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13. ABSTRACT (Maximum 200 words) An advanced laser-Doppler velocimeter (LDV), deemed the 'comprehensive LDV', is designed to acquire fully-resolved turbulence structural measurements in high Reynolds number two- and three-dimensional turbulent boundary layers. The new instrument combines, for the first time, new techniques allowing for the direct measurement of particle acceleration and sub-measurement-volume-scale position resolution so that second-order 3D particle trajectories may be measured at high repetitions. Using these measurements, several terms in the Reynolds stress transport equations may be directly estimated, giving new data for modeling and understanding the processes leading to the transport of turbulence in boundary layer flows. Measurements utilizing the CompLDV technologies are presented and include turbulence dissipation rate and fluctuating velocity-pressure gradient correlations that have been measured in 2D and 3D turbulent boundary layers using the unique capabilities of the 'comprehensive' LDV—many of these measurements are the first of their kind ever acquired in high Reynolds number turbulent flows. The flat-plate turbulent boundary layer is studied at several Reynolds numbers up to $Re_\theta \approx 7500$ to examine Reynolds numbers effects on terms such as the velocity-pressure gradient correlation and the dissipation rate in the Reynolds transport equations. Measurements are also presented in a pressure-driven three-dimensional turbulent boundary layer created upstream from a wing-body junction. The current results complement the extensive data from previous studies and provide even richer depth of knowledge on the most-completely-documented 3D boundary layer flow in existence. Further measurements include the effects of high free-stream turbulence on flat plate turbulent boundary layers and a wing/body junction flow that is similar to a gas turbine flow.				
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## 1 Introduction

Most practical mechanical systems which are measurably affected by fluid flow encounter or produce turbulent flow fields. These flow fields are characterized by a chaotic and time-dependent fluid motion that has a wide range of characteristic space- and time-scales (Simpson 2003). The existence of many scales of chaotic motion makes turbulent flow phenomena very complex and difficult to predict, even for the most basic situations. Fundamental turbulence research continues to be necessary in order to advance our understanding of turbulence and how it affects important phenomena such as wall skin friction, flow separation, and acoustic noise.

Fundamental turbulence research may be generally classified into two approaches. Theoretical and computational research seeks to reveal aspects of turbulence by mathematically dissecting the Navier-Stokes (N-S) equations which govern all flow of a sufficiently continuous medium. Studies either attempt to simplify the equations to obtain analytical approximations to ideal problems such as in asymptotic analyses or to numerically solve some form of the N-S equations such as done with direct numerical simulation (DNS). Experimental turbulence research involves obtaining measurements in real turbulent flows in hopes of characterizing the interactions of the flow parameters. The two approaches are certainly not totally separate endeavors. Only through well-conducted experiments can theoretical assertions be verified. Likewise, experimental studies expose important behaviors which may later be explained by theoretical analysis.

The complex, broad-scale nature of turbulence makes solutions to general problems impossible, particularly at practical Reynolds numbers (Pope 2000). Therefore, useful computational techniques must somehow simplify the complexity of the phenomena so that solutions are obtained within reasonable times. The two methods commonly used to do this involve obtaining solutions of the Reynolds-averaged N-S (RANS) equations or by large-eddy simulation (LES). The RANS equations are obtained by substituting the decomposed velocity,  $U = \bar{U} + u$ , where  $U$  is the instantaneous velocity,  $\bar{U}$  is the mean velocity, and  $u$  is the instantaneous velocity fluctuation from the mean velocity, into the N-S equations and time-averaging the result so that the dependent variables in the equations are the expected values of statistical quantities. The resulting RANS equations for incompressible, constant temperature flow are

$$\frac{D\bar{U}_j}{Dt} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_j} + \nu \nabla^2 \bar{U}_j - \frac{\partial \overline{u_i u_j}}{\partial x_i} \quad (1.1)$$

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0 \quad (1.2)$$

where the usual subscript summing conventions are employed,  $\rho$  is the fluid density,  $\nu$  is the fluid viscosity and  $\bar{P}$  is the mean static pressure. The fundamental problem with



this set of equations is that there are more unknowns than equations thanks to the introduction of the Reynolds stress tensor,  $\overline{u_i u_j}$ . In comparison to RANS, in LES the N-S equations are spatially filtered so that only scales larger than some chosen size are directly solved (see Sagaut 2001 for a more detailed discussion of LES filtering). This results in a loss of information from the unresolved small scales which must be taken into account for a proper solution. In order to solve the RANS equations or to obtain meaningful results from LES, some mathematical models for the information lost due to time-averaging (contained in the Reynolds stress tensor for RANS and the sub-grid stresses in LES) must be used to obtain the additional relationships needed for the Reynolds stresses (RANS) or the effects of the subgrid scales (LES). These models must be devised based upon experimental results, theoretical developments, or DNS from flows of practical applicability to the problem at hand.

Of particular interest in the current research is the mechanics of turbulence within the thin boundary layer region adjacent to solid surfaces. Attached boundary layer flows are characterized by very large gradients in the normal-to-wall direction accompanied with small velocity magnitudes in the same direction. This leads to a simplification of the equations (1.1) and (1.2) since only gradients normal to the wall are significant (Schetz 1993). Unfortunately, many flows are not fully-attached, but have some degree of boundary layer separation where the usual assumptions fail. Furthermore, three-dimensionality introduced by span-wise pressure gradients results in the generation of streamwise vorticity which also causes deviations from the boundary layer assumptions. In this research we wish to characterize the flow physics by obtaining and analyzing new kinematics data on the velocity fields within two- and three-dimensional turbulent boundary layers.

## 1.1 Motivation for advanced turbulence measurements

In the past, measurement techniques such as hot-wire anemometry (HWA) and laser-Doppler velocimetry (LDV) have proved useful for obtaining statistics directly related to the velocity field. These measurements generate statistical information about the velocity at discreet regions in the flow. From the measurements, the values of mean velocities, Reynolds stresses and higher-order statistics may be determined along with some time- and space-correlations such as velocity-fluctuation frequency spectra or two-point velocity correlations. While these measurements have certainly advanced the understanding of turbulent flow phenomena tremendously, more information in the form of novel measurements, such as flow gradient statistics and acceleration statistics, is needed to accurately model the processes involved in unsteady, vortical flows.

To begin to examine the need for more-advanced measurements, in particular the instantaneous velocity gradient and acceleration measurements, the Reynolds-stress transport (RST) equations are given below in tensor notation:

$$\frac{Du_i u_j}{Dt} = P_{ij} + \Pi_{ij} + \nu \nabla^2 \overline{u_i u_j} - \varepsilon_{ij} - \frac{\partial \overline{u_i u_j u_k}}{\partial x_k} \quad (1.3)$$



where the production rate of the Reynolds stress tensor  $\overline{u_i u_j}$  is  $P_{ij} = -\overline{u_i u_k} \frac{\partial \overline{U_j}}{\partial x_k} - \overline{u_j u_k} \frac{\partial \overline{U_i}}{\partial x_k}$ , the velocity-pressure-gradient tensor is

$$\Pi_{ij} = -\frac{1}{\rho} \overline{u_i \frac{\partial p}{\partial x_j}} + \overline{u_j \frac{\partial p}{\partial x_i}}$$

with  $p$  being the fluctuating static pressure, and the dissipation-

rate tensor is  $\varepsilon_{ij} = 2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}}$ . These equations are developed directly from the N-S

momentum equations with the motivation of distilling the individual processes that result in the transport of the Reynolds stresses models. When RST modeling is employed during RANS solutions, the system of equations (1.3) is used for closure of the system of equations (1.1) by providing additional equations for each Reynolds stress gradient,

$\frac{\partial \overline{u_i u_j}}{\partial x_j}$ . This technique is attractive since the Reynolds-stress terms are determined by

solving physically-significant differential equations, which unlike simpler eddy-viscosity models, can account for the Reynolds stress anisotropy which exists in all real turbulent flows (Durbin and Petterson Reif 2001).

