spectral energy. Interestingly, the weakly nonlinear two-equation model compares very well here, better than the weakly and fully nonlinear wave equation models. At h=0.075m, wave breaking is predominant, and the two-equation model overpredicts the tail of the spectrum, quite possibly because of the rather *ad-hoc* way the dissipation was added to the momentum equation. In this case the wave equation models, expressed entirely in terms of the free surface elevation, fare better, with the fully nonlinear model comparing best. However, it is remarkable that these extended Boussinesq models, which are enhancements of the weakly dispersive, weakly nonlinear classical Boussinesq model, are able to compare well to the data in a depth regime well outside the weakly-dispersive limit.

While the models seem to compare well to each other, it should be noted that the two-equation model, with its resonant triad nonlinear summations, required O(1 day) to simulate this experiment, while the wave equation models (with the hybrid FFT approach for calculating the nonlinear terms) required ~17 minutes for the fully-nonlinear model on a Dell Precision 530 (2GHz processor). The computational advantage of the hybrid FFT approach is clear.

STEEPNESS-LIMITED WAVE BREAKING

It has been hypothesized (Kirby and Kaihatu 1996) that the dissipation model of Thornton and Guza (1983) was not designed to replicate the local effects of breaking, and that various frequency dependencies of the distribution over the frequency range are insufficient attempts at incorporating the spectral signature of these local effects. It was also hypothesized that the momentum transfer to the water column from the overlying breaker (be it spilling or plunging) acted like an impulsive force on the water column, leading to a larger initial dissipation at the point of breaking with a gradual falloff as the wave height decreases. Kirby (personal communication, 1996) then developed a dissipation function which involved the frequency domain transformation of the steepness-limited function included in the FUNWAVE model. One feature of the dissipation function involves the evaluation of the coefficients of the function in the time domain, thus requiring toggling between time and frequency domains. This makes it a natural fit for the hybrid FFT model, which involves this toggling anyway. Initial tests of the dissipation model have proved promising but more work is required.

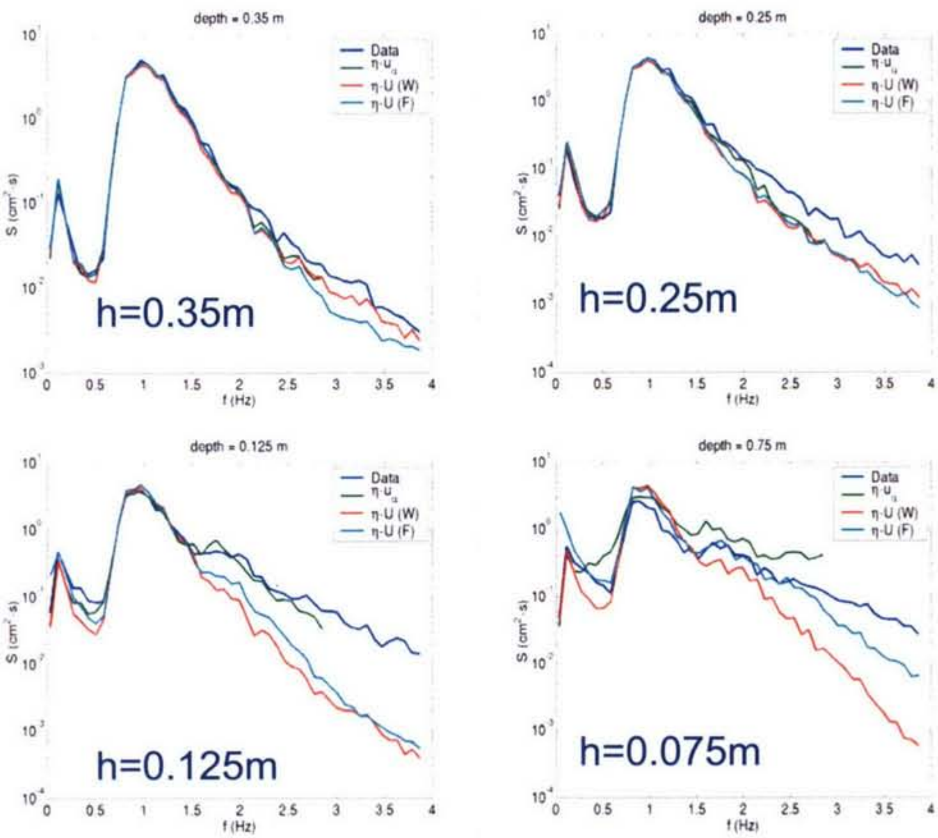

Figure 3. Comparison of wave spectra from wave equation models (cyan: fully nonlinear; red: weakly nonlinear) to two-equation model (green) and to Case 2 data of Mase and Kirby (blue).

