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MICRO UAV PATH PLANNING FOR RECONNAISSANCE IN WIND (PREPRINT)

Nicola Ceccarelli, John J. Enright, Emilio Frazzoli,
Steven J. Rasmussen, and Corey J. Schumacher

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14. ABSTRACT
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Micro UAV Path Planning for Reconnaissance in Wind

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***Abstract*— The problem addressed in this paper is the control of a Micro Unmanned Aerial Vehicle (MAV) for the purpose of obtaining video footage of a set of known ground targets with preferred azimuthal viewing angles, using fixed onboard cameras. Control is exercised only through the selection of waypoints, without modification of the MAV's pre-existing autopilot and waypoint following capability. Specifically, we investigate problems and potential solutions of performing this task in the presence of a known constant wind. Simulations are provided in presence of randomly perturbed wind, based on the Air Force Research Laboratory equipment and the high fidelity simulator MultiUAV2.**

I. Introduction

The problem addressed in this paper is the control of a Micro Unmanned Aerial Vehicle (MAV) for the purpose of obtaining video footage of a set of known ground targets with preferred azimuthal viewing angles, using onboard body frame fixed cameras. Control is exercised only through the selection of waypoints, without modification of the pre-existing autopilot and waypoint following capability. Specifically, we investigate problems and potential solutions of performing this task in the presence of a known constant wind field.

Algorithms for flight path guidance and synchronous target observations have been developed in several works [3], [19], [17], [18], [15]. In particular in [17], [18], [15] the wind scenario has been explicitly taken into account by developing control laws based on actuated cameras. The path planner we develop generates a waypoint sequence with the primary objective of giving rise to a trajectory such that each target is viewed by one of the onboard cameras from the preferred viewing angle. As a secondary priority, the path planner should minimize the total flight time of the resulting trajectory in order that intelligence is gathered in a timely manner. Moreover it is our aim to design a path planner which reflects the underlying behavior of the onboard autopilot, allowing for easy path following, especially in consideration of the known wind field [13].

This work is motivated by the ongoing research at the Air Force Research Laboratory (AFRL) [5] on flight operations with MAVs. The limitation that path planning must be done through a sequence of waypoints alone is not a simplification of the problem, but rather a requirement to be met due to the current testbed. This approach should be considered an alternative to methods for minimum time path generation in wind where the trajectories are provided in terms of pose time profiles [9],[10].

Although we assume a known constant wind field, we aim to design a path planner robust to variations in the wind field. Our objective is to provide reliable quality of service to the end users of the MAVs. The quality of the video provided can be measured with several metrics including

- duration the target is within the camera footprint,
- distance from center of camera footprint,
- difference between actual and preferred viewing angle.

Moreover, the robustness of the path planner can be measured by the rate at which targets are missed.

In the following sections we develop our path planner in a step-by-step manner, beginning with the local geometry of a single target and the MAV, continuing with the method of moving from one target to the next, and ending with methods for choosing the overall order of targets in the construction of the tour.

II. Assumptions and Problem Statement

We assume that the autopilot follows a line segment by "crabbing", i.e., maintaining level wings, but a heading different than the ground track. The MAV could fly with non-zero bank and sideslip angles, with heading equal to the ground track. However, for small fixed wing UAVs, the autopilot is usually designed with two independent longitudinal and lateral controllers [1], preventing the possibility mentioned above. It is also possible that the pitch of the aircraft is non-zero, as the angle of attack may depend on airspeed. However, since we command a constant airspeed throughout the mission, the steady-state pitch can be evaluated a priori, and thus the change in geometry between the MAV's position and heading, and the camera footprints position, can be easily built into the planner. Moreover, our observation of the real MAVs behavior suggests that all of these factors are in fact negligible.

Formally, we state our problem as follows. Given n targets $\{\tau_1, \tau_2, \dots, \tau_n\}$ each defined by a unique position and viewing angle,

$$\tau_i = (x_i, y_i, \theta_i) \quad (i = 1 \dots n),$$

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plan a finite sequence of waypoints $X = \{X_1, X_2, \ldots, X_N\}$ defined by positions,

$$X_j = (x_j, y_j) \qquad (j = 1 \ldots N)$$

and sequence order in increasing j beginning at $j = 1$, such that the resulting trajectory of the MAV allows reconnaissance of each of the n targets at its preferred viewing angle θ_i, with one of the available onboard cameras.

III. Geometric Considerations

In this section, we address the following problem. Given a target with fixed position and preferred azimuthal viewing angle, $\tau = (x_T, y_T, \theta_T)$, how do we plan a path such that the MAV's camera will view the target at the correct angle?