To solve the system in (1.3), it is necessary to model several of the terms using either experimental findings or theoretical and computational results. Velocity field measurements, as done with HWA or traditional LDV, allow for direct estimation of each of the terms in equations (1.3) except for the dissipation rate,  $\varepsilon_{ij}$ , and the velocity-fluctuation pressure gradient-fluctuation correlation,  $\Pi_{ij}$ . The dissipation rate is determined when the instantaneous velocity gradients are measured, as the current system design allows. The velocity-fluctuation pressure gradient-fluctuation correlation can be determined by considering the *linear* form of the instantaneous N-S equations in the Lagrangian reference frame:

$$A_i = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \nabla^2 U_i \quad (1.4)$$

where  $A_i$  is the Lagrangian fluid particle acceleration. Since this equation is linear, the fluctuating form is analogous. By multiplying the fluctuating form of equation (1.4) by the fluctuating velocity  $u_j$  and Reynolds-averaging one obtains

$$\overline{a_i u_j} = -\frac{1}{\rho} \overline{u_j \frac{\partial P}{\partial x_i}} + \nu \overline{u_j \nabla^2 u_i} \quad (1.5)$$

By switching the indices in equation (1.5) and adding the result back with the original equation, the follow form results:

$$\overline{a_i u_j} + \overline{a_j u_i} = -\frac{1}{\rho} \overline{u_j \frac{\partial P}{\partial x_i}} + \overline{u_i \frac{\partial P}{\partial x_j}} + \nu \overline{u_j \nabla^2 u_i + u_i \nabla^2 u_j} \quad (1.6)$$

In equation (1.6) we note that the original form of the velocity-fluctuation pressure gradient-fluctuation correlation is regained. In order to find this term, coincident velocity-acceleration measurements allow for direct measurement of the left hand side while the



dissipative/diffusive Laplacian terms may be decomposed into the following measured quantities, as shown by Pope (2001):

$$\overline{\nu u_j \nabla^2 u_i + u_i \nabla^2 u_j} = -2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} + \nu \nabla^2 \overline{u_i u_j} \quad (1.7)$$

where we note that the right-hand-side of equation (1.7) is given by the difference between the Reynolds stress diffusion rate and the Reynolds stress dissipation rate. Therefore, the velocity-fluctuation pressure gradient-fluctuation correlation which appears directly in the RST equations may be decomposed of measurable kinematics quantities,

$$\frac{1}{\rho} \overline{u_j \frac{\partial P}{\partial x_i} + u_i \frac{\partial P}{\partial x_j}} = -(\overline{a_i u_j} + \overline{a_j u_i}) - 2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} + \nu \nabla^2 \overline{u_i u_j} \quad (1.8)$$

In addition to improvements to RST models, velocity gradient measurements allow for the study of vorticity, a fundamental quantity in turbulent flows. The vorticity equations offer an advantage over the basic N-S equations in that the pressure terms vanish. The vorticity is defined as the curl of the velocity:  $\vec{\Omega} \equiv \nabla \times \vec{U}$ . By taking the curl of equation (1.1), we may obtain the instantaneous vorticity equations which then may be Reynolds-averaged to obtain

$$\frac{D\overline{\Omega}_i}{Dt} = \overline{\Omega}_j \frac{\partial U_i}{\partial x_j} + \nu \nabla^2 \overline{\Omega}_i - \overline{\frac{\partial u_j \omega_i}{\partial x_i}} - \overline{\frac{\partial u_i \omega_j}{\partial x_j}} \quad (1.9)$$

$$\frac{\partial \overline{\Omega}_i}{\partial x_i} = 0 \quad (1.10)$$

where  $\overline{\Omega}$  is the mean vorticity and  $\omega$  is the fluctuating vorticity. In an analogous manner as the RANS equations, the fluctuating velocity-fluctuating vorticity tensor,  $\overline{u_j \omega_i}$ , must be modeled in order to close the Reynolds averaged vorticity equations in (1.9) and (1.10).

For purposes of analysis, one half of the  $L^2$ -norm of the vorticity, known as the enstrophy, is often studied (Tennekes and Lumley 1972). By multiplying equation (1.9) by  $\overline{\Omega}$ , the result becomes a single transport equation for the mean enstrophy,  $\frac{1}{2} \overline{\Omega_i \Omega_i}$ ,

$$\frac{D\left(\frac{1}{2} \overline{\Omega_i \Omega_i}\right)}{Dt} = \overline{\Omega_i \omega_i s_{ij}} + \nu \frac{\partial^2 \left(\frac{1}{2} \overline{\Omega_i \Omega_i}\right)}{\partial x_j \partial x_j} - \nu \frac{\partial \overline{\Omega}_i}{\partial x_j} \frac{\partial \overline{\Omega}_i}{\partial x_j} - \frac{\partial (\overline{\Omega_i \omega_i u_j})}{\partial x_j} \quad (1.11)$$

where the fluctuating rate-of-strain tensor is given by  $s_{ij} \equiv \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ . An analogous equation to the TKE equation (i.e., one half times equation (1.3) with  $i=j$ ) is formed by considering the fluctuating enstrophy,  $\frac{1}{2} \overline{\omega_i \omega_i}$ ,



$$\frac{D\left(\frac{1}{2}\overline{\omega_i\omega_i}\right)}{Dt} = \overline{\omega_i\omega_j s_{ij}} + \overline{\omega_i\omega_j S_{ij}} + \overline{\Omega_j\omega_i s_{ij}} + \nu \frac{\partial^2\left(\frac{1}{2}\overline{\omega_i\omega_i}\right)}{\partial x_j\partial x_j} - \nu \frac{\partial\overline{\omega_i}}{\partial x_j} \frac{\partial\overline{\omega_i}}{\partial x_j} - \overline{u_j\omega_i} \frac{\partial\overline{\Omega_i}}{\partial x_j} - \frac{1}{2} \frac{\partial(\overline{u_j\omega_i\omega_i})}{\partial x_j} \quad (1.12)$$

where the mean rate-of-strain tensor is  $\overline{S_{ij}} \equiv \frac{1}{2} \left( \frac{\partial\overline{U_i}}{\partial x_j} + \frac{\partial\overline{U_j}}{\partial x_i} \right)$ .

A measurement technique which can resolve instantaneous velocity gradients, and thus vorticity, offers the possibility of measuring all the processes involved in the Reynolds-averaged vorticity equations and the mean-square vorticity fluctuation transport equations with the exception of the dissipation of mean-square vorticity fluctuations which may be determined by subtraction. A more thorough coverage of vorticity dynamics may be found in the works by Tennekes and Lumley (1972) and Morton (1984).

## 1.2 Review of advanced measurement techniques

### 1.2.1 Velocity gradient measurement

Much interest in experimental fluid mechanics has been placed in advancing the state-of-the-art of instantaneous velocity gradient measurements in turbulent flows. Several researchers have utilized HWA in multiple-wire configurations to achieve velocity gradient measurements (Balint et al. 1991, Wallace and Foss 1995, Honkan and Andreopoulos 1997, Wang and Sen 1999, Honkan and Andreopoulos 2001, and Kholmyansky et al. 2001). Although able to measure three components of vorticity, these techniques are intrusive, thus preventing near-wall measurements, and spatially unresolved, with probe sizes no smaller than twice the size of the smallest scales of turbulence in their flows.

Stereo-, holographic-, and micro-Particle Imaging Velocimetry (PIV) offer the possibility for high-spatial-resolution velocity-gradient measurements (Zhang et al. 1997, Santiago et al. 1998, Meneveau and Katz 2000, Tao et al. 2000, Hu et al. 2001, Hu et al. 2002, van der Bos et al. 2002, and Mullin and Dahm 2004, 2006). Of these techniques, micro-PIV offers the greatest spatial resolution, with  $O(1\mu m)$  interrogation volumes possible. Even in two-dimensions, this technique often suffers from having very short working distances and is mostly limited to micro-fluidic studies (Santiago 1998), although some work on so-called 'long-distance' micro-PIV has shown limited success (Kähler et al. 2006). To the author's knowledge, micro-PIV has yet to be implemented in dual-plane or holographic forms, so measurement of the complete velocity gradient tensor has not been realized. The holographic techniques afford measurements of three components of velocity and position, but suffer from significant resolution penalties due to depth-of-focus issues in the volume (Mullin and Dahm 2004), and thus fully-resolved velocity gradients have not been reported. Only recently have fully-resolved velocity gradient measurements in three-dimensions been realized; Mullin and Dahm (2004) utilized a stereo-PIV arrangement to achieve fully-resolved gradient measurements in a self-similar



axisymmetric co-flowing jet. The technique utilizing stereo-PIV is very complicated since the streamwise gradients require two complete stereo-PIV systems (i.e., dual-plane stereo-PIV). Though no fewer than three groups have implemented such systems (Mullin and Dahm 2004, Kähler et al. 2002, and Hu et al. 2001), only Mullin and Dahm (2004) have reported velocity gradient field measurements, apparently due to the complexities of obtaining low-uncertainty/unbiased gradient measurements. These systems also suffer from the well-known increased out-of-plane velocity uncertainty, which Mullin and Dahm (2004, 2006) quantify as twice that of the in-plane components, which are still somewhat greater than the uncertainties expected for point-wise techniques. Further difficulties arise when trying to obtain near-wall measurements due to the large dynamic range of velocities, flare from the wall, and the relatively low number of particles compared to the outer layer (Somandepalli and Mungal 2004).

