REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*
13-07-2007

2. REPORT TYPE
Conference Proceeding

3. DATES COVERED *(From - To)*

4. TITLE AND SUBTITLE
A Hybrid Model for Nearshore Nonlinear Wave Evolution

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
0601153N

6. AUTHOR(S)
Kacey L. Ewards, Jayaram Veeramony, James M. Kaihatu

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER
73-6729-A6-5

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
Oceanography Division
Stennis Space Center, MS 39529-5004

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL/PP/7320-06-7018

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217-5660

10. SPONSOR/MONITOR'S ACRONYM(S)
ONR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Field and laboratory data show the presence of equilibrium in the high frequencies of surf zone frequency spectra. In this region, dissipation of high frequency energy equilibrates with nonlinear interactions that transfer energy primarily from low frequencies to high frequencies. The nonlinear interactions that describe nearshore wave evolution increase model complexity, which results in an increase in computational expense. To reduce the computational burden of a nonlinear wave model, it is combined with a parameterization for surf zone wave spectra, and different implementations of the parameterization are tested and the results show that the predicted spectra and H_{rms} compare well with data and the full model nonlinear parameters skewness and asymmetry are not accurately predicted. Several ideas for future improvements are suggested.

15. SUBJECT TERMS
modeling, shoaling, surf zones, domain

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	UL	13	Kacey L. Edwards
					19b. TELEPHONE NUMBER *(Include area code)* 228-688-5077

A HYBRID MODEL FOR NEARSHORE NONLINEAR WAVE EVOLUTION

Kacey L. Edwards[1] Jayaram Veeramony[2], and James M. Kaihatu[3]

Field and laboratory data show the presence of an equilibrium in the high frequencies of surf zone frequency spectra. In this region, dissipation of high frequency energy equilibrates with nonlinear interactions that transfer energy primarily from low frequencies to high frequencies. The nonlinear interactions that describe nearshore wave evolution increase model complexity, which results in an increase in computational expense. To reduce the computational burden of a nonlinear wave model, it is combined with a parameterization for surf zone wave spectra, and ifferent implementations of the parameterizations are tested and the results show that the predicted spectra and H_{rms} compare well with data and the full model nonlinear parameters skewness and asymmetry are not accurately predicted. Several ideas for future improvements are suggested.

INTRODUCTION

It is well known that as waves propagate through the nearshore environment, they change shape. In the shoaling zone, simple sinusoidal waves transform into waves with narrow, peaked crests and shallow, broad troughs. Furthermore, near the area of breaking the waves have a "sawtoothed" (Kirby, 1997) shape where the front face of the wave is steep and the back of the wave is more gentle sloping.

Wave transformation in the nearshore environment occurs as a result of energy transfers between different frequency components of the wave. Although energy is transferred between all frequencies, the dominant transfer is from low to high frequency. In addition, dissipation preferentially removes energy from the high frequency components. In the surf zone, the transfer of energy to high frequencies and the dissipation of energy from higher frequencies reaches an equilibrium; spectra evolve toward a broad, featureless shape for a range of high frequencies.

The transfer of energy and resulting equilibrium range of high frequency waves is easily seen in frequency and wavenumber spectra. As depth decreases, energy at the peak frequency (f_p) or peak wavenumber (k_p) of a

[1]Oceanography Division, Naval Research Laboratory, 1009 Balch Blvd., Stennis Space Center, MS 39529, USA
[2]Jacobs Engineering, 1009 Balch Blvd., Stennis Space Center, MS 39529, USA
[3]Zachry Department of Civil Engineering, Texas A&M University, 3136 TAMU, [illegible]

spectrum decreases and the tail of the spectrum obtains a gentler slope. In the surf zone, the slope or shape of the spectral tail remains relatively constant, although the amount of energy in the spectral tail may decrease. The transformation of spectral shape in the nearshore environment has been observed in field and laboratory data, as well as model results (Smith & Vincent, 2003; Herbers & Burton, 1997).

Laboratory Experiments

Two such laboratory experiments are the experiments of Mase & Kirby (1992) (referred to as MK92) and Bowen & Kirby (1994) (referred to as BK94). MK92 measured random waves propagating up a 1 : 20 bottom slope. At the wavemaker, the Pierson-Moskowitz spectrum was characterized by $H_{rms} = 0.04$ m and the $f_p = 1.0$ Hz. Therefore, MK92 is an example of mostly deep water waves, and the data are characterized by weak nonlinear interactions and a narrow surf zone. BK94 measured random waves propagating up a 1 : 35 bottom slope. At the wavemaker, $H_{rms} = 0.08$ m and $f_p = 0.225$ Hz. Therefore, their case is an example of mostly shallow water waves. Nonlinear interactions have more of an effect on the transformation of wave spectra, and the surf zone is wider, relative to MK92.

