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13. ABSTRACT (Maximum 200 words)

This project supplies support for an additional graduate student on experimental investigations on quantum coherence, entanglement, and quantum computation in a solid-state, electronic realization of quantum bits based on superconducting single-electron devices, namely the single Cooper-pair box. Since these qubits can be microfabricated in large numbers on a single chip, addressed and coherently manipulated electronically, and then entangled with each other, they represent one of the most scalable implementations for possible quantum computers. A radio-frequency single electron transistor (RF-SET) is used for readout of the charge state of the qubit. Of particular importance in this scheme is an understanding and the control of the backaction of the measurement system, which can affect the lifetime and coherence of the qubit. Therefore, this backaction is studied with the goal of optimizing the sensitivity and lifetime in order to attain single-shot readout of the qubit.

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**REPORT DOCUMENTATION PAGE (SF298)**
**(Continuation Sheet)**

**Final Report (8/31/04-8/31/05):**
Student Support for Quantum Computing with Single Cooper-pair Electronics: RF-SET Readout and Backaction (DAAD-19-01-1-0611) PIs: R.J. Schoelkopf.

This is the final report on a supplementary grant, which began on June 1, 2001. The funds provide support for an additional graduate student, Mr. Ben Turek, related to our continuing grants studying quantum coherence and computation in superconducting single-charge circuits, namely the Cooper-pair box (CPB). Mr. Turek has participated in many phases of our experimental and theoretical program on quantum computing, but has especially concentrated on studies of the CPB qubit with the superconducting SSET, as well as studies of the SET backaction in the normal state. We have made substantial progress in several areas, listed below.

To date Mr. Turek has been a co-author on three papers, and the first author on a Physical Review B publication on backaction of the SET in the normal state. Mr. Turek is working with a postdoc, Dr. Hannes Majer, to produce and measure a final sequence of RF-SET plus Cooper-pair box samples, to measure the backaction effects on the lifetime and excited state population of the qubit when measured. He will probably complete his PhD in late 2006 or early 2007.

## I. Results from 2002:

Mr. Turek has been working with a nanoelectronic quantum system, the single Cooper-pair box, which is fabricated using small-area Al/AlOx/Al tunnel junctions, and is a leading candidate for realizing a solid-state quantum computer. We have successfully integrated a Cooper-pair box qubit with a high-speed, time-gated quantum readout amplifier, using the Radio-Frequency Single-Electron Transistor (RF-SET) invented by our group(see Figure 1). This has allowed us to observe the coherence and relaxation of a single qubit, and to make initial determinations of both the ensemble decoherence time ($T_2$*) and the energy relaxation time ($T_1$).

During this specified period, Mr. Turek completed his required coursework and exams. He began doing laboratory research fulltime as of May '02. During that period, Ben has made rapid progress, learning how to run both the dilution refrigerator, and all of the microwave and other electronics associated with our experiments using the RF-SET. Ben has also refined the data acquisition software for the experiment, writing code in Labview so that there is now a "virtual front panel" via which the entire experiment is computer controlled. Because of the complexity of the experiment, this is highly beneficial, as now the experimenter can control the variable of the experiment in the sensible physical units, and see charge data displayed in real time as it is acquired. In addition, one can use various tabs to change experimental protocols, switching from a standard Coulomb staircase measurement to Rabi, T1, or Ramsey type experiments. Ben has also included an automatic routine which checks for offset charge jumps on slow timescales (10 minutes) and corrects all the gates for any changing offsets. With the postdoc, Dr. Konrad Lehnert, he has developed ways to calibrate the voltage sources used for gate control and this has resulted in the ability to measure charging of a box with a SNR of 1,000, and a repeatability at the ½ of a milli-electron level.

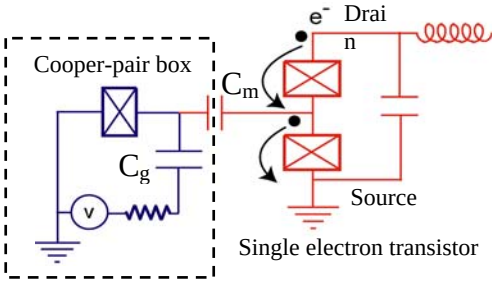



Figure 1: Schematic of Cooper-pair box qubit (CPB)and single electron transistor (SET) readout amplifier. The boxes with crosses denote small-capacitance Josephson junctions. The

Ben then focused on a series of experiments studying the single-electron box, when a large magnetic field is applied to keep the aluminum superconductors of our circuit in the normal state. He focussed on some discrepancies we noticed when trying to compare these measurements to our studies of the box in the superconducting state. In particular, we found that the shape, breadth, and location of the staircases varied in a subtle, but repeatable manner, depending on the operating point of the SET electrometer (see Figure 2).

The changes in the Coulomb staircase in fact depend on the intrinsic backaction of the SET on the box, and the apparent shape of the staircase, as measured by the SET, contains interesting information about the backaction. The backaction arises directly from the capacitive coupling to the SET, through which the box is measured. When current flows through the SET, electrons move from drain to source, tunneling onto and off of the SET's central island (see Fig. 1). These fluctuations of charge state of the SET lead to a fluctuating potential, which then couples to the box via the measurement capacitor, $C_m$. By sensitively measuring the apparent shape of the staircase, we can discern several properties of the SET's noise. In addition, working with a post-doc, Dr. Aash Clerk, in co-investigator Steve Girvin's group, Ben has developed a model of the coupled box-SET system, which can be solved analytically using so-called "orthodox theory." By other independent measurements, all of the parameters of this model, such as the capacitance and resistance of the various junctions, can be measured with reasonable precision. We find that the shapes of the staircases can not only be qualitatively understood, but also quantitatively predicted using this model. A comparison of the first derivative of the Coulomb

staircases in the box for two different operating points in the electrometer are shown in Figure 2, and compared to the predictions of the model. There is quite good agreement, with no adjustable parameters. This implies that the backaction of the SET in the normal state is now understood, and that it is predominantly due to the theoretically expected process, coupled to the system via the same mechanism as the measurement. This is also the first definitive measurement of the backaction in an SET device.

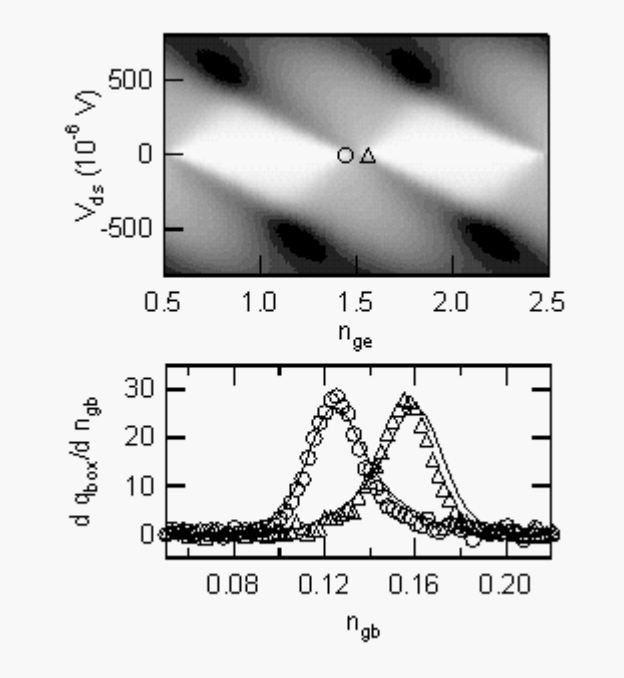



**Figure 2:** Top: measured RF conductance of a normal SET, as a function of electrometer gate and drain bias. Bottom: Measured derivative of the "Coulomb staircase" of a normal electron box measured at two different operating points (denoted by the corresponding symbols in the top panel ) (B. Turek et al., ref. 1, compared to theory (solid curves). The effect of the detector back action causes a shift, a broadening, and an asymmetry of the

Ben has made such measurements at over 30 different operating points for the SET (i.e. different locations in the greyscale plot of Fig. 2), and is in the process of analyzing the results using our model. A manuscript is also being prepared containing these results, for submission to Physical Review Letters. The next steps involve performing these measurements in the superconducting state, where the box behaves as a coherent two-level system, which can measure both the dephasing due to the backaction, the approach of the SET to the standard quantum limit, and the full positive and negative frequency spectrum of the SET backaction noise. Our goal is to compare these results in a continuous measurement with the theory developed by Dr. Clerk and Prof. Girvin for the measurement using the "DJQP" feature of the superconducting SET. These results will allow us to make further inferences about the dephasing and relaxation of our Cooper-pair box qubits, as well as to optimize the system and obtain high SNR single-shot measurements of our charge qubits. In the next few months Ben will also work with a new postdoc, Dr. Andreas Wallraff, to perform time-domain measurements of coherence in the CPB qubits and demonstrate coherent state control.

**II. Results from 2003:**

During this period, Mr. Turek has concentrated on studies of the CPB qubit with the superconducting SSET, as well as studies of the SET backaction in the normal state. We have made substantial progress in several areas, listed below.

- Obtained and tested new set of qubits from Chalmers with decreased charging energy to eliminate parity problems/quasiparticle generation
- Observed full Cooper-pair staircase and performed spectroscopy down to 1/f insensitive charge degeneracy point of CPB.
- Demonstrated controlled tuning of tunnel coupling of qubit Hamiltonian
- Remeasured relaxation times (T1) of a Chalmers qubit, and found systematic variation due to SET readout.
- Found that cause of fast relaxation in Chalmers coherent oscillation experiments is the SET backaction.
- Confirmed theoretical description of qubit relaxation and SET backaction, and observed predicted population inversion due to measurement.
- Observed activation of 1/f noise by SET readout, and very low levels of 1/f charge noise on long timescales.
- Two Physical Review Letters on this work appeared this year, one on measurements of relaxation and decoherence time of CPB as measured by SET, the second on renormalization of charging energy in the box by quantum charge fluctuations.

As described above, we now have the capability to fabricate our own CPB qubits and SETs, but up until now we have been forced to rely on samples obtained through our collaboration with Chalmers University in Sweden. We found in our earlier experiments that it was detrimental to have too large a charging energy for the CPB, as this leads both to relatively strong dephasing due to 1/f noise, and creates problems with excess quasiparticles occupying the island, since the charging energy can be comparable with the superconducting gap. This second problem is particularly detrimental, since it means that the CPB cannot be operated near the charge degeneracy point in gate voltage, where the results of Saclay have shown that 1/f dephasing can be minimized, leading to coherence times approaching one microsecond. Finally, the large charging energy raises all the energy scales of the box, meaning that much higher frequency (and much more difficult to generate and couple) microwaves are required for control, and shortening the relaxation times available for the control and readout of the states. Our goal was therefore to investigate CPB qubits with a more optimal choice of energies, namely a lower charging energy so that charging energy is equal to the Josephson coupling energy, $E_C \sim E_J$.

We obtained such samples this summer from Chalmers University (a turn-around time of about one year). Most of our expectations for improvement have been borne out by measurements carried out in the fall and winter of 2003. First, the problem of excess quasiparticles ("poisoning") was reduced, allowing an observation of the fully two-electron periodic Cooper-pair staircase. We were also able to observe the modulation of the ground state of the box by tuning the flux through the SQUID loop, which modulates the Josephson coupling, or the tunnel splitting (sigma-x term) in the Hamiltonian. A comparison of the charge signals obtained with the previous samples and the improved devices this year is shown in Figure 5.