There have also been recent efforts to utilize LDV for gradient measurements. A dual-parallel beam LDV method for measuring the velocity difference of two particles has been developed and verified by multiple groups (Tarau et al. 2002, Yao et al. 2001). This technique has been successfully employed for one component of vorticity measurement with spatial resolution as small as  $420\ \mu\text{m}$ . Perhaps the biggest drawback in this technique is that the velocity is not measured along with the difference, making velocity/velocity gradient correlations impossible and the necessity for an additional technique to document the velocity statistics of the flow. Another drawback is the difficulty in achieving a system with high signal-to-noise ratio (SNR). The Doppler equation describing this technique contains a dependence of the Doppler frequency on the light receiving direction, requiring a small angle for received light (Durst et al. 1981). This reduces the SNR of the signal and increases uncertainty, particularly when trying to extend the method to multiple dimensions. Agui and Andreopoulos (2002) utilized the more common dual-beam LDV (also called differential) for velocity difference measurements. This technique allows for measurement of the instantaneous velocity as well as velocity difference by receiving light from two cones within the measurement volume. The resolution achieved was 3 times the smallest scales of the flow. The resolution of this and the dual-parallel beam techniques are limited by finite size of the receiving cone and laser beams diameters.

## 1.2.2 Lagrangian acceleration measurement

Interest in Lagrangian acceleration measurement has been growing with the advent of some new optical particle tracking technologies and the increased computational and storage capacities of modern computers and digital signal processors. Due to the complexity of the measurements, very little information exists about the acceleration structure in turbulent flows. Published techniques include indirect measurement via the isotropy assumption by measuring the fourth-order velocity structure functions (Hill and Thoroddsen 1997), as well as direct studies using DNS (Vedula and Yeung 1999), particle tracking velocimetry techniques (Virant and Dracos 1997; LaPorta et al. 2001; Voth et al. 1998, 2002), particle image velocimetry (PIV) (Christensen and Adrian 2002), and LDV (Lehmann et al. 2002).



DNS is a useful tool for studies utilizing instantaneous pressure values as well as the difficult-to-measure particle acceleration. Vedula and Yeung (1999) used the technique to observe the Eulerian spatial structure of acceleration in homogeneous turbulence simulated up to  $R_\lambda = 230$ . The Reynolds number restrictions of such studies hinder their ultimate usefulness since Reynolds number effects separate them from most practical real flows.

Significant progress has been made in this subject using variants of particle tracking velocimetry (PTV). In particular, improved two-dimensional photodetectors have allowed important advances in measuring three-dimensional particle trajectories. Virant and Dracos (1997) presented PTV in the more traditional sense using CCD cameras as photodetectors for measurements of particle trajectories. The most impressive work utilizing particle tracking has been done by a group at the Laboratory of Atomic and Solid State Physics at Cornell University (LaPorta et al. 2001; Voth et al. 1998, 2002). This group has utilized instrumentation developed for the study of high-energy particle physics to obtain resolved particle trajectories in quasi-homogeneous mixing flows. Two silicon strip detectors are used to obtain two-components of particle position each. The measurement region was projected onto the strips such that each strip represented about  $7.8\mu m$ . The position could be interpolated to about an order of magnitude better than that. Though these studies have yielded some of the best data ever obtained for resolved particle trajectories, the technique is primarily limited to somewhat homogeneous flows with small mean velocities where particle residence times are large.

PIV has successfully been used to evaluate two components of particle acceleration (Christensen and Adrian 2002). Similar to stereo-PIV methods for gradient measurements, these measurements require two PIV systems to work in together. Polarization is used to discriminate the light from two different dual-cavity Nd-YAG lasers which are double-pulsed some tens of microseconds apart due to the much large shutter times of the cameras used. This arrangement was used to study mean two-dimensional channel flow. Unfortunately, the system was unable to resolve the out-of-plane motions necessary for studying mean three-dimensional flows.

In the current study, LDV is chosen primarily due to its exceptional resolution in the near-wall region. Previous work has shown the potential for estimating instantaneous particle accelerations using LDV. The differential LDV technique can be directly extended to make acceleration measurements by simply adjusting the signal processing. In work reported by Lehmann et al. (2002), the authors compared three signal processing methods for estimating particle accelerations and used one of the techniques in a flow situation. The results validated that LDV could successfully be extended to acquire acceleration measurements in turbulent flows. Of particular interest in the current study is the role of the correlation between the fluctuating velocity and fluctuating acceleration in the Reynolds stresses transport (RST) equations. This term is chosen for two reasons, first because it appears directly in the RST equations as a combination of up-to-now difficult to measure terms. Second because the correlation results in low uncertainties relative to the individual uncertainties of the velocities and the accelerations, since the random noise content will not result in any net correlation.



### 1.3 Structure of the report

This report is organized in four additional chapters following the current one. Chapter 2 describes the instrumentation designed, the associated uncertainties, and the techniques used to validate the newly-developed ‘comprehensive’ LDV (CompLDV) instrumentation. In chapter 3, the problem of signal processing for LDV is tackled to identify methods available and quantify their effectiveness for estimating burst frequency and rate-of-change of frequency as well as data processing techniques for the extraction of time-delay correlations and frequency spectra from non-equidistantly-sampled LDV data. Chapter 4 contains the results obtained using the newly-developed CompLDV instrumentation for turbulent boundary layer measurements in both mean-2-D and 3-D flows. Finally chapter 5 wraps up the discussion of the results by stating the conclusions of the study.



## **2 Instrumentation and apparatus**

### **2.1 Advanced turbulent flow instrumentation development**

In this chapter the recent work that has been done to advance the state-of-the-art for laser-Doppler velocimetry (LDV) is discussed. These recent improvements have motivated the development of a novel instrument that combines several of these concepts. The advanced laser-Doppler instruments used for this study are briefly discussed including the uncertainties for these devices.

### **2.2 Recent work**

Several recent developments have significantly increased the capabilities of the LDV technique. These include methods for estimating the particle acceleration (Lehmann et al. 1990, Lehmann and Helbig, 2000, Lehmann, Nobach, and Tropea 2002 and Nobach, Kinzel, and Tropea 2005), obtaining sub-measurement volume particle position resolution (Czarske et al. 2001, 2002, and 2003), processing very closely-spaced-in-time Doppler bursts (Nobach 2002a), the ability to store vast amounts of burst data to be processed offline using high-speed PC-based digitizers, and robust, efficient estimation of signal time-delay correlation and spectra for non-evenly sampled data (Benedict, Nobach, and Tropea 2000, Nobach 2002b).

