AFRL-SN-WP-TR-2007-1124

APERTURE AND RECEIVER TECHNOLOGY

Delivery Order 0002: Bandwidth Invariant Spatial Processing

Volume 1 - Computational Requirement Analysis of Wide-band Direction of Arrival (DOA) Algorithms

Mohsin M. Jamali

The University of Toledo
Department of Electrical Engineering and Computer Science
Toledo, OH 43606

MAY 2007

Final Report for 21 December 2005 – 31 May 2007

SENSORS DIRECTORATE
AIR FORCE RESEARCH LABORATORY
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AFB, OH 45433-7320

NOTICE

THIS REPORT HAS BEEN REVIEWED AND IS APPROVED FOR PUBLICATION IN ACCORDANCE WITH ASSIGNED DISTRIBUTION STATEMENT.

*//signature//	//signature//
CHRISTOPHER R. REHM, Capt, USAF	JILL E. JOHNSON

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*
May 2007

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
12/21/2005 – 05/31/2007

4. TITLE AND SUBTITLE
APERTURE AND RECEIVER TECHNOLOGY
Delivery Order 0002: Bandwidth Invariant Spatial Processing
Volume 1 - Computational Requirement Analysis of Wide-band Direction of Arrival (DOA) Algorithms

5a. CONTRACT NUMBER
FA8650-05-D-1848-0002

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
62204F

5d. PROJECT NUMBER
7622

5e. TASK NUMBER
11

5f. WORK UNIT NUMBER
7622110L

6. AUTHOR(S)
Mohsin M. Jamali

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The University of Toledo
Department of Electrical Engineering and Computer Science
Toledo, OH 43606

8. PERFORMING ORGANIZATION REPORT NUMBER
DSPH-10

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Sensors Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson AFB, OH 45433-7320

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL-SN-WP

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-SN-WP-TR-2007-1124

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
Report contains color. PAO Case Number: AFRL/WS-07-1443, 18 June 2007. Report contains color.

14. ABSTRACT
This work performed a review of wide-band DOA algorithms in the literature which was accumulated for more than 30 years. We have reviewed the most relevant ones with the goal of implementing them in hardware for real time applications. We have discovered a class of computational requirements that would be required in all these algorithms. We have also given reviews and challenges. We were able to cut through all the mathematics and convert algorithms into simple arithmetic operations. This step is very useful in visualizing an architecture. We have filled a gap between the design of computer hardware especially special purpose parallel architectures and available algorithms for various wide-band DOA algorithms. This work was the first step in sorting out which algorithm is appropriate for further study and for its hardware implementation for real time applications.

15. SUBJECT TERMS
Wide-band, Direction of Arrival, Array Processing

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
Unclassified	Unclassified	Unclassified	SAR	130	Capt. Christopher Rehm **19b. TELEPHONE NUMBER** *(Include Area Code)* (937) 255-5579, ext. 3728

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

Contents

Acknowledgement

I would like to thank the generous support of AFRL SN and MacAulay-Brown, Inc. Dayton for their financial support under contract Contract # FA 8650-05-D-1848/2. I would like to thank Mr. Nathan Wilkins and Captain Christopher Rehm for supporting and helping in this work. I would also like to extend my thanks Mr. Ken Hahn of MacAulay Brown to be there at every step in the course of this work and be anxious to help any time.

I would like to thank and offer our sincere gratitude to all authors whose work has been reviewed in positive way and to extend their work to the next level of excellence and for the good of the country and mankind. Some of the language in Chapter 2 was borrowed from their original work as it was very difficult to re-write their excellent work in our own words. I have acknowledged their work by names and in references. I also apologize to authors whose work was not reviewed or referenced in this report. It was just impossible to put thirty years of excellent literature in one concise report and in finite time.

Executive Summary

This work performed a review of wide-band DOA algorithms in the literature which was accumulated for more than 30 years. There are more than fifty publications for wide-band detection of Direction of Arrival (DOA) algorithms which are available in the literature. We have reviewed the most relevant one and have not reviewed others which will not be applicable or suitable for hardware implementation. These algorithms were generally presented in a very complex or condense form, which are not easily understandable for people who are outside that narrow field. One of the reasons for their complex representation is due to their publication in IEEE transactions and conferences. These transactions generally prefer highly mathematical papers and sometimes authors insert mathematics so chances of their papers are increased. Another reason for condense reporting is that these papers face a page limit. Therefore algorithms need to be accommodated within those guidelines and also comply with the reviewers comments. One unfortunate thing happens in this process that essential information does not get into the papers and there is always missing information. This missing information is acute in our case as we are looking from hardware implementation point of view and we are ignoring details of statistical results and errors which are irrelevant in our case. We are willing to sacrifice small amount of error in order to accomplish the goal of implementing them in hardware for real time applications.

We have discovered a class of computational requirements that would be required in all these algorithms as was summarized. We have also given reviews and challenges. We were able to cut through all the mathematics and convert algorithms into simple arithmetic operations. This step is very useful in visualizing an architecture. We have filled a gap between the design of computer hardware especially special purpose parallel architectures and available algorithms for various wide-band DOA algorithms.

This work was the first step in sorting out which algorithm is appropriate for further study and for its hardware implementation for real time applications. We have developed hardware as described in Chapter 5 and Volume 2 of this report. Work is in progress for implementation of identified computational steps in FPGAs.

This work can be extended to develop re-configurable test-bed environment for investigative studies for various algorithm. The re-configurable test-bed would be useful to study timing, memory, hardware requirement and accuracy of results for various algorithms. This test-bed would be useful in evaluating different number of sensors and different kind of sensors. This test-bed would also be a technology scalable system and would become useful in deployment hardware.

Chapter 1

Introduction

Array processing has been an important part of signal processing in the past few decades [1-40]. The array consists of sensors located at different points in space with respect to a reference point. Direction of Arrival (DOA) denotes the direction from which the wave fields arrive at the sensor array. The goal in DOA detection and estimation is to accurately determine the number of sources producing waveforms and the locations of those sources. The passive detection of objects has become important in the military applications as it evades detection by others. The estimation of Direction-Of-Arrival (DOA) from energy wavefield has many applications both inside and outside of the military use. Some civilian applications are in the areas of communications, air traffic control, seismology, sonar, and bioengineering. Generally passive detection approaches are computed intensive. Their hardware implementation could be cumbersome and hard to meet the real time computational requirements.

With the growth in technology and increase in processing power, it is now possible to develop real time hardware for many applications. There are three types of techniques generally available in the literature for detection of DOA [1-6]. They are Power Spectral (PSD), maximum likelihood, maximum entropy and signal subspace methods. PSD techniques are ineffective and maximum likelihood techniques are considered accurate, they are very computationally expensive and also yield non-linear equations. Maximum entropy methods introduce bias and are very sensitive to various parameters. Signal subspace techniques are accurate, give high resolution and reasonably compute intensive. Subspace-based detection techniques that yield high resolution for estimating DOA through passive sensors have been popular and have become an intensive research area in array signal processing. These techniques utilize eigenstructure of the covariance matrix. The eigenvalues are then used to estimate the number of sources.

This work considers that there are D sources in far field and wavefront is considered planar. There are M sensors arranged in a Uniform Linear Array. We also assume that number of sensors is greater than number of sources. The distance between two sensors is d. Signals under consideration could be narrow-band or wide-band. In the narrow-band case the carrier frequency is considered very large as compared to the bandwidth of the signal. If the carrier frequency is comparable to the bandwidth of the signal then the signal is considered as wide-band signal. The propagation time from one sensor to another sensor is constant and is generally approximated by a pure phase delay in a narrow-band case. This approximation can not be made whenever there is a wide-band signal. The wavefront impinge on the sensor with azimuth angle θ. An ULA is shown in Figure 1.

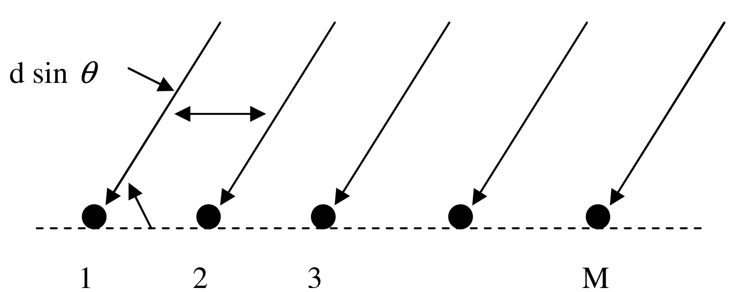

Figure 1: Uniform Linear Array of M sensors

We consider an array of M sensors and D point sources. Signals from these sources impinge on the array from directions $\theta_1, \theta_2, ..., \theta_D$. The narrow-band signals can be represented as time shifted version of each other and are expressed as:

$$Y(t) = \begin{bmatrix} x_1(t-\tau_1) \\ x_2(t-\tau_2) \\ \vdots \\ x_M(t-\tau_M) \end{bmatrix} \tag{1.1}$$

Where τ_k is the time it takes the planar wavefront to travel from the source to the origin of the array through the mth sensor. The signal is generally considered as a complex signal and can be expressed as

$$y_k(t) = x(t-\tau)\, e^{-j(w(t-\tau)+\varphi)} \tag{1.2}$$

where φ is an arbitrary phase.

This could then be expressed as

$$Y(t) = x(t-\tau) \begin{bmatrix} e^{-jw\tau_1} \\ e^{-jw\tau_2} \\ \vdots \\ e^{-jw\tau_M} \end{bmatrix} \tag{1.3}$$

The Mx1 vector in the above equation is generally referred as the steering vector. This steering vector includes the DOA parameter. The signal model can then be written as:

$$Y(t) = x(t-\tau)\begin{bmatrix} e^{-jw\tau_1} \\ e^{-jw\tau_2} \\ \vdots \\ e^{-jw\tau_M} \end{bmatrix} + \begin{bmatrix} n(t)_1 \\ n(t)_2 \\ \vdots \\ n(t)_M \end{bmatrix} \tag{1.4}$$

If there are D sources and M sensors then the sensor output model can be written as:

$$Y(t) = \sum_{k=1}^{D} a(\theta_k)\, x_k(t-\tau) + n(t) \tag{1.5}$$

where x_k(t), i=1,…,D are planar wavefronts corresponding to each received source signal. *n(t)* is the noise which is considered as zero mean and white Gaussian, a(θ) is the steering vector

$$\text{and } \tau = d\frac{\sin\theta}{c} \tag{1.6}$$

where d is the distance between two sensors and is considered as the half of the wavelength.

Define

$$\mathbf{A}(\theta) = \left[\mathbf{a}(\theta_1),\quad \mathbf{a}(\theta_2),\quad \cdots \quad \mathbf{a}(\theta_D)\right] \tag{1.7}$$

$$\mathbf{a}(\theta_K) = \begin{bmatrix} a_1(\theta_k)e^{-jw_0\tau_1(\theta_k)} \\ \vdots \\ a_M(\theta_k)e^{-jw_0\tau_M(\theta_k)} \end{bmatrix} \tag{1.8}$$

This equation is also commonly known as an array manifold. The output can be expressed in the matrix form as:

$$Y(t) = A(\theta)\, X(t) + N(t) \tag{1.9}$$

where

Y(t), N(t) $\in$ C^{M}, X(t) $\in$ C^{D} and A(θ) $\in$ C^{MxD}

Matrix **A** is called the direction matrix and each of the column vectors are the direction vectors of the sources. Assuming that the noise is independent of the signal with zero mean, the covariance matrix can be expressed as:

$$\mathbf{R}_{yy} = E\left[Y(t)Y^{H}\left(t\right)\right] \tag{1.10}$$

If the output array vector $Y(t)$ is observed over K subintervals of duration ΔT seconds each, the covariance matrix can be expressed as the snapshot averaged cross-product of $y_k(t)$:

$$\mathbf{R}_{yy} = \frac{1}{K}\sum_{k=1}^{K} y_k\left(t\right) y_k^{H}\left(t\right) \tag{1.11}$$

1.1 Narrow-band Multiple Signal Classification algorithm:

The Multiple Signal Classification (MUSIC) algorithm is a high resolution technique. It uses signal subspace approach and separates signal and noise subspaces. It finds the array manifold orthogonal to the noise space. Signal and noise subspaces are orthogonal to each other.

This means that the eigenvectors associated with the M - D smallest eigenvalues are orthogonal to the direction vectors making up $\mathbf{A}$. These observations form the basis of the MUSIC algorithm. We can estimate the direction of arrival by finding direction vectors which lie in the signal subspace. These vectors are the direction vectors that are orthogonal to the noise subspace. To search the noise subspace, we form a matrix containing the noise eigenvectors:

$$\mathbf{V}_n = \left[\mathbf{q}_{D+1} \quad \cdots \quad \mathbf{q}_D\right] \tag{1.12}$$

Since the direction vectors of the incoming signals are orthogonal to the noise subspace eigenvectors, we can say:

$$\mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\theta)=0 \tag{1.13}$$

where θ is the direction of arrival of a signal component.

The direction of arrival can then be estimated by finding the peaks of the MUSIC spectrum given by:

$$p(\theta)=\frac{1}{\mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\theta)},\theta\in[0,2\pi] \tag{1.14}$$

The *D* largest peaks in the spectrum will correspond to the directions of arrival of the signals impinging on the sensor array.

The MUSIC algorithm can be summarized as follow:

1. Collect input samples $y_k(t)$
2. Estimate the covariance matrix given by:

$$\mathbf{R}_{yy}=\frac{1}{K}\sum_{k=1}^{K}y_k(t)\,y_k^H(t)$$

3. Compute eigenvalues and eigenvectors of the covariance matrix.
4. Find number of sources *D*.
5. Find the DOA estimates by finding the *D* largest peaks of the MUSIC spectrum given by:

$$p(\theta)=\frac{1}{\mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\theta)}$$

1.2 Estimating number of sources:

The MUSIC algorithm depends on the parameter *D*, the numbers of wave fronts impinging on the sensor array, for estimating the DOA of multiple targets. The Minimum Description Length (MDL) was used in order to achieve this objective [1-7]. The MDL is specified by:

$$\text{MDL}(D) = -2\log\left(\frac{\prod_{k=D+1}^{M} \lambda_k^{\frac{1}{(M-D)}}}{\frac{1}{M-D}\sum_{k=D+1}^{M}\lambda_k}\right) + \frac{1}{2}(2M-D)\log(N) \quad (1.30)$$

where *N* is the number of samples, λ represents the eigenvalues of the covariance matrix and *M* is the number of sensors. The number of sources is determined by finding a value of *D* which minimizes the MDL criterion. The maximum number of sources that can be estimated is *M* - 1.

1.3 Wide-band sources

Signal model

Assume that is a Uniform Linear Array of M sensors and there are *D* wide-band sources with identical bandwidth *B* impinging on the array from directions $\theta_1, \theta_2, ..., \theta_D$. In the wide-band case the time delay of planar wave propagating from one sensor to another cannot be approximated as a phase shift. Assuming that the signals are observed over a finite interval *T*, we can represent the signal x_i by a Fourier series [1-7]:

$$x_i(t) = \sum_{n=l}^{l+m} X_i(w_n) e^{jw_n t} \quad (1.31)$$

where $X_i(w_n)$ are the Fourier coefficients given by:

$$X_i\left(w_n\right)=\frac{1}{T^{1/2}}\int_{-T/2}^{T/2} x_i\left(t\right)e^{jw_nt}dt \tag{1.32}$$

$$w_n=\frac{2\pi}{T}n, \quad n=l,\ldots,l+m \tag{1.33}$$

where w_l is the lowest frequency and w_{l+m} is the highest frequency included in the bandwidth *B*. Assuming that the observation time *T* is much greater than the propagation delay across elements of the array, we can use a phase shift as an approximation of the time delay in the Fourier domain.

$$y_i\left(w_n\right)=\sum_{k=1}^{D}\mathbf{a}_{ik}e^{-jw_n\tau_{ik}}x_k\left(w_n\right)+\mathbf{n}_i\left(w_n\right) \tag{1.34}$$

The model used to represent the output vector is:

$$Y\left(w_n\right)=\mathbf{A}\left(w_n\right)X\left(w_n\right)+\mathbf{N}\left(w_n\right) \tag{1.35}$$

$$\mathbf{A}\left(w_n\right)=\left[\mathbf{a}_{\theta 1}\left(w_n\right), \quad \mathbf{a}_{\theta 2}\left(w_n\right), \quad \cdots \quad \mathbf{a}_{\theta D}\left(w_n\right)\right] \tag{1.36}$$

$$\mathbf{a}_{\theta k}\left(w_n\right)=\begin{bmatrix} a_1\left(\theta_k\right)e^{-jw\tau_1\left(\theta_k\right)} \\ \vdots \\ a_M\left(\theta_k\right)e^{-jw\tau_M\left(\theta_k\right)} \end{bmatrix} \tag{1.37}$$

As a result of the Fourier transform applied over a time segment ΔT, the array output vector is decomposed into non-overlapping narrow-band components. The covariance matrix for component w_n can be expressed as:

$$\mathbf{R}_{yy}\left(w_n\right)=E\left[y\left(w_n\right)y^H\left(w_n\right)\right] \tag{1.38}$$

This covariance matrix can also be expressed as:

$$\mathbf{R}_{yy}\left(w_n\right)=\frac{1}{K}\sum_{k=1}^{K}y_k\left(w_n\right)y_k^H\left(w_n\right) \tag{1.40}$$

Chapter 2

Review of Wide-band DOA Algorithms

There are number of wide-band DOA algorithms currently available in literature mainly from IEEE. Number of searches was performed to locate wide-band DOA algorithms. These searches were conducted at the IEEE digital library website commonly known as IEEE Explorer and Google. Searches at Google extend beyond IEEE and hence proved very useful. This chapter provides review of wide-band DOA algorithms. Each review contains introduction, algorithm development, simulation information and our conclusions. Algorithmic equations have modified as much as possible to keep uniformity and increase the readability of the work. This work assumes that there are D wide-band sources, a Uniform Linear Array of M sensors, J beamformers (where applicable) and N pieces of data. Sources are assumed to be in far field so waves are impinging on the array as planar wave. Moreover sources are also Omni-directional.

Initially wide-band DOA estimation was performed by estimating narrow-band DOA at each frequency and then combined to obtain the wide-band frequency. This approach is called incoherent approach [4-5]. Comparison of computational requirements for all wide-band DOA algorithms reviewed in this chapter will be discussed in the next chapter.

2.1 Coherent signal subspace method for wide-band sources by Wang & Kaveh

It is possible to combine the signal subspaces of different frequencies in order to generate a single subspace that will allow us to determine the correct number of sources and directions of arrival [6-7]. The matrix $\mathbf{R}$ can be used to find the final DOA estimates. This matrix can be formed by:

$$\mathbf{R} = \sum_{j=1}^{J} \mathbf{T}(w_j)\mathbf{R}_{yy}(w_j)\mathbf{T}^H(w_j) \tag{2.1}$$

where J is the number of narrow-band components. The matrices $\mathbf{T}$ are called transformation matrices and can be expressed as

$$\mathbf{T}(w_j) = \begin{bmatrix} a_{1\beta}(w_o)/a_{1\beta}(w_j) & & & \\ & a_{2\beta}(w_o)/a_{2\beta}(w_j) & & \\ & & \ddots & \\ & & & a_{M\beta}(w_o)/a_{M\beta}(w_j) \end{bmatrix} \tag{2.2}$$

where w_o is the central frequency of bandwidth B, β is the initial DOA value,

and $a_{i\beta}(w_j)$ is the i^{th} element of the direction vector $\mathbf{a}_{\beta}(w_j)$.

The coherent signal subspace method for computation of DOA (wide-band sources) as proposed by Wang & Kaveh [10] can be summarized as follow:

1. Collect data samples and convert the samples into frequency domain using FFT.
2. Estimate the covariance matrix for each frequency component given by:

$$\mathbf{R}_{yy}(w_n) = \frac{1}{K}\sum_{k=1}^{K} y_k(w_n)\, y_k^H(w_n)$$

3. Compute eigenvalues and eigenvectors of the covariance matrix.
4. Find initial estimates of direction of arrival by computing the MUSIC spectrum given by:

$$p(\phi)=\frac{1}{\mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_n\mathbf{a}(\theta)}$$

5. Compute transformation matrix focusing on central frequency.
6. Compute eigenvalues and eigenvectors of the focus matrix.
7. Find number of sources *D*.
8. Find the final DOA estimates by finding the *D* largest peaks of the MUSIC spectrum.

2.2 Efficient Wide-band Source Localization Using Beamforming Invariance Technique by Ta-Sung Lee (Review)

A beamspace wide-band source localization scheme is proposed which exploits the concept of beamspace manifold invariance [19]. A principle of Least Squares (LS) fit is employed to construct a beamforming matrix for each of the narrow-band frequency bins extracted from the wide-band array data. The beamforming matrices perform the same operation as do the focusing matrices of Wang and Kaveh [6-7]. In this case beamforming matrices are chosen in such a way as the resulting beamspace DOA matrices are essentially the same for all frequencies. The focused beamspace data/noise correlation matrix pencil can then be readily formed with the respective narrow-band beamspace correlation matrices without any additional preliminary processing.

A computationally efficient implementation of the beamspace Root-MUSIC algorithm via subarray beamforming and banded transformation is developed. By subarray beamforming, the large order Root-MUSIC [1-8] signal polynomial is first reduced to one with the order equal to the beamspace dimension. The algorithm is further simplified by transforming the matrix representing the element space noise subspace into a banded form and converting the reduced-order signal polynomial into several polynomials with the order equal to the number of sources. These polynomials are then rooted in parallel to determine the DOA's.

CSSM Model Formulation

This paper assumes that there are D wide-band sources and M identical sensor elements which have a common passband width B_w centered at frequency f_c. First of all data model needs to be defined which uses following two approaches:

- The received data is decomposed into J narrow-band components using a bank of J bandpass filters. They are centered at f_j followed by the conventional I-Q demodulation and sampling. The model has J sets of Mx1 complex array data snapshot vectors.

$$X(n;f) = A(f_j)s(n;f_j) + v(n;f_j) \tag{2.2}$$

where

$s(n; f_j)$ represents the data received at some reference point of the array (Dx1)

$v(n; f_j)$ represents the noise present at the M elements. (Mx1)

$A(f_j)$ accounts for the phase variation across the array due to the wavefront (MxD).

- This model can also be created by forming narrow-band components using Fourier transform as done by Wang & Kaveh [6].

Invariant means that it is unaffected by the group of mathematical operations under consideration and invariance means the quality or state of being invariant. Therefore beamforming-invariance means that beamforming is unaffected by other factors. In a beam space beamforming, outputs from the array elements are first processed by a multiple-beam beamformer to form a suite of orthogonal beams. The output of each beam can then be weighted and the result combined to produce a desired output. The beamformer can be implemented using the FFT. For an M-element array, the overlapped orthogonal beam can be formed [28].

Beamspace beamforming requires a set of beamspace combiners to generate weighted outputs. The digital signal streams from the antenna elements are passed to the FFT processor, which generates K simultaneous orthogonal beams. The purpose of the beam selection function is to choose a subset of these orthogonal beams that need to be weighted to form a desired output.

The approach in this paper performs beamspace transformation at each of the J frequency bins and choosing beamforming matrices with some kind of criterion. Therefore beam patterns are identical for all frequencies. Consider first the patterns associated with a single beam:

$$w(\vec{r};f_j) = w_j^H a(\vec{r};f_j) \tag{2.3}$$

where *j* is from 1 to J and w_j is the Mx1 complex weight vector employed at f_j.

It would be difficult to find two weight vectors that would produce identical beam patterns at two different frequencies. Due to the discrete nature of the array, it is not possible to find two weight vectors that produce completely identical beam patterns at two different frequencies. A Least Square (LS) fit method is used as an approximation to construct weight vectors that nearly provide frequency-invariant beam patterns. A simple measure of the proximity between the two patterns $w(\vec{r};f_i)$ and $w(\vec{r};f_j)$ is the generalized L_2 distance:

$$d_{ij} = \int_{\Omega} \rho(\vec{r}) \left| w(\vec{r}; f_i) - w(\vec{r}; f_j) \right|^2 d\vec{r} \tag{2.4}$$

$$d_{ij} = \int_{\Omega} \rho(\vec{r}) \left| w_i^H a(\vec{r}; f_i) - w_j^H a(\vec{r}; f_j) \right|^2 d\vec{r} \tag{2.5}$$

where Ω is a sector of the unit sphere representing the Field of View (FOV) of the array.

The weighting function $\rho(\vec{r})$ is enhancing the approximation within a pre-selected angular region. The weight vector computation for "beamforming-invariance" is calculated using the following minimum-distance criterion

$$\underbrace{\min}_{w_i, w_j} \int_{\Omega} \rho(\vec{r}) \left| w_i^H a(\vec{r}; f_i) - w_j^H a(\vec{r}; f_j) \right|^2 d\vec{r} \tag{2.6}$$

Above condition is subject to that w_i and w_j are not equal to zero. One consideration that is used as a constraint is to obtain good SNR gain and low side lobes. The weight selection process would first require a desired weight vector $w_\circ$ that would be associated with a pre-selected reference frequency $f_\circ$ within the passband of the array. The Beamforming Invariance (BI) weight vectors associated with the J frequencies are calculated as:

$$\underbrace{\min}_{w_j} \int_{\Omega} \rho(\vec{r}) \left| w_\circ^H a(\vec{r}; f_\circ) - w_j^H a(\vec{r}; f_j) \right|^2 d\vec{r} \tag{2.7}$$

It may be noted that $f_\circ$ need not be one of the frequencies f_j. Following solutions are given by:

$$w_j = U_j^{-1} S_j w_\circ \tag{2.8}$$

where

$$U_j = \int_{\Omega} \rho(\vec{r}) a(\vec{r}, f_j) a^H(\vec{r}, f_j) d\vec{r}$$

$$S_j = \int_{\Omega} \rho(\vec{r}) a(\vec{r}, f_j) a^H(\vec{r}, f_\circ) d\vec{r}$$

This technique is reducing the size of data by using a low-dimension beamspace narrow-band data associated with $f_\circ$. It is arguing that if wide-band source moves across the Field of View (FOV) of the array then this set of data would allow observation of output waveforms at the J beamformer outputs.

Beamspace focusing with BI Transformation

In order to achieve effective reception of the source signals, multiple beams can be formed over the spatial band of interest. A set of K beamforming weight vectors w_{jk} are used to simultaneously form K linear combinations of the array data at f_j. Therefore Mx1 element space data snapshot vectors are converted into Kx1 beamspace data snapshot vectors using the following relation:

$$x_B(n; f_j) = W_j^H x(n; f_j) \tag{2.9}$$

where W_j are the respective MxK beamforming matrices for J frequencies. K is chosen such that it is greater than number of sources and less than number of sensors. The beamspace data snapshot vectors will have a similar form as the original array snapshot vectors and is given as:

$$x_B(n; f_j) = B(f_j) s(n; f_j) + v_B(n; f_j) \tag{2.10}$$

where $B(f_j) = W_j^H A(f_j)$ and $v_B(n; f_j) = W_j^H v(n; f_j)$

B and *v* are the beamspace DOA matrices and noise vectors respectively.

