Background and context
A coupled cell system is a dynamical system distributed over the nodes (or ‘cells’) of a network. Each cell has an internal ‘local’ dynamics (either discrete or continuous evolution in time), and the couplings between cells may reflect spatial or relational organisation in the system that the network is trying to describe. Dynamical systems viewed as coupled cell systems have a natural collection of ‘observables’ - the individual cell states. In this way the partition imposed by the network structure leads to the notions of symmetry and synchrony in the dynamics. Examples of coupled cell systems include central pattern generators, genetic regulatory networks, IT networks such as the internet, neural circuits, swarms of fireflies, ecological food webs and social networks.

In many cases the network structure is highly complicated and only recently has data become available that gives detailed information about many biological networks and plausible information about the statistics of such networks. The challenge now is to complement new biological understanding of network structures with a mathematical understanding of their dynamics.

Coupled cell systems fitted naturally into the general theme of the Isaac Newton Institute (INI) programme Pattern Formation in Large Domains (PFD) that was held from August – December 2005. In addition to the workshop reported on here, the programme contained a training course aimed at research students and recent post-docs and three further workshops, on experimental results in physical systems, theoretical aspects, and patterns in fluid dynamics, broadly interpreted. The programme, and associated workshops had a substantial international profile, indicating the wide-ranging interest in the themes and challenges under discussion.

In summary, the main themes of the workshop were:

- **The dynamics of coupled cell networks** (synchronisation, pattern formation, clustering, chaotic behaviour, heteroclinic dynamics)

- **The role of network architecture and topology on network dynamics** (symmetry groups, groupoids, motifs, layers of different cell types)

- **Application to and inspiration from physics and biology** (neural dynamics, neural field theories, continuum vs discrete models, existence and stability of travelling waves.)

Research achievements and outcomes
The workshop was a resounding success. A total of 86 participants attended, from a total of 16 countries. The major geographical division of participants was between those from the UK (35), USA (19) and continental Europe (25); the remainder were from Japan (3), Canada (2), Mexico (1) and Brazil (1).

A brief summary of the timetable for the workshop follows below. Invited speakers are highlighted in **bold.**
Workshop: Theory and Applications of Coupled Cell Networks

P.C. Matthews, S. Coombes, P. Ashwin and J.H.P. Dawes

26 - 30 September 2005, Isaac Newton Institute, Cambridge

Timetable

Monday 26 September

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Speaker(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>08:30-09:45</td>
<td>Registration</td>
<td></td>
</tr>
<tr>
<td>09:45-10:00</td>
<td>Welcome</td>
<td></td>
</tr>
<tr>
<td>10:00-11:00</td>
<td>Golubitsky, M (Houston)</td>
<td>Dynamics in coupled identical cell systems</td>
</tr>
<tr>
<td>11:00-11:30</td>
<td>Coffee</td>
<td></td>
</tr>
<tr>
<td>11:30-12:30</td>
<td>Rabinovich, M (San Diego)</td>
<td>Winnerless competition (WLC) principle in neuroscience</td>
</tr>
<tr>
<td>12:30-13:30</td>
<td>Lunch at Wolfson Court</td>
<td></td>
</tr>
<tr>
<td>14:00-15:00</td>
<td>Popovych, O (Jülich)</td>
<td>Development of novel deep brain stimulation technique: coordinated reset and nonlinear delay feedback</td>
</tr>
<tr>
<td>15:00-15:30</td>
<td>Chawanya, T (Osaka)</td>
<td>Long time correlation due to high-dimensional chaos in globally coupled tent map system</td>
</tr>
<tr>
<td>15:30-16:00</td>
<td>Tea</td>
<td></td>
</tr>
<tr>
<td>16:00-17:00</td>
<td>Coombes, S (Nottingham)</td>
<td>Bumps, breathers and waves in a neural network with threshold accommodation</td>
</tr>
<tr>
<td>17:00-18:30</td>
<td>Poster Session</td>
<td></td>
</tr>
</tbody>
</table>

Tuesday 27 September

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Speaker(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00-10:00</td>
<td>Hofbauer, J (UCL)</td>
<td>On the repressilator</td>
</tr>
<tr>
<td>10:00-11:00</td>
<td>Ashwin, P (Exeter)</td>
<td>Oscillator dynamics and discrete computation</td>
</tr>
<tr>
<td>11:00-11:30</td>
<td>Coffee</td>
<td></td>
</tr>
<tr>
<td>11:30-12:30</td>
<td>Cross, M (Caltech)</td>
<td>Models of coupled nanomechanical oscillators</td>
</tr>
<tr>
<td>12:30-13:30</td>
<td>Lunch at Wolfson Court</td>
<td></td>
</tr>
<tr>
<td>14:00-15:00</td>
<td>Longtin, A (Ottawa)</td>
<td>How spatiotemporal correlations of stimuli drive network dynamics and information transfer</td>
</tr>
<tr>
<td>15:00-15:30</td>
<td>Huerta, R (San Diego)</td>
<td>Dynamical gain control for odor recognition in insect olfaction</td>
</tr>
<tr>
<td>15:30-16:00</td>
<td>Tea</td>
<td></td>
</tr>
<tr>
<td>16:00-16:30</td>
<td>Hansel, D (Paris 5)</td>
<td>Rate models with delays and the dynamics of large networks of spiking neurons</td>
</tr>
<tr>
<td>16:30-17:00</td>
<td>Castro, S (Porto)</td>
<td>Interaction of economic agents and coupled cell systems</td>
</tr>
<tr>
<td>17:00-17:30</td>
<td>Dias, A (Porto)</td>
<td>Linear and ODE-equivalence for coupled cell networks</td>
</tr>
</tbody>
</table>

Wednesday 28 September

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Speaker(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00-10:00</td>
<td>Glendinning, P (Manchester)</td>
<td>Minimal dynamics and one way coupling</td>
</tr>
<tr>
<td>10:00-11:00</td>
<td>Tyson, J (Virginia)</td>
<td>Network dynamics and cell physiology</td>
</tr>
<tr>
<td>11:00-11:30</td>
<td>Coffee</td>
<td></td>
</tr>
<tr>
<td>11:30-12:30</td>
<td>Kaneko, K (Tokyo)</td>
<td>Magic number 7 ± 2 in dynamical systems and Milnor attractors; relevance to cell differentiation and development?</td>
</tr>
<tr>
<td>12:30-13:30</td>
<td>Lunch at Wolfson Court</td>
<td></td>
</tr>
<tr>
<td>14:00-15:00</td>
<td>Mischalkow, K (Georgia Tech)</td>
<td>Computational homology in nonlinear dynamics</td>
</tr>
<tr>
<td>15:00-15:30</td>
<td>Antoneli, FM (Sao Paulo)</td>
<td>Patterns of synchrony in lattice dynamical systems</td>
</tr>
<tr>
<td>15:30-16:00</td>
<td>Tea</td>
<td></td>
</tr>
<tr>
<td>16:00-16:30</td>
<td>Buriko, O (NAS Ukraine)</td>
<td>Bifurcation of heteroclinic cycles in Kuramoto model of globally coupled oscillators</td>
</tr>
<tr>
<td>16:30-17:00</td>
<td>Monk, N (Sheffield)</td>
<td>The effects of time delays in models of cellular pattern formation</td>
</tr>
<tr>
<td>17:00-17:30</td>
<td>Abreu, S (Porto)</td>
<td>Symmetric chaos in a local codimension two bifurcation</td>
</tr>
</tbody>
</table>
Thursday 29 September