#### **2.2.1 Laser-Doppler acceleration measurement**

Previous work has shown the power of LDV for estimating instantaneous particle accelerations. The differential LDV technique can be directly extended to make acceleration measurements by simply adjusting the signal processing. In work reported by Lehmann et al. (2002), the authors compared three methods for estimating particle accelerations from linearly-chirped Doppler signals and validated one of the techniques in a flow situation. The acceleration extraction capabilities of the comprehensive LDV (CompLDV) developed for the present study are derived from the work discussed by Lehmann et al. (2002). The primary concerns addressed in previous work have been in signal processing, particle lag, and beam alignment (Lehmann et al. 2002, Nobach et al. 2005). Signal processing methods discussed in those works and expanded upon in Chapter 1. The key aspect of particle lag was examined by Lehmann et al. (2002). In that work the authors showed that the same relative lag effects are present for acceleration as



for velocity—the first-order system-type time constant for either the velocity or acceleration is given as

$$\tau = \frac{d_p^2 \left( \rho_p + \frac{1}{2} \rho_f \right)}{18\mu} \quad (2.1)$$

where the time constant is  $\tau$ ,  $d_p^2$  is the particle diameter,  $\rho_p$  is the particle density,  $\rho_f$  is the fluid density, and  $\mu$  is the fluid viscosity. It then follows that a particle which can faithfully follow the meaningful frequencies for the velocity will do the same for the accelerations. For 0.6  $\mu\text{m}$  diameter DOP particles in air at lab conditions, we compute a time constant of 1  $\mu\text{s}$  indicating that Lagrangian fluctuations out to 160kHz are attenuated by no more than 3dB.

Both the works of Lehmann et al (2002) and Nobach et al. (2005) mention the importance of minimizing aberrations which result in variation of the fringe space. These anomalies cause additional uncertainties and result in chirp signals systematically in error with the linear acceleration/chirp Doppler equation. Nobach et al. (2005) also mentions a device for analyzing the quality of the fringe pattern utilizing a traversable solid particle on a stepper motor. The methods for extracting the Doppler chirp and for optimizing the LDV system for these measurements are still research topics and are addressed in part in the current work.

## 2.2.2 Enhanced-spatial-resolution laser-Doppler velocimetry

The sub-measurement-volume resolution in LDV is a topic that a few researchers have addressed in earnest in recent years (Czarske, Büttner, and Razik 2002; Büttner and Czarske 2003; Pfister, Büttner, and Czarske 2005; Czarske, J. 2001; Büttner and Czarske 2001). The methodologies for achieving enhanced spatial resolution includes measurement volume size reduction by employing limited coherency light (Büttner and Czarske 2001), using a ‘tilted’ fringe pattern measuring nearly the same velocity component (Büttner and Czarske 2003), and by employing interference fringe patterns with calibrated spatial variations (Czarske, Büttner, and Razik 2002; Pfister, Büttner, and Czarske 2005; Czarske, J. 2001). The advantages of achieving the enhanced spatial resolution are many. In particular, very small spatial scales of motion may potentially be resolved, flows that rapidly change in space may be characterized very precisely, broadening effects due to the finite measurement volume size may be greatly reduced, spatial resolution may be enhanced in long optical access systems, and of interest in the present study, the precise position information may be used to track relative particle trajectories for estimation of velocity gradient measurements. Of the high-resolution methods mentioned, the only reported efforts have been one-component velocity and position resolution until the current work. Thus, an important aspect of the current work is extending these methods and developing new ones to enable three components of simultaneous velocity and position measurement that will allow for full-vector velocity and position measurements.

### 2.2.3 Velocity gradient laser-Doppler measurements

As discussed in the introduction, velocity gradient measurement has been realized, at least to a limited degree, with LDV in previous work. Two methods have been employed for these measurements, a parallel beam method (Ötügen, M.V., Su, W-J., and Papadopoulos, G. 1998; Tarau, T., Stepaniuk, V.P., and Ötügen, M.V., 2002; and Yao, S., Tong, P., and Ackerson, B. J., 2001) and a dual-receiving-cone method (Agui, J.H. and Andreopoulos, Y., 2002). Each of these techniques was validated in flow situations, although to-date no significant results have been reported. The parallel beam method was considered in early applications for the current project and the results from that study are discussed in following sections.

### 2.2.4 Laser-Doppler signal processing

A detailed review of the aspects of laser-Doppler signal processing and non-equidistantly sampled velocity spectral estimation as well as the contributions from this work is reserved for chapter 1 to follow.

## 2.3 Comprehensive laser-Doppler velocimetry

The goal of the comprehensive laser-Doppler velocimetry (CompLDV) program is to produce *novel* results for turbulence structural quantities never before measured in wall-bounded turbulent flows. The CompLDV was designed to leverage the technologies mentioned in section 2.1 to enable near-wall, non-intrusive, spatio-temporally resolved instantaneous measurements of all components of velocity, acceleration, and velocity gradients in high Reynolds number laboratory flows. The power of such a measurement system is illustrated by the abundance of useful measurement quantities which may be realized. Table 2.1 lists some of the quantities of primary interest which may be measured or inferred from the data.

**Table 2.1 CompLDV measurement quantities**

- Instantaneous direct measurements:
  - 3 components of velocity
  - 3 components of particle position
  - 3 components of acceleration
- Instantaneous calculated quantities:
  - Reynolds stress tensor
  - Triple product tensor
  - Velocity gradient tensor
  - Vorticity tensor
  - Rate-of-strain tensor
  - Reynolds stress gradient tensor
  - Acceleration gradient tensor
  - Velocity-acceleration fluctuation tensor
  - Dissipation
  - Skin friction velocity



- Additional time-averaged quantities:
  - Velocity-pressure gradient fluctuation tensor
  - Laplacian of Reynolds stress tensor

To be useful for interpretation, each of these measurements is needed at the lowest-possible uncertainties. Thus, the overwhelming driver for system design was obtaining low-uncertainty, unbiased measurements of the desired quantities. Despite the design uncertainties driving many parts of the system, some aspects of the final design were not predicted by simple uncertainty analyses. The design process, including formal uncertainty analysis as well as practical implementation considerations, for obtaining the optimal ComplDV configuration is described to follow.

### 2.3.1 Basic ComplDV Principles of Operation

The ComplDV technique combines advanced LDV techniques to simultaneously measure three components of velocity, acceleration and crossing position of a sub-micron seed particle following the local motions in test flows. The quantities measured allow a good approximation—to second order—for the trajectory of each particle crossing the volume:

$$\vec{x}(t) = \vec{x}_0 + \vec{V}(t - t_0) + \frac{\vec{a}}{2}(t - t_0)^2 \quad (2.2)$$

where  $\vec{x}(t)$  is the time-dependent three-dimensional particle trajectory,  $\vec{x}_0$  is the position vector at  $t = t_0$ ,  $\vec{V}$  is the velocity vector at  $t = t_0$ , and  $\vec{a}$  is the acceleration at  $t = t_0$ . With this information, the rate of strain tensor may be estimated by a linear fit to the velocity field measured from four or more particles. This will be discussed in detail in section 2.3.2.1.1.2.

A technique for obtaining spatial resolution within LDV measurement volumes, already briefly discussed, was developed by Czarske et al. (2002). This technique utilizes two measurements of the same component of velocity to achieve particle position resolution two orders of magnitude smaller than the measurement volume diameter, at absolute scales unobtainable for any other spatially-resolving flow velocity measurement technique with a dynamic velocity range approaching that of LDV. The method can be easily understood looking at the Doppler equation for differential LDV,

$$U_{\perp} = f_D d \quad (2.3)$$

where  $U_{\perp}$  is the velocity of the particle perpendicular to the fringes,  $f_D$  is the Doppler frequency and  $d$  is the interference fringe spacing. Figure 2.1 gives a schematic of the basic geometry of such a LDV measurement volume. Now consider the case when two parallel measurement volumes are overlapped, the ratio of the Doppler equations for the two systems becomes

$$\frac{U_1}{U_2} = \frac{f_{D1} d_1}{f_{D2} d_2} = 1 \quad (2.4)$$

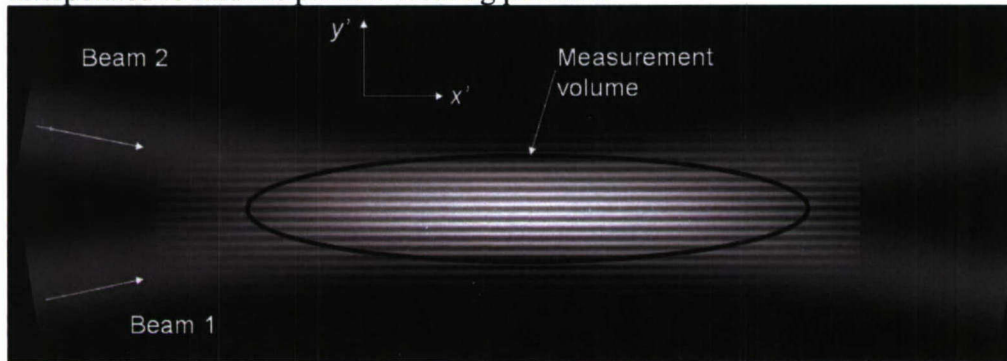
since  $U_1 = U_2$  for parallel measurement volumes. It follows then that

$$\frac{f_{D1}}{f_{D2}} = \frac{d_2}{d_1} \quad (2.5)$$

Now if the fringe spacing is non-uniform along the measurement volume, as is possible when beams are crossed off waist (Miles 1996), then the ratio of the two Doppler frequencies is a function of the crossing position within the measurement volume. Also, if the fringe spacings vary monotonically along the measurement volume, as is the case if one set of fringes are converging while the other is diverging as illustrated schematically in Figure 2.2, there will exist a monotonic function,  $q$ , such that

