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Enhancement of Stochastic Resonance by Tuning System Parameters and Adding Noise Simultaneously

Xingxing Wu
Zhong-Ping Jiang

Polytechnic University
Brooklyn, NY 11201

Daniel Repperger

Human Effectiveness Directorate
Warfighter Interface Division
Wright-Patterson AFB OH 45433-7022

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Human Effectiveness Directorate
Warfighter Interface Division
Collaborative Interfaces Branch
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Enhancement of Stochastic Resonance by Tuning System Parameters and Adding Noise Simultaneously

Xingxing Wu, Zhong-Ping Jiang, and Daniel W. Repperger

***Abstract*— The stochastic resonance effect can be realized by tuning system parameters or by adding noise. This paper investigates the possibility to enhance the stochastic resonance effect by tuning system parameters and adding noise simultaneously. First, we use some examples to demonstrate the situation where only the system parameters or noise can be adjusted for maximizing the stochastic resonance effect. Then, it is shown using standard optimization theory that the normalized power normal $<C_1>$ of the bistable double-well system with aperiodic input signal can reach a larger maximal value by tuning the system parameter and adding noise simultaneously. Finally, for the purpose of practical implementation, searching for the optimal system parameter and noise intensity is realised by an on-line fast-converging optimization algorithm.**

Index Terms— stochastic resonance, signal processing, and optimization.

I. INTRODUCTION

Stochastic resonance (SR) is the phenomenon that the noise can be used to enhance rather than hinder the system performance. The noise can excite the richness of the nonlinearities and provides improved dynamics which better enables the system to increase signal-to-noise ratio (SNR) or mutual information. The concept of stochastic resonance was first proposed by Benzi in 1981, addressing the problem of the periodically recurrent ice ages [1]. Over the last two decades, stochastic resonance has been continuously attracting considerable attention. It is a ubiquitous and conspicuous phenomenon. Many nonlinear systems have demonstrated the stochastic resonance effects, such as discrete systems [4], dynamic systems [2], static systems [5], coupled systems [6] and random systems [7]. The signal can be periodic [2], aperiodic [8], subthreshold [2] or suprathreshold [9]. In order to quantify the stochastic resonance phenomena and reveal the synchronization between signals and noise, different measures have been adopted. For the periodic signals, the most commonly used quantifier is signal-to-noise ratio (SNR) [2]. For aperiodic signals, cross-correlation measures [10], power norm [8] and information-based measures, such as mutual information [11], are used instead. Over the years, stochastic resonance has been applied in wide-range of areas, such as physics, chemistry, biomedical sciences, and engineering [2-3]. One of the important applications of stochastic resonance is in signal processing. As a nonlinear signal processor, it has been used for signal detection [12-13], signal transmission [14-15] and signal estimation [16]. In order to realize the stochastic resonance so as to make the chosen quantifier, e.g. the output signal-to-noise ratio (SNR), reach its maximal value, certain conditions must be satisfied. The traditional way is to adjust the noise intensity by adding optimal amount of noise. Recently, tuning system parameters have been demonstrated to be a better method to realize stochastic resonance, especially when the initial input noise level already exceeds the resonance region [17-19]. The output SNR will reach a higher maximal value by tuning system parameters than by adjusting noise intensity [18]. Among this research, either the noise intensity is adjusted or the system parameters are tuned in order to maximize the chosen measure, but not both. This paper will investigate the possibility to further increase the maximum by tuning the system parameters and by adding noise simultaneously. This will in turn improve the system performance when used as the nonlinear signal processor for signal detection, signal transmission or signal estimation.

The rest of this paper is organized as follows. In Section 2, we demonstrate the cases when only system parameters or noise intensity can be adjusted. Section 3 will prove the possibility to further increase the maximal value of normalized power norm [8] of the aperiodic stochastic resonance in bistable double-well system by adjusting system parameters and noise intensity, based on the conventional first-order necessary condition and second-order sufficient condition in optimization theory. Section 4 will provide an on-line fast-convergent optimization algorithm to search the optimal system parameters and noise intensity. Section 5 is devoted to verify, via computer simulations, the improvement of maximal $<C_1>$ by comparing the maximal normalized power norm obtained by tuning system parameters, by adding noise and by both. Finally, Section 6 closes the paper with brief concluding remarks.

