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EFFECT OF ADHESIVE MATERIAL PROPERTIES ON INDUCED STRESSES IN BONDED SENSORS (PREPRINT)

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14. ABSTRACT

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EFFECT OF ADHESIVE MATERIAL PROPERTIES ON INDUCED STRESSES IN BONDED SENSORS

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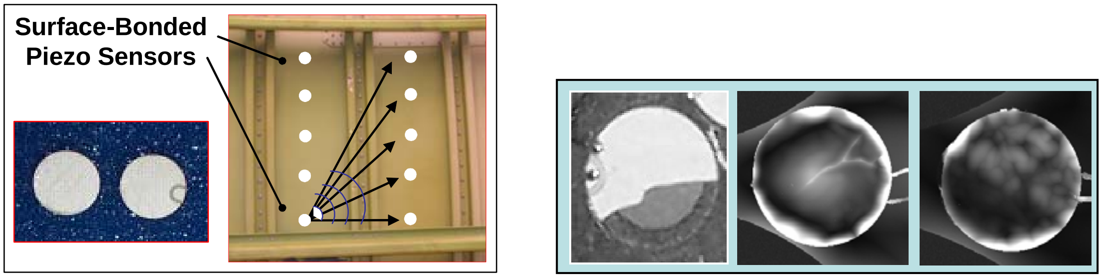
ABSTRACT. An analysis of the stresses induced in adhesively bonded sensors from a biaxial stress field in the underlying substrate is presented. Recent Structural Health Monitoring work has looked at using surface bonded sensors to detect and characterize damage in aircraft structures. In addition to the proper design of these systems, it is important that they be able to survive in a sometimes hostile operating environment in terms of weather, vibration, temperature, and mechanical loading of the structural members of the airframe. The analysis first considers the load transfer mechanism from the substrate through the adhesive layer into the sensor. The partitioning of the load between the substrate and sensor is found to depend on the substrate stiffness, the sensor thickness, and the shear modulus and thickness of the adhesive. The analysis then shows that for an elliptically shaped sensor whose maximal dimension is small compared to the substrate in-plane dimensions the stress induced by a biaxial state of stress can be determined using inclusion theory. It is further shown that the stresses in a circular sensor on a substrate subjected to a hydrostatic state of stress can be calculated using equations derived from those used to determine the interfacial pressure for an interference fit between annular cylinders. Finally, the analysis considers induced bending stresses caused by the asymmetric change in thickness in the region where the bonded sensor resides.

Keywords: adhesively bonded sensors, induced stress, structural health monitoring.
PACS: 43.38.Fx

INTRODUCTION

Recent work on maintaining structural integrity of structures has focused on Structural Health Monitoring. This work has looked at the use of integral or surface bonded sensors to detect and characterize defects in aircraft structures. Figure 1 shows a typical application in an airframe. A proper design of these systems must include the ability to operate in a non-ideal environment which can mean exposure to weather, vibration, temperature, and mechanical loading of the structural members of the airframe. These issues were examined experimentally in Blackshire and Cooney[1]. Stresses in bonded sensors are induced by biaxial loading, bending, and thermally induced stresses of the substrate. Biaxial and bending stresses in the substrate result from externally applied loadings. The thermally induced stresses arise from differences in the thermal coefficient of expansion between the substrate and sensor material. The current work looks at the



(a) (b)

FIGURE 1. (a) Typical application of bonded sensors for structural health monitoring (b) sensors damaged by loading of the substrate and thermal cycling

stresses in bonded sensors arising from biaxial stress in the substrate and the effect the adhesive bond has on them.

The bonded sensor is analogous to bonded composite reinforcements, 'doublers', that are used for airframe repairs, as shown in figure 2, in regard to its' stress behavior. These reinforcing patches are bonded to plates over a cracked area. There are two differences when comparing the bonded ‘doubler’ and bonded sensor cases. First, the ‘doubler’ stress problem requires accounting for the stress effects of the crack while there should be no crack in the bonded sensor case. Second, the objective when using the composite reinforcement is to transfer as much of the load from the underlying substrate as possible. The exact opposite is true when using bonded sensors, i.e. it is desired to expose the sensor to as little of the underlying substrate load as possible.