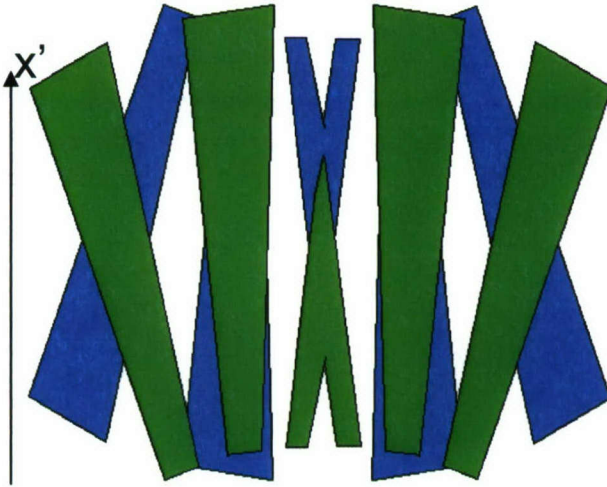
$$\frac{f_2}{f_1} = \frac{d_1(x'_{cross})}{d_2(x'_{cross})} = q(x'_{cross}) \quad (2.6)$$

where  $x'_{cross}$  is the particle crossing position measured axially along the measurement volume, as in Figure 2.1. The function  $q(x')$  is a constant for a given beam alignment and may be calibrated by measuring the fringe spacing along the measurement volume. Using the instantaneous measurements of the Doppler frequency ratios, the calibration may be interpolated to find the particle crossing position.



**Figure 2.1. Schematic of the intersection of two coherent laser beams forming an interference fringe pattern.**





**Figure 2.2. Schematic of overlapping fringe sets with varying fringe spacings along the measurement volume beam-bisector.**

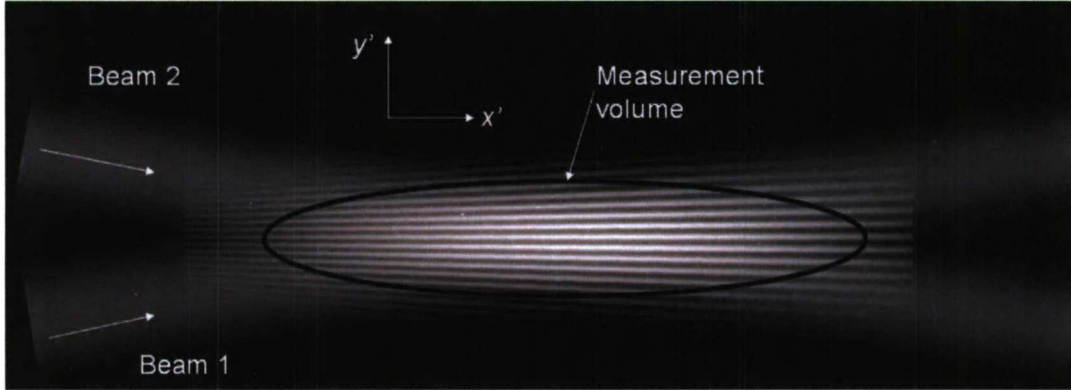
Analysis of the interference fringe patterns formed by pairs of Gaussian-intensity distribution light beams has been examined by Miles (1996). Of interest in this application is the special case when a pair of laser beams has waists that are spaced equal distances from the center of the beam overlap region. In this case, an axial variation of fringe spacing occurs for which Miles (1996) derived the following expression:

$$d(x') = \frac{\lambda}{2 \sin \theta} \left( 1 + \frac{x' \cos^2 \theta (x' \cos^2 \theta - x'_w)}{x'_R \cos^2 \theta - x'_w (x' \cos^2 \theta - x'_w)} \right) \quad (2.7)$$

where  $x'$  is measured along the centerline of the measurement volume,  $x'_w$  is the distance between the waist of the beam and the center of the measurement volume,  $\theta$  is the angle of the intersection of the beams,  $\lambda$  is the nominal wavelength of the light, and  $x'_R$  is the

Rayleigh number of the beams as given by  $x'_R = \frac{\pi \omega_0^2}{\lambda}$  with  $\omega_0$  being the radius of the

beam at the waist. A schematic example of such an interference fringe pattern arising from two Gaussian beams is given in Figure 2.3. While deviations from this model situation may occur in practice, this equation may be employed by the instrument designer to achieve optimal fringe variations to satisfy the position resolution needs.



**Figure 2.3. Schematic of the off-waist intersection of two coherent laser beams forming interference fringes with  $\frac{\partial d}{\partial x'} > 0$ .**

The principle for acceleration extraction is also based upon the Doppler model for the differential LDV. Taking the time-derivative of equation (2.3) yields

$$\frac{df_D}{dt} = \frac{1}{d} \left( \frac{dU_{\perp}}{dt} - \frac{1}{d} \frac{d(d)}{dt} f_D \right) \quad (2.8)$$

Thus the rate of change of the Doppler frequency is proportional to the Lagrangian acceleration. In order to find the acceleration of each particle, the rate-of-change of frequency or chirp rate is determined using the power spectral density distribution in the orthogonal frequency-chirp-rate space. This PSD map is computed from individual burst signals by way of the discrete chirp Fourier transform (DCFT)—further details of the signal processing will be presented in the following chapter.

To interpret the chirp rate results, one must take into account the effects of the fringe gradients—both in the longitudinal and transverse directions of the measurement volume. By differentiating equation (2.3), this is quantified as

$$\left. \frac{\partial f}{\partial t} \right)_{\text{FringeGradient}} = - \frac{U_{\perp}}{d(x'_{\text{cross}})^2} \frac{Dd}{Dt} \quad (2.9)$$

where  $\frac{Dd}{Dt} = U_{\parallel} \frac{\partial d}{\partial x'} + U_{\perp} \frac{\partial d}{\partial y'} + U_z \frac{\partial d}{\partial z'}$ . The quantity in equation (2.9) can be calculated at

low uncertainties since the uncertainties for all velocity components are low and the fringe gradients are constant. Note that in equations (2.6) and (2.9), the fringe space itself is only considered a function of the direction  $x'$ , yet fringe gradients in the  $y'$  and  $z'$  directions are also considered for the chirp rate correction. This is explained due to the scaling of quantities making up the total derivative of the fringe spacing passed by the particle in equation (2.9). Since the contributions to the chirp rate due to fringe gradient perpendicular to the fringes themselves scale on the square of the Doppler velocity,  $U_{\perp}$ , very small values of  $\frac{\partial d}{\partial y'}$  can contribute significantly to the value of  $\left. \frac{\partial f}{\partial t} \right)_{\text{FringeGradient}}$ .