To get the highest quality picture possible, we attempt to place the center of the footprint directly on the target, rather than simply requiring the target to be inside the footprint. In steady level flight, the distance in the horizontal plane between the MAV and the center of the footprint is $d = \frac{h}{\tan \phi_{\text{cam}}}$, where h is the altitude of the MAV and ϕ_{cam} is the depression angle of the camera, i.e., the polar angle measured down from the horizontal plane at the MAV's altitude. Thus, at the time of video capture, the MAV must be a distance d, in the horizontal plane, from the target. In order to view the target at the correct angle, the relative angular position of MAV to the target must be θ_T. Combining these facts, the video capture must take place at a location:

$$\begin{bmatrix} x_{\text{cap}} \\ y_{\text{cap}} \end{bmatrix} = \begin{bmatrix} x_T + d \cos \theta_T \\ y_T + d \sin \theta_T \end{bmatrix}.$$

Denote θ_H as the aircraft heading and θ_{cam} as the relative azimuthal angle of the camera measured from the nose of the aircraft, we know that the line of sight of the camera is $\theta_{\text{LOS}} = \theta_H + \theta_{\text{cam}}$. We set our desired line of sight as $\theta_{\text{LOS}} = \theta_T + \pi$ in order to look *toward* the target *from* a relative angular position θ_T. Combining we find our desired heading, $\theta_H = \theta_T + \pi - \theta_{\text{cam}}$.

The question remains: given a constant wind field, how do we choose a ground track, i.e., pair of waypoints, such that the resulting heading will be the desired one? Assuming a constant wind speed, wind direction, and vehicle air speed, v_W, θ_W, and v_a, and considering that in steady level flight, the air velocity direction is aligned to the heading of the aircraft, the ground velocity is the resultant of the air velocity and wind velocity

$$V_g = \begin{bmatrix} v_a \cos \theta_H + v_W \cos \theta_W \\ v_a \sin \theta_H + v_W \sin \theta_W \end{bmatrix}.$$

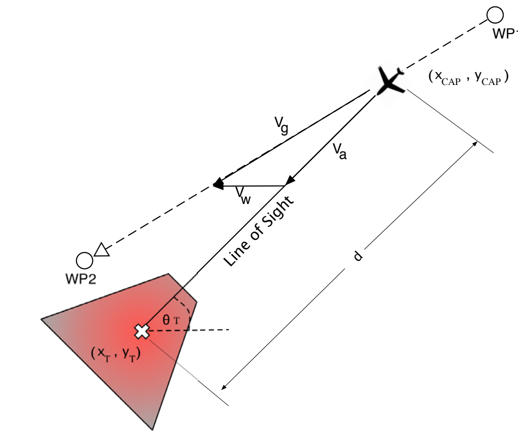
Hence, by denoting with $\measuredangle : \mathbb{R}^2 \to [-\pi, \pi]$ the vector phase, we obtain the desired ground track direction as

$$\theta_g = \measuredangle Vg.$$

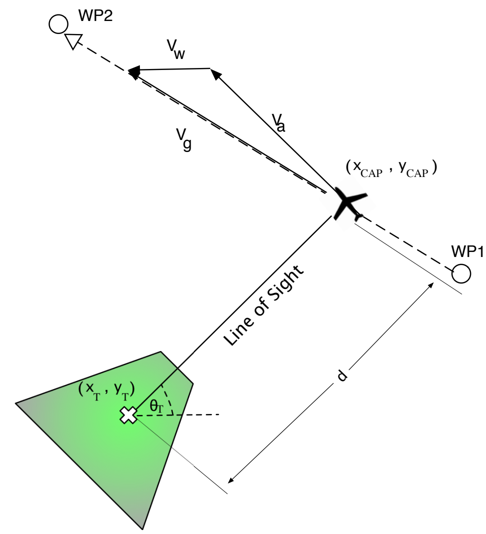
Thus, we choose a pair of waypoints such that the line between them intersects the point (x_{cap}, y_{cap}), and has a direction of θ_g.

In the equipment and simulations considered, there is a front camera, for which $\theta_{\text{cam}} = 0$, and a left-side camera, for which $\theta_{\text{cam}} = \pi/2$. Thus, as shown in Figures 1(a) and 1(b), the MAV may use either the "front" or "left" camera to view a given target, and each of these choices corresponds to its own ordered waypoint pair that the MAV must fly. Our problem is now defined by a set of waypoint pairs. Each waypoint pair corresponds to the reconnaissance of a specific target, each with a pre-defined choice of cameras.

In Section IV, we begin with two targets, i.e., each target simply corresponds to a single ordered pair of waypoints. Given these two waypoint pairs, we develop a path planner that places waypoints connecting the two pairs.



(a) front camera



(b) left-side camera

Fig. 1. Schematic for waypoint for different choice to view the target.

IV. Path Planner

In this section we tackle the problem of finding a feasible path for visiting two subsequent targets, each with their own preferred viewing angle. First we develop the structure of our planner without considering wind, and then show how the planner adapts to a known constant wind field.