The challenge would be the selection of $\boldsymbol{W_j}$ and this could be done similar to procedure given earlier and is also provided for the convenience.

$$\underbrace{\min}_{w_j}\int_{\Omega}\rho(\vec{r})\left|w_{\circ}^{H}a(\vec{r};f_{\circ})-w_{j}^{H}a(\vec{r};f_{j})\right|^{2}d\vec{r}$$

It may be noted that $f_{\circ}$ need not be one of the frequencies f_j. Following solutions are given by:

$$w_j = U_j^{-1}S_j w_{\circ} \qquad (2.11)$$

where

$$U_j = \int_{\Omega}\rho(\vec{r})a(\vec{r},f_j)a^{H}(\vec{r},f_j)d\vec{r}$$

$$S_j = \int_{\Omega}\rho(\vec{r})a(\vec{r},f_j)a^{H}(\vec{r},f_{\circ})d\vec{r}$$

- Determine a set of K reference weight vectors w_{0k} associated with $f_{\circ}$.
- Construct the K weight vectors associated with the J frequencies.

$$w_{jk} = U_j^{-1}S_j w_{\circ k}$$

This can be written in matrix form

$$W_j = U_j^{-1}S_j W_{\circ}$$

where $\boldsymbol{W_0}$ is the reference beamforming matrix. The beamforming matrices for all values of j's would have the following form due to the BI property:

$$B(f_j) \approx B(f_0) \qquad (2.12)$$

The beamspace data snapshot vectors would be fully characterized by a single beamspace DOA matrix $B(f_0)$ associated with $f_{\circ}$. This will represent the beamspace signal subspace associated with $f_{\circ}$. Now CSSM approach can be applied and obtain a focused beamspace data correlation matrix.

$$\overline{Q}_{xx} = \sum_{j=1}^{J} \alpha_j E\{x_B(n;f_j)x_B^H(n;f_j)\}$$

$$\overline{Q}_{xx} \approx B(f_0)\overline{R}_{ss}B^H(f_0) + \overline{Q}_{vv}$$

$$\overline{Q}_{vv} = \sum_{j=1}^{J} \alpha_j E\{v_B(n;f_j)v_B^H(n;f_j)\}$$

The above operation is named as BI-CSSM and would produce an effective narrow-band beamspace data/noise correlation matrix pencil associated with associated with $f_\circ$ giving an effective source correlation matrix R_{ss}. The following section would address the issue of obtaining beamforming data matrix.

Design of Reference Beamforming Matrix

There are number of publications and techniques for forming a reference beamforming matrix and their references are provided in this paper. A Chebyshev beamformer can be used which exhibits low side lobes for LES arrays. One of the goals would be to design a beamforming matrix that is optimal and also has a small focusing error.

Determine an orthonormal basis for the subspace of minimum focusing error. Let E_w be an MxK' matrix satisfying $E_W^H E_W = I$ and $K \le K' \le M$. E_w is used as the reference beamforming matrix then from $W_j = U_j^{-1} S_j W_\circ$ the resulting total focusing error is given by

$$E_f = \frac{1}{J}\sum_{j=1}^{J}\int_{\Omega}\rho(\vec{r})\left\| E_w^H a(\vec{r};f_0) - E_w^H S_j^H U_j^{-1} a(\vec{r};f_j) \right\|^2 d\vec{r} = tr\{E_w^H S_U E_w\} \tag{2.13}$$

and

$$S_U = \frac{1}{J}\sum_{j=1}^{J} \left(S_0 - S_j^H U_j^{-1} S_j\right)$$

$$\overline{Q}_{xx} = \sum_{j=1}^{J} \alpha_j E\{x_B(n; f_j) x_B^H(n; f_j)\}$$

$$\overline{Q}_{xx} \approx B(f_0)\overline{R}_{ss}B^H(f_0) + \overline{Q}_{vv}$$

$$\overline{Q}_{vv} = \sum_{j=1}^{J} \alpha_j E\{v_B(n; f_j) v_B^H(n; f_j)\}$$

The above operation is named as BI-CSSM and would produce an effective narrow-band beamspace data/noise correlation matrix pencil associated with associated with $f_\circ$ giving an effective source correlation matrix R_{ss}. The following section would address the issue of obtaining beamforming data matrix.

Design of Reference Beamforming Matrix

There are number of publications and techniques for forming a reference beamforming matrix and their references are provided in this paper. A Chebyshev beamformer can be used which exhibits low side lobes for LES arrays. One of the goals would be to design a beamforming matrix that is optimal and also has a small focusing error.

Determine an orthonormal basis for the subspace of minimum focusing error. Let E_w be an MxK' matrix satisfying $E_W^H E_W = I$ and $K \le K' \le M$. E_w is used as the reference beamforming matrix then from $W_j = U_j^{-1} S_j W_\circ$ the resulting total focusing error is given by

$$E_f = \frac{1}{J}\sum_{j=1}^{J} \int_\Omega \rho(\vec{r}) \left\| E_w^H a(\vec{r}; f_0) - E_w^H S_j^H U_j^{-1} a(\vec{r}; f_j) \right\|^2 d\vec{r} = tr\{E_w^H S_U E_w\} \qquad (2.13)$$

and

$$S_U = \frac{1}{J}\sum_{j=1}^{J} (S_0 - S_j^H U_j^{-1} S_j)$$

Design Example

An array manifold vector associated with an LES array consisting of M identical elements and operating with frequency *f* is given by:

$$a(u;f) = \left[1, e^{j\tau fu}, e^{j2\tau fu}, ..., e^{j(M-1)\tau fu]}\right]^T \qquad (2.14)$$

where $\tau = 2\pi d / c$ and d is the spacing between two adjacent elements. The sine-space angle u is defined as u=sin(θ) and θ is the angle measured with respect to the broadside of the array. First sensor element is assumed to be the reference point of the array.

The frequency band is decomposed into J=33 uniformly distributed subbands. The reference beamforming matrix W_0 is constructed at $f_\circ = f_1$. by first formulating W_d and using following equations:

$$\underbrace{\min}_{W_0 = E_w \psi} \left\| W_0 - W_d \right\|_F^2$$

$$W_0 = E_w (E_w^H E_w)^{-1} E_w^H W_d$$

Use K'=9 and W_d is composed of weight vectors associated with k=7 Chebyshev beams with -30 dB sidelobles point at 0, $\pm$7.7, $\pm$15.5 and $\pm$23.6 in degrees. To alleviate the grating lobe problem $f_\circ = f_1$ is chosen. Remaining thirty two beamforming matrices are computed via $W_j = U_j^{-1} S_j W_\circ$,. The FOV was set to be -1.0 to 1.0 in u domain. The weighting function was chosen as 1.0 and 0.5. These assumptions were verified by computing the normalized focusing error spectrum.

BI-CSSM/ROOT –Form Eigen-Based DOA Estimation

Now consider only the operation associated with $f_\circ$ and omit the argument of frequency in the relevant terms. The beamspace eigen-based methods can be applied on the BI_CSSM focused data dictates. It is known that the source DOA's should be determined via the null spectrum

$$\Phi(\vec{r}) = a^H(\vec{r})W_0\overline{E}_B P\overline{E}_B^H W_0^H a(\vec{r}) \qquad (2.15)$$

where

$\overline{E}_B$ is referred to as the beamspace noise eigenvector matrix which is Kx(K-D). It contains generalized eigenvectors.

P is a positive semi-definite matrix serving to weight the respective columns of $\overline{E}_B$.

The null spectrum is converted into the $2(M-1)^{th}$ order signal polynomial

$$\Phi(z) = a^T(z^{-1})W_0\overline{E}_B P\overline{E}_B^H W_0^H a(z)$$

where $a(z)=[1,z,\ldots z^{M-1}]^T$ and $z = e^{j\pi f_0 u}$.

There are D signal roots z_i extracted from $\Phi(z)$. Their DOA's can be determined by $\vec{u}_i = \arg\{\hat{z}_i\}/\pi f_0$ where i is from 1 to D. These coefficients of $\Phi(z)$ exhibit conjugate symmetry such that the corresponding 2(M-1) roots from M-1 conjugate reciprocal pairs. As a result only M-1 distinct values are observed regarding the phase angles of 2(M-1) roots.

The root-form methods may be computationally expensive due to the need of large order polynomial rooting. A new method is proposed which may be more computationally efficient and has following three stages:

1. The signal polynomial is reduced from order 2(M-1) to 2(K-1) via judiciously performed subarray beamforming.

2. The reduced order signal polynomial is converted into several 2Dth or (Dth) order polynomials via a banded transformation of the corresponding reduced noise EV matrix.
3. Signal roots are extracted by rooting these polynomials in parallel.

Polynomial-Order Reduction via Subarray Beamforming

Consider an M-element LES array which consists of L=M-K+1 and overlaps K-element subarrays. Define the KxM selection matrices that select from the full array data snapshot vectors the respective subarray data snapshot vectors

$$\Gamma_l x(n) = x_K^{(l)}(n)$$

where $x_K^{(l)}(n)$ denotes the Kx1 data snapshot vectors received at the lth subarray and Γ_l is given by

$$\Gamma_l = [O_{KX(l-1)} \mid I_{KXK} \mid O_{KX(M-K-l+1)}]$$

Subscripts indicate the sizes of the respective identity and zero matrices. Assume that the same Kx1 weight vector g is applied at each of the subarrays that would produce a set of Lx1 vectors.

$$x_L(n) = \begin{bmatrix} g^H x_K^{(1)}(n) \\ g^H x_K^{(2)}(n) \\ . \\ . \\ g^H x_k^{(L)}(n) \end{bmatrix} = G^H x(n) \qquad (2.16)$$

It represents the data snapshots received at the L subarray beamformer outputs. $x_L(n)$ is the data snapshot vector of L-element LES array which is also denoted as A_L and apply an Lx1 vector c to form:

$$x_B(n) = c^H x_L(n) = (Gc)^H x(n) = w^H x(n) \qquad (2.17)$$

It can be seen that w=Gc and is the effective weight vector acting on the entire array. We may consider the beamforming to be performed first on A_L and treating the L subarray as super elements. We may also apply an Lx1 weight vector c to produce a set of Kx1 vector data snapshots.

$$x_K(n) = \sum_{l=1}^{L} c_l^* x_K^{(l)}(n) = C^H x(n)$$

where $C = \sum_{l=1}^{L} c_i \Gamma_l^T$

These vector data snapshots can be regarded as “data snapshot vectors” obtained from A_K. Applying a Kx1 weight vector g forms:

$$x_B(n) = g^H x_L(n) = (Cg)^H x(n) = w^H x(n) \tag{2.18}$$

It can be concluded that the full weight vector exhibits two types of decomposition:

W=Gc=Cg

Using the banded, Toeplitz structure of C or G

$$w^H a(z) = \{g^H a_k(z)\}\{c^H a_L(z)\}$$

Similarly we can get

$$a^T(z^{-1})w = \{a_K^T(z^{-1})g\}\{a_L^T(z^{-1})c\}$$

A beamspace transformation scheme based on the concept of weight vector decomposition can be developed. K reference weight vector can be constructed using the following form:

$$w_{0k} = G_k c = Cg_k$$

The w contains the same weight factor c. Now putting in matrix form and using above equations:

$$W_{\circ}^{H} a(z) = \{\overline{G}^{H} a_{k}(z)\}\{c^{H} a_{L}(z)\}$$

$$a^{T}(z^{-1}) W_{\circ} = \{a_{K}^{T}(z^{-1})\overline{G}\}\{a_{L}^{T}(z^{-1}) c\}$$

Substituting these equations in $\Phi(z) = a^{T}(z^{-1}) W_{0} \overline{E}_{B} P \overline{E}_{B}^{H} W_{0}^{H} a(z)$ yields

$$\Phi(z) = \{a_{L}^{T}(z^{-1}) c c^{H} a_{L}(z)\}\{a_{K}^{T}(z^{-1}) \overline{G}\overline{E}_{B} P \overline{E}_{B}^{H} \overline{G}^{H} a_{K}(z)\} \tag{2.19}$$

It can be seen that this is decomposed into two individual factors accounting for c and G. The factor involving c is known a priori and thus there is no information about the DOAs. The DOA estimates can be determined with the 2(K-1)th order polynomial.

$$\Phi_{K}(z) = a_{K}^{T}(z^{-1}) \overline{G}\overline{E}_{B} P \overline{E}_{B}^{H} \overline{G}^{H} a_{K}(z)\}$$

This equation suggests that this is the signal polynomial associated with K-element LES array generated by the noise EV matrix $\overline{G}\overline{E}_{B}$. This equation leads to substantial reduction in computational load if K<<M.

Parallel Processing via Banded Transformation

Subarray beamforming somewhat simplifies the polynomial rooting. The difficulty remain with the procedure of choosing D signal roots out of the 2(K-1) roots of $\Phi_{K}(z)$. A scheme has been proposed by others to convert the DOA estimation problem into that of rooting a Dth order polynomial. They exploit the fact that the ideal noise subspace associated with an LES array is spanned by the columns of a banded Toeplitz matrix with bandwidth D+1.

Performing some algebraic manipulation which leads to:

$$\Phi_{K}(z) = a_{D+1}^{T}(z^{-1}) F D(z^{-1}) T_{B} P T_{B}^{H} D(z) F^{H} a_{D+1}(z)\} \tag{2.20}$$

where a $^{D+1}$(z)=a(z) and D(z) is the diag {1,z,…..,z^{K-D+1}}

The polynomial is associated with a (D+1) element LES array except for the z-dependent term D (z). In order to fully exploit this reduction in dimension, replace D (z) with a constant matrix by fixing z=z_0.

$$\Phi_K(z \mid z_0) = a_{D+1}^T(z^{-1}) F D({z_0}^{-1}) T_B P T_B^H D(z_0) F^H a_{D+1}(z) \qquad (2.21)$$

Comparing two $\Phi_K(z)$ equations, it can be seen that in order to achieve the performance of working with original signal polynomial, we must choose $z_0 \approx z_i = e^{j\pi f_0 u_i}$ in estimating u_i. It indicates that a set of reduced order signal polynomial be constructed with z_i, i=1 ,…, D. The problem with this approach is that it requires the knowledge of the DOA's that is being estimated. The effective spatial passband of the reference beamforming matrix can be decomposed into I_s disjointing sectors centered at u_m, m=1,…., I_s and construct the following:

$$\Phi_K(z \mid \bar{z}_m) = a_{D+1}^T(z^{-1}) F D({\bar{z}_m}^{-1}) T_B P T_B^H D(\bar{z}_m) F^H a_{D+1}(z) \qquad (2.22)$$

with $\bar{z}_m = e^{j\pi f_0 \bar{u}_m}$ and m=1,…., I_s . $\Phi_K(z)$ can be approximated with the above equation for the m^{th} sector with high accuracy for a small sector size. This equation can be rooted in parallel and obtain I_s set of roots of z_{im}.

Roots need to be picked and we should pick roots which are closer to the unit circle. This DOA estimator is suboptimum according to the author. It would also result in degradation in estimation accuracy at low SNR. Even with the true DOA, the DOA estimates may not be exactly identical as signal roots may not lay on the unit circle. The parallelized estimator behaves as a mixture of the root-form and spectral-form estimator.

We note that under no noise/error conditions is the banded matrix is Toelplitz. We may assume that F is approximately rank one. This could be done using following replacement:

$$FD(\bar{z}_m^{\;-1})T_B P T_B^H D(\bar{z}_m)F^H a_{D+1}(z) \;=>\; FD(\bar{z}_m^{\;-1})T_B P_m P_m^H T_B^H D(\bar{z}_m)F^H \tag{2.23}$$

where P_m is a (K-D)x1 vector. Subscript m is used to emphasize the dependence on different sectors. With the above structure we need only work with the set of D^{th} order polynomials:

$$\Phi_K(z \mid \bar{z}_m) = P_m^H T_B^H D(\bar{z}_m) F^H a_{D+1}(z)$$

Algorithm Summary

- Construct beamforming matrices and the BI transformations.
- Perform a KxK generalized eigen-decomposition
- Solve in parallel K-D systems of equations of size K-D.
- Rooting in parallel I_s $2D^{th}$ (or D^{th}) order polynomials.

Design of Subarray-Based Reference beamforming Matrix

The factorization of the equation $w_{0k} = G_k c = C g_k$ does not hold for a particular desired beamforming matrix W_d. One way to retain the merits of W_d using subarray beamforming is to selectively choose c and g_k so that W_0 is close to W_d. Using the LS fit technique leads to the following problem:

$$\overbrace{c, g_1, \ldots, g_k}^{\min} \left\| W_\circ - W_d \right\|_F^2 \equiv \overbrace{c, g_1, .. g_k}^{\min} \sum_{k=1}^{K} \left\| w_{\circ k} - w_{dk} \right\|^2 \tag{2.24}$$

Invoking the structure of $w_{0k} = G_k c = C g_k$ we can re-write above equation as:

$$\overbrace{C, \overline{G}}^{\min} \left\| C\overline{G} - W_d \right\|_F^2 \equiv \overbrace{C, \overline{G}}^{\min} \left\| \widetilde{G} c - \widetilde{w}_d \right\|^2$$

This equation has no closed form solution in general. The problem should be decomposed into two individual stages for which in one stage we solve for the common weight factor c whereas in the other we solve for the uncommon weight factor g_k.

Assuming that the initial guess of c is available and solving the left hand side of G we get:

$$\tilde{G} = (C^H C)^{-1} C^H W_d$$

Constructing G and then solving the right hand side for c yields:

$$c = (\tilde{G}^H \tilde{G})^{-1} \tilde{G}^H \tilde{w}_d$$

Now construct C and solve for new G. This procedure is then alternatively executed until the solution converges.

Summary

The algorithm decomposes data using bandpass filters into J frequency beams. It performs beamspace transformation and computes weights using least square method. It then computes beamspace data matrix and focuses on single reference frequency which would be something similar to CSSM method. It performs transformation into K beamspaces and forms beamspace data matrix. This beamspace data matrix then focused on a single reference frequency out of J frequency bands. The design of beamspace data matrix is described which requires first design of beamforming matrices. They are again designed in a least square sense. The problem is then reduced to beamspace data correlation and noise matrices. Authors then apply their own derived root MUSIC algorithm which could be substituted with the MUSIC algorithm. The algorithm does not require any preliminary DOA estimates.

This DOA estimator is suboptimum according to the author. It would also result in degradation in estimation accuracy at low SNR. Even with the true DOA, the DOA estimates may not be exactly identical as signal roots may not lay on the unit circle. The proposed parallelized estimator behaves as a mixture of the root-form and spectral-form estimator. This approach provides an excellent alternative to CSSM but frequency decomposition and calculations of weight and beamspace data matrices. There may be some alternative ways to calculate weight and compute data beamspace matrices. In the end it again uses something similar to MUSIC algorithm to compute DOA.

2.3 Multiple Broad-Band Source Location Using Steered Covariance Matrices Jefrey Krolik and David Swingler (Review) [17]

This paper is based on space time statistics called the Steered Covariance Matrix (STCM). It is obtained by measuring the covariance of the time-domain array outputs after delays have been inserted to steer a conventional Delay-and-Sum (DS) beamformer beam [17]. This technique avoids source localization problem by defining a broad band covariance matrix having a rank one characterization regardless of source spectral content or source location. The drawback of this approach is that it is compute intensive.

Model Formulation and the steered covariance matrix

We consider an array of M wide-band sensors and D wide-band point sources. The output of a sensor located at x_m is denoted by $y_m(t)$ observed over a time interval of T seconds and is expressed as:

$$y_m(t) = \sum_{i=1}^{D} u_i(t - \tau_m(\theta_i)) + v_m(t) \qquad (2.25)$$

where $u_i(t)$, i=1,…,D are stationary, zero-mean, random processes corresponding to each received source signal and $v_m(t)$ is the noise. $\tau_m(\theta_i)$ is the signal propagation delay to the mth sensor when a source is located at θ_i.

The locations θ_i are the parameters that need to be estimated from a finite-time observation of the sensor outputs. The STCM can be defined using the complex analytic representation of the sensor outputs. The delay and sum beamformer enhances the reception of signals emanating from location θ by inserting a delay of $\tau_m(\theta)$ at the output of each sensor. The complex analytic representation of the delay and sum beamformer output $b(t, \theta)$ with steering in direction θ is formed by taking the weighted sum of the elements of $y(t, \theta)$ and is expressed as:

$$b(t,\theta) = w^T y(t,\theta)$$

where *w's* are a vector of real-valued array shading weights. The expected beam power would be:

$$z(\theta) = E\{|b(t,\theta)|^2\}$$

This could then be expressed as $z(\theta) = w^T R(\theta) w$

where $R(\theta) = \mathrm{E}\{y(t, \theta)\, y(t, \theta)^{\mathrm{H}}\}$ is the Steered Covariance Matrix (STCM) and is corresponding to direction θ.

The well known Cross Spectral Density Matrix (CSDM) is a function of temporal frequency and its corresponding weighting vector depends both on the array shading and steering angle. $R(\theta)$ is a function of steering θ and w is a constant shading vector. The main advantage of using $R(\theta)$ in wide-band case is that the number of statistical degrees of freedom available to estimate $R(\theta)$ is approximately equal to the time-bandwidth product of the sources rather than the usually much smaller number of snapshots used in estimating each narrow-band CSDM.

Structure of $R\ (\theta\ )$ can be used for a wide variety of high resolution source location methods. Consider a Uniform Linear Array (ULA) with M sensors at d distance apart at positions x_m=md, m=0,… , M-1.

$$\tau_m(\theta) = m.\tau(\theta) \tag{2.26}$$

where $\tau(\theta) = d / c \sin(\theta)$, c is the propagation speed and θ is the bearing of the source relative to array broadside. Using the model of $y_m(t) = \sum_{i=1}^{D} u_i(t - \tau_m(\theta_i)) + v_m(t)$, the jkth element of $R\ (\theta\ )$ is a function of m=j-k given by:

$$r_{jk}(\theta) = \sum_{i=1}^{D} \rho_i(m.(\tau(\theta) - \tau(\theta_i)) + \eta_m(m.\tau(\theta)) \tag{2.27}$$

where $\rho_i(\tau)$ is autocorrelation function of the ith source and $\eta_m(\tau)$ is the cross-correlation function of the noise received at the array coordinate origin and the mth sensor.

When the steering direction is the direction of the source and the source is aligned with the steering direction then the STCM contains a constant. This perfectly coherent component is equal to the source power regardless of its spectral signature. One approach to determine the power level of a point source in the steering direction θ is to accurately estimate the level of the dc constant term. This estimation is performed for a closely spaced set of steering directions yields a wide-band spatial power spectral estimate. It can also be noted that DS beamforming corresponds to making a conventional Blackman-Tukey estimate of the dc component of r (m, θ). Later on minimum variance and linear predictive spectral estimation are examined as methods of estimating the dc component for each steering direction.

Estimation of the Steered Covariance Matrix (STCM)

An estimate of the Steered Covariance Matrix (STCM) is obtained using a simple relationship between the STCM and CSDM. Statistics of the estimated STCM can be expressed in terms of the Wishart characteristic function. Complex time domain vector of M sensor outputs can be expressed as $[y_0^c(t), y_1^c(t),..., y_{M-1}^c(t)\}^T$ over the time interval (-T/2,T/2) in terms of the frequency-domain vectors as

$$Y(k) = [Y_0(k), Y_1(k),...Y_{M-1}(k)]^T \quad (2.28)$$

With elements $Y_m(k)$ corresponds to the Fourier series coefficients of $y_m(t)$ at frequency $\omega_k = 2\pi k / T$ The sensor outputs are approximately band-limited to $\omega_l \le \omega \le \omega_h$. The steered sensor output vector $y(t, \theta)$ can be expressed as:

$$y(t,\theta) = \sum_{k=l}^{h} T_k(\theta) Y(k) e^{j\omega_k t} \quad (2.29)$$

where

$$T_k(\theta) = \begin{bmatrix} e^{j\omega k \tau_0(\theta)} \ldots\ldots\ldots\ldots 0 \\ 0 \ldots e^{j\omega k \tau_1(\theta)} \ldots\ldots\ldots\ldots \\ . \\ . \\ . \\ 0 \ldots\ldots\ldots\ldots e^{j\omega k \tau_{M-1}(\theta)} \end{bmatrix} \quad (2.30)$$

Substituting above equation of $y(t, \theta)$ into $R(\theta)$ and under the assumption of large T, the STCM can be expressed as:

$$R(\theta) = \sum_{k=l}^{h} T_k(\theta) K(\omega_k) T_k(\theta)^H \quad (2.31)$$

where $K(\omega_k) = E\{Y(k)Y(k)^H\}$ is the conventional un-steered CSDM at frequency ω_k.

This $R\ (\theta)$ matrix is similar to coherently focused covariance matrix of CSS method proposed by Wang and Kaveh [2] for the case where all sources in the field are in a single group unresolved by a conventional delay and sum beamformer. In STCM the $R\ (\theta)$ matrix is computed for each steering angle θ making it computationally intensive. However, it eliminates the source location bias resulting from errors made in forming focusing matrices required by the CSS method.

The relationship between matrices K and $R\ (\theta)$ as given in $R(\theta) = \sum_{k=l}^{h} T_k(\theta) K(\omega_k) T_k(\theta)^H$ suggests a natural way of estimating $R\ (\theta)$ by using finite-time CSDM estimates for $\hat{K}$. A common method of forming $\hat{K}$ from discrete-time sensor outputs is to divide the T second observation into N non-overlapping segments of ΔT seconds each and then apply the FFT to obtain uncorrelated frequency domain vectors Y_n(k) for each segment n=1,…,N. The cross spectral density matrix at each frequency ω_k is then estimated and then substituted to estimate of $R\ (\theta)$ matrix. An attractive feature of the STCM estimate given is that its statistics can be expressed in terms of the Wishart characteristic function.

Steered Covariance Source Location Methods

It is derived by finding the beamformer weight vector w which minimizes the beam power given by $z(\theta) = w^T R(\theta) w$ subject to the constraint that the processor gain is unity for a broad-band plane wave in direction θ. The problem is viewed as one of estimating the dc component of the STCM steered in direction θ by means of a Minimum Variance (MV) approach. In either case, this technique has the effect of choosing w to minimize the power contribution from sources and noise not propagating from direction θ. The resulting STCM-based spatial spectral estimate is given by

$$Z_{STMV} = [1^H R(\theta)^{-1} 1]^{-1} \tag{2.32}$$

where 1 is an Mx1 vector of ones.