We have also performed microwave spectroscopy on these improved qubits, and studied the relaxation times. This spectroscopy confirmed that the design goal $E_C \sim E_J$ was achieved. Due to the improvements in the qubits, we could now study the transitions and energy spectrum of the box all the way down to the desired operation point, at the charge degeneracy point. This is the point where the qubit is minimally affected by 1/f noise. The data in Figure 6 requires approximately 8 hours to acquire, and indicates the excellent stability of the offset charge and the parity which was obtained. Because the microwave coupling at the chip level (see discussion in Section I on design of qubits with microwave coupling) is not possible so far with the Chalmers samples, the power coupled to the qubit, and its expected lifetime are not yet well controlled versus frequency. We were therefore not yet able to search for spurious resonances or interactions with impurities suggested by Martinis and co-workers.

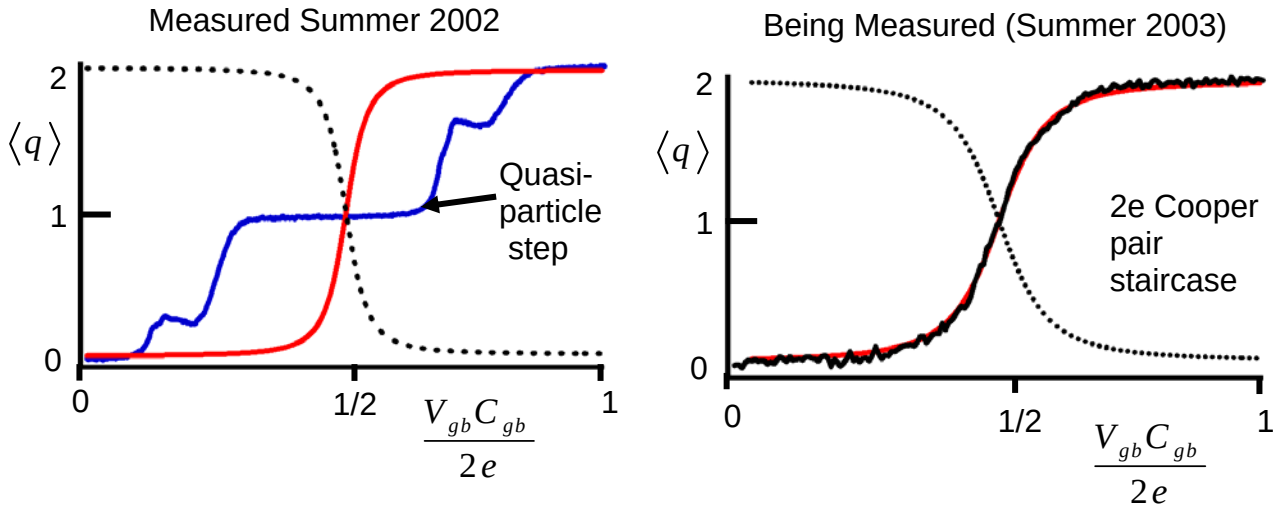



Figure 5: Comparison of charging staircases for two CPB qubits with different charging energies. On left is the old design with $E_C \sim \Delta$, causing non-equilibrium quasiparticles to be generated in the qubit, and obscuring the ground state of the qubit. On right is a recent measurement of a device with reduced charging energy, showing elimination of the quasiparticle feature, and a good measurement of the qubit, including at the charge degeneracy point (center of staircase) where 1/f noise effects are minimized.

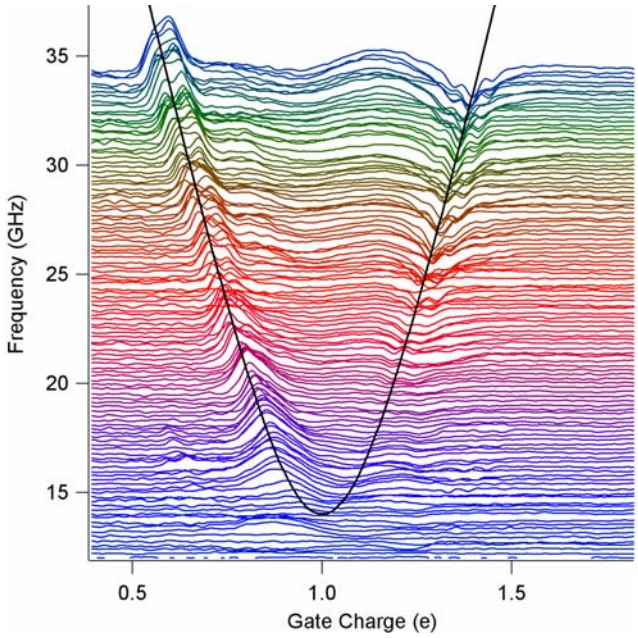



Figure 6: Spectroscopy of an improved CPB qubit as measured by an RF-SET. The photoresponse (i.e. the difference in the Cooper pair staircase with and without CW microwaves applied to the gate) is displayed as a function of both gate charge and microwave frequency. Shown is a fit to the simple hyperbolic energy spectrum of the box, which yields the energies of the qubit, $E_C$ = 18.9 GHz and $E_J$ = 14.9 GHz, and indicates that the design goal $E_C \sim E_J$ was achieved. Due to the improvement in the parity described above, the transition between the two lowest

states can now be followed down to the degeneracy point, $n_g$=1, shown at the center of the plot. This is the point where the qubit is minimally affected by 1/f noise. This data requires approximately 8 hours to acquire, and indicates the excellent stability of the offset charge and the parity which was obtained.

Following our suggestions for improved qubit parameters, and also using our techniques and experience gained from extensive spectroscopy of CPB qubits using the SET, the Chalmers team sought to observe coherent oscillations and repeat the original observation of Nakamura and co-workers. Using one of the improved samples described above, and the expensive picosecond pulse generator used by Nakamura (not available in our lab) they succeeded in observing these oscillations. These measurements, however, still used a continuous measurement by the SET, and operated away from the charge degeneracy point, thus inducing maximal 1/f dephasing, so that the observed coherence times were only on the order of one nanosecond.

This fall we have made extensive studies of the relaxation times (T1) of the new Chalmers samples. We have found that the dominant effect on the qubit lifetime can be the backaction of the SSET, and shown that this time can sometimes be reduced by more than an order of magnitude by the SSET. This observation is very important for our goals of using the SET as a qubit readout, and also for the superconducting quantum computing community as a whole. Though not always emphasized, the problem of short and/or variable relaxation times has been observed to be a major challenge for all groups, working either with charge, flux, or phase qubits. An example of this variability of relaxation times in shown in Figure 7. The relaxation of two different spectroscopy peaks in the CPB qubit are acquired simultaneously, with the only difference being the precise biasing point of the electrometer. The transitions of the box which are measured with a higher current and higher conductance thru the SET are observed to have much faster relaxation, in qualitative agreement with our model of the backaction of the SET. We have again observed, for appropriate operating conditions, relaxation times in excess of 1.2 microseconds, which is consistent with our expectations for spontaneous emission of the box into its 50 Ohm electromagnetic environment (here there is no cavity to suppress this decay!). We find that there are many areas of operation of the electrometer which strongly damp the qubit and cause the state to reset" in times of less than 100 nanoseconds. The theory of the backaction of the SSET, developed by our collaborators Aash Clerk and Steve Girvin under this project earlier, suggests that this fast relaxation can even occur under conditions in which the SET is nominally "off," and very little current flows. It also suggests an explanation for the observation of uniformly fast relaxation in the experiments at Chalmers using fast pulse generators. Since the fast pulses couple strongly to their RF-SET, they sample many of the "forbidden" regions of bias where the SET gives fast relaxation. This suggest to us that fast pulses, though capable of showing basic coherence, are not desirable in the long run for quantum computation, and that one must carefully control the qubit in order not to perturb the SET too strongly, which will lead to excess dephasing and relaxation in the control phase before the measurement.

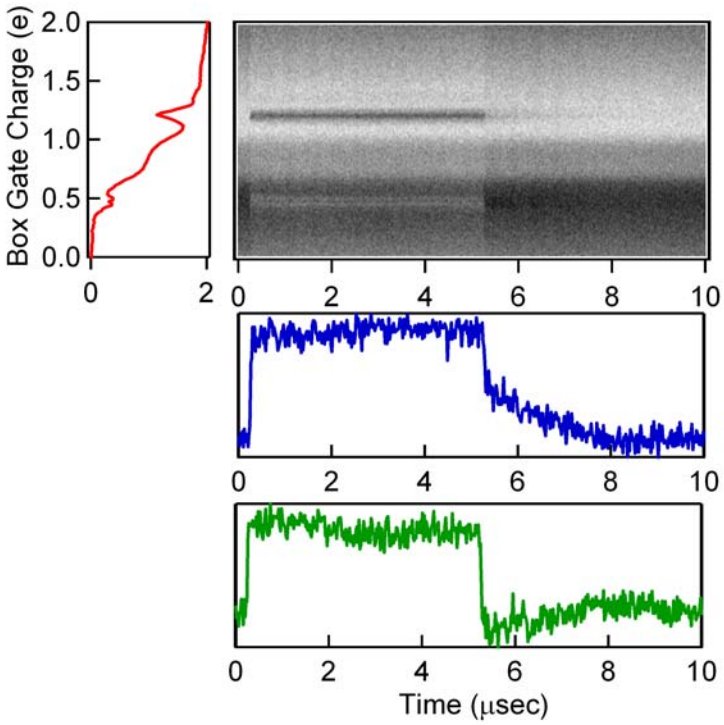



Figure 7: Measured dependence of relaxation time (T1) on the SET backaction. Top panel shows a grayscale image of the average charge on the CPB qubit as a function of time (horizontal axis) and gate charge (vertical axis). Each vertical slice of the image consists of a measurement of the Cooper-pair staircase, as shown by the line cut in upper left. In this measurement, a microwave pulse at 30 GHz is applied during the first 5 microseconds, and then switched off with a fall time of ~ 20 nanoseconds. The streaks in the greyscale image correspond resonant absorption of the microwaves by the qubit, i.e. the peaks shown in the line cut when the qubit transition frequency match the microwave stimulus. After the microwaves are switched off, the decay to the ground state of the box is measured using the RF-SET with a timing resolution of < 50 nanoseconds. The first of the lower linecuts shows the T1 decay of one of the peaks, with a slow component having a time constant of ~ 2 microseconds. The lower curve shows the second peak, which decays in a time less than the timing resolution of the experiment, i.e. over an order of magnitude

faster. The only difference in the two decays is caused by the differing backaction of the readout SSET at the two points, in agreement with theoretical predictions.

Another observation which confirms the importance of the SET's backaction to the dephasing and relaxation of the CPB qubit is that the measurement can actually induce a population inversion in our qubits. This effect was again predicted by our theory collaborators. It comes from the fact that the SSET backaction represents a coupling to a non-equilibrium noise source, which can preferentially excite, rather than relax the qubit. The population inversion is manifested as the observation that the system is more likely to occupy the excited state in the presence of a continuous measurement, and of an average charge greater than electron. Such a measurement is shown in Figure 8. Again, the qualitative behavior of this effect is in good agreement with theory, and induced only for certain specific qubit parameters and SET bias conditions. It also gives direct evidence that the interaction with the measurement circuitry can dominate over the passive portions of the circuitry, and over the intrinsic losses in the qubit. In other words, it indicates that the qubit may be of high fidelity if the readout is properly operated and optimized.

