Army Research Laboratory

The Application of Models of Decision Making During Uncertainty to Simulations of Military Command and Control Systems

by Sam E. Middlebrooks and Brian J. Stankiewicz

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14. ABSTRACT

Most military decision making requires a sequence of actions. These actions may include aspects of intelligence gathering, troop movement, artillery fire, etc. Typically, these actions are tied to a specific goal that might include securing a region or disrupting the enemy forces. Furthermore, when one is making the necessary decisions to reach the specific goal, there may be much uncertainty about the situation. Where exactly are the enemy troops? What is their objective? The use of Bayesian statistics makes it possible to compute optimal performance for military-like situations. This research develops a model that provides the theoretical best performance that can be achieved in the task. A sample task that can illustrate this condition is a seek-and-destroy mission in which there is an enemy at an unknown location within a certain region. With reconnaissance that is imprecise and artillery that is not always accurate the mission is to destroy the enemy. Each action (intelligence gathering and artillery) comes at a specific cost. Furthermore, succeeding at destroying the enemy generates a reward, and declaring "mission accomplished" when the enemy is still alive generates a significant cost.

The current paradigm for the description and understanding of the nature of command and control (C2) system (C2S) operations and performance within the U.S. Army is undergoing a radical change. Tactical battlefield C2 is extremely complicated to orchestrate and conduct in an effective manner. With the introduction of a myriad of new information systems, sophisticated new weapons with unprecedented capabilities for lethality, new requirements for battlefield integration, and the total reorganization of force structures into a new modular concept, the need for effective understanding of how this force structure can work effectively as a system entity increases dramatically. The C2S has become complicated to the point as to escape the ability for intuitive understanding of how individual components or subsystems can improve or degrade the operation of the overall system. The goal of this research is to understand the cognitive limitations associated with sequential decision making with uncertainty in these types of situations through predictive computer simulation. When empirical research investigating optimal decision making during uncertainty is combined with evolving simulations of military C2, the potential now exists to correlate optimal decision making performance with actual results on the battlefield. The benefit of these simulations will be to enable the evaluations of new C2 subsystems that take into account optimal decision-making performance as an evaluation metric.

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1. Introduction

The traditional paradigm for the description and understanding of the nature of command and control (C2) system (C2S) operations and performance within the U.S. Army is currently undergoing a radical change. The U.S. Army Field Manual (FM) 6-0 (Army, 2003) defines the C2S as "the arrangement of personnel, information management, procedures, and equipment and facilities essential for the commander to conduct operations." Tactical battlefield C2 is an extremely complicated action to orchestrate and conduct in an effective manner in its own right. However, with the introduction of new information systems such as the Army Battle Command System (ABCS) (Army, 2002), sophisticated new weapons now exist with unprecedented capabilities for lethality and requirements for battlefield integration. As they are contributing to a total reorganization of force structures into the new modularity concept, the need for effective understanding of how this system can work effectively as a system entity increases exponentially. The fact is that the complexity of the modern C2S has surpassed the ability for an intuitive understanding of how individual components or subsystems can improve or degrade the operation of the overall system. This situation poses the question of how to further develop and improve the performance of the C2S without making changes that might actually degrade its effectiveness. From this it becomes apparent that some systematic approach is needed to predict and evaluate the effects that changes, additions, and improvements in this system will have on its overall ability to conduct battle space management.

1.1 The Command and Control System's Demands for Decision Making

Previous research on this topic (Middlebrooks, 2003; Middlebrooks et al., 1999a; Wojciechowski, Plott, & Kilduff, 2005) has developed a paradigm for the systematic evaluation of the C2S from the system level viewpoint. A basic premise of this approach is that all observable characteristics of live tactical operations centers (TOCs) in the field can be used in the development of quantitative predictive models of various system components for use in a simulation of the complete C2S. Some of these characteristics include such things as the quantity and quality of communications messages through the TOC from various digital systems, quality and timeliness of intelligence information about the enemy, numbers and expertise levels of team members present in the TOC at any given time, individual and group interactions, physical setting of the TOC, and environmental conditions, to name a few. However, a striking limitation of this approach is in its inability to simulate and predict cognitive performance of individuals and teams in areas such as situation awareness, knowledge elicitation, error generation, individual versus team performance, and decision making. An ability to predict the optimal decision required for success can be extremely useful as a performance measure for use in describing the overall effectiveness of the system.

1.2 The Model of Optimal Decision Making

This research integrates basic research in decision making that is being conducted at the Psychology Department of the University of Texas with applied research in unit of action (UA) TOC operations being conducted at the Fort Hood Field Element of the U.S. Army Research Laboratory's (ARL's) Human Research and Engineering Directorate. Initial work on this topic (Middlebrooks & Stankiewicz, 2005) was supported by a grant from the Congressionally funded University XXI program in a partnership between the faculty and staff of The University of Texas at Austin through the Institute for Advanced Technology and ARL. It is the goal of this research to develop predictive simulations of the C2S UA performance that can be used in the evaluations of changes in the system or the addition or modification of system subcomponents. For example, what might the effect be on the overall ability of the UA TOC to conduct battle space management from the addition of a new intelligence system that allows information about the enemy to have a maximum age of 1 hour before it becomes obsolete versus a maximum age of 4 hours? One intuitive conclusion that could be deduced from this new capability is that it would significantly increase the commander's understanding of the enemy situation because the information is always more current than before. However, this intense stream of information might cause the commander to become more focused on the instantaneous situation on the battlefield and lose situation awareness of longer term developments with a resulting degradation in the ability to make effective decisions about how to react to the threat. The effective ability of predictive simulations of these types of environments is based on how well they account for the myriad of variables stemming from physical activities and the human's cognitive ability to react to those variables. This current research is a first step in allowing simulations of system performance to account for limitations in human cognitive performance abilities.

2. Method

The methodology used in the operation of this model involves two components. The first is defined as the "belief vector" (BV). This represents the current knowledge that the operator has about the system. Using this knowledge, the operator decides what to do next or what action to take in the pursuit of the mission goal. This model can be generalized to many different decision-oriented situations where the operator is confronted with a goal-directed task. In the pursuit of this task, the operator may decide to seek information about the condition of the current situation, s/he may decide to take some action to achieve the goal, and at some point, s/he will decide that s/he has achieved the goal, thus terminating the decision mission sequence. An example of this scenario is provided in the hospital emergency room where a doctor is faced with the mission goal of successfully treating a sick patient. The doctor may administer a medication (take an action) or may perform a medical test (seek information) to attempt to identify the patient's condition. A complete series of medications and tests may be performed before the doctor achieves the belief that the patient has been cured. If the doctor terminates the sequence before the patient is cured, the patient may die. If the doctor prolongs the sequence beyond the point when the patient is cured, a substantial unjustified cost is the result. This medical sequence is an illustration of gathering information and taking actions until a belief is achieved that the goal has been reached. Another example is provided in the military context where a ground force commander is given an order to seek, find, and destroy an enemy that is at an unknown location. The commander may seek information (e.g., by flying an unmanned aerial vehicle [UAV] reconnaissance mission) or s/he may take a direct action to destroy the enemy by firing artillery at a suspected location occupied by the enemy. An entire sequence of UAV and artillery missions may be performed in some goal-directed order until the commander believes that the enemy has been destroyed. At this point, the commander decides that the mission is complete because the enemy is believed to be destroyed and decides to terminate the action. This process is modeled through the use of the Markov decision process analysis to determine the belief vector and the use of conditional probability logic to determine the next action to take, based on an evaluation of the current BV.

2.1 Determination of the Belief Vector

It is important to recognize that most decisions that are made are not "one off" decisions in which the decision is made and then the rewards reaped or the punishment endured. Instead, most decisions that are made have future ramifications and affect the options and decisions that are available later. One challenge faced by any decision maker is the uncertainty that s/he has about the true state of the system. In most circumstances, the true state of the system is unknown or hidden. That is, it cannot be directly observed. For example, in military decisions, often there is uncertainty about an enemy's position, strength, and morale. Given that the true state is hidden, there are things that can be done to reduce the decision maker's uncertainty about these states. For example, the decision maker may try to determine the enemy's position by sending reconnaissance to a location where the enemy is believed to be located. When the reconnaissance returns with either an "enemy sighted" or "enemy not sighted" report, decision makers must update their belief about the location of the enemy.

If the observations and actions were all deterministic, revising a belief would be relatively simple. However, in almost all conditions, the observations and actions are probabilistic. That is, the probability of getting an observation, given the true state of the environment, is not necessarily 0.0 or 1.0, or in the example given, there is a certain non-zero probability that the reconnaissance mission was sent to the correct location and will miss the enemy and send a report of "enemy not sighted". Furthermore, there may be a non-zero probability that the reconnaissance mission falsely sent a report of "enemy sighted" (or false alarmed) when the enemy was not actually at the location.

Given that the observation and actions are probabilistic, revising a belief, given an observation and an action, can become cognitively difficult. Furthermore, evaluating the added benefit of a specific piece of equipment that changes these probabilities can also become difficult. This research focuses on a task that is commonly faced by decision makers in the military, namely, the seek-and-destroy task. In this task, the decision maker is trying to localize and destroy an enemy within a specific region. At the decision maker's disposal are actions that allow information to be gained about the true state of the system (i.e., the location of the enemy) in addition to changing the state of the system (e.g., moving the enemy from a specific location to the state of destroyed). The former actions are reconnaissance actions and the latter are artillery actions. The outcomes of these actions are probabilistic. That is, reconnaissance actions will not always detect the enemy when a sensor is sent to the enemy's location. Furthermore, the reconnaissance may also falsely report that the enemy is seen at a location in which the enemy is not located. In addition, the artillery will not always kill the enemy when striking it, which is characterized as moving the enemy from being alive at a certain location to the "destroyed" state.

2.1.1 The Optimal Observer

To best evaluate performance in a task that leads to uncertainty and probabilistic actions, it is useful to define the optimal performance within the task. The optimal performance can be calculated with Bayesian statistics. However, because of the nature of the current type of task, simple Bayesian statistics are insufficient. That is, with simple Bayesian statistics, the likelihood of the true state of the system can be optimally estimated. However, this likelihood does not indicate what action should be selected. In order to select action, not only must the current state be calculated, given the previous actions and observations, but the optimal action to be performed in a given belief state must also be calculated. This is done to verify whether a belief state has a particular probability distribution across all the possible states in the environment.

A variation of classical Bayesian statistics that may well add some more predictive power for sequential decision making during uncertainty is the *Partially Observable Markov Decision Processes* (POMDP) (Cassandra, 1998; Cassandra, Kaelbling, & Kurien, 1996; Cassandra, Kaelbling, & Littman, 1994; Kaelbling, Littman, & Cassandra, 1998; Legge, Klitz, & Tjan, 1997). By defining the *State Space, Observation Vector, Transition Matrix,* and the *Reward Structure*, we can compute the expected reward for a particular action. In the following sections, a description of these actions is provided. In addition, a description of how to optimally update an individual's belief (*Belief Updating*), given these definitions, is provided.

An ideal observer model provides optimal performance, given the information available in the task. Typically, ideal observers <u>are not proposed as models of human cognition</u>. Instead, the ideal observer provides a benchmark by which to compare human performance. More specifically, these models illustrate what optimal performance should look like. When human performance matches that of the ideal observer model, it can be concluded that the human is effectively processing all of the information in the task. When the human under-performs the ideal observer, specific discrepancies between the human data and the ideal data may identify the constraints imposed by the human information-processing system.

Ideal observer analysis is not new to this research and has been previously used to help us understand perceptual functions from the quantum limits of light detection (Hecht, Shlaer, & Pirenne, 1942) to many forms of visual pattern detection and discrimination (Geisler, 1989) to reading Legge, Hooven, Klitz, Mansfield, & Tjan, 2002; Legge et al., 1997), object recognition (Liu, Knill, & Kersten, 1995; Tjan, Braje, Legge, & Kersten, 1995; Tjan & Legge, 1998), eye movements Najemnik & Geisler, 2005), and in reaching tasks (Trommershäuser, Gepshtein, Maloney, Landy, & Banks, 2005).

2.1.2 Defining the State Space[1]

In all problems that are solved with a POMDP architecture, there is a set of possible states that defines the state space. In a POMDP problem, the true state ($State_{True}$) is not directly observable (i.e., it is hidden). For the work in this project, the hidden state is defined as the enemy's current position within a two-dimensional grid. This grid of location state spaces is supplanted by an additional "destroyed" state that the enemy could transition into following an action to destroy it such as an artillery strike at its current position. Thus, the dimensions of the grid can be characterized as

$$(X \text{ x } Y) + z, \text{ where both } X \text{ and } Y > 1 \text{ and } z = 1.$$

In this case, X is the number of locations of the location grid in the X dimension, Y is the number of locations of the location grid in the Y dimension, and Z is the dead state which is always equal to 1. With this nomenclature, a 5x5 location grid state space yields a 26-state space. A 4x4 location state grid gives a 17-state space, a 3x3 location state gives a 10-state space, a 2x2 location state gives a 5-state space, and so on. These different state space dimensions are illustrated in figures 1 through 3.