In [2], Rose pointed out that the problem of a symmetrical bonded reinforcement patch on a plate may be transformed into the inclusion problem to solve for the stress in the reinforcement and the plate. The overall analysis is divided into two stages:

- consider the stress redistribution caused by reinforcement of the un-cracked plate
- introduce a cut in the plate to model the crack and consider the change induced in the stress field

Only the first stage need be considered for the bonded sensor case. The three steps outlined by Rose to analyze the stresses in the un-cracked, reinforced plate are:

- determine the equivalent stiffness of the reinforced area
- find the stresses resulting from the in-plane loading for the reinforced area
- determine the out of plane bending stresses if the reinforcement is asymmetrical

To determine the equivalent stiffness of the reinforced area the effects of the adhesive bonding layer must be taken into account. Rose gives an outline of the adhesive model, inclusion model, and out of plane bending effects, but is concerned with the stresses in the plate in the reinforced region. In our case we are concerned with the load in the reinforcement (sensor). The results of this stress analysis are applied to an example cylindrical piezoelectric sensor bonded onto an aluminum plate. The sensor is .01 m

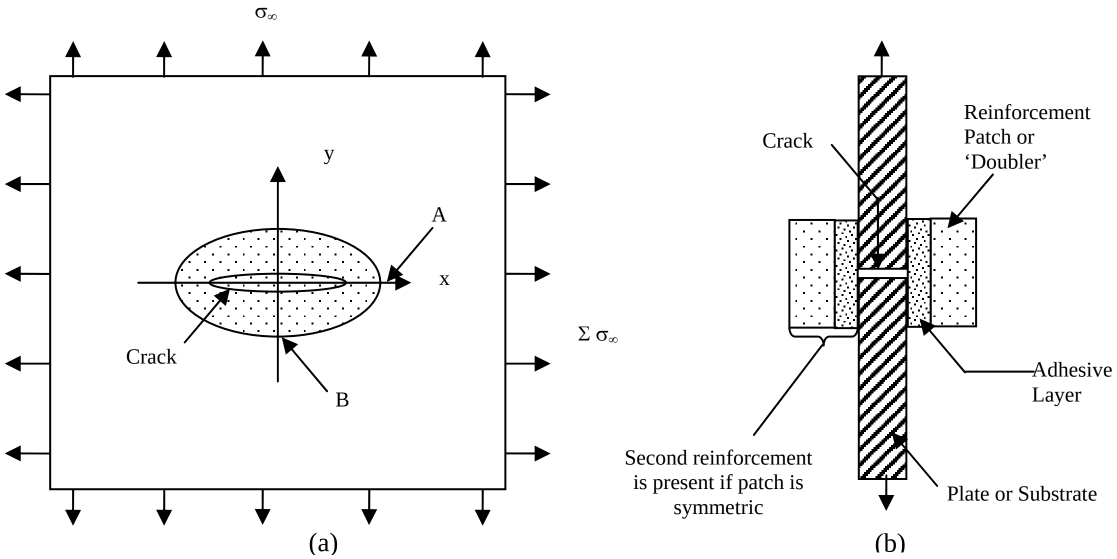


FIGURE 2. Bonded reinforcement patch over a crack (a) Plan view (b) Cross section along x = 0.

in diameter and .0001m thick with a Modulus of Elasticity equal to 84.0 GPa. The substrate is .001m thick with a Modulus of Elasticity equal to 73.1 GPa. The adhesive layer is .0001m thick. with an adhesive shear modulus of 700.0 MPa. The adhesive shear modulus of 700.0 MPa, representative of composite patches adhesives, is considered to be a rigid bond while a value of 7.0 MPa is used as typical of a compliant bond.

ADHESIVE MODEL

The bonding of the sensor to the substrate determines the effectiveness with which the load is transferred into the sensor. The load transfer mechanism between members in a bonded joint can be explained using the one-dimensional theory of bonded joints[3]. The load transfer is considered to arise as from a shear loading through the adhesive layer as shown in figure 3. In this theory, each adherend is treated as a one-dimensional continuum with a corresponding Modulus of Elasticity, *E*, and thickness, *t*. The adherend deformation is given by a longitudinal displacement, *u*, and a corresponding longitudinal stress, σ. The stress-displacement relation for a member is:

$$\sigma(y) = Eu'(y) = F/t$$

where *F* is the load per unit length and the dash(') indicates differentiation. The adhesive