Despite weak nonlinear interactions in the MK92 data, changes in the spectral shape are apparent (Figure 1). In deeper water, where nonlinear interactions are the weakest, there is little change in the spectral shape. However, as depth decreases and the waves shoal, energy at the low frequencies decreases, and energy at higher frequencies increases. Therefore, the slope of the spectral tail decreases. In the surf zone, significant energy is lost from the spectral peak, and the spectrum is much broader. The slope of the spectral tail is flatter.

Modeling Nonlinear Waves

Because waves do not retain a simple, sinusoidal shape throughout the shoaling and surf zones, modeling the evolution of waves through the nearshore environment requires a more complex approach than linear theory. The transfer of energy that causes the changes in the shapes of waves and spectra is facilitated by nonlinear, triad interactions. These three-wave interactions must be included in a model of nearshore wave evolution to accurately predict wave shape statistics, which in turn are necessary for sediment transport problems. Unfortunately, increasing the complexity of a nearshore wave model by including triad interactions significantly increases the computational expense of the wave model.

To reduce the computational burden of a nonlinear wave model, we present a hybrid model that combines a weakly nonlinear, dispersive wave model (Kaihatu & Kirby, 1995) with a parameterization of the surf zone equilibrium range (Smith & Vincent, 2003). In addition to reducing the

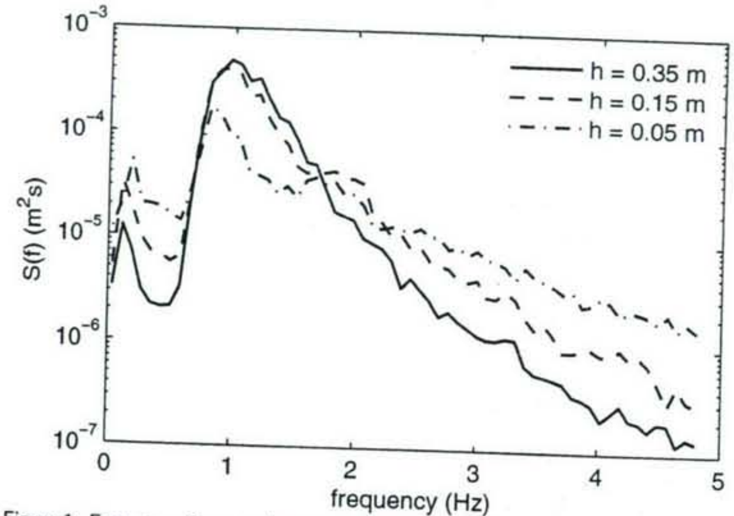

Figure 1: Frequency Spectra determined from measurments collected during the MK92 laboratory experiment.

computational burden, we anticipate that the hybrid model's estimation of spectral tails is an improvement over a full implementation of the weakly nonlinear model. We will examine model results in the form of frequency spectra, as well as wave properties computed from the frequency spectra including H_{rms}, skewness, and asymmetry.

HYBRID MODEL

Nonlinear Wave Model

Many types of models are capable of evolving waves through the nearshore environment, and each model has its advantages and disadvantages. Described herein is a frequency domain, weakly nonlinear, dispersive, one-dimensional wave model (Kaihatu & Kirby, 1995) (referred to as NLMSE). Derivation of the model begins with the boundary value problem expressed in terms of the velocity potential (ϕ) in only one horizontal dimension. The governing equation and boundary conditions are

$$\phi_{xx} + \phi_{zz} = 0; \qquad -h \le z \le 0, \qquad (1)$$

$$\phi_z = -h_x \phi_x; \qquad z = -h, \qquad (2)$$

$$g\eta + \phi_t + \frac{1}{2}(\phi_x^2 + \phi_z^2) + \eta\phi_{zt} = O(\epsilon^3); \qquad z = 0, \qquad (3)$$

$$\eta_t - \phi_z + \eta_x\phi_x - \eta\phi_{zz} = O(\epsilon^3); \qquad z = 0, \qquad (4)$$

where ϕ is the velocity potential, ϵ ($= ka$, $\ll 1$) scales the nonlinear terms tion, and the subscripts refer to derivatives in space (x and z) and time (t) respectively.