2.1.3 Defining the Belief Vector

Although the true state is hidden, the operator typically employs actions and observations that provide information about the true state of the problem. In a 26-state space as shown in figure 1, the operator can fire artillery at a specific position or conduct reconnaissance at a particular location within the environment (i.e., one of the 25 location states). In this model, a reconnaissance action provides two possible observations: "*Enemy Sighted*" or "*Enemy Not Sighted*". An artillery strike is defined to only provide an observation of "*No Information,*" meaning that although the artillery strike was conducted, no information is provided about the resulting condition of the enemy resulting from it because only a reconnaissance mission can observe the condition of the enemy. This replicates the fact that the artillery firing unit does not see the effects of its fires because it is an indirect firing unit and is not able to see where the artillery rounds fall.

[1]Note that the state space being described for this model presentation represents a very primitive enemy state space; only one enemy exists in this state space and the enemy can transition only to the dead state or stay where it is. Clearly, a state space of a likely enemy in a current real-world battle space will have multiple target types (and number) as well as associated probabilities of transitioning to the dead state. Probabilities will depend on the type of target as well as the method of engagement.

It must rely on forward observer (FO) assets to report what is termed “battle damage assessment” (BDA) in military jargon. An illustration of a reconnaissance asset might be an FO on the ground or a UAV that provides the BDA.

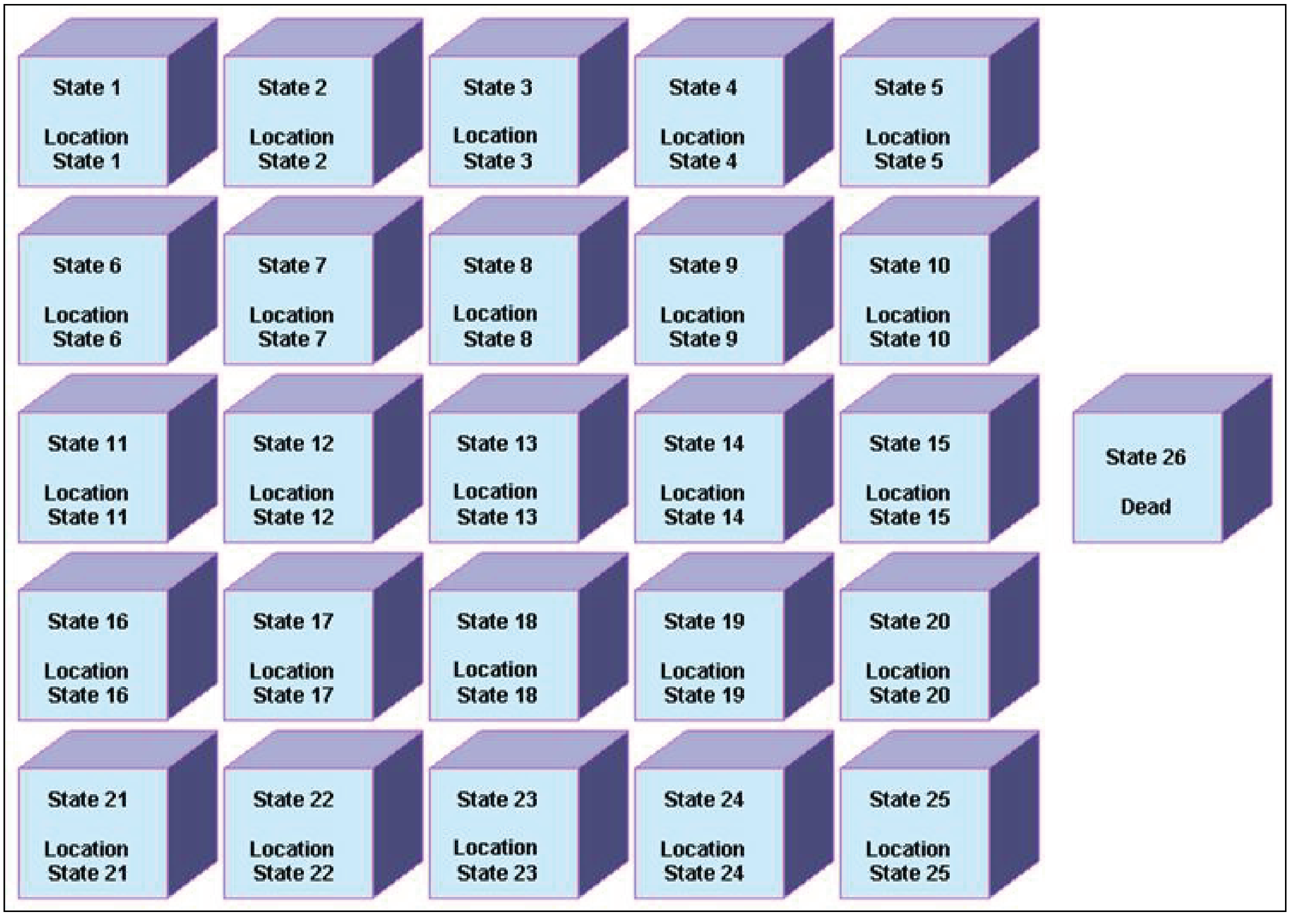

Figure 1. 5x5 location state space.

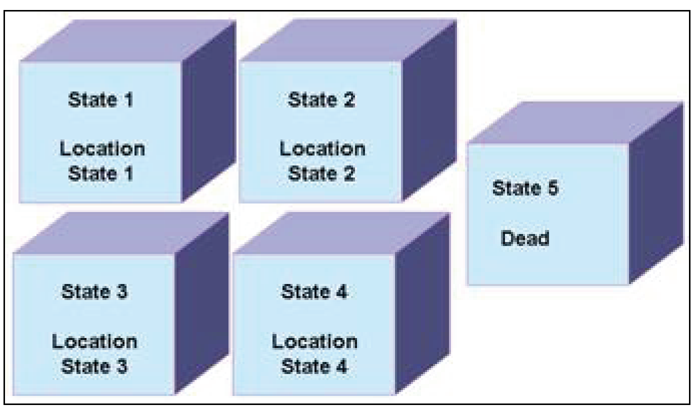

Figure 2. 2x2 location state space.

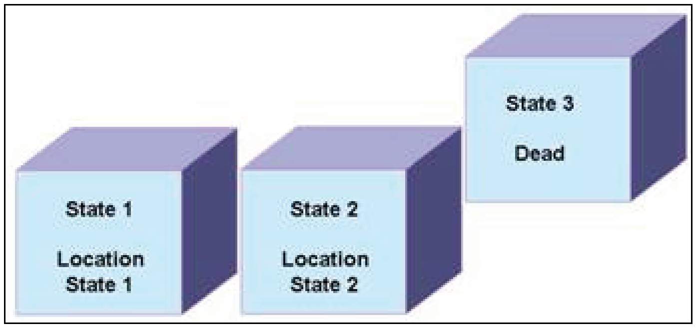

Figure 3. 2x1 location state space.

This model assumes that the observer has a belief probability between 0% and 100% that the enemy exists in one of the states within the total state space at any given period in time. It is noted that residing in one of the location states is mutually exclusive from residing in the destroyed state. That is, if the enemy is "alive" in one of the location states, it cannot be "dead" in the destroyed state and vice versa. The destroyed state is considered to be an "absorbing state" in that once the enemy transitions from being alive in a location state to being dead in the destroyed state, it cannot return to a live location state. The set of the belief probabilities for all the states in the state space is termed the BV. For the simple three-state space example in figure 3, the BV could be represented as

$$[B_{\text{Location 1}}, B_{\text{Location 2}}, B_{\text{Destroyed}}]$$

For this case, assume that the enemy is alive with an equal probability of residing in one of the location states. Thus, the BV becomes

$$[0.5, 0.5, 0.0].$$

2.1.4 Defining the Transition Matrix

In this seek-and-destroy problem, the observer (in this case, the military commander) has a number of possible actions available. For actions in a 5x5 location grid state space, there are 25 possible reconnaissance actions (one for each of the 25 locations in the environment), 25 possible artillery actions (again, one for each of the 25 locations within the environment), and the action to declare "mission complete" when it is believed that the enemy has been destroyed, making a total of 51 possible actions. The transition matrix defines the probability of the resulting state, given that the observer executes a particular action in a specified state (i.e., $p(s'|s,a)$) where s' is the resulting state, s is the existing state the observer chooses to act on (by reconnaissance or artillery), and a is the action taken. In the static form of the seek-and-destroy problem (i.e., where the enemy is not moving), there is only one state transition that can occur for any of the possible actions.

When the commander fires artillery to where the enemy is located, the enemy may be killed with a certain probability, which will cause it to transition into the "destroyed" state. Sample probability estimations for use in this discussion are shown in tables 1 through 3. These values are estimates only and are not to be construed as factual. The probabilities used in actual analyses depend on the scenario conditions at the time of the actual action sequence and are left as model input parameters to be employed during the course of simulation studies that use this model.

Table 1. The set of actions and their observations for the current seek and destroy task.

Action	Observation	Condition	Probability
Recon	Enemy Sighted	Enemy Present	0.75
Recon	Enemy Not Sighted	Enemy Present	0.25
Recon	Enemy Sighted	Enemy Not Present	0.2
Recon	Enemy Not Sighted	Enemy Not Present	0.8

The observations for the reconnaissance action depend on whether the enemy is actually within the viewing region of the reconnaissance. Thus, the two possible states are "enemy present" and "enemy not present".

Table 2. Probabilities for observation from artillery strike.

Action	Observation	Condition	Probability
Strike	NoInfo	Enemy Present	1.0
Strike	NoInfo	Enemy Not Present	1.0

Table 3. Probabilities for killing the enemy from artillery strike.

Action	Result	Condition	Probability of Dead
Strike	Probability of Enemy being killed.	Enemy Present	0.75
Strike	Probability of Enemy not being killed.	Enemy Present	0.25
Strike	Probability of Enemy being killed.	Enemy Not Present	0.0

2.1.5 Updating the Belief Vector (BV)

Given an initial probability distribution over the state space, the observation matrix, and the transition matrix, hypotheses can be generated about the current state of the problem after an action and the returned observation. The general form of Bayes' rule (Trueman, 1977; Walpole, Myers, & Myers, 1998; Wine, 1964), as shown in equation 1, is used as a basis to develop this relationship.

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1,k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1,k} P(B_i)P(A|B_i)} \qquad \text{for } r = 1,2, \ldots k$$

Equation 1 – General Form of Bayes' Rule

in which

P - probability
A - $State_A$
B - $State_B$
| - "so that" or "given"
$\in$ - Probability theory - all state spaces
$\cap$ - Boolean AND
$\cup$ - Boolean OR

2.1.5.1 Bayesian Updating Rule

Using Bayes' rule, POMDP expressions are derived to simulate sequential decision making with uncertainty. We compute an updated BV by performing a particular action to account for the current condition of the state space, a transition matrix for moving from one state to the next, the application of a BV generated by the results of past actions, and an observation vector of past information elements obtained from previous observations of the state space. This Bayesian updating rule is expressed as

$$p\left(s'\middle|b,o,a\right)=\frac{p\left(o\middle|s',b,a\right)p\left(s'\middle|b,a\right)}{p\left(o\middle|b,a\right)}$$

Equation 2 – Bayesian Updating Rule

in which

s'	=	true state (of the condition being present within the total of all states, S), represented as s' $\in$ S
b	=	prior belief vector
o	=	observation
a	=	action that was generated

nomenclature. The term p(s'|b,o,a), is read as

The probability of s" being true, "so that" or "given" the Boolean conditions of "b" AND "o" AND "a".

Equation 2 specifies how the ideal observer would update the belief that *s′* is the true state, given the prior belief (*b*), the observation (*o*), and the action that was generated (*a*).

2.1.5.2 Update the Belief Vector for First Action: Perform $Recon_1$ at $State_1$

To illustrate the process of belief updating, the simple 2x1 location state space of figure 3 will be used. Here, the enemy will be associated with one of three states: $State_1$, $State_2$, or $State_{Dead}$. For this case, assume that the enemy is alive with an equal probability of residing in one of the location states. Thus, the BV becomes

[0.5, 0.5, 0.0]

meaning that there is a 50% probability of the enemy being believed to be in $State_1$, a 50% probability of the enemy being believed to be in $State_2$, and a 0% probability of the enemy being believed to be in $State_{Dead}$. Assume that the enemy actually is located in $State_1$ and that the observer decides to do a reconnaissance of $State_1$ and receives an "enemy sighted" observation. The first task is to determine what the observer's belief is resulting from this action for the enemy being located in $State_1$, $State_2$, and $State_{Dead}$.

With equation 2, the belief likelihood that the enemy is in $State_1$ is computed. That is, the desire is to compute the belief probability that the enemy is in $State_1$, given the current BV, the current observation, and the current action, or *p($State_1$| [0.5, 0.5, 0.0], "EnemySighted",$Recon_1$).*

Computing the separate components of equation 2, first, compute $p(o|s',b,a)$ or p("Enemy Sighted"|$State_1$, [0.5, 0.5, 0.0],$Recon_1$). To do this, the likelihood of obtaining an observation of "Enemy Sighted" if $State_1$ were the true state is needed. From table 1, the likelihood of correctly identifying the enemy as 0.75 is selected.

Next, compute $p(s'|b,a)$ or the likelihood of the true state being $State_1$, given the previous belief and the action of $Recon_1$. Because there is no transition possible to $State_{Dead}$ from a recon mission, these remain at the previous probabilities of 0.5.

Finally, compute $p(o|b,a)$ the likelihood of receiving the observation "EnemySighted" when reconnaissance is made at $State_1$ or p('EnemySighted'|[0.5,0.5, 0],$Recon_1$).