TWO-DIMENSIONAL PARABOLIC MODEL

The extended Boussinesq equation model of Nwogu (1993) was also transformed into the frequency domain using the parabolic approximation (Radder 1979). This then involves three equations (one continuity and two momentum equations), with the normal scaling of the parabolic approximation (which limits the alongshore wave phase representation of the incident wave field) assumed to be sufficient for sizing the alongshore horizontal velocity v_α. The equations are written schematically as:

$$
\begin{aligned}
&\lambda_1 A_n + \lambda_2 B_n + \lambda_3 D_n + \lambda_4 B_{nx} + \lambda_5 D_{nx} + \lambda_6 B_{ny} + \lambda_7 D_{ny} + \lambda_8 B_{nxy} + \lambda_9 D_{nxy} + \lambda_{10} B_{nyy} \\
&+ \lambda_{11} D_{nyy} + \lambda_{12} B_{nyyx} \\
&= -\frac{i}{4}\left[\sum_{l=1}^{n-1}(k_l + k_{n-l})(A_l B_{n-l} + B_l A_{n-l})e^{i\Theta} + 2\sum_{l=1}^{N-n}(k_{n+l} - k_l)(A_l^* B_{n+l} + B_l^* A_{n+l})e^{i\Omega}\right]
\end{aligned} \tag{11}
$$

$$\Psi_1 A_n + \Psi_2 B_n + \Psi_3 D_n + \Psi_4 B_{nx} + \Psi_5 A_{nx} + \Psi_6 D_{ny} + \Psi_7 D_{nxy}$$
$$= -\frac{i}{4}\left(\sum_{l=1}^{n-1} (k_l + k_{n-l}) B_l B_{n-l} e^{i\Theta} + 2 \sum_{l=1}^{N-n} (k_{n+l} - k_l) B_l^* B_{n+l} e^{i\Omega} \right) \tag{12}$$

$$\Phi_1 A_{ny} + \Phi_2 B_n + \Phi_3 D_n + \Phi_4 B_{ny} + \Phi_5 B_{nx} + \Phi_6 B_{nxy} + \Phi_7 D_{nyy} + \Phi_8 D_{ny}$$
$$= -\frac{i}{4}\left(\sum_{l=1}^{n-1} (k_{n-l} B_l D_{n-l} + k_l B_{n-l} D_l) e^{i\Theta} + 2 \sum_{l=1}^{N-n} (k_{n+l} B_l^* D_{n+l} - k_l B_{n+l} D_l^*) e^{i\Omega} \right) \tag{13}$$

where:

$$\Theta = \int (k_l + k_{n-l} - k_n) dx$$
$$\Omega = \int (k_{n+l} - k_l - k_n) dx \tag{14}$$

λ, Ψ, and Φ are complicated transformation coefficients, and D_n is the complex amplitude of the alongshore horizontal velocity v_α. Optimum numerical techniques are being investigated at present, with Crank-Nicholson discretization with multiple level iteration among one of the most straightforward (if cumbersome) to implement.

CONCLUSION

For a stationary wave field, nonlinear spectral models have been proven to represent the surface wave motion sufficiently accurately. In this paper, we try to address two drawbacks in using nonlinear spectral models. In time domain modeling, it has been shown that retention of higher-order nonlinear terms is essential to model waves which are close to breaking. However, with the traditional formulation, the computational time involved when simulating a random wave field is high even for weakly nonlinear formulations that contain quadratic nonlinearity. The second concern is that spectral models are usually derived from two-equation time domain models which are then reduced to one-equation. This reduction is not unique and each of the resulting one-equation models does not then have the same characteristics of the original model. Here we explore two approaches to separately solve these problems.

The fully nonlinear extended Boussinesq equations given by Madsen and Schaffer (1998) and Veeramony (1999) was transformed into frequency domain. The nonlinear terms are solved using the FFT method described by Bredmose et al. (2004). In addition to significantly reducing the computational time, this allows us to retain higher-order nonlinear terms in the frequency domain model. The model performs significantly better than the weakly nonlinear one-equation formulation when compared to the data of Mase and Kirby (1992). The frequency domain models, which are formulated assuming weak dispersion are nevertheless able to compare well to the data in a domain far outside the weakly dispersive regime.

To address the second problem, we look at the frequency domain transformation of the continuity and momentum equations separately. The two-dimensional parabolic model was derived from the extended Boussinesq equations of Nwogu (1993), without the traditional reduction to a one-equation model. Optimal numerical techniques to solve this system of equations are currently being investigated. This model is expected to perform better than the one-equation models for shoaling, refraction and higher-order moments such as skewness and asymmetry.

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