10:00-11:00  Lord, G (Heriot-Watt)  A spike-diffuse-spike model of a dendritic cable with active spines
11:00-11:30  Coffee
11:30-12:30  Alon, U (Weizmann Inst)  Design principles of biological systems
12:30-13:30  Lunch at Wolfson Court
14:00-15:00  Rubin, J (Pittsburgh)  A geometric analysis of oscillator recruitment
15:00-15:30  Maistrenko, Y (Jülich)  High-dimensional chaos versus synchronization in the Kuramoto model
15:30-16:00  Tea
16:00-16:30  Kori, H (Fritz Haber Inst)  Entrainment of randomly coupled oscillator networks
16:30-17:00  Viktorov, E (Free U, Brussels)  Synchronisation and clustering in a multimode quantum dot laser
17:00-17:30  Aguiar, M (Porto)  Minimal coupled cell networks

Friday 30 September

10:00-11:00  Pikovsky, A (Potsdam)  Phase compactons in nonlinear oscillator lattices
11:00-11:30  Coffee
11:30-12:30  Field, M (Houston)  Heteroclinic cycles and dynamics in coupled cell systems
12:30-13:30  Lunch at Wolfson Court
14:30-15:30  Josic, K (Houston)  The impact of architecture on the structure of solutions in networks of phase oscillators
15:30-16:00  Tea
16:00-16:30  Ruediger, S (Hahn-Meitner)  Numerical simulation of intracellular Ca²⁺ dynamics
16:30-17:00  Atay, F (MPI Leipzig)  Network topology and delay-induced oscillator death

The full programme, including abstracts, can be reviewed on the INI website at

http://www.newton.cam.ac.uk/.

The INI maintains an archive of presentations, both slides and audio recordings: this provides a permanent record and summary of the workshop activity.

It is clear from the schedule above that the topics covered included aspects of biological science (developmental biology, neuroscience, intracellular dynamics), and physics (lasers and nanoscale mechanics) as well as problems of substantial mathematical interest, as detailed above in the workshop themes. It should be emphasised that the mathematical techniques presented range from traditionally ‘pure’ areas such as group and representation theory, through dynamical systems and statistical mechanics, to ‘applied’ techniques such as multiple scale asymptotics. Thus the level of mathematical interaction provoked by the problems under discussion is substantial. Twelve posters were presented by younger researchers, on a similarly wide range of subjects.

Project Plan Review

Of our original list of 22 proposed invited speakers, seven were not able to attend, for various reasons beyond our control such as illness and adverse weather conditions (including Hurricane Katrina): L. Pecora, B. Ermentrout, J. Rinzel, M. Mackey, I. Stewart, Y. Kuramoto and P. Tass. As a result we were able to invite replacement figures of similar seniority in the field, and also allow more time for informal discussion in the programme for the week: this discussion time was welcomed by many participants.

After submitting our applications to EOARD (and to the Office of Naval Research Global) we were informed of our success in a substantial application for funds to the UK Engineering and Physical Sciences Research Council (EPSRC).

As we had hoped, the Isaac Newton Institute provided an outstanding environment for the workshop. We were delighted with physical location and environment and the scientific interactions with the longer term participants in the PFD programme. Furthermore, the Institute
staff undertook a great deal of the administrative tasks, including organising accommodation, travel advice, publicity and web material. We felt that this level of administrative support was more than justified by the level of fixed costs imposed by the INI.

Research Impact
The workshop succeeded in generating a tangible air of excitement about the challenges posed by coupled cell systems, both in terms of the mathematical questions, and in the potential to understand, predict and control real biological and physical systems of this kind.

On the questionnaires returned at the end of the week, many participants commented that the varied nature of the presentations had given them new ideas and research directions. In response to the question ‘Evaluate the scientific content (Excellent/Good/Adequate/Poor)’, all responses were either ‘Excellent’ (62%) or ‘Good’ (38%). General comments included ‘Uniformly high quality talks and stimulating discussion in between’ and ‘I especially enjoyed the variety of topics of the lectures’.

The international nature of the workshop was a further benefit, and participants commented very favourably on the excellent networking opportunities.

As well as the scientific exchange and discussion we succeeded in attracting a large number of younger researchers working in related scientific fields. A total of 33 post-doctoral and research students attended, of which 13 are based in the UK. Informal feedback suggests they enjoyed the workshop tremendously, finding the scientific ideas relevant to their research projects and exposing them to new methods and techniques. Many were very grateful for the financial support we were able to offer them; we are sure this contributed to their enthusiastic participation. There is no doubt that the workshop has met our objectives in

- generating a wider appreciation of the mathematical interest and importance of new challenges laid down by bioscience, and
- encouraging younger researchers to work on these, and related, problems.

The workshop fitted very well into the five month PFD programme: the longer term visitors interacted with the workshop participants, and by focusing on a slightly different collection of themes, the workshop participants brought fresh perspectives and challenges to discuss. Many of the workshop participants remained at the INI for three or four weeks after the workshop and continued discussions, making full use of the INI and its facilities.

Explanation of expenditure
The funds provided by EOARD were spent in line with our original proposal. Overall the workshop was funded from five sources. One of these was the Leverhulme Trust who provided £350 towards the travel costs of A. Longtin, one of our invited speakers. The other four funding sources were EPSRC, ONRG, EOARD and the PFD programme budget from the Isaac Newton Institute. The table below indicates which items in our original proposal were funded from these sources. The INI funding is itself broken down into funding of several different kinds, for example for Research Students through the INI Junior Member scheme, and this detailed breakdown is not shown. The INI figures below indicate the level of support for workshop participants: the INI programme also supported the longer-term participants who were resident during the workshop and this support is not included in the INI figures.