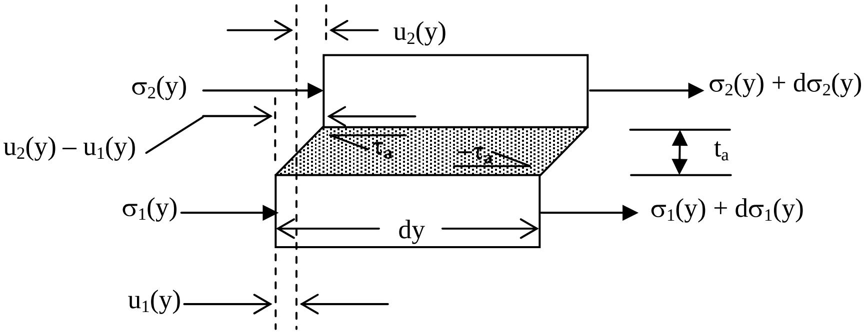


FIGURE 3. Differential section of an adhesively bonded joint in a deformed state.

layer acts as a shear spring with the relationship for the adhesive shear stress given by

$$\tau_a(y) = G_a\gamma_a = (G_a/t_a)[u1(y) - u2(y)]$$

where G_a is the adhesive shear modulus, γ_a is the adhesive shear strain, and t_a is the adhesive layer thickness.

The shear tractions exerted by the adhesive can be replaced by an equivalent body force distributed uniformly across the thickness of each adherend, leading to the differential equilibrium equations:

$$t_p\sigma'_p = -t_r\sigma'_r(y) = \tau_a(y).$$

The forgoing assumptions lead to the differential equation

$$\tau''_a(y) - \beta^2\tau_a(y) = 0 \quad \text{where } \beta^2 = (G_a/t_a)\{(1/E_pt_p) + (1/E_rt_r)\}.$$

The asymmetric bonded doubler case is shown in figure 4. The substrate adherend is not disjoint and the full load is not carried by the doubler. The difference between the symmetric and asymmetric doubler case is that there are out of plane bending stresses induced by the loading. These will be treated later separately. The solution for this problem is found over the domain $(-L <= y <= L)$. The equation for β^2 is:

$$\beta^2 = (G_a/t_a)\{(1/E_pt_p) + (1/E_rt_r)\}.$$

The boundary conditions for this problem are:

$$\tau'_a(-L) = (G_a/t_a)\{(\sigma_p(0)/E_p) - (\sigma_r(0)/E_r)\} = (G_a/t_a)\{(F/ E_pt_p) - 0\}$$

$$\tau'_a(L) = (G_a/t_a)\{(\sigma_p(L)/E_p) - (\sigma_r(L)/E_r)\} = (G_a/t_a)\{(F/ E_pt_p) - 0\}.$$

The load is asymptotically transferred into the sensor and approaches a value determined by stiffness partitioning between the sensor and the substrate. This stiffness partitioning value is found from:

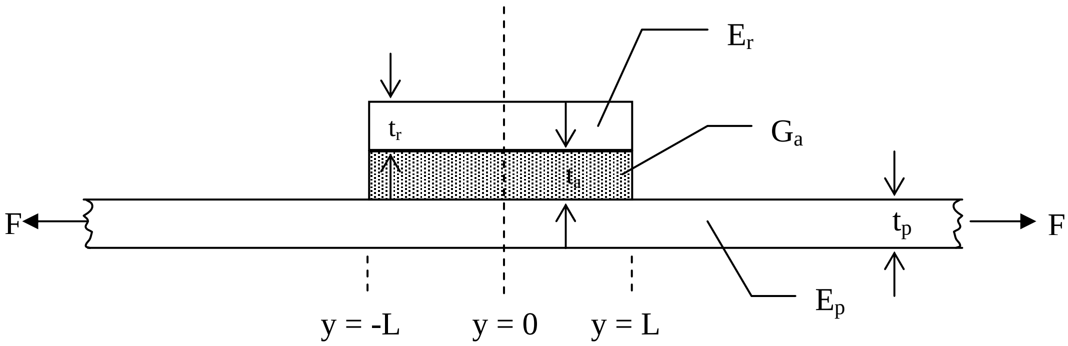


FIGURE 4. Adhesively bonded reinforced plate geometry.

$$\sigma_r(sp) = FS/((1+S)t_r)$$

where S is the stiffness ratio defined by $S = (E_r t_r / E_p t_p)$.