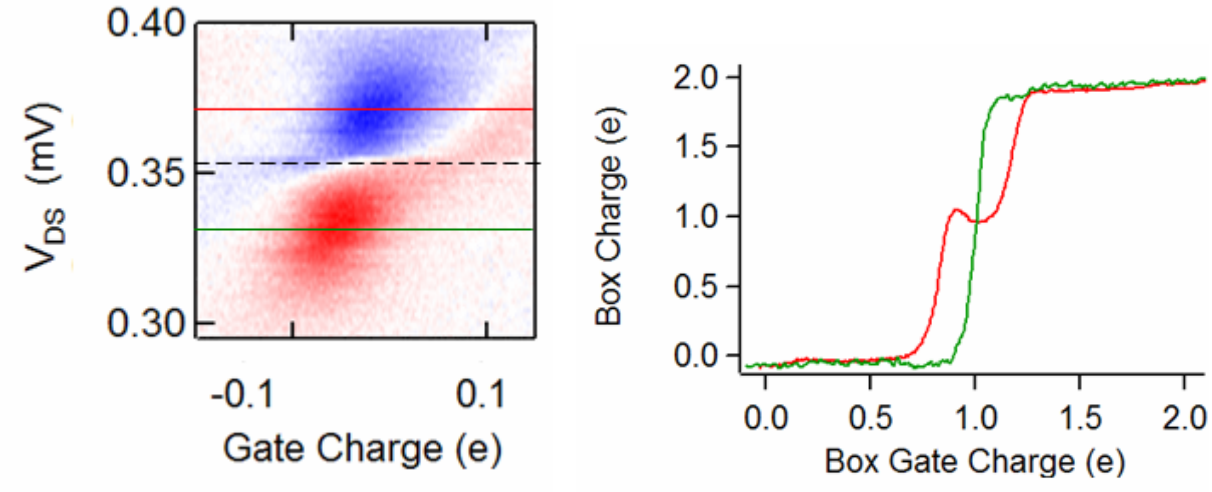



Figure 8: Observation of population inversion of the qubit due to SET backaction. Left panel shows the observed reflected RF power from the SSET within a narrow range of the DJQP resonance used for the measurement. The dashed line indicates the center of the DJQP resonance. The solid lines indicate the operating drain-source bias voltage used for two measurements of the Cooper-pair staircase of the box, shown in the right panel. These measurements are performed with the Josephson energy, $E_J$, tuned to a small value of about 5 GHz. Under these conditions, the ground state of the box is uniformly observed (in green, smooth step-like behavior) when the SET is biased below the DJQP resonance, and the SET tends to mostly absorb energy from the qubit. In contrast, the red trace, obtained with SSET biased above resonance, shows a prominent extra bump, with a charge greater than 1 electron, i.e. with a higher probability to be in the excited, rather than ground, state, which would correspond to a population inversion or a negative temperature of the qubit. This effect was independently predicted by earlier theory work under this project, and indicates that the SET is the dominant source of relaxation or excitation for the qubit.

We have therefore confirmed the detailed mechanism of the effect of the readout on the qubit, which was the next item on our flow chart for the SET/CPB project. This fact, and our detailed understanding, means that we can now design and fabricate (using device at Yale) a CPB qubit and SET readout which are optimized and capable of providing a single-shot readout and long coherence times. It emphasizes a continuing theme found in our experiments: one cannot simply design a qubit and expect to achieve long coherence times, without first understanding in detail the effects of the readout circuitry and how to operate it.

A final interesting observation regards the 1/f charge noise in our system. We often observe that the level of charge noise in the qubit seems much lower than that in the measuring SET. By controlling the bias on the SET, we studied how the low-frequency 1/f noise in the SET varied with the bias and power levels dissipated by the SET. We found a very striking effect, in which the 1/f noise could be dramatically increased when more power is dissipated. This suggests that the 1/f noise in superconducting qubits may be in part or in whole an "activated" process. By avoiding all dissipation on the chip, using for example the dispersive measurement techniques also being developed in our group, one may hope to reduce the 1/f noise and gain a corresponding increase in coherence times.

**III. Results from Jan – August 2004**

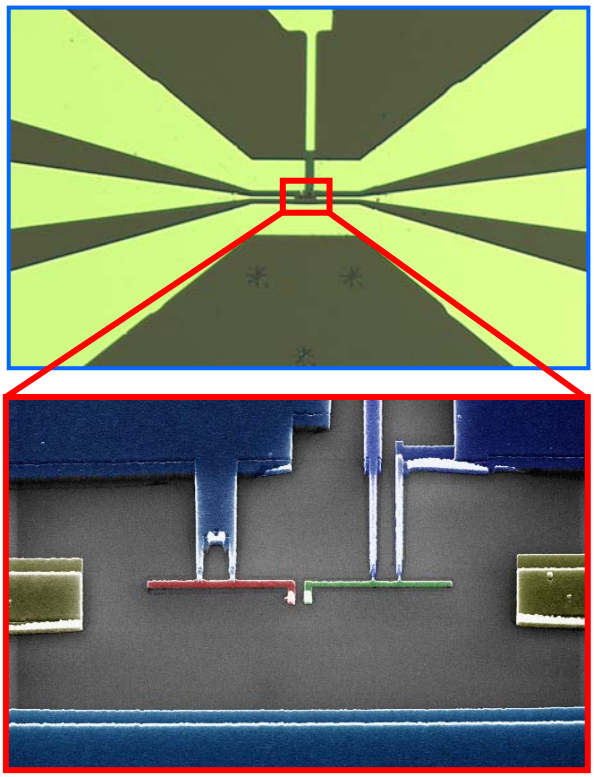
During the second half of the 2003 calendar year, we made several advances in our understanding of the SET as a measurement system for superconducting charge qubits. In particular, we observed that the backaction of the SET can have dramatic effects on both the polarization and relaxation time (T1) of the qubit during the measurement phase. We succeeded in measuring near-perfect 2e-periodic Cooper-pair staircases, indicating that the SET need not create quasiparticles in the qubit, and that the qubit can remain in the ground state, even in the presence of a continuous measurement. However, we also observed, under other conditions, that the SET can dramatically affect the steady-state population of the qubit, and can even create a measurement-induced population inversion in the qubit, as previously predicted by the theoretical part of our collaboration.

These results were extensively detailed in the 2003 calendar year interim progress report. During 2004, we have undertaken a detailed modeling effort in an attempt to determine the optimum parameters for an SSET readout of the

CPB, based on the backaction model. A redesign of the SET-CPB samples was undertaken, and a process which allows a control of the electromagnetic environment for the qubit on chip was implemented at Yale. Samples for the next experiments along this approach have been fabricated. SETs with sufficiently large charging energies to realize the double Josephson quasiparticle feature (DJQP) used for the SET measurements have been made and tested at 250 mK. Several papers detailing the preliminary results on the SET backaction and its suitability as a quantum readout of CPB qubits have been submitted for publication (see papers 1 & 2 listed below)

**IV. Results from final period: Aug 2004-Oct 2005**

During the Aug 2004-Aug 2005 period, we have continued with our efforts to measure the backaction of the RF-SET on the Cooper-pair box qubit, which is essentially to determining whether this is a viable method for performing single-shot readout of qubits. The present goal was to fabricate samples with a well-controlled electromagnetic environment, in order to study the effects of SET backaction on qubit lifetime. These devices turned out to be more challenging to fabricate than we had anticipated, but we have now been successful and final measurements are underway. A chip showing the Cooper-pair box qubit and RF-SET readout are shown in Figure 1, along with a measurement of the current-voltage characteristic of a qubit configured as a SQUID. The measurement of the IV curve displays resonances corresponding to points where the environment of the qubit changes abruptly, i.e. we are able to use the Josephson effect as a sort of “network analyzer” which probes the microwave engineering of the wiring. This confirms that we have succeeded in our goal of controlling the electromagnetic environment, at least up to ~ 30 GHz which is much higher than the transition frequency (about 6-8 GHz) planned for the qubits and recently measured. Characterization of a box has shown the desired energy level splittings in the qubit have been achieved, and that a working SET with proper characteristics to have a DJQP resonance used for readout is obtained. This work will conclude with a final set of measurements testing the backaction theory of the SET developed earlier in this project by our theory collaborators. A paper detailing preliminary measurements of the qubit ground state and showing full tenability of the qubit Hamiltonian have appeared this year in IEEE Transactions on Applied Superconductivity (Paper #3). An earlier paper quantitatively testing the backaction of the SET in the normal state also appeared in Physical Review B (Turek et al.; Paper #2).



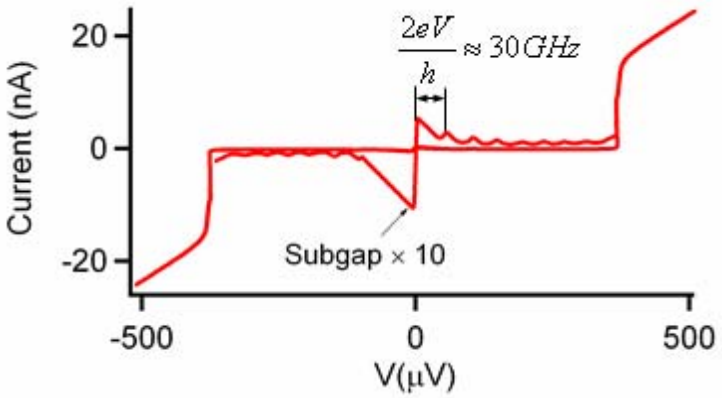



Figure 1: Left: Optical (top) and zoomed SEM micrographs of a completed CPB qubit –RF-SET readout chip with controlled electromagnetic environment. Right: Current-voltage characteristic of a “qubit” wired up as a SQUID for DC transport measurements, showing that, except for resonances seen by the qubit at relatively high frequencies of 30 and 50 GHz, the qubit should see a well controlled electromagnetic environment with 50 Ohm impedance. This sample is in use to test SET backaction limits on readout fidelity.

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1) “Quantum Fluctuations of Charge and the Polarizability of the Single-Electron Box,” K. W. Lehnert, B.A. Turek, K. Bladh, D. Gunnarsson, P. Delsing, and R.J. Schoelkopf, Phys. Rev. Lett 91, 106801 (2003).
2) “Single-Electron Transistor Backaction on the Single-Electron Box,” B.A. Turek, K. W. Lehnert, D. Gunnarsson, K. Bladh, P. Delsing, and R.J. Schoelkopf, Phys. Rev. B. 71, 193304 (2005).
3) “Backaction Effects of an SSET Measuring a Qubit: Spectroscopy and Ground State Measurements,” B. Turek, H. Majer, A. Clerk, S.M. Girvin, A. Wallraff, K. Bladh, D. Gunnarson, T. Duty, P. Delsing, and R.J. Schoelkopf, IEEE Trans. on Appl. Superconductivity **15**, 880 (2005).

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# Quantum Charge Fluctuations and the Polarizability of the Single-Electron Box

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We measure the average charge on the island of a single-electron box, with an accuracy of two thousandths of an electron. Thermal fluctuations alone cannot account for the dependence of the average charge on temperature, on external potential, or on the quasiparticle density of states in the metal from which the box is formed. In contrast, we find excellent agreement between these measurements and a theory that treats the quantum fluctuations of charge perturbatively.

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A general feature of quantum many-body phenomena is the screening of a single degree of freedom by a bath of virtual excitations. The Lamb shift and the Kondo effect are well-known examples, where the discrete states of a hydrogen atom or a magnetic impurity are renormalized by the quantum fluctuations of an environment of virtual photons or virtual spin flips. In single-electron circuits [1], such as the single-electron transistor (SET) [2], the charge pump [3], or the single-electron box [4], the same sort of quantum fluctuations exist in a system which can be controlled and measured electrically. These fluctuations arise from the virtual tunneling of electrons between the metal islands and the metal leads that comprise single-electron devices. Electron-hole pairs, generated by the virtual tunneling, partially screen the charge on the islands and modify the discrete spectrum of charge states. The single-electron box, the simplest single-electron circuit, is the ideal system in which to test the theory of quantum charge fluctuations.