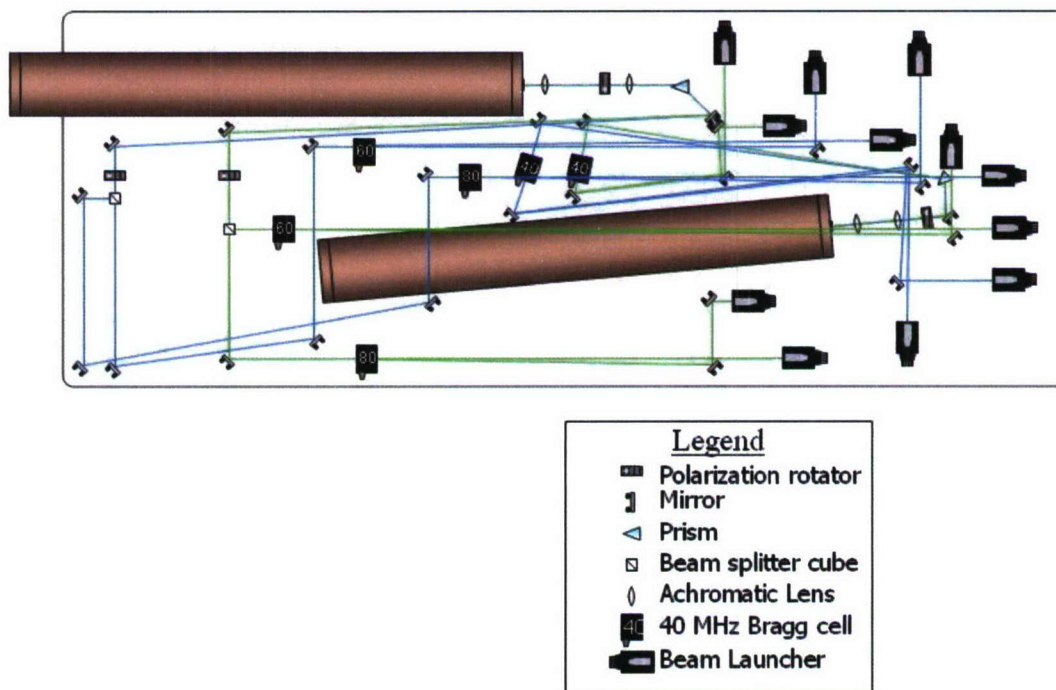


Further since a stress-like term,  $U_{\parallel}U_{\perp}$ , is required to correlate to result in a longitudinal fringe gradient contribution to  $\left. \frac{\partial f}{\partial t} \right)_{\text{FringeGradient}}$ ,  $\frac{\partial d}{\partial y'}$  can be more than an order of magnitude smaller than  $\frac{\partial d}{\partial x'}$  and still be significant.

### 2.3.1.1 Optical table

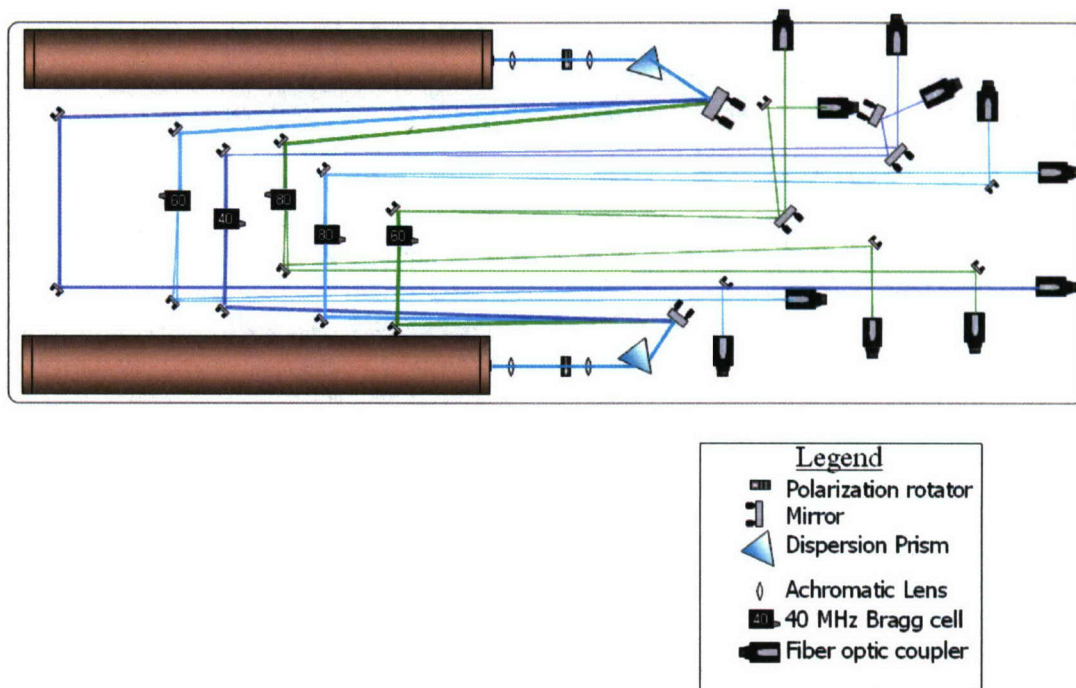
A schematic of the optical table configurations used for the CompLDV systems are given in Figure 2.4 and Figure 2.5. The argon-ion lasers (Coherent Innova I90C-5) were run in multiline mode with no etalon such that all wavelength lines were emitting and the coherence length of single wavelengths was about 20mm. The multimode light was first focused using a pair of achromatic lenses, one with a focal length of about 100 mm and another with about 50 mm focal length. By optimizing the diameter of the beam at the point of fiber optic coupling, the efficiency of the coupling system is maximized. Prior to chromatic separation via a dispersion prism, the polarization of the multiline beam is rotated 90° from vertical to horizontal using a mica half-wave plate in a rotating mount. This step is necessary to minimize reflective losses at the prism since such light with horizontal polarization will nearly satisfy the Bragg condition of zero-reflectance upon entering the prism. The beam is steered upon the table using front surface protected aluminum mirrors with 98% or greater reflectivity. The mirrors are mounted on kinematical mounts with two-degree of freedom angular adjustments. After passing through the dispersion prism, the light is given ample distance to separate to allow the desired wavelengths to be ‘picked off’. The 476.5nm, 488nm, and 514.5 nm lines are chosen due to the optical power that is achievable for these lines using a standard argon ion laser. Note that in Figure 2.4, only the 488nm and 514.5nm lines are used, as this was the first configuration for the CompLDV setup. In that configuration, additional mica half-wave plates are used to adjust the beam polarization prior to being split with a polarizing beam-splitting cube. The power balance between the two split beams is dependent upon the input beam polarization. In the later implementations utilizing the 476.5nm line, these beam-splitting components were omitted since a sufficient number of beams could result without splitting. This three-wavelength optical table configuration is illustrated in Figure 2.5. With the required number of beams, mirrors are used to direct the individual beams of the desired wavelengths into acousto-optical modulators (Bragg cells) with varying carrier frequencies. These devices are used to impart a velocity to the interference fringes in the measurement volume by heterodyning light of very slightly different wavelengths. This fringe velocity solves the zero-velocity ambiguity that can occur for static fringes. In this way, a zero-velocity particle will result in a Doppler signal frequency equal to the carrier frequency of the Bragg cell. Only when particles are moving at the fringe velocity will ambiguities again arise, but such a condition is out of the realm of possibility for the fringe spacings and flow velocities expected. The Bragg cell carrier frequencies chosen also allow frequency-domain separation of the signals from different measurement volumes of the same color. The Bragg cells and corresponding radio frequency (RF) drivers are purchased from Intra-Action in carrier frequencies of 30, 40, 50, 60, and 80 MHz. The desired output from the Bragg cells is a

pair of beams of equal intensity, one at the original frequency of the laser and the other shifted in frequency (positive or negative) by the carrier frequency of the Bragg cell. The powers in the beams are adjusted in two ways: first the Bragg cell is mounted on a horizontal rotation stage that allows angular adjustment of the device; and second the power of the Bragg cell carrier signal is adjusted. To maximize the efficiency of this process, first the mechanical adjustment is used to maximize the intensity of the first-order shifted beam (the one shifted by the Bragg cell carrier frequency). Next, the power of the carrier signal is adjusted electronically via a potentiometer knob on the Bragg cell driver unit. This adjustment allows much finer and predictable balance of the power instead of simply adjusting the angular direction of the Bragg cell. After exiting the Bragg cell the beams are allowed distance to separate sufficiently and are steered into the fiber optic coupling units. These units are obtained from Newport (model F-91-C1 coupler, FPH-CA6 ST-style fiber optic chuck, and M-20X objective lens) and consist of a 20X objective lens for focusing the beam to approximately the diameter of the fiber optic and a 5 degree-of-freedom precision fiber optic alignment system. Using these couplers, coupling efficiencies of 45-60% are achieved.