A finite estimate of Z_{STMV} can be obtained by substituting the estimate of $R(\theta)$ matrix similarly estimated by $R(\theta) = \sum_{k=l}^{h} T_k(\theta) K(\omega_k) T_k(\theta)^H$. A complete broad-band spatial power spectral estimate is then formed by computing $Z_{STMV}(\theta)$ for a set of steering angles which span the locations of interest.

For wide-band sources, the Minimum Variance Distortionless Response (MVDR) beam power is obtained by summing narrow-band beam powers over the band of interest and is given as:

$$Z_{mvdr}(\theta) = \sum_{k=l}^{h} [D_k(\theta)^H K(\omega_k)^{-1} D_k(\theta)]^{-1} \tag{2.33}$$

The steps in the STMV method are as follows:

1. Form estimated Cross-Spectral Density Matrices k over the frequency band of interest.

$$\hat{K}(\omega_k) = \frac{1}{N} \sum_{n=1}^{N} Y_n(k) Y_n(k)^H \tag{2.34}$$

2. Compute estimated steered covariance matrices for each steering direction θ of interest.

$$\hat{R}(\theta) = \sum_{k=l}^{h} T_k(\theta) \hat{K}(\omega_k) T_k(\theta)^H \tag{2.35}$$

3. Compute $R(\theta)$ in the following equation and form Z for each steering direction θ to obtain a broad-band spatial power spectral estimate.

$$\hat{Z}_{STMV} = [1^H \hat{R}(\theta)^{-1} 1]^{-1} \tag{2.36}$$

Compute linear prediction coefficients from the steered sensor outputs and approximate or model them for the Autoregressive (AR) model. Compute forward and backward prediction error sequence and linear prediction coefficients. The linear prediction coefficients which minimize forward and backward prediction error sequence are simply complex conjugates of each other. Exploiting this property the Steered Linear Prediction (STLP) method is simply the minimizing the sum of forward and backward squared prediction errors. The steered linear predictive estimate of the spatial power spectrum in direction θ is given by

$$\hat{Z}_{stlp}(\theta) = \frac{\hat{\rho}_L(\theta)}{\left|1 - \sum_{m=1}^{L} \hat{a}_{L,m}(\theta)\right|^2} \tag{2.37}$$

This is also known as a maximum entropy spectral estimate evaluated at dc. The computation of above equation and as the minimization of average forward and backward prediction squared error can be obtained as the solution to the following equation.

$$\hat{R}_L(\theta)\hat{a}_L(\theta) = \begin{bmatrix} \hat{\rho}_L(\alpha) \\ 0_L \end{bmatrix}$$

Summary of the STLP algorithm

- Form spatially averaged cross spectral density matrix estimates K over the frequency band of interest.
- Compute spatially averaged steered covariance matrices R for each steering direction θ of interest.

- Solve the augmented normal equation $\hat{R}_L(\theta)\hat{a}_L(\theta) = \begin{bmatrix} \hat{\rho}_L(\alpha) \\ 0_L \end{bmatrix}$ to obtain vector STLP coefficients $\hat{a}_L(\theta)$ and $\hat{\rho}_L(\theta)$ for each steering direction θ.

- Compute equation $\hat{Z}_{stlp}(\theta) = \dfrac{\hat{\rho}_L(\theta)}{\left|1 - \sum_{m=1}^{L} \hat{a}_{L,m}(\theta)\right|^2}$

Simulation Study

This study simulates using a ULA with 16 sensors with inter-element spacing of 7.5 m corresponding to a half wavelength frequency of 100 Hz is considered. The location parameter θ is the bearing of the source relative to endfire. The Rayleigh limit of angular resolution for this array is approximately 2/(M-1)=0.133 radians or 7.62 degrees. Two source signals were modeled as temporally stationary and mutually uncorrelated zero mean Gaussian processes with bandpass auto-spectra.

$S_i(f)$ Li for $f_0 - BW/2 \le f \le f_0 + BW/2$

0 otherwise

where each source has the same frequency f_0=100Hz and bandwidth, BW=40Hz. The noises $v_m(t)$, m=1..M were taken to be stationary and mutually uncorrelated zero-mean Gaussian bandpass processes independent from the signals and watch with the auto-spectrum:

$S_v(f)$ Lv for $f_0 - BW/2 \le f \le f_0 + BW/2$

1 otherwise

defined over the same frequency band as the source signals. The SNR of the ith source denoted SNR_i is defined from the above by $SNR_i = L_i/L_v$. Estimated cross-spectral density matrices were formed using

$$\hat{K}(\omega_k) = \frac{1}{N}\sum_{n=1}^{N} Y_n(k) Y_n(k)^H \qquad (2.38)$$

From sensor outputs where a segment length ΔT =0.8 seconds was used. For each segment the array output is decomposed into B=h-l+1=33 narrow-band DFT bins. The number of segments or snapshots N employed to estimate each k is varied in this simulation study.

Summary

We have used our simulation program with previously generated data outlined in [6]. Our program did not give two peaks as expected. Problems could be in two different areas:

1. There may be some missing information regarding transformation matrix that could lead to not getting appropriate result.
2. The paper uses random processes as data which may contribute to getting two peaks in their case and not in our case.
3. The third conclusion we have is that this technique requires computation for each steering direction θ increasing the computation requirements but eliminates the selection of initial focusing angle.

2.4 Focused Wide-Band Array Processing by Spatial Re-sampling by Jefrey Krolik and David Swingler (Review) [18]

This focusing technique reduces each wide-band source in multigroup scenarios to a rank one representation. This approach does not require preliminary estimates of the source locations. The method is based on simply adjusting the spatial sampling rate or “spatially re-sampling” the array outputs as a function of temporal frequency so that wide-band sources are aligned in the spatial frequency domain. [18]

In this case an output of a discrete array of M sensors is considered as the result of spatially sampled a continuous linear array. Let $y(x,\omega)$ denote the field incident upon a line array positioned along the x axis at temporal frequency ω. The $y(x,\omega)$ is expressed as:

$$y(x,\omega)=\sum_{i=1}^{D} S_i(\omega)e^{j\omega\alpha_i x}+v(x,\omega) \tag{2.39}$$

where S is the temporal Fourier transform

$\alpha_i=\sin(\theta_i)/c$ is the slowness

θ is the bearing angle.

Assume a Uniform Linear Array (ULA) with M sensors is spaced at a uniform distance d meters apart. The $y_d(m,\omega)$ is the output of the sensor located at x=md and is expressed as:

$$y_d(x,\omega)=\sum_{i=1}^{D} S_i(\omega)e^{j\omega\alpha_i md}+v(md,\omega) \tag{2.40}$$

A narrow-band covariance matrix $R(\omega)$ can be computed from sensor array outputs. This narrow-band covariance matrix R (ω) can be expressed as:

$$R(\omega)=A(\omega,\alpha)P_s(\omega)A(\omega,\alpha)^H+R_v(\omega) \tag{2.41}$$

where A is the MxD source direction matrix of the i^{th} source, P is an unknown DxD source spectral density matrix R_v is the noise matrix.

The goal of focusing methods is to transform the sensor outputs $y_d(m,\omega)$ in such a way that it results in a source direction matrix which is constant for all frequencies within the common bandwidth of the sources. A direction matrix A is obtained for all ω by adjusting the spatial sampling interval d as a function of temporal frequency ω. A focusing frequency ω_o is also selected.

The frequency dependent spatial sampling interval is denoted by d (ω). The wide-band focusing via spatial re-sampling can be achieved by letting d (ω) =d ω_0 / ω. If we substitute d (ω) in the above expression we get following focused array outputs:

$$\tilde{y}_d(x,\omega) = \sum_{i=1}^{D} S_i(\omega) e^{j\omega\alpha_i md} + v(\frac{md\omega_0}{\omega}, \omega) \tag{2.42}$$

In order to avoid any spatial aliasing, d (ω) must be chosen such that $\omega / c < \pi / d(\omega)$. This implies $\omega_0 < \pi c / d$ relation should be true. It is known that the sensor spacing should be half wavelength of the highest source frequency $\omega_{\max}$ then this will result in focusing frequency ω_o should be less than highest source frequency giving $\omega_0 < \omega_{\max}$. The re-sampling of y(x, ω) from the outputs of a discrete line array y_d(m, ω) at a finite set of frequencies ω_n (where n is from 1 to B) should ensure $\omega_0 / \omega_n = L_n / K_n$. Therefore the sampling interval should follow this relation

$$d(\omega_n) = d.L_n/K_n$$

Therefore spatially re-sampling the data at frequency ω_n involves changing the spatial sampling rate by a factor of K_n/L_n. This is also achieved by selecting the focusing frequency ω_0 equal to the minimum frequency of the sources $\omega_{\min}$. If elements are spaced a half wavelength apart at $\omega_{\max}$ ($d = \pi c / \omega_{\max}$) this implies $L_n \le K_n$ for all n. Hence re-sampling becomes an interpolation by a factor of K_n/L_n. The y_d(m, ω) is spatially re-sampled by performing the following steps.

1. Insert (K_n-1) zeros between each pair of measured spatial samples giving

$$z(m,\omega_{n)} = \begin{cases} y_d(\frac{m}{K_n},\omega_{n,}) for - m = 0, \pm K_n, \pm 2K_n, ... \end{cases}$$

0 otherwise

2. Filter z (m, ω_n) using a low-pass filter with FIR $h_n(m)$ designed to approximate the frequency response. Convolve z_{lp}(m, ω_n) with $h_n(m)$ yields the low-pass filtered output z_{lp}(m, ω_n).

3. Decimate z_{lp}(m, ω_n) by a factor of L_n to obtain the focused spatial data sequence

$$\tilde{y}_d(m,\omega_n) = z_{lp}(L_n m,\omega_n)$$

After forming the spatially re-sampled Mx1 data vectors

$$\tilde{Y}_d(\omega_n) = [\tilde{y}_d(0,\omega_n), \tilde{y}_d(1,\omega_n),..., \tilde{y}_d(M-1,\omega_n)]^T$$

For n=1…,B, a single focused covariance matrix at frequency ω_0 is estimated by

$$\tilde{R}(\omega_0) = \frac{1}{B}\sum_{n=1}^{B}\tilde{Y}_d(\omega_n)\tilde{Y}_d(\omega_n)^H$$

The wide-band source location can be computed using MUSIC operated at frequency ω_0.

Simulation Method

A 16 element line array with sensors spaced a half wavelength apart at $\omega_{\max} = \pi$ was used. The array focused to $\omega_0 = \omega_{\min} = \pi/2$ where the Rayleigh resolution limit corresponds to sin(θ)=0.25. For each trial, 32 independent realization of $y_d(\omega_n)$ were generated for each $\omega_n = K_n\omega_0 / L_n$. Where K_n=32 and L_n=16, 17,…, 32. In these experiments the up-sampling factor K_n factor was fixed for convenience, which led to

ω_n, n=1, …., 17 non-uniformly spaced frequency samples spanning the band from $\pi/2$ to π. The spectral MUSIC was to process the focused covariance matrix.

Summary

This method requires interpolation and then decimation of input data and then computation of covariance matrix. The interpolation filter was designed using the minimum mean square error filter design method. An FIR filter of length 2.J. K_n +1 points was used to approximate an ideal low pass filter with cutoff frequency $\omega_c = \beta\pi / K_n$. β is between 0 and 1 (It was chosen as 0.4). The filter length factor J is chosen as 4. The computation of covariance matrix is done on B length of data which is done across various frequencies. So the value of B could be large in the case of wide-band data. The MUSIC algorithm is applied to resolve various DOAs. Technique looks good some work need to be done to resolve value of B, K and L which are also assumed. Selection of B, K, and L may be tricky and this approach then will not be useful in a generalized case. No attempt was made to run MATLAB program due to above reasons. One advantage of this approach is that it does not require preliminary DOA estimates and iterations.

2.5 New Signal Subspace Direction Of Arrival Estimator for Wide-band Sources by Yeo-Sun Yoon, Lance M. Kaplan and James H. McClellan (Review) [20]

This work considers a uniform linear array with M sensors and assumes that the frequency bands of the D sources are known [20, 38, 40]. They are also overlapped. The sensor outputs are decomposed either using a filter bank or FFT. The sensor output at frequency ω_i is given as

$$x(\omega_i) = A_i(\theta)S(\omega_i) + \eta(\omega_i) \tag{2.43}$$

The relation between array manifolds of different frequencies and DOAs is as follows:

$$a(\omega_x,\theta_x)=\Phi(\omega_y,\theta_y)a(\omega_z,\theta_z) \tag{2.44}$$

where $\Phi(\omega_y,\theta_y)=diag\{e^{-j\omega_y\tau_y},e^{-j\omega_y 2\tau_y},...,e^{-j\omega_y M\tau_y}\}$

The relation between frequencies and DOAs is as follows:

$$\omega_x=\omega_y+\omega_z$$
$$\sin\theta_x=\frac{\omega_y}{\omega_x}\sin\theta_y+\frac{\omega_z}{\omega_x}\sin\theta_z$$

Let x_i=x(ω_i) where ω_i for i=1,2,..K is within the frequency bands of all sources. The number of frequency bins K is constrained by

$$K\geq\max\left\{\frac{M}{M-D},3\right\}$$

The covariance matrix R_i can be defined as:

$$R_i=E[x_i x_i^H]=A_i R_{s,i} A_i^H+\sigma^2 I \tag{2.45}$$

where $R_{s,i}=E[s(\omega_i)s(\omega_i^H]$ and $\boldsymbol{R}$ is a full rank matrix.

There are D largest eigenvectors corresponding to D sources and their range is the same as the range of A_i. Let $\boldsymbol{e}$ be the ordered eigenvectors of $\boldsymbol{R_i}$ from the largest to the smallest. They define matrices $\boldsymbol{F_i}$ as signal range space and $\boldsymbol{W_i}$ is the null (noise) space

$$F_i=[e_{i,1},e_{i,2},...e_{i,D}] \tag{2.46}$$

$$W_i=[e_{i,D+1},e_{i,D+2},...e_{i,M}] \tag{2.47}$$

Then following can be specified for signal and null range spaces:

Range $\{F_i\}$= Range $\{A_i\}$

Range $\{W_i\}$=Null Range $\{A_i^H\}$

Let $\Delta\omega = \omega_j - \omega_i$ then,

$$Range\{\Phi(\Delta\omega, \theta_0)F_i\} = Range\{A_j(\hat{\theta})\}$$

where $\hat{\theta} = [\hat{\theta}_1, ...\hat{\theta}_D]^T$ and $\hat{\theta}_d = \arcsin\{\frac{\omega_i}{\omega_j}\sin\theta_d + \frac{\Delta\omega}{\omega_j}\sin\theta_0\}$

This theorem informs that a signal subspace of one frequency bin can be linearly transformed into that of other frequency with modified DOAs in

$$\sin\theta_x = \frac{\omega_y}{\omega_x}\sin\theta_y + \frac{\omega_z}{\omega_x}\sin\theta_z$$

If θ_0 in $Range\{\Phi(\Delta\omega, \theta_0)F_i\} = Range\{A_j(\hat{\theta})\}$ is the same as one of the θ_d in the original signal subspace, this DOA is preserved in the new signal subspace.

Assume that $K \geq \max\left\{\frac{M}{M-D}, 3\right\}$ holds.

Let E_i for i=1,2…K be Dx(M-D) matrices such that

$$E_i = F_1^H \Phi(\Delta\omega_i, \theta)^H W_i$$

where $\Delta\omega = \omega_i - \omega_1$. Define the Dx(M-D) matrix B such that

B=[E_2 E_3 … E_K]

Then

$$rank\{B(\theta)B(\theta)^H\} = \begin{cases} D-1....if\theta = \theta_d \\ D.........if\theta \neq \theta_d \end{cases}$$

Proof: As Range {W_i}=Null Range {A_i^H} has been specified earlier then

$$a_d(\omega_i)^H W_i = 0^T$$

This is true for all d. We also know that

$$F_1^H \Phi(\Delta\omega_i, \theta)^H = T_1^H A_i(\hat{\theta}_i)^H$$

where $\hat{\theta}_{i,d} = \arcsin\left\{\frac{\omega_1}{\omega_i}\sin\theta_d + \frac{\omega_i - \omega_1}{\omega_i}\sin\theta\right\}$

If $\theta = \theta_d$ and $\hat{\theta}_{2,d} = ... = \hat{\theta}_{k,d} = \theta_d$

Therefore,

$$E_i = T_1^H \begin{bmatrix} a^H(\omega_i, \hat{\theta}_1)W_i \\ . \\ . \\ . \\ a^H(\omega_i, \theta_d)W_i \\ . \\ . \\ . \\ a^H(\omega_i, \hat{\theta}_d)W_i \end{bmatrix} \tag{2.48}$$

Giving the following relation

$$E_i = T_1^H \begin{bmatrix} * \\ 0^T \\ * \end{bmatrix}$$

This is the dth row where we have 0^T.

Since there are multiple sources then there is a possibility that one of the $\hat{\theta}'s$ would be the same as θ_j. If we use atleast three frequency bins then this ambiguity can be removed. Therefore only for $\theta = \theta_d$, B becomes

$$B = [E_2..E_K] = T_1^H \begin{bmatrix} * \\ 0^T \\ * \end{bmatrix} \tag{2.49}$$

and it loses rank.

The estimation process is

1. Divide the sensor output into J identical blocks
2. Compute FFT of each block
3. Find $\hat{x}_i$ for pre-selected ω_i
4. Find the signal subspace $\hat{F}_1$ and noise subspace $\hat{W}_i$ by eigendecomposition of the covariance matrix $\hat{R}_i$
5. Find $\hat{E}_i$ using $E_i = F_1^H \Phi(\Delta\omega_i, \theta)^H W_i$
6. Find $\hat{\theta}$ such that

$$\hat{\theta} = \arg\max K\{\hat{B}(\theta)\hat{B}(\theta)^H\}$$

Summary

This method requires that preliminary estimate of the DOA that could be one of the angles in the estimation and number of sources need also be estimated. There is missing information in this work so no further study is planned at this time for this work. We may come back later on and have a second look at it as things progress with other work. Authors have extended their work for arbitrary shaped multidimensional arrays. They have also named this technique as Test of Orthogonality of Projected Spaces (TOPS) [20, 38-40].

2. 6 A Method for Wide-band Direction of Arrival Estimation Using Frequency-Domain Frequency-Invariant Beamformers by Tuan Do-Hong, Franz Demmel, Peter Russer (Review) [21-25]

A new method for wide-band DOA estimation using arbitrary antenna array based on Frequency-Domain Frequency-Invariant Beamformers (FDFIB) is proposed by these authors [21-25]. Earlier a beam-space processing using Time-Domain Frequency-

Invariant Beamformers (TDFIB) was proposed [22]. This method does not require preliminary DOA estimates and has less computational complexity than beamspace CSS [19]. However the frequency invariant characteristics depend on the number of antenna elements within the arrays, on the geometry of arrays and on the design of filters in TDFIB.

In this work, Frequency-Domain Beamformers (FDBs) are used with appropriately designed weights at different frequencies. This ensures that beam-patterns of FDBs remain constant over the frequency band. This approach is then termed as frequency-invariant beamformer as beam patterns are independent of the frequency. This technique transforms the element-space into the beam-space and acts as spatial processor. DOA is then estimated using the well known narrow-band MUSIC that is applied in beam-space at single selected frequency. The selected frequency should also be within the bandwidth.

Frequency-Domain Frequency-Invariant Beamformers (FDFIB)

An array of M identical elements and a beamforming network of J frequency domain beamformers are considered and it is assumed that there are D wide-band sources located in the far field. The j^{th} beam pattern is given by:

$$B_j(\omega_k) = w_j^H(\omega_k) b(\omega_k, \Omega_b) \tag{2.50}$$

where $b(\omega_k, \Omega_b)$ is the steering vector and $\Omega_b = [\cos\phi_b \sin\theta_b, \sin\phi_b \sin\theta_b, \cos\theta_b]$ with azimuth and elevation at the b^{th} direction. Azimuth is between $-\pi$ to π and elevation is between 0 and $\pi/2$. $w_j^H(\omega_k)$ is the weighting vector with weight at frequency ω_k of the j^{th} beamformer at m^{th} antenna element.

The weights at each frequency are chosen such that $B_j(\omega_k) = B_j(\omega_0)$. Where ω_0 is the focusing frequency? The weights for frequency-invariant beamformer at each frequency are determined as

$$\omega_{jm}(\omega_k) = \alpha_m e^{-j\frac{\omega_k}{c} d_m^T \Omega_b + j\frac{\omega_0}{c} d_m^T \Omega_b} \tag{2.51}$$

Where α_m is the amplitude weighting coefficients and its value as unity is suggested in this work?

Wide-band Signal Model in Element and Beam-Space

Denote x (n) as the output of the sensor arrays which is Mx1 vector. Sensor array output in frequency domain can be written as:

$$X(\omega_k) = \sum_{d=1}^{D} a(\omega_k, \Gamma_d) S_d(\omega_k) + N(\omega_k) \tag{2.52}$$

where $\Gamma_p = [\cos\phi_d \sin\theta_d, \sin\phi_d \sin\theta_d, \cos\theta_d]^T$ and in matrix notation above equation becomes:

$$X(\omega_k) = A(\omega_k) S(\omega_k) + N(\omega_k) \tag{2.53}$$

where *A* is MxD source direction matrix, *S* is the Dx1 vector of signals at inputs of the array and *N* is the noise matrix.

A J-beamforming network is used and it is assumed that D is less than equal to J and J is less than equal to M. Output of J beamforming network in frequency domain can be written as:

$$Y(\omega_k) = [Y_1(\omega_k),...Y_j(\omega_k),..Y_J(\omega_k)]^T = C^H(\omega_k) X(\omega_k) = A_C(\omega_k) S(\omega_k) + N_C(\omega_k) \tag{2.54}$$

where Y is Fourier coefficient of beam-space signal. $A_C(\omega_k) = C^H(\omega_k)A(\omega_k)$ is source direction matrix in beam space, $C(\omega_k) = [w_1(\omega_k),...w_j(\omega_k),..w_J(\omega_k)]^T$ is called the beamforming matrix and w is weighting vector of jth frequency-invariant beamformer.

The source direction matrix in beam-space is constant for all frequencies within the signal bandwidth as this beamformer is designed to be frequency-invariant. A single direction matrix $A_C(\omega_0)$ can characterize the DOA for the wide-band case. It is customary to assume that the signals and the noise are uncorrelated. The cross spectral density matrix in beam-space is given by:

$$R_Y(\omega_k) = E\{Y(\omega_k)Y^H(\omega_k)\} = C^H(\omega_k)R_x(\omega_k)C(\omega_k)$$

where $R_X(\omega_k) = E\{X(\omega_k)X^H(\omega_k)\}$ is the cross-spectral density matrix in element-space. The wide-band covariance matrix in beam-space can then be written as:

$$R_Y = \sum_{k=l}^{h} R_Y(\omega_k) = \sum_{k=l}^{h} C^H(\omega_k)R_x(\omega_k)C(\omega_k) \qquad (2.55)$$

A dense grid of I angle points (or spatial frequency points) of azimuth and elevation is defined. For the MUSIC algorithm the DOAs are determined by searching the peak positions of the spatial spectrum.

$$\hat{S}_{FDFIB-MUSIC}(\Gamma_i) = \frac{a_C^H(\Gamma_i)a_C(\Gamma_i)}{a_C^H(\Gamma_i)U_N U_N^H a_C(\Gamma_i)} \qquad (2.56)$$

Where U is the noise subspace matrix and is obtained from the eigenvalue computation of R matrix and a_C^H is the Jx1 source direction vector in beam space.

Summary of the algorithm:

Compute the FFT of the data to go to frequency domain

Frequency domain data is fed to J beamformers

Compute output of the J beamformers in parallel using steering vector and weights

Compute covariance matrix from the outputs of J beamformers

Compute MUSIC algorithm using a selected frequency on a dense grid of I angle points.

The coordinates of the array (d_m) are not known. Azimuth estimation using a uniform circular array of 9 elements is used. The co-ordinates of the elements are normalized over $\lambda = c / f_h$ and the radius of array r= λ. Three uncorrelated wide-band sources with normalized frequencies are considered. The focusing frequency ω_0 is selected at ω_h.

2.7 Wide-band Direction of Arrival Estimation and Beamforming for Smart Antennas System by Tuan Do-Hong, Peter Russer (Review) [21-25]

This paper is similar to the previous paper from these authors [21] and their two other papers [24-25] with following differences. It assumes an arbitrary Uniform Linear Array. It uses a wide-band beamforming method with prescribed narrow main-beam width and Low Sidelobe Level (SLL) using spatial interpolation. It uses a spatial interpolation process consisting of two FDFIBs. The first FDFIB is based on a (prototype) FDFIB when the inter-element spacing of d is replaced by Nd. In this case N is an integer referred to as expansion factor. The required main-beam width can be obtained by adjusting the number N in the combination with spatial widows. The second FDFIB is applied at the output of the first FDFIB to attenuate grating lobes, which appear due to changing of the spacing. The attenuated level depends on the required SLL. As a result, the beam-pattern with narrow main-beam width (due to larger spacing) and low SLL (due to the attenuation of the second FDFIB) are simultaneously obtained without increasing the number of antenna elements. The use of spatial interpolation provides for the specification of main-beam width while at the same time allowing for the specification of SLL, one can reduce number of antenna elements and the corresponding RF modules, A/D converters etc., while still retaining same main-beam width and SLL as traditional beamforming method that requires larger number of elements. Moreover due to larger inter-element spacing, the mutual coupling between antenna elements can be eliminated.

Summary

Four papers were reviewed from these authors [21-25] and they are similar in their work with notational changes in equations. That makes it difficult to separate their work. Simulation results are also somewhat similar. They have not provided a step by step mechanism for their algorithm. The bright part of their work is that it does not require preliminary estimate of the DOA. It does require array geometry and guessing of common frequency so their algorithm could be based on it.

2.8 Theory and Design of Broadband Sensor Arrays with Frequency Invariant Beam Patterns by Darren B. Ward, Rodney A. Kennedy, Robert C. Williamson (Review) [26]

This work deals with the problem of designing a uniformly spaced array for wide band applications. This group of authors has published three papers in this series and they will be reviewed one at a time [26,29-30]. A summary of all these work will be provided at the end of the third review.

Consider broad band arrays in which there is little or no frequency variation in the far-field array beam pattern over an arbitrarily wide desired bandwidth. The asymptotic theory of unequally spaced arrays is used to derive relationships between beam pattern properties and array design. Theory in this work assumes planar arrays and sources are in far field. Broadband FI array is defined in terms of the array beam pattern. These beam patterns are assumed to be identical at different frequencies and require a compound array of k subarrays. Subarrays are identical and their spatial coordinates are expressed in wavelength. There infinite number of subarrays would be required for producing an identical beam for wide range of frequencies. First of all a continuous sensor is developed which will produce frequency invariant beam pattern and then it is approximated with group of discrete sensors.