The inner-loop of the autopilot controller works by following the line segment between sequential waypoints. Each time the MAV reaches a waypoint, it begins to follow the next line segment, and exhibits some transient behavior characterized by a damped oscillation that decays to the desired steady state flight along the new line. In some cases, the line segment may not be sufficiently long for the MAV to reach steady state, or the change in desired heading is too drastic, and the MAV may miss the next waypoint. Rather than relying on this transient response in our path planning, we attempt to choose waypoints in such a way that, upon reaching each waypoint, the MAV has reached steady state flight along the previous line segment. This is done by placing an upper bound on the heading difference between sequential line segments, and a lower bound on line segment length.

As shown previously, finding a feasible path between targets corresponds to finding a path connecting two pairs of waypoints. Given an initial and a final pair of waypoints,

$$X_{\text{init}} = [X_{\text{init}_1}, X_{\text{init}_2}] \quad X_{\text{final}} = [X_{\text{final}_1}, X_{\text{final}_2}],$$

find a number of steps $N \in \mathbb{N}$, and an input sequence $\mathbf{u} = [u(1) \dots u(N-1)]$ such that

$$\begin{aligned} X(k+1) &= X(k) + u(k) \qquad (k = 1 \dots N-1) \\ \text{s.t.} & \\ X(0) &= X_{\text{init}_1} \\ X(1) &= X_{\text{init}_2} \\ X(N-1) &= X_{\text{final}_1} \\ X(N) &= X_{\text{final}_2}. \end{aligned} \tag{1}$$

Once a path has been found, the autopilot inner control loop will be responsible for flying it, based on the sequence, $X(k)$, of waypoints. Hence, we introduce additional constraints in order that the resulting path may be followed by the autopilot easily. Specifically, we wish to minimize the transient response to direction change in terms of settling time and peak error between the actual trajectory and the desired path. In Fig.2, possible transient responses of the MAV for different changes of direction are depicted. In particular let us define

$$\theta(k) = (\measuredangle u(k) - \measuredangle u(k-1)) \bmod \pi \tag{2}$$

as the change in direction performed at the step k. We introduce the angular constraints

$$|\theta(k)| \leq \alpha \quad (k = 2 \dots N-1), \tag{3}$$

where α is the maximum magnitude allowed of the change in direction at each step of the waypoint path. This constraint is meant to keep the peak error within a reasonable level. Furthermore, we introduce a requirement on the minimum length of each step. This is designed to allow the MAV to reach steady-level flight before the next change in direction occurs:

$$\|u(k)\| \geq L \quad (k = 1 \dots N-1). \tag{4}$$

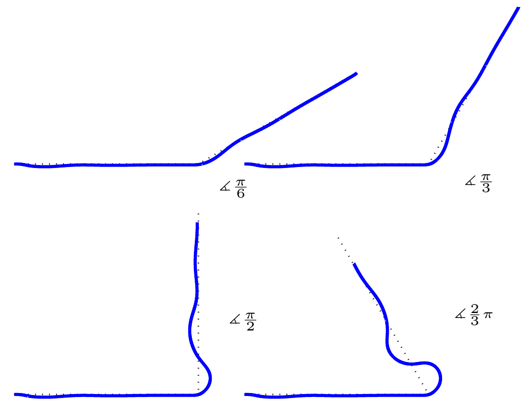


Fig. 2. Transient response due to direction changes.

In many ways, the building blocks of our path planner are line segments, not waypoints.

We search for a minimum length path, feasible with respect to system (1) and for which constraints (3) and (4) hold. Taking inspiration from work on Dubins vehicle [8],[2], we restrict the search to the following class of control sequences:

$$\bar{\mathcal{U}} \in [u_{\text{init}} \ \mathcal{T}_{\text{init}} \ \mathcal{C} \ \mathcal{T}_{\text{final}} \ u_{\text{final}}], \tag{5}$$

where $\mathcal{T}_{\text{init}}, \mathcal{T}_{\text{final}} \in \mathcal{T}$ represent input sequences performing a right or left turn at the maximum rotational rate, i.e., constraint (3) is saturated; and $\mathcal{C}$ represents the connecting step between the initial and final turn. In particular we define

$$\begin{aligned} u_{\text{init}} &= X_{\text{init}_2} - X_{\text{init}_1}, && (6) \\ u_{\text{final}} &= X_{\text{final}_2} - X_{\text{final}_1}, && (7) \\ \mathcal{T} &\triangleq \Big\{ [u_{\mathcal{T}} \dots u_{\mathcal{T}}] \in \mathbb{R}^{2 \times N_{\mathcal{T}}} : N_{\mathcal{T}} \in \mathbb{N}, \\ & \quad u_{\mathcal{T}} = L \cdot \begin{bmatrix} cos(\alpha_{\mathcal{T}}) \\ sin(\alpha_{\mathcal{T}}) \end{bmatrix}, \alpha_{\mathcal{T}} \in \{\alpha, -\alpha\} \Big\}, && (8) \\ \mathcal{C} &\triangleq \big\{ u \in \mathbb{R}^2 : \|u\| \geq L \vee \|u\| = 0 \big\}. && (9) \end{aligned}$$

Although the issue of the existence of such a path for any two pairs of waypoints has been neglected, we should report that counterexamples can be generated by choosing arbitrarily close initial and final waypoint pairs. Simulations have shown this event to be rare, and when it does occur, it simply means that the tour designer does not consider the possibility of placing those two pairs of waypoints in sequence.