These calculations are

<u>p (o|s',b,a)</u> , for $State_1$ = p('Enemy Sighted' | True State Belief, [0.5, 0.5, 0.0], $Recon_1$)
= Probability of Enemy sighted, given belief that enemy was at $State_1$ and $Recon_1$ was performed at $State_1$ =
= <u>0.75</u>,
from table 1

<u>p(s'|b,a)</u> , for $State_1$ = p(State1 is true state | ([0.5, 0.5, 0.0], $Recon_1$))
= Probability of $State_1$ being the true state, given belief that State1 is true state and $Recon_1$ showed enemy present in $State_1$ = s' =
= <u>0.5</u>,
from assumption of equal probability that the enemy has an initial probability of being at one of the two location states.

p(o|b,a) , for $Stat_1$ = p('Enemy Sighted' | ([0.5, 0.5, 0.0], $Recon_1$))
= (Probability of Enemy in $State_1$ x Probability of Enemy Sighted When Present) + (Probability of Enemy in $State_2$ x Probability of Enemy Sighted When Not Present) + (Probability of Enemy Being Dead x Probability of Enemy Sighted When Not Present)
= 0.5x0.75 + 0.5x0.2 + 0.0x0.2 = .375 + 0.1 = 0<u>.475</u>

Thus, *p(State₁ | [0.5, 0.5, 0.0], "EnemySighted," Recon₁)* =

$$p(s'|b,o,a) = \frac{p(o|s',b,a)\,p(s'|b,a)}{p(o|b,a)} = \frac{0.75 \times 0.5}{0.475} = \underline{0.7895}$$

Likewise, *p($State_2$ | [0.5, 0.5, 0.0], "EnemySighted," $Recon_1$)* =

$$= \frac{0.2 \times 0.5}{0.475} = \underline{0.2105}$$

and *p($State_{Dead}$ | [0.5, 0.5, 0.0], "EnemySighted," $Recon_1$)* =

$$\frac{0.2 \times 0.0}{0.475} = \underline{0.0}$$

Thus, if the first action is to observe, i.e., perform a UAV reconnaissance mission, at *State*$_1$, the new belief vector would be

$$[0.7895, 0.2105, 0.0].$$

The interpretation of this BV is that the enemy has a 0.7895 probability of being believed to be at State$_1$ (present in Cell$_1$), a 0.2105 probability of being believed to be at State$_2$ (present in Cell$_2$), and a 0.0000 probability of being believed to be at State$_3$ (dead). Since these probabilities must account for the total belief state of the operator, they must therefore sum to 1.0. Performing this check sum,

CS = 0.7895 + 0.2105 + 0.0000 = 1.0000; therefore, Checksum verification passed.

2.1.5.3 Update the Belief Vector for Second Action: Perform Strike$_1$ at State$_1$

Now assume that the second action is to conduct an artillery strike at State$_1$ which is represented as *Strike*$_1$. In order to update the BV with the belief that the enemy is in State$_1$ as a result of this new action, determine the probability that the enemy is at State$_1$, given the BV from the first action (Recon$_1$), [0.7895, 0.2105, 0.0], and the new action, Strike$_1$, recognizing that the only observation from an artillery strike is that the strike was fired which provides the observation “NoInfo”. Thus, the new probability

$$p(\text{State}_1|[0.7895, 0.2105, 0.0], \text{“NoInfo,” Strike}_1)$$

is computed. Calculating the components for the updated BV component for State$_1$ from equation 2,

p (o|s’,b,a) = p (‘NoInfo’ | True State Belief, [0.7895, 0.2105, 0.0], Strike1) =
= Probability of ‘NoInfo’ given current BV and Strike1 was performed at State$_1$ =
= 1.0,
because an artillery strike will always return a report of “NoInfo” simply meaning that the artillery strike was fired with no other information provided.

p(s’|b,a) = p(State$_1$ | [0.7895, 0.2105, 0.0], Strike$_1$)
= Probability of “Enemy Not Dead” being the true state, given the current BV and the action of Strike$_1$ being fired.
From table 3, the probability of the enemy being transitioned to dead if artillery is fired at the location containing the enemy, or in this case, Strike$_1$ being to State$_1$ is = 0.75.
Conversely, if the artillery strike, Strike$_1$, into State$_1$ does not kill the enemy with the enemy remaining in a state of “Enemy Not Dead,” from table 3 the probability becomes (because of the three states in the state space which are State$_1$ (present or not present in cell 1), State$_2$ (present or not present in cell 2), and State$_3$ (belief that the enemy is dead or not dead), the deduction is that the enemy is believed to be alive (i.e., not dead) in State$_3$) = 0.25.

Thus, the probability that the enemy's state will not change or that the enemy will remain alive in $State_1$ is equal to 0.25 times the probability that the previous $Recon_1$ sighted the enemy in $State_1$, or 0.7895 from p($State_1$| [0.5,0.5,0.0], "Enemy Sighted," $Recon_1$).

Therefore, p(s'|b,a) , for $State_1$, following $Strike_1$ is

p(s'|b,a) = 0.7895 x 0.25

= 0.1974

p(o|b,a) = p(('NoInfo' | ([0.7895, 0.2105, 0.0], Strike1))

= Probability of 'NoInfo' given current BV and $Strike_1$ being performed at $State_1$ =

= 1.0, because an artillery strike will always return a report of "NoInfo" simply meaning that the artillery strike was fired with no other information provided.

Employing equation 2 to determine the BV,

$$p(s'|b,o,a) = \frac{p(o|s',b,a)\, p(s'|b,a)}{p(o|b,a)} = \frac{1.0000 \times 0.1974}{1.0} = \underline{0.1974}$$

Thus,	*p($State_1$ | [0.7895, 0.2105, 0.0], "NoInfo",$Strike_1$)*	=	0.1974
Likewise,	*p($State_2$ | [0.7895, 0.2105, 0.0], "NoInfo",$Strike_1$)*	=	0.2105
Likewise,	*p($State_{Dead}$ | [0.7895, 0.2105, 0.0], "NoInfo",$Strike_1$)*	=	0.5921

Thus, after the second iteration, where the action was to fire artillery strike 1 into $Cell_1$ ($State_1$), called $Strike_1$,

the BV now becomes = [0.1974, 0.2105, 0.5921]

which is interpreted to mean a 0.1974 probability of the enemy being believed to be alive in $State_1$ ($Cell_1$), a 0.2105 probability of the enemy being believed to be alive in $State_2$ ($Cell_2$), and a 0.5921 probability of the enemy being believed to be dead or in $State_{Dead}$. Performing the check sum verification,

CS = 0.1974 + 0.2105 + 0.5921 = 1.0000; therefore, Checksum verification passed.

See appendix A for a complete set of BV sample calculations for all components of the state space for a selected set of five action sequences.

2.2 Determination of the Action Sequence

The determination of which action to take at each iteration of the model is made in order to create the statistical optimal end state effect. This state is defined as the end state reached by the fewest action sequences with the highest optimal reward value possible. Reward structure is discussed later in this report. Thus, the selection of the action sequence is achieved through a deterministic evaluation of the previous Bayesian state space and BV with the use of conditional probability logic. To begin this discussion, the following definitions are made:

Δ	≡	(Delta) Declare Threshold; if the Belief Probability of Enemy Destroyed ≥ Δ, then Declare; Δ only refers to the DEAD state.
σ	≡	(Sigma) Shoot Threshold; if the Belief Probability of Enemy Destroyed ≥ σ, then Shoot; otherwise, perform Recon; σ only refers to the LOCATION states.
Contrast	≡	The probability of the enemy being in one state relative to all other (location) states (dead state is therefore excluded from the Contrast determination). This is a calculated value referred to as a Conditional Probability (CP).

Thus, if the belief probability of the enemy being destroyed ≥ Δ, then the action decision will be to declare the mission complete and end it. However, if Δ ≤ 0.9, then go through Recon versus Shoot logic according to the σ threshold. Note that the Δ threshold only applies to $State_3$, the dead state, and the σ threshold only applies to the location states, $State_1$ and $State_2$. Therefore, the Δ and σ thresholds are independent and not directly related. For the purposes of discussion, the following assignments are made. These are for reference only as were previous assignments in tables 1 through 3 and are left as input parameters to the model to set its level of tolerance and to reflect the scenario conditions in effect at the time of the model invocation.

Assume the following assignments:

$$\Delta = 0.9.$$
$$\sigma = 0.75.$$

2.2.1 Conditional Probability Logic

For the case undergoing investigation, the action decision logic can be viewed as a CP. The probability of the enemy being in one of the two location states ($State_1$ or $State_2$), given that the enemy is not destroyed, i.e., not present in $State_3$, can be represented as

$P\ (S_1 \mid \text{Enemy Not Destroyed}) = P(S_1 \mid !S_3)$, where the ! symbol represents Boolean 'NOT'

Employing the form of Bayes' Theorem (equation 3), this becomes

$$P(S_1 \mid !S_3) = \frac{P(!S_3 \mid S_1)\ P(S_1)}{P(!S_3)}$$

Equation 3 – Conditional Probability Initial Form

Since the probability of the enemy not being dead if in $State_1$ is equal to 1.0, meaning that the enemy is alive, is represented as

$$P(!S_3 \mid S_1) = 1.0,$$

and the probability of the enemy not being dead, $P(!S_3)$ is equal to the sum of the probabilities of being in one of the location states, or in this case $[P(S_1) + P(S_2)]$, equation 3 now becomes

$$P(S_1 \mid !S_3) = \frac{1.0 \text{ x } P(S_1)}{P(S_1) + P(S_2)} = \frac{P(S_1)}{P(S_1) + P(S_2)}$$

Equation 4 – Conditional Probability Expression

2.2.2 Calculation of Sample Action Sequence Using Conditional Probability

By evaluating the CP using the σ and Δ thresholds, the model makes the action decisions during each iteration of the logic. After each action, a new BV is calculated to be used in the next action decision. For the sample σ and Δ threshold values of 0.75 and 0.90, the action sequence for the first five actions is computed for verification of the computer simulation runs that will be made with this model. These parameter choices provide for an action sequence for the first five actions to be

1) $Recon_1$ to S_1.
2) $Strike_1$ to S_1.
3) $Recon_2$ to S_2.
4) $Strike_2$ to S_1.
5) $Recon_3$ to S_2.

The BV and action values manually calculated for this five-action sequence will be used as test parameters to develop a computer simulation modeling this process.

See appendix B for tables that illustrate the action calculations that determine this sequence.

2.3 Implementation in C3TRACE

To implement this model in a computer simulation, the programming environment of command, control, and communications: techniques for the reliable assessment of concept execution (C3TRACE) (Kilduff, Swoboda, & Barnette, 2005; Plott, 2002; Plott, Quesada, Kilduff, Swoboda, & Allender, 2004) is employed. C3TRACE, developed through funding by ARL's Human Research and Engineering Directorate, is an adaptation of the commercial discrete event programming language Micro Saint Sharp[2] (Schunk & Plott, 2004). Although the basic Micro Saint Sharp programming language allows task-based computer simulations of real-world systems and processes to be represented, C3TRACE has embedded data structures that augment Micro Saint Sharp to allow for detailed representation of U.S. Army C2 systems.

The optimal decision-making model described in this report allows existing computer simulations of C2 systems configured around task performance analysis (Cassandra et al., 1996; Hancock & Meshkati, 1988; Middlebrooks, 2003, 2004; Middlebrooks et al., 1999a, 1999b; Middlebrooks & Williges, 2002) to now be structured to incorporate optimal decision making as a performance metric with the use of the belief updating model. The steps in this process resemble the well-known observe-orient-decide-act (OODA) model (Belknap, 1996; Boyd, 1982; Morgan, Glickman, Woodward, Blaiwes, & Salas, 1986). The decision actions in this model consist of gathering information, updating the belief about the environment or state space, taking an action to accomplish an objective in the state space, and then making a decision whether to continue the mission or terminate it with an assessment of mission success or failure. An example in a military C2 scenario employs a UAV to gather the intelligence, artillery to take an action to destroy an enemy somewhere within the state

[2]Micro Saint Sharp is a trademark of MicroAnalysis and Design, Inc.

space, and belief updating to evaluate the situation after each action and then repeat the sequence or declare “mission complete”.

2.3.1 Design of the C3TRACE Simulation

C3TRACE programs are implemented with discrete event language constructs common to any Micro Saint Sharp simulation program. The top level of a C2 sub-workgroup within a sample organization is shown in the example depicted in figure 4. Here, messages received by the radio operator are distributed according to their subject content. Situation reports (SITREPs) are passed to the S3 operations officer, logistics reports are passed to the S4 logistics officer for action, and so on. If, for example, a mission directive such as seek and destroy an enemy is received, it is passed to the commander for action. There are different reactions that might be invoked in response to such a directive. The commander might communicate to the originating authority to clarify information, an initial estimate of the situation before taking action might be performed, an updating of the situational awareness before taking action might be performed, or the mission might be undertaken as directed. In this case, as depicted in the green box in figure 4, what is referred to as the decision making during uncertainty (DMDUC) process would be initiated to execute the mission.