In summary there were no unforeseen major difficulties in supporting the workshop participants, and we were able to allocate financial support in line with our original proposal aims.
<table>
<thead>
<tr>
<th>Item</th>
<th>EPSRC</th>
<th>EOARD</th>
<th>ONR</th>
<th>INI</th>
<th>Total</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. UK speakers</td>
<td>1613</td>
<td>1613</td>
<td></td>
<td></td>
<td>1675</td>
<td></td>
</tr>
<tr>
<td>2. Europe speakers</td>
<td>138</td>
<td>2238</td>
<td>2376</td>
<td></td>
<td>1380</td>
<td></td>
</tr>
<tr>
<td>3. USA speakers</td>
<td>2074</td>
<td>2501</td>
<td>1557</td>
<td></td>
<td>6133</td>
<td>6100</td>
</tr>
<tr>
<td>4. Canada speakers</td>
<td>410</td>
<td>**760</td>
<td>1220</td>
<td></td>
<td>3500</td>
<td>1220</td>
</tr>
<tr>
<td>5. Japan speakers</td>
<td>1560</td>
<td></td>
<td>1560</td>
<td></td>
<td>1560</td>
<td>1220</td>
</tr>
<tr>
<td>6. UK participants</td>
<td>2879</td>
<td>445</td>
<td>2678</td>
<td></td>
<td>6003</td>
<td>9045</td>
</tr>
<tr>
<td>7. Social events</td>
<td></td>
<td></td>
<td>1500</td>
<td></td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>8. INI lecture room *</td>
<td>3500</td>
<td></td>
<td>3500</td>
<td></td>
<td>3500</td>
<td>3500</td>
</tr>
<tr>
<td>9. INI staff costs *</td>
<td>3500</td>
<td></td>
<td>3500</td>
<td></td>
<td>3500</td>
<td>3500</td>
</tr>
<tr>
<td>10. Consumables *</td>
<td>500</td>
<td></td>
<td>500</td>
<td></td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>14561</td>
<td>2501</td>
<td>3615</td>
<td>5417</td>
<td>25944</td>
<td>29740</td>
</tr>
</tbody>
</table>

Grants obtained

Table 1: Actual workshop costs in GBP. Items 1–6 are for travel and subsistence. * indicates INI standard fixed costs for workshops. ** indicates includes £350 travel from Leverhulme Trust for A. Longtin. Last column gives budget estimates from original EPSRC proposal for comparison.

**Future dissemination and impact**

The INI has a formal framework for assessing the impact of research programmes and their component workshops, taking place eighteen months to two years after the end of each programme. As part of this review we expect to form a more coherent view of the effectiveness of particular workshops.

The workshop more than fulfilled our expectations in terms of the quality and enthusiasm of the participants and the breadth of research discussed. One does not expect immediate tangible benefits, but our belief is that the research of the majority of participants will have been affected by the workshop either through particular presentations or through informal discussions. Certainly the prevailing mood of optimism and excitement matches our individual views of its success.
Appendix

This appendix contains the titles and abstracts of the workshop speakers and poster presenters. The workshop speakers are divided into two sections: invited speakers, and shorter contributed presentations.

Invited speakers

Dynamics in coupled identical cell systems
Martin Golubitsky (Houston)
This lecture will focus on some of the interesting dynamics and bifurcations that can occur generically in two and three identical cell systems. Topics will include the types of high codimension Hopf bifurcations that are forced by network architecture in three-cell systems and some curious types of bursting that can occur in two-cell systems.

Winnerless competition principle in neuroscience
M.I. Rabinovich (UCSD)
It is well known that the traditional way for neural computations is computation with attractors. This means transformation of a given input - an initial state inside of the basin of attraction of one attractor, to a fixed desired output. Five years ago Rabinovich et al. (2000) have introduced a new concept based on the Winnerless Competition (WLC) principle. According to this principle the incoming stimulus is transformed into a complex temporal output based on the intrinsic sequential dynamics of a network with WLC. They have shown that the neural circuits with non-symmetric inhibition demonstrate sequentially switching WLC dynamics that is stable and uniquely depends on the incoming information. The WLC principle is able to solve the contradiction between stability and flexibility in neural circuits. An appropriate set of mathematical models for competition phenomena is based on the generalized Lotka-Volterra models. These models describe the cooperative dynamics of an arbitrary number of competitive agents that can be dynamical elements themselves. Generally, the evolution of the system follows complicated or even chaotic trajectory in the phase space. In particular, the authors have analyzed the chaotic (strange attractor), periodic (limit cycle), and transient (Stable Heteroclinic Sequence (SHS)) cooperative sequential dynamics of simple and complex (random) networks. The transient sequential representation of the olfactory information (SHS) has been observed in the locust and bee’s antennal lobe. The interaction of excitatory and inhibitory neural ensembles in a model of brain microcircuits also leads to the SHS. The authors have also shown that the irregular hunting swimming of the marine mollusk Clione is the result of the chaotic dynamics of the gravimetric sensory network, consisting of interconnected inhibitory neurons (chaotic WLC).

Bumps, breathers and waves in a neural network with threshold accommodation
Stephen Coombes (University of Nottingham), Markus R Owen (University of Nottingham)
I will discuss the dynamics of synaptically coupled model neurons that undergo a form of accommodation in the presence of sustained activity. The basic model is an integral equation for synaptic activity that depends upon the non-local network connectivity, synaptic response, and firing rate of a single neuron. A phenomenological model of accommodation is examined whereby the firing rate is taken to be a simple state-dependent threshold function. As in the case without threshold accommodation classical Mexican-Hat connectivity is shown to allow for the existence of spatially localised states (bumps). Importantly an analysis of bump stability (in both one and two spatial dimensions) using recent Evans function techniques shows that bumps may undergo instabilities leading to the emergence of both breathers and travelling waves. Numerical simulations show that bifurcations in this model have the same generic properties as those seen in many other dissipative systems that support localised structures, and in particular those of coupled cubic complex Ginzburg-Landau equations, and three component reaction diffusion equations. Interestingly, travelling pulses in this model truly have a discrete character in the sense that they scatter as auto-solitons.

Related Links


**On the repressilator**

**Josef Hofbauer (UCL),**

St. Mueller and P. Schuster (Theoretical Biochemistry Group, U. Vienna)

The repressilator is a simple gene regulatory network consisting of a loop of three proteins that repress the transcription of the following gene. Such a synthetic network was constructed by Elowitz and Leibler (2000) in *E. coli* and shown to oscillate. We propose a new dynamic model for this gene network and analyse its dynamic features.

**Oscillator dynamics and discrete computation**

**Peter Ashwin (Exeter)**

This talk will discuss aspects of the dynamics and structure of robust heteroclinic networks in globally coupled phase oscillator systems. As recognised by a number of authors, one can readily find dynamics of ‘slow switching’ or ‘slow oscillations’ between cluster states in cases where these heteroclinic networks are attracting. The detailed dynamics becomes rapidly very complicated on increasing the number of oscillators in the system, but it can be modelled by transitions between a number of symmetrically related states. We suggest how this allows one to construct and model coupled dynamical systems that perform reliable discrete computations given very small perturbations to the the system, and we highlight a number of theoretical and practical questions about such systems. (Work with Jon Borresen and Marc Timme).