The above formulation does not consider the heavy effect of a wind field on the flight performance of the MAV. Constraints (3) and (4) correspond to performance requirements in a zero wind field, based on the transient response of the MAV controlled by the autopilot. It is possible to transform the path planning problem in the presence of a known constant wind, to one with zero wind, by considering a moving target, with velocity opposite to the actual wind velocity [9]. In order to maintain consistency with the original path planning problem, initial and final constraints on (1) must be generated as follows

$$\tilde{X}(0) = \tilde{X}_{\text{init}_1} \quad (10)$$
$$\tilde{X}(1) = \tilde{X}_{\text{init}_2} = X_{\text{init}_2} - T_2 \cdot V_W \quad (11)$$
$$\tilde{X}(N-1) = \tilde{X}_{\text{final}_1} = X_{\text{final}_1} - T_{N-1} \cdot V_W \quad (12)$$
$$\tilde{X}(N) = \tilde{X}_{\text{final}_2} = X_{\text{final}_2} - T_N \cdot V_W, \quad (13)$$

where V_W is the wind velocity vector, while T_k is the time required to arrive at the k-th waypoint. It should be noted that the value of T_k depends on the particular path considered and its computation require the solution of a quadratic equation.

Now we look for an input sequence $\tilde{\mathbf{u}}^* \in \bar{\mathcal{U}}$ that gives the minimum time path. In practice we bound the maximum number of subsequent turn steps of (8) to $N_{\mathcal{T}} \leq \lceil \frac{2\pi}{\alpha} \rceil$, and we consider the shortest among the resulting feasible paths. Once $\tilde{\mathbf{u}}^*$ and the associated path $\tilde{X}^*$ have been determined, we transform back to the constant wind fixed target scenario by considering the control input sequence $\mathbf{u}^*$:

$$u^*(k) = \tilde{u}^*(k) + T_{u^*(k)} \cdot V_W \quad \forall (k = 1 \dots N-1), \quad (14)$$

where, by denoting with v_a the MAV air speed,

$$T_{u^*(k)} = \frac{\|\tilde{u}^*(k)\|}{v_a} \quad (15)$$

is the time to execute step $u^*(k)$, assuming perfect path following. Using (10)-(13) it is trivial to show that $\mathbf{u}^*$ is feasible w.r.t. (1). Moreover, by (15), the tour time $T = \sum_{k=1}^{N-1} T_{u^*(k)}$, to execute the path X^* generated by $\mathbf{u}^*$, is exactly the same time required to accomplish the associated path $\tilde{X}^*$ in the scenario of a moving target and null wind; this property holds in general for any path generated in the moving target scenario and transformed to the wind scenario by (14). Hence it follows that X^* is of minimum time among the paths generated according to (5), in the scenario of zero wind and moving target, and transformed by (14).

In Fig.3 the effect of transformation (15) is depicted. Note that that while turning against the wind results in a shorter step and a more gentle turn w.r.t. the original input $\tilde{u}^*(k)$, turning with the wind results in a longer step and a sharper turn. This effect accommodates the transient response of the MAV in wind (solid blue line). This behavior is less damped when turning with the direction of the wind, and when turning against wind, it exhibits a a highly damped response, allowing the next step to take effect sooner.

In Fig.4 an example of the path planning method is depicted. The red dashed path is the minimum time path generated with input of class (5), computed for the system in a zero wind field and a moving target. The black solid line is the minimum time path transformed back to the system of constant wind and fixed target. The dotted regular octagon segments represent the maximum rate right turns $\mathcal{T}_{\text{init}}$ and $\mathcal{T}_{\text{final}}$.

V. Designing the Tour

There are n targets, each of whose reconnaissance can be satisfied by executing either of two waypoint pairs. The

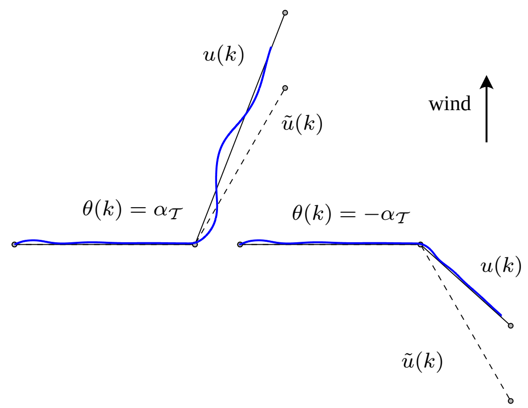


Fig. 3. Transformation of inputs from no wind moving target scenario ($\tilde{u}(k)$) to wind fixed target scenario ($u(k)$) and associated transient response