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X. Wu and Z. P. Jiang are with the Control and Telecommunications Research Lab, Electrical and Computer Engineering Department, Polytechnic University, Brooklyn, NY 11201 USA, (email: xwu03@utopia.poly.edu, zjiang@control.poly.edu).

D. W. Repperger is with the Air Force Research Laboratory, AFRL/HECP, Wright-Patterson AFB, OH 45433 USA. (email: Daniel.Repperger@wpafb.af.mil).

II. STOCHASTIC RESONANCE VIA TUNING SYSTEM PARAMETERS OR ADDING NOISE

In some stochastic resonance systems, the chosen measures can be described as a function of both the system parameters and the noise. This, however, does not ensure that both system parameters and noise can be adjusted at the same time to maximize or minimize these quantifiers. In [20], the output signal-to-noise ratio is affected by the ratio of noise variance and system parameter (threshold), rather than by the parameter value and noise individually. The paper [17] shows that the noise intensity cannot be adjusted and must be fixed at the initial value (no noise will be added) in order to minimize the bit error rate (BER) for the aperiodic stochastic resonance (ASR). In this case, only system parameters tuning is meaningful. In what follows we will demonstrate this is also true for the *periodic* stochastic resonance in double-well bistable dynamic systems.

The double-well bistable system is described by the equation [21]:

$$\tau_a \dot{x}(t) = x(t) - \frac{x^3(t)}{X_b^2} + s(t) + \eta(t), \tag{1}$$

where system parameters $\tau_a > 0$, $X_b > 0$. The periodic input is $s(t) = A\cos(2\pi t / T_s)$. $\eta(t)$ is an additive Gaussian white noise with zero mean average and autocorrelation of $< \eta(t)\eta(0) >= 2D\delta(t)$.

For small and slow input signal, the output signal-to-noise ratio is given by [21]:

$$SNR = \sqrt{2}\frac{A^2 X_b^2 / 4}{(D/\tau_a)^2}\exp(-\frac{X_b^2/4}{D/\tau_a}), \tag{2}$$

Assuming parameter τ_a is fixed, the output SNR is a function of both system parameter X_b and noise intensity D. One constraint on the noise intensity is that it should not be less than the initial value D_0. There are also constraints on the parameter X_b. For example, $X_b > \sqrt{27}A/2$ for the subthreshold system [17].

To prove our above claim, assume the SNR is maximized at the optimal values (X_b^*, D^*). Obviously, there will be a local maximizer for this optimization problem if both system parameter X_b and noise intensity D can be adjusted at the same time. According to the first-order necessary condition for a local maximizer, we have $\nabla SNR(X_b, D) = 0$, and thus:

$$X_b^2 = 8D, X_b^2 = 4D \quad , \tag{3}$$

The derived solution ($X_b = 0$ and $D = 0$) does not meet the constraint requirements. In other words, this constrained optimization problem has no local maximizer. This means either system parameter X_b or noise D, but not both, can be adjusted to maximize the output SNR.

In order to determine which one will take the extremum and which one is not adjustable, we can rescale the variables as:

$$\tau = t/T_s, \quad y = x\sqrt{T_s/D}, \quad \overline{X_b} = X_b\sqrt{T_s/D},$$
$$\overline{A} = A\sqrt{T_s/D}, \quad \overline{\tau_a} = \tau_a/T_s. \tag{4}$$

For the rescaled y-system, its noise density becomes unit and the output SNR is:

$$SNR = \sqrt{2}\frac{\overline{A}^2\overline{X_b}^2/4}{(1/\overline{\tau_a})^2}\exp(-\frac{\overline{X_b}^2/4}{1/\overline{\tau_a}}), \tag{5}$$

Now, the optimization parameters are $\overline{A}$ and $\overline{X_b}$. Obviously, $\overline{A}$ should take the maximal value in order to maximize SNR. From (4), this means that the noise intensity should be fixed to its initial value D_0 which is also the minimal noise intensity, assuming the signal amplitude A is not changeable. Again, only system parameter X_b can be adjusted.

In the next section, we will examine some interesting situations where both the system parameters and noise intensity can be adjusted simultaneously to improve the stochastic resonance effect.