**Models of coupled nanomechanical oscillators**

**Michael Cross (Caltech)**

In this talk I will discuss the behavior of models of coupled oscillators motivated by experimental work on nanomechanical systems. I will describe the collective behavior of parametrically driven arrays, which connects with issues of pattern formation in parametrically driven fluids, and the synchronization of self sustained oscillators with a range of individual frequencies, which leads to a new model for synchronization involving reactive coupling and nonlinear frequency pulling.
How spatiotemporal correlations of stimuli drive network dynamics and information transfer

Andre Longtin (U. Ottawa), Benjamin Lindner (U. Ottawa), Brent Doiron (U. Ottawa), Jan Benda (U. Ottawa) and L. Maler (U. Ottawa)

We present two experimental studies with accompanying theory in which the spatial correlations of naturalistic stimuli induce temporal correlations in neural network activity. Both situations arise in the electric sense, which can be seen as a combination of vision and audition. Spatiotemporal variations in the electric field at the skin of weakly electric fish are a signature of specific stimuli, such as navigational cues, food, or inter-fish communication. The geometries of random stimuli which have long range spatial correlations are shown to induce regular oscillations in the network, which are carved out of the noisy activity of the cells. This relies on delayed feedback from higher brain centers. These results, manifested e.g. in single cell spike train spectra and cross-correlations between cells, can be explained using a slightly non-linear version of linear response theory applied to the network with global feedback. Extensions to localized feedback are also discussed.

In a second study, we study the combined effect of positive and negative delayed feedback on the information transfer about stimuli into spike trains. We further relate this transfer to network dynamics. We finally present experimental results and theoretical support on how feedback modifies the information transfer via a noise shaping effect.

Minimal dynamics and one way coupling

Paul Glendinning (Manchester)

I will review recent results on blowout bifurcations for globally coupled maps, and extend some results to the case of random maps at sites. I will also describe some results on cluster states and their stability.

Related Links

- http://www.ma.umist.ac.uk/pag/ - my home page

Network dynamics and cell physiology

John J. Tyson (Virginia Tech), Kathy Chen (Virginia Tech) and Bela Novak (Budapest Univ of Technology & Economics)

Complex networks of interacting proteins control the physiological properties of a cell (metabolism, reproduction, motility, signaling, etc.). Intuitive reasoning about these networks is often sufficient to guide the next experiment, and a cartoon drawing of a network can be useful in codifying the results of hundreds of observations. But what tools are available for understanding the rich dynamical repertoire of such control systems? Why does a control system behave the way it does? What other behaviors are possible? How do these behaviors depend on the genetic and biochemical parameters of the system (gene dosage, enzymatic rate constants, equilibrium binding constants, etc)? Using basic principles of biochemical kinetics, we convert network diagrams into sets of ordinary differential equations and then explore their solutions by analytical and computational methods. We illustrate this approach with a mathematical model of cell cycle transitions in eukaryotes, based on a molecular network controlling the activity of cyclin-dependent kinase (Cdk). In this model, arrest points in the cell cycle correspond to stable steady states of the control system. As biochemical parameters of the control system change, these arrest points are imposed or lifted by transitions called 'bifurcations.' During normal growth and division, cell size is the critical parameter that
drives progression from G1 to S/G2 to M phase and back to G1. Simple diagrams, which correlate Cdk activity with cell growth, give a new way of thinking about cell cycle control, particularly the role of checkpoint pathways in arresting the cycle. The method is generally applicable to any complex gene-protein network that regulates some behavior of a living cell.

Related Links
- [http://jigcell.biol.vt.edu/generic_model/GenericUC.html](http://jigcell.biol.vt.edu/generic_model/GenericUC.html) - Generic model of cell cycle in eukaryotes
- [http://cellcycle.mkt.bme.hu/](http://cellcycle.mkt.bme.hu/) - Molecular network dynamics

Magic number 7+-2 in dynamical systems and Milnor attractors; relevance to cell differentiation and development?

Kunihiko Kaneko (U. Tokyo)

In some examples in coupled dynamical systems, stability of attractors is lost with the increase of the number of coupled elements, where the threshold number for the instability is found to often lie at around 5-10. Numerical results on globally coupled maps, feed-forward neural networks, and genetic networks are described, where prevalence of Milnor attractors as a result of instability is noted. The origin of this critical number 5-10 for instability is discussed as the dominance of 'factorial' over 'exponential'. Following this general argument, we report some results on hierarchical cell differentiation, based on coupled cell dynamics with internal catalytic reactions. We show that Milnor attractors provide a state of a stem cell that can both proliferate and differentiate, i.e., have both stability for reproduction and instability to switch to other cell states.

Related Links
- [http://chaos.c.u-tokyo.ac.jp](http://chaos.c.u-tokyo.ac.jp)

Computational homology in nonlinear dynamics

Konstantin Mischaikow (Georgia Tech)

Much of the fascination and challenge of studying infinite dimensional nonlinear dynamical systems arises from the complicated spatial and/or temporal behavior that they exhibit. On the mathematical side, this complicated behavior can occur on all scales both in phase space and parameter space. Somewhat paradoxically, from a scientific perspective this points to the need for a coherent set of mathematical techniques that is capable of extracting coarse but robust information about the structure of these systems. Furthermore, since most of our understanding of specific systems comes from experimental observation or numerical simulations, it is important that these techniques can be applied in a computationally efficient manner. Finally, it is important that the experimentally observed patterns be quantified in an efficient manner both for purposes of parameter identification and for modeling purposes. In this talk it will be argued that computational homology has an important role to play in these endeavors.

Related Links
- [http://www.math.gatech.edu/~mischaik](http://www.math.gatech.edu/~mischaik)
Design principles of biological systems
Uri Alon (Weizmann Institute)
Biological networks seem to show an inherent simplicity: They contain only very few of the possible interaction patterns. These patterns are called network motifs. We will present theoretical and experimental studies on the dynamical functions of network motifs in the transcription network of the bacterium E. coli. We will also discuss how motifs and modularity in the network may have evolved.

- www.weizmann.ac.il/MCB/UriAlon

A geometric analysis of oscillator recruitment
Jonathan Rubin (University of Pittsburgh), Amitabha Bose (New Jersey Institute of Technology)
In a coupled network of excitable elements that are initially at rest, the application of a transient, spatially localized stimulus can lead to the propagation of activity throughout the network, a small burst of activity that dies out, or an intermediate state of sustained, localized activity. For propagation to result, silent elements must be recruited to become active. This recruitment can fail through the build-up of inhibitory coupling, but even without inhibition, it can fail through the desynchronization of active cells. We give a geometric analysis of the degree of desynchronization for which recruitment fails in networks composed of certain types of elements, corresponding to some particular neuronal models.