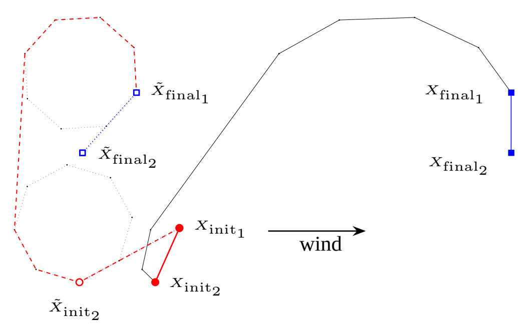


Fig. 4. Path planner between two pairs of waypoints

travel between any two waypoint pairs has an associated cost which is in general asymmetric. In this section, we address the coupled problem of choosing which camera to use for each target, and the order in which the targets should be visited. For the sake of clarity, we first address a simplified version of this problem, in which each target has an a priori camera assignment, and the remaining task is to choose the order of the targets.

A. A priori camera choice

In this scenario, the reconnaissance of each target is satisfied by the MAV's flight through a single pair of waypoints. Each pair of waypoints has an associated asymmetric cost of travel between them as calculated by the path planner. To gather intelligence in a timely manner, and save fuel for future missions, we aim to minimize the time elapsed throughout the tour of all targets. This is clearly a realization of the Asymmetric Traveling Salesman Problem (ATSP).

To solve the ATSP, we used a mixed-integer linear program formulation first presented in [11]. The following is a summary of this method. Let $d_{i,j}$ be the time required to fly the path from the pair of waypoints associated to target τ_i to the pair of waypoints associated to target τ_j where $1 \leq i, j \leq n$. Minimize:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} \qquad (i \neq j) \tag{16}$$

s.t.

$$\sum_{i=1}^{n} x_{ij} = 1 \quad (i \neq j = 1 \ldots n) \tag{17}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad (j \neq i = 1 \ldots n) \tag{18}$$

$$y_i - y_j + n x_{i,j} \leq n - 1 \quad (2 \leq i \neq j \leq n) \tag{19}$$

where $x_{i,j}$ are non-negative integers and y_i are arbitrary real numbers.

The constraints (17)-(18) require $x_{i,j}$ to be 0 or 1, and the solution to the ATSP is interpreted as follows: the MAV flies from target τ_i to target τ_j if and only if $x_{i,j} = 1$. The y_i variables indicate the order in which the targets are visited. The constraints (19) serve to eliminate the possibility of disjoint sub-tours. We refer the reader to [11] for further details of the method.

Since the above formulation is a mixed integer linear program, it is NP-hard [7]. Furthermore, it is well known that there is no efficient approximation algorithm of the ATSP, unless P $=$NP [16]. With the restriction that the distances satisfy the triangle inequality, there are algorithms that achieve $\log n$ approximations [4]. The instance of the ATSP that we address can be formulated in such a way that the distances satisfy the triangle inequality, and the possibility of using such an approximation scheme is left for future work.

B. General problem

The above formulation of our tour design problem is useful if some desirable property of the tour is produced by following an explicit rule to make the a priori camera choices for each target. However, consider the output of such an approach. It is an ordering of the targets, along with the camera choices for each target, and an associated cost of the entire tour. If the camera choice for a single target was changed from front to left or vice versa, the waypoint pair associated with this target would change, and two "legs" of the tour would be altered, possibly decreasing the length of the tour as a whole. Thus it is clear that a minimization of the tour time for our problem is not captured by an a priori camera choice and the ATSP. However, there is a variation on the ATSP that does suffice.

The Generalized Traveling Salesman Problem (GTSP) [6] is defined on a directed graph whose nodes are grouped into m mutually exclusive and jointly exhaustive nodesets. The nodesets need not contain equal numbers of nodes. The arcs are defined only between nodes of different sets, i.e., there are no intraset arcs, and each defined arc has an associated non-negative cost. The GTSP is the problem of finding a minimum cost m-arc cycle that visits exactly one node from each nodeset.

For our scenario, the GTSP has n nodesets (targets), each of which has a cardinality of two (each target has two available camera choices, i.e., waypoint pairs), thus there are $2n$ nodes (waypoint pairs). We wish to find the minimum time trajectory that flies exactly one of the two waypoint pair associated with each target.

To solve the GTSP, we implemented a GTSP to ATSP transformation first presented in [12]. The method takes the graph of $2n$ nodes on which the GTSP is defined, swaps the weights of certain edges, increases the weight of all edges by an amount greater than the sum of all edge weights, and then adds certain zero weight intraset arcs. The special structuring of the new graph is such that the ATSP solution will always visit all nodes within a single cluster before moving to another. Once this solution has been obtained, [12] presents a method to discern the optimal solution to the associated GTSP. This solution, of course, only visits one node per cluster.