**Figure 2.4. Schematic of the optical table configuration used for the first ComplDV optical system. Schematic is credited to Devin Stewart.**





**Figure 2.5. Schematic of the optical table configuration used for the three-color ComplDV optical systems.**

### 2.3.1.2 Data Acquisition System and Signal Conditioning

The signal processing complexity and the rate of particle arrivals for the ComplDV makes real-time frequency-domain processing infeasible. For this reason, data acquisition units acquire and record *all* of the time-series data for a given location in space and over the sampling duration. Due to the bandwidth of the Doppler signals for obtaining all the signals on two channels of analog-to-digital conversion, very high sampling frequencies are necessary. Therefore, the data acquisition specified was chosen for maximum throughput of data and storage capabilities.

#### 2.3.1.2.1 A/D Converter Card

In choosing an A/D device for this application, two requirements are of high importance: the sampling rate of the device and the onboard storage capabilities. The sampling frequency limits the maximum Doppler frequency that may be measured due to the Nyquist criterion. On the other end of the frequency spectrum, the onboard storage capacity limits the lowest observable frequencies for a given sampling rate.

The ComplDV utilizes Bragg frequencies up to 80 MHz, requiring sampling at no less than 160 MS/s. One must add to this the potential Doppler shift of the signal up to 10 MHz for flows in the Virginia Tech Boundary Layer Wind Tunnel with fringe spacings of the ComplDV1. Thus, a minimum sample rate is 170 MS/s. Commercial devices with

the order of sampling capabilities sample at 250 MS/s, giving practical bandwidths of 100 MS/s. Due to the ease of application and the efficiency of data transfer, a PCI-board from Strategic Test (model UF.258) was chosen. This device samples one channel at 500 MS/s or two simultaneous channels at 250 MS/s on a single card with a resolution of 8 bits. At the maximum sampling rate, continuous data samples of 0.54 s may be obtained, which adequately defines low frequency flow phenomena.

#### ***2.3.1.2.2 DAQ Computer***

The DAQ computer used was a standard top-end PC. Both immediate and long-term storage of raw burst data is achieved on high capacity IDE hard disks. During data acquisition, measurement run times are limited by the transfer rates between the PCI A/D card and the IDE hard disks. To reduce this bottle neck, pairs of RAID disks running in the striping (0) mode have been used, however, the non-portability of such pairs of disks led to using single IDE disks. The portability is an important issue, as the processing-intensive data must be transported to other computers to free-up resources on the DAQ computer so that measurements are not delayed.

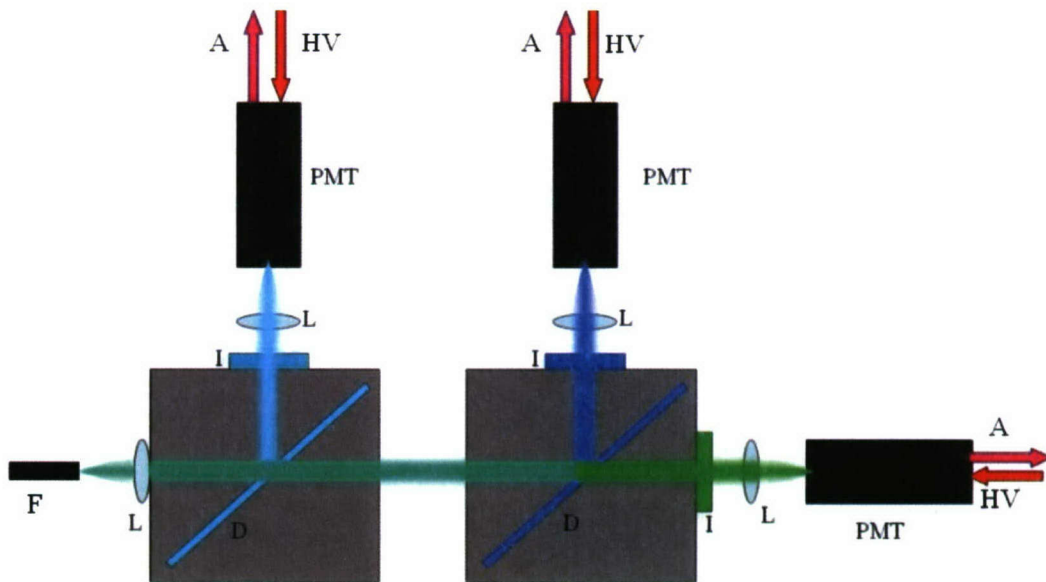
#### ***2.3.1.2.3 Signal Conditioning***

The light gathered by the receiving system exits the multimode optical fiber into the chromatic separation optics (Figure 2.6). That light is collimated and passes through successive filters that direct a maximum amount of monochromatic light to three photomultiplier tubes (PMTs). Dichroic filters from Omega Optical are used to separate the light of different wavelengths. The first dichroic (model XF-2039) is a narrow band blocking filter that reflects light in the region of 485-497nm. The only wavelength reflected by this filter is the 488nm blue signal that then passes through an interference filter to further protect against cross-talk before being focused upon the PMT face. Next, the remaining light consisting primarily of 476.5 nm and 514.5 nm burst signals pass a second dichroic filter (model XF-2010) with a single transition in the wavelengths of interest. This filter is essentially a long-pass filter allowing light above 505nm in wavelength to pass while reflecting shorter wavelengths. The 476.5nm and 514.5nm signals then each pass through narrow-pass interference filters to avoid cross talk and are focused upon the PMT faces. The efficiency of this separation process is acceptable, with the dichroic filters sending more than 80% of the light in the correct directions. The interference filters contribute a loss of 40-50% of the light signal in order to sharply limit unwanted wavelengths from reaching the PMT faces.

The voltage signals which the DAQ system record originate as current signals from the PMTs. The PMTs used are Hamamatsu model R4124, high frequency response (low single electron transit time) tubes. The very low current signals leaving the PMT anodes are converted to voltage signals using the input impedance of the low noise, high bandwidth, high gain RF amplifiers (Sonoma Instruments Model 315, 10kHz-1GHz response, 355V/V gain). These amplifiers have 50 Ohm impedance which limits the filtering effects that can occur before current-to-voltage conversion. To use the optimal amount of the amplifiers dynamic range, the PMT high voltage power supply is set to a



value of about -520V, yielding signals of  $O([-10mV\ 500mV])$  from the amplifier. Since the DAQ unit used has only two input channels, the three PMT signals must be combined to two channels for compatibility. To do this, the Bragg frequencies of each of the channels is considered, since the electronic filters that will allow combination of the signals must be specified based upon these frequencies. The pre-combining filters are necessary to avoid the addition of noise from spectral regions of the channels that are known to have no useful signals. In the case of the 488nm channel, the Bragg cell carrier signals used are 60MHz and 80MHz. The 476.5 nm has Bragg cell carrier frequencies of 30MHz and 40MHz, allowing ample separation of these channels so that filtering may be used to condition the signals prior to combining. The amplified signal of the 476.5nm channel is filtered using a Mini-Circuits brand BLP-50 low pass filter that has a cut-off frequency of 55MHz. The 488nm channel is filtered with a 55 MHz high pass filter from KR Electronics model 2290-55. Next the signals are combined using a Mini-Circuits model ZSC-3-1 power combiner. The 514.5nm channel continues to the digitizer card independently. Prior to digitization both the electronic channels are passed through anti-aliasing filters (Mini-Circuits model BLP-100) with cut-off frequencies of 108MHz. This value is chosen over any greater frequency up to the Nyquist to allow for ample roll-off of the anti-aliasing filter to develop. The practical rule-of-thumb for such an anti-aliasing cut-off is 2.56 divided by the sampling frequency rather than 2 as defined by the Nyquist criterion (Doebelin 2001).