The equation for $\tau_a(y)$ can be solved by using the boundary conditions. By utilizing the relations between the derivatives of the normal stresses and the shear stress, integration is performed to give the reinforcement stress in terms of the stiffness ratio as:

$$\sigma_r(y) = [FS/((1+S)t_r)][\,1 - \{\cosh(\beta y)/\cosh(\beta L)\}] = \sigma_r(sp)[\,1 - \{\cosh(\beta y)/\cosh(\beta L)\}].$$

The reinforcement stress for a compliant, rigid, and infinitely rigid bond is shown in figure 5. The adhesive shear stress for a compliant and rigid bond is shown in figure 6.

INCLUSION MODEL

The state of stress in an infinite plate with a biaxial stress state and with different mechanical properties has been solved in terms of integral equations[4]. For the case of an ellipsoidal inclusion the stress field is constant throughout and can be calculated by relatively simple means. The problem for the inclusion analogy is shown in figure 7. The load transmitted across $y = 0$ within the reinforced region ($|x| < A$, $|y| < B$), with $\Sigma = (A/B)$ and $\nu =$ Poisson's ratio, amounts to the force F per unit length along the x-axis, given by:

$$F = \sigma^{\infty} t_p\{\,1 + (S/D)[\,1 + 2(1 + S)(B/A)(1 - \nu\Sigma) + (1 + S - \nu S)(\,\Sigma - \nu)]\}$$

where $D = 3(1 + S)^2 + 2(1 + S)((B/A) + (A/B) + \nu S) + 1 - \nu^2 S^2$.

The load induces a normal stress in the plate of:

$$\sigma_0^{\,p} = \sigma_{yy}^{\,p}(|x| < A, y = 0) = F/\{t_p(1+S)\}$$

and a normal stress in the reinforcement of:

$$\sigma_0^{\,r} = \sigma_{yy}^{\,r}(|x| < A, y = 0) = FS/\{t_r(1+S)\}.$$

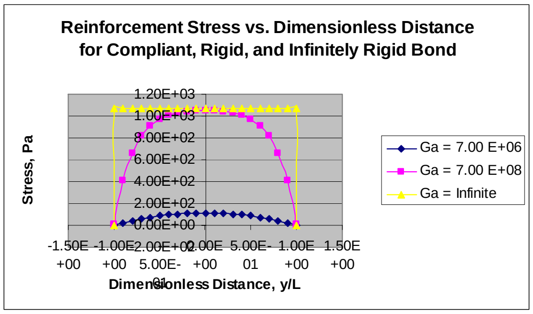


FIGURE 5. Reinforcement stress distribution for 1.0 N/m unit lineal load in a reinforced plate with a compliant, rigid, and infinitely rigid bond.

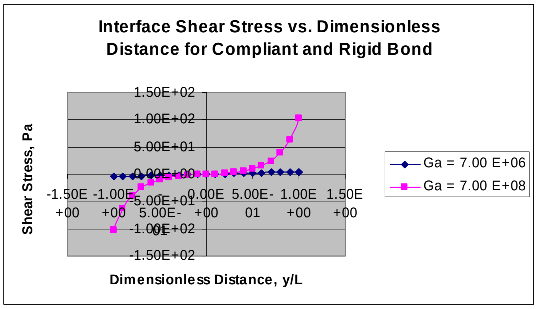


FIGURE 6. Interface shear stress distribution for 1.0 N/m unit lineal load in a reinforced plate with a compliant and rigid bond.