The above method can be used to obtain the minimum time path which satisfies the reconnaissance requirements of all targets in our problem. However, it requires the computation of an ATSP with $2n$ nodes. This is a substantial increase in computational burden as compared to the method of making a priori camera choices. Furthermore, the distances in the specially structured ATSP resulting from the transformation of the GTSP do not satisfy the triangle inequality, and so there can be no efficient approximation of the optimal solution. In Section VI we explore the implementation and performance of both methods mentioned.

VI. SIMULATIONS

This section is devoted to analysis of simulations. The purpose of these simulations is to test the proposed approach on scenarios of practical interest for the AFRL's MAV equipment.

A. Setup

The simulation test bed is based on the MultiUAV2simulator [14]. In particular, we considered the scenario of one MAV, moving in a wind field, on a reconnaissance mission with 5 targets positioned in an area of size $1km \times 1km$. The desired cruise air speed was set to $v_a = 24$ knots, and the desired altitude to 200 feet. For the path planner presented in Section IV, the following parameters were chosen

- $\alpha = \frac{\pi}{4}$ rad,
- $L = 500$ft.

Although a constant wind was assumed in the development of Section IV, a variable wind is included in the simulation to obtain more realistic results. In particular, we considered a wind field with a main component West-East of 15 knots perturbed by additive variable wind gusts in all three directions (North,East,Up), uniformly spatially distributed and with random magnitudes and constant spatial extension. The magnitude of the wind gusts was generated as a zero mean random variable with standard deviation of 5 knots.

The MAV model used in the simulations presented is based on the Applied Research Associates' Nighthawk, a very small (370 grams), short endurance aircraft powered by a 55 Watt electric motor. It can operate at air speeds of 15-40 knots and at altitudes up to 1,000 feet mean sea level. The airframe is a carbon fiber V-tail with a 53cm span flexible wing and a 50cm fuselage length. The Nighthawk has no control surfaces along the wings, and its control relies solely on the V-tail's two rudder/elevators.

The Nighthawk is equipped with a front and a left side camera. For the purpose of this investigation the same parameters have been assumed for both of the two cameras:

- horizontal field of view 64deg,
- vertical field of view 64deg,
- depression angle -45deg.