III. STOCHASTIC RESONANCE ENHANCEMENT

In [22], the aperiodic stochastic resonance (ASR) was demonstrated in the bistable-well system. The cross-correlation measures (power norm C_0 and normalized power norm C_1) were adopted for characterizing the ASR behaviour:

$$C_0 = \max\{\overline{S(t)R(t+\tau)}\}$$
$$C_1 = \frac{C_0}{[\overline{S^2(t)}]^{1/2}\{\overline{[R(t)-\overline{R(t)}]^2}\}^{1/2}}, \tag{6}$$

where S(t) is the zero-mean aperiodic input signal, R(t) is the mean transition rate of the system.

The symmetric bistable-well system with a fluctuating barrier is given by [22]:

$$\frac{dx}{dt} = -\frac{\partial U}{\partial x} + \xi(t), \tag{7}$$

where $U(x) = -[A - S(t)]\frac{x^2}{2} + \frac{x^4}{4}$ is the potential function. Usually, A is a constant. Here A will be taken as a system parameter. $\xi(t)$ is Gaussian white noise with zero mean average and autocorrelation of $< \xi(t)\xi(s) >= 2D\delta(t-s)$. The angular brackets denote an ensemble average.

In general, the power norm does not have an explicit expression. For the specific case where the signal amplitude is small compared with the barrier height, i.e., $\overline{S(t)^2} << A^2$, and S(t) is a Gaussian-distributed signal, $<C_0>$ and $<C_1>$ are given as [22]:

$$<C_0> \approx Q\Delta\exp[-\Theta + \Delta^2\overline{S^2(t)}/2]\overline{S^2(t)}, \tag{8}$$

$$< C_1 > \approx \frac{\Delta[\overline{S^2(t)}]^{1/2}}{\{\exp[\Delta^2\overline{S^2(t)}] - 1 + \sigma(D)Q^{-2}\exp[2\Theta - \Delta^2\overline{S^2(t)}]\}^{1/2}}, \tag{9}$$

where $\sigma(D) = K_1 < \overline{R(t)} >, < \overline{R(t)} > \approx Q\exp[-\Theta + \Delta^2\overline{S^2(t)}/2]$

$$Q = k_0 A/\sqrt{2}\pi, \Theta = A^2/4D, \Delta = A/2D$$

We will choose $<C_1>$ as the objective function to be maximized. The theoretic expression of $<C_1>$ can predict its real shape, even when the noise intensity is outside the range of its validity [22]. So, we can form the following constrained optimization problem:

$$\max < C_1 >, \quad \text{subject to } \overline{S(t)^2} << A^2,\ D \geq D_0 \tag{10}$$

The optimization parameters are the system parameter A and the noise intensity D. From the expressions of Q, Θ and Δ, we notice that Θ is a function of Q and Δ:

$$\Theta = \frac{\sqrt{2}\pi\Delta Q}{2K_0} = cQ\Delta \tag{11}$$

where $c=\sqrt{2}\pi/2k_0$ is a constant, Q and Δ are functions of A and D.

The $<C_1>$ can be expressed in term of Q and Δ:

$$< C_1 >= \frac{\Delta s}{\{\exp[\Delta^2 s^2]-1+k_1 Q^{-1}\exp[c\Delta Q+d\Delta^2]\}^{1/2}} \tag{12}$$

where $s=[\overline{S^2(t)}]^{1/2}$ and $d=\overline{S^2(t)}/2-\overline{S^2(t)}=-\frac{s^2}{2}$.

Therefore we will be interested in maximizing $<C_1>$ at some nonzero optimal values Q^* and Δ^*.

From the simulation, we find that there is a unique local maximizer for the unconstrained optimization problem, i.e. (10) without the constraints. Unfortunately, the local maximizer (Q^* and Δ^*) cannot meet the constraint requirements for some input signal. For example, when s=0.01, we will have Δ^*=142.791, Q^*=0.0032, A*=0.014. It cannot meet the requirement of small signal ($\overline{S(t)^2} << A^2$).

So we introduce two additional parameters (a>0, b>0) into the system by defining:

$$U(x) = -[A - S(t)]\frac{x^2}{2a} + \frac{x^4}{4b} \tag{13}$$

We can then get:

$$< C_1 >= \frac{s\Delta}{(\exp[s^2\Delta^2]-1+k_1 Q^{-1}\exp[ca\Delta Q+d\Delta^2])^{1/2}} \tag{14}$$

where: $Q = k_0 A/\sqrt{2}a\pi, \Theta = bA^2/4a^2D, \Delta = bA/2a^2D, \Theta = ca\Delta Q$.