The boundary value problem, governed by the Laplace equation (1), describes the movement of an inviscid, irrotational fluid. Furthermore, the problem is restricted to progressive waves

$$\phi = \sum_{n=1}^{N} -\frac{ig}{\omega_n} f_n A_n(x,y) e^{i(\int k_n \, dx - \omega_n t)} + \text{c.c.}, \tag{5}$$

where N is the number of frequency components.

The evolution equation for the amplitudes A_n is

$$\begin{aligned} A_{n_x} + \frac{(kCC_g)_{n_x}}{2(kCC_g)_n} A_n + \alpha_n A_n = \\ -\frac{i}{8(kCC_g)_n} \Bigg[\sum_{l=1}^{n-1} R A_l A_{n-l} e^{i\int (k_l + k_{n-l} - k_n) dx} \\ + 2 \sum_{l=1}^{N-n} S A_l A_{n+l} e^{i\int (k_{n+l} - k_l - k_n) dx} \Bigg]. \end{aligned} \tag{6}$$

$C = \omega_n / k_n$ is phase speed, and $C_g = \partial \omega_n / \partial k_n$ is group velocity. The coefficients R and S are the subharmonic and superharmonic triad interactions given by

$$\begin{aligned} R = \frac{g}{\omega_l \omega_{n-l}} \left[\omega_n^2 k_l k_{n-l} + (k_l + k_{n-l})(\omega_{n-l} k_l + \omega_l k_{n-l}) \omega_n \right] \\ - \frac{\omega_n^2}{g} (\omega_l^2 + \omega_l \omega_{n-l} + \omega_{n-l}^2), \end{aligned} \tag{7}$$

$$\begin{aligned} S = \frac{g}{\omega_l \omega_{n+l}} \left[\omega_n^2 k_l k_{n+l} + (k_{n+l} - k_l)(\omega_{n+l} k_l + \omega_l k_{n+l}) \omega_n \right] \\ - \frac{\omega_n^2}{g} (\omega_l^2 - \omega_l \omega_{n+l} + \omega_{n+l}^2). \end{aligned} \tag{8}$$

The one dimensional model is initialized offshore with a wave spectrum. It moves forward in space determining complex amplitudes for a given number of frequency components (N).

Because the evolution equation is derived assuming weak nonlinearity and mild slope, the model retains the linear depth dependence and linear dispersion relation given by

$$f_n = \frac{\cosh k_n (h+z)}{\cosh k_n h} \quad \text{and} \tag{9}$$

$$\omega^2 = gk \tanh k_n h \tag{10}$$

Although the NLMSE includes terms on the left hand side that combine shoaling, refraction and diffraction, it does not account for energy lost to friction or breaking. Therefore, Kaihatu & Kirby (1995) add the third term (dissipation) to the left hand side of the evolution equation. The dissipation term is based on the dissipation model of Thornton & Guza (1983). They define the total energy dissipation as

$$\epsilon_b = \frac{3\sqrt{\pi}}{16} \rho g \frac{B^3 f_p}{\gamma^4 h^5} H_{rms}^7, \tag{11}$$

where B and γ are free parameters with values 1.0 and 0.6, respectively, and f_p is peak frequency.

Using the approach of Mase & Kirby (1992) the dissipation model (11) is distributed between frequency independent and frequency dependent components.

$$\alpha_n = \alpha_{n_0} + \left(\frac{f_n}{f_p} \right)^2 \alpha_{n1}, \tag{12}$$

$$\alpha_{n_0} = F \epsilon_b', \tag{13}$$

$$\alpha_{n_1} = [\epsilon_b' - \alpha_{n_0}] \frac{f_p^2 \sum_{n=1}^{N} |A_n|^2}{\sum_{n=1}^{N} f_n^2 |A_n|^2}, \tag{14}$$

$$\epsilon_b' = \frac{\epsilon_b}{\rho g \sum_{n=1}^{N} C_g |A_n|^2}. \tag{15}$$

N is the number of frequency components, and F is a free parameter that determines what percentage of the dissipation model (11) is distributed to the frequency independent function (13).