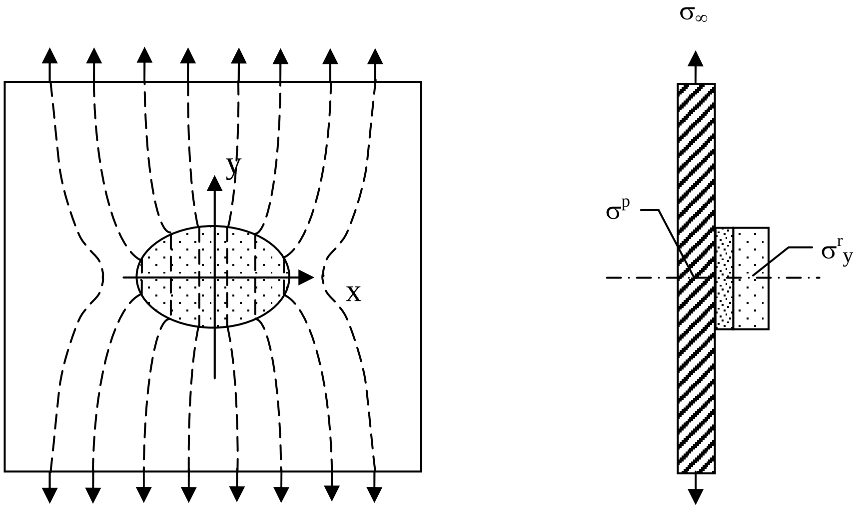
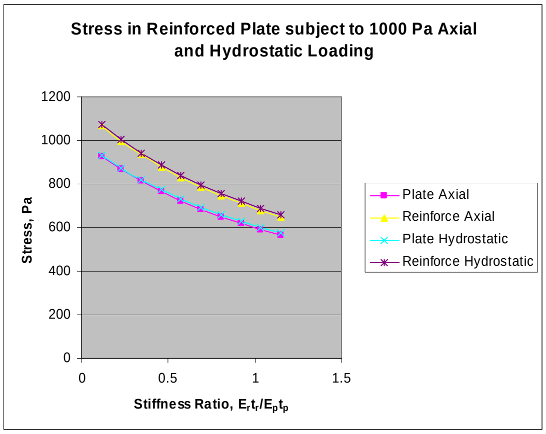


FIGURE 7. Stress field in an adhesively bonded reinforced plate geometry for uniaxial loading.

The results of the stress model for our sensor are shown in Figure 8. A second line of reasoning was followed to check the results of the inclusion analogy. The Lame' thick cylinder equations give the stress and displacement distributions when an annular cylinder (pipe) is exposed to an internal and external pressure. These equations can be used to find the interfacial pressure for an interference fit between thick wall cylindrical pipes[5] as shown in figure 9. They are capable of handling inner and outer pipes with different material properties.

The primary idea is to treat the inclusion problem with a circular inclusion and equal biaxial stresses as the interference fit between an outer pipe of infinite outer radius and an inner pipe with a zero inner radius (i.e. a solid cylinder). The difference in stiffness between the plate and reinforced plate areas is handled by adjusting the Young's Modulus of the inner cylinder. The equations for a circular inclusion in a infinite circular plate can be generated by taking the interference fit equations and finding their limits as the outer radius, *c*, becomes infinite and the inner radius, *a*, becomes zero. This procedure yields



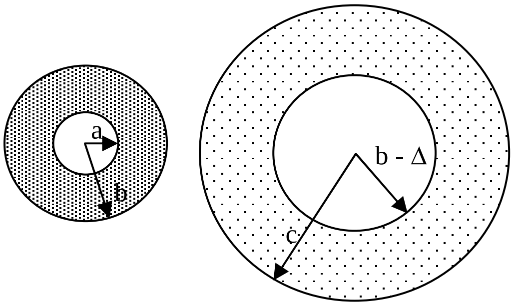


FIGURE 8. Plate and reinforcement stress distribution for 1,000 Pa axial and hydrostatic stress state in a rigidly bonded reinforced plate with an infinitely rigid bond.

FIGURE 9. Compound Cylinders with interference fit problem solved using the Lame' thick cylinder equation.

the following relationship between the pressure at infinity, p_∞, and the interfacial pressure, p_f :

$$\left(\frac{p_f}{p_\infty}\right) = \left[\frac{\left(\frac{1-\nu_2}{1-\nu_1}\right)E_1 - E_2}{\left(\frac{1+\nu_2}{1-\nu_1}\right)E_1 + E_2}\right]$$

where E is Young's modulus and ν is Poisson's ratio. A comparison of the results for the inclusion theory and Lame' generated equations is given in figure 10.

OUT OF PLANE BENDING

When the reinforcement is asymmetrical out of plane bending stresses result. The correction in Rose is based on a result given in Appendix B of Fraser[6]. The correction is based upon the analysis of induced bending in a stretched beam with a thickness discontinuity. The stresses resulting from out of plane bending are shown in figure 11.

CONCLUSIONS AND FUTURE RESEARCH

The stress analysis presented here has shown that the stress behaviour of bonded sensors is analogous to that of bonded reinforcement patches. The amount of stress experienced in a bonded sensor from a biaxial loading on the substrate is dependent upon the stiffness ratio, the adhesive layer thickness, adhesive shear modulus, and the geometry of the sensor. The theory of bonded joints gives a model to understand the effect these parameters have on the stress experienced in the adhesive layer and the sensor by load transfer from the substrate. The reinforcement stress exponentially increases towards its maximum value as approximately $(1 - exp(-\beta y))$. If $(\beta L < 5)$ then the entire domain is boundary layer and fully developed stress transfer as given by the stiffness partitioning value can not occur. The stress values predicted allow for prediction of failures in the adhesive layer (debonding) and/or the sensor(fracture).