From (14), one sees that the bigger the parameter *a*, the smaller the $<C_1>$ will be. Here parameter *a* will be taken as a supporting parameter used to adjust the local maximizer (Q^*,Δ^*) to enable A* and D* to meet the constraints. Parameter b is also taken as a supporting parameter which will be used to match parameter *a* to keep the potential function in good shape and make the optimal noise intensity D* reasonable. For example, we can let b/ $(2a^2)$ be a proper constant.

Proposition 1: There exists one and only one pair of parameters (Q^*,Δ^*) satisfying the first-order necessary condition of this unconstrained optimization problem.

Proof: First, it is shown that the first-order necessary condition has at least one solution:

From the first-order necessary condition:

$$\frac{\partial < C_1 >}{\partial\Delta} = 0 \text{ and } \frac{\partial < C_1 >}{\partial Q} = 0. \tag{15}$$

We get:

$$\begin{aligned} &ca\Delta Q = 1, \\ &(2-2s^2\Delta^2)\exp[s^2\Delta^2]-2 \\ &+k_1(2Q^{-1}-ca\Delta-2d\Delta^2 Q^{-1})\exp[caQ\Delta+d\Delta^2]=0 \end{aligned} \tag{16}$$

Letting $Q^{-1} = ca\Delta$, we have

$$\begin{aligned} &(2-2s^2\Delta^2)\exp[s^2\Delta^2]-2 \\ &+cak_1(\Delta-2d\Delta^3)\exp[1+d\Delta^2]=0 \end{aligned} \tag{17}$$

Let $f(\Delta)$ denote the left hand of equation (17). Obviously, f (0) = 0 and f (+ ∞) = - ∞ . Also:

$$\begin{aligned} \frac{\partial f}{\partial\Delta} &= (-4s^2\Delta-4s^4\Delta^3)\exp[s^2\Delta^2] \\ &+k_1ca(1+2s^2\Delta^2-s^4\Delta^4)\exp[1+d\Delta^2] \end{aligned} \tag{18}$$

If $\Delta \to 0^+$, we get $\frac{\partial f}{\partial\Delta} > 0$. We can conclude that there is at least one $\Delta > 0$ satisfying eq. (17).

Now, we need to prove that the solution is unique for fixed parameters *a* and b.

Let $f_1(\Delta) = (2-2s^2\Delta^2)\exp[s^2\Delta^2]$. The function $f_1(\Delta)$ will decrease monotonically to $-\infty$ as $\Delta \to \infty$, starting from $f_1(0) = 2$. We denote the rest part of the LHS of equation (17) as function $f_2(\Delta) = cak_1\Delta(1+s^2\Delta^2)\exp[1-s^2\Delta^2/2]$. It will first increases from zero, and then decreases to zero. From these special characteristics of $f_1(\Delta)$ and $f_2(\Delta)$, it follows readily that equation (17) can only have one positive solution.

Proposition 2: The parameter *a* can be used to continuously adjust the values of Q^* and Δ^* satisfying the first-order necessary condition of the unconstrained optimization problem to ensure A* and D* will meet the constraint requirements.

Proof: From (17) and definitions of $f_1(\Delta)$ and $f_2(\Delta)$, we notice that parameter *a* only affects $f_2(\Delta)$, but not $f_1(\Delta)$ which is a decreasing function of Δ. The increase of parameter *a* will increase the value of $f_2(\Delta)$ for the same Δ. This will in turn increase Δ^*. Also, Δ^* will approach zero when parameter *a* approaches zero. This means that Δ^* can be changed continuously by adjusting parameter *a*.

Proposition 3: There is one and only one local maximizer for the unconstrained optimization problem with small input (s<<1) and properly chosen parameters *a* and b.

Proof: Assume the pair (Q^*, Δ^*) is the only solution satisfying the first-order necessary condition. We need to prove the pair will also satisfy the sufficient condition of the unconstrained optimization problem, that is, the Hessian matrix $\nabla^2 < C_1 > (\Delta^*, Q^*)$ is negative definite.