The right hand side of the evolution equation contains nonlinear sums, which replicate the nonlinear interactions between waves of different frequencies. Unfortunately, the complexity of the nonlinear sums makes the NLMSE model computationally expensive. The nonlinear sums require a number of computations on the order of N^2. In addition, the NLMSE model can potentially overpredict energy in very high frequencies (Figure 2).

Surf Zone Parameterization

To reach our goals of reducing the computational expense of modeling wave evolution in the nearshore environment and improving the ability to replicate high frequency energy, we employ the surf zone parameterization of Smith & Vincent (2003)

Until 2003, little effort was put into parameterizing surf zone waves. Following the development of deep water wave parameterizations (Resio, Pihl, Tracy & Vincent, 2001) and previous work by Thornton (1977) and Zakharov (1999), Smith & Vincent (2003) described a parameterization for surf zone($H_{rms}/h > 0.4$) wave spectra.

Relying on wavenumber spectra from many laboratory and field experiments, Smith and Vincent (2003) found that the range of frequencies for which energy transfer and dissipation equilibrated was composed of two subranges. They call the two subranges the Zakaharov range and the Toba range. The Zakharov range is valid for low wavenumbers ($k_p < k < 1/h$), and the Toba range is valid for high wavenumbers ($k \geq 1/h$). Therefore, both ranges exist only if $k_p h << 1$.

Each subrange has its own shape and energy level. The ranges are defined mathematically as power laws in wavenumber space and are written

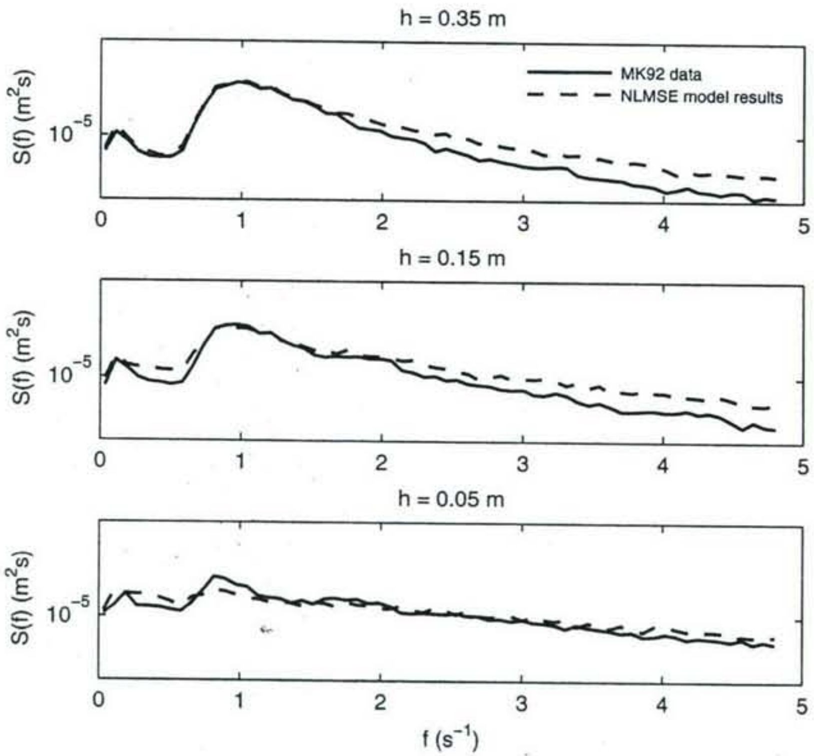

Figure 2: Results from the NLMSE model.

$$S(k) = \beta_{Toba} k^{-5/2}; \qquad kh > 1 \tag{16}$$

$$S(k) = \beta_{Zak} k^{-4/3}; \qquad k_p < k \leq 1/h \tag{17}$$

$$\beta_{Toba} = \alpha_{Toba} h^{0.5} \tag{18}$$

$$\beta_{Zak} = \alpha_{Zak} h^{1.67} \tag{19}$$

Because the subranges intersect at $k = 1/h$, $\alpha_{Toba} = \alpha_{Zak} = 0.0103 = \alpha$. In addition, the dependence on depth requires that the energy level decreases as depth decreases.

Combining the Two Components

Use of the parameterization in conjuction with the NLMSE model insures a reduction in computation expense because the number of frequency components explicitly determined by the NLMSE model decreases. To combine the two, use the relation

$$S(f_n)df = S(k_n)dk_n. \tag{20}$$

In addition, complex amplitudes are determined for the equilibrium range using the relation

$$A_n = \sqrt{\frac{S(f_n)df}{2}} e^{i\theta_n} \tag{21}$$

where θ_n is a random phase produced by a random number generator.