Related Links
- www.math.pitt.edu/rubin - homepage

Phase compactons in nonlinear oscillator lattices
Arkady Pikovsky (Potsdam University), Philip Rosenau (Tel Aviv University)
We study the phase dynamics of a chain of autonomous oscillators with a dispersive coupling. In the quasicontinuum limit the basic discrete model reduces to a Korteveg–de Vries-like equation, but with a nonlinear dispersion. The system supports compactons: solitary waves with a compact support and kovatons which are compact formations of glued together kink-antikink pairs that may assume an arbitrary width. These robust objects seem to collide elastically and, together with wave trains, are the building blocks of the dynamics for typical initial conditions. Numerical studies of the complex Ginzburg-Landau and Van der Pol lattices show that the presence of a nondispersive coupling does not affect kovatons, but causes a damping and deceleration or growth and acceleration of compactons.

Heteroclinic cycles and dynamics in coupled cell systems
Michael Field (University of Houston)
We describe recent work and results on heteroclinic phenomena in coupled cell systems. We also indicate some potential applications. Some of this work is joint with Peter Ashwin (Exeter).

The impact of architecture on the structure of solutions in networks of phase oscillators

Kresimir Josic (University of Houston), Martin Golubitsky (University of Houston), Eric Shea-Brown (New York University)

Network architecture can have a large impact on the dynamics of coupled phase oscillators. It will be shown that architecture can force relations between average frequencies of the different oscillators in such networks. The main tool in this analysis is coupled cell theory which provides precise relations between network architecture and the flow-invariance of certain polydiagonal subspaces. Architecture also imposes restrictions on the spatiotemporal symmetries of periodic solutions that a network of phase oscillators can support. These obstructions are related, but distinct from those observed in equivariant differential equations.

Contributed presentations

In addition there were a further 16 contributions on more specialised topics related to the workshop themes:

Development of novel deep brain stimulation techniques
O. Popovych Jülich

Long time correlation due to high-dimensional chaos in globally coupled tent map system
Tsuyoshi Chawanya (Osaka University)

This talk will focus on the statistical property of 2-band intermittency observed in globally coupled tent map system and reveal its peculiarity, and the observed features will be related to the way of the loss of ergodicity in a limit of large number of degrees of freedom.

Dynamical gain control for odor recognition in insect olfaction
Ramon Huerta (UCSD)

Insect olfaction is known to use sparse activity in the odor recognition areas of the brain. Prior to this module for recognition purposes there is a first relay station that regulates, modulates and controls the flow of information into the recognition module. This first relay station is named the Antennal Lobe (AL). Here, we discuss a hypothesis about one of the possible functions of the AL, that is, to regulate the amount of activity delivered into the recognition module. We hypothesized that the regulation or gain control of the activity is effectively determined by specific inhibitory connectivity to both the excitatory and inhibitory population. We show that without this gain control condition the system is unable to carry out odor recognition.

Rate models with delays and the dynamics of large networks of spiking neurons
D. Hansel (U. Rene Descartes)

We investigate the dynamics of a one-dimensional network of spiking neurons with spatially modulated excitatory and inhibitory interactions through extensive numerical simulations. We
find that the network displays a very rich phase diagram as a function of the interaction parameters. Dynamical states include homogeneous oscillations, oscillatory bumps, traveling waves, lurching waves, standing waves arising via a period doubling bifurcation, aperiodic regimes and regimes of multistability. We show that a similar diversity is found in the framework of a reduced rate model in which the interactions are delayed. A great deal of this study can be performed analytically. Our work suggests that the delay in the response of the neurons to their synaptic inputs due to the non-linearity underlying spike generation have important consequences for the collective dynamics of large neuronal systems.

**Interaction of economic agents and coupled cell systems**

**Sofia Castro (Faculdade de Economia and Centro de Matematica, Universidade do Porto)**

Application of coupled cell systems have been made to a wide range of problems in the physical and biological sciences. Recent developments in the study of coupled cell systems include the description of possible synchronous states in networks with many cells.

One of the open problems in economics, which has been solved in a wide variety of specific contexts, concerns the quest for a price which clears (i.e., supply equals demand) the markets. One of the difficulties is closely related to the vast number of agents and goods in any realistic formulation of the problem.

We propose the use of coupled cell systems as a useful tool to, at least partly, overcome the difficulties in dealing with many agents or goods. This will raise questions concerning non-synchronous solutions, that is, synchrony-breaking bifurcation for coupled cell systems.

We shall illustrate our point with a simple example using well-known results in coupled cell systems.

**Linear and ODE-equivalence for coupled cell networks**

**A. Dias (Universidade do Porto)**

Coupled cell systems are systems of ODEs, defined by ‘admissible’ vector fields, associated with a network whose nodes represent variables and whose edges specify couplings between nodes. It is known that non-isomorphic networks can correspond to the same space of admissible vector fields. Such networks are said to be ‘ODE-equivalent’. We prove that two networks are ODE-equivalent if and only if they determine the same space of linear vector fields; moreover, the variable associated with each node may be assumed 1-dimensional for that purpose.

This is a joint work with Ian Stewart (Warwick, UK)

**Patterns of synchrony in lattice dynamical systems**

**Fernando Antoneli (Sao Paulo), Ana Paula S. Dias (Porto), Martin Golubitsky (Houston), Yunjiao Wang (Houston)**

From the point of view of coupled systems developed by Stewart, Golubitsky, and Pivato, lattice differential equations consist of choosing a phase space for each point in a lattice and a system of differential equations on each of these spaces such that the whole system is translation invariant. The architecture of a lattice differential equation is the specification of which sites are coupled to which (nearest neighbor coupling is a standard example). A polydiagonal is a finite-dimensional subspace of phase space obtained by setting coordinates in different phase spaces equal. There is a coloring of the network associated to each polydiagonal that is obtained by coloring any two cells that have equal coordinates with the same color. A pattern of synchrony is a coloring associated to a polydiagonal that is flow-invariant for every lattice differential equation with a given architecture. We prove that
every pattern of synchrony for a fixed architecture in planar lattice differential equations is spatially doubly periodic assuming that the couplings are sufficiently extensive. For example, nearest and next nearest neighbor couplings are needed for square and hexagonal couplings, and a third level of coupling is needed for the corresponding result to hold in rhombic and primitive cubic lattices. On planar lattices this result is known to fail if the network architecture consists only of nearest neighbor coupling. The techniques we develop to prove spatial periodicity and finiteness can be applied to other lattices.

**Bifurcation of heteroclinic cycles in Kuramoto model of globally coupled oscillators**

Oleksande Burilko *(Institute of mathematics of NAS of Ukraine)*

Nowadays there is very big interest in cooperative phenomena in ensembles of globally coupled limit cycle oscillators because of many possible applications in physics, chemistry, biology and medicine. Many works are dedicated to model of globally coupled oscillators proposed by Y. Kuramoto in 1984. One of the most important and interesting thing in studying such a model is to understand mechanisms that cause synchronization of oscillators. We consider finite dimensional Kuramoto model with coupling function \( g(x) = -\sin(x-a) + r \sin 2x \), where \( a \) and \( r \) are parameters (Kuramoto-Hansel-Mato-Meunier model). We discuss mechanisms of appearance of different types of heteroclinic cycles when parameters change and influence of such bifurcations on phase synchronization of the system in 3 and 4 dimensional cases. We obtained some new types of heteroclinic bifurcation.