The box has been studied theoretically [5–9] because it is a model system for understanding electron-electron interactions and because the quantum fluctuations in the box are analogous to both the Kondo effect [5] and the Lamb shift. In spite of the extensive theoretical work, few experiments have probed the fluctuations described by Refs. [5–9]. Those experiments that have done so are mostly in semiconductor dots [10–12]; whereas the theory of Refs. [6–9] describes metallic systems, such as our box or Refs. [13,14], in which the tunnel junctions comprise many nearly opaque channels. Because the quantum fluctuations screen an electron with a polarization charge much less than one electron, very sensitive charge measurements are required to resolve the fine structure associated with these fluctuations.

In this Letter, we measure the time-averaged charge on the island of a single-electron box with an accuracy much better than one electron using a radio-frequency SET (rf-SET) [15]. We observe quantum fluctuations of charge, and we modify the strength of these quantum fluctuations by changing the temperature, the external potential, and the quasiparticle density of states of the metal in which the tunnel junction is embedded. In each case, we find quantitative agreement between our results and the theory of quantum fluctuations.

Our single-electron box is composed of an isolated aluminum island attached to an aluminum lead through a thin insulating layer across which electrons can tunnel. A 1 T magnetic field is applied to keep the aluminum in its normal (nonsuperconducting) state. An additional lead, called the gate lead, lies near the island and changes the electrostatic potential of the island with the application of a voltage $V_g$ to the gate lead through the gate capacitance $C_g$. The total island capacitance $C_\Sigma^0$ is small enough that the addition of a single electron to the island requires a large electrostatic energy

$$U_n = E_C^0(n - n_g)^2, \tag{1}$$

where $E_C^0 = e^2/2C_\Sigma^0$ is the charging energy, $n$ is the number of excess electrons on the box, and $n_g = C_g V_g/e$. The minimum energy is clearly achieved when $n$ is the integer nearest $n_g$; when $n_g = 0.5$ the two lowest-energy charge states are degenerate.

Equation (1) ignores the quantum fluctuations, or the effects of the coupling of island and lead through the tunnel junction. The junction couples the charge states to each other and to quasiparticle excitations in the metal on either side of the junction. This alters the spectrum of states in Eq. (1) in three ways. First, the charging energy is reduced ($C_\Sigma^0$ is enhanced) from its bare value $E_C^0$ to a renormalized value $E_C^N = e^2/2C_\Sigma^N$ in the normal state or $E_C^S = e^2/2C_\Sigma^S$ in the superconducting state. Second, when a pair of states are nearly degenerate their energy difference becomes temperature dependent. Finally, the electrostatic energy $U_n$ of the charge states is no longer quadratic in $n_g$. The magnitude of these three effects is calculated with a theory perturbative in the dimensionless conductance $g = R_K/(4\pi^2 R_j) = (h/e^2)/(4\pi^2 R_j)$ [6,7,9], where $R_j$ is the box junction resistance. By measuring the average charge on the box island $Q_{\mathrm{box}}/e$ versus $n_g$ (Coulomb staircase) with an uncertainty less than $g$, we can compare all three effects with theory.

  

We measure $Q_{\rm box}$ as a function of $n_g$ by coupling the box island to an rf-SET electrometer through a capacitor $C_C$ [15] [Fig. 1(a)]. The quantity that we measure directly is the charge coupled to the electrometer $Q_{\rm elec}$ versus $n_g$, which approximates a sawtooth function (Coulomb sawtooth). We infer the Coulomb staircase as $Q_{\rm box} = (n_g e\kappa - Q_{\rm elec})/\kappa$ [Fig. 1(c)], where $\kappa = C_C/C_\Sigma^N$ is the fraction of the charge on the box that couples to the electrometer. The value of $e\kappa$ is not an independently known parameter; rather, it is determined as the slope of the Coulomb sawtooth at $n_g = 0$, *assuming* that $Q_{\rm box}$ is independent of $n_g$ at $n_g = 0$. Following this procedure, we extract a value of $\kappa = (3.35 \pm 0.05) \times 10^{-2}$. Our assumption is valid if we interpret $C_\Sigma^N$ (and $E_C^N$) determined in our experiments as a value renormalized by tunneling, not the bare, geometrical value $C_\Sigma^0$, which is a parameter in the theory of Refs. [6,7]. While we cannot prevent tunneling and measure $C_\Sigma^0$, we can suppress tunneling and observe a variation in the total box capacitance.

We find the first evidence of quantum fluctuations by examining the temperature dependence of the Coulomb staircase. We measure the staircase at a high temperature ($T = 500$ mK) and extract a value of $E_C^N/k_B = 1.57 \pm 0.05$ K ($C_\Sigma^N = 590 \pm 20$ aF) by assuming thermal broadening, that is, a Boltzmann occupation of the states in [Eq. (1)]. In the range 200–500 mK, we find excellent agreement [Fig. 2(a)] between the measured staircase and thermal broadening in a comparison with no adjustable parameters. Below 200 mK, thermal fluctuations characterized by any single temperature cannot account for the measured staircase [Fig. 2(b)]. As the temperature of the cryogenic apparatus is reduced, away from $n_g = 0.5$ the staircase remains rounded as if the box's temperature were saturating around 130 mK. Nevertheless, the staircase grows continually sharper at

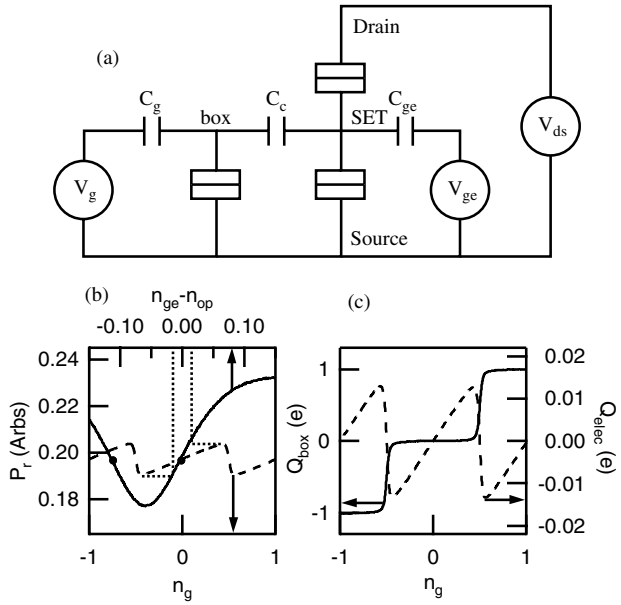


FIG. 1. (a) Circuit diagram of the single-electron box capacitively coupled to an rf-SET electrometer. The tunnel junctions are represented by boxes divided by a horizontal line. The junction capacitance $C_J$ is the dominant component of the total box capacitance, $C_\Sigma^0 = C_J + C_g + C_C$. Additional circuit elements (not shown) apply an rf signal between the SET's drain and source and detect the amount of rf power reflected $P_r$ from the rf-SET [15]. (b) Calibration of the Coulomb sawtooth is accomplished by varying the SET's control gate voltage $V_{\rm ge} = n_{\rm ge}(e/C_{\rm ge})$ about a fixed operating voltage $V_{\rm op} = n_{\rm op}(e/C_{\rm ge})$ while the box gate is held at $n_g = 0$. This applies a known charge signal $Q_{\rm elec} = e(n_{\rm ge} - n_{\rm op})$ to the SET. The plot $P_r$ versus $n_{\rm ge} - n_{\rm op} = Q_{\rm elec}/e$ (solid line, top axis) is a nonlinear map (implied by dotted lines) that converts $P_r$ versus $n_g$ (dashed lines, bottom axis) into $Q_{\rm elec}$ versus $n_g$. The electrometer's operating point $n_{\rm op} = 0.44$ and an alternative $n_{\rm op} = 0.56$ and are indicated (two dots). (c) The Coulomb sawtooth, $Q_{\rm elec}$ versus $n_g$, (dashed line) on the right axis, and the Coulomb staircase, $Q_{\rm box}$ vs $n_g$, (solid line) on the left axis at $T = 30$ mK.

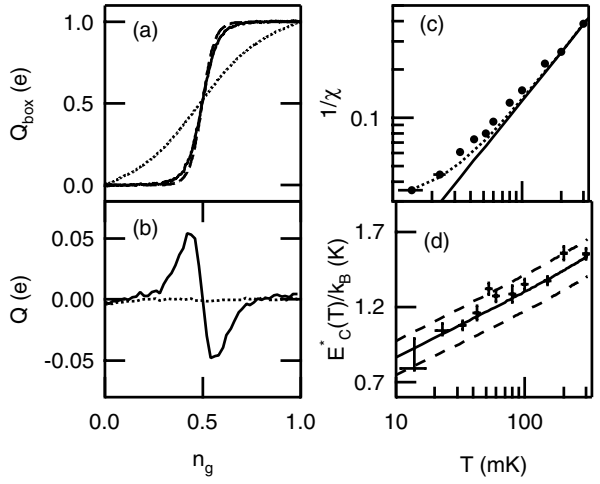


FIG. 2. (a) Coulomb staircases at $T = 500$ mK (dotted line) and $T = 100$ mK (solid line). The charging energy $E_C^N$ is extracted by fitting to the 500 mK data a theoretical staircase (not shown) broadened only by thermal fluctuations. The 100 mK staircase is compared to the thermal fluctuation theory with no adjustable parameters (dashed line). (b) The residuals of the 500 mK fit (dotted line) and of the 100 mK comparison (solid line). (c) Plotted versus $T$ on logarithmic scales are the measured value of $(1/\chi)$ (points), the expression $1/\chi = 2k_BT/E_C^N$ (line) showing the expected behavior in the absence of quantum fluctuations, and the expression $1/\chi = 2k_B\sqrt{(T^2 + T_{\rm sp}^2)}/E_C^N$ showing the expected behavior in the absence of quantum fluctuations but in the presence of a spurious broadening characterized by a phenomenological effective temperature $T_{\rm sp} = 25$ mK (dotted line). A model of temperature-independent spurious broadening does not contain the observed behavior of $\chi$ versus $T$. (d) The quantity $E_C^*(T) = 2k_BT\chi$ (points) and the prediction of [9] (line) versus $T$ with no adjustable parameters. Dashed lines indicate the range of theory consistent with the uncertainties in $g$ and $E_C^N$. The error bars of two lowest $T$ points account for a systematic rounding introduced by the SET's backaction [16].

electrostatic degeneracy, $n_g = 0.5$, consistent with temperatures below 30 mK. Because the box is most sensitive to external noise at degeneracy, this surprising behavior is both inconsistent with an external source of noise and a qualitative hallmark of quantum fluctuations [10].

The theories of Refs. [5,8,9] predict the slope $\chi = (1/e)dQ_{\rm box}/dn_g$ of the Coulomb staircase, essentially the polarizability, at $n_g = 0.5$ as a function of temperature [8,9]. Because $\chi = E_C^N/2k_BT$ in the absence of quantum fluctuations, a plot of $1/\chi$ versus $T$ reveals the quantum fluctuations in its deviation from a line with slope $2k_B/E_C^N$ [Fig. 2(c)]. Near $n_g = 0.5$, where the two lowest charges states are nearly degenerate, quantum fluctuations cause a temperature-dependent reduction in the energy separation, $U_1 - U_0 = E_C^*(0.5 - n_g)$, of the levels, described by a reduced $E_C^* = 2k_B\chi T < E_C^N$. Reference [9] implies $E_C^* = E_C^N[1 - 2g(3.154 + \ln(E_C^0/\pi k_BT))] + \mathcal{O}[g^2, (k_BT/E_C^N)^2]$, where $E_C^0 = E_C^N(1 + 4g + \mathcal{O}(g^2))$. Note the similarity to the Kondo effect where the screening of a localized magnetic impurity by itinerant spins leads to a logarithmic in $T$ correction of the impurity's magnetic moment [5]. In Fig. 2(d), we plot $E_C^*(T)$ versus $T$ and find good agreement with Ref. [9], in a comparison with no adjustable parameters. This same effect was observed in SET's by Joyez *et al.* [13].