ALTERNATIVE β_{Toba}

Because the parameterization was developed for surf zone wave spectra, it does not represent offshore wave spectra well. At offshore locations, the slope of the parameterization compares reasonably to data; however, the β coefficients overestimate the energy of the equilibrium range (Figure 3). Limiting use of the parameterization to the surf zone region works well, but hinders any computational savings. This is especially true for cases similar to MK92 where the surf zone is relatively narrow.

An alternative approach to determine the beta coefficient was explored. β_{Toba} was written not only as a function of water depth but also as a function of the nonlinearity parameter H_{rms}/h. This was accomplished by writing

$$\alpha = C\frac{H_{rms}}{h}. \tag{22}$$

C is a constant that defines α as a fraction of the nonlinearity parameter. It is given by the relation

$$C = \alpha_0 \frac{h_0}{H_{rms,0}} \tag{23}$$

where α_0, h_0, and $H_{rms,0}$ represent offshore values, and

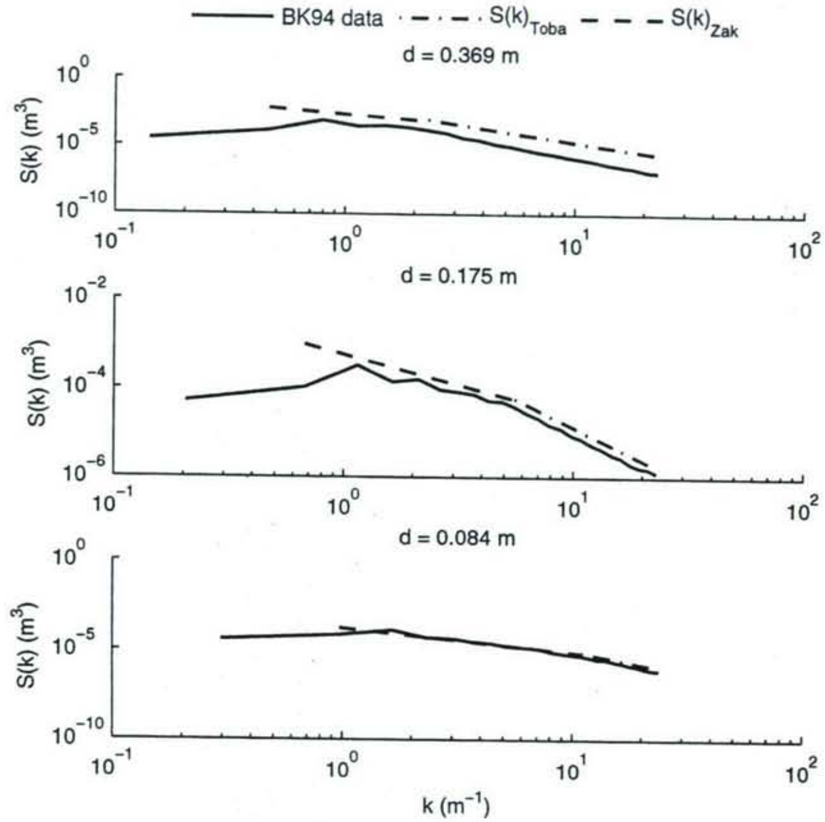

Figure 3: Comparison of Smith & Vincent (2003) parameterization to data from the BK94 experiment.

$$\alpha_{Toba,0} = \frac{\displaystyle\int_{k=1/h}^{k_N} S(k)_0\, dk}{\displaystyle h_0^{0.5} \int_{k=1/h}^{k_N} k^{-5/2}\, dk} \tag{24}$$

where $S(k)_0$ is the wavenumber spectrum at the offshore boundary.

Frequency spectra resulting from the NLMSE model, the hybrid model using $\alpha = 0.0103$ (referred to as SNV) and the hybrid model using (22) (referred to as NALF) are compared in Figure 4. As expected, offshore the SNV hybrid model provides the same results as the NLMSE model. In addition, the NALF hybrid model compares well; the results overestimate energy at the higher frequencies slightly less than the NLMSE and SNV

hybrid models do. However, as depth decreases, the NALF hybrid model underestimates the frequency spectrum. The SNV hybrid model compares to the data well in the parameterized range, butoverestimates low frequency energy. Well into the surf zone, both hybrid models compare well.