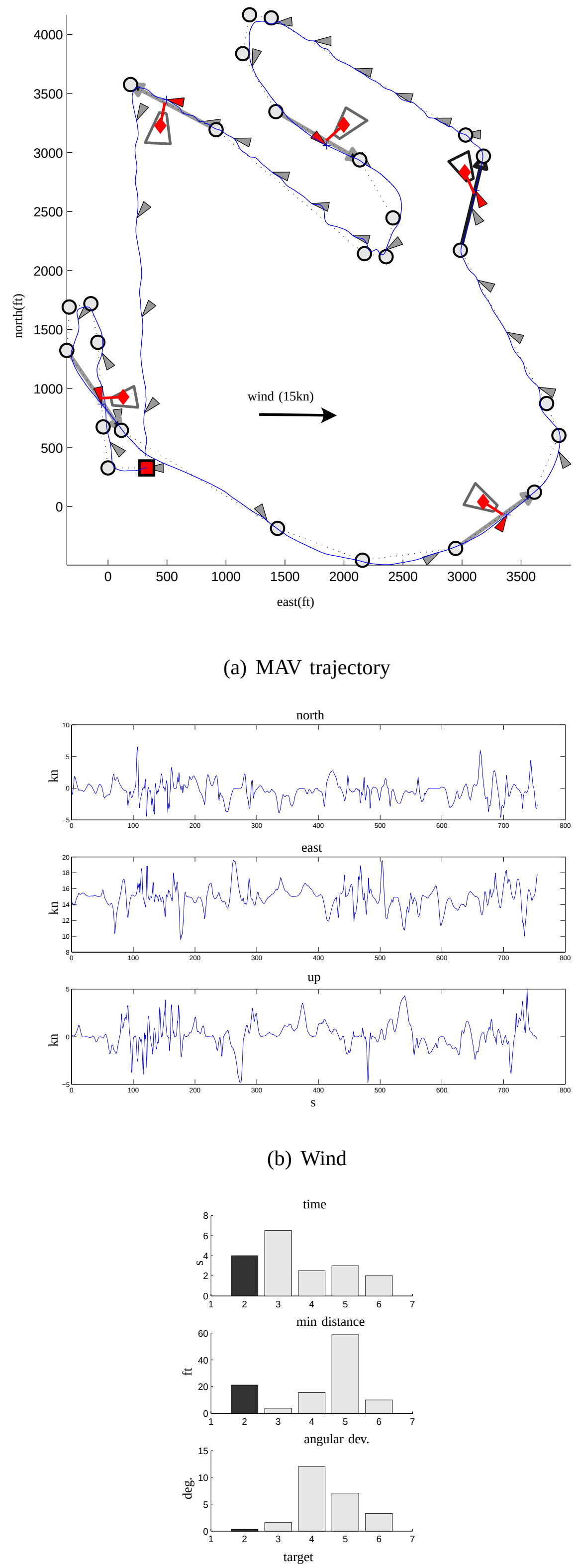
B. Results

In Fig. 5 a simulation of a 5 target scenario is reported. In Fig. 5(a) the red square is the take-off point. The red diamonds and the associated red solid lines represent the targets and the preferred directions of sight. The tour sequence and the associated camera choice was found as a solution of the GTSP, using the formulation of Sec. V, after the planner from Sec. IV computed the inter-target paths and their associate costs, with the assumption of a West-East constant wind of 15 knots. The arrows represent the pairs of waypoints associated with each target, as evaluated in Sec. III, where dark (light) indicates that the front (left) camera has been selected. The black dotted line and the circles represent the resulting path of waypoints associated with the solution sequence. The solid blue line is the actual ground track as output of MultiUAV2, while simulating the model of the Nighthawk MAV following the waypoint sequence. The gray triangles represent the pose trajectory sampled every ~ 40 seconds, while the red triangles and the associated footprints are the poses of the MAV at the snapshot instants.

Figure 5(b) refers to three components of the wind velocity blowing along the actual trajectory of the vehicle. In Fig.5(c), for each target, results are presented in terms of dwell time of the target inside the footprint, target minimum distance from the center of the footprint and relative angular deviation w.r.t. the preferred line of sight. Dark (light) bars stand for the front (left) camera.

C. Monte Carlo Simulations

In order to validate the proposed approach, Monte Carlo simulations have been performed. Each run refers to a 5 target scenario similar to the one proposed in the previous subsection, for which randomly generated wind fields, target positions and desired angles of view are generated. We propose a comparison between four different approaches. We consider two possible path planners: the one proposed in Section IV and the discretized Dubins path, currently implemented in the MultiUAV2simulator. We consider two tour design formulations: the GTSP as presented in Sec. V, and a simpler ATSP formulation where for each target we



(a) MAV trajectory

(b) Wind

(c) Targets statistics

Fig. 5. Scenario with 5 targets

TABLE I

METHODS USED FOR THE COMPARISON

	problem formulation	cost matrix
A	GTSP	Proposed Path Planner
B	GTSP	Dubins Path Planner
A_s	TSP	Proposed Path Planner
B_s	TSP	Dubins Path Planner

TABLE II

SUMMARIZED AVERAGE DATA FROM 100 SIMULATIONS

	A	B	A_s	B_s
tour time (min)	10.5	15.3	11.7	17.1
missed target	83	145	60	81
number of waypoints	30	38	32	43
time over target (s)	5.54	5.97	6.21	6.4
minimum distance (ft)	27.9	24.3	26.4	23.5
angular deviation (deg)	8.2	10.2	8.5	6.6
tour with missed targets	60	86	45	52
tour with missed targets ≥ 2	16	41	13	18

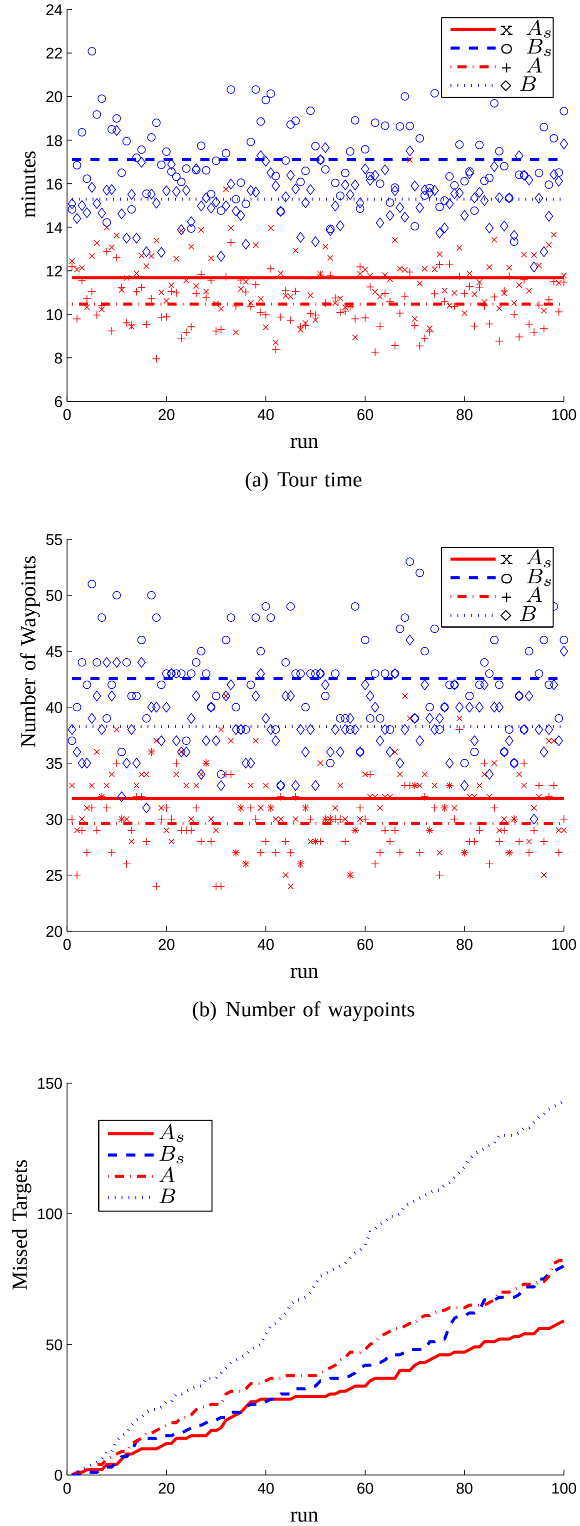
choose between "front" and "left" camera by selecting the one resulting in the slowest ground speed for that "viewing segment". This choice was made in agreement with the intuition that a slower ground speed might result in a better quality image taken by the camera. Table I summarizes the different methods considered.