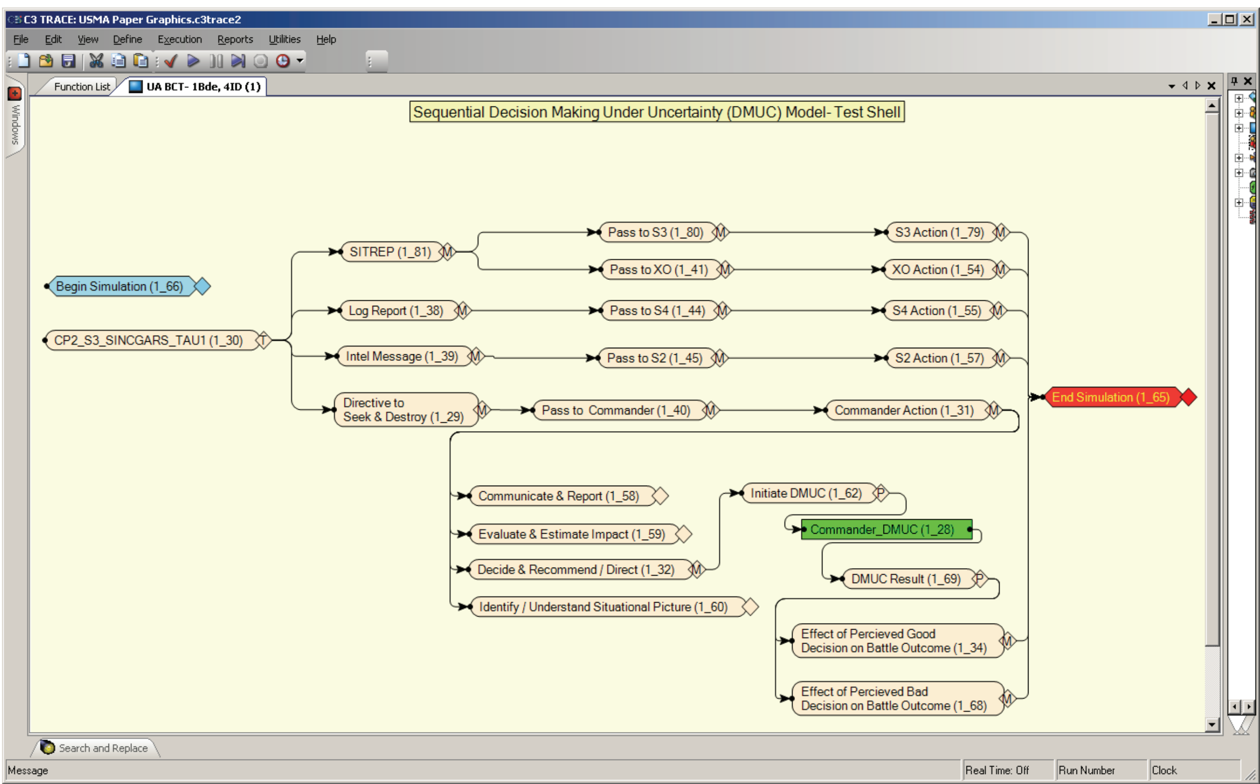

Figure 4. C3TRACE command and control simulation vignette.

Figure 5 illustrates the optimal decision process that is modeled. As stated, this process is very similar to the OODA model. This diagram represents an iterative process where the decision maker makes an initial estimate of the situation and then begins an iterative process of gathering additional information (flying a UAV mission) or taking an action to destroy the enemy (firing artillery). When the commander believes that the enemy has been destroyed, a mission complete decision is made and the results of the decision are realized. If the enemy was destroyed and the decision maker made that correct assessment, then a positive reward resulting from a good decision is applied to the performance of the overall system. If the enemy was not destroyed and the decision maker believed that it was destroyed, then a negative battlefield outcome is applied to the simulation. Likewise, if the enemy was destroyed but the decision maker believed it was not, then the results of poor decision making are applied. This process of iterative action can be generalized to similar scenarios where information is gathered (observe), belief updating occurs (orient), decisions are made for mission success (Decide), and actions are taken to accomplish the mission (act). The examples of employing a UAV and firing artillery are used here to provide a tangible example of how this type of activity might occur.

Referring to figure 5, the top-level logic for this model can be examined. After initiating the decision sequence and performing an initial estimate of the situation, the commander updates the BV, defined as the belief about the current situation regarding the enemy, and then begins an iterative process of looking for information or taking an action to accomplish the mission. When this process has reached some level of belief that the mission is accomplished, the commander terminates the action and completes the decision process by declaring that the mission is a "success" or a "failure".

If the initial desire is to obtain additional information, a UAV is sent to a specified location to try to locate the enemy. The UAV is the information-gathering or BDA tool available to the commander to update the BV about the enemy. If the target is already dead from previous artillery action, then there is no correct location for the enemy because it does not exist. If the enemy is alive and the UAV is sent to the correct location, then it has a probability, according to table 1, of detecting or not detecting the enemy according to the accuracy of the UAV. From this it will correctly or incorrectly report that the enemy was found. Likewise, if it is sent to a location where the enemy is not located or if the enemy is already dead, it may correctly or incorrectly report the enemy sighted, again according to table 1. The values in table 1 are only sample estimates for use in the development of this model and do not represent any actual system currently in existence. During the actual use of this model, these parameters are set to represent the actual detection characteristics of the information gathering entity being evaluated. After the UAV mission is flown, the commander evaluates the report from the UAV through the process of updating the BV and using this new information, decides what process to invoke next.

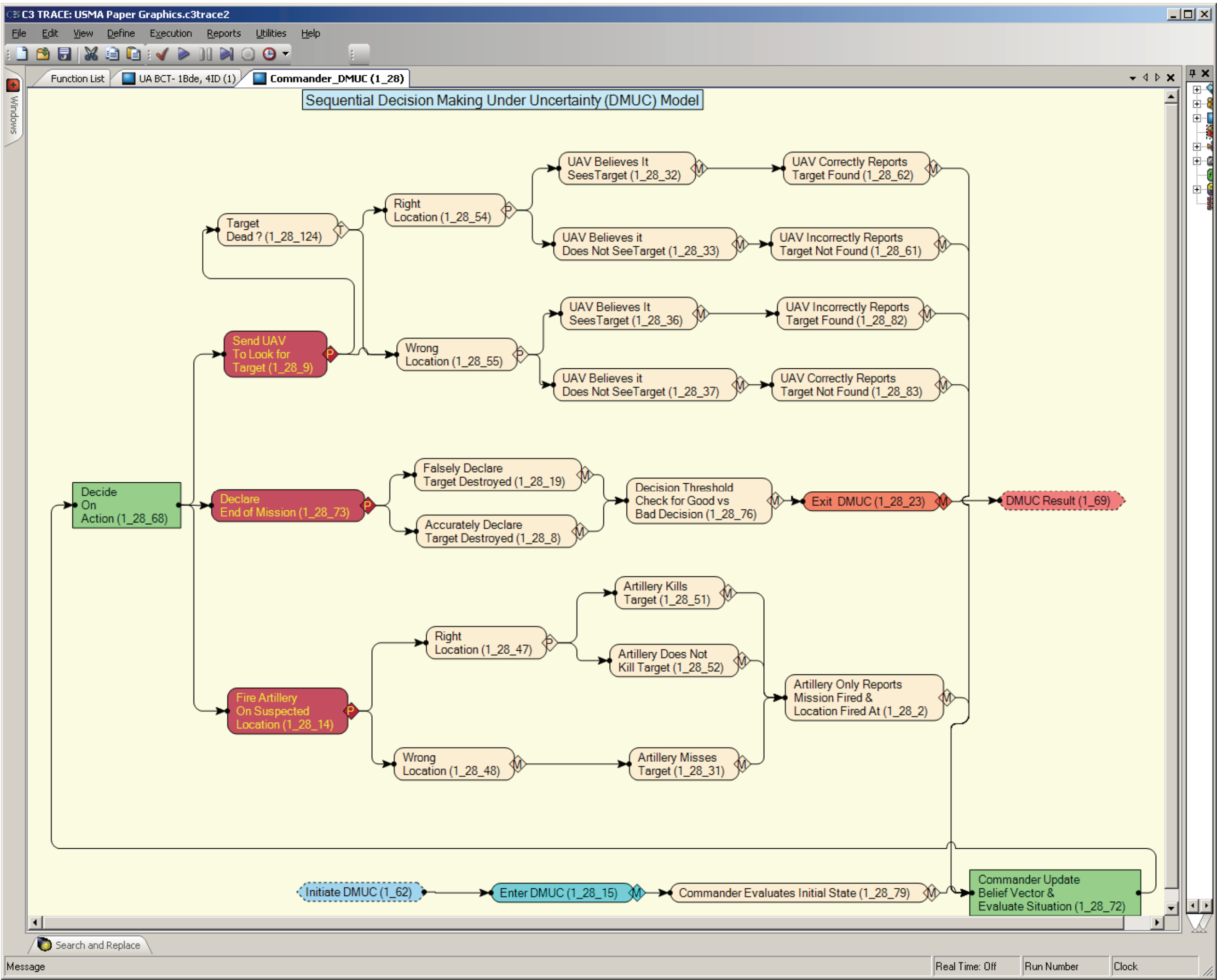

Figure 5. Optimal decision making during uncertainty model.

If the commander decides to fire artillery (which is representative of taking a positive action to do something to accomplish the mission), then the probability exists that the right or wrong location will be fired upon. If the artillery fires on the wrong location, then the only outcome will be to miss the target. If the correct location is fired upon, then the artillery will kill or not kill the enemy according to the circular area of probability for the type of artillery fired. Independent of where the artillery is fired, the only report that is sent to the commander is that the artillery fired upon the location specified or “no information” concerning BDA. This represents the fact that artillery is an indirect fire weapon and the firing unit never actually sees the target. The forward observer, or in this case, the UAV, must report the actual target situation, i.e., to provide the BDA. The commander must than evaluate the firing data and information from previous UAV reconnaissance missions to decide whether to continue the mission or declare the enemy is dead and end the mission.

When the commander believes that the enemy has been destroyed, then mission complete is declared. Then the commander is faced with the rewards of a successful, i.e., good decision sequence where the enemy was killed, meaning that the mission accomplished, or the effects of a bad decision where the enemy was not killed, meaning that the mission was not accomplished.

2.3.2 Belief Updating Logic

Figure 6 illustrates the input feeding the sequence of evaluating the current situation and updating of the BV and the resulting choice for the next action.

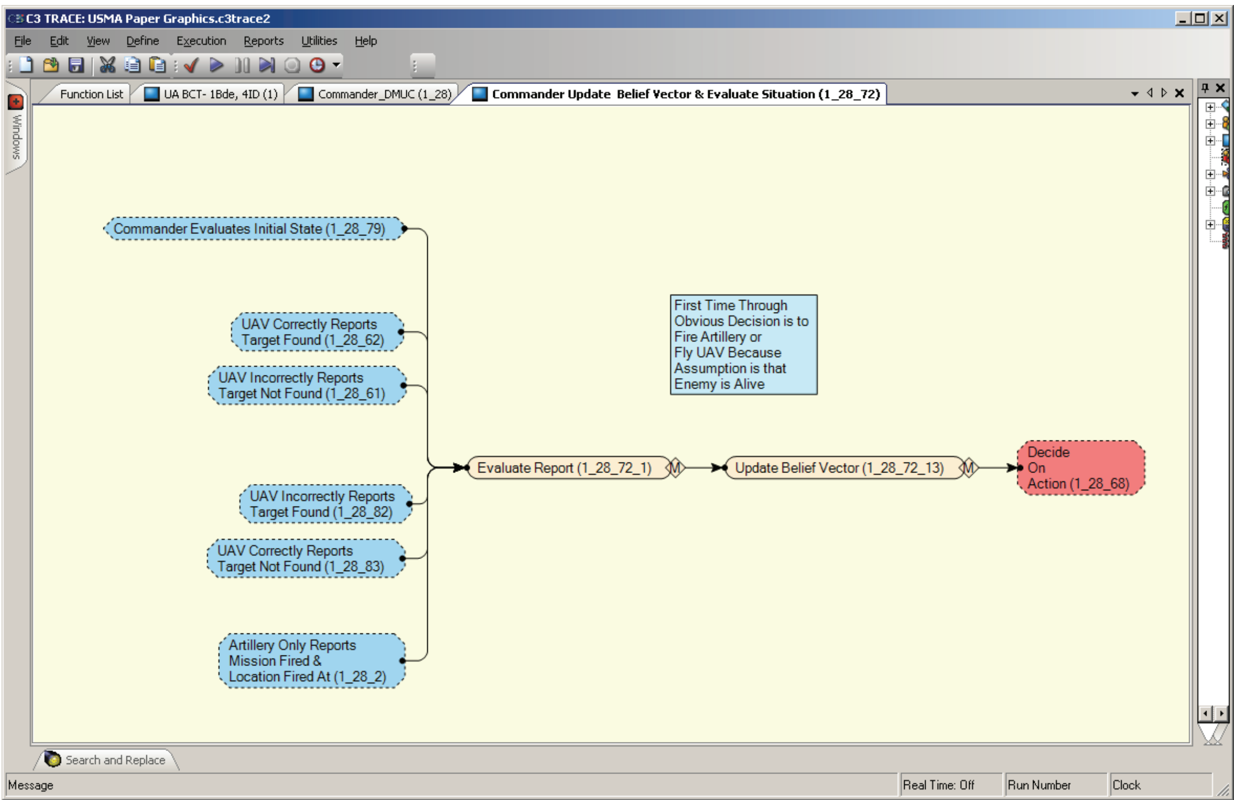

Figure 6. Task diagram for belief updating.