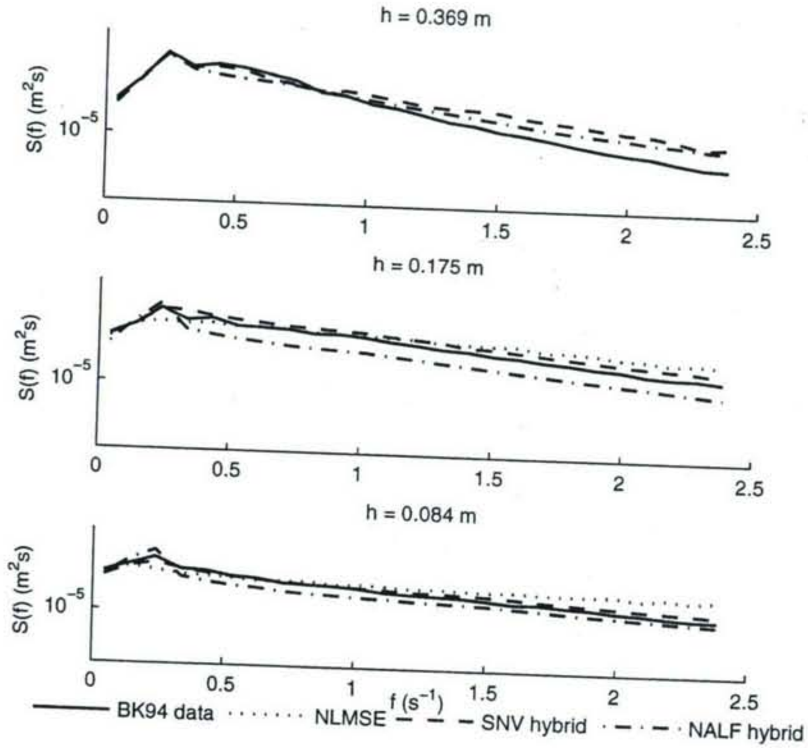

Figure 4: Frequency spectra resulting from the hybrid models and the NLMSE model.

The SNV hybrid model compares well through the modeled domain, but the computational expense is not lessened. The NALF hybrid model required only fifteen percent of the NLMSE model's computational time. The NALF hybrid model compares well to the data at the offshore location because the constant, C, depends on offshore parameters. This fraction, however, is not constant through the entire domain.

Comparisons of wave parameters determined from the data and the various model results reflect what is observed in the frequency spectra comparisons (Figure 5). The SNV hybrid model replicates H_{rms} outside the surf zone, but once the surf zone is reached and the parameterization i tling into agreement with the data. The NALF hybrid model significantly underestimates H_{rms} until well into the surf zone.