The output data of each run, produced by MultiUAV2, are summarized in Table II and Fig.6. The red solid lines and symbol "x" refer to the results obtained for the presented path planner by solving the ATSP problem (method A_S); blue dashed lines and symbol "○" refer to the discretized Dubins paths solving the ATSP (method B_s); red dash-dot lines and symbol "+" refer to the results obtained for the presented path planner by solving the GTSP (method A); blue dotted lines and symbol "◇" refer to the discretized Dubins paths solving the GTSP (method B).

Figure 6(a) refers to the actual tour time required to complete the path. The proposed path planner significatively reduced the average tour time by about 30% with respect to the Dubins' planner, for both TSP and GTSP formulations. This aspect is particulary critical for the MAVs as they have short endurance.

Figure 6(b) refers to the number of waypoints composing the whole tour. Again, the proposed approach reduces the average number of waypoints required to complete the tour by 25% with respect the Dubins path planner for both ATSP and GTSP formulations. This aspect can be considered critical by noticing that the maximum number of waypoints is limited by communication protocol packet size (100 waypoints for the Nighthawk).

Finally in Fig. 6(c) the incremental evolution of missed target along the 100 runs is presented. A target has been considered missed if its dwell time inside the footprint is less than 2 seconds. In this case the two ATSP formulations result in better performance with respect to GTSP formulations. This follows intuition that a slower ground speed reduces the probability of missing a target. The rate of missed targets has been reduced from 30% to 16% for the Dubins path planner and from 16% to 12% for the proposed path planner.



(a) Tour time

(b) Number of waypoints

(c) Incremental evolution of missed Targets

Fig. 6. Comparison over 100 runs for the approaches of Table I

The average dwell time of the targets inside footprints (Table II) is $\sim$ 6s for all the approaches, the average minimum distance, of the targets from the center of the footprint, is ~ 25ft and the average angular deviation is ~ 10deg.

Several aspects have been investigated in this simulation analysis. From the tour time data we conclude that different path planning methods which explicitly take the wind into account should be considered in order to significatively reduce the fuel consumption and the number of tour waypoints needed. Moreover the missing targets data show that the camera choice associated with the slower ground speed can significatively reduce the probability of missing a target, hence the ATSP formulation is preferred with respect to the GTSP in that respect. Finally one should notice that although method B_s showed a degradation of the tour time with respect to method B of about 10%, this value could be easily compensated by the possible mission requirements of a second tour in the case of missed target.

VII. Conclusions

This paper deals with a reconnaissance problem for an MAV flying in a constant wind field, using fixed onboard cameras to take video footage of a set of targets with known positions and desired viewing angles. The MAV is assumed to be equipped with an autopilot capable of performing path following of a waypoint sequence. The challenge is to generate a waypoint path that explicitly takes the wind and the autopilot path following module into account, so that the camera footprint is more likely to pass over each target in spite of the wind.

First we address the problem of placing a pair of waypoints such that, if flown by the MAV, the camera footprint passes over the target. Next, we introduce a waypoint path planner that chooses minimum time paths joining two given pairs of waypoints, among a restricted path family which takes the wind and the MAV's behavior into account. We then identify the problem of minimizing the duration of the tour viewing all targets, where each target may be viewed by one among several cameras, as a realization of the GTSP. Extensive simulations in the presence of randomly perturbed wind have been provided, along with a comparison among different approaches. In particular, the path planner proposed here has been compared with a discretized Dubins path planner, previously implemented in MultiUAV2, and the GTSP formulation has been compared to an ATSP formulation where between the two cameras available for the MAV we choose the one resulting in the slowest ground speed. All the simulations use a model of the Nighthawk MAV, a component of the AFRL equipment. The analysis and study conducted in this work have shown that path planners which explicitly take the wind into account result in a significant reduction in fuel consumption and a lowered probability of missing targets.

Future research may involve the implementation and testing of this method on the real Nighthawk MAV testbed.

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