To make the comparison with theory, we must have an independent determination of the dimensionless conductance of the box, $g = (4.2 \pm 0.2) \times 10^{-2}$, which can be obtained by studying the box in its superconducting state. With no applied magnetic field, the aluminum superconducts, and the parameters of the box, $C_\Sigma^S = 518 \pm 6$ aF and $R_j = 15.4 \pm 0.9\ \mathrm{k}\Omega$, can be extracted by microwave spectroscopy of the coherent two-level system formed by the coupling of Cooper pairs between the lead and the island [18]. What is directly measured is the charging energy in the superconducting state $E_C^S/k_B = 1.79 \pm 0.02$ K and the Josephson energy $E_J/k_B = (h\Delta/8e^2R_jk_B)F(E_C^0/\Delta) = 0.62 \pm 0.01$K, where $\Delta/k_B = 2.4 \pm 0.1$ K is the BCS gap in aluminum and $F(E_C^0/\Delta)$ is a function that accounts for Coulomb blockade effects by modifying the usual Ambegakor-Baratoff relation [19]. For our sample $F(E_C^0/\Delta) = 1.25$. Note that in the superconducting state $E_C^S = e^2/2C_\Sigma^S$ is not the same as $E_C^N = e^2/2C_\Sigma^N$ in the normal state. This difference reflects the different quantum fluctuations of a metal with a superconducting or with a normal quasiparticle density of states (DOS).

We are able to tune this influence of the DOS by continuously reducing $\Delta$ in the aluminum with an applied magnetic field $B_{\rm app}$. We observe that $C_\Sigma^S$ is a function of $\Delta(B_{\rm app})$ by measuring $\kappa^S = C_C/C_\Sigma^S$, the slope of the Coulomb sawtooth at $n_g = 0$, as the aluminum is driven from the fully superconducting state to the normal state. With increasing $B_{\rm app}$, $\kappa^S$ is reduced continuously from a value $\kappa^S = (3.70 \pm 0.05) \times 10^{-2}$ with $B_{\rm app} = 0$ to $\kappa = (3.35 \pm 0.05) \times 10^{-2}$ in the normal state (Fig. 3). Because both $\kappa^S$ and $E_C^S$ are proportional to $1/C_\Sigma^S(B_{\rm app})$ we infer $E_C^N/k_B = [E_C^S(B_{\rm app})/k_B][\kappa/\kappa^S(B_{\rm app})] = 1.62 \pm 0.04$ K, which is consistent with the value $1.57 \pm 0.05$ K extracted from the broadening of the Coulomb staircase at high temperatures.

The theory of the normal box [7] predicts that the effects of tunneling can be treated around $n_g = 0$ as a renormalization of $C_\Sigma^0$ to a value larger by the factor $[1 + 4g + 10.93g^2 + \mathcal{O}(g^3) = 1.18]$. The renormalization of the bare capacitance in the superconducting state $C_\Sigma^S/C_\Sigma^0$ is predicted to be smaller than in the normal state because the quasiparticle excitations have a minimum energy $\Delta$, which suppresses the virtual tunneling. The bare capacitance is not an experimentally accessible parameter; however, the perturbative techniques of Ref. [7] can be used [20] to calculate the renormalization of $C_\Sigma^0$ for a metal with a BCS, rather than constant, DOS. Inverting this, we infer from the normal state ($C_\Sigma^0 = 498 \pm 16$ aF) and from the superconducting state ($C_\Sigma^0 = 478 \pm 7$ aF) values for the bare capacitance that are consistent with each other. By altering the DOS, we have observed that the capacitance of a tunnel junction is not a property of tunnel junction alone, but also of the spectrum of low-energy excitation in the metal from which it is made.

We have already seen that the electrostatic energy of the box is both a function of the temperature and of the

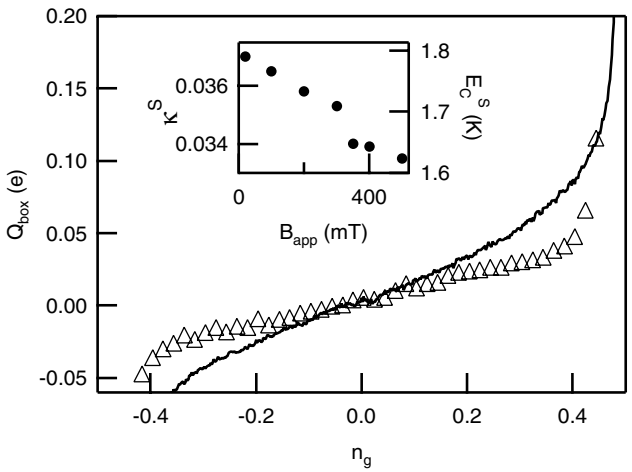


FIG. 3. Coulomb staircases with the box in its normal state (line) and in its superconducting state, $B_{\rm app} = 100$ mT (triangles). For both, the Coulomb sawtooths have been converted to staircases using $\kappa^0 = C_C/C_\Sigma^0 = 3.9 \times 10^{-2}$, which would be the slope of the Coulomb sawtooth around $n_g = 0$ in the absence of tunneling. The renormalization of $C_\Sigma^0$ is visible as the nonzero slope $(1/e)dQ_{\rm box}/dn_g$ of these plots at $n_g = 0$. The inset shows $\kappa^S(B_{\rm app})$ versus $B_{\rm app}$ and the value of $E_C^S(B_{\rm app})$ inferred from $\kappa^S$, as the aluminum is driven from its superconducting to its normal state. In the superconducting state, a single out-of-equilibrium quasiparticle on the box's island keeps the Coulomb staircase $e$ periodic.

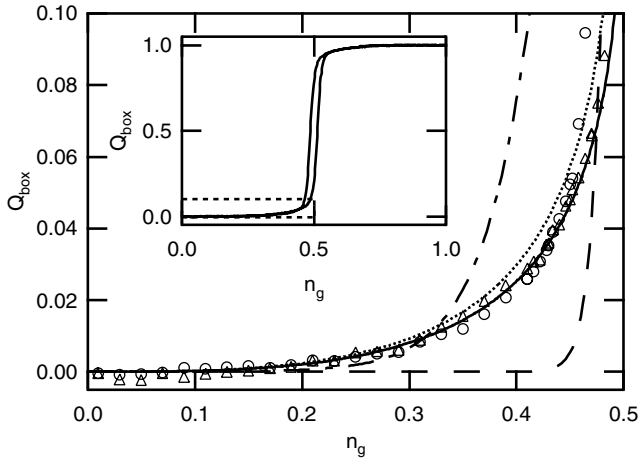


FIG. 4. Coulomb staircases with the box at two different electrometer operating points, $n_{op} = 0.44$ (triangles) and $n_{op} = 0.56$ (circles); $V_{ds} = 0$ for both. The SET's backaction causes these two curves to deviate from each other around $n_g = 0.5$. The theory plots are the charge on the box predicted for thermal fluctuations but no quantum fluctuations at $T = 125$ mK (dashed dotted line) and $T = 29$ mK (dashed line), and for the quantum fluctuations calculated to first order in $g$ (dotted line) and second order in $g$ (solid line) [6,7]. The second order calculation is fit to the data with the adjustable parameter $\kappa = (3.375 \pm 0.001) \times 10^{-2}$, which is better constrained by this fit than by extracting the slope of the Coulomb sawtooth. Note we have plotted both data and theory in the form with $dQ_{box}/dn_g = 0$ at $n_g = 0$ unlike Refs. [6,7]. Inset: the same data over a range $0 < n_g < 1$. The dashed box indicates the region plotted in the main figure.

quasiparticle DOS in metal lead and island. We now show that the ground-state energy deviates from the parabolas of Eq. (1). Because $Q_{box}/e = n_g - (1/2E_C^0)(dU_n/dn_g)$ at $T = 0$ [6], Eq. (1) implies perfectly flat steps in the Coulomb staircase, whereas we observe some curvature around $n_g = 0$ even at $T \ll E_C^N$. The ground-state energy cannot be quadratic in $n_g$. This modification of the ground-state energy is the Lamb shift in the single-electron box.

It is precisely the detailed shape of the Coulomb staircase at $T = 0$ that is predicted by [6,7] and which provides the most stringent test of the theory (Fig. 4). We find that $Q_{box}$ deviates from a perfect step function by several percent in the range $0 < n_g < 0.45$ [16]. In this region at the base temperature of our cryogenic apparatus, we may consider the box to be in a zero temperature limit and ignore the influence of the electrometer (Fig. 4). To an accuracy of $2 \times 10^{-3}$ e, limited by the linearity of the applied gate voltage, we find agreement with this theory. Our measurement is sufficiently accurate and sensitive that the perturbative calculation of Ref. [7] must be carried out to second order to show agreement with our experiment, even for the relatively small value of $g = 4.2 \times 10^{-2}$.

In these experiments, we have used an rf-SET electrometer to measure the polarizibility of a mesoscopic electrical circuit. We have chosen to apply this technique to the single-electron box, a model system for understanding electron-electron interactions whose Hamiltonian is analogous to the Kondo Hamiltonian. We find excellent agreement between our measurements and a perturbative treatment of the quantum fluctuations. The excellent agreement between our measurements and theory both supports this theory and demonstrates the precision electrometry possible with the rf-SET. The technique we demonstrate would be an ideal method for exploring the equilibrium behavior of more complicated mesoscopic circuits, such as semiconductor quantum dots or carbon nanotubes.

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# Single-electron transistor backaction on the single-electron box

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We report an experimental observation of the backaction of a single-electron transistor (SET) measuring the Coulomb staircase of a single-electron box. As current flows through the SET, the charge state of the SET island fluctuates. These fluctuations capacitively couple to the box and cause changes in the position, width, and asymmetry of the Coulomb staircase. A sequential tunneling model accurately recreates these effects, confirming this mechanism of the backaction of a SET. This is a first step toward understanding the effects of quantum measurement on solid-state qubits.

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In the recent work toward the goal of quantum computing, and in the study of single quantum systems in general, the single-electron transistor (SET) is often used as a measurement device. It has been proposed as a readout device for mechanical,[1] spin,[2] and charge[3] quantum systems, and has been successfully used to measure superconducting charge qubits.[4] As with any amplifier, the SET must produce electrical noise on its input, perturbing the measured system and causing the unavoidable backaction of a quantum measurement.

SET backaction on a two-level system has been studied extensively in the theoretical literature. It has been determined that the SET should be able to approach the quantum limit of backaction, where it dephases a qubit as rapidly as it is reads the qubit state.[5] Spectral components of the SET backaction at the two-level system transition frequency can also contribute to transitions between two qubit states.[6,7] A qubit could thus form a spectrum analyzer capable of probing previously inaccessible frequencies.[8] These theoretical analyses presume SET backaction results from fluctuations in the charge state of the SET island caused by the drain-source current, but no experimental measurements exist confirming that this is the dominant or the sole mechanism of the SET's backaction. Indeed, it often appears that the SET can poison the Cooper-pair box, inducing nonequilibrium quasiparticles through other mechanisms.[9,10]

As a first quantitative test of SET backaction, we consider the SET and box operated in the normal (nonsuperconducting) state, created with the application of a 1-T magnetic field. Analysis of the normal box is simpler than in the superconducting state because the box is no longer sensitive to parity and quasiparticle generation. The normal SET can also be simply described by a sequential tunneling model, which avoids the complication of the many possible quasiparticle-pair tunneling cycles[11] in the superconducting SET. Nevertheless, the primary mechanism of SET backaction is still the capacitive electromagnetic coupling between the box and SET, and the box remains a mesoscopic device that is sensitive to this backaction. Just as with the SSET-Cooper-pair box system, sensitive measurements of the Coulomb staircase of the normal box can reveal the dynamics of the coupled system, and probe the nature of SET backaction.