First consider a one-dimensional (linear) continuous sensor aligned with the x-axis. The output of this continuous sensor is

$$Z_f = \int_{-\infty}^{\infty} S(x,f)\rho(x,f)dx \tag{2.57}$$

where S is the signal received and ρ is the sensitivity distribution

The ρ (x,f) is considered as the aperture distribution. It is assumed that the sensitivity distribution is absolutely integrable and the integral exists for finite power signals. The output of the sensor when subject to plane waves arriving from an angle θ is given by

$$S(x,f) = e^{-j2\pi c^{-1} fx\sin\theta}$$

where c is the speed of wave propagation.

The output of the sensor is a function of θ and the sensor beam pattern at frequency f is as follows:

$$b_f(\theta) = \int_{-\infty}^{\infty} e^{-j2\pi c^{-1} fx\sin\theta}\rho(x,f)dx$$

A wide-band frequency invariant (FI) sensor will have pattern as frequently invariant and is defined as , $b_f(\theta) = b(\theta)$, for all *f*>0.

The sensitivity distribution of a one-dimensional sensor, which is a function of distance *x* along the sensor and frequency *f* is given by

$$\rho(x, f) = fG(xf) \text{ for all f>0}$$

where *G* is an arbitrary absolutely integrable complex function of a single real variable. Then the far-field beam pattern $b_f(\theta)$ which is a function of the angle θ measured relative to broadside and frequency *f*, will be frequency invariant.

$$b_f(\theta) = b(\theta) = \int_{-\infty}^{\infty} e^{-j2\pi c^{-1}\xi \sin\theta} G(\xi)d\xi \tag{2.58}$$

where ξ=*xf*.

Above mathematical derivation was presented as a theorem by authors. A sensitivity distribution theorem was also developed by authors. Assume $b(\theta)$ as an arbitrary continuous square integrable frequency invariant far field beam pattern which is specified for $\theta \in (-\pi/2, \pi/2)$ and it determines ρ(x,f) uniquely. Then the sensitivity distribution ρ(x,f) of a linear sensor which realizes this beam pattern must satisfy the following conditions:

1. $\rho(x, f) = fG(xf)$ for some function *G*.

2. *G* has a Fourier transform Γ satisfying

Γ(s)=B(s)=b{sin $^{-1}$ (sc)}, $s \in (-1/c, 1/c)$

Γ(s)=A(s), $s \notin (-1/c, 1/c)$

where *c* is the speed of wave propagation and *A*(.) is an arbitrary square integrable function such that

$$A[(-1)^i / c] = \lim_{s \to \frac{(-1)^i}{c}} B(s)$$

for i=0,1

Thus the only freedom in choosing ρ(x,f) for a desired FI beam pattern is in the sufficiently high spatial frequency behavior of *G*. $b(\theta)$ for $\theta \in (-\pi/2, \pi/2)$ determines ρ(x,f) uniquely.

Assume D-dimensional continuous sensor. Let the output of the sensor be given by

$$Z_f = \int_R S(x,f)\rho(x,f)dx$$

The sensor has a frequency invariant far-field beam pattern if

$$\rho(x,f) = f^D G(xf)$$

If *G* is an arbitrary absolutely integrable complex-valued function then it can be expressed as:

$$G(xf) = A_f(x) = H_x(f)$$

A defines the aperture distribution function at a nominally fixed frequency *f* and *H* defines the primary filter at a single point *x* on the sensor. The total filtering required at a fixed point *x* can be expressed as

$$\rho(x,f) = f^D H_x(f)$$

where f^D is considered as a secondary filter. It is independent of the sensor spatial vector *x* and it is a function of the sensor dimensions D only.

G is a symmetric function of spatial variable *x* and of the frequency variable *f*. This implies that *f* and *x* can be interchanged without affecting the value of the function.

Their values can be varied one at a time and keeping the other constant. The primary filter response takes the same shape as the aperture distribution.

If *H* denotes the frequency response of the primary filter at a point *x* and *A* denotes the aperture distribution for a given frequency then *H=A. (fx)* (ignoring some mathematical notations).

The primary filter response required at point *x* can be obtained by taking a slice through the aperture distribution from the origin in the direction of *x*. The aperture distribution can be determined from the desired beam pattern and vice versa. The correspondence between aperture distribution and primary filter response is for both magnitude and phase. All primary filter responses in a D-dimensional frequency invariant broadband sensor for a given x are identical up to frequency dilation.

If we concentrate on single sided one dimensional array apertures with the first element located at *x*=0. An array of sensors can only approximate the ideal broadband continuous sensor. This reduces to a numerical approximation uniformly in *f* to the following integral representing the output of the ideal continuous sensor for an arbitrary signal *S*.

$$Z_f = \int_{-\infty}^{\infty} S(x, f) fG(xf) dx \qquad (2.59)$$

This is for *f* greater than 0. Assume $\{x_i\}$ denote a finite set of N discrete sensor locations. In approximating a finite number of sensors and limiting the range of frequency to $[f_L, f_U]$ then we have:

$$\overline{Z}_f = f \sum_{i=0}^{N-1} g_i S(x_i, f) G(x_i f) \qquad (2.60)$$

S is the complex signal received at point x_i.

$G(x_i f)$ is the sampled value of $G(xf)$ at $x = x_i$

g_i is a frequency independent weighting function to compensate for the possibly non-uniform sensor locations.

An important aspect of this broadband array design is that the array design comes from approximating an integral describing a broadband FI continuous sensor. A trapezoid integration method is used.

Using $G(xf) = A_f(x) = H_x(f)$ write the output of the primary filter attached to the ith sensor as

$$y_i(f) = H_{x_i}(f) S(x_i, f)$$

This can also written using filter dilation theorem as

$$y_i(f) = H_{x_i}(x_i / x_1 f) S(x_i, f)$$

This emphasizes that only one primary filter shape is required in the numerical integration approximation and is written as

$$\tilde{Z}_f = f y(f)' Tx$$

The weighting function g can be seen to relate to *Tx* via an un-illuminating formula. The weighting functions can be a function of one or more discrete sensor locations but are independent of the frequency.

It is assumed that the aperture distribution is a slowly varying function with respect to x compared to the exponential term in $b_f(\theta) = \int_{-\infty}^{\infty} e^{-j2\pi c^{-1} fx \sin\theta} \rho(x, f) dx$.

The block diagram of general single sided one dimensional broadband FI array would look like a FIR block diagram.

1. The primary filters are simple dilations of a single frequency response.

2. $H(f) = H_{x_1(f)}$

3. The primary filter frequency response *H(f)* is similar to the continuous aperture distribution shape both in magnitude and phase.

4. The primary filter outputs can be combined via frequency independent weight *g* that depends only on the sensor locations generating a scalar output.

5. All sensors share a common secondary filtering response *f* to generate the final output.

2.9 FIR Filter Design for Frequency Invariant Beamformers by Darren B. Ward, Rodney A. Kennedy and Robert C. Williamson (Review) [29]

This work uses a Frequency Invariant Beamformer (FIB) approach and in this case the response of the beamformer is constant over an arbitrarily wide design bandwidth. Previously these authors have presented an analog technique based on approximating an ideal continuous aperture [26]. This filter design approach just described in this report. Based on this theory, two methods namely multirate sampling and single sampling rate of designing FIR filters for use in an FIB are proposed in this paper [26, 29-30].

The response of a linear continuous aperture to planar waves from an angle θ measured to broadside was given as:

$$\tau(\theta, f) = \int_0^{x_{\max}} e \frac{j2\pi fx \sin\theta}{c} \rho(x, f) dx)$$

where $\rho(x, f)$ is the aperture illumination.

It is a continuous function of both location *x* and frequency *f* and c is the speed of wave propagation. The response remains constant if the aperture illumination is given by

$f.G(xf)$, where $G(.)$ is an arbitrary absolutely integrable function. This allows breaking of filtering of an FIB into two parts namely primary filter response and the secondary filter response.

1. The primary filter response is $H_x(f)=G(xf)$
2. The secondary filter response f is independent of position.

An important feature of the FI aperture is that all primary filters are related by dilation, i.e. if $H_x(f)$ is the primary filter response at an arbitrary point on the aperture, then the primary filter response at a point $\gamma x, \gamma > 0$ is given by

$$H_{\gamma x}(f) = G(\gamma x f) = H_x(\gamma f)\,. \tag{2.61}$$

The continuous aperture is approximated using numerical approximation of the integral described in the above equation to be able to design a practical frequency invariant beamformer. Let x_n denote a set of N+1 sensor locations with the zeroth sensor located at x_0=0. If we limit the frequency to the range $[f_L, f_U]$, we can use the following approximations:

$$\hat{r}(\theta) = f\sum_{n=0}^{N} g_n H_n(f) e^{j2\pi f x_n c^{-1}\sin\theta} \tag{2.62}$$

where $\hat{r}(\theta)$ is the approximate frequency invariant response

g is a spatial weighting term

$H_n(f) = G(x_n f)$ is the primary filter response of the nth sensor.

In other work [26], it was shown how to obtain g for the case corresponding to the trapezoidal integration method. The set of sensor locations can be determined by minimizing the number of sensors required while avoiding spatial aliasing.

The sensor locations are given by

$$x_n = \begin{Bmatrix} \frac{\lambda_U}{2} n, \\ P\frac{\lambda_U}{2}(\frac{P}{P-1})^{n-P} \end{Bmatrix} \quad (2.63)$$

where P is the aperture length measured in half wavelength. λ_U is the wavelength corresponding to the upper frequency of operation and $N = P + \left[\log\left(\frac{f_U}{f_L}\right) \log\left(\frac{P}{P-1}\right) \right]$

Design of the primary filters

The primary filters of an FIB have an important property of frequency dilation. It means that all primary filters are derived from a single reference frequency response, and hence all primary filter coefficients may be derived from a single set of coefficients. There are two design techniques described by this work and they are multirate method and single rate method.

Multirate Method

A primary filter response $H_{ref}(f)$ at a reference location x_{ref} having a sampling period T will have $h_{ref}[k]$ as a set of l filter coefficients. The primary filters needs to have the required dilation property if the n^th^ primary filter response is given by

$$H_n(f) = \sum_{k=-(L-1)/2}^{(L-1)/2} h_{ref}[k] e^{-j2\pi f T_n k} \quad (2.64)$$

where $T_n = Tx_n / x_{ref}$ is the sampling period of the n^{th} sensor.

Multirate sampling is generally achieved by sampling every sensor at the highest rate required and then use decimation to bring down the sampling rate to the desired sampling rate. Therefore each of the primary filters would be implemented by

- Downsampling by $\gamma_n = x_n / x_{ref}$

- Applying the reference primary filter
- Upsampling by γ_n.

The aperture length is defined to be P half-wavelengths at all frequencies within the design band. Therefore, the n[th] primary filter is band limited with $H_n(f)=0$ for $|f| > Pc/(2x_n)$. If we ignore the zeroth primary filter (which has a constant response), the primary filter with the widest bandwidth will be located at $x_1 = c/(2f_U)$. It will have the effective bandwidth as Pf_U, requiring a sampling rate of $f_s=2Pf_U$. The reference primary filter is located at $x_{ref}=c/(2f_U)$.

Single Rate Method

A desired primary response at some reference location will have a set of reference coefficients $h_{ref}[k]$. The n[th] set of primary filter coefficients can be obtained by first reconstructing the continuous time impulse response of $h_{ref}[k]$, applying the scaling property of the Fourier transform and resampling the scaled impulse response. The primary filter coefficients are given by:

$$h_n[m] = \frac{1}{\gamma_n} \sum_{k=-(L-1)/2}^{(L-1)/2} h_{ref}[k] \operatorname{sin} c(\frac{m}{\gamma_n} - k)$$

where $\gamma_n = x_n / x_{ref}$ and $\operatorname{sin} c(x) = \sin(\pi x)/(\pi x)$.

The reference set of coefficients must first be convolved with the coefficients of a low pass filter having a cutoff of $\gamma_n f_s / 2$ to avoid temporal aliasing. For $\gamma_n = 0$, the reference coefficients are simply an impulse. The length of the n[th] primary filter should be L γ_n a predefined number of coefficients should be used to give best results.

If the input signal is band-limited to f_U, the minimum sampling rate is $f_s=2f_U$. The location of the reference sensor should be such that $H_{ref}(f)=0$. Hence for the single sampling rate FIB, the reference sensor is located at $x_{ref}=Pc/(2f_U)$.

Determining the Coefficients of the Reference Primary Filter

With the use of $fG(xf)$ and the change of variables in the first equation in this work. The Fourier transform relationship between the desired response and the aperture distribution is apparent. Thus given a desired response and the visible region the required distribution is given. The coefficient of the primary filter can be calculated with a vague method described by the author. The secondary filter is a differentiator and its design could be a Type 4 FIR filter, even length with odd symmetric coefficients.

Simulation Example:

Authors used an aperture length of P=4 half-wavelengths, and the design bandwidth of 200-3400 Hz, requiring 17 sensors and a total array size of 3.4 m. A secondary filter with 12 coefficients and a uniform aperture illumination were used in both cases. They show response of the multirate FIB with a maximum sampling rate of 30 kHz and a reference filter with nine coefficients. The single sampling rate FIB with a sampling rate of 8 kHz is also shown. The reference filter had nine coefficients but each of the primary filters had a minimum of 51 coefficients (with the larges primary filter having 151 coefficients).

Summary

The technique presented is for acoustic wave in the air. This approach requires design of set of primary filters and a secondary filter. Design process is described vaguely and its implementation in MATLAB would be a difficult task as of missing information. No further study is planned for this work.

2.10 Broadband DOA Estimation Using Frequency Invariant Beamforming by Darren B. Ward, Zhi Ding, and Rodney A. Kennedy (Review) [30]

This technique is for wide-band focusing using time domain processing. It performs beamspace processing using frequency-invariant beamformers i.e. beamformers whose beampatterns are constant over a wide frequency band. This approach uses a set of appropriately designed beam-shaping filters. These filters ensure that the same array manifold is produced for all frequencies within the design band. The proposed estimator does not perform frequency decomposition. This approach exploits the FIR filtering based approach to implicitly perform focusing over a wide frequency band. It is a filter and sum approach. Authors have provided design of filters for this scheme in their earlier work [26, 29].

Frequency Invariant Beamforming

This approach uses a filter and sum structure. The FIR filter at the mth sensor has response as $H_m(f)$. $H_s(f)$ is an optional normalization FIR filter. The beam shape is constant as a function of frequency and beam shaping is performed by sensor filters. The response of this beamformer to plane waves arriving from an angle θ is

$$r(\theta, f) = H_s(f)\sum_{m=1}^{M} H_n(f)e^{j2\pi f \tau_n(\theta)} \qquad (2.65)$$

where $\tau_n(\theta)$ is the propagation delay to the mth sensor. Above equation may also be written as:

$$r(\theta, f) = b^H(f)a(\theta, f)$$

Assume that each row of $b(f)$ is an FIR filter with filter coefficients $b_m[nc]$ and nc=0, ….. NC-1. These filters need to be properly design so the response of the beamformer is approximately constant with respect to frequency over the design bandwidth giving a frequency invariant response.

Frequency Invariant Beamspace Processing for DOA Estimation

Consider a linear array of M sensors that are not necessarily uniformly spaced. Assume that D<M far field broadband signals arriving from D sources. The time series received at the m[th] sensor is

$$y_m[k] = \sum_{d=1}^{D} s_d[k - \tau_m(\theta)] + v_m[k] \tag{2.66}$$

Its frequency response can be written as

$$y_m[f] = \sum_{d=1}^{D} e^{j2\pi f \tau_m(\theta)} s_d[f] + v_m[f] \tag{2.67}$$

We can define an N-dimensional vector of stacked array data and its frequency response can be written as:

$$y(f) = A(\Theta, f)s(f) + v(f) \tag{2.68}$$

where *s(f)* is the Dx1 source signal vector. *A* is the MxD source direction matrix and *v* is the noise vector.

The source signals and noise have finite bandwidth $[f_L, f_U]$. We want to determine the source direction from the observed array data vector y[k] over a finite time period n=1….N.

Assume we apply an FIB to the received array data. The beam former output is

$$z[n] = \sum_{m=1}^{M} \sum_{nc=0}^{NC-1} b_m[nc] y_m[n - nc] \tag{2.69}$$

where b is the set of FIR filter coefficients on the m[th] sensor. The frequency response of the beamformer output is

$$z(f) = b^H(f)y(f) \tag{2.70}$$

Assume we now form J ($D < J \le M$) such beamformers using J different sets of filtering vectors. Denote the stacked vector of beamformer response as z(f) where b is the i[th] set of beam shaping filter responses. These beamformers are designed to cover a spatial vector in which the sources are assumed to lie, and they should have uniformly low side lobes to attenuate unwanted out-of-sector sources e.g. Chebyshev beamformers.

Let C(f) be the MxJ beamforming matrix. Because the beamformers are designed to satisfy the frequency invariant property, the FIBs source direction matrix is approximately constant for all frequencies with the design band. Hence the broadband source directions are completely characterized by a single beamspace source matrix Ac.

Assuming the source signals and the noise are uncorrelated the FIBS data covariance matrix is

$$R_z(f) = E\{z(f)z^H(f)\} = A_c(\Theta)R_s(f)A_c(\Theta)^H + R_v(f) \tag{2.71}$$

The broad band FIBS data covariance matrix is now in a form in which conventional eigenbased DOA estimator may be applied. The eigensubspaces of noise and signals can be obtained and MUSIC algorithm can then be applied.

Summary of the proposed algorithm

Design J FIB's that cover the selected spatial region.

Calculate the broadband FIBs noise covariance matrix Rv (why and how in real life can this be calculated)

Collect data from each of the J beamformers over the observation period n=1,…N and estimate the broadband FIBS data covariance matrix $\hat{R}_z$ using the following relation.

$$\hat{R}_z = \frac{1}{N}\sum_{n=1}^{N} z[n]z^H[n]$$

where z is computed using the following relation:

$$z[n] = \sum_{m=1}^{M}\sum_{nc=0}^{NC-1} b_m[nc]y_m[n-nc]$$

Using $\hat{R}_z$, we can form signal and noise subspace and use MUSIC algorithm to find DOAs.

This work use 27 uniformly spaced elements with d spacing. Three FIB's were designed (using FIR filters with 201 taps) to be frequency invariant over the normalized frequency band [0.2, 0.4] and to cover the spatial sector {80 to 100 degrees}. The corresponding beampattern with center frequency of 0.3 were calculated. Authors show results and claim that they are better than CSS and Lee's paper.

Summary

This approach looks very attractive and needs some more work. It is still not clear as how to design these FIR filters which require more calculation and experimentation. We need to find a way to design these FIR filter coefficients need to be more focused and find a way to compute them.

2.11 Cyclostationarity Based Coherent Methods for Wide-band-Signal Source Location by Giacinto Gelli and Luciano Izzo (Review) [39]

This work exploits cyclostationarity properties existing in modulated signals and uses for the computation of direction of arrival and spatial filtering in congested areas. Signals which do not have similar cyclostationary properties do not pose problems for cyclic methods [39]. These methods can also be used where number of sources are greater than or equal to the number of sensors. This work extends the cyclic approach to wide-band case and uses Cyclic Spectral Density Matrix (CSDM) for the estimating DOAs at different frequency values.

Cyclic wide-band signal source location method should have following properties:

1. Exhibit high spatial resolution
2. Suitable to work in a multiple access scenario
3. Capable of also working in a fully correlated environment
4. Can potentially exploit all the Signal of Interest (SOI) bandwidths.

This work is based on the concept of focusing transformations [2, 20-21]. Different array CSDMs are coherently combined into a single matrix using a common frequency. This is done by means of appropriate linear transformations. The resulting matrix will condense all the information contained in the frequency dependent array CSDMs. Then its signal subspace properties are used to obtain high resolution estimates of the DOAs.

The scope of this paper is two fold:

1. To show theoretically how the coherent or focusing approach can be applied to the cyclic case.
2. To extend practical focusing techniques from the conventional to the cyclic case.

Two techniques are proposed in this paper.

1. It is an extension of the Wang & Kaveh's CSS method [6-7]. It is called Cyclic Coherent Signal Subspace (CCSS) method.
2. The second part uses a class of spatial resampling method and it is an extension of the Array Manifold Interpolation (AMI) method of Krolik [17-18]. It is renamed as Cyclic AMI (CAMI)

Both these methods can perform high-resolution wide-band signal source location even in fully correlated environment. They exploit all the bandwidth of the SOIs and are able to separate SOIs with similar characteristics.

Background on Frequency Domain Cyclostationarity

Second order wide sense joint characterization of two scalar zero mean discrete time complex valued signals requires knowledge of the cross correlations.

$$R_{x1x2}(n,m) \cong E[x_1(n+m)x_2^*(n)]$$

$$R_{x1x2}{}^*(n,m) \cong E[x_1(n+m)x_2(n)]$$

where E denotes statistical averaging.

If x_1 and x_2 are jointly second order cyclostationary then $R_{x1x2}(n,m)$ is a periodic function of n. It then allows a generalized Fourier series representation with respect to n for every value of m.

$$R_{x1x2}{}^{[*]}(n,m) = \sum_{\alpha} R_{x1x2}^{\alpha}(m)e^{j2\pi\alpha n}$$

where each Fourier coefficient

$$R_{x1x2*}^{\alpha}(m) = \left\langle R_{x1x2[*]}(n,m)e^{-j2\pi\alpha n} \right\rangle$$

It denotes infinite time averaging, represents the cyclic cross correlation function and the values of α are referred to as the cyclic frequencies. When $x_1(n)=x_2(n)$, the above functions *R* reduce to the Cyclic Autocorrelation Function (CAF) and the Cyclic Conjugate Correlation Function (CCCF). The Fourier transform S of R is referred to as the cyclic cross spectral density function and cyclic conjugate cross spectral density function. When $\alpha=0$, both *R* and *S* will reduce to the conventional cross correlation function and the power cross spectral density functions respectively.

If we assume that $x_1(n)$ and $x_2(n)$ satisfy appropriate mixing conditions, the functions *S* can be given an alternative interpretation in terms of spectral correlation. Then $X_1(v)_N$ and $X_2(v)_N$ represents their Fourier transform of finite sequences. It results that

$$S^{\alpha}_{x1x2[*]}(v) = \lim \frac{1}{N} E[X_1(v)_N X_2^{*[*]}([-](v-\alpha))_N] \tag{2.72}$$

S is interpreted as the limit spectral correlation between pairs of spectral components of $x_1(n)$ and $x_2(n)$ spaced α cycles apart. Such a correlation is zero for a wide-sense stationary signal.

Let an array of M sensors receiving D signals and D<M which are bandlimited zero mean SOIs. It is assumed that sources are in far field and wavefronts are planar. They have common frequency spectral support and exhibit cyclostationarity with a common cycle frequency. Continuous time signal received at the mth array sensor will have the following form.

$$x_m(t) = \sum_{d=1}^{D} s_d[t - \tau_m(\theta_d)] + i_m(t) \tag{2.73}$$

It is assumed that sensor outputs are first down converted to baseband and successively sampled and processed digitally. Denoting f_s=1/T$_s$ $\geq$2W the sampling frequency. Then the discrete time complex envelope at the mth sensor is expressed as:

$$x_k(n) = \sum_{d=1}^{D} s_d[n - m_k(\theta_d)]e^{-j2\pi v0mk(\theta d)} + i_k(n) \tag{2.74}$$

where $v_0 = f_0 / f_s$ and $m_k(\theta_d) = \tau_k(\theta_d)/Ts$. We also assume that m term is integer.

Moreover the discrete time SOIs will exhibit cyclostationarity with discrete time cycle frequency

α. It turns out that

$$X(v)_N \approx A(\theta, v+v_0)S(v)_N + I(v)_N \tag{2.75}$$

which are Fourier transform and expressed in frequency domain. If we assume that the interfering and noise signals are uncorrelated with the SOIs and do not exhibit cyclostationarity with the considered cycle frequency α, the array CSDM defined by S is given by

$$S^{\alpha}_{xx[*]}(v) = A(\theta, v+v_0)S^{\alpha}_{ss[*]}(v)A^{H[*]}(\theta,[-](v-\alpha)+v_0) \tag{2.76}$$

where S is the CSDM of the SOIs and we have accounted for the Hermitian property of A with respect to v. Note that for α=0. The CSDM given by above equation reduces to the conventional array spectral density matrix (SDM). Wide-band cyclic methods which are based on this above equation are expected to perform well even in unknown or time varying interference environments.

If we assume that there exists at least a value of v such that the array manifold is known and unambiguous and that the SOI CSDM has full rank D, the signal subspace approach can be applied to obtain high resolution estimates of the signal DOAs. The wide-band cyclic MUSIC method estimates the signals DOAs by resorting to the SVD decomposition of the array CSDM evaluated at a single value of v.

Coherent Cyclic Methods for DOA Estimation

Authors propose to perform a coherent combination of the array CSDMs evaluated in correspondence of J distinct frequency values vj, j=1,…J. Let

$$Y(v)_N = T(v)X(v)_N$$

This represents a linear transformation of X and T is a nonsingular MxM matrix. T should satisfy the following focusing condition [6-7]:

$$T(v)A(\theta, v+v_0) = A(\theta, v_f + v_0) \tag{2.77}$$

Where v_f is a suitable focusing frequency belonging to referred focusing bandwidth which is symmetric to v=0. Denoting *y(n)* as the inverse Fourier transform of *Y(v)* the array CSDM of *y(n)* is given by

$$S^{\alpha}_{yy[*]}(v) = \lim_{N\to\infty} \frac{1}{N} E[Y(v)_N Y^{H[*]}([-](v-\alpha))_N]$$

$$S^{\alpha}_{yy[*]}(v) = T(v) \lim_{N\to\infty} \frac{1}{N} E[X(v)_N X^{H[*]}([-](v-\alpha))_N] * T^{H[*]}([-](v-\alpha))$$

$$S^{\alpha}_{yy[*]}(v) = T(v) S^{\alpha}_{xx[*]}(v) * T^{H[*]}([-](v-\alpha))$$

Modifying this by using Equation 12

$$S^{\alpha}_{yy[*]}(v) = T(v)A(\theta, v+v_0) S^{\alpha}_{ss[*]}(v) A^{H[*]}(\theta, [-](v-\alpha)+v_0) T^{H[*]}(\theta, v_f + v_0) \tag{2.78}$$

$$S^{\alpha}_{yy[*]}(v) = A(\theta, v_f + v_0) S^{\alpha}_{ss[*]}(v) A^{H[*]}(\theta, v_f + v_0) \tag{2.79}$$

Note that the array CSDM at frequency v has been focused to frequency v_f by means of the transformation *T(v)*. Therefore all the different frequency contributions can be coherently combined to obtain the matrix

$$R^{\alpha}_{xx[*]} = \sum_{v\in\Omega_f} g(v) S^{\alpha}_{yy[*]}(v) \tag{2.80}$$

where *g(v)* is a complex weight function to be suitably chosen and the discrete sum ranges over J frequency values belonging to Ω_f. By substituting (2.79) into (2.80) we get the following:

$$R^{\alpha}_{xx[*]} = A(\theta, v_f + v_0) R^{\alpha}_{ss[*]}(v) A^{H[*]}(\theta, v_f + v_0)$$

where *R* is the DxD matrix that combines all the focused SOI contributions.