The code script in the beginning effect of C3TRACE task "Update Belief Vector" of figure 6 closely follows the BV updating logic described before. An annotated description of this logic as implemented in the C3TRACE computer simulation takes the form

Definitions:

BV - Belief Vector

Variables in Equation 2:

- s' - True state within the total state space.
- b - Prior belief for that state.
- o - Current observation.
- a - Action that was generated.
- In-State - Probability of a state being transferred in to. This is equal to 0 for location states because for a static enemy condition a location state can never be transferred in to.
- Out-State - Probability of a state being transferred out from. This is equal to 0 for the dead state because once the enemy is dead it can not be transferred back to alive.

Components of Equation 2:

- PoGs'ba - Probability of o Given s' & b & a.

Ps'Gba - Probability of s' Given b & a.
PoGba - Probability of o Given b & a.
Ps'Gboa - Probability of s' Given b & o & a = (PoGs'ba * Ps'Gba) / PoGba => Eq. 2.

If the action generated was to recon, then update the BV from the recon mission:
Do for each state:
Calculate PoGs'ba from Table 1 lookup.
Calculate Ps'Gba from previous belief probability of the enemy in the state occupied.
Calculate PoGba from:
(Probability of Enemy in State Action To * Table 1 Lookup for that State) +
(Probability of Enemy in State Action Not To * Table 1 Lookup for that State) +
(Probability of Enemy in State Dead * Table 1 Lookup for condition applicable to State Dead)
Calculate Ps'Gboa from: (PoGs'ba * Ps'Gba) / PoGba
Equate the BV component for that state = Ps'Gboa.
End Do

Else if the action generated was to Shoot, then update BV from the shoot mission:
Do for each state:
Calculate PoGs'ba from Table 2 lookup which will always = 1.0.
Calculate Ps'Gba from:
[Probability (In-State) x (previous probability for State Struck] +
[(1.0- Probability (Out-State) x (previous probability for State Occupied)]
Calculate PoGba from Table 2 lookup which will always = 1.0.
Calculate Ps'Gboa from: (PoGs'ba * Ps'Gba) / PoGba
Equate the BV component for that state = Ps'Gboa.
End Do

End

See figures 12 through 16 for sample calculations performed to test this logic.

2.3.3 Action Decision-Making Logic

The code script in the beginning effect of C3TRACE task "Evaluate Report" of figure 6 closely follows the CP logic described previously. An annotated description of this logic as implemented in the C3TRACE computer simulation takes the form

Definitions:
CP - Conditional Probability
Sigma - σ, Shoot Threshold; if the Belief Probability of Enemy Destroyed $\geq \sigma$, then Shoot, otherwise recon. σ only refers to the LOCATION states.
Delta - Δ, Declare Threshold; if the Belief Probability of Enemy Destroyed $\geq \Delta$, then Declare. Δ only refers to the DEAD state.

Variables in Equation 6:
PS_1 - Previous BV component for S_1.
PS_2 - Previous BV component for S_2.
Ps_1GNs_3 -Probability of S_1 given NOT S_3 = probability of the enemy not being dead
- PS_1 / (PS_1 + PS_2) => Eq. 6.

Do
Calculate $CP_1 = PS_1 / (PS_1 + PS_2)$
Calculate $CP_2 = PS_2 / (PS_1 + PS_2)$
End Do
If $((CP_1 \,\&\, CP_2) < \sigma)$ then

If $(CP_1 > CP_2)$ then Recon at S_1
If $(CP_1 < CP_2)$ then Recon at S_2
If $(CP_1 = CP_2)$ then Recon at Random pick of S_1 or S_2
End If
If $(CP_1 > \sigma)$ then SHOOT at S_1
If $(CP_2 > \sigma)$ then SHOOT at S_2
If $((CP_1 > \sigma)$ & $(CP_2 > \sigma))$ then SHOOT at Random pick of S_1 or S_2
Calculate new BV
If ((BV component for S_3) $> \Delta$) then
Declare mission complete
Else continue processing and go to next iteration

See tables 8 through 12 for sample calculations performed to test this logic.

3. Discussion and Results

The simplest version of the POMDP model state space design as shown in figure 3 is used in this report to evaluate and demonstrate the logic through the computer simulation in C3TRACE. Although the state space that consists of two location states, $State_1$ and $State_2$, and one status state, $State_{Dead}$, can seem trivial and unrelated to any actual human performance condition, even this simple arrangement can relate to actual performance. The seek-and-destroy mission, looking to destroy an enemy residing at some unknown location, can be characterized as looking or shooting at the enemy at the right or wrong location before declaring that the enemy has been destroyed. Thus, a simple form of the state space such as this can form the basis for developing logic that can be expanded after verification to much larger location state spaces.

A means for evaluating the performance of the simulation is to implement a reward structure (RS) consisting of an explicit cost for taking different actions. There would be a certain cost for conducting a reconnaissance and another greater cost for conducting an artillery strike. There would also be a reward if the mission complete declaration is made when the enemy has actually been destroyed and a corresponding large cost assessed when mission complete is called when the enemy has not been destroyed.

3.1 Control Parameters

The Δ and σ control parameters for the BV and CP calculations allow the model to respond to settings for aggressiveness by the operator in making decisions to perform reconnaissance or to shoot and to reflect the operator's confidence when a successful mission has occurred. The σ control parameter is used to set the reconnaissance versus shoot threshold criteria for the performance of the simulation. Values of σ during analytical runs of the simulations can be varied to represent the complexity and decision threshold conditions of the scenario being simulated. The Δ control parameter is used to set the decision threshold criteria for the performance of the simulation. Values of Δ during analytical runs of the simulations can be varied to represent the complexity and decision threshold conditions of the scenario being simulated.

3.1.1 Sigma Parameter

The σ control parameter for the BV and CP calculations is used to set the look versus shoot threshold criteria for the performance of the simulation. Values of σ during analytical runs of the simulations can be varied to represent the complexity and decision threshold conditions of the scenario being simulated.

3.1.2 Delta Parameter

The Δ control parameter is used to set the decision threshold criteria for the performance of the simulation. Values of Δ during analytical runs of the simulations can be varied to represent the complexity and decision threshold conditions of the scenario being simulated.

3.2 Action Sequence Assessment

In order to examine the CP logic associated with actions in this state space, an example of actions and the resulting belief vectors are examined. The assumptions are that the enemy is located in $State_1$ and that it is static, i.e., not moving. Initially, there is an equal probability in the belief of the commander that the enemy could be in either of the location states and a belief that the enemy is alive. The initial belief vector is thus [0.5, 0.5, 0.0], meaning a 50% chance of being in location $State_1$, a 50% chance of being in location $State_2$, and a 0.0% chance of being in $State_{Dead}$, i.e., the enemy is alive.

3.2.1 Simulation Action Sequence With Δ = 0.90 and σ = 0.75

Assume that the control parameter values are initially set to

σ =	0.75,	i.e., if the Belief Probability of Enemy Destroyed (in regard to a location state) $\geq \sigma$, then shoot, otherwise recon.
Δ =	0.90,	i.e., if the Belief Probability of Enemy Destroyed (in regard to the dead state) $\geq \Delta$, then declare mission complete.

These assumed values are for example only and are not to be construed to represent any actual system.

Applying these parameters to the BV and CP logic generates the sequence of actions as shown in figure 7 and table 4. Even though the BV component for $State_{Dead}$ exceeds Δ at iteration 5, the model run was continued for 20 iterations to illustrate the action sequence asymptotic relationships.

Activating the Δ control parameter causes the simulation to declare mission complete after iteration 5 shown in table 9 with a $State_{Dead}$ BV component = 0.9137 which is just over the Δ threshold of 0.90. This results in a five-action sequence of recon-shoot-recon-shoot-recon to declare. If an RS is implemented with a cost of 10 combat power points to recon and 100 combat power points to shoot, then the cost of this action sequence would be (3x10) + (2x100) = 230.

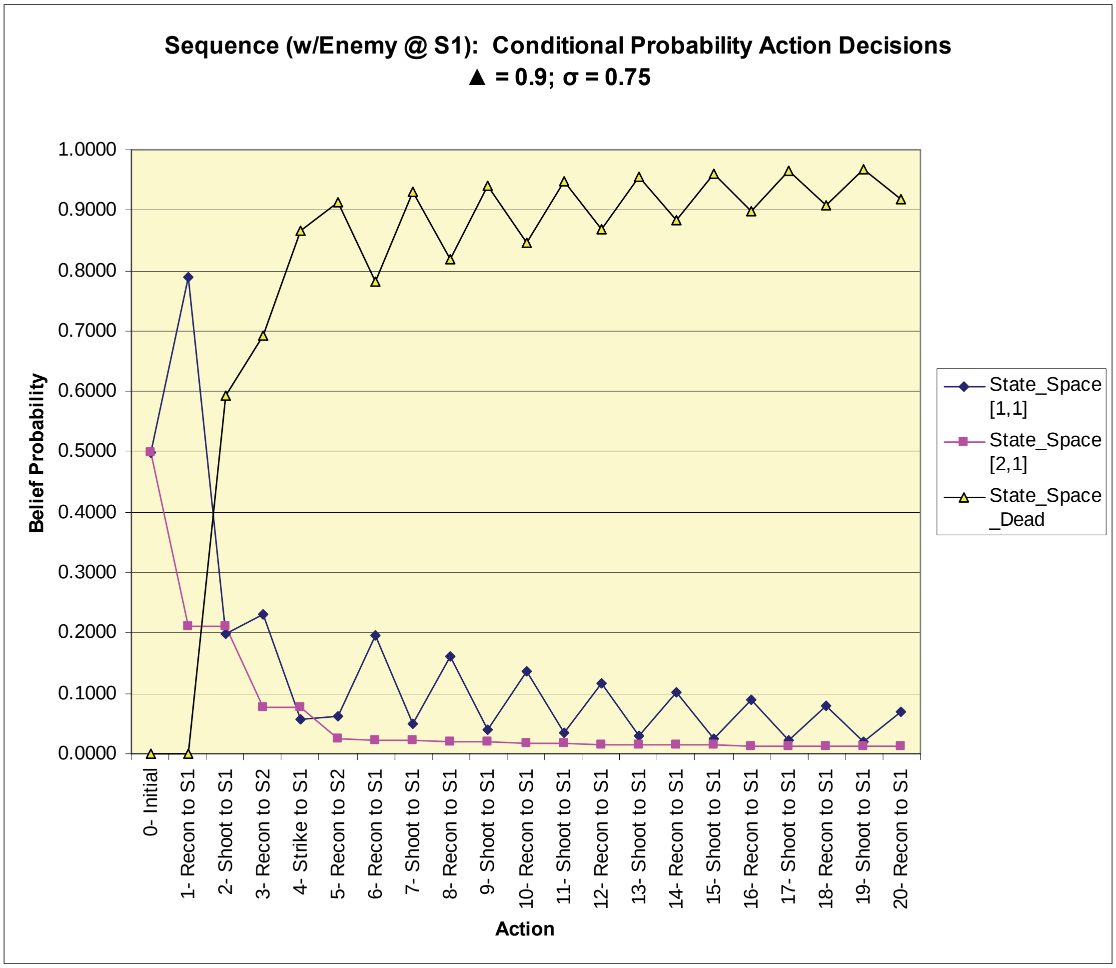

Figure 7. First 20 action decisions for Δ = 0.90 and σ = 0.75.

Table 4. First 20 action decisions for Δ = 0.90 and σ = 0.75.

Action	State_Space [1,1]	State_Space [2,1]	State_Space _Dead	
0- Initial	0.5000	0.5000	0.0000	
1- Recon to S1	0.7895	0.2105	0.0000	
2- Shoot to S1	0.1974	0.2105	0.5921	
3- Recon to S2	0.2308	0.0769	0.6923	
4- Shoot to S1	0.0577	0.0769	0.8654	
5- Recon to S2	0.0609	0.0254	0.9137	Declare Threshold
6- Recon to S1	0.1957	0.0217	0.7826	
7- Shoot to S1	0.0489	0.0217	0.9293	
8- Recon to S1	0.1617	0.0192	0.8192	
9- Shoot to S1	0.0404	0.0192	0.9404	
10- Recon to S1	0.1364	0.0172	0.8463	
11- Shoot to S1	0.0341	0.0172	0.9487	
12- Recon to S1	0.1169	0.0158	0.8673	
13- Shoot to S1	0.0292	0.0158	0.9550	
14- Recon to S1	0.1015	0.0146	0.8840	
15- Shoot to S1	0.0254	0.0146	0.9600	
16- Recon to S1	0.0889	0.0136	0.8974	
17- Shoot to S1	0.0222	0.0136	0.9641	
18- Recon to S1	0.0786	0.0129	0.9086	
19- Shoot to S1	0.0196	0.0129	0.9675	
20- Recon to S1	0.0699	0.0122	0.9179	

Table 5. First 20 action decisions for Δ = 0.90 and σ = 0.75.