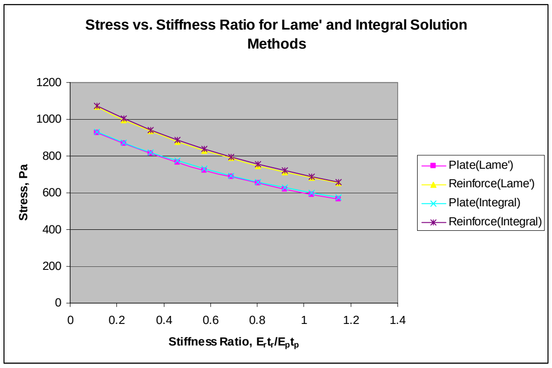


FIGURE 10. Plate and reinforcement stress distribution for 1,000 Pa hydrostatic stress state in a rigidly bonded reinforced plate with an infinitely rigid bond.

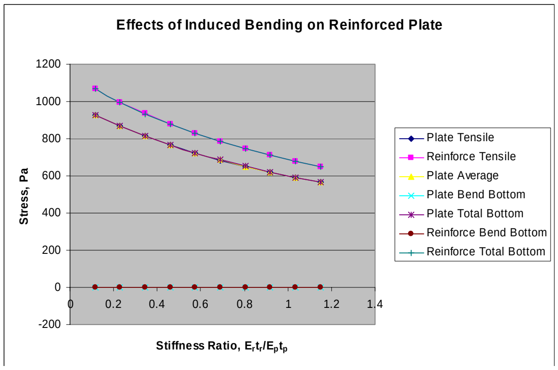


FIGURE 11. Plate and reinforcement bending and tensile stress distribution for 1,000 Pa axial stress state in a bonded reinforcement plate with an infinitely rigid bond.

The actual load carried through an elliptical reinforced area for a general biaxial state of stress when the patch dimension was small compared to the plate in-plane dimensions was shown to be predicted by the inclusion theory. A solution specific to the case of a circular patch exposed to a hydrostatic state of stress was derived from the solution for an interference fit between annular cylinders with different elastic properties. The derived equations were solved for our example sensor with different stiffness ratios and the two solutions demonstrated excellent correlation. The bending induced by the asymmetry of the geometry was modeled and for the particular case examined, the stress arising from induced bending was negligible.

The areas to be pursued for further work include analyzing stresses in bonded sensors that are generated by bending and thermally induced stresses of the substrate. The results of the analysis using the theory of bonded joints suggested the possibility of decoupling the sensor from the stress field of the underlying substrate through use of a compliant adhesive. Since the biaxial, bending and thermally induced stresses in the substrate are generally static or low frequency in nature, this leads to the idea of using an adhesive with strong viscoelastic properties to decouple the sensor from the loading in the substrate while allowing the sensor to be strongly coupled to the substrate when generating ultrasonic waves.

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REFERENCES

1. J. L. Blackshire and A. T. Cooney, “Characterization of Bonded Piezoelectric Sensor Performance and Durability in Simulated Aircraft Environments,” in *Review of Progress in Quantitative Nondestructive Evaluation 25B, edited by D. O. Thompson and D. E. Chimenti, AIP Conference Proceedings vol. 820, American Institute of Physics, Melville, NY, 2005, pp. 1694-1701.*
2. L.R.F. Rose, “Theoretical Analysis of Crack Patching” in *Bonded Repair of Aircraft Structures,* edited by A.A. Baker and A. Jones, Martinus Nihoff Publishers, Hingham, MA, USA, 1988.
3. L.J. Hart-Smith, “Analysis and design of advanced composite bonded joints”, NASA Contractor Rep. CR-2218 (1974).
4. J.D. Eschelby, *Proc. R. Soc. London,* **A241**, 376-396 (1957).
5. R. C. Juvinall, *Engineering Considerations of Stress, Strain, and Strength,* McGraw-Hill, U.S.A., pp. 128-130 (1967).
6. W.B. Fraser, *Int. J. Solids Structures*, **11**, 1245-1256 (1975).