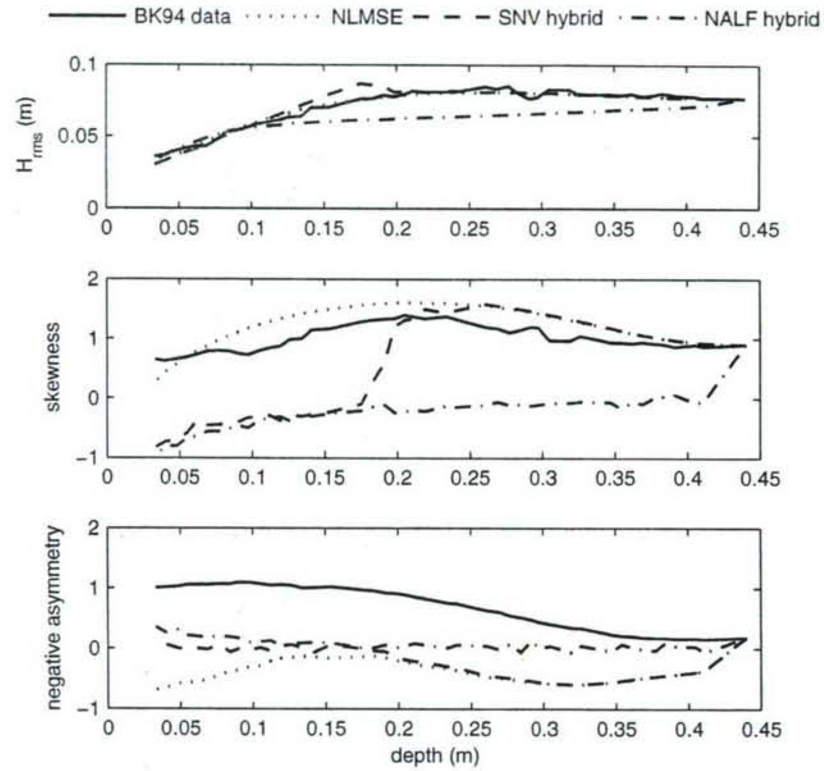

Figure 5: Wave parameters determined from NLMSE and hybrid model results. Top: H_{rms}, Middle: skewness, Bottom: asymmetry.

Comparisons of the parameters skewness and asymmetry show that neither of the hybrid models reproduce the nonlinear properties. Accurate energy transfers require correct phase locking. The random phases, θ_n, used to determine complex amplitudes in the equilibrium range do not ensure the correct phase locking.

PHASE REPRESENTATION

In an effort to improve the nonlinear properties, two additional phase representations were explored. First, the offshore phases were held constant through the domain. Then, the evolution equation was used to evolve the phase through the domain.

To obtain the evolution equation for the phases, we began by assuming that

$$A_n = a_n e^{i\phi_n(x)}. \tag{25}$$

After substituting (25) into (6), we solved for the partial derivative of ϕ_n and used the result to specify phase for the equilibrium range so that

$$\phi_{n_x} = -\frac{1}{8\omega_n C_{g,n} a_n} \cdot \left[\sum_{l=1}^{n-1} R a_l a_{n-l} e^{i\int (k_l + k_{n-l} - k_n + \phi_l + \phi_{n-1} - \phi_n) dx} + 2\sum_{l=1}^{N-n} S a_l a_{n+l} e^{i\int (k_{n+l} - k_l - k_n + \phi_{n+l} - \phi_l - \phi_n) dx} \right]. \tag{26}$$

Figure 6 shows skewness and asymmetry results from the SNV hybrid model using random, constant, and evolved phases. Neither the constant phase nor the evolved phase appears to improve the hybrid model's ability to produce the nonlinear parameters or H_{rms}.

SUMMARY

Modeling wave evolution through the shoaling and surf zones requires a complex model that is computationally expensive. In this study, the complex model is a frequency domain, weakly nonlinear, dispersive one dimensional wave model (Kaihatu & Kirby, 1995). The computational expense is reduced by limiting the number of frequency components explicitly determined by the nonlinear model and determining the complex amplitudes at the excluded frequencies with a parameterization.

The surf zone parameterization as described by Smith & Vincent (2003) was combined with the NLMSE model (Kaihatu & Kirby, 1995) to form a hybrid model. Because the parameterization applies to only the surf zone, no computational savings was acheived. However, the hybrid model produced a better replication of high frequency energy.

Using an alternative descripton of β_{Toba} allows the parameterization to be used outside the surf zone. Therefore, the computational burden of evolving waves through a domain is lessened. However, β_{Toba}'s depth dependence dominates the parameterization, and the hybrid model does not compare well in the surf zone.

Model results are reiterated in the computation of H_{rms}. The SNV hybrid model provides H_{rms} estimates that compare well offshore and inside the surf zone. However, there is a region of overestimation because for an area of the domain, the SNV hybrid overestimates the low frequency energy. For most of the BK94 domain