**The effects of time delays in models of cellular pattern formation**

Nick Monk *(University of Sheffield)*, Siren Veilingstad, Erik Plahte *(Norwegian University of Life Sciences)*

Key processes in genetic regulatory networks (notably transcription and translation) are associated with irreducible time delays. Models of delayed networks operating in single cells show that delays can induce oscillatory dynamics that are not present in corresponding non-delayed models. The associated oscillatory gene expression has been observed in a number of biological settings, confirming the potential importance of delays in determining the dynamics of gene networks. These studies show that the impact of delays is particularly important when the delay occurs within a negative feedback loop, and when the magnitude of the delay is comparable to other key time scales in the network (particularly the half-lives of the regulated components). The networks underlying pattern formation during multicellular development are characterised by the presence of feedback loops and by time scales that are of the same order of magnitude as the delays expected from the processes of transcription and translation. In this talk, I will discuss the effects of time delays in some simple models of cellular pattern formation, based on intercellular signalling through proteins of the Notch and Delta families. I will show how inclusion of delays can have very striking effects on the dynamics of model solutions, resulting in a competition between oscillatory modes and spatial pattern formation. I will also discuss the potential implications of these results for the mechanisms that underlie some of the most basic patterning events in development.

**Symmetric chaos in a local codimension two bifurcation**

Stella Abreu *(Univ. Portucalense)*, Philip Aston, Ian Melbourne *(Univ. Surrey)*

We study a codimension two steady-state/steady-state mode interaction with \( D_4 \) symmetry, where the center manifold is three-dimensional. Primary branches of equilibria undergo secondary Hopf bifurcations to periodic solution which undergo further bifurcations leading to chaotic dynamics. This is not an exponentially small effect, and the chaos obtained in simulations using DsTool is large-scale, in contrast to the ”weak” chaos associated with
Shilnikov theory. Moreover, there is an abundance of symmetric chaotic attractors and symmetry-increasing bifurcations. The local bifurcation studied by us is the simplest (in terms of dimension of the center manifold and codimension of the bifurcation) in which such phenomena have been identified. Numerical investigations demonstrate that the symmetric chaos is part of the local codimension two bifurcation. The two dimensional parameter space is mapped out in detail for a specific choice of Taylor coefficients for the center manifold vector field. We use AUTO to compute the transitions involving periodic solutions, Lyapunov exponents to determine the chaotic region, and symmetry detectives to determine the symmetries of the various attractors.

High-dimensional chaos versus synchronization in the Kuramoto model
Yuri Maistrenko (Forschungszentrum Jülich)
A complex high-dimensional chaotic behavior is found in the finite-dimensional Kuramoto model of globally coupled phase oscillators. This type of chaos extends over small and intermediate coupling strength up to the synchronization transition, and is characterized by half of the spectrum of Lyapunov exponents being positive and the Lyapunov dimension equaling almost the total system dimension. We analyze the case of uniform distribution of the natural frequencies and find that the intensity of the phase chaos, as given by the maximal Lyapunov exponent, decays quadratically with coupling strength and inverse proportionally to ensemble size. Intriguingly, strongest phase chaos occurs for intermediate-size ensembles. We argue that phase chaos is caused by intrinsic nonlinear phase interactions and is a common property of networks of oscillators of very different nature, such as phase oscillators with different coupling matrix and different frequency distributions, e.g. Gaussian. The phenomenon is also inherent in networks of limit-cycle oscillators and chaotic oscillators, e.g., Roessler systems. In the case of coupled chaotic oscillators, phase chaos manifests itself in the appearance of additional chaotic "dimensions", where additional positive LEs (with respect to those of individual oscillators) emerge.

Entrainment of randomly coupled oscillator networks
H. Kori Fritz Haber Institute

Synchronisation and clustering in a multimode quantum dot laser
E Viktorov (Universite Libre de Bruxelles), P Mandel (Universite Libre de Bruxelles), Y Tanguy, J Houlihan, G Huyet (Tyndall National Institute, Ireland)
The global coupling predicts the appearance of one or more clusters where each cluster consists of a subset of synchronized oscillators. The aim of our work is to show the first experimental evidence to our knowledge, of clustering effects associated with synchronization. This was achieved by analyzing the intensity oscillations of the longitudinal modes of quantum dot semiconductor lasers. We find that the modal intensities can oscillate chaotically with different average frequencies and observe that groups of modes may have the same frequency indicating the appearance of clustering. We also observe a highly organized antiphase dynamics leading to a constant total output power and the propagation of perturbations across the optical spectrum.

Minimal coupled cell networks
M. Aguiar (Faculdade de Economia do Porto), A.P.S. Dias (Departamento de Matematica Pura da Faculdade de Ciencias do Porto)
As pointed by [1], in the class of coupled cell networks that permits self-coupling and multiarrows, it is possible for two different coupled cell networks to generate the same space of admissible vector fields, i.e., to be ODE-equivalent.
In [2] it is shown that two coupled cell networks are ODE-equivalent if and only if they are linearly equivalent. Basically, the ODE-equivalence reduces to ‘linear equivalence’, where two networks (with suitably identified phase spaces) are linearly equivalent if they determine the same space of linear admissible vector fields.

In this work we address the question: Given an ODE-equivalence class of coupled cell networks, is there a subclass of networks such that the number of edges is minimal among all the networks of that ODE-class?


Related Links
- http://www.fep.up.pt/docentes/maguiar/ - Manuela Aguiar Homepage

**Numerical simulation of intracellular Ca2+ dynamics**

**Sten Ruediger (Hahn-Meitner Institut), Martin Falcke (Hahn-Meitner Institut)**

Calcium is an important second messenger in cell communication. The dynamics of intracellular calcium is determined by the liberation and uptake by cellular stores and reactions with buffers. We develop models and numerical tools to study the liberation of calcium from the endoplasmic reticulum. This process is characterized by the existence of multiple length scales. Local events, Ca2+ puffs at length scales of nanometers, originate at clusters of the inositol-1,4,5 trisphosphate (IP3) receptor channels. The interaction of these local stochastic events leads to Ca2+ waves, typically of micrometer size, which travel through the whole cell. We use a finite-element software package to implement the reaction-diffusion processes for concentration fields of Ca2+ and buffer proteins. In our description the dynamics of IP3-controlled channels remains discrete and stochastic and is implemented in the numerical simulations by a stochastic source term in the reaction diffusion equation. The strongly localized temporal behavior due to the on-off behavior of channels as well as their spatial localization is treated by an adaptive numerical method. We characterize phenomena such as nucleation and pinning of waves.

