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The main theme of the grant has been to study design of input probe signals for robust system identification. We have used these methods for design of pulse sequences in NMR spectroscopy that are robust to inhomogeneity and dispersion in the parameters of the system of interest. We have developed control theoretic methods which allow control of nuclear spins in presence of extreme inhomogeneities.  
We have shown that these new nonlinear control methods for probe design and system identification are an enabling technology for NMR and MRI in inhomogeneous fields, making mobile NMR and MRI systems practical. Such a system can serve multiple roles in a combat situation including diagnosis of head injury and trauma, detection of explosives and scanning metabolites in the living tissue.

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# Intelligent Sensing and Probing with Applications to Protein NMR Spectroscopy and Laser Chemistry

## 1 Main Theme

In many military and biological applications of signal processing, one is faced with the situation that the signals recorded by the sensors are largely influenced by how the system of interest is probed or excited. Therefore, besides design of optimal estimators, it is possible to acquire information about the system more efficiently by design of input signals and waveforms that probe the system of interest. These input signals should be so designed that they attempt to reduce the uncertainty of the unknown parameters the most. In this proposed work, we aim to develop a mathematical theory which will lead to optimal probing procedures with applications ranging from military applications including radar technology to biological applications like protein NMR spectroscopy. Since the inception of the grant, our main focus has been to study design of input probe signals (pulse sequences) in NMR spectroscopy that are robust to inhomogeneity and dispersion in the parameters of the spin systems that we are probing.

## 2 Applications

### 2.1 Robust NMR Spectroscopy

In NMR spectroscopy, probe signals (radio frequency pulses) are used to manipulate the dynamics of the nuclear spins. The goal is to disturb the system from its equilibrium position and by observing the return dynamics of the system one can infer something about the system parameters. A major challenge in these problems is to design a probe signal that produces a uniform excitation of an inhomogeneous ensemble of systems with large dispersion in their parameters. For example, in magnetic resonance experiments, the spins of an ensemble may have large dispersion in their natural frequencies (Larmor dispersion), strength of applied rf-field (rf-inhomogeneity) and the relaxation rates of the spins. In solid state NMR spectroscopy of powders, the random distribution of orientations of inter-nuclear vectors of coupled spins within an ensemble leads to a distribution of coupling strengths. A canonical problem in control of quantum ensembles and quantum information processing is to develop external excitations that can simultaneously steer the ensemble of systems with variation in their internal parameters from an initial state to a desired final state. From the standpoint of mathematical control theory, the challenge is to simultaneously steer a continuum of systems between points of interest with the same control signal. In many cases of practical interest, one wants to find a control field that prepares the final state as some desired function of the parameter. For example, slice selective excitation and inversion pulses in magnetic resonance imaging. The problem of designing excitations that can compensate for dispersion in the dynamics is a very important problem in NMR spectroscopy and is of fundamental importance in developing a theory of optimal probe signal design.

In last couple of years we have had a major conceptual breakthrough in understanding of what aspects of system dynamics makes it possible to design probe signals signal that can compensate for dispersion in the system dynamics. We have used these ideas for design of pulse sequences in both liquid and solid state NMR spectroscopy which give far superior performance than state of the art

methods [6-14]. This has resulted in over two dozen publications in refereed journals over the period of last 2 years. We sketch the main ideas of our work in the following section. **Then we show how we have used these ideas in developing methods for NMR and MRI in Inhomogeneous fields which is the major breakthrough in our research in the past year.**

### 3 Main Ideas: Synthesis of higher powers of dispersion parameters by Lie Bracketing

To fix ideas, consider the following equations that model evolution of spin  $\frac{1}{2}$  in a magnetic field. Let  $X$  denote the unit vector pointing in the direction of magnetization of the spin, and  $v, u$  represent strength of applied magnetic field along  $x$  and  $y$  axis. In practice the magnetic field applied to the system is not uniform across the sample and therefore different spins see a different magnetic field (rf-inhomogeneity) modeled by a scale parameter  $\epsilon$ .

$$\dot{X} = \epsilon(u(t)\Omega_y + v(t)\Omega_x)X$$

where

$$\Omega_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \Omega_y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad \Omega_z = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are the generators of rotation around  $x$ ,  $y$  and  $z$  axis, respectively. Consider now the problem of designing controls  $u(t)$  and  $v(t)$  that simultaneously steer an ensemble of such systems with dispersion in their natural frequency and strength of rf-field from an initial state  $X(0) = (0, 0, 1)$  to a final state  $X_F = (1, 0, 0)$ .

Observe for small  $dt$ , the evolution

$$U_1(\sqrt{dt}) = \exp(-\epsilon\Omega_y\sqrt{dt}) \exp(-\epsilon\Omega_x\sqrt{dt}) \exp(\epsilon\Omega_y\sqrt{dt}) \exp(\epsilon\Omega_x\sqrt{dt})$$

to leading order in  $dt$  is given by  $I + (dt)[\epsilon\Omega_y, \epsilon\Omega_x]$ , i.e., we can synthesize the generator  $[\epsilon\Omega_x, \epsilon\Omega_y] = \epsilon^2\Omega_z$ , by back and forth maneuver in the directly accessible directions  $\Omega_x$  and  $\Omega_y$ .