The possibility of SET backaction on a single-electron box was proposed with experiments in the field,[12] but has proven difficult to quantify. The signature of SET electrical backaction is difficult to separate from simple heating of the sample.[13,14] The backaction has been measured with very strong coupling between the SET and the box,[15] but few measurements exist in systems that are as weakly coupled as the proposed Cooper pair box-SET experiments. In this Brief Report, we present an experimental analysis of a SET weakly coupled to a single-electron box. We vary the operating point of the SET, measure the Coulomb staircase of the box, and find the variations in the shift, width, and asymmetry of the staircases to be in agreement with a model that includes backaction caused by the charge-state fluctuations of the SET island. These variations in the measured staircases allow us to measure average properties of the noise of the SET.

The SET [Fig. 1(a)] consists of a aluminum island connected through tunnel junctions to two leads (the drain and the source) and capacitively coupled to a third (the gate). A SET is described by its charging energy ($E_c = e^2/2C_\Sigma$, the energy to add an additional electron to the island), by the tunneling resistance of the junctions on the drain and the source leads ($R_j$), and by the size of the capacitors coupling it to the external control voltage ($C_{ge}$) and to the measured

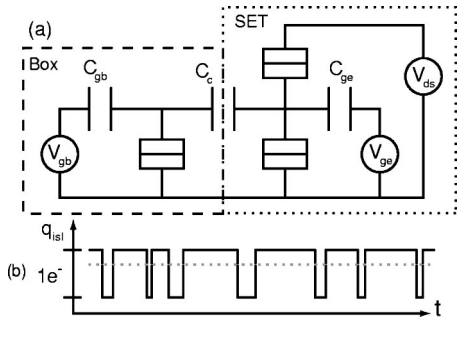


FIG. 1. (a) Circuit diagram of the single-electron box (dashed box) capacitively coupled to the SET (dotted box). The normal-state tunnel junctions are represented by boxes with a single line through them. (b) Plot of the charge state on the SET island vs time. The dotted line shows the mean value of the charge on the SET island.

 

system ($C_c$). A high tunneling resistance ($R_j > h/e^2$) and large charging energy ($E_c > k_B T$) suppress the addition of charge to the island by quantum or thermal fluctuations, so the island may be considered confined to a discrete set of charge states. A bias voltage ($V_{ds}$) provides the energy necessary for the system to switch between charge states, allowing current to flow from the drain to the source. The amount of current is controlled by the rate of transition between accessible charge states, which is a function of the potential of the island. Thus the SET forms a very sensitive electrometer, where changes in the total charge capacitively coupled to the island modulate the current flowing through the transistor. The SET is operated by fixing the values of the externally applied $V_{ds}$ and $V_{ge}$, and observing variations in the conductance as the charge coupled to the SET from the measured system changes. The point at which $V_{ds}$ and $V_{ge}$ are fixed is termed the operating point; the same measurement can be performed by observing conductance variations about many different operating points.

The box [Fig. 1(a)] consists of another island capacitively gated by an external lead ($V_{gb}$) and connected through a tunnel junction to ground. As with the SET, the gate lead controls the potential of the box and changes the relative electrostatic energies of the available charge states. We express the gate voltages for both the box and the electrometer in terms of the number of electrons on the corresponding gate capacitors: $n_{gb} = C_{gb} V_{gb}/e$ and $n_{ge} = C_{ge} V_{ge}/e$. When $n_{gb}$ is raised by one electron, the island charge state of minimum energy changes, and a single electron tunnels on to the island to keep it in its ground state. Plotting the time-averaged number of additional electrons on the island as a function of $n_{gb}$ gives the familiar "Coulomb staircase" [Fig. 2(b)].[12] The width of this staircase is normally a function only of the temperature of the sample. In this Brief Report we quantify SET backaction by observing additional variations in the Coulomb staircase that are systematic with the SET operating point.

The coupling capacitor [$C_c$ in Fig. 1(a)] couples together the potential on the two islands, allowing the SET to measure the box and also allowing the potential on the SET island to affect the box. The strength of this coupling is expressed either as the fraction of the electrometer charge coupled to the box ($\kappa = C_c/C_{\Sigma_{SET}}$) or as the temperature necessary to cause changes in a Coulomb staircase comparable to those caused by backaction ($T_\kappa = \kappa E_{c_{box}}/k_B$). As the polarization charge on $C_c$ changes, the total charge coupled to the SET changes, changing the tunneling rates in the SET and modulating the current that flows from the drain to the source. The charge on the box is then inferred from the change in current through the SET. $C_c$ also couples the charge on the SET island to the box, and in doing so creates the effects that we see as the SET's backaction.

The discrete nature of charge causes two kinds of noise in the SET. The drain-source current flows not as a continuous fluid, but as individual charges, causing an uncertainty in the SET's measurement due to shot noise. In addition to shot noise on the output (the drain-source current), there is also charge noise on the SET input (the gate capacitor) that affects the measured system. Electrons tunneling on and off the island cause both the charge state and the potential of the SET island to fluctuate between two values [Fig. 1(b)]. The fluctuating potential on the SET island coupled through $C_c$ is found to be the source of the SET's backaction. Three averaged properties of the fluctuating potential have effects visible on the Coulomb staircase and can be varied with the operating point of the SET. The mean charge on the SET island varies by as much as one electron, and leads to shifts in the position of the Coulomb staircase by as much as $\kappa e$. The rms magnitude of the charge fluctuations on the SET island broaden the measured Coulomb staircase by an amount that varies with $n_{ge}$. Finally, the telegraph-noise nature of the charge-state fluctuations on the SET island causes the staircases to be asymmetric; the magnitude and direction of that asymmetry varies with the SET's operating point.

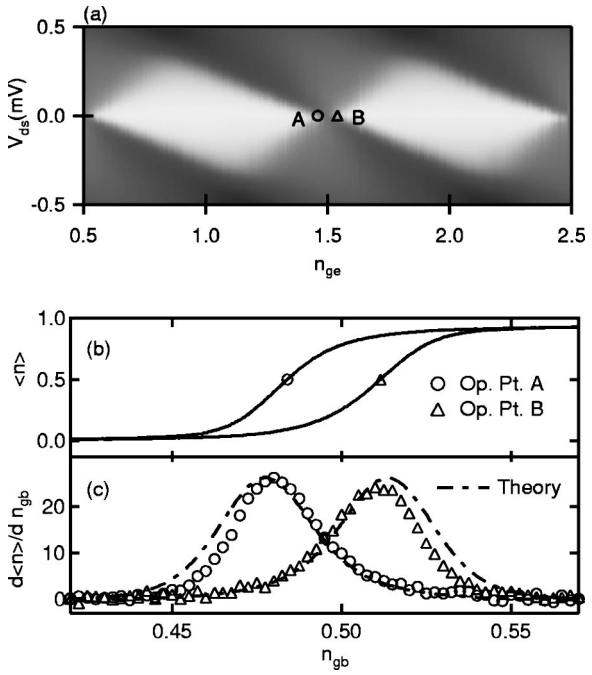


FIG. 2. (a) Plot of the reflected power from the SET as a function of gate ($n_{ge}$) and drain-source ($V_{ds}$) voltage. (b) Coulomb staircases measured at the operating points marked in (a) as a function of the box gate voltage $n_{gb}$. The time-averaged number of electrons on the box is measured with a precision of $\pm 1 \times 10^{-3}$ and an accuracy of $\pm 2 \times 10^{-3}$. (c) Derivatives of these Coulomb staircases and of the corresponding Coulomb staircases generated with a sequential tunneling model with $E_{C_{SET}}/k_B = 2.3$ K, $E_{C_{box}}/k_B = 1.6$ K, $R_{j_{SET}} = 47$ kΩ, $R_{j_{box}} = 15.4$ kΩ, and $C_c/C_{\Sigma_{SET}} = 0.048$. The derivative of the Coulomb staircase is reported with an accuracy of ±0.4.

A sequential tunneling model for the full SET-box system accurately recreates both the measurement and the backaction. The tunneling rates between any two box and SET charge states are calculated as a function of $n_{ge}$, $n_{gb}$, and $V_{ds}$ (for details, see Ref. 16). The time-averaged charge state of the SET-box system corresponds to the steady state of these coupled rates. The current through the transistor is calculated as the product of the time-averaged charge on the SET island and the rate at which charge tunnels off the island. This model allows us to replicate the Coulomb staircases taken at various operating points with only the electron temperature

as a free parameter. The elevated temperature of the best-fit model steps ($T=27\pm1$ mK in a fridge at $T=13$ mK) reflected the broadening of the measured steps due to quantum fluctuations of charge,[17] and is well understood. Theoretical curves also correctly account for higher-order effects in the box-SET system. At certain operating points (e.g., $n_{ge}=\frac{1}{2}$, $V_{ds}=0$, $n_{gb}=\frac{1}{2}$), the SET's backaction is a sensitive function of the state of the box. Changes in the Coulomb staircase measured at such operating points can only be understood by a sequential tunneling model for the full coupled box-SET system.

Coulomb staircases were measured in a dilution refrigerator at 13 mK, where the available thermal energy was far less than the charging energy of either the SET or the box island. The SET was operated as a rf-SET,[18] with a LC resonant circuit reflecting an amount of microwave power that varied as the oscillator was damped by the varying conductance of the SET. Staircases were measured by sweeping $n_{gb}$ over a range corresponding to $1/4e$. While the box gate was swept, the SET gate was swept in the opposite direction to cancel the parasitic capacitance of the box lead to the electrometer's island. Before each Coulomb staircase was measured, $n_{ge}$ was swept to find the reflected microwave power as a function of charge coupled to the SET island. Variations in reflected power with $n_{gb}$ were then converted (via this lookup table) to charge on $C_c$ (for a more detailed description, see Ref. 17). The measured charge on the box is thus reported from the amount of charge on $C_{ge}$ necessary to cause an equivalent electrometer response.

Backaction effects were found to be very sensitive to variations in $n_{gb}$ and $n_{ge}$ and our experiment therefore required that these voltages be set with high precision. Drifts were removed by referencing the steps to a fiducial step every 20 min. First, $n_{ge}$ was swept at $V_{ds}=0$ and the value of $n_{ge}$ that maximized SET conductance was determined as $n_{ge}=\frac{1}{2}$ [see Fig. 2(a)]. Next, a Coulomb staircase was measured with the SET operated at $n_{ge}=0.44$, $V_{ds}=0$. The value of $n_{gb}$ at the center of this step was determined. Charge offset noise and $1/f$ noise drifts add constant offsets to either $n_{ge}$ or $n_{gb}$; measuring the fiducial step as described here allows us to quantify the change in these offsets on both the box and the SET. Measurements found to contain large charge jumps in $n_{ge}$ or $n_{gb}$ were discarded. This procedure allowed measurement of Coulomb staircases with an uncertainty of $1\times10^{-3}e$ in the charge and an uncertainty of $5.5\times10^{-4}e$ in the horizontal position of the steps. The uncertainty in the applied $n_{ge}$ was found to be $5\times10^{-3}e$.

The differences in Coulomb staircases measured at different operating points allow us to measure average properties of the fluctuating potential of the SET island. Staircases measured at different operating points are shifted in $n_{gb}$ [Fig. 2(b)]. The shift of each staircase is proportional to the mean charge on the SET island. The mean charge on the SET island varies by as much as one electron with SET operating point, and the corresponding charge that couples to the box and adds to $n_{gb}$ varies by as much as $\kappa e$. We measure staircase shift by reporting the value of $n_{gb}$ at each step's midpoint, measured relative to the center of a fiducial step [Fig. 3(a)]. The sequential tunneling model accurately recreates these variations in the step position.