**Network topology and delay-induced oscillator death**

**Fatihcan M. Atay (Max Planck Institute, Leipzig)**

The fact that transmission delays can suppress oscillatory behavior of coupled systems has recently attracted attention in diverse scientific fields. The implications for biological systems are easily appreciated if one considers the possibility of cessation of oscillations in a group of cardiac cells or neurons as a result of delayed coupling. The relation of this phenomenon to coupling topology was so far unknown. In this talk, I will consider systems near Hopf bifurcation as oscillators, and give a complete characterization of delay-induced stability in terms of the spectrum of the graph Laplacian, for both directed or undirected networks.
Silencing a neuronal pacemaker with noise

John Clay (NIH), David Paydarfar (U Mass), Danny Forger (Michigan State)

Excitable cells usually exhibit a single type of behavior in response to a given set of experimental conditions, such as a repetitive train of action potentials in response to a sustained depolarizing current pulse or a stable rest state in the absence of a stimulus. In some instances both types of behavior occur for the same set of conditions - a stable rest state and repetitive firing in the absence of a stimulus, i.e., pacemaker activity. Either behavior is stable. Hence the preparation is referred to as being bistable. A brief duration current pulse can switch the preparation from quiescence to repetitive firing. Moreover, a similar pulse, appropriately timed, can annihilate repetitive firing thereby resulting in quiescence. Switching between these modes may be an important control mechanism governing a variety of normal and pathological states of excitable cells. Transitions between the two states may also be produced by noisy perturbations in membrane potential that are ubiquitous in the nervous system. Multisynaptic input and channel noise induce stochastic fluctuations over a broad time scale. Low level noise input to neurons can have large effects on output patterns which may be relevant to transduction and encoding of information in neocortex, thalamus, motoneurons, and sensory receptors. The manner in which neurons process information that is inherently noisy is a topic of considerable interest with a large body of experimental and modeling work on spike timing that assumes each neuron is intrinsically quiescent and monostable. Little is known about how bistable neurons respond to noisy fluctuations in membrane potential. We have examined this issue experimentally using squid giant axons made bistable with a moderately alkaline intracellular milieu. We have found that low level noise can annihilate repetitive firing with the cell remaining quiescent thereafter except for small subthreshold fluctuations. Moderate noise intensities can induce bursts of action potentials that are interrupted.

Understanding control of connexin levels in the hepatic cell plate.

James Hetherington (UCL), Anne Warner, Rob Seymour, Linzhong Li, Marta Varela-Rey (UCL)

Hepatocytes in the liver cell plate communicate through gap junctions. This permits sharing of signal molecules and metabolic resources. The degree of intercellular communication is regulated, with increased production of gap junction proteins occurring in response, for example, to glucagon stimulus. We are using simple mathematical models to further understand the role of gap junctions in the liver, and of control of gap junction levels. Gap junctions are important in the presence of cellular variation, including both systematic variation due to metabolic zonation and random variation due to cellular individuality. We seek to understand the relative costs and benefits which accrue from changes in the level of intercellular communication- changes to the predictability, robustness, overall performance, and control behaviour of the organ. Our motivation, however, is not solely to understand function. We hope that in turn, this understanding will allow us to identify which are the critical phenomena that our models of hepatocytes cooperating to help regulate blood glucose levels must reproduce in order to be useful. Preliminary results indicate that communication is particularly important in reducing variation in organ behaviour in the presence of cellular variation. This increases organ robustness and system reliability. Research is ongoing.

Related Links

- [http://www.ucl.ac.uk/CoMPLEX/](http://www.ucl.ac.uk/CoMPLEX/) - UCL CoMPLEX
Analysis of Coupled Logistic Map Network Adele Peel (Imperial College London), Henrik Jensen (Imperial College London)

We undertake an analysis of a dynamical network model that consists of coupled logistic maps, in order to clarify the behaviour across the boundary between the distinct regimes that have been found to exist in the parameter space. The model consists of nodes whose state evolves in time, dependent on the connections it has and its previous state. The connection weights also evolve in time dependant on the similarity of the two nodes’ states at either end of the link. We looked at the power spectra of the mean node state time series as well as the variance time series. We find some interesting behaviour occurs across the boundary between the coherent and ordered regime.

Wavelet and Graph Theoretic Analysis of Human MEG Data
Danielle Smith Perry (University of Cambridge), Thomas Duke, Ed Bullmore (University of Cambridge), Andreas Meyer–Lindenberg (National Institute of Mental Health)

The current project encompasses an analysis of data acquired using magnetoencephalography (MEG), functional magnetic resonance imaging (fMRI), and structural magnetic resonance imaging (MRI), in a model human system to understand the structure of, dynamics of, and transitions between large-scale neuronal assemblies in the human brain. We report a new methodology that can be applied to time series of such dynamical systems to provide information on the spatial and temporal frequency-dependent dynamics of the system using a wavelet decomposition as well as connectivity relations between time series using graph theory. We show the application of this methodology on human MEG data in two experimental conditions: resting eyes open and right index finger tapping synchronized with a visual stimulus at 1.2 Hz.

Two Central Pattern Generators for Bipedal Locomotion
Carla M.A. Pinto (CMUP, Portugal), Martin Golubitsky (UI, USA)

We use symmetry to study two central pattern generator (CPG) models for biped locomotion. The first one is a coupled four-cell network, proposed by Golubitsky, Stewart, Buono, and Collins, that models rhythms associated to legs. A classification based on symmetry shows that this network can produce 11 different periodic solutions, two of these with rhythms corresponding to the standard bipedal gaits of walk and run. Other periodic solution types can be identified with the (two-legged) hop, the gallop, and the skip. Moreover, the four-cell model can produce two types of hop, and two types of gallop. The second locomotor CPG network is an attempt to model interlimb coordination in bipeds (arms+legs). It is obtained by breaking the symmetry between fore and hind legs in an eight-cell CPG network for quadruped gaits, also proposed by Golubitsky et al. We base this derivation on literature concerning human evolution theory and human interlimb coordination during locomotion. We compare patterns of oscillation of gaits of the eight-cell model with results on bipedal interlimb coordination in the literature, showing that the eight-cell model may be thought as an embrionary state of the actual CPG network modeling human interlimb coordination.

We show numerical simulations of periodic solutions corresponding to the biped gaits in the two CPG models.