Action	State_Space [1,1]	State_Space [2,1]	State_Space _Dead	
0- Initial	0.5000	0.5000	0.0000	
1- Recon to S1	0.7895	0.2105	0.0000	
2- Shoot to S1	0.1974	0.2105	0.5921	
3- Recon to S2	0.2308	0.0769	0.6923	
4- Shoot to S1	0.0577	0.0769	0.8654	
5- Recon to S2	0.0609	0.0254	0.9137	Declare Threshold
6- Recon to S1	0.1957	0.0217	0.7826	
7- Shoot to S1	0.0489	0.0217	0.9293	
8- Recon to S1	0.1617	0.0192	0.8192	
9- Shoot to S1	0.0404	0.0192	0.9404	
10- Recon to S1	0.1364	0.0172	0.8463	
11- Shoot to S1	0.0341	0.0172	0.9487	
12- Recon to S1	0.1169	0.0158	0.8673	
13- Shoot to S1	0.0292	0.0158	0.9550	
14- Recon to S1	0.1015	0.0146	0.8840	
15- Shoot to S1	0.0254	0.0146	0.9600	
16- Recon to S1	0.0889	0.0136	0.8974	
17- Shoot to S1	0.0222	0.0136	0.9641	
18- Recon to S1	0.0786	0.0129	0.9086	
19- Shoot to S1	0.0196	0.0129	0.9675	
20- Recon to S1	0.0699	0.0122	0.9179	

3.2.2 Simulation Action Sequence With Δ = 0.90 and σ = 0.89

Applying these parameters to the BV and CP logic generates the sequence of actions as shown in figure 8 and table 6. Even though the BV component for $State_{Dead}$ exceeds Δ at iteration 9, the model run was again continued for 20 iterations to illustrate the action sequence asymptotic relationships.

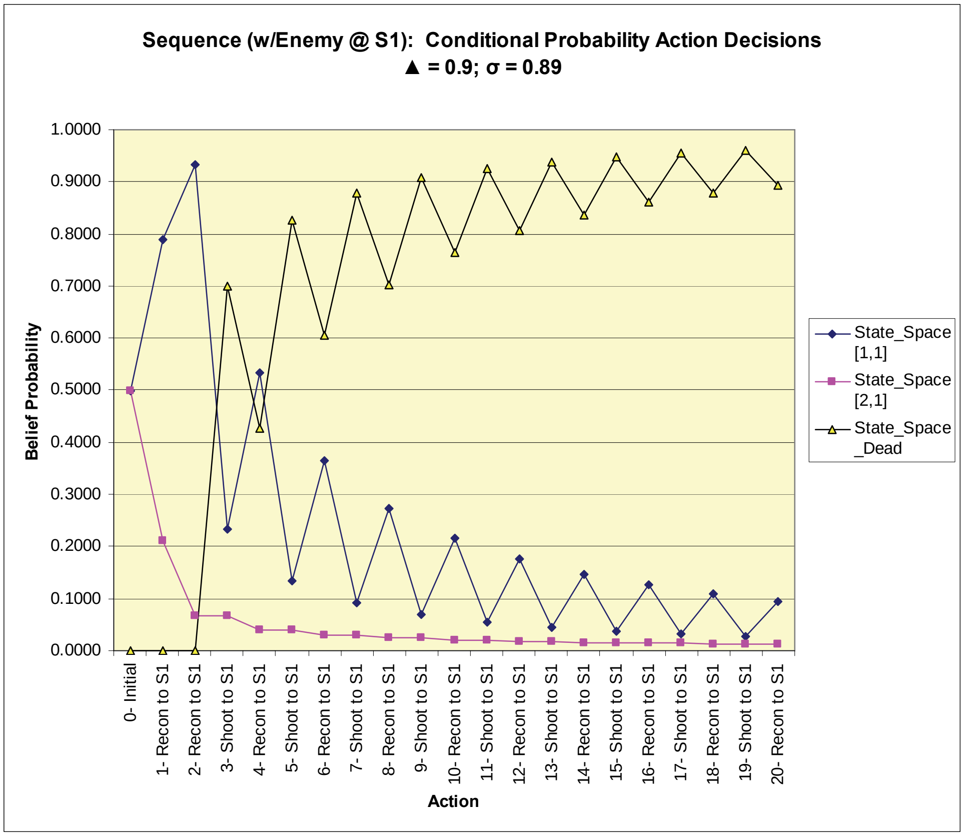

Figure 8. First 20 action decisions for Δ = 0.90 and σ = 0.89.

Table 6. First 20 action decisions for Δ = 0.90 and σ = 0.89.

Action	State_Space [1,1]	State_Space [2,1]	State_Space _Dead
0- Initial	0.5000	0.5000	0.0000
1- Recon to S1	0.7895	0.2105	0.0000
2- Recon to S1	0.9336	0.0664	0.0000
3- Shoot to S1	0.2334	0.0664	0.7002
4- Recon to S1	0.5331	0.0404	0.4265
5- Shoot to S1	0.1333	0.0404	0.8263
6- Recon to S1	0.3657	0.0296	0.6047
7- Shoot to S1	0.0914	0.0296	0.8790
8- Recon to S1	0.2740	0.0236	0.7024
9- Shoot to S1	0.0685	0.0236	0.9079
10- Recon to S1	0.2161	0.0199	0.7640
11- Shoot to S1	0.0540	0.0199	0.9261
12- Recon to S1	0.1764	0.0173	0.8063
13- Shoot to S1	0.0441	0.0173	0.9386
14- Recon to S1	0.1475	0.0154	0.8370
15- Shoot to S1	0.0369	0.0154	0.9477
16- Recon to S1	0.1256	0.0140	0.8604
17- Shoot to S1	0.0314	0.0140	0.9546
18- Recon to S1	0.1084	0.0129	0.8787
19- Shoot to S1	0.0271	0.0129	0.9600
20- Recon to S1	0.0945	0.0120	0.8934

For this action sequence, the Δ control parameter causes the simulation to declare mission complete after iteration 9 shown in table 10 with a $State_{Dead}$ BV component = 0.9079. This results in a nine-action sequence of R-R-S-R-S-R-S-R-S to declare. If we evaluate this sequence with the RS with a cost of 10 combat power points to Recon and 100 combat power points to shoot, the cost of the action sequence is (5x10) + (4x100) = 450.

3.2.3 Simulation Action Sequence With Δ = 0.90 and σ = 0.55

Applying these parameter values to the BV and CP logic generates the sequence of actions as shown in figure 9 and table 7. Even though the BV component for $State_{Dead}$ exceeds Δ at iteration 5, the model run was also continued for 20 iterations to illustrate the action sequence asymptotic relationships.

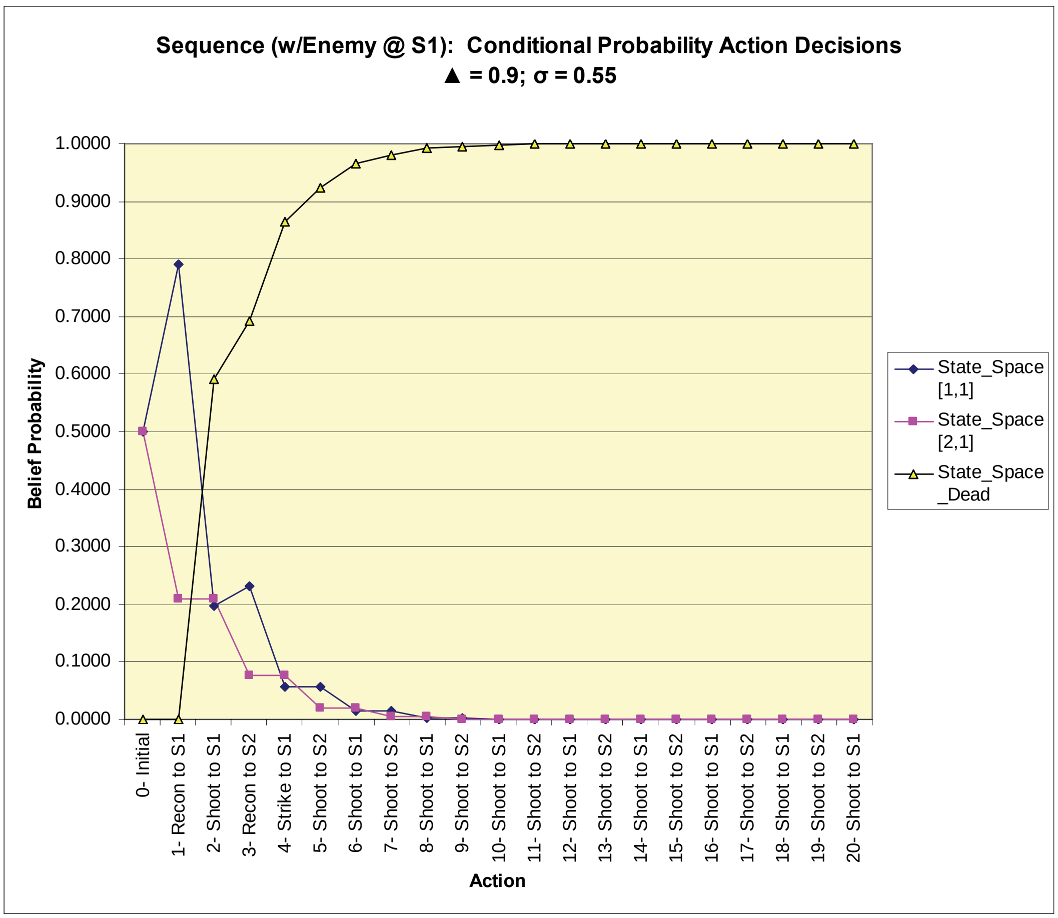

Figure 9. First 20 action decisions for Δ = 0.90 and σ = 0.55.

Table 7. First 20 action decisions for $\Delta = 0.90$ and $\sigma = 0.55$.

Action	State_Space [1,1]	State_Space [2,1]	State_Space _Dead
0- Initial	0.5000	0.5000	0.0000
1- Recon to S1	0.7895	0.2105	0.0000
2- Shoot to S1	0.1974	0.2105	0.5921
3- Recon to S2	0.2308	0.0769	0.6923
4- Shoot to S1	0.0577	0.0769	0.8654
5- Shoot to S2	0.0577	0.0192	0.9231
6- Shoot to S1	0.0144	0.0192	0.9663
7- Shoot to S2	0.0144	0.0048	0.9808
8- Shoot to S1	0.0036	0.0048	0.9916
9- Shoot to S2	0.0036	0.0012	0.9952
10- Shoot to S1	0.0009	0.0012	0.9979
11- Shoot to S2	0.0009	0.0003	0.9988
12- Shoot to S1	0.0002	0.0003	0.9995
13- Shoot to S2	0.0002	0.0001	0.9997
14- Shoot to S1	0.0001	0.0001	0.9999
15- Shoot to S2	0.0001	0.0000	0.9999
16- Shoot to S1	0.0000	0.0000	1.0000
17- Shoot to S2	0.0000	0.0000	1.0000
18- Shoot to S1	0.0000	0.0000	1.0000
19- Shoot to S2	0.0000	0.0000	1.0000
20- Shoot to S1	0.0000	0.0000	1.0000

For this action sequence, the Δ control parameter causes the simulation to declare mission complete after iteration 5 shown in table 8 with a $State_{Dead}$ BV component = 0.9231. This results in a five-action sequence of R-S-R-S-S to declare. If we evaluate this sequence with the RS with a cost of 10 combat power points to Recon and 100 combat power points to shoot, then the cost of the action sequence is (2x10) + (3x100) = 320.

3.3 Reward Structure

Evaluation of the RS as σ is varied from 0.00 to 1.00 provides an indication of the "cost of doing business" based on how aggressive the decision maker is in making action choices. Figure 10 shows a profile of the RS over this range with σ in increments of 0.10. Table 9 shows an expanded view of the information in the X axis.

The intent is to minimize the cost of doing business by performing the least costly sequence of actions to achieve the desired belief that the enemy has been destroyed. In an analytical use of this model, tailoring the reconnaissance and strike asset capabilities so that they support an action sequence of recon-shoot-recon-shoot-recon to achieve the belief threshold specified would allow the system to be tailored for optimal performance along this parameter. Here, the optimal performance occurs over the range of $\sigma \cong 0.59$ to 0.75 with a resulting action cost of 230.

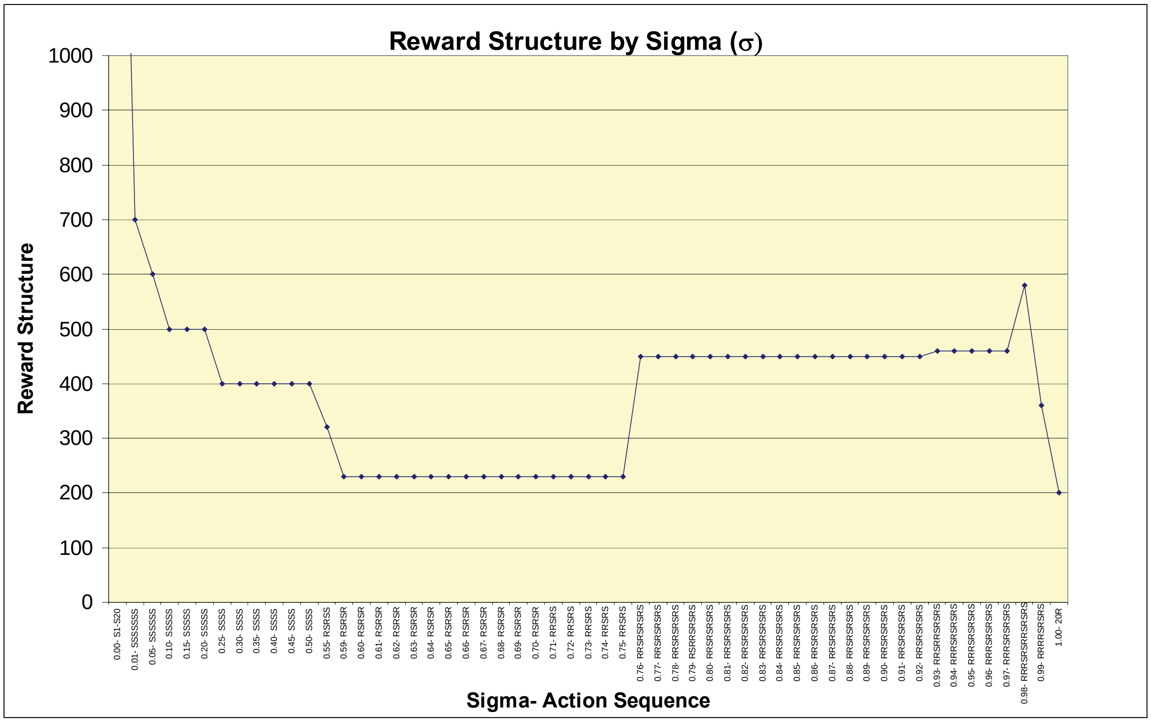

Figure 10. Action cost from reward structure varied by σ.