At the point (Q^*, Δ^*), we have:

$$\frac{\partial^2 < C_1 >}{\partial \Delta^{*2}} = \tag{19}$$

$$\frac{2(-s^3\Delta^{*2})[s\Delta^* \exp[s^2\Delta^{*2}] + k_1 ca(-\frac{1}{2} + \frac{s^2\Delta^{*2}}{4})]\exp[1 - s^2\Delta^{*2}/2]}{(\exp[s^2\Delta^{*2}] - 1 + k_1 Q^{*-1}\exp[1 + d\Delta^{*2}])^{3/2}},$$

$$\frac{\partial^2 < C_1 >}{\partial Q^{*2}} = \frac{(-k_1 c^3 a^3 s \Delta^{*4})}{2(\exp[s^2\Delta^{*2}] - 1 + k_1 Q^{*-1}\exp[1 + d\Delta^{*2}])^{3/2}}, \tag{20}$$

$$\frac{\partial^2 < C_1 >}{\partial \Delta^* \partial Q^*} = \frac{\partial^2 < C_1 >}{\partial Q^* \partial \Delta^*} = , \tag{21}$$

$$\frac{(-k_1 c^2 a^2 s \Delta^{*2})\exp[1 + d\Delta^{*2}]}{2(\exp[s^2\Delta^{*2}] - 1 + k_1 Q^{*-1}\exp[1 + d\Delta^{*2}])^{3/2}}.$$

We have: (for s<<1)

$$\begin{aligned}
&2s^2\Delta^{*2}\exp[s^2\Delta^{*2}] \\
&+ 2k_1 cas\Delta^*(-1/2 + s^2\Delta^{*2}/4)\exp[1 - s^2\Delta^{*2}/2] \\
&= 2\exp[s^2\Delta^{*2}] - 2 \\
&+ k_1 ca\Delta^*[(1 - s) + s^2\Delta^{*2}(1 + s/2)]\exp[1 - s^2\Delta^{*2}/2] \\
&\approx 2s^2\Delta^{*2}\exp[s^2\Delta^{*2}] > 0
\end{aligned}$$

So $\frac{\partial^2 < C_1 >}{\partial \Delta^{*2}} < 0$ for Q^*, Δ^*, and small input signal (s<<1).

If the parameter a is adjusted properly such that s(Δ*)^2>>1, the numerator of the Hessian matrix determinant value is:

$$\begin{aligned}
&2\exp[s^2\Delta^{*2}] - 2 \\
&+ k_1 ca(\Delta^* - \frac{1}{s\Delta^*} + s^2\Delta^{*3})\exp[1 - s^2\Delta^{*2}/2] \\
&\approx 2\exp[s^2\Delta^{*2}] - 2 \\
&+ k_1 ca(\Delta^* + s^2\Delta^{*3})\exp[1 - s^2\Delta^{*2}/2] \\
&= 2s^2\Delta^{*2}\exp[s^2\Delta^{*2}] > 0
\end{aligned}$$

From the standard test on negative-definiteness of a symmetric matrix, it follows that the Hessian matrix is negative definite. This completes the proof of Proposition 3.

Proposition 4: There is one and only one local maximizer for the original *constrained* optimization problem (10) with small input and properly chosen parameter a and b.

Proof: From above, we can get A*=2/Δ^*. In order to satisfy the constraint: (A*)^2 >>s^2, we should have s^2(Δ^*)^2<<4. Combined with the requirement s(Δ^*)^2>>1, we should have: $s << s^2(\Delta^*)^2 << 4$ for small input signal (s<<1). This can be satisfied by adjusting parameter a. Also, D* will be greater than D_0 for the properly chosen parameter b. Therefore, Proposition 4 follows directly from Proposition 3.

Proposition 5: The local maximizer (*A**, *D**) is also the global maximizer of the constrained optimization problem (10).

Proof: It follows from Proposition 4 and the fact that the first-order necessary condition only has one solution.

From the above analysis, the normalized power norm $<C_1>$ of the double-well bistable system with Gaussian-distribution input signal can be maximized by tuning system parameter A and adding noise simultaneously and will reach a higher maximal value than that of adjusting only the system parameter or noise intensity.

IV. OPTIMIZATION ALGORITHM

There is no closed-form solution for the constrained optimization problem in Section 3. The maximizer can be obtained by solving the nonlinear equation (17) with the aid of standard optimization algorithms. In some situations, however, this optimization problem should be solved on-line with changing input signals, such as the case of high-speed target detection when it is used as the nonlinear signal processor. The speed is a critical requirement in these situations. This makes the development of an on-line fast convergent optimization algorithm an issue of crucial importance.