Related Links
• http://www.fc.up.pt/pessoas/cpinto/publications.html - preprint
Stability and Diversity in Collective Adaptation
Yuzuru Sato (Santa Fe Inst), Eizo Akiyama (University of Tsukuba), James P. Crutchfield (University of California, Davis)
We derive a class of macroscopic differential equations that describe collective adaptation, starting from a discrete-time stochastic microscopic model. The behavior of each agent is a dynamic balance between adaptation that locally achieves the best action and memory loss that leads to randomized behavior. We show that, although individual agents interact with their environment and other agents in a purely self-interested way, macroscopic behavior can be interpreted as game dynamics. Application to several explicit network interactions shows that the adaptation dynamics exhibits a diversity of collective behaviors. We also analyze the adaptation dynamics from an information-theoretic viewpoint and discuss self-organization induced by dynamics of uncertainties, giving a novel view for problems of collective adaptation and signaling behavior.
Related Links

On the phase reduction and response dynamics of neural oscillator populations
Eric Shea-Brown (Courant Inst., New York Univ.), Jeff Moehlis (UC Santa Barbara), Philip Holmes (Princeton)
We undertake a probabilistic analysis of the response of repetitively firing neural populations to simple pulselike stimuli. Recalling and extending results from the literature, we compute phase response curves (PRCs) valid near bifurcations to periodic firing for Hindmarsh-Rose, Hodgkin-Huxley, FitzHugh-Nagumo, and Morris-Lecar models. Phase density equations are then used to analyze the role of the bifurcation, and the resulting PRC, in responses to stimuli. In particular, we explore the interplay among stimulus duration, baseline firing frequency, and population level response patterns.
Related Links
- www.math.nyu.edu/~ebrown - Author’s site; paper available here

Bursting in Coupled Systems
LieJune Shiau (University of Houston), Marty Golubitsky, Kresimir Josic (University of Houston)
Periodic bursting in fast-slow systems can be viewed as closed paths through the unfolding parameters of degenerate singularities. Using this approach we show that bursting in coupled systems can have interesting behavior. We focus on two identical cell systems and use the $Z_2$ symmetry present in such systems to illustrate interesting bursting phenomena. In particular, we show that Hopf/Hopf mode interactions can lead to bursting between in phase and out of phase periodic solutions and symmetry-breaking Takens-Bogdanov singularities can lead to bursting that randomly chooses between two (symmetrically related) limit cycles.

Multiple Time-Scale Structure in Assembly of Oscillators with Plastic Frequencies
Masashi Tachikawa (ERATO Complex Systems Biology Project JST), Koichi Fujimoto (Univ. of Tokyo, ERATO Complex Systems Biology Project JST)
Multiple time-scale structures are ubiquitous in complex systems. Chemical, biological or geophysical processes contain the wide variety of time scales. Though these structures are often described with distributions of external time-constant parameters, the origin of distributions are also interesting problems to be discussed.

In this poster we demonstrate that a multiple time-scale structure emerges through a self-organizing process in an assembly of identical oscillators. In the system the interaction is designed to modify the frequencies of oscillators, which is derived from diffusive coupling among limit cycle oscillators with some assumptions. It is found that initial conditions close to a one-cluster state self-organize into multiple clusters with different frequencies, i.e. hierarchization of the time scales emerges through the multi clustering process in the phases. Analyses for the clusters solution, the stability of the solution and the mechanism determining the time scales are also reported.

Related Links


Effects of time delay on pattern formation: competition between homogenisation and patterning.

Siren R. Veflingstad (Norwegian University of Life Sciences), Erik Plahte (Norwegian University of Life Sciences), Nicholas A. M. Monk (Sheffield)

An important issue in developmental biology is the formation of fine-grained spatial patterns. These patterns are likely to arise through contact-dependent signalling in a lattice of cells. One possible mechanism is juxtacrine signalling in which signalling molecules anchored in the cell membrane bind to and activate receptors on the surface of cells with which they are in direct physical contact. There are several mathematical models for juxtacrine signalling, all considering the changes in the state variables to be instantaneous. However, the signalling mechanism is believed to be regulated at the level of transcription, a process in which there is an irreducible time delay between the initial binding of transcription factors and appearance of mature mRNA. To model this accurately within a framework of continuous time, it is imperative to introduce delays in the system of equations. Here we study a general model for juxtacrine signalling incorporating a time delay. We consider a discrete lattice of cells in which there is an overall positive feedback loop between neighbouring cells. Our main objective of the analysis is how time delays affect the pattern-forming potential of the model.

The analysis reveals a transient competition between two distinct dynamic modes: spatially homogenous oscillations and spatial patterning. A fine-grained pattern will eventually appear over the whole lattice, but the time taken grows rapidly with the delay. The only way to achieve direct patterning, although relatively slow, is by introducing an initial bias in accordance with the final pattern.

The results illustrate both the importance of including the known delays in a model and of studying transients, as these may not be favourable to the system. In addition, the results may suggest that there are other mechanisms than transcription regulating juxtacrine signalling. Post-translational protein networks do not suffer from the delays inherent in transcriptional regulation, and may thus provide more rapid and stable pattern-generators.

Spatiotemporal pattern formation in a cellular neural network chip

Mustak E. Yalcin (Istanbul Technical University), Johan Suykens, Joos Vandewalle (K.U.Leuven, Belgium)

In this poster, we report on pattern formation occurring on the ACE16k Cellular Neural Network (CNN) chip. The CNN chip can be programmed with a cloning template in order to
generate spiral waves and autowaves. The waves diffract from \textit{internal sources} which cannot be relocated on the network. However, by using initial and/or input images, sources (\textit{external sources}) can be located at any place on the network. Furthermore, a competition between autowaves generated by external and internal sources is observed. Propagation of autowaves on the inhomogeneous CNN array, formed by the fixed-state map, is presented.

Related Links

- www2.itu.edu.tr/yalcinmust - Web page of Mustak E. Yalcin
- www2.itu.edu.tr/yalcinmust/bare_conf_JS.pdf - One of our papers which is related to this poster
- www.esat.kuleuven.ac.be/sista/chaoslab/pictures.html - you can see the observed waves in this URL
### Title and Subtitle

Theory and Applications of Coupled Cell Networks

### Authors

Conference Committee

### Performing Organization Name(s) and Address(es)

University of Cambridge  
20 Clarkson Road  
Cambridge CB3 0EH  
United Kingdom

### Distribution/Availability Statement

Approved for public release; distribution is unlimited. (approval given by local Public Affairs Office)

### Abstract

The Final Proceedings for Theory an Applications of Coupled Cell Networks, 26 September 2005 - 30 September 2005

The main areas to be covered by the workshop are:

a) the dynamics of coupled cell networks (synchronisation, pattern formation, clustering, chaotic behaviour, heteroclinic dynamics)

b) the role of network architecture and topology on network dynamics (symmetry groups, groupoids, motifs, layers of different cell types)

c) application to and inspiration from physics and biology (neural dynamics, neural field theories, continuum vs discrete models, existence and stability of travelling waves).

### Subject Terms

EOARD, Dynamical Systems, Coupled Cell Networks, Neural Networks

### Security Classification of:

a. Report  
UNCLASSIFIED

b. Abstract  
UNCLASSIFIED

c. This Page  
UNCLASSIFIED

### Limitation of Abstract

UL

### Number of Pages

21

### Telephone Number

+44 (0)20 7514 4922