Table 8. Sigma, σ, action sequence (R- recon, S- strike, RS- reward structure value).

Recon Cost	10			
Shoot Cost	100			
σ, Sigma- Sequence	RS		σ, Sigma- Sequence	RS
0.00- S1-S20	2000		0.74- RRSRS	230
0.01- SSSSSSS	700		0.75- RRSRS	230
0.05- SSSSSS	600		0.76- RRSRSRSRS	450
0.10- SSSSS	500		0.77- RRSRSRSRS	450
0.15- SSSSS	500		0.78- RRSRSRSRS	450
0.20- SSSSS	500		0.79- RSRRSRSRS	450
0.25- SSSS	400		0.80- RRSRSRSRS	450
0.30- SSSS	400		0.81- RRSRSRSRS	450
0.35- SSSS	400		0.82- RRSRSRSRS	450
0.40- SSSS	400		0.83- RRSRSRSRS	450
0.45- SSSS	400		0.84- RRSRSRSRS	450
0.50- SSSS	400		0.85- RRSRSRSRS	450
0.55- RSRSS	320		0.86- RRSRSRSRS	450
0.59- RSRSR	230		0.87- RRSRSRSRS	450
0.60- RSRSR	230		0.88- RRSRSRSRS	450
0.61- RSRSR	230		0.89- RRSRSRSRS	450
0.62- RSRSR	230		0.90- RRSRSRSRS	450
0.63- RSRSR	230		0.91- RRSRSRSRS	450
0.64- RSRSR	230		0.92- RRSRSRSRS	450
0.65- RSRSR	230		0.93- RRSRRSRSRS	460
0.66- RSRSR	230		0.94- RRRSRSRSRS	460
0.67- RSRSR	230		0.95- RRRSRSRSRS	460
0.68- RSRSR	230		0.96- RRRSRSRSRS	460
0.69- RSRSR	230		0.97- RRRSRSRSRS	460
0.70- RSRSR	230		0.98- RRRSRSRRSRSRS	580
0.71- RRSRS	230		0.99- RRRRSRSRS	360
0.72- RRSRS	230		1.00- 20R	200
0.73- RRSRS	230			

Table 9. Sigma, σ, action sequence (R- recon, S- strike, RS- reward structure value).

Recon Cost	10		
Shoot Cost	100		
σ, Sigma- Sequence	RS	σ, Sigma- Sequence	RS
0.00- S1-S20	2000	0.74- RRSRS	230
0.01- SSSSSSS	700	0.75- RRSRS	230
0.05- SSSSSS	600	0.76- RRSRSRSRS	450
0.10- SSSSS	500	0.77- RRSRSRSRS	450
0.15- SSSSS	500	0.78- RRSRSRSRS	450
0.20- SSSSS	500	0.79- RSRRSRSRS	450
0.25- SSSS	400	0.80- RRSRSRSRS	450
0.30- SSSS	400	0.81- RRSRSRSRS	450
0.35- SSSS	400	0.82- RRSRSRSRS	450
0.40- SSSS	400	0.83- RRSRSRSRS	450
0.45- SSSS	400	0.84- RRSRSRSRS	450
0.50- SSSS	400	0.85- RRSRSRSRS	450
0.55- RSRSS	320	0.86- RRSRSRSRS	450
0.59- RSRSR	230	0.87- RRSRSRSRS	450
0.60- RSRSR	230	0.88- RRSRSRSRS	450
0.61- RSRSR	230	0.89- RRSRSRSRS	450
0.62- RSRSR	230	0.90- RRSRSRSRS	450
0.63- RSRSR	230	0.91- RRSRSRSRS	450
0.64- RSRSR	230	0.92- RRSRSRSRS	450
0.65- RSRSR	230	0.93- RRSRRSRSRS	460
0.66- RSRSR	230	0.94- RRRSRSRSRS	460
0.67- RSRSR	230	0.95- RRRSRSRSRS	460
0.68- RSRSR	230	0.96- RRRSRSRSRS	460
0.69- RSRSR	230	0.97- RRRSRSRSRS	460
0.70- RSRSR	230	0.98- RRRSRSRRSRSRS	580
0.71- RRSRS	230	0.99- RRRRSRSRS	360
0.72- RRSRS	230	1.00- 20R	200
0.73- RRSRS	230		

4. Conclusions and Future Work

The current work has established a model that supports a computer simulation capable of determining and optimizing optimal decisions during conditions of uncertainty according to evaluations of the BV about the current state, action decisions based on CP logic, and optimal performance determination through the evaluation of action cost from the RS. Thus, the C3TRACE simulation employing this model can make action decisions based on conditional probability evaluations of the belief state representing the current situation. These action decisions are oriented toward a goal-directed optimal outcome, and subsequently recognize when the belief has been achieved and the outcome reached.

This report demonstrates the logic of this model through the evaluation of the most simple of state spaces. This sample state space consists of a 2x1 location state matrix and a single status state of the dead condition for a total three-state space system. Future work will expand the location state space matrix to 2x2 and 5x5 as shown in figures 1 and 2, along with more sophisticated enemy actions for moving versus static operations and goal-directed movement activities.

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Appendix A. Belief Vector Calculations for a Selected Five-Action Sequence

Belief vector calculations for the first five action sequences are presented here.

Parameter Definitions

Tables 1 through 3 are repeated in figure A-1 with parameter identifications to clarify which parameter is being used in which calculation. These parameter identifications are shown with a number inside a circle (e.g., ③④). These are used to identify original constants and the results of each calculation to eliminate confusion as to which parameter constant is being applied where.

This is a matrix of the POMDP conditional probability conditions for each combination of Location State occupied, Location State Reconed, and Location State Struck

General Form of Bayes rule:

$P(B_r \mid A)$

$$P(B_r \mid A) = \frac{P(B_r \cap A)}{\Sigma_{i=1,k} P(B_i \cap A)} = \frac{P(B_r)\,P(A \mid B_r)}{\Sigma_{i=1,k} P(B_i)\,P(A \mid B_i)} \quad \text{for } r = 1,2, \ldots k$$

Bayesian Updating rule for this application:

p(s'|b,o,a)

$$p(s'|b,o,a) = \frac{p(o|s',b,a)\;p(s'|b,a)}{p(o|b,a)}$$

Definition:
'State Destroyed' is an "Absorbing State" meaning that once the enemy is there they cannot get out, i.e., returned to 'alive'.

where,

s'	=	true state (of the condition being present within the total of all states, S), represented as: s' Î S
b	=	prior belief
o	=	observation
a	=	action that was generated

Table 1 - The set of actions and their observations for the Seek & Destroy task.

Action	Observation	State	Probability	
Recon	Enemy Sighted	Enemy Present	0.75	①
Recon	Enemy Not Sighted	Enemy Present	0.25	②
Recon	Enemy Sighted	Enemy Not Present	0.20	③
Recon	Enemy Not Sighted	Enemy Not Present	0.80	④

Initial State Probabilities

Enemy in State 1	=	0.50	①⓪
Enemy in State 2	=	0.50	①①
Enemy in State Dead	=	0.00	①②

The 2 cases for Enemy Sighted vs. Enemy Not Sighted are:

	Recon to State1	Recon to State2
Recon Sighted	[.75, .2, .2]	[.2, .75, .2]
Recon Not Sighted	[.25, .8, .8]	[.8, .25, .8]
Arty- NoInfo	[1.0, 1.0, 1.0]	[1.0, 1.0, 1.0]

Table 2 - Probabilities for Observation from Artillery Strike

Action	Observation	State	Probability	
Strike	NoInfo	All Cases	1.00	⑤

Table 3 - Probabilities for Killing Enemy from Artillery Strike

Action	Result	State	Probability of Dead	
Strike	SUCCESS	Enemy Present	0.75	⑦
Strike	FAILURE	Enemy Present	0.25	⑧
Strike	SUCCESS	Enemy Not Present	0.00	⑨

Transitioning Probabilities:

Transition Into Loc. State	=	0.00	⑥ for static enemy case
Trans. Out Of Loc.State	=	Prev. Prob for Killing	
Trans. Out Of Dead State	=	0.00	⓪⓪ as enemy cannot transition out of the dead state once there

Figure A-1. Initial constants for belief vector calculations.

Recon1 to S1													
Action	Observation	State Action To	State Occupied		Effect		[p (o|s',b,a)	x	p(s'|b,a)]	/	p(o|b,a)	=	p(s'|b,o,a)
	Enemy Sighted			Description:			Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prob of En in State Action To * Table 1 Lookup for that State) + (Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.5x0.75 + 0.5x0.2 + 0.0x0.2 = .375 + 0.1 = .475	=	= ①③
							→①		→①⓪		= ①⓪x① + ①①x③ + ①②x③		
Recon1	0.75	1	1	Sample	Calculation:		0.75	x	0.5000	/	0.4750	=	0.7895
	Enemy Sighted			Description:			Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prob of En in State Action To * Table 1 Lookup for that State) + (Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.5x0.75 + 0.5x0.2 + 0.0x0.2 = .375 + 0.1 = .475	=	= ①④
							→③		→①①		= ①⓪x① + ①①x③ + ①②x③		
Recon1	0.20	1	2	Sample	Calculation:		0.20	x	0.5000	/	0.4750	=	0.2105
	Enemy Sighted			Description:			Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prob of En in State Action To * Table 1 Lookup for that State) + (Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.5x0.75 + 0.5x0.2 + 0.0x0.2 = .375 + 0.1 = .475	=	= ①⑤
							→ ④		→①②		= ①⓪x① + ①①x③ + ①②x③		
Recon1	0.20	1	3	Sample	Calculation:		0.80	x	0.0000	/	0.4750	=	0.0000
											Check Sum	=	1.0000

Figure A-2. Belief vector calculations for first action: $recon_1$ to $state_1$.

Strike1 to S1													
Action	Result	State Action To	State Occupied		Effect		[p (o|s',b,a)	x	p(s'|b,a)]	/	p(o|b,a)	=	p(s'|b,o,a)
	Kill Success			Description:			Table 2 Lookup	x	[p(In-State) x (prev. prob. for State Struck] + [(1.0-p(Out-State) x (prev. prob for State Occupied)] [(0x.7895) + ((1-.75)x.7895)]	/	Table 2 Lookup	=	= ②⑤
							→⑤		⑥x①③ + [(1.0-⑦) x (①③)]		→⑤		
Strike1	1.00	1	1	Sample	Calculation:		1.00	x	0.1974	/	1.00	=	0.1974
	Kill Success			Description:			Table 2 Lookup	x	[p(In-State) x (prev. prob. for State Struck] + [(1.0-p(Out-State) x (prev. prob for State Occupied)] [(0x.7895) + ((1-0)x.2105)]	/	Table 2 Lookup	=	= ②⑥
							→⑤		⑥x①③ + [(1.0-⑨) x (①④)]		→⑤		
Strike1	1.00	1	2	Sample	Calculation:		1.00	x	0.2105	/	1.00	=	0.2105
	Kill Success			Description:			Table 2 Lookup	x	[p(In-State) x (prev. prob. for State Struck] + [(1.0-p(Out-State) x (prev. prob for State Occupied)] [(.75x.7895) + ((1-0)x0)]	/	Table 2 Lookup	=	= ②⑦
							→⑤		⑦x①③ + [(1.0-⑩) x (①⑤)]		→⑤		
Strike1	1.00	1	3	Sample	Calculation:		1.00	x	0.5921	/	1.00	=	0.5921
											Check Sum	=	1.0000

Figure A-3. Belief vector calculations for second action: $strike_1$ to $state_1$.