If the array manifold is known and unambiguous for each *v*, the matrix *A* has full rank D. Then under the assumption that *R* also has full rank D which must be assured by an appropriate choice of *g(v)*, it results that *R* has rank D<M and hence any signal subspace method can be applied to obtain high-resolution estimates of the DOAs.

$$R_{xx[*]}^{\alpha} = USV^H = \begin{pmatrix} U_s & U_0 \\ & \end{pmatrix} \begin{pmatrix} S_s & 0 \\ 0 & S_0 \end{pmatrix} \begin{bmatrix} V_S^H \\ V_0^H \end{bmatrix}$$

This denotes the SVD decomposition of *R*. where *U* and *V* are unitary matrices and *S* is a real non-negative diagonal matrix. Since *R* has rank D and accounting for (15) it turns out that $A^H(\theta, v_f + v_0)U_0 = 0$ and hence the maxima of the spatial spectrum

$$P(\theta) = \frac{1}{a^H(\theta, v_f + v_0)U_0 U_0^H a(\theta, v_f + v_0)}$$

It can be utilized to determine the unknown DOAs.

Conventional focusing methods are able to work in a multipath environment. This property is also extended to the cyclic case. S_{ss} is singular at every frequency, the frequency averaging performed in (16) to obtain R_{ss} removes the singularity. If we denote Δv as the spacing between two consecutive frequency values, for $\Delta v << 1$ and $\Omega_f \approx \Omega$ we can write:

$$R_{ss[*]}^{\alpha} = \frac{1}{\Delta v} \sum_{v \in \Omega_{f_f}} g(v) S_{ss[*]}^{\alpha}(v) \Delta v$$

$$R_{ss[*]}^{\alpha} \approx \frac{1}{\Delta v} \int_{\Omega_f} g(v) S_{ss[*]}^{\alpha}(v) \Delta v$$

Hence choosing $g(v) = e^{j2\pi v m_0}$, we get

$$R_{ss[*]}^{\alpha} \approx \frac{1}{\Delta v} R_{ss[*]}^{\alpha}(m_0)$$

which has rank D and m_0 is suitably chosen according to the signal characteristics. Since this approach is a sub space based approach and it requires estimation of D. Generally this can be done using AIC or MDL algorithms but there is an absence of noise term in R_{xx} and therefore other techniques will be required.

Practical Focusing Strategies

Authors suggest that T should be estimated either using the Wang & Kaveh's approach [6] for single group case or use Hung & Kaveh's approach [7-8]for multigroup case. In both cases preliminary estimates of DOA are required.

A different approach which does not require preliminary DOA estimate is the class of spatial resampling method of Krolik and Swingler [17-18]. Their technique exploits the separability of the array manifold with respect to the unknown DOA and the frequency. By exploiting the series of expansion of a plane wave in polar coordinates, it can be shown [32] that the k[th] element of the steering vector $\{a(\theta_d, v+v_0)\}$ in $X(v)_N \approx A(\theta, v+v_0)S(v)_N + I(v)_N$ can be expressed as:

$$\{a(\theta_d, v+v_0)\}k = e^{j2\pi(v+v_0)f_s(r_k/c)\cos(\phi_k-\theta_d)}$$

$$\{a(\theta_d, v+v_0)\}k = \sum_{n=-\infty}^{\infty} j^n J_n[2\pi(v+v_0)f_s \frac{r_k}{c}] \times e^{-jn\phi_k} e^{jn\theta_d}$$

where (r_k, ϕ_k) are polar coordinates of the k[th] sensor and J_n is the n[th] order Bessel function of the first kind. The function J decays faster than exponentially, we can truncate the infinite sum as:

$$\{a(\theta_d, v+v_0)\}k \approx \sum_{n=-n\varepsilon}^{n_\varepsilon} j^n J_n[2\pi(v+v_0)f_s \frac{r_k}{c}] \times e^{-jn\phi_k} e^{jn\theta_d} \quad (2.81)$$

which allows one to approximately express the steering vector $\{a(\theta_d, v+v_0)\}$ in separable form as:

$$\{a(\theta_d, v+v_0)\} \approx G(v+v_0)w(\theta_d) \qquad (2.82)$$

where G is an Mx2N_e matrix whose (k,n)th element is $\{G(v+v_0)\}kn = j^n J_n[2\pi(v+v_0)f_s \frac{r_k}{c}] \times e^{-jn\phi_k}$ for k =1,..M,

where as w is the 2n_e column vector whose nth element is $\{w(\theta_d)\}n = e^{jn\theta_d}$

Therefore by substitution of (2.82), the focusing condition reduces to

$$T(v)G(v+v_0) = G(v_f + v_0)$$

Now *T(v)* can be solved without requiring any preliminary DOA estimate using *G* matrices. This method is referred as CAMI.

Note: Number of terms in summation increases with increasing size of the array and increasing focusing bandwidth.

1. For a fixed number of M it puts a limitation on the maximum number of terms.
2. The accuracy will degrade as the size of the array is increased.

Since focusing transformations basically exploit the spatial properties of the array, it is clear that almost every focusing technique can be extended with minor modification to the cyclic one.

Summary

This approach is based on previously reviewed two approaches of Wang & Kaveh [2,20-21] and Krolik [17]. They proposed ways to compute DOA using above mentioned approach. Authors specify another focusing matrix for use in their algorithm. One drawback of their work is that they are restricting their signal to cyclostationary signals which is not a good assumption. We are looking more a general approach and hence we are not pursuing this approach at this time.

2.12 Fabrizio Sellone,"Robust Auto-Focusing Wide-band DOA Estimation (Review) [31]

Authors propose a new way of designing focusing matrices which has some robustness. It does not need initial estimates of DOA. He also claims that computational requirement is also reduced when compared to RSS and SST approach [32,33].

Chapter 3

Simulation

Simulation is the best way to verify the validity of any algorithm. Simulation program first of all requires generation of appropriate data which is reasonable for the algorithm under the test. The input data should also have any assumption or initial condition that are necessary and should be spelled out. Algorithm then needs to be coded properly and debugged. Obtained results should be analyzed to check validity. Results can then be shown in tabular form and or graphically in one dimensional or three dimensional forms. Plotting of results also requires appropriate plotting mechanism. In this work it is best to test our algorithm in MATLAB which provides mechanism to generate data, commands to execute any algorithm at various levels. It also has various ways to plot results from one dimensional to three dimensional plots.

3.1 DOA estimation for narrow-band sources

A Uniform Linear Array of sixteen equally spaced Omni-directional sensors was used as shown in Figure 3.1.

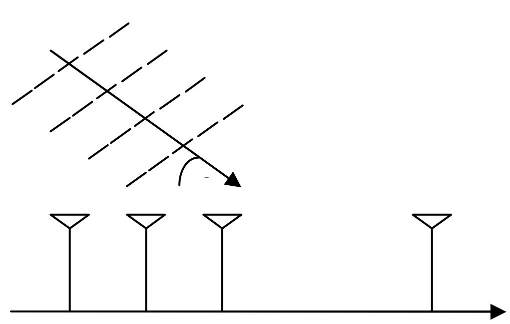

Figure 3.1 : Uniform Linear Array of 16 elements

The spacing between sensors is $\frac{c}{2f_o}$, where c is the velocity of propagation and f_o is the central frequency. The signal sample size is 4096. The MATLAB simulation program first of all generates random data using two sources located at 9 and 12 degrees. It assumes 16 sensors located at equidistance in an Uniform Linear Array. Gaussian noise has been added to the signals. The signal to noise ratio is 10 dB. MATLAB simulation using above mentioned data generation was performed Figure 3.2 shows DOA for two sources in narrow-band.

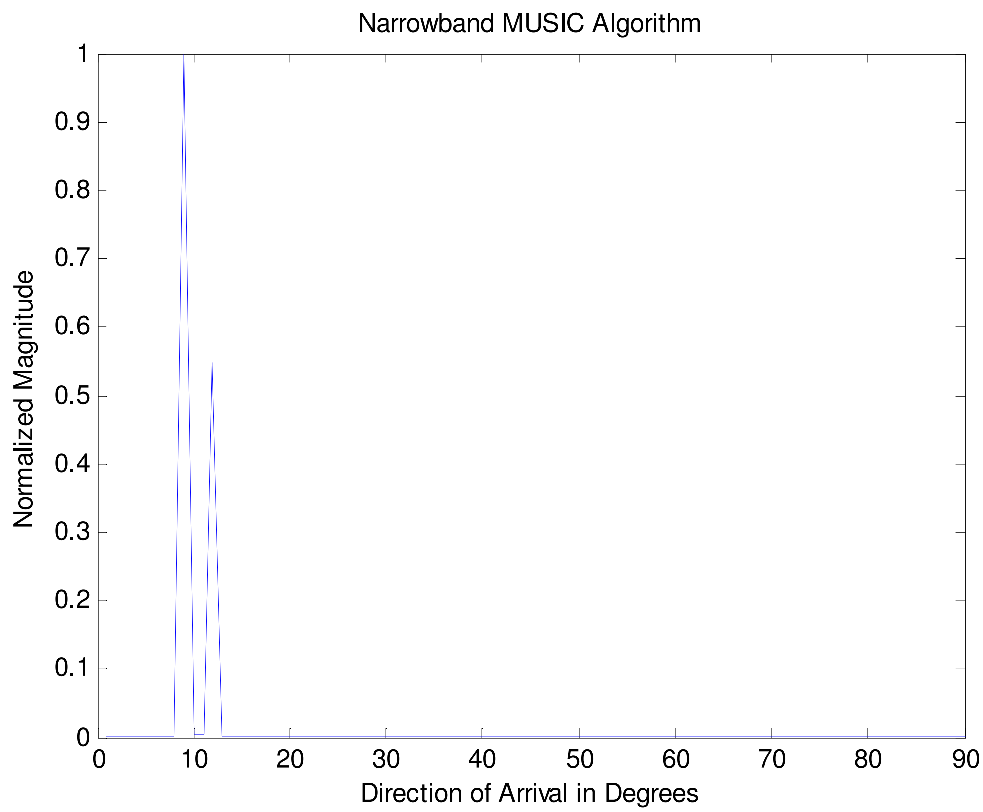

Figure 3.2: DOA estimation for two narrow-band sources

3.2 DOA estimation for wide-band sources

There are number of wide-band DOA algorithms that have been discussed in the previous chapter. Most popular wide-band DOA algorithms are signal subspace based algorithms. These subspace algorithms can further be divided into incoherent and coherent signal. MATLAB programs have been written for both these approaches.

Wang and Kaveh [6, 7] proposed Coherent Signal-Subspace (CSS) method for detection of DOA for wide-band sources. This technique separates the wide frequency band into narrow-band components. The data set for this algorithm is divided into 64 segments and each segment contains 64 samples. It also assumes an Uniform Linear Array of 16 sensors. The time domain samples will be transformed into frequency domain by applying a 64 point FFT to each of the 64 segments. The Coherent Signal Subspace approach proposed by Wang & Kaveh [6] follows following computational steps.

1. Compute 64 sets of 64-point FFT
2. Compute 33 Covariance matrices (16 by 16)
3. Computation of initial DOA estimate using MUSIC algorithm [6]
4. Computation of 33 Focusing matrices
5. Computation of Focus matrix
6. Computation of number of sources
7. Separation of Signal & Noise subspaces
8. Compute DOA using MUSIC algorithm

In order to demonstrate the performance of the DOA algorithm for wide-band signals, an Uniform Linear Array of sixteen equally spaced Omni-directional sensors was used. The spacing between sensors is $\frac{c}{2f_o}$, where c is the velocity of propagation and

f_o is the central frequency. Two wide-band sources at DOA θ_1 and θ_2 were assumed. The signals are stationary zero mean band pass white Gaussian processes with central frequency $f_o = 100Hz$ and bandwidth $B = 40Hz$. The array noise is also stationary zero mean band pass with the same pass band as the signal with a SNR of 10dB at each sensor. Source signals and the noise are random processes with a bandwidth of 40 Hz. The sampling frequency is chosen to be 300 Hz. The signal will be observed over a period of T_o seconds and T_o will be divided into $k = 64$ segments. On each of those segments, the array output along with corresponding noise will be decomposed into narrow-band components using a fast Fourier transform. The total number of samples taken by each sensor will be 4096.

The MATLAB simulation program first of all generates random data using two sources located at 9 and 12 degrees. It assumes 16 sensors located at equidistance in an Uniform Linear Array. Gaussian noise has been added to the signals. The signal to noise ratio is 10 dB. The data set consists of 64 segments of 64 sets of data [6]. The data is bandpass filtered using a Butterworth filter and has wide-band characteristics of containing frequencies from 80 Hz to 120 Hz. MATLAB simulation using above mentioned data generation was performed Figure 3.3 shows initial estimate of DOA. It can be seen that it is sort of pointing towards an angle of 10.5 degrees. Using this initial estimate, simulation continues to perform other operations and was able to show two peaks at angles of 9 and 12 degrees very clearly. These two DOA peaks are shown in Figure 3.4.

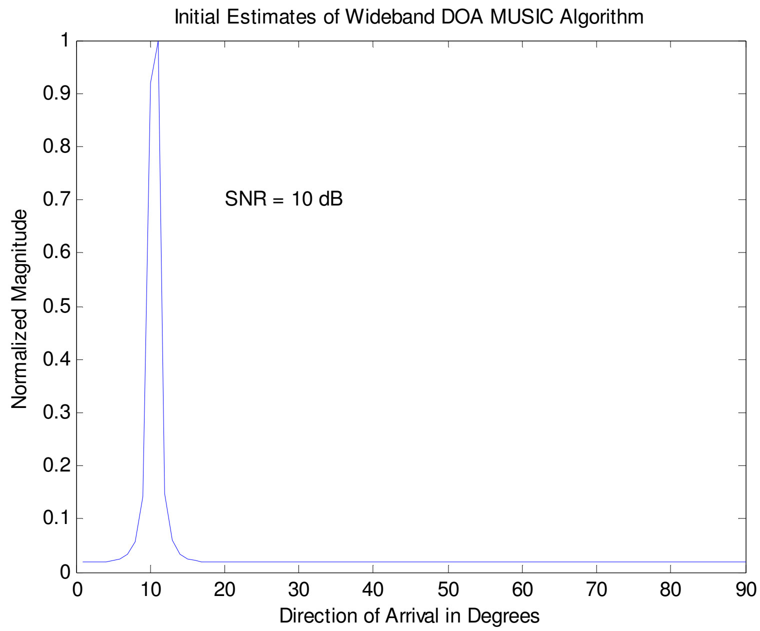

Figure 3.3: Initial estimate of wide-band DOA CSS algorithm

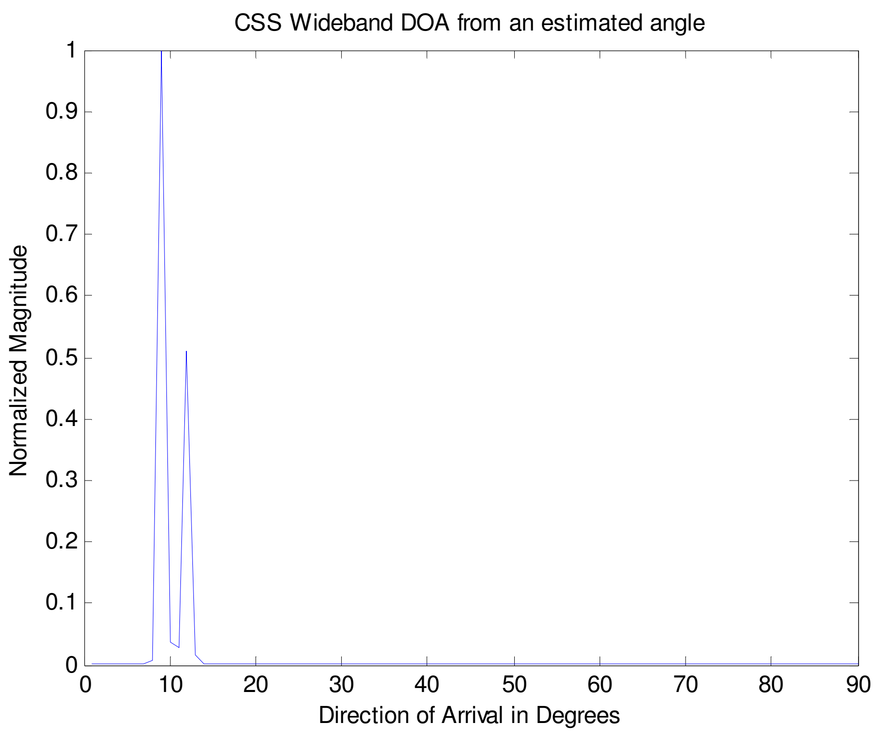

Figure 3.4: Final estimate of wide-band DOA CSS algorithm

Simulation program which was generated earlier for the Coherent Signal Subspace method has been modified to incorporate incoherent signal subspace method proposed by Wax [5]. The simulation program uses similar input data as was used in the case of CSS. Figure 3.5 shows DOA plots showing two peaks at 9 and 12 degrees as was obtained earlier. The only difficult part is that this approach uses lot more computation as MUSIC algorithm was used for each frequency and then it was averaged.

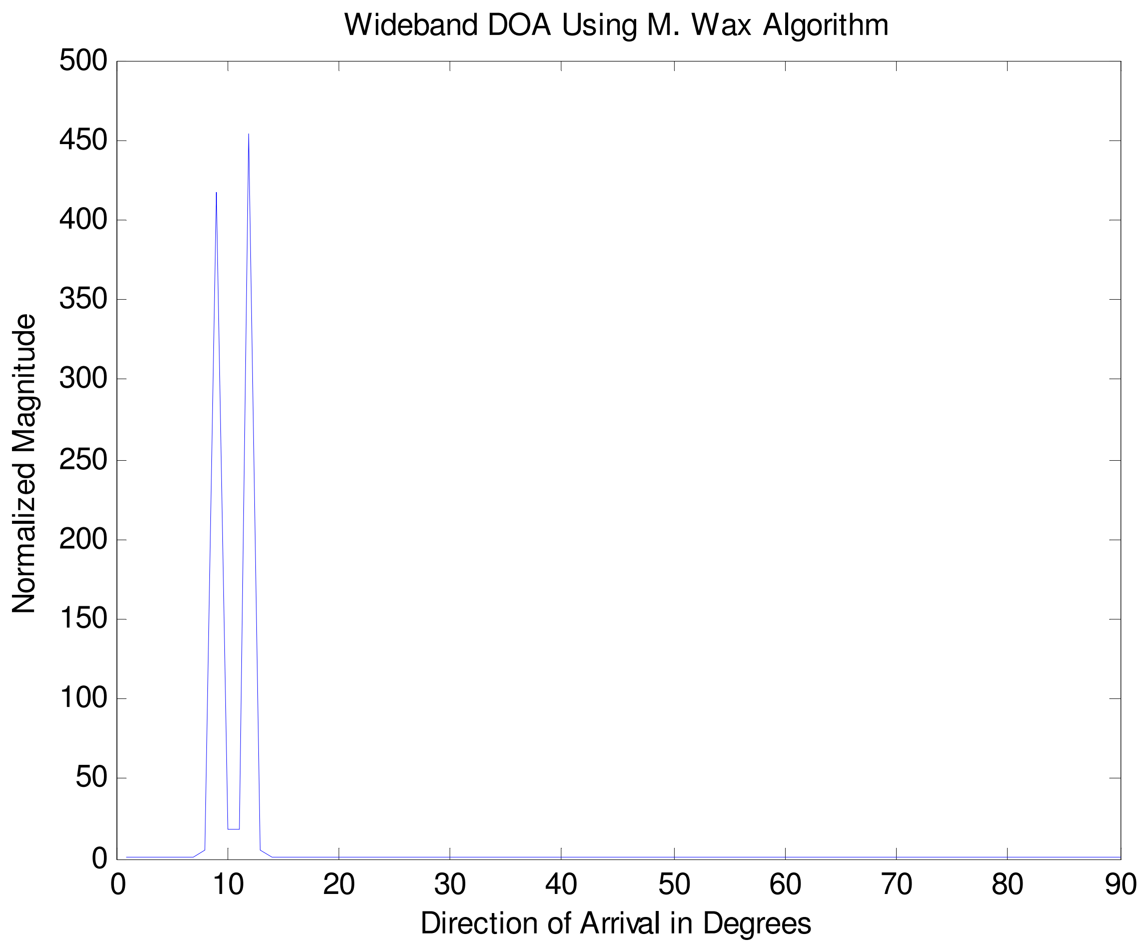

Figure 3.5 : DOA estimate using incoherent signal subspace method

Simulation program which was generated earlier for the Coherent Signal Subspace method has been modified to incorporate bilinear transformation algorithm as proposed by Shaw [8]. The simulation program uses similar input data as was used in the case of CSS. Figure 3.6 shows DOA plots showing two peaks at 9 and 12 degrees as was obtained earlier.

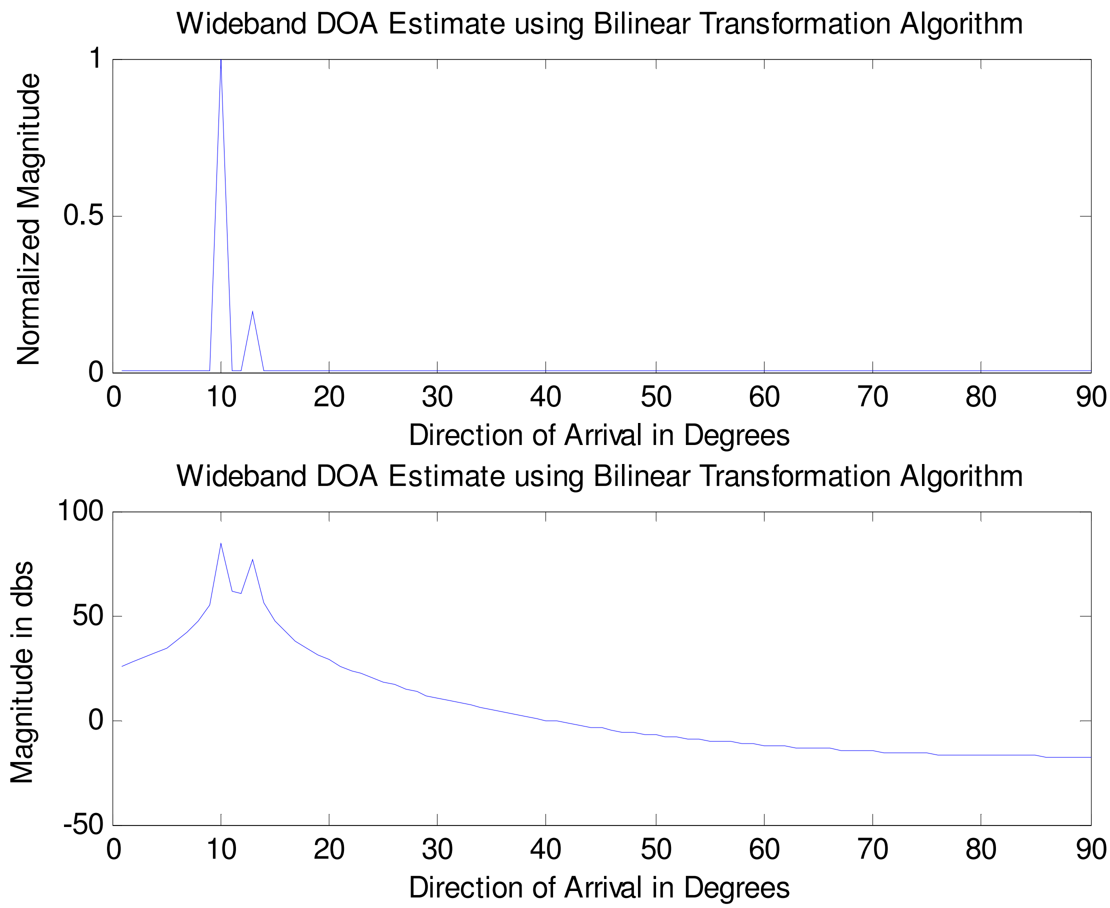

Figure 3.6: DOA estimation for wide-band sources using Bilinear Transformation

Simulation program which was generated earlier for the Coherent Signal Subspace method has been modified to incorporate BASS-ALE algorithm as proposed by Buckley [41]. The simulation program uses similar input data as was used in the case of CSS. Figure 3.7 and 3.8 shows DOA plots showing two peaks at 9 and 12 degrees as was obtained earlier. The only difficult part is that this approach uses lot more computation as MUSIC algorithm was used for each frequency and then it was averaged.