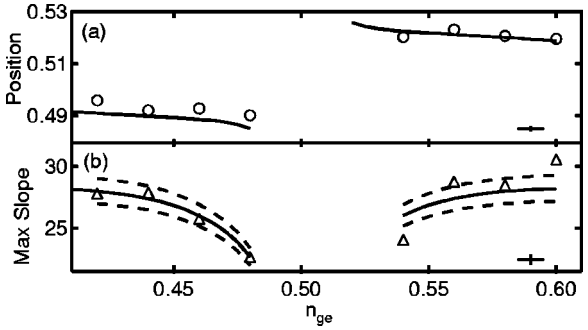


FIG. 3. (a) The horizontal position of the center of the Coulomb staircase for various operating points of the SET. The model is a solid line, and circles are experimental measurements. No measurements exist near $n_{ge}=\frac{1}{2}$ where the electrometer had no gain. Representative error bars are shown in the bottom right-hand side of the plot. Horizontal uncertainty reflects the measured instability of the SET operating point due to charge noise. (b) The maximum slope of the staircases measured with the SET at the same series of operating points. Confidence bands show the model curve for $27\pm1$ mK.

The measured Coulomb staircases also exhibit variations in width that change with operating point [Fig. 3(b)]. Three different mechanisms broaden the Coulomb staircase: quantum fluctuations, thermal excitation, and SET backaction. Quantum fluctuations of charge on the box cause broadening, but only away from the center of the step.[17] Our measurement, which quantifies broadening as the maximum slope at the center of each Coulomb step, is therefore insensitive to quantum broadening. Thermal excitations of the box also broaden the Coulomb staircase. SET heating varies with operating point, and, for large values of $V_{ds}$, can produce a trend in staircase width similar to the effects of backaction. All of our data were taken, however, at $V_{ds}=0$, where heating from the SET was negligible. Finally, SET backaction broadens the Coulomb staircase when the charge-state fluctuations of the SET island cause the box to switch between charge states. SET backaction broadens staircases by as much as $\kappa e$, and broadens staircases most at operating points where the

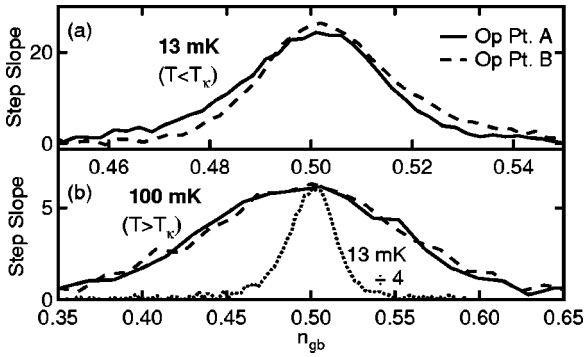


FIG. 4. (a) Derivatives of steps measured at the operating points in Fig. 2(a), offset in $n_{gb}$ to eliminate the shift in position of the steps. Note that the tails of the two steps are asymmetric. (b) Steps measured at the same operating points with the sample at 100 mK. The asymmetry is no longer visible. The inset demonstrates the thermal broadening by showing a $1/4$ scale curve from the top graph plotted on the $x$ axis for the bottom graph.

rms magnitude of the SET charge-state fluctuations is largest. The observed variations in staircase broadening with operating point [Fig. 3(b)] are fully accounted for with our sequential tunneling model.

The staircases are also asymmetric in a manner that varies predictably with operating point. Each staircase was found to have a longer tail in the direction away from which the staircase was shifted. The asymmetry of the Coulomb staircase is best viewed in the derivative of the steps [Fig. 2(b), or with the curves shifted to overlay in Fig. 4(a)], where it clearly follows the same trend as the model produces. Unfortunately, differentiating our data increased the noise and made it difficult to quantify the asymmetry; qualitatively, however, the model reproduces the experimentally observed trends. The staircase asymmetry is caused by the nature of the charge-state fluctuations on the SET island. The potential of the SET island lies preferentially to one side of the mean potential, with infrequent fluctuations far to the other side [Fig. 1(b)]. The staircases are thus broadened asymmetrically in the $+n_{gb}$ and $-n_{gb}$ directions. The preferred charge state, and thus the asymmetry of the measured staircase, is found to switch at $n_{ge}=\frac{1}{2}$.

The model also shows good agreement with our data at higher temperatures, where the various effects of the backaction change predictably. At higher temperatures, the mean potential of the SET island still changes with $n_{ge}$, and thus step shifts are still visible. For $T>T_{\kappa}$, however, the range (in $n_{gb}$) of thermal broadening is greater than the range of the backaction broadening or the asymmetry, and neither of these effects are therefore visible [Fig. 4(b)].

In these experiments we confirm that charge-state fluctuations of the SET island are the primary source of SET backaction. We observe the differences in Coulomb staircases measured with the SET biased at a variety of different operating points, and note changes in the shift, width, and asymmetry of the steps that are accurately recreated by a sequential tunneling model. This confirms that electromagnetic coupling to the fluctuating SET island potential can provide the ultimate lower bound on SET backaction.

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# Backaction Effects of a SSET Measuring a Qubit Spectroscopy and Ground State Measurement

Benjamin Turek, Johannes Majer, Aashish Clerk, Steve Girvin, Andreas Wallraff, Kevin Bladh, David Gunnarsson, Tim Duty, Per Delsing, and Robert Schoelkopf

***Abstract*—We investigate the backaction of superconducting single-electron transistor (SSET) continuously measuring a Cooper-pair box. Due to the minimized backaction of the SSET, we observe a $2e$ periodic Coulomb staircase according to the two-level system Hamiltonian of the Cooper-pair box. We demonstrate that we can control the quantum broadening of the ground state in-situ. We perform spectroscopy measurements and demonstrate that we have full control over the Cooper-pair box Hamiltonian. The ability to reduce the backaction is a necessary condition to use the SSET as a quantum state readout for the CPB as a qubit.**

***Index Terms*—Quantum computing, superconducting devices.**

## I. Introduction

An interesting question in solid-state quantum computation is how to measure a qubit and how the measurement process influences the qubit. Recently there has been considerable experimental progress using superconducting circuits to realize the qubit and the meter that measures the qubit. Many of the these devices are based on the single-Cooper-pair box [1], [2]. Coherent oscillations in such a Cooper-pair box have been observed [3]–[5] as well as Rabi oscillations [6] and Ramsey oscillation [7]. Despite these encouraging results, the measuring device and the influence of the measurement on the qubit are not yet completely understood.

In this article we report measurements where the Cooper-pair box is measured using a superconducting single-electron transistor (SSET). The SSET can be operated such that it continuously and weakly measures the charge of the Cooper-pair box. However, it also couples noise to the Cooper-pair box. This effect is called backaction and influences the states of the box in different ways. One can divide this influence in four categories, in order of decreasing severity, as follows:

First, the SSET can create nonequilibrium quasiparticles in the box. Therefore the states of the box are not described by Cooper-pair tunneling alone, and the box is no longer a simple two-level system. Quasiparticle poisoning is often [5], [8], [9], but not always [2], [4] observed in SSET measurements of the box, but its origin is not well understood. Second, even if there are no quasiparticles present, the SSET can excite the qubit. Now the box is still described by a two-level Hamiltonian, however the system does not stay in the ground state but is in a mixture of ground and excited state. Third, SSET's backaction can cause increased relaxation. After the box has been brought to the first excited state, the noise of the SSET can destroy the state of the box by extracting energy and bringing the system back to the ground state. The fourth category is the dephasing caused by the measurement process [10]. By the fundamental laws of quantum mechanics, measuring a system perturbs its state, and specifically destroys the phase of a superposition. Therefore this form of backaction is the fundamental limit.

The purpose of this article is to investigate the first two manifestations of backaction and to demonstrate that we were able to reduce them to observe the box in the $2e$-periodic ground state. Furthermore, we show that the box obeys a simple spin-1/2 Hamiltonian in which both terms can be controlled in-situ. We perform continuous-wave spectroscopy on the box, measuring the energy level separation and the avoided crossing of the charge states. We find that the quantum broadening of the Coulomb staircase is consistent with the level repulsion observed in spectroscopy.

## II. The Cooper-Pair Box

The Cooper-pair box consists of a superconducting island which is connected to a superconducting lead via a Josephson junction (Fig. 1). Another gate lead allows one to change the electrostatic potential of the island with the application of a voltage $V_{gb}$ through the capacitance $C_{gb}$. The state of the island is described by the number of Cooper-pairs on the island. Because the Josephson energy $E_J$ is smaller than four times the charging energy $E_c = e^2/2C_\Sigma$, one has to consider only two charging states. The Cooper-pair box is described by the following Hamiltonian

$$\hat{H} = \frac{E_{el}}{2}\hat{\sigma}_z + \frac{E_J}{2}\hat{\sigma}_x \quad E_{el} = 4E_c(n_{gb} - 1) \tag{1}$$

where $\sigma_z$ and $\sigma_x$ are the Pauli matrices. $n_{gb}$ is the number of electrons induced by the gate electrode $n_{gb} = C_{gb}V_{gb}/e$. The Josephson junction consists of two junctions in parallel, forming a SQUID loop [2] (see Fig. 1). The effective Josephson coupling $E_J$ of these two junctions can be tuned with the magnetic flux $\Phi$ through the loop: $E_J = E_J^{\max}|\cos(\pi\Phi/\Phi_0)|$. Here $\Phi_0 = h/2e$ is the superconducting flux quantum. Therefore the split Cooper-pair box is described by the two-level Hamiltonian (1), where both terms can be controlled during the experiment.

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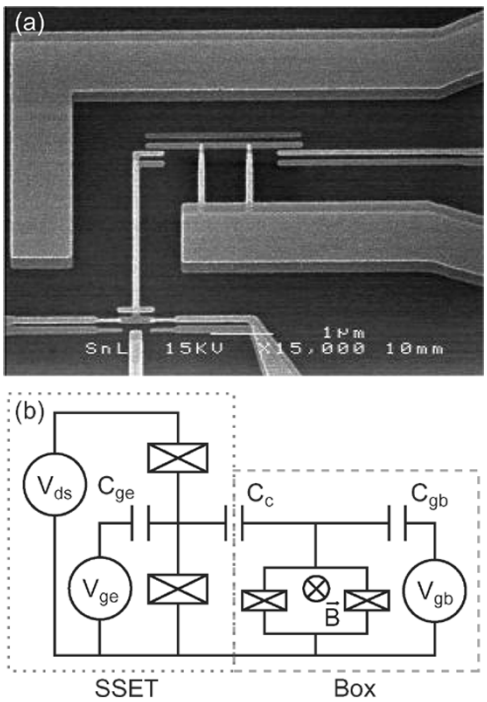


Fig. 1. (a) A scanning electron micrograph (SEM) of the device. The device is fabricated using shadow evaporation technique [11]. (b) The circuit diagram for the Cooper-pair box coupled with $C_C$ to the superconducting-single-electron transistor.

The states and the energy levels of the system can by found by diagonalizing the Hamiltonian (1). The energy difference between the ground state and the excited state is given by

$$\Delta E = \sqrt{E_J^2 + (4E_c)^2(n_{gb} - 1)^2} \tag{2}$$

Far away from the degeneracy point ($|n_{gb} - 1| \gg 0$) the eigenstates are given by pure charge states. However in the vicinity of the degeneracy point ($|n_{gb} - 1| \approx E_J/4E_c$) the eigenstates are superpositions of charge states and the energy levels show an avoided crossing.