Recon2 to S2											
Action	Observation	State Action To	State Occupied	Effect	[p (o|s',b,a)	x	p(s'|b,a)]	/	p(o|b,a)	=	p(s'|b,o,a)
	Enemy Not Sighted			Description:	Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prev Prob of En in State Action To * Table 1 Lookup for that State) + (Prev Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prev Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.2105x0.25 + 0.1974x0.8 + 0.5921x0.8 = .6842	=	= ②⑧
					→ ④		= ②⑤		= ②⑥x② + ②⑤x④ + ②⑦x④		
Recon2		2	1	Sample Calculation:	0.80	x	0.1974	/	0.6842	=	0.2308
	Enemy Not Sighted			Description:	Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prev Prob of En in State Action To * Table 1 Lookup for that State) + (Prev Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prev Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.2105x0.25 + 0.1974x0.8 + 0.5921x0.8 = .6842	=	= ②⑨
					→ ②		= ②⑥		= ②⑥x② + ②⑤x④ + ②⑦x④		
Recon2		2	2	Sample Calculation:	0.25	x	0.2105	/	0.6842	=	0.0769
	Enemy Not Sighted			Description:	Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prev Prob of En in State Action To * Table 1 Lookup for that State) + (Prev Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prev Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.2105x0.25 + 0.1974x0.8 + 0.5921x0.8 = .6842	=	= ③⓪
					→ ④		= ②⑦		= ②⑥x② + ②⑤x④ + ②⑦x④		
Recon2		2	3	Sample Calculation:	0.80	x	0.5921	/	0.6842	=	0.6923
									Check Sum	=	1.0000

Figure A-4. Belief vector calculations for third action: recon_2 to state_2.

Strike2 to S1													
Action	Result	State Action To	State Occupied		Effect		[p (o|s',b,a)	x	p(s'|b,a)]	/	p(o|b,a)	=	p(s'|b,o,a)
	Kill Success			Description:			Table 2 Lookup	x	[p(In-State) x (prev. prob. for State Struck] + [(1.0-p(Out-State) x (prev. prob for State Occupied)] [(0x.2308) + ((1-.75)x.1468)]	/	Table 2 Lookup	=	= ③①
							→⑤		= (⑥x②⑧) + ((1.0 - ⑦) x ②⑧)		→⑤		
Strike2	1.00	1	1	Sample Calculation:			1.00	x	0.0577	/	1.00	=	0.0577
	Kill Success			Description:			Table 2 Lookup	x	[p(In-State) x (prev. prob. for State Struck] + [(1.0-p(Out-State) x (prev. prob for State Occupied)] [(0x.7895) + ((1-0)x.0769)]	/	Table 2 Lookup	=	= ③②
							→⑤		= (⑥x②⑧) + ((1.0 - ⑨) x ②⑨)		→⑤		
Strike2	1.00	1	2	Sample Calculation:			1.00	x	0.0769	/	1.00	=	0.0769
	Kill Success			Description:			Table 2 Lookup	x	[p(In-State) x (prev. prob. for State Struck] + [(1.0-p(Out-State) x (prev. prob for State Occupied)] [(.75x.2308) + ((1-0)x.6923)]	/	Table 2 Lookup	=	= ③③
							→⑤		= (⑦x②⑧) + ((1.0 - ⓪⓪) x ③⓪)		→⑤		
Strike2	1.00	1	3	Sample Calculation:			1.00	x	0.8654	/	1.00	=	0.8654
											Check Sum	=	1.0000

Figure A-5. Belief vector calculations for fourth action: strike$_2$ to state$_2$.

Recon3 to S2											
Action	Observation	State Action To	State Occupied	Effect	[p (o|s',b,a)	x	p(s'|b,a)]	/	p(o|b,a)	=	p(s'|b,o,a)
	Enemy Not Sighted			Description:	Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prev Prob of En in State Action To * Table 1 Lookup for that State) + (Prev Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prev Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.2105x0.25 + 0.1974x0.8 + 0.5921x0.8 = .6842	=	= ③④
					→ ④		= ③①		= ③②x② + ③①x④ + ③③x④		
Recon2		2	1	Sample Calculation:	0.80	x	0.0577	/	0.7577	=	0.0609
	Enemy Not Sighted			Description:	Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prev Prob of En in State Action To * Table 1 Lookup for that State) + (Prev Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prev Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.2105x0.25 + 0.1974x0.8 + 0.5921x0.8 = .6842	=	= ③⑤
					→ ②		= ③②		= ③②x② + ③①x④ + ③③x④		
Recon2		2	2	Sample Calculation:	0.25	x	0.0769	/	0.7577	=	0.0254
	Enemy Not Sighted			Description:	Table 1 Lookup	x	Previous Prob of Enemy in State Occupied	/	*** <This denominator factor is the normalizing factor> (Prev Prob of En in State Action To * Table 1 Lookup for that State) + (Prev Prob of En in State Action Not To * Table 1 Lookup for that State) + (Prev Prob of En in State Dead * Table 1 Lookup for condition applicable to State Dead) (i.e., The Table 1 Lookup related to State3 is the prob of a false alarm) = 0.2105x0.25 + 0.1974x0.8 + 0.5921x0.8 = .6842	=	= ③⑥
					→ ④		= ③③		= ③②x② + ③①x④ + ③③x④		
Recon2		2	3	Sample Calculation:	0.80	x	0.8654	/	0.7577	=	0.9137
									Check Sum	=	1.0000

Figure A-6. Belief vector calculations for fifth action: $recon_3$ to $state_3$.

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Appendix B. Conditional Probability Calculations for First Five Action Sequences

The following 5 tables illustrate the conditional probability calculations for the sequence $RECON_1$ to S_1, $STRIKE_1$ to S_1, $RECON_2$ to S_2, $STRIKE_2$ to S_1, and $RECON_3$ to S_2.

Action 1: Make Choice to Perform $Recon_1$ to S_1.

The first action decision to be made by the model is whether to recon or shoot at S_1 or S_2. This is based on the initial conditions of the state space where the BV has been previously defined as [0.5, 0.5, 0.0]. It is also noted that the enemy is occupying $State_1$. Using Equation 6 the numerical evaluation of the CP, based on the limitations of the σ and Δ thresholds, is performed resulting in a first action to perform a reconnaissance mission to $State_1$. The results of the action is to generate a new BV = [0.7895, 0.2105, 0.0000].

Table B-1. Conditional probability calculations for first action: $Recon_1$ to S_1.

Initial Belief Vector	CP	Exceed Δ Threshold ?	Exceed σ Threshold ?	Choice	Observation
[0.5,0.5,0.0]	$\left\lceil \frac{S_1}{S_1+S_2} , \frac{S_2}{S_1+S_2} \right\rceil$ $\left\lceil \frac{0.5}{0.5+0.5} , \frac{0.5}{0.5+0.5} \right\rceil$ = [0.5 , 0.5] ∴For Action where Dead = 0.0, the denominator must sum to 1.0, and the total contrast must sum to 1.0.	NO Because DEAD state with a BV value = 0.0 is less than Δ which is equal to 0.9.	NO Because neither location cell has a value > 0.75 as each cell has a contrast ratio value = 0.5.	As both S_1 & S_2 meet recon criteria with value = 0.5, Select, RAN (i) Assuming Random Pick=1. ∴ Choice = RAND recon (S_1), or perform a recon to S_1, or perform $RECON_1$.	ENEMY SIGHTED From this action of $RECON_1$ to S_1 when enemy @ S_1. Generating a Belief Vector = [0.7895,0.2105,0.0] .

Action 2: Make Choice to Perform $Strike_1$ to S_1.

Using the BV from Action #1 of [0.7895, 0.2105, 0.0000], the CP is now evaluated to select Action #2 to be to perform an artillery strike to S_1 generating a new BV = [0.1974, 0.2105, 0.5921].

Table B-2. Conditional probability calculations for second action: $Strike_1$ to S_1.

Previous Belief Vector	CP	Exceed Δ Threshold ?	Exceed σ Threshold ?	Choice	Observation
[0.7895, 0.2105, 0.0]	$\left[\frac{S_1}{S_1+S_2}, \frac{S_2}{S_1+S_2}\right]$ $\left[\frac{0.7895}{0.7895+0.2105}, \frac{0.2105}{0.7895+0.2105}\right]$ = [0.7895 , 0.2105] ∴For Action where Dead Belief = 0.0, the denominator must sum to 1.0, and the total contrast must sum to 1.0.	NO Because DEAD state with a Belief Vector value = 0.0 is less than Δ which is equal to 0.9.	YES Because cell S_1 has a Contrast Ratio value > 0.75	Because S_1 and only S_1 meets SHOOT criteria, Select, Shoot S_1 because S_2 @ 0.2105 is < σ at 0.75. For future cases where there might be multiple location states exceeding σ, set up the general selection of **<u>RAND Shoot (S_1).</u>** In this case the choice is to SHOOT at S_1.	NO INFO (From this action of STRIKE1 to S_1.) Generating a Belief Vector = [0.1974, 0.2105, 0.5921]

Action 3: Make Choice to Perform $Recon_2$ to S_2.

Table B-3. Conditional probability calculations for third ction: $Recon_2$ to S_2.

Previous Belief Vector	CP	Exceed Δ Threshold ?	Exceed σ Threshold ?	Choice	Observation
[0.1974, 0.2105, 0.5921]	$\left[\frac{S_1}{S_1+S_2}, \frac{S_2}{S_1+S_2}\right]$ $\left[\frac{0.1974}{0.1974+0.2105}, \frac{0.2105}{0.1974+0.2105}\right]$ = [0.4839 , 0.5161] ∴For Action where Dead Belief ≠ 0.0, the denominator will not sum to 1.0, but the total contrast must still sum to 1.0., or 0.4839 + .5160 = 1.0	NO Because DEAD state with a Belief Vector value = 0.5921 is less than Δ which is equal to 0.9.	NO Because neither cell has a CR value > 0.75 with S_1= 0.4839 and with S_2= 0.5160	Because neither S_1 or S_2 meets σ criteria, select, RAN (i) of set of cells w/ largest contrast value, in this case only S_2 @ 0.5160 is in the set of cells containing the largest contrast value, thus select a recon into S_2, i.e., perform $RECON_2$ into S_2.	ENEMY NOT SIGHTED From this action of $RECON_2$ to S_2 when enemy @ S_1. Generating a Belief Vector = [0.2308, 0.0769, 0.6923]

Action 4: Make Choice to Perform $Strike_2$ to S_1.

Using the BV from Action #3 of [0.2308, 0.0769, 0.6923], the CP is now evaluated to determine Action #4 to be to perform an artillery strike to S_1 generating a new BV = [0.0577, 0.0769, 0.8654].

Table B-4. Conditional probability calculations for fourth action: $Strike_2$ to S_1.

Previous Belief Vector	CP	Exceed Δ Threshold ?	Exceed σ Threshold ?	Choice	Observation
[0.2308, 0.0769, 0.6923]	$\left[\frac{S_1}{S_1+S_2}, \frac{S_2}{S_1+S_2}\right]$ $\left[\frac{0.2308}{0.2308+0.0769}, \frac{0.0769}{0.2368+0.0769}\right]$ = [0.75008 , 0.2499] $\therefore$ For Action where Dead Belief $\neq$ 0.0, the denominator will not sum to 1.0, but the total contrast must still sum to 1.0., or 0.750081 + .249919 = 1.0	NO Because DEAD state with a Belief Vector value = 0.6923 is less than Δ which is equal to 0.9.	YES Because cell S_1 has a Contrast Ratio value = 0.750081 which is > σ at 0.75.	Because S_1 and only S_1 meets SHOOT criteria, Select, Shoot S_1 because S_2 @ 0.0769 is < σ at 0.75. For future cases where there might be multiple location states exceeding σ, set up the general selection of **<u>RAND Shoot (S_1).</u>** Thus select SHOOT at S_1, i.e., perform STRIKE2 at S_1.	NO INFO (From this action of STRIKE2 to S_1.) Generating a Belief Vector = [0.0577, 0.0769, 0.8654]

Action 5: Make Choice to Perform $Recon_3$ to S_2.

Using the BV from Action #4 of [0.0577, 0.0769, 0.8654]., the CP is now evaluated to determine Action #5 to be to perform a reconnaissance mission to S_2 generating a new BV = [0.0609, 0.0254, 0.9137]. As the new BV component for $State_3$ at 0.9137 now exceeds the Δ threshold of 0.90, the model makes the decision to terminate with a declaration of 'Mission Complete' with the belief that the enemy has been destroyed.

Table B-5. Conditional probability calculations for fifth action: $Recon_3$ to S_2

Previous Belief Vector	CP	Exceed Δ Threshold ?	Exceed σ Threshold ?	Choice	Observation
[0.0577, 0.0769, 0.8654]	$\left[\frac{S_1}{S_1+S_2}, \frac{S_2}{S_1+S_2}\right]$ $\left[\frac{0.0577}{0.0577+0.0769}, \frac{0.0769}{0.0577+0.0769}\right]$ = [0.4287 , 0.5713] ∴For Action where Dead Belief $\neq 0.0$, the denominator will not sum to 1.0, but the total contrast must still sum to 1.0., or 0.4287 + .5713 = 1.0	NO Because DEAD state with a Belief Vector value = 0.8654 is less than Δ which is equal to 0.9.	NO Because neither cell has a CR value > 0.75 with S_1= 0.4287 and with S_2= 0.5713	Because neither S_1 or S_2 meets σ criteria, select, RAN (i) of set of cells w/ largest contrast value, in this case only S_2 @ 0.5713 is in the set of cells containing the largest contrast value, thus select a recon into S_2, i.e., perform $RECON_3$ into S_2.	ENEMY NOT SIGHTED From this action of $RECON_3$ to S_2 when enemy @ S_1. Generating a Belief Vector = [0.0609, 0.0254, 0.9137]

Note: Δ threshold now exceeded with DEAD state Belief Vector value = 0.9137 which is greater than Δ at 0.9, therefore next action will be to DECLARE.

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