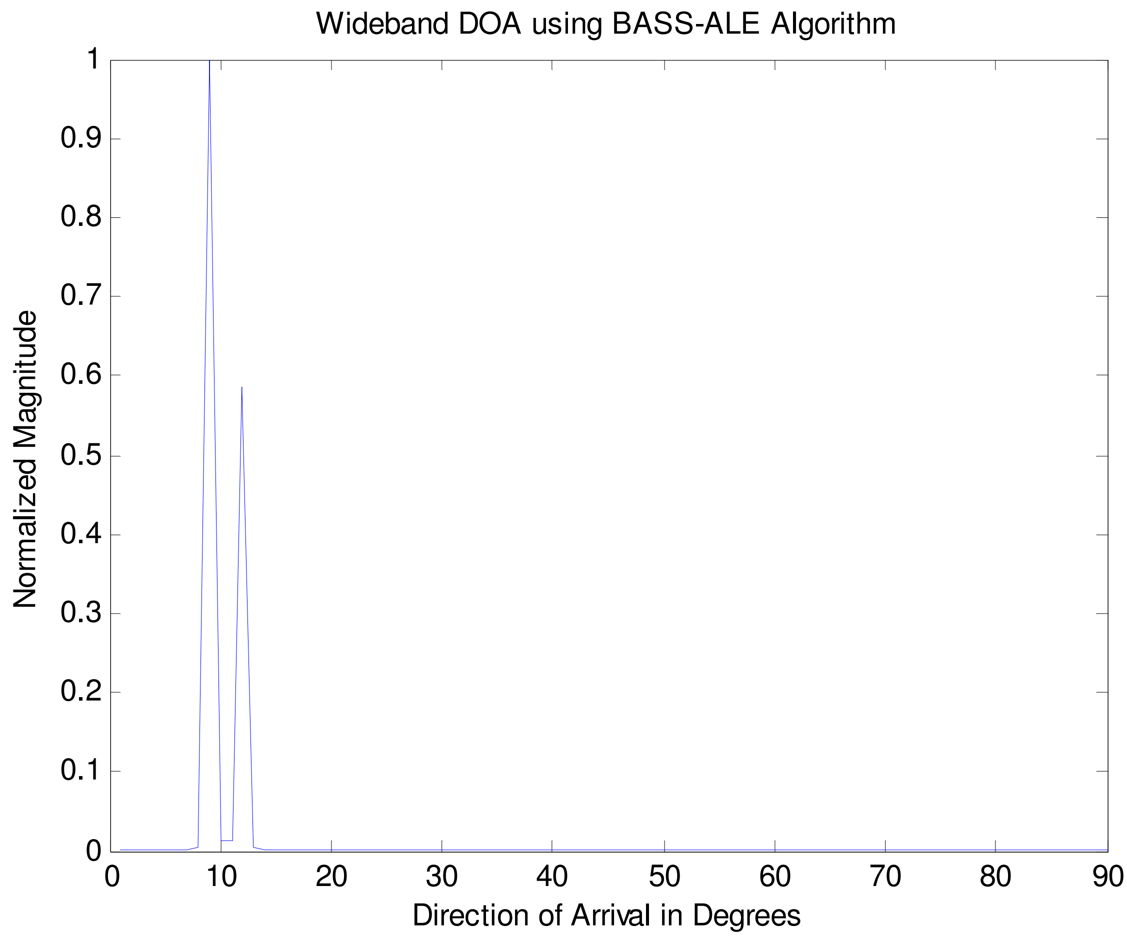

Figure 3.7 : DOA estimation for wide-band sources using BASS-ALE algorithm

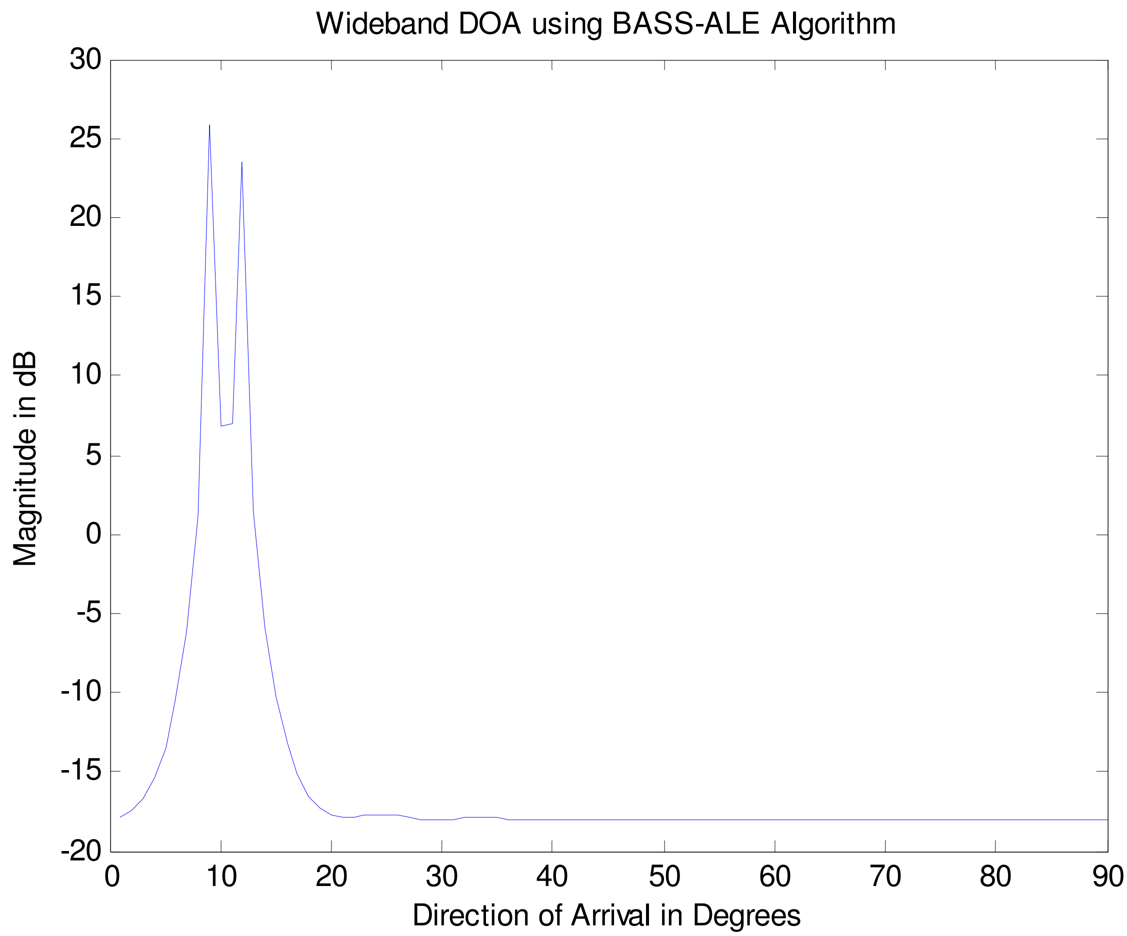

Figure 3.8: DOA estimation for wide-band sources using BASS-ALE algorithm

Simulation program which was generated earlier for the Coherent Signal Subspace method has been modified to incorporate wide-band DOA in time domain. We inserted 16 delays in each sensor data and formed a data vector of 256. This then translated into a covariance matrix of size 256*256. Eigendecomposition was again on a matrix size of 256 giving 256 eigenvalues. MUSIC algorithm was then applied to get similar DOA plots and two peaks.

Chapter 4

Analysis of Computational Requirements of Wide-band DOA Algorithms

Passive detection of objects has been studied for more than 20 years [1-14]. This approach evades detection by others and has many applications. There are more than thirty publications for wide-band detection of Direction of Arrival (DOA) algorithms which are available in the literature. These algorithms are generally presented in a very complex or condense form, which are not easily understandable for people who are outside that narrow field. It is not known which class of techniques would be appropriate for implementing them in hardware and would be useful for real time applications. One has to cut through all the mathematics and convert algorithms into simple arithmetic operations before architecture can be visualized. There is a need to bridge a gap between the design of computer hardware especially special purpose parallel architectures and available algorithms for various interdisciplinary problems.

This work is the first step in sorting out which algorithm is appropriate for further study and for its hardware implementation for real time applications. Wide-band DOA algorithms available in the literature have been reviewed in Chapters 1 and 2. They are also simulated in MATLAB and their results are provided in Chapter 3. These wide-band algorithms can be divided into following categories:

- Modal Decomposition Signal Subspace (In-coherent signal subspace)
- Coherent Signal Subspace Method
- Rotational Signal Subspace Method

- Wide-band DOA detection using focusing matrices
- Wide-band DOA detection using beamforming approaches
- Combination of beamforming and focusing approaches restricted to chirp or cyclostationary signals.
- Use of ARMA model and Bayesian approaches
- Use of maximum likelihood algorithms
- Bilinear Transformation Method
- Wide-band DOA in time domain

Signal subspace approaches are very popular for computing DOA for both narrow-band and wide-band sources. One problem with them is that they may not be optimal but produce computationally efficient algorithms. Signal subspace based approaches are further subdivided into coherent based and incoherent based signal subspace approaches. Incoherent signal subspace approach decomposes signals into individual narrow-band frequencies and then combined them to produce final results. They are computationally expensive and are unable to resolve coherent sources [2, 18-19]. Some of the subspace approaches require initial estimates. If the initial estimate is not accurate then final DOA will also not be accurate or may have some issues with it. There will also be issues with bias and variances.

In order to implement DOA algorithm in hardware for real time applications, it is important to use a computationally efficient algorithm. One approach is to evaluate computational requirements of currently available wide-band DOA algorithms and select one of them for hardware implementation. An interdisciplinary research is performed for the development of Parallel Architectures suitable for real time wide-band digital receiver applications.

Development of wide-band DOA algorithm started with the incoherent signal subspace approach which is a brute force extension of narrow-band case. This approach works but computationally expensive. Wang & Kaveh [6] proposed a Coherent Signal Subspace approach which creates a focusing matrix using a single frequency. This has a simple transformation scheme and requires initial estimates of DOA. Results are reasonable and produce two peaks. Hung & Kaveh [7] extended their previous work with a promise of statistically better results. They introduce concept of rotational signal subspace making it more complex or accurate focusing matrix. The cost to their approach is that they added a computational step of singular value decomposition. There may be little improvement in accuracy of results with additional computational cost. Their work was simulated in MATLAB and results were obtained and were shown in Chapter 3.

Shaw [8] extended work of Wang & Kaveh [6] and Hung & Kaveh [7] and introduced a bilinear transformation approach with certain approximation. Advantage of their work is that it does not require initial estimate of DOA. Their work was simulated in MATLAB and results were obtained. Their algorithm is very sensitive to certain assumptions and parameter values which make it unattractive for hardware implementation.

The algorithm proposed by Ta-Sung Lee [19] and he decomposes data using bandpass filters into J frequency beams. It performs beamspace transformation and computes weights using least square method. It then computes beamspace data matrix and focuses on single reference frequency which would be something similar to CSSM method. It performs transformation into K beamspaces and forms beamspace data matrix.

This beamspace data matrix then focused on a single reference frequency out of J frequency bands. The design of beamspace data matrix is described which requires first design of beamforming matrices. They are again designed in a least square sense. The problem is then reduced to beamspace data correlation and noise matrices. Authors then apply their own derived root MUSIC algorithm which could be substituted with the MUSIC algorithm. The algorithm does not require any preliminary DOA estimates.

This DOA estimator is suboptimum according to the author. It would also result in degradation in estimation accuracy at low SNR. Even with the true DOA, the DOA estimates may not be exactly identical as signal roots may not lay on the unit circle. The proposed parallelized estimator behaves as a mixture of the root-form and spectral-form estimator. This approach provides an excellent alternative to CSSM but frequency decomposition and calculations of weight and beamspace data matrices. There may be some alternative ways to calculate weight and compute data beamspace matrices. In the end it again uses something similar to MUSIC algorithm to compute DOA. This algorithm was not implemented due to lack of information and we are looking into filling those gaps. The previously MATLAB program can easily be extended to accommodate this algorithm.

We have investigated work of Krolik and its extension [17-18]. Our program did not give two peaks as expected. Problems could be in missing information regarding transformation matrix that could lead to not getting appropriate result. Moreover their paper uses random processes as data which may contribute to getting two peaks in their case and not in our case. This technique requires computation for each steering direction

θ increasing the computation requirements but eliminates the selection of initial focusing angle.

The second of Krolik work uses interpolation and then decimation of input data and then computation of covariance matrix. Technique looks good some work need to be done to resolve value of B, K and L which are also assumed. Selection of B, K, and L may be tricky and this approach then will not be useful in a generalized case. No attempt was made to run MATLAB program due to above reasons. One advantage of this approach is that it does not require preliminary DOA estimates and iterations.

The work of Yoon [20, 38-40] requires that preliminary estimate of the DOA that could be one of the angles in the estimation and number of sources need also be estimated. There is missing information in this work so no further study is planned at this time for this work. They have also named this technique as Test of Orthogonality of Projected Spaces (TOPS).

Four papers were reviewed from Tuan Do-Hong and Peter Russer [21-25] and they are similar in their work with notational changes in equations. That makes it difficult to separate their work. Simulation results are also somewhat similar. They have not provided a step by step mechanism for their algorithm. The bright part of their work is that it does not require preliminary estimate of the DOA. It does require array geometry and guessing of common frequency so their algorithm could be based on it.

Darren Ward et al [26, 29-30] produced a group of three papers and performs filter and sum beamforming in frequency invariant fashion. They proposed a design of FIR filter and then computed covariance matrices. Three FIB's were designed (using FIR filters with 201 taps) to be frequency invariant over the normalized frequency band [0.2, 0.4] and to cover the spatial sector {80 to 100 degrees}. The corresponding beampattern with center frequency of 0.3 were calculated. This approach looks very attractive and needs some more work. It is still not clear as how to design these FIR filters which require more calculation and experimentation. Filter specifications were not provided. We need to find a way to design these FIR filter coefficients which needs to be more focused and find a way to compute them.

Gelli and Izzo [39] work is based on previously reviewed two approaches of Wang & Kaveh [2,20-21] and Krolik [17]. They proposed ways to compute DOA using above mentioned approach. Authors specify another focusing matrix for use in their algorithm. One drawback of their work is that they are restricting their signal to cyclostationary signals which is not a good assumption. We are looking more a general approach and hence we are not pursuing this approach at this time.

Sellone [31] proposed a new way of designing focusing matrices which has some robustness. It does not need initial estimates of DOA. He also claims that computational requirement is also reduced when compared to RSS and SST approach [32,33].

Wide-band DOA algorithms reviewed in Chapter 2 and simulated in Chapter 3 are compiled together. This work evaluated computational requirement for various DOA algorithms using common data and assumptions. Computational requirements for these algorithms are presented in Table 4.1. A review and challenges of these algorithms are summarized in Table 4.2.

These wide-band DOA algorithms use following computational steps:

- Generation of wide-band signals
- Conversion of time domain signals into frequency domain via FFT.
- Computation of covariance matrices in frequency domain.

- Computation of eigenvalues and eigenvectors
- Computation of initial estimates of number of sources
- Computation of initial estimates of DOA
- Computation of transformation matrices and focusing on central frequency
- Computation of eigenvalues and eigenvectors
- Computation of number of sources
- Computation of final estimates of DOA

In this work we have identified common computational steps and they can be implemented in hardware. Most of these algorithms follow similar computational steps with some variations. These variations can be adopted if we use re-configurable approach and implement these algorithms in FPGAs. Chapter 5 presents hardware implementation of these algorithms.

NAME OF THE TECHNIQUE	COHERENT SIGNAL SUBSPACE	ROTATIONAL SIGNAL SUBSPACE	BINLINEAR TRANSFORMATION	BEAMFORMING INVARIANCE	STEERED COVARIANCE	SPATIAL RESAMPLING	TOPS- DOA ESTIMATOR	FDFIB-BEAMFORMER
Authors	Wang & Kaveh [6]	Hung & Kaveh [7]	Shaw [8]	Ta-Sung Lee [19]	Krolik & Swingler [17]	Krolik & Swingler [18]	Yoon [20,38,40]	Do-Hong, Russer [21,23-24]
Computational steps	Compute 64 point FFT	Compute 64 point FFT	Compute 64 point FFT	Beamspace transformation into J frequency bins	Separate frequencies using FFT	Insert K_n -1 zeros Perform interpolation	FFT for each block (J)	Compute FFT
	Compute 33 Covariance matrices	Compute 33 Covariance matrices	Compute 33 Covariance matrices	Compute weight using Least Square fit method	Cross spectral density matrix **K** for each frequency	Convolve with the low pass FIR filter	Compute sensor output for pre-selected frequencies	J beamforming network operation
	initial DOA estimate using MUSIC	initial DOA estimate using MUSIC		J Beamspace data matrix focused on single ref. frequency	Steered cov. matrix **R** for each angle	Perform decimation operation	K Covariance matrices	Compute covariance matrix
	Computation of Focusing matrices	Computation of Focusing matrices	Computation of Focusing matrices	Beamspace data matrix is designed in least square sense	Inverse of steered covariance matrix **R** for each angle	Compute covariance matrix for B samples	Compute eigenvalues & eigenvectors for each frequency	Eigen-decomposition
	Eigen-decomposition	Eigen-decomposition	Eigen-decomposition	Eigen-decomposition	Spatial power spectral estimate **Z** for each angle	Operations are performed for each frequency	Define signal & noise subspace for each frequency	Number of sources
	Number of sources	Number of sources	Number of sources	Number of sources	Determine peak positions of the power	Eigen-decomposition	Compute rotational signal subspace focusing matrix	Perform MUSIC on selected frequency
	Compute DOA using MUSIC	Compute DOA using MUSIC	Compute DOA using MUSIC	Proposes a root MUSIC algorithm for DOA		Number of sources	Eigen-decomposition	
						Perform MUSIC to compute DOA	Number of sources	
							Perform MUSIC to compute DOA	

Table 4.1: Computational requirements of various wideband DOA algorithms

NAME OF THE TECHNIQUE	TIME DOMAIN FIB USING FIR	CYCLOSTATIONARY BASED COHERENT METHOD
Authors	Ward [26,29-30]	Gelli & Izzo [39]
Computational steps	Design FIR filters (primary & secondary)	Compute 64 point FFT
	Form J beamforming networks	Compute Covariance matrices
	Covariance matrix	Focusing matrices
	Eigen-decomposition	Find weight function
	Number of sources	R matrix
	Perform MUSIC	Singular value decomposition
		Eigen-decomposition
		Number of sources
		Perform MUSIC

Table 4.1: Computational requirements of various wideband DOA algorithms (continued from the previous page)

Name	Authors	Review	Challenges
Coherent signal subspace method	Wang & Kaveh	Utilizes linear transformation using focusing matrices Coherent sources can be resolved, superior detection and accuracy	Vulnerable to source location bias Initial DOA estimates are required
Rotational signal subspace method	Hung and Kaveh	Uses rotational transformation matrices Improves bias and performance	Additional computation steps (SVD) Initial DOA estimates are required
Coherent interpolation	Bienvenu	Interpolates the wavefield to emulate a spatial sampling which is adopted to individual frequency	It is applicable to linear arrays and error increases with the increase in the spatial frequency. It will limit the range of DOA.
Efficient Wide-band Source Localization Using Beamforming Invariance Technique	Lee	Forms J beamforming matrices Computes weight using least square. Focusses on single frequency. Uses root MUSIC Utilizes parallel algorithm to offset heavy polynomial computations.	DOA estimator is suboptimum Iterative Too many computations and transformations from one computational step to another. Difficult to implement.
Signal subspace DOA estimator	Yoon, Kaplan, McClellan	Assumes that the J frequency bands of the sources are known Computes FFT for J blocks, compute K covariance matrices Compute eigenvalues & eigenvectors. Define signal & noise subspaces, compute focusing matrices Compute DOA	Initial DOA estimates are required There is some missing information in this work
Cyclostationary based coherent method	Gelli & Izzo	Uses focusing, transformation matrices, weight function, and SVD	Restricts to cyclostationary signals
Spatial Resampling	Krolik & Swingler	Perform operation for each frequency bin (33) Insert zeros to interpolate Low pass filter using convolution Decimate signals Form covariance matrix Compute DOA with MUSIC	Design of interpolator filter. Stability of various parameters and their selection Processing is done for each frequency bin
Steered Covariance matrices	Krolik & Swingler	Performs FFT on input data. Computes covariance matrix for each frequency. Computes steered covariance matrix for each angle. Inverse all steered covariance matrices. Finds peak of the power. No eigenvalues or eigenvectors	33 different covariance matrices 90 different steered covariance matrices (2 matrix multiplication for each angles) 90 inverse matrices Compute maximum power Extensive computations for each steered angle
FDFIB Beamformer	Do-Hong & Russer	Compute FFT Form J beamforming networks Compute covariance matrices Perform MUSIC	Requires array geometry and guessing of common frequency. Computational steps are not very clear.

Table 4.2: Reviews and challenges of various wideband DOA algorithms

Chapter 5

Hardware Implementation

Various wide-band DOA algorithms available in the literature have been investigated. Comparative studies were performed from the computational requirement point of view. The goal of this study to select an algorithm or class of algorithm which will be suitable for further study and will be a candidate for hardware implementation for real time application. It is clear from this study that there are common computational modules in these algorithms and most of them use following computational steps:

- Generation of wide-band signals
- Conversion of time domain signals into frequency domain via FFT.
- Computation of covariance matrices in frequency domain.
- Computation of eigenvalues and eigenvectors
- Computation of initial estimates of number of sources
- Computation of initial estimates of DOA
- Computation of transformation matrices and focusing on central frequency
- Computation of eigenvalues and eigenvectors
- Computation of number of sources
- Computation of final estimates of DOA

It can be seen that these computation requirement would require special purpose hardware in order that it would get executed in real time. There would also be a need to exploit parallel processing to speed up the computation process. There are number of ways to build hardware for these applications and some of the options are described as:

- Commercially available Digital Signal Processor
- Field Programmable Gate Arrays (FPGAs)
- Application Specific Integrated Circuits (ASIC)

Use of commercially available DSP chips is a viable way to design DSP hardware. These DSP devices can either be programmed in C or Assembly language. DSPs have general purpose instruction set which are mapped to the architecture in an optimal. They may not be suitable for a specific application. Modern DSPs offer on-chip multiply accumulate unit, multiple memories, specialized instruction set for signal processing applications. Generally support software is available from the DSP manufacturer. Some times there is access to design libraries and also design boards are also available. One drawback with this approach is that the algorithms are executed in sequential fashion and programs are stored in the memory along with the data. This could create a bottleneck in terms of speed of execution. DSPs are also clocked with certain clock frequency. This approach provides a viable way to have a proof of concept hardware. In order to gain more speed parallel processing can be exploited and multiple chips can be used.

Second option of developing application specific hardware is to use FPGAs. They contain thousands of look up tables to store logic, hundreds of I/O blocks, on-chip memory, on-chip multiplier and very flexible programmable multi-standard I/O pins. There are number of manufacturer namely Actel, Alterra, Xilinx and others who provides these devices along with synthesis and design tools. Different manufacturers use different technologies and operational philosophies for their devices. They provide their own proprietary design tools. The choice of use of a particular FPGA device family and manufacturer depends on availability of experimental devices, design tools, training facilities, personal preferences, application dependency, customer specification and most of all available expertise in an organization. There is a steep learning curve for these devices. Switching design from one family to another family of FPGA devices may again require additional training and gaining experience.

FPGAs are capable of implementing high performance DSP algorithm as they can provide multiple Giga Operation Per Second (GOPS). FPGAs are flexible, programmable and can be re-configured infinite number of times. Another advantages of using FPGAs is that they can be used to implement true parallel processing. A parallel algorithm can be implemented easily in FPGAs. This can help to gain speed and cut down the execution. This kind of facility or flexibility is unavailable in commercially available DSPs as we have to worry about scheduling of the parallel tasks. The operating system should also be capable of handling parallel processing operations and be capable of handling scheduling and load balancing to truly achieve benefits of parallel processing. This is not a problem in FPGAs as custom parallel algorithms can be configured and mapped on the FPGAs. Various designs containing parallel algorithms can be experimented with the FPGAs to achieve high speed operations and optimize need for FPGA resources. This kind of approach can totally avoid need of schedulers and operating system with parallel processing capabilities. There FPGAs offer a good rapid prototyping option for computer intensive DOA estimation applications.

Third option of developing special purpose hardware is with the use of Application Specific Integrated Circuits (ASIC). This approach is capable of producing design which are custom designed for speed and area. They could be hand crafted to achieve desired goal. Drawbacks for this approach are high design cost, very high level of design expertise, and one time programmability. Other drawback would be that each ASIC would then need to be handcrafted for each application. Moreover device needs to be manufactured by the manufacturer themselves. This will take away flexibility of user programmability and ability to change designs at the last minute. Moreover this option is a very expensive one compared to the previous one. This approach is not pursued in this work.

Hardware Design using Digital Signal Processors

A general purpose DSP was selected as an appropriate platform for implementation because of its ease of programming. Also, the DSP is the best suited for matrix and floating points computations. The DSP used in our implementation is a DIOPSIS™ 740 by Atmel. DIOPSIS™ 740 (D740) is a high performance dual-core processing platform for real time applications. The D740 is optimally suited for floating point applications complex domain computations. The ARM7TDMI embedded microcontroller core is equipped with several peripherals and on-chip memories. The main components of the DSP subsystem are the core processor, the on-chip memories and the interfaces to and from the ARM subsystem. The mAgic DSP has four on-chip memory blocks: the program memory, the data memory, the data buffer, and the dual ported memory shared with the ARM processor. An external memory interface multiplexes the data accesses and the program accesses to and from the external memory. The program memory stores the Very Long Instruction Word (VLIW) program to be executed by mAgic.

Multicore Application Development Environment (MADE) is an Integrated Development Environment (IDE) that can be used to develop D740 applications [43-45]. It includes the C compilers for both ARM and mAgic DSP based on GNU compiling tools named as GCC. The magic C compiler contains a DSP library composed of over 220 functions such as Fast Fourier Transform and IIR and FIR filter creation. The JTST board [43-45] is low-cost, stand-alone, general-purpose module that provides the appropriate resources in order to evaluate D740 DSP performances in a wide range of applications. The JTST board provides several memories and other peripherals.

DIOPSIS™ 740 (DSP) was used as an implementation vehicle for the Coherent Signal-Subspace algorithm proposed by Wang & Kaveh [6]. This technique separates the wide frequency band into narrowband components. The data set for this algorithm is divided into 64 segments and each segment contains 64 samples. It also assumes a uniform linear array of 16 sensors. The Coherent Signal Subspace approach proposed by Wang & Kaveh [6] follows following computational steps.

- Compute 64 sets of 64-point FFT
- Compute 33 Covariance matrices (16 by 16)
- Computation of initial DOA estimate using MUSIC algorithm [14]
- Computation of Focus matrix
- Computation of number of sources
- Separation of Signal & Noise subspaces
- Compute DOA using MUSIC algorithm

The covariance matrices for all the frequency components are estimated and combined to form a single focused covariance matrix. The narrowband MUSIC algorithm can then be applied to the resulting focused covariance matrix. It can be seen that the CSS algorithm is based on matrix computations and orthogonal transformations, which are computationally intensive. The eigendecomposition problem is a very important part of this DOA estimation algorithm. Finding the eigenvalues and eigenvectors of the covariance matrix is needed to construct the signal and noise subspaces that the CSS algorithm will use. The Householder and QR algorithms [43] can be used to compute the eigenvalues and eigenvectors of the symmetric covariance matrix. The Householder algorithm is used to reduce the bandwidth of the covariance matrix by transforming it into tridiagonal form. The eigenvalues and eigenvectors can then be computed using the QR algorithm. The computational cost needed for computing the eigenvalues and eigenvectors of a tridiagonal matrix will be much smaller than that of the original symmetric matrix.

A parallel architecture capable of improving the performance of the coherent signal subspace algorithm was proposed. The computation time of the proposed architecture was measured and compared with the single DSP implementation. The results showed that the parallel architecture yielded the same results, while providing superior performance. One of the limitations of the proposed architecture was the use of static matrices. This implied that the number of sensors and sources was known in advance. It also implied that the system would not be able to detect a greater number of sources without major modifications in the source code. Using dynamic matrices would allow the system to easily adapt to the number of sources to be detected. Table 5.1 shows the comparison between the computation time of the single DSP and the computation time of the parallel architecture.