The charge of the box in the ground state is given by the expectation value of the charge operator $\hat{q} = e(1 + \hat{\sigma}_z)$ in the ground state:

$$\langle q \rangle_{\substack{\text{ground} \\ \text{state}}} = e\left(1 + \frac{n_{gb} - 1}{\sqrt{\left(\frac{E_J}{4E_c}\right)^2 + (n_{gb} - 1)^2}}\right) \tag{3}$$

Without any Josephson coupling ($E_J = 0$) the box charge versus applied gate voltage (i.e. Coulomb staircase) is just a simple step function, which is 0 for $n_{gb} < 1$ and 2 for $n_{gb} > 1$. However with a finite Josephson energy $E_J$, a superposition of charge states exists in the region where $E_J \approx E_{el}$ and therefore the step becomes broadened. The larger the value of $E_J$ gets, the broader the step function. The charge of the box in the excited state is given by $\langle q \rangle_{\text{excited state}} = 2e - \langle q \rangle_{\text{ground state}}$.

## III. Measurements

The sample was measured in a dilution refrigerator at 13 mK. This thermal energy is far less than the relevant energy scales

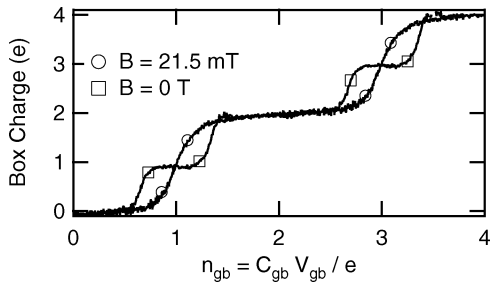


Fig. 2. The Coulomb staircase, i.e. box charge versus induced gate charge $n_{gb}$, without (square symbols) and with (circular symbols) magnetic field applied. The magnetic field affects the superconducting gap and removes the quasiparticle poisoning.

of either the SSET or the box. The charge of the box is measured with a SSET placed nearby (Fig. 1). Via the capacitor $C_C$, a small fraction ($C_C/C_\Sigma = 2.5\%$) of the charge of the box island couples to the SSET island. The SSET is operated as an radio-frequency single-electron transistor (RF-SET) [12], which is an outstanding electrometer. The sensitivity of the electrometer as well as the backaction noise [13] depend strongly on the operation point of the SSET, which can be tuned with the drain-source voltage $V_{DS}$ and the SSET gate voltage $V_{ge}$.

First we biased the SSET on the gap rise. The measured box charge as a function of the applied gate voltage $V_{gb}$, is completely $1e$ periodic, similar to the observations by Männik *et al.* [9]. At this bias point ($V_{ds} = 4\Delta/e \approx 1$ mV), the current through the SSET breaks many pairs in the drain and source lead of the SSET, though the power dissipated is only 1–10 pW. This apparently induces nonequilibrium quasiparticles in the box, though there is no direct connection between them and the mechanism is not known. This backaction noise is of the first kind as described above.

We then bias the SSET on the double Josephson quasiparticle process (DJQP) [13], [14], which occurs at lower SSET drain-source voltage. Fig. 2 shows with square symbols the measured Coulomb staircase. The Coulomb staircase is $2e$-periodic, however at odd number of electrons an intermediate step occurs due to quasiparticle poisoning. However after applying a magnetic field of 20 mT perpendicular to the substrate, the small step disappears (Fig. 2 circular symbols). The Coulomb staircase follows exactly the theoretical prediction (3). Applying a magnetic field lowers the superconducting gap in the aluminum, and could reduce the gap in the larger leads more than in the thin island [15]. The quasiparticle states in the leads would have a lower energy than on the island and therefore the quasiparticles can not tunnel on the island and poison the Coulomb staircase, as observed in SSET's [16]. A similar method has been used by Duty *et al.* [4] where a large magnetic field parallel to the device is applied. In contrast to previous experiments by Lehnert *et al.* [8] the box measured here has a smaller charging energy and therefore the quasiparticle states are more separated. In conclusion, using the optimal bias point of the SSET, applying a magnetic field and reducing the charging energy of the box allows us to measure a full Coulomb staircase that is not quasiparticle poisoned. Hence, we can avoid the backaction of the first category.

We measured the Coulomb staircase as function of the applied magnetic field (Fig. 3). One observes that the Coulomb

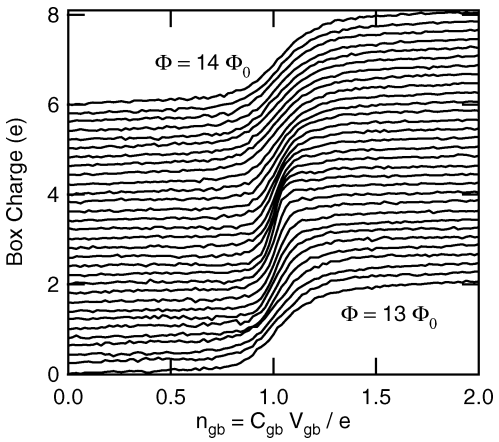


Fig. 3. Coulomb staircase for different magnetic field values. The staircase is periodically broadened and sharpened as a function of the applied flux. For integer flux quanta, i.e. large $E_J$, the staircase is maximally broadened and for half integer flux quanta, i.e. small $E_J$, step-like. Note: 13 $\Phi_0$ correspond to 27 mT.

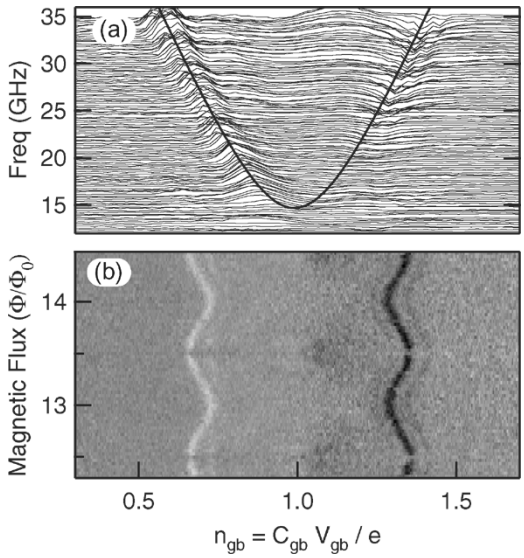


Fig. 4. Microwave spectroscopy results. Obtained by measuring the Coulomb staircase with microwaves applied and subtracting the staircase without microwaves. (a) Spectroscopy versus microwave frequency. From the fit with the hyperbola (Eq. (2)) we obtain $E_J = 14.9$ GHz and $E_c = 18.9$ GHz. (b) Spectroscopy versus magnetic field. A microwave signal with a frequency of 25 GHz is applied. The peak and dip oscillate as a function of the applied flux. For integer flux quanta, i.e. large $E_J$, the avoided energy level crossing is large and the resonances move inwards, toward the degeneracy point ($n_{gb} = 1$). Note: each plot is 10 hours of data without any offset charge jump.

staircase periodically sharpens and broadens. The period is consistent with the number of flux quanta in the split box junction. At integer flux quanta the Josephson energy is maximal and the Coulomb staircase is maximally broadened. At half integer flux, the Josephson energy is suppressed and the Coulomb staircase approaches a step-like function. We fit the theoretical expression (3) to the staircases, which allows us to extract the energy ratio between Josephson and charging energy $(E_J/4E_c)$. This ratio as a function of magnetic field is plotted in Fig. 5.

The shape of the Coulomb staircase in Fig. 3 is not generic. In order to observe these curves the SSET has to be biased slightly below the DJQP resonance. At this bias point theory [13] predicts that the backaction noise is primarily relaxing the box, i.e. there is backaction noise of the third category, but no backaction of the second kind. The box is therefore forced into the ground state by the noise of the SSET.

To observe the excited state and the energy spectrum of the box, we perform spectroscopy by applying a continuous microwave signal to the gate of the Cooper-pair box. When the microwave energy $h\nu$ (where $h$ is Planck's constant) matches the energy difference between the ground and excited state, the microwaves induce a transition from the ground state to the excited state. The system can be put in a mixture of ground and excited state at two discrete points in gate charge where the excitation is resonant. This microwave response appears as an extra peak and dip in the Coulomb staircase. We measured the Coulomb staircase with and without microwave signal and subtracted them to separate out the microwave induced response. Fig. 4(a) shows the microwave peak and dip as a function of the applied microwave frequency. The position of peak and dip follow the expected hyperbolic behavior with an avoided crossing of about 15 GHz. By fitting the positions with the expression for the energy level difference (2) we can extract the Josephson energy $E_J = 14.9$ GHz and the charging energy $E_c = 18.9$ GHz. This measurement was performed at integer flux quanta applied and therefore maximal $E_J$. One observes that the peak and dip height disappear toward the degeneracy point. This is due to the fact that the eigenstates are superposition of charge states and the difference of the box charge between the ground state and the excited state becomes small and disappears at the degeneracy point. The fact that our Coulomb staircase is not quasiparticle poisoned is very important, because it allows us to observe the spectroscopy signal down to the degeneracy point.

We also performed spectroscopy for a constant frequency of 25 GHz and varying magnetic fields (Fig. 4(b)). One observes that the peak and dip positions oscillate periodically with the applied the flux. As the $E_J$ becomes larger, the avoided crossing is larger and therefore the energy levels are more rounded. Hence the position, where the microwave frequency is in resonance with the energy level difference, moves toward the degeneracy point $(n_{gb} = 1)$. One can also observe that the signal disappears at the positions where $E_J$ is minimal (i.e. half integer flux quanta). When $E_J$ is zero, the eigenstates are pure charge states. Our microwave excitation is applied to the gate and is therefore a charge excitation. Since only a perpendicular component can induce transitions between states, the microwave signal is not able to drive the transition.

The spectroscopically obtained values of $E_J$ and $E_c$ versus magnetic field are shown in Fig. 5 (empty symbols) and can be compared with the values derived from the ground state. One observes that the two curves, obtained in completely different measurements, agree very well. This confirms that the measured Coulomb staircase is indeed the ground state of the two-level system and that the broadening is only due to quantum fluctuations. Hence we demonstrate that the Cooper-pair box is not affected by backaction of the second category. One observes a small discrepancy of the two measurements at low values of $E_J$. This is possibly due to $1/f$ charge noise which additionally broadens the step.

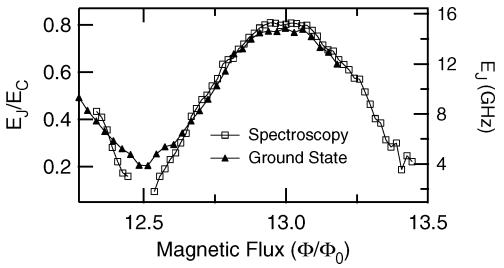


Fig. 5. Energy ratio $E_J/E_c$ versus magnetic field, obtained from the broadening of the ground state (solid symbols) and from the spectroscopy (open symbols).

## IV. Conclusion

In these experiments we demonstrate that we are able to eliminate the two most severe forms of backaction on the Cooper-pair box: The backaction that creates nonequilibrium quasiparticles on the Cooper-pair box and the backaction noise that excites the box from the ground to the excited state. With biasing the SSET at the optimal position and applying a small field, we are able to observe the ground state of the Cooper-pair box without quasiparticle poisoning. Spectroscopy and ground state measurements demonstrate that the Cooper-pair box is behaving according to the simple two-level Hamiltonian (1) and that we are able to control both terms in-situ. The good agreement between spectroscopy and ground state results shows that we are indeed observing the ground state of the system.

However, as discussed above, the SSET may still be relaxing the box, i.e. backaction of the third category. Further measurements will address the backaction induced contribution to the relaxation.

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