Let $f(\Delta)$ be the left part of equation (17). Noting that $\nabla f(\Delta^*)$ is nonsingular for small input signals, the following result can be proved using standard arguments from [23]:

Proposition 6: The Newton's Method for Nonlinear Equations, when applied to solving $f(\Delta^*) = 0$, gives a local Q-quadratic convergence, if Δ_0 is sufficiently close to Δ^*.

Our proposed optimization algorithm is based on the Newton Algorithm. The optimization algorithm is divided into two categories. The fist case is when parameters a and b are fixed. They are properly chosen so that the pair of optimal solutions (A*, D*) for the unconstrained optimization problem can meet the given constraints. The second case is when parameter a should also be adjusted on-line. Here we assume parameter b is a pre-defined function of parameter a. For example, $b/(2a^2)$ = const.

Case 1:

The convergence speed of the Newton algorithm depends on the initial value Δ_0. We propose a way to estimate the initial value for different input signals on-line, based on a table or function constructed off-line. The table or the function describes the relationship between input signal average amplitude s and the optimal value Δ^* which can be got off-line. For a given input signal s, Δ^* will first be estimated using interpolation for the table or direct calculation for the function constructed off-line. This Δ^* will then be used as the initial value Δ_0 for this input signal. It will be close to the optimal value to ensure the required convergence speed, if the table or the function is constructed properly.

Algorithm 1:

Step 1:

Calculate its average amplitude value s=$\overline{s^2(t)}$ for the given input signal;

Step 2:

Estimate initial Δ_0, using the constructed table or function;

Step 3:

Solve $f(\Delta^*) = 0$ using normal Newton Algorithm [23];

Step4:

Calculate Q*, A* and D* and stop.

Case 2:

If adjustable, the smallest parameter *a* ensuring the satisfaction of constraints will maximize $<C_1>$.

Similarly we can rely on the off-line work to increase the convergence of the on-line algorithm. Two tables or two functions will be constructed off-line. The first table or function describes the relationship of Δ^* with input s and parameter *a*. The second table or function is the relationship of input signal s with *a**, where *a** is the smallest parameter *a* for the above-stated constrained optimization problem with input signal s.

Algorithm 2:

Step 1:

Calculate s=$\overline{s^2(t)}$, set $x_0 = 0, y_0 = +\infty$

Step 2:

Estimate a*, take it as the initial value a_0;

Step 3: Estimate the initial value Δ_0;

Step 4:

Solve $f(\Delta^*) = 0$ using normal Newton Algorithm [23];

Step 5:

If $s << s^2(\Delta^*)$ **:**

$y_{k+1} = a_k, x_{k+1} = x_k, a_{k+1} = (x_{k+1} + y_{k+1})/2$

else:

$y_{k+1} = y_k, x_{k+1} = a_k$

if $y_{k+1} = +\infty$ **:**

$a_{k+1} = 2a_k$

else:

$a_{k+1} = (x_{k+1} + y_{k+1})/2$

Step 6:

If $|a_{k+1} - a_k| < \varepsilon$:

Calculate Q*, A* and D* and stop.

else:

Go back to Step 3

V. SIMULATION RESULTS

In order to verify the improvement of the maximal value of normalized power norm $<C_1>$ by adjusting system parameter A and noise intensity D simultaneously over that by adjusting system parameter A or noise intensity D alone, simulation is performed. The following is the simulation result: (K_0=1, K_1=0.019, a=0.001, and A=1 when adjusting D, and D=0.1 when adjusting A)

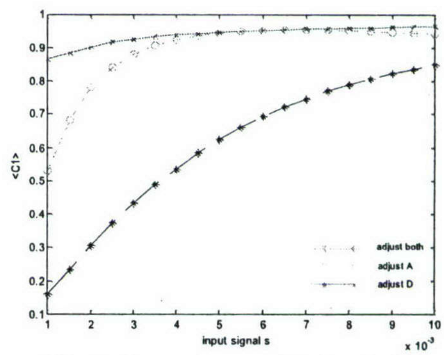

Fig. 1. Comparison of Maximal $<C_1>$

V. CONCLUSION

This paper demonstrates the possibility to further enhance the stochastic resonance effect if the system parameter and the noise intensity can be adjusted at the same time. The enhancement of the stochastic resonance effect will in turn improve the system performance and have wide application in signal and image related engineering problems such as target detection. Specifically, the nonlinear signal processor based on stochastic resonance will increase the target detection performance if it has a higher output signal-to-noise ratio.

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