Task	Single DSP		Parallel architecture	
	Cycles	Seconds	Cycles	Seconds
Covariance matrix	1400000	0.014	42000	0.00042
Householder	2600000	0.026	30000	0.0003
QR	100000000	1	2100000	0.021
Power spectrum	1400000	0.014	47000	0.00047
Total	210000000	2.1	5300000	0.053

Table 5.1 : Performance results for single DSP and parallel architecture

Details of DSP based implementation are provided in the second volume of this report. A conference paper based on this design has been accepted for presentation at the 2007 IEEE Radar Conference. A copy of the paper is provided in Appendix I.

FPGA based Design

FPGAs can be used to implement wide-band DOA algorithms. This approach will provide shorter time to design, chance to exploit parallel processing to squeeze all the processing power and a chance to reconfigure to accommodate other designs. In this work, we have chosen to use Xilinx's FPGAs for multiple reasons. Most of all, it was easy to access design tools to work with. A Field Programmable Gate Array (FPGA) is a general purpose integrated circuit. It is user programmable and provides flexibility and reconfigurability. Xilinx's FPGAs [46] use static RAM to keep their configuration environment. A bitstream file is created and downloaded into RAM for FPGAs. They are used to provide their configuration. If another design need to be used then another file can be downloaded as bitstream file and FPGA will use that configuration.

Xilinix provide excellent design tools and these tools are also available for research work. Main design tool consists of behavioral simulation, synthesizer to perform functional simulation, implementation for timing simulation and download feature for in-circuit verification. There are two mechanisms to provide initial design specifications to the design tool.

First of all we need to describe any design in a hardware descriptive language. In our case we use Very High Speed Integrated Circuit Hardware Descriptive Language (VHDL) which is an industry and IEEE standard. The VHDL code is fed to Xilinx's synthesizer which performs synthesis and provides mapping, timing, placement in a bit stream file. Xilinx tool [46] will transform VHDL codes into standard FPGA components such as Look Up Tables (LUTs), outputs F and Gs from the LUTs and FFs. It would yield a fuse file to program the FPGA. Synthesis tools should be also be fast, cost effective and technology independent. This approach reduces risk and last minute changes can be made to the design. It also optimizes for area or speed. If one is satisfied with the synthesis then one can move on to downloading and perform hardware development using actual FPGA chip.

One drawback of this approach is that the Xilinx synthesizer does not recognize floating point and complex numbers. Wide-band DOA algorithms heavily rely on floating point and complex numbers. It would require translating of all these data types to integer using other software available else where. The problem with this approach is that we can quickly loose control of our design and get bog down with so many details. The other problem is that this approach requires very high skill level of VHDL programming. It is not advisable to pursue this approach due to inefficiencies in the synthesizer.

There is an alternate mechanism which is the result of partnership between Mathworks [47] and Xilinx [46] Corporations. They provide a System Generator software. This software would take a design in MATLAB/Simulink code and system generator would provide a VHDL code which that can be used with the Xilinx's design tools (synthesizer). This will by pass writing of VHDL code. System generator also provides converter blocks for converting floating point numbers to standard bit-vectors and vice versa. This feature eliminates the need for special routines for dealing and redefining the floating point and complex numbers.

System Generator [47] is used to create design in Simulink, simulation capabilities in a graphical environment. It allows connection of subsystems to form a larger system. System Generator has a library of blocks some times called cores and allows incorporation of user defined blocks in VHDL. Library provides blocks for multipliers, arithmetic operations, memories, buffers, up/down data rate converters, counters, input/output ports and other blocks etc. It provides a capability to implement the designing onto FPGAs. The output of the System Generator is VHDL which is used by the synthesizer.

System generator approach is very attractive and is an excellent technique for designing specialized hardware for DSP application. It would be extremely beneficial for us to convert our wide-band DOA algorithm code and re-develop Simulink code. System generator also offers some pre-developed cores which can be used for initial rapid prototyping. This work is using System Generator software for developing FPGA based hardware. The hardware design work using FPGAs is in progress.

FFT Implementation

First computational block in most of the wide-band DOA algorithms is computation of Fast Fourier Transform (FFT). Most of algorithms and our MATLAB program use 64-point FFT. This FFT block is used continuously. It would be appropriate to investigate best way to design FFT block. Its implementation is well researched, documented and widely available in the literature. There are number of DSP and other chips are also available for its computation. Most of these high-performance FFT chips employ parallel arithmetic units and cascaded structures with varying processing time. Cascaded structures provide pipelining and parallel processing capabilities and provide good cost-performance tradeoff. First of all it is proven and documented that FFT computation using radix-4 would provide efficient implementation when compared to radix-2 implementation. It is proposed that we should use radix-4 implementation which would be appropriate in our case as the only restriction in radix-4 algorithm that the number of points should be power of 4. It is true in our case as we would like to compute 64-point FFT and it is power of 4. Literature search was conducted and two following interesting work were found:

- COBRA: A 100-MOPS Single-Chip Programmable and Expandable FFT by Tom Chen, Glen Sunada, and Jian Jin [27]
- An Expandable Column FFT Architecture Using Circuit Switching Networks by Tom Chen and Li Zhu [28]

These techniques offer interesting structure that need to be modified and adopted for FPGA implementation. Following building blocks would be required.

1. An array of 16 bit radix-4 butterfly processors.
2. 128*128 crossbar switch matrix
3. 128-element data exchange block
4. 128-element input/output (I/O) memory
5. Controller

This approach would require design of highly parallel butterfly processor and an interconnection mechanism using crossbar switch matrix concept. The I/O memory block should be divided into two sub-blocks: input memory block and the output memory block. The memory sub-block is further subdivided into two sections: one for real part of a 64-point complex input vector and another for the imaginary part. Loading of input data, circulating of intermediate data and sending out the output data should be considered and design in an optimal way. This detailed design work has not been pursued and at this time we will use available block from the System Generator for FFT computation.

The work on other computational block is in progress and would be reported at the end of this contract.

Chapter 6

Conclusions

This work performed a review of wide-band DOA algorithms in the literature which was accumulated for more than 30 years [1-41]. There are more than fifty publications for wide-band detection of Direction of Arrival (DOA) algorithms which are available in the literature. We have reviewed the most relevant one and have not reviewed others which will not be applicable or suitable from hardware implementation. These algorithms were generally presented in a very complex or condense form, which are not easily understandable for people who are outside that narrow field. One of the reason for their complex representation is due to their publication in IEEE transactions and conference. These transactions generally prefer highly mathematical papers and sometimes authors insert mathematics so chances of their papers are increased. Another reason for condense reporting is that these papers face a page limit. Therefore algorithms need to be accommodated within those guidelines and also comply with the reviewers comments. One unfortunate thing happens in this process that essential information does not get into the papers and there is always missing information. This missing information is acute in our case as we are looking for hardware implementation point of view and we are ignoring details of statistical results and errors which are irrelevant in our case. We are willing to sacrifice small amount of error in order to accomplish the goal of implementing them in hardware for real time applications.

We have discovered a class of computational requirements that would be required in all these algorithms as was summarized in Table 4.1. We have also given reviews and challenges in Table 4.2.

We were able to cut through all the mathematics and converted algorithms into simple arithmetic operations. This step is very useful in visualizing an architecture. We have filled a gap between the design of computer hardware especially special purpose parallel architectures and available algorithms for various interdisciplinary problems.

This work was the first step in sorting out which algorithm is appropriate for further study and for its hardware implementation for real time applications. We have developed some hardware as described in Chapter 5 and Volume 2 of this report. Work is in progress for implementation of identified computational steps.

This work can be extended to develop re-configurable test-bed environment for investigative studies for various algorithm. The re-configurable test-bed would be useful to study timing, memory, hardware requirement and accuracy of results from various algorithms. This test-bed would also be useful in evaluating different number of sensors and different kind of sensors. This test-bed would also be a technology scalable system and would become useful in deployment hardware.

Chapter 7

References

1. J. Liberti and T. Rappaport, “Smart antennas for wireless communications”, Prentice Hall, 1999

2. Peter Stoica and Randolf Moses,” Spectral Analysis of Signals,” Prentice Hall, 2005

3. 8. John Litva and T. Kwok-Yeung Lo,”Digital Beamforming in Wireless Communications,” Artech House Publishers, Boston (TK5103.2.L58), 1996.

4. G. Su and M. Morf,”Signal subspace approach for multiple wide-band emitter locations, IEEE Transactions on Acoustic Speech and Signal Processing, Vol 31, December 1983, pp. 1502-1522.

5.. M. Wax, T. J. Shan and T. Kailath,”Spatio-temporal spectral analysis by eignestructure methods,” IEEE Transactions on Acoustic Speech and Signal Processing, Vol 32, August 1984, pp. 817-827.

6. H. Wang, M. Kaveh, ”Coherent Signal-Subspace Processing for the Detection and Estimation of Angles of Arrival of Multiple Wide-band Sources,” IEEE Transactions on Acoustic, Speech and Signal Processing, Vol –ASSP-33, No. 4, August 1985, pp 823-831.

7. H. S. Hung, M. Kaveh,”Focussing matrices for coherent signal subspace processing,” IEEE Transactions on Acoustic Speech and Signal Processing, Vol 36, August 1988, pp. 1272-1281.

8. A. K. Shaw and R. Kmaresan,”Estimation of Angles of Arrivals of Broadband Signals,” ICASSP 1987, PP. 2296-2299.

9. J. Bohme, “Estimation of Source Parameters by Maximum Likelihood and Nonlinear Regression”, Proc. IEEE International Conference on Acoustic, Speech and Signal Processing, pp.7.3.1-7.3.4, 1984.

10. B. D. Van Veen and K. M. Buckley, “Beamforming: a versatile approach to spatial filtering,” IEEE ASSP Mag., vol. 5, pp. 4–24, Apr.1988

11. B. Porat and B. Friedlander, “Estimation of spatial and spectral parameters of multiple sources.” IEEE Transactions on Information Theory, vol. IT-29, pp. 412-425, May 1983.

12. A. Nehorai, G. Su, and M. Morf, “Estimation of time differences of arrival by pole decomposition,” IEEE Transactions on Acoustic, Speech and Signal Processing, vol. ASSP-31, pp. 1478-1491, December 1983.

13. M. Coker and E. Ferrara, “A new method for multiple source location,” in Proc. IEEE International Conference on Acoustic, Speech and Signal Processing, Paris, France, Apr. 1982.

14. G. Bienvenu, “Eigensystem properties of the sampled space correlation matrix,” IEEE International Conference on Acoustic, Speech and Signal Processing, April. 1983.

15. M. Wax, T. San and T. Kailath, “Spatio-Temporal spectral analysis by eigenstructure methods”, IEEE International Conference on Acoustic, Speech and Signal Processing, Vol 32, No.4, pp 817-827.

16. M. Wax and T. Kailath, “Determining the number of signals by information theoretical criteria,” in Proc. ASSP Spectrum Estimation Workshop 11, Tampa, Florida, pp. 192-196, 1983

17. Jefrey Krolik and David Swingler, “Multiple Broad-Band Source Location Using Steered Covariance Matrices,” IEEE Transactions on Acoustic, Speech and Signal Processing Vol 37, No. 10, October 1989. PP. 1481-1494.

18. Jefrey Krolik and David Swingler, “Focused Wide-Band Array Processing by Spatial Resampling,” IEEE Transactions on Acoustic, Speech and Signal Processing, Vol 38, No. 2, February 1990, pp. 356-360.

19 . Ta-Sung Lee,” Efficient Wide-band Source Localization Using Beamforming Invariance Technique,” IEEE Transactions on Signal processing, vol. 42, No. 6, June 1994, PP. 1376-187.

20. Yeo-Sun Yoon, Lance M. Kaplan and James H. McClellan, “New Signal Subspace Direction Of Arrival Estimator for Wide-band Sources”, Proceedings of International Conference on Acoustic, Speech and Signal Processing, pp. V-225-228, Hong Kong, 2003

21. Tuan Do-Hong, Franz Demmel, Peter Russer, “A Method for Wide-band Direction of Arrival Estimation Using Frequency-Domain Frequency-Invariant Beamformers,” IEEE Symposium on Antennas and Propagation, June 2003.

22. R. O. Schmidt,”Multiple emitter location and Signal Parameter Estimation,” IEEE Transaction on Antenna Propagation, Vol. AP-34, March 1986, pp. 276-280.

23. Tuan Do-Hong, Peter Russer, “Wide-band Direction of Arrival Estimation and Beamforming for Smart Antennas System,” 2003.

24. Tuan Do-Hong, Peter Russer, “Beam-Space Processing with Spatially Interpolated Wide-band Beamformers for Wide-band Direction of Arrival Estimation,” European Conference on Wireless Technology, Amsterdam, 2004.

25. Tuan Do-Hong, Peter Russer, “Wide-band Direction of Arrival Estimation Using Frequency-Domain Frequency-Invariant Beamformers: An Analysis of Performance,” IEEE Microwave and Wireless Components Letters, Vol 14, No. 8, August 2004, pp. 383-385.

26. Darren B. Ward, Rodney A. Kennedy, Robert C. Williamson,”Theory and Design of Broadband Sensor Arrays with Frequency Invariant Beam Patterns,” Journal of Acoustic Society of America, February 1995, PP. 1023-1034.

27. Tom Chen, Glen Sunada, and Jian Jin, "COBRA: A 100-MOPS Single-Chip Programmable and Expandable FFT," IEEE Transactions on Very Large Scale Integration (VLSI) Systems, Vol. 7, No. 2. June 1999, PP. 174-182

28. Tom Chen and Li Zhu, "An Expandable Column FFT Architecture Using Circuit Switching Networks," Journal of VLSI Signal Processing 6, pp. 243-257, 1993

29. Darren B. Ward, Rodney A. Kennedy and Robert C. Williamson, "FIR Filter Design for Frequency Invariant Beamformers," IEEE Signal Processing Letters, Vol. 3, No. 3., March 1996, pp 69-71

30. Darren B. Ward, Zhi Ding, and Rodney A. Kennedy, "Broadband DOA Estimation Using Frequency Invariant Beamforming," IEEE Transactions of Signal Processing, Vol 46, No. 5, May 1998, pp. 1463-1469.

31. Fabrizio Sellone,"Robust Auto-Focusing Wide-band DOA Estimation," Signal Processing 86, 2006, Elsevier

32. M. A. Doron, A. J. Weiss,"On Focusing Matrices for Wide-band Array Processing," IEEE Transactions on Signal Processing, Vol 40, No. 6, June 1992, pp. 1295-1302.

33. H. S. Hung and C. Y. Mao, "Robust coherent signal subspace processing for directions of arrival estimation of wide-band sources," IEE Proceedings of Radar, sonar and Navigation, October 1994, pp. 256-262.

34. G. Bienvenu, P. Fuerxer, G. Vezzosi, L. Kopp, F. Florin,"Coherent wide band high resolution processing for linear array," International Conference on Acoustics, Speech, and Signal Processing, ICASSP-89, Vol 4., may 1989, pp. 2799-2802.

35. Y. H. Chen, F.P. Yu,"A comparison of spatial interpolation and transformation matrices in CSS for beafing estimation of multiple broadband signals," Journal of Acoustic Society of America, Decemeber 1991, pp. 3149-3154.

36. Y. H. Chen, R. H. Chen," Directions of arrival estimation of multiple coherent broadband signals," IEEE Transactions Aerospace Electronic Systems, Vol 29, July 1993, pp 1035-1043.

37. M. A. Doron, E. Doron, A. J. Weiss,"Coherent wide band processing for arbitrary array geometry," IEEE Transactions on Signal Processing, Vol 41, No. 1, January 1993, pp. 414-417.

38. Yeo-Sun Yoon, Lance M. Kaplan and James H. McClellan, "Direction Of Arrival Estimation of Wide-band Sources Using Arbitrary shaped multidimensional Arrays", Proceedings of International Conference on Acoustic, Speech and Signal Processing, pp. II-221-224, Montreal, 2004

39. Giacinto Gelli and Luciano Izzo, “Cyclostationarity Based Coherent Methods for Wide-band-Signal Source Location”, IEEE Transactions on Signal Processing, Vol. 51, No. 10, October 2003, pp. 2471-2482.

40. Yeo-Sun Yoon, Lance M. Kaplan and James H. McClellan, “TOPS: New DOA Estimator for Wide-band Signals”, IEEE Transactions on Signal Processing, Vol. 54, No. 6, June 2006, pp. 1977-1989.

41. Kevin Buckley, Lloyd Griffiths,”Broad-Band Signal-Subspace Spatial-Spectrum (BASS-ALE) Estimation,” IEEE Transactions on Acoustic Speech and Signal Processing, Vol. 16, No. 7, July 1988, pp. 953-964.

42. N.S. Asaithambi, “Numerical Analysis”, Saunders College Publishing, 1995.

43. W. Vetterling and B. Flannery, “Numerical Recipes in C++”, Cambridge University Press, 2002

44. “Diopsis 740 Dual Core user manual”, Atmel, http://www.atmel.com

45. D. Grover and J. Deller, “Digital Signal Processing and the Microcontroller”, Prentice Hall, 1999.

46. Xilinx website www.xilinx.com

47 Mathworks website www.mathworks.com

Appendix I

Paper presented at the 2007 IEEE Radar Conference Boston, April 17-20, 2007

DSP Based Implementation of Direction of Arrival for Wideband Sources

Mohsin M. Jamali[1], Abdel Affo[1], Nathan Wilkins[2], Philip D. Mumford[2] and Ken Hahn[3]

[1.] Department of Electrical Engineering & Computer Science,
The University of Toledo, Toledo, OH 43606
Email: mjamali@utnet.utoledo.edu

[2.] AFRL-SNRR/RE, Wright Patterson AFB, OH 45433
[3.] MacAulay-Brown, Inc. Dayton, OH 45430-1062

Abstract

There is a need for the computation of Direction of Arrival (DOA) for wideband sources for number of applications. There are number of algorithms available in the literature for wideband case however we focus on Coherent Signal Subspace method proposed by Wang and Kaveh. Most algorithms available in the literature follow some variation of CSS algorithm thereby increasing computational complexity in an effort to get more accurate, statistically stable and unbiased DOA. We are interested in computing DOA for wideband sources in real time. We chose CSS algorithm and investigated possibility of implementing it using commercially available Digital Signal Processor (DSP) in an effort to achieve real time capability. DSPs offer flexibility, ease of development of embedded system, reduces design cost and offers use of high-level programming language such as C. In this work, we propose a DSP based architecture for detecting and estimating the DOA of wideband sources. It is known that DOA algorithms require computation of eigenvalues and eigenvectors. It would be best to find computational friendly algorithm for the computation of eigenvalues and eigenvectors. In this work eigenvalues and eigenvectors are computed using well known Householder and QR algorithms. CSS algorithm is then implemented in C and executed on DIOPSIS™ 740 by Atmel. DIOPSIS™ 740 (D740) is a high performance dual-core processing platform for real time applications. The CSS algorithm was then parallelized and a parallel architecture was then developed. This paper presents parallel architecture using DIOPSIS™ 740 (D740) and computes performance parameters.

I. Introduction

Array processing has been an important part of signal processing in the past few decades [1-2]. The array consists of sensors located at different points in space with respect to a reference point. Direction of Arrival (DOA) denotes the direction from which the wave fields arrive at the sensor array. The goal in DOA detection and estimation is to accurately determine the number of sources producing waveforms and the locations of those sources. There are number of publications available in the literature for detection of directional of arrival for wideband sources. [1-15]. Wang and Kaveh [1] proposed Coherent Signal-Subspace (CSS) method for detection of DOA for wideband sources. This technique separates the wide frequency band into narrowband components. The data set for this algorithm is divided into 64 segments and each segment contains 64 samples. We also assume a uniform linear array of 16 sensors. The Coherent Signal Subspace approach proposed by Wang & Kaveh [1] follows following computational steps.

9. Compute 64 sets of 64-point FFT
10. Compute 33 Covariance matrices (16 by 16)
11. Computation of initial DOA estimate using MUSIC algorithm [14]
12. Computation of Focus matrix
13. Computation of number of sources
14. Separation of Signal & Noise subspaces
15. Compute DOA using MUSIC algorithm

The covariance matrices for all the frequency components are estimated and combined to form a single focused covariance matrix. The narrowband MUSIC algorithm can then be applied to the resulting focused covariance matrix. It can be seen that the CSS algorithm is based on matrix computations and orthogonal transformations, which are computationally intensive. The eigendecomposition problem is a very important part of this DOA estimation algorithm. Finding the eigenvalues and eigenvectors of the covariance matrix is needed to construct the signal and noise subspaces that the CSS algorithm will use. The Householder and QR algorithms [16-17] can be used to compute the eigenvalues and eigenvectors of the symmetric covariance matrix. The Householder algorithm is used to reduce the bandwidth of the covariance matrix by transforming it into tridiagonal form. The eigenvalues and eigenvectors can then be computed using the QR algorithm. The computational cost needed for computing the eigenvalues and eigenvectors of a tridiagonal matrix will be much smaller than that of the original symmetric matrix.

II. DSP Implementation

To implement this algorithm in real-time, hardware capable of executing millions of operations per second is required. A general purpose DSP was selected as an appropriate platform for implementation because of its ease of programming. Also, the DSP is suitable for matrix and floating point computations. The DSP used in our implementation is a DIOPSIS™ 740 by Atmel. DIOPSIS™ 740 (D740) is a high performance dual-core processing platform for real time applications [18]. The D740 is optimally suited for floating point applications complex domain computations. The ARM7TDMI embedded microcontroller core is equipped with several peripherals and on-chip memories. The main components of the DSP subsystem are the core processor, the on-chip memories and the interfaces to and from the ARM subsystem. The mAgic DSP has four on-chip memory blocks: the program memory, the data memory, the data buffer, and the dual ported memory shared with the ARM processor. An external memory interface multiplexes the data accesses and the program accesses to and from the external memory. The program memory stores the Very Long Instruction Word (VLIW) program to be executed by mAgic.

Multicore Application Development Environment (MADE) is an Integrated Development Environment (IDE) that can be used to develop D740 applications [18]. It includes the C compilers for both ARM and mAgic DSP based on GNU compiling tools named as GCC. The magic C compiler contains a DSP library composed of over 220 functions such as Fast Fourier Transform, IIR and FIR filter creation. The JTST board [18] is low-cost, stand-alone, general-purpose module that provides the appropriate resources in order to evaluate D740 DSP performances in a wide range of applications. The JTST board provides several memories and other peripherals.

CSS computation

There are 33 covariance matrices in this algorithm and the covariance matrix for each frequency component is computed. The main issue resides in forming the matrix **X** containing all samples of one frequency components. Samples related to each frequency are spread across the 64 matrices and need to be put in the same matrix. This can be done by using the memory transfer function to transfer each of the data matrices and then store the row corresponding to the frequency needed. Figure 1 shows that computation process for the frequency component w_o.

Using the periodicity and symmetric properties of the FFT, it is possible to have all the information needed by selecting frequencies w_o to w_{32}. The process above is repeated for each of the 33 covariance matrices. The eigenvalues and eigenvectors of a symmetric matrix are computed using the Householder and QR algorithms [16-17]. However, these algorithms cannot be directly applied to the covariance matrix as it is a Hermitian matrix. Solving this problem requires the conversion of the Hermitian matrix into a real symmetric matrix. It should be noted that Hermitian matrices have real eigenvalues. The power spectrum is computed to obtain an initial estimate for the DOA. The column vectors forming the noise matrix are the eigenvectors associated with the M-D lowest eigenvalues, where M is the number of sensors and D is the number of sources. The number of sources has been determined using the MDL algorithm and initial DOA estimates are obtained [14-15]. The computational process continues for the computation of focus matrix, computation of eigenvalues & eigenvectors, computation of number of sources and then final computation of DOA estimate.

III. Parallel Architecture for Coherent Signal Subspace Algorithm

A method for computing the covariance matrices and other modules using a single DSP was

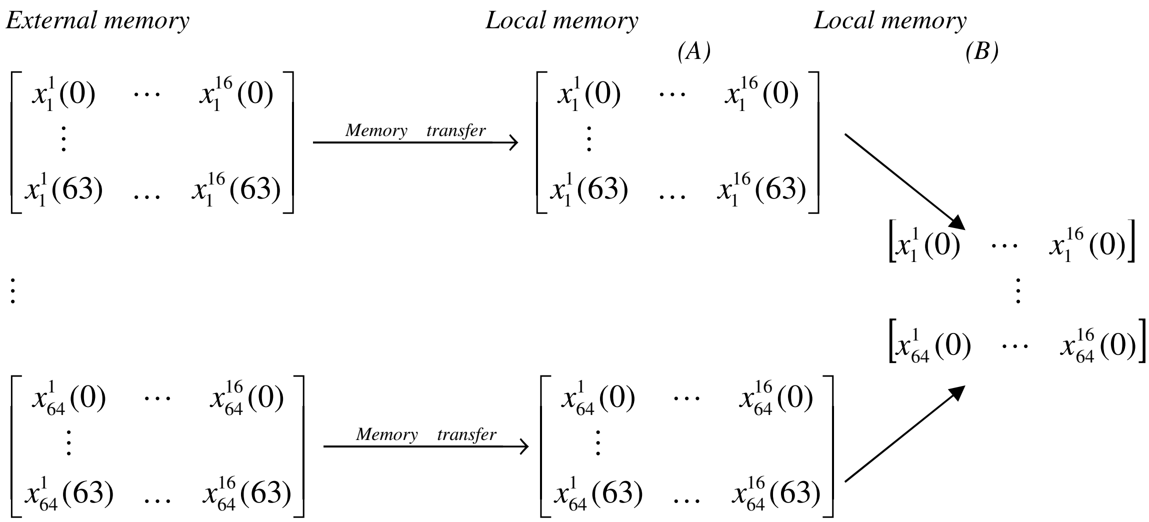

Figure 1: Data transfer scheme from external memory to local memories

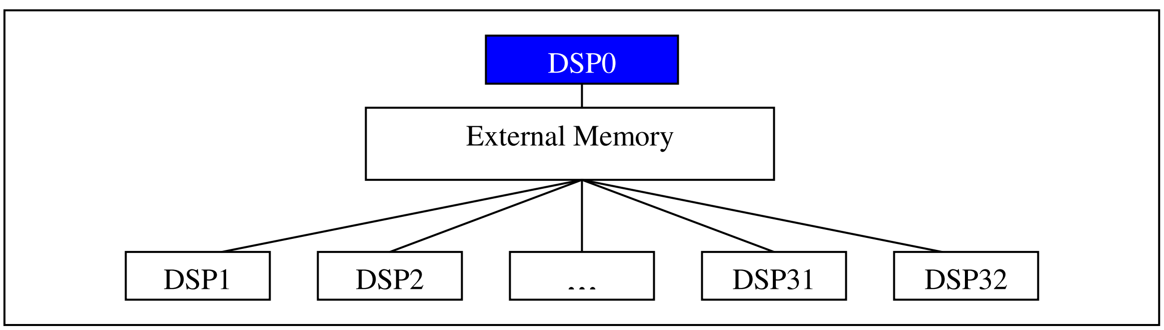

Figure 2: Parallel architecture for coherent signal subspace algorithm

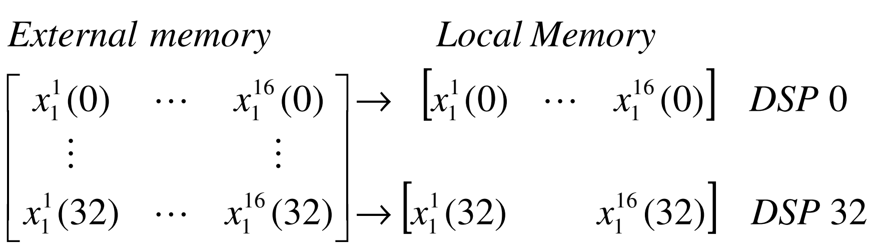

Figure 3: DMA transfer mechanism

presented earlier. This method can be improved upon. by using one DSP to perform computation related to each frequency bins. This would require computation of 33 covariance matrices. All 33 DSPs share the same external memory. Figure 2 shows its parallel architecture. Processors can initiate a DMA transfer from the external memory to their local memory. The address used during the transfer is the address of the row containing the samples at a particular frequency and the number of elements transferred is the number of elements contained in the row. The DMA transfer for the first matrix is shown above in Figure 3.