Similarly, the leading order term in the evolution

$$U_2 = U_1(-\sqrt{dt}) \exp(-\epsilon\Omega_y dt) U_1(\sqrt{dt}) \exp(\epsilon\Omega_y dt).$$

is  $[\epsilon\Omega_y, [\epsilon\Omega_x, \epsilon\Omega_y]] = \epsilon^3\Omega_x$ . Therefore by successive Lie brackets, we can synthesize terms of the type  $\epsilon^{2k+1}\Omega_x$ . Now using  $\{\epsilon\Omega_x, \epsilon^3\Omega_x, \dots, \epsilon^{2n+1}\Omega_x\}$  as generators, we can produce an evolution

$$\exp\left\{\sum_{k=0}^n c_k \epsilon^{2k+1} \Omega_x\right\},$$

where  $n$  and the coefficients  $c_k$  can be chosen so that

$$\sum_{k=0}^n c_k \epsilon^{2k+1} \approx \theta \tag{1}$$

for all  $\epsilon \in [1-\delta, 1+\delta]$ . Hence we can generate an evolution  $\exp(\theta\Omega_x)$  for all  $\epsilon$  to any desired accuracy. Therefore, we achieve robustness with dispersion to  $\epsilon$  by generating suitable Lie brackets. Similar arguments show that we can generate any evolution  $\exp(\beta\Omega_y)$  and as a result any three dimensional rotation in a robust way. It is also now easy to see that we can synthesize rotation  $\Theta$  with a desired functional dependency on the parameter  $\epsilon$ . Parametrize a rotation in  $\Theta \in SO(3)$  by the Euler angles  $(\alpha, \beta, \gamma)$  such that  $\Theta = \exp(\alpha\Omega_x) \exp(\beta\Omega_y) \exp(\gamma\Omega_x)$ . Given continuous function  $(\alpha(\epsilon), \beta(\epsilon), \gamma(\epsilon))$  of  $\epsilon$ , we can find polynomials that approximate  $\alpha(\epsilon)$ ,  $\beta(\epsilon)$  and  $\gamma(\epsilon)$  arbitrarily well and use these to generate a desired rotation  $\Theta(\epsilon)$  as a function of  $\epsilon$ . Hence there exists a control field that maps a smooth initial distribution  $X_0(\epsilon)$  to a target distribution  $X_F(\epsilon)$ .

## 4 Mobile Magnetic Resonance Imaging and Sensing

Modern magnetic resonance imaging and spectroscopy systems typically demand the placement of the sample or subject inside the bore of a large superconducting magnet producing a homogeneous magnetic field. This requirement makes high-field magnets bulky and expensive. For many applications, it would be useful if a mobile magnet could be scanned over an otherwise inaccessible object in order to acquire magnetic resonance information. The advantage of such ex-situ analysis is that limitations in sample size and portability no longer prevail. Mobile NMR/MRI systems essentially about bringing the magnet to the object rather than bringing the object to the magnet. Such a system will serve multiple roles in a combat situation including diagnosing head injury and trauma, detection of explosives and scanning metabolites in the living tissue.

One principal problem with portable or one-sided magnets is that the magnetic fields are necessarily spatially inhomogeneous. As a consequence, the NMR spectra become broadened to the extent that resolution is lost and the chemical shift information is hidden. We are developing new methods of controlling spin dynamics that allows the observation of resolved NMR spectra and MRI pictures of an object or subject in the inhomogeneous field. Our work focuses on pulse and field schemes that enable the retrieval of spectroscopic information even when the static field ( $B_0$ ) and/or the rf field ( $B_1$ ) are strongly inhomogeneous. These methods rely on the fact that the response of the spins to driving excitation is nonlinear which helps to develop external excitations that compensate for their own inhomogeneities.

In past year we have developed control algorithms based on Fourier synthesis for designing phase compensating radio-frequency (rf) pulse sequences for high-resolution nuclear magnetic resonance (NMR) spectroscopy in an inhomogeneous  $B_0$  field. We show that using radio frequency pulses and time varying linear gradients in three dimensions, it is possible to impart the transverse magnetization of spins a phase, which is a desired function of the spatial  $(x, y, z)$  location. Such a sequence can be used to precompensate the phase that will be acquired by spins at different spatial locations due to inhomogeneous magnetic fields. With this precompensation, the chemical shift information of the spins can be reliably extracted and high resolution NMR spectrum can be obtained. This makes NMR in inhomogeneous fields possible [2].

In next 3 years, we will develop a light weight mobile magnetic resonance imaging and spectroscopy system suitable for battle field operation. Such a imaging system with help diagnose mental trauma, injury and fracture in a battle field scenario. Magnetic resonance sensing will be used for detection of explosives and chemicals and analysis of metabolites in the living tissue. The enabling technology for developing such a system is novel control techniques for manipulation of spin dynamics in inhomogeneous fields in such portable imaging systems. These control techniques exploit in a fundamental way noncommutativity and nonlinearity in the dynamics of spins as a function of external excitations [1, 2].

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