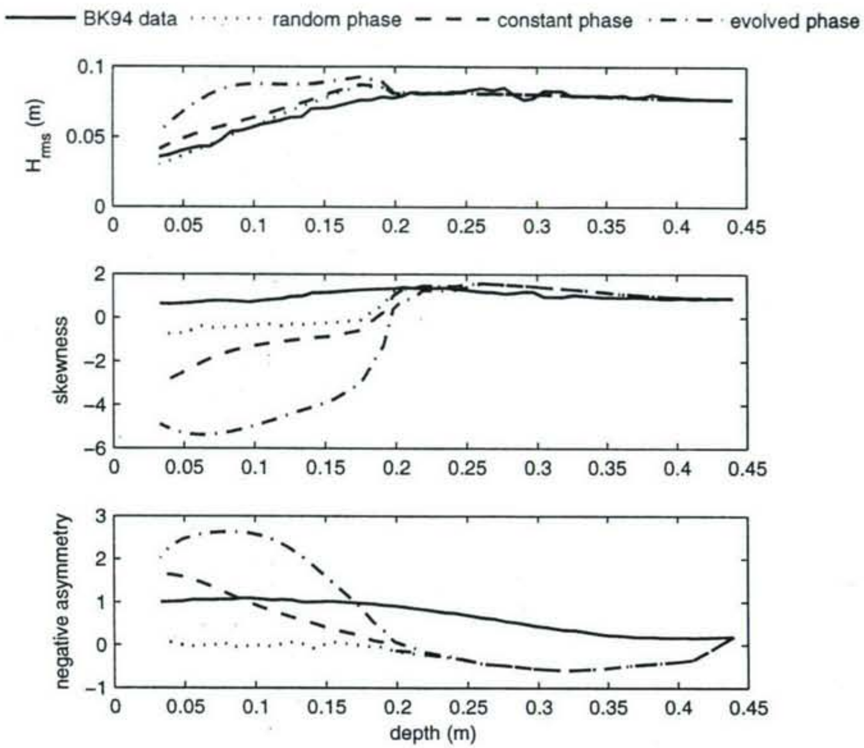

Figure 6: Wave parameters determined from the SNV hybrid model results using different phase representations. Top: H_{rms}, Middle: skewness, Bottom: asymmetry.

Although the hybrid models give reasonable H_{rms} values, the skewness and asymmetry are clearly inaccurate. Using a random phase to determine complex amplitudes in the equilibrium range interferes with the phase locking of the triad interactions. However, neither of the two alternative approaches of determining phase for the parameterized range provided any improvement over the use of the random phase.

Both aspects of this study (the energy level and the phase representation) require additional investigation. To improve the β coefficient, the $k = 1/h$ limit will be explored as well as a more reliable method to determine the constant C (23). Furthermore, perhaps a gentler approach to employing the parameterization will improve the results and reduce the computational expense. Rather than forcing the spectrum to the parameterization, maybe it would be better to guide it toward the parameteriztion.

ACKNOWLEDGMENTS

Special thanks to Dr. Jane Smith of the US Army Engineer Research and Development Center's Coastal and Hydraulics Laboratory and Dr. James Kirby at the University of Delaware for providing data needed for this study. The work was supported by the Office of Naval Research through the 6.1 NRL Core Project "Nonlinear Interactions in Nearshore Environments," Program Element 61153N.

REFERENCES

Bowen, G. D. & Kirby, J. T. (1994). Shoaling and breaking random waves on a 1:35 laboratory beach. Technical report, Center for Applied Coastal Research, Department of Civil Engineering, University of Delaware, Delaware.

Herbers, T. H. C. & Burton, M. C. (1997). Nonlinear shoaling of directionally spread waves on a beach. *J. Geophys. Res.*, *102*, 21101–21114.

Kaihatu, J. M. & Kirby, J. T. (1995). Nonlinear transformation of waves in finite water depth. *Physics of Fluids*, *7*, 1903–1914.

Kirby, J. T. (1997). Nonlinear, dispersive long waves in water of variable depth. In J. N. Hunt (Ed.), *Gravity Waves in Water of Finite Depth*, volume 10 of *Advances in Fluid Mechanics* (pp. 55–126). Great Britain: Computational Mechanics Publications.

Mase, H. & Kirby, J. T. (1992). Hybrid frequency-domain kdv equation for random wave transformation. In Edge, B. L. (Ed.), *Proceedings for the 23rd International Conference of Coastal Engineering*, (pp. 474–478)., Reston, VA.

Resio, D. T., Pihl, J. H., Tracy, B. A., & Vincent, C. L. (2001). Nonlinear energy fluxes and the finite depth equilibrium range in wave spectra. *J. Fluid Mech.*, *95*, 159–176.

Smith, J. M. & Vincent, L. V. (2003). Equilibrium ranges in surf zone wave spectra. *J. Geophys. Res.*, *108*, 3366–3376.

Thornton, E. B. (1977). Rederivation of the saturation range in the frequency spectrum of wind-generated gravity waves. *J. Phys. Oceanogr.*, *7*, 137–140.

Thornton, E. B. & Guza, R. T. (1983). Transformation of wave height distribution. *J. Geophys. Res.*, *88*, 5925–5938.

Zakharov, V. (1999). Statistical theory of gravity and capillary waves on the surface of a finite-depth fluid. *J. Geophys. Res.*, *88*, 5925–5938.