After a DMA transfer is complete, another transfer will be initiated for a different segment matrix. This process is repeated for all 64 matrices (segments) in external memory. The execution time will be the same for each processor. This execution time can be further reduced by computing only the lower triangular part of the covariance matrix. Since the covariance matrix is Hermitian, the information contained in the lower triangular part is sufficient to form the entire matrix. Using a single DSP approach, a total of 64 x 33 DMA transfers of 256 elements each were required. The parallelized process only requires 64 transfers of 16 elements each, which should result in increased performance.

Using certain properties of the Householder matrices, it is possible to parallelize the process. This observation will allow us to create parallel architectures for both the Householder and QR methods. The Householder method consists of transforming a symmetric matrix into tridiagonal form using the following transformations

$$\mathbf{B} = \mathbf{H}_{n-2} \ldots \mathbf{H}_2 \mathbf{H}_1 \mathbf{A} \mathbf{H}_1 \mathbf{H}_2 \cdots \mathbf{H}_{n-2}$$

The orthogonal transformation will be accumulated in a matrix **Q** in order to recover the eigenvectors of the original matrix **A**. This can then be written as $\mathbf{B} = \mathbf{QAQ}$. It can be shown that the product **HA** could be computed using n parallel processors if **H** was a Householder matrix. As a result, $\mathbf{H}_2\mathbf{H}_1\mathbf{A}$ and $\mathbf{H}_{n-2} \ldots \mathbf{H}_2\mathbf{H}_1\mathbf{A}$ can also be computed with n processors. The tridiagonal matrix will be the result of the product of two sequences. After execution of the Householder algorithm, matrix **A** is the tridiagonal matrix and matrix **Q** contains the product of all the orthogonal transformations.

The QR method is based of the use of the following orthogonal transformations

$$\mathbf{R} = \mathbf{H}_{n-1} \ldots \mathbf{H}_2 \mathbf{H}_1 \mathbf{A} \quad \text{and}$$

$$\mathbf{Q} = \mathbf{H}_{n-1} \ldots \mathbf{H}_2 \mathbf{H}_1$$

We can compute **R** and **Q** using n parallel processors. The next step involves the computation of the matrix **A** = **RQ** and the computation of a matrix **X** containing the product of the orthogonal transformations $\mathbf{Q}_1\mathbf{Q}_2 \cdots \mathbf{Q}_m$, where m is the number of iterations. These computations can be done using a single DSP. This process will start over until the number of iterations required for a good approximation has been reached. After execution of the QR algorithm, the matrix **A** contains the eigenvalues of the original matrix along its diagonal and matrix **X** contains the eigenvectors of the tridiagonal matrix. The eigenvectors of the original matrix can be obtained be multiplying the matrix **Q** obtained in the Householder process and matrix **X**.

The power spectrum needs to be computed for every angle between 0 and 90 degrees. Each spectrum value can be computed by sending the matrix containing the eigenvectors, the number of sources and the angle of arrival to a specific processor. The DSP can then compute the power spectrum and send the results back to external memory. By sending 3 angles values to 30 of the 33 available DSPs, it is possible to reduce the time needed to compute the power spectrum by a factor of 30.

VI. Simulation

In order to demonstrate the performance of the DOA algorithm for wideband signals, a uniform linear array of sixteen equally spaced Omni-directional sensors was used. The spacing between sensors is $\frac{c}{2f_o}$, where c is the velocity of propagation and f_o is the central frequency. Two wideband sources at θ_1 and θ_2 were assumed. The signals are stationary zero mean band pass white Gaussian processes with central frequency $f_o = 100Hz$ and bandwidth $B = 40Hz$. The array noise is also stationary zero mean band pass with the same pass band as the signal with a SNR of 10dB at each sensor. Source signals and the noise are random processes with a bandwidth of 40 Hz. The sampling frequency is chosen to be 300 Hz. The signal will be observed over a period of T_o seconds and T_o will be divided into $k = 64$ segments. On each of those segments, the array output along with corresponding noise will be decomposed into narrowband components using a fast Fourier transform. The total number of samples taken by each sensor will be 4096. Simulation data is similar to the method described by Wang & Kaveh [1].

The time domain samples will be transformed into frequency domain by applying a 64 point FFT to each of the 64 segments. Due to the memory limitation of the DSP, the output data will be saved in the external memory as sixty four 64x16 matrices, where each matrix represents the output for each of the 64 segments. This is done using a memory transfer from local to external memory.

The previous simulation created for single DSP was used again for the case of the parallel DSP architecture. This was done to measure DOA and compare their performances. The parallel architecture was simulated using a single DSP by executing the parallel processes sequentially. However, the performance would be measured using the longest running process in that sequence in terms of number of clock cycles. The purpose of the simulations was to detect and estimate the DOA of two sources located at 20° and 60° using 40 iterations for the QR algorithm. Figure 4 shows the simulations results. It can be seen that both techniques yielded the same results. The performance analysis showed that the most computationally intensive tasks were the QR algorithm, the Householder algorithm and the power spectrum computation. Table 1 shows the comparison between the computation time of the single DSP and the computation time of the parallel architecture.

The total number of cycles for both the single DSP and the parallel architecture includes tasks that could not be parallelized and are not listed in the table. The total execution time for the parallel architecture is 0.05 seconds, compared to 2.1 seconds for the single DSP. This is due to the fact that the execution time for QR algorithm, which accounts for approximately 95% of the whole process, was significantly reduced using the parallel architecture.

VII. Conclusion

The coherent signal subspace algorithm was chosen for the implementation because of its high resolution; the platform selected was an Atmel Diopsis740 DSP. A parallel architecture capable of improving the computational time of the coherent signal subspace algorithm was proposed. The computation time of the proposed architecture was measured and compared with the single DSP implementation. The results showed that the parallel architecture yielded the same results, while cutting computational time. One of the limitations of the proposed architecture was the use of static matrices. This implied that the number of sensors and sources was known in advance. It also implied that the system would not be able to detect a greater number of sources without major modifications in the source code. Using dynamic matrices would allow the system to easily adapt to the number of sources to be detected.

VIII. Acknowledgement

Authors would like to thank the generous support of Atmel Corporation for donating DSP hardware and software used in this work. Authors would also like to thank WPAFB AFRL SNRR/SNRP and MacAulay-Brown, Inc. Dayton for their financial support under contract # FA 8650-05-D-1848/2.

IX. References

1. H. Wang, M. Kaveh, "Coherent Signal-Subspace Processing for the Detection and Estimation of Angles of Arrival of Multiple Wideband Sources," IEEE Transactions on Acoustic, Speech and Signal Processing, Vol – ASSP-33, No. 4, August 1985, pp 823-831.

2. Hamid Krim and Mats Viberg," Two Decades of Array Signal Processing Research," IEEE Signal Processing Magazine, July 1996. PP. 67-94.

3. Kevin Buckley, Lloyd J. Griffiths,"Broad-Band Signal-Subspace Spatial- Spectrum (BASS-ALE) Estimation," IEEE Transactions on Acoustic, Speech and Signal Processing, Vol – ASSP-36, No. 7, August 1988, pp 953-959.

4. H. Hung and M. Kaveh, "Focussing Matrices for Coherent Signal-Subspace Processing," IEEE Transactions on Acoustic, Speech and

Signal Processing, Vol –36, No. 8, August 1988, pp 1272-1281.

5. Jefrey Krolik and David Swingleer, “ Multiple Broad-Band Source Location Using Steered Covariance Matrices,” IEEE Transactions on Acoustic, Speech and Signal Processing Vol 37, No. 10, October 1989. PP. 1481-1494.

6. Shahrokh Valaee, Benoit Champagne and Peter Kabal,” Localization of Wideband Signals Using Least-Squares and Total Least-Squares Approaches,” IEEE Transactions on Signal Processing, Vol 47, No. 5, May 1999, PP. 1213-1222.

7. Ilan Zisking and Mati Wax,”Maximum Likelihood Localization of Multiple Sources by Alternating Projection,” IEEE Transactions on Acoustic, Speech and Signal Processing Vol 36, No. 10, October 1988. PP. 1553-1560.

8. Miriam A. Doron and Anthony J. Weiss,” On Focussing Matrices for Wide-Band Array Processing,” IEEE Transactions on Signal Processing, Vol 40, No. 6, June 1992, PP. 1295-1302.

9. Giacinto Gelli and Luciano Izzo,”Cyclostationarity-Based Coherent Methods for Wideband-Signal Source Location,” IEEE Transactions on Signal Processing, Vol 51, No. 10, October 2003, PP. 2471-2482.

10. A. K. Shaw and R. Kmaresan,”Estimation of Angles of Arrivals of Broadband Signals,” ICASSP 1987, PP. 2296-2299.

11. Tuan Do-Hong, Peter Russer, “Wideband Direction of Arrival Estimation and Beamforming for Smart Antennas System,” 2003.

12. Fabrizio Sellone,”Robust Auto-Focusing Wideband DOA Estimation,” Signal Processing 86, 2006, Elsevier

13. R. O. Schmidt, “Multiple emitter location and signal parameter estimation” IEEE Transactions on Antennas and Propagation, vol. AP-34, No. 3, pp. 276-280, March 1986.

14. J. Liberti and T. Rappaport, “Smart antennas for wireless communications”, Prentice Hall, 1999

15. M. Wax and T. Kailath, “Determining the number of signals by information theoretical criteria,” in Proc. ASSP Spectrum Estimation Workshop 11, Tampa, Florida, pp. 192-196, 1983

16. N.S. Asaithambi, “Numerical Analysis”,Saunders College Publishing, 1995.

17. W. Vetterling and B. Flannery, “Numerical Recipes in C++”, Cambridge University Press, 2002

18. “Diopsis 740 Dual Core user manual”, Atmel, http://www.atmel.com

19. D. Grover and J. Deller, “Digital Signal Processing and the Microcontroller”, Prentice Hall, 1999.

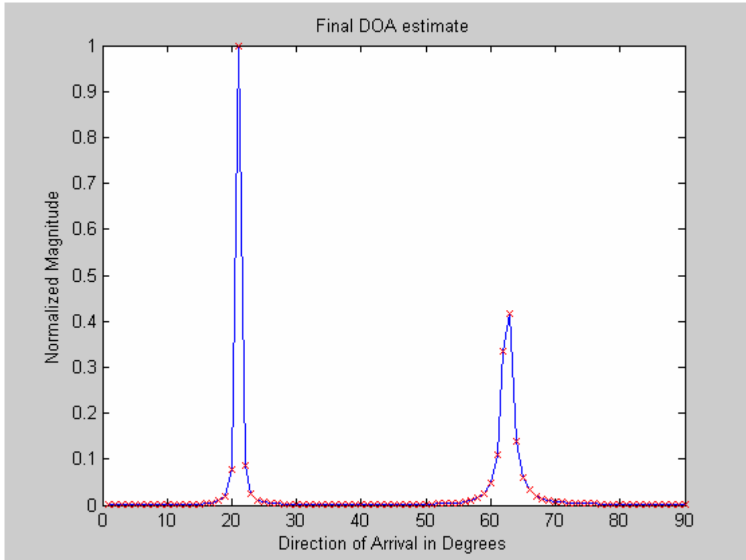

Figure 4: Power spectrum for final DOA estimate of angles $\theta_1 = 20^\circ$ and $\theta_2 = 60^\circ$ using single and parallel DSP approach

Task	Single DSP		Parallel architecture	
	Cycles	Seconds	Cycles	Seconds
Covariance matrix	1400000	0.014	42000	0.00042
Householder	2600000	0.026	30000	0.0003
QR	100000000	1	2100000	0.021
Power spectrum	1400000	0.014	47000	0.00047
Total	210000000	2.1	5300000	0.053

Table 1 : Performance results for single DSP and parallel architecture

Appendix II

Paper is submitted to The Second International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP 2007) to be held at St. Thomas, U. S. Virgin Islands from December 12-14, 2007

Analysis of Computational Requirements of Wide-band DOA Algorithms

Mohsin M. Jamali[1], Nathan Wilkins[2], Christopher R. Rehm[2] , and Ken Hahn[3]
[1.] Department of Electrical Engineering & Computer Science,
The University of Toledo, Toledo, OH 43606
Email: mjamali@utnet.utoledo.edu

[2.] AFRL/SNRE, Wright-Patterson AFB, OH 45433-4318
[3.] MacAulay-Brown, Inc. Dayton, OH 45430-1062

Abstract

There are an extensive number of wide-band DOA algorithms available in the literature. If one needs to implement embedded hardware for a real-time application, there is a daunting task of identifying an appropriate algorithm. We can accept a small amount of error in the hope of getting hardware which can execute these algorithms and provide results in real-time for a given application. This work is a first step in sorting out which algorithm will be appropriate for hardware implementation and presents some quantitative comparisons of these algorithms.

Introduction

Passive detection of objects has been studied for more than 30 years [1-13] and references within them. There are more than sixty publications for wide-band detection of Direction of Arrival (DOA) algorithms which are available in the literature. These algorithms are generally presented in a very complex or condensed form, which are not easily understood by people who are outside that narrow field. It is not known which class of techniques would be appropriate for implementing in hardware and would be useful for real-time applications. One has to cut through all the mathematics and convert algorithms into simple arithmetic operations before the architecture can be visualized. There is a need to bridge a gap between the design of computer hardware, especially special purpose parallel architectures and available algorithms for various interdisciplinary problems.

This work is the first step in sorting out which algorithm is appropriate for further study and its hardware implementation in real-time applications. Wide-band DOA algorithms appropriate to our needs have been reviewed and have been simulated in MATLAB. These algorithms can be divided into following categories:

- In-coherent Signal Subspace
- Coherent/rotational Signal Subspace (CSS)
- CSS using other focusing matrices
- DOA detection using beamforming
- Combination of beamforming and focusing approaches
- Use of ARMA model and Bayesian approaches
- Use of maximum likelihood algorithms
- Bilinear Transformation Method

Signal subspace approaches are very popular for computing DOA for both narrow-band and wide-band sources [1-6]. One problem with them is that they may not be optimal but they produce computationally efficient algorithms. Signal subspace based approaches are further subdivided into coherent based and incoherent based signal subspace approaches. Incoherent signal subspace approaches decompose signals into individual narrow-band frequencies and then combine them to produce the final results. They are computationally expensive. Some of the subspace approaches require initial estimates. If the initial estimate is not accurate then the final DOA will also not be accurate and there will be issues with bias and variances.

In order to implement a DOA algorithm in hardware for real-time applications, it is important to use a computationally efficient algorithm. One approach is to evaluate the computational requirements of currently available wide-band DOA algorithms and select one of them for hardware implementation.

Wide-band DOA Algorithms

Development of wide-band DOA algorithm started with the incoherent signal subspace approach which is a brute force extension of the narrow-band case and is computationally expensive. Wang & Kaveh [5] proposed a Coherent Signal Subspace Method (CSSM) which creates a focusing matrix using a single frequency. This has a simple transformation scheme and requires an initial estimate of the DOA. Results are reasonable and produced peaks at the appropriate DOA. Hung &

Kaveh [6] extended their previous work with a promise of statistically better results. They introduce the concept of a Rotational Signal Subspace (RSS) producing a more complex or accurate focusing matrix. This required the additional computational step of singular value decomposition. There may be little improvement in accuracy of the results. Shaw [7] extended the work of Wang/Hung & Kaveh [5-6] and introduced a bilinear transformation approach with certain approximation. The advantage of their work is that it does not require an initial DOA estimate. Their algorithm is very sensitive to certain assumptions and parameter values which make it unattractive for hardware implementation.

Krolik et al. [8] introduced the computation of a steered covariance matrix for each angle, then inverted the covariance matrices to find peaks of the power. He eliminated the need of eigen-decomposition and selection of an initial focusing angle. This technique requires computation for each steering direction θ increasing the computation requirements. His extended work [9] uses interpolation and then decimation of the input data followed by computation of the covariance matrix. The technique looks good but some work needs to be done to resolve some of the parameters as their selection may be tricky. This approach may not be useful in a generalized case. It also eliminates preliminary DOA estimates.

Ta-Sung Lee [10] decomposes received data using bandpass filters into J frequency beams. His algorithm performs beamspace transformation and computes weights using a least square method. It then computes a beamspace data matrix and focuses on a single reference frequency which would be something similar to CSSM method. It performs transformation into K beamspaces and forms the beamspace data matrix. This beamspace data matrix is then focused on a single reference frequency out of J frequency bands. The design of the beamspace data matrix is described which first requires design of beamforming matrices. The weights are again designed in a least square sense. The problem is then reduced to beamspace data correlation and noise matrices. The authors then apply their own derived root MUSIC algorithm which could be substituted with the MUSIC algorithm. The algorithm does not require any preliminary DOA estimates. This DOA estimator is suboptimum according to the author. It would also result in degradation in estimation accuracy at low SNR. Even with the true DOA, the DOA estimates may not be exactly identical as the signal roots may not lay on the unit circle. This approach provides an excellent alternative to CSSM except for the frequency decomposition and calculations of weight and beamspace data matrices.

Tuan Do-Hong et al. [11] first compute the FFT then form beamforming networks, compute covariance matrices, and then perform MUSIC. They require array geometry and guessing of a common frequency. Daren et al. [12] performs filter and sum beamforming in a frequency invariant fashion. They proposed a FIR filter design and then computed covariance matrices. Three FIB's were designed using FIR filters in a frequency invariant fashion covering the normalized frequency band and the spatial sector. This approach looks very attractive and needs some more work. Filter specifications were not provided. We need to find a way to design these FIR filter coefficients and a more focused way to compute them. Sellone [13] proposed a new way of designing focusing matrices which has the same robustness. It does not need initial estimates of DOA. He also claims that the computational requirement is also reduced when compared to the RSS and SST approach. The work of Yoon [14] requires a preliminary estimate of the DOA that could be one of the angles in the estimation and the number of sources need also needs to be estimated. They compute K covariance matrices, eigenvalues and eigenvectors. They form signal & noise subspaces and compute focusing matrices. Finally they find the DOA. They have also named this technique as Test of Orthogonality of Projected Spaces (TOPS).

Algorithm	Challenges
CSSM [5]	Initial DOA estimates are required
RSS [6]	Additional SVD step and initial DOA is required
Beamforming Invariance [10]	DOA estimator is suboptimum and iterative
DOA estimator [14]	Initial DOA estimates are required
Spatial Resampling [9]	Design of interpolator filter. Stability of parameters and their selection Processing is done for each frequency bin.
Steered Covariance matrices [8]	Steered covariance matrices for each angle. Compute maximum power for each steered angle
FDFIB Beamformer [11]	Requires array geometry and guessing of common frequency.

Table 2: Challenges of wide-band DOA algorithms

Computational Requirements

This work evaluated the computational requirement for various DOA algorithms using common data and assumptions. Computational requirements for these algorithms are presented in Table 1. Their implementation was also explored. These wide-band DOA algorithms use following computational steps:

- Generation of wide-band signals
- Conversion of time domain signals into frequency domain via FFT.
- Computation of covariance matrices in the frequency domain.
- Computation of eigenvalues and eigenvectors
- Computation of initial estimates of the number of sources
- Computation of initial DOA estimates
- Computation of transformation matrices and focusing on central frequency
- Computation of eigenvalues and eigenvectors
- Computation of number of sources and final estimates of DOA

Conclusions

In this work we have identified common computational steps and they can be implemented in hardware. Most of these algorithms follow similar computational steps with some variations. These variations can be adopted if we use a re-configurable approach and implement these algorithms in FPGAs.

Acknowledgement

Authors would like to thank the generous support of AFRL/SNRE and MacAulay-Brown, Inc. Dayton. Contract # FA 8650-05-D-1848/2.

References

1. Peter Stoica and Randolf Moses," Spectral Analysis of Signals," Prentice Hall, 2005
2. G. Su and M. Morf,"Signal subspace approach for multiple wide-band emitter locations, IEEE Transactions on Acoustic Speech and Signal Processing, Vol 31, December 1983, pp. 1502-1522.
3. R. O. Schmidt,"Multiple emitter location and Signal Parameter Estimation," IEEE Transaction on Antenna Propagation, Vol. AP-34, March 1986, pp. 276-280.
4. M. Wax, T. J. Shan and T. Kailath,"Spatio-temporal spectral analysis by eignestructure methods," IEEE Transactions on Acoustic Speech and Signal Processing, Vol 32, August 1984, pp. 817-827.
5. H. Wang, M. Kaveh, "Coherent Signal-Subspace Processing for the Detection and Estimation of Angles of Arrival of Multiple Wide-band Sources," IEEE Transactions on Acoustic, Speech and Signal Processing, Vol –ASSP-33, No. 4, August 1985, pp 823-831.
6. H. S. Hung, M. Kaveh,"Focussing matrices for coherent signal subspace processing," IEEE Transactions on Acoustic Speech and Signal Processing, Vol 36, August 1988, pp. 1272-1281.
7. A. K. Shaw and R. Kmaresan,"Estimation of Angles of Arrivals of Broadband Signals," ICASSP 1987, PP. 2296-2299.
8. Jefrey Krolik and David Swingler, "Multiple Broad-Band Source Location Using Steered Covariance Matrices," IEEE Transactions on Acoustic, Speech and Signal Processing Vol 37, No. 10, October 1989. PP. 1481-1494.
9. Jefrey Krolik and David Swingler, "Focused Wide-Band Array Processing by Spatial Resampling," IEEE Transactions on Acoustic, Speech and Signal Processing, Vol 38, No. 2, February 1990, pp. 356-360.
10. Ta-Sung Lee," Efficient Wide-band Source Localization Using Beamforming Invariance Technique," IEEE Transactions on Signal processing, vol. 42, No. 6, June 1994, PP. 1376-187.
11. Tuan Do-Hong, Peter Russer, "Wide-band Direction of Arrival Estimation Using Frequency-Domain Frequency-Invariant Beamformers: An Analysis of Performance," IEEE Microwave and Wireless Components Letters, Vol 14, No. 8, August 2004, pp. 383-385.
12. Darren B. Ward, Zhi Ding, and Rodney A. Kennedy, "Broadband DOA Estimation Using Frequency Invariant Beamforming," IEEE Transactions of Signal Processing, Vol 46, No. 5, May 1998, pp. 1463-1469.
13. Fabrizio Sellone,"Robust Auto-Focusing Wide-band DOA Estimation," Signal Processing 86, 2006, Elsevier
14. Yeo-Sun Yoon, Lance M. Kaplan and James H. McClellan, "TOPS: New DOA Estimator for Wide-band Signals", IEEE Transactions on Signal Processing, Vol. 54, No. 6, June 2006, pp. 1977-1989.
15. Kevin Buckley, Lloyd Griffiths,"Broad-Band Signal-Subspace Spatial-Spectrum (BASS-ALE) Estimation," IEEE Transactions on Acoustic Speech and Signal Processing, Vol. 16, No. 7, July 1988, pp. 953-964.

DOA ALGORITHM	COHERENT SIGNAL SUBSPACE	ROTATIONAL SIGNAL SUBSPACE	BINLINEAR TRANSFORM.	BEAMFORMING INVARIANCE	STEERED COVARIANCE	SPATIAL RESAMPLING	TOPS- DOA ESTIMATOR	FDFIB BEAMFORMER	FIB USING FIR	CYCLOSTATIONARY
Authors	Wang & Kaveh [6]	Hung & Kaveh [7]	Shaw [8]	Ta-Sung Lee [19]	Krolik [17]	Krolik [18]	Yoon [20,38,40]	Do-Hong [21,23-24]	Ward [26,29-30]	Gelli & Izzo [39]
Compute	64 point FFT	64 point FFT	64 point FFT	Beamspace transformation into J frequency bins	Separate frequencies using FFT	Insert K_n -1 zeros Perform interpolation	FFT for each block (J)	Compute FFT	Design FIR filters	64 point FFT
Compute	Compute 33 Covariance matrices	Compute 33 Covariance matrices	33 Covariance matrices	Compute weight using Least Square fit method	Cross spectral density matrix **K** for each frequency	Convolve with the low pass FIR filter	Compute sensor output for pre-selected frequencies	J beamforming network operation	Form J beamforming networks	Covariance matrices
Compute	initial DOA estimate using MUSIC	initial DOA estimate using MUSIC		J Beamspace data matrix focused on single ref. frequency	Steered cov. matrix **R** for each angle	Perform decimation operation	K Covariance matrices	Compute covariance matrix	Covariance matrix	Focusing matrices
Compute	Focusing matrices	Focusing matrices	Focusing matrices	Beamspace data matrix is designed in least square sense	Inverse of steered covariance matrix **R** for each angle	covariance matrix for B samples	eigenvalues & eigenvectors for each frequency	Eigen-decomposition	Eigen-decomposition	weight function
Compute	Eigen-decomposition	Eigen-decomposition	Eigen-decomposition	Eigen-decomposition	Spectral estimate **Z** for each angle	Operations for each frequency	subspaces for each frequency	# of sources	# of sources	*R* matrix
Compute	# of sources	# of sources	# of sources	# of sources s	Determine peak positions of the power	Eigen-decomposition	RSS focusing matrix	MUSIC on selected frequency	Perform MUSIC	SVD
Compute	MUSIC	MUSIC	MUSIC	Root MUSIC		# of sources	Eigen-decomp			Eigen-decomp
Compute						MUSIC	MUSIC			MUSIC

Table 1: Computational requirements of various wideband DOA algorithms