**Figure 2.6. Diagram of the light separation and detection optics and photonics. F, multimode fiber optic transmitting received light; L, 19mm focal length lens; D, dichroic filter; I, interference narrow bandpass filter; PMT, photomultiplier tube; A, anode signal out; HV, high voltage in.**

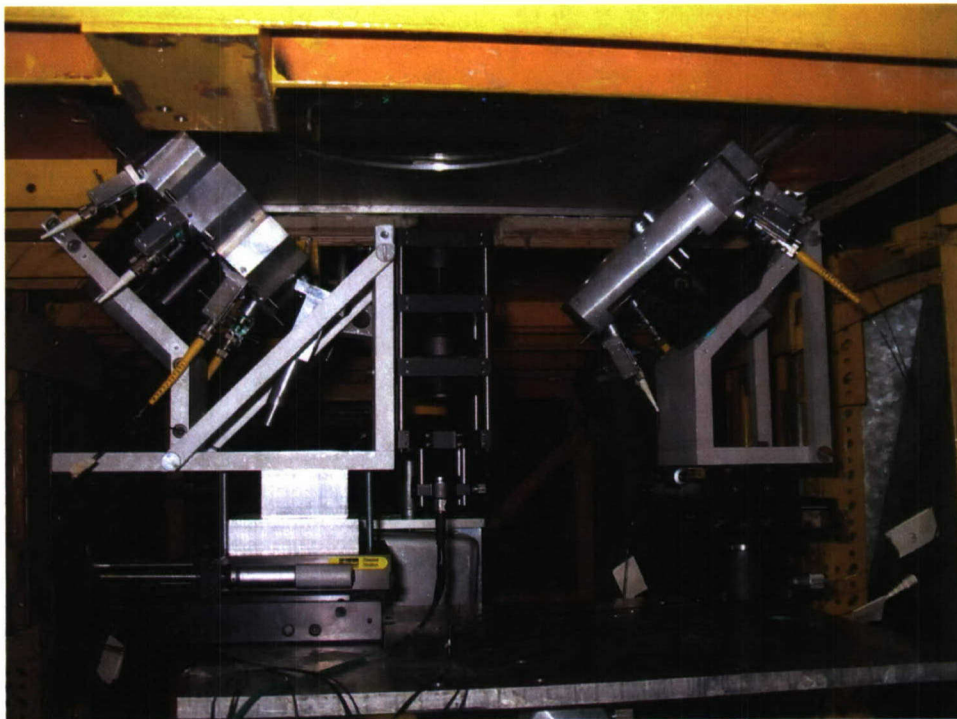
### 2.3.2 Third generation CompLDV

The framework for realizing a comprehensive instrument for studying the structure of near-wall turbulent flows was formed during the development of the first two generations of the CompLDV already discussed. During this work many aspects of conventional LDV were significantly improved by implementing new technologies. Among these improvements have included the development of

- completely PC-based burst acquisition methods
- low-uncertainty adaptive signal processing
- much-improved signal-to-noise ratio signals by optimal optics design and low noise detection and electronics
- high valid burst rates due to easily-obtainable, nearly-monodisperse seeding
- sub-measurement-volume particle position resolution
- particle acceleration sensing

During the course of implementing these improvements, much was learned about the nature of the uncertainties encountered in the advanced measurements. As the first generations were primarily combinations of parallel ideas discussed but never fully integrated in the literature, the third generation makes use of the experience gained to produce a laser instrument which is significantly different from previously designs. A complete description of the CompLDV3 may be found in Lowe (2006).

For visual reference, Figure 2.7 is a photograph of the CompLDV3 probe and Figure 2.8 is a photograph of the laser beam configuration taken during a test.



**Figure 2.7. Photograph of CompLDV generation 3 configuration.**





**Figure 2.8. Photograph of laser beam configuration for the CompLDV3.**

### 2.3.2.1 CompLDV3 Uncertainties

The uncertainties for the CompLDV3 have been analyzed based upon the optics, geometry, and noise levels of the system. The first step in determining the uncertainties for statistical quantities measured using the CompLDV3 was to estimate uncertainties in the instantaneous measurements of velocity and position. To do this, a Monte Carlo uncertainty simulation was run to propagate the known Doppler frequency uncertainties as determined by the FFT signal processing simulations through the CompLDV3 system of equations (see Lowe 2006) relating the Doppler frequency measurements to the geometry and desired physical quantities.

For the simulation, a single representative velocity vector was utilized— $\vec{U} = (18\hat{i} + 0.01\hat{j} - 0.5\hat{k})\text{m/s}$ , and a position measured from the center of the volume was used— $\vec{x} = (0\hat{i} + 150\hat{j} - 50\hat{k})\mu\text{m}$ . Note that the simulation was run for a range of other velocity-position sets and they produce the same quantitative results when the velocities are normalized by the magnitude of the input velocity. The geometric constants,  $a_i$ ,  $b_i$ ,  $c_i$ ,  $e_i$ , and  $g_i$ , are all given from the CompLDV3 design as are the fringe space gradients as determined by from the analysis of Miles (1996). From this information, the expected Doppler frequencies,  $f_i$ , were determined. The work of Shinpaugh et al. (1992) as well as the current work has quantified the expected uncertainties in Doppler frequency estimates

using digital signal processing techniques. From this work, the uncertainty levels for the lowest signal-to-noise ratio that allows reliable Doppler frequency estimates ( $SNR_I=18-20dB$ ) was used,  $\delta f = 0.1/T$  where  $T$  is the Doppler signal duration. The Monte Carlo simulation sampled 20,000 frequencies around the nominal values determined from the given velocity and position vector. The PDFs of the frequencies sampled were Gaussian with standard deviations of magnitude  $\delta f = 0.1/T$ . The signal duration was determined using the transit time of the particle estimated as  $T \approx D/|\vec{U}|$  where  $D$  is the diameter of the measurement volume, nominally  $200\mu m$ . In the simulation,  $T = 11\mu s$ . For coordinate system reference, the CompLDV3 is designed to operate with the mean  $U$  flow velocity being approximately aligned with the  $x$ -axis of the optical coordinate system which is defined by the normal to the plane of the laser light sheet. The statistics for the simulated velocities and positions propagated through the LDV system equations discussed by Lowe (2006), which give the representative uncertainties for the quantities, are given in Table 2.2. Note that the uncertainties in the velocity components are exceptional compared with other measurement systems used for similar measurements. For instance, the particle-image velocimetry (PIV) technique for direct measurements of the velocity gradient tensor described by Mullin and Dahm (2006) exhibits 2-4% uncertainties for in-plane velocity components and 6-8% for the out-of-plane velocity component for 20:1 odds. For the CompLDV3 the stream-wise velocity component is measured at an uncertainty that is 25% lower than for the span-wise and normal-to-wall values and each is less than 0.2% uncertainty at 20:1 odds. The 20:1 absolute uncertainties in the  $y$ - and  $z$ -particle positions are each about  $5.8\mu m$  for all velocity magnitudes. Although Czarske et al. (2002) report an uncertainty smaller than this in their work with a similar LDV technique, the probe implemented therein has limited general applicability since it operates in a forward scatter mode which requires optical access on both sides of the flow facility. This difference results in stronger signals and reduces the frequency uncertainty considerably if noise sources are properly managed. For practical back-scattering systems such as the CompLDV3, the frequency metric used is typical (Albrecht et al. 2003). Otherwise, the incident optical arrangement of the CompLDV3 is seen to be superior to the one discussed by Czarske et al. (2002), since despite a factor of 10 improvement in the frequency estimation variance in that work, the particle position uncertainty is only 50% lower.

**Table 2.2. 20:1 uncertainties for instantaneously-measured velocities and positions as determined by the sensitivities for these quantities.**

Term	Uncertainty
$\delta U / \ \vec{U}\ $	0.075%
$\delta V / \ \vec{U}\ $	0.11%
$\delta W / \ \vec{U}\ $	0.12%
$\delta x / D$ Entire volume used	0.6%
$\delta x / D$ Narrow light sheet used	0.2%
$\delta y / D$	2.9%
$\delta z / D$	2.9%



$\delta A_x T / \ \bar{U}\ $	0.38%
$\delta A_y T / \ \bar{U}\ $	0.56%
$\delta A_z T / \ \bar{U}\ $	0.61%

The x-position uncertainty was determined in an independent simulation of the process for estimating the particle arrival time. To estimate the arrival time, a fit of the low-frequency data for the burst envelope is performed. The non-high-pass-filtered electronic burst signal is analyzed. The digitized data are low-pass-filtered to remove the broadband noise and narrowband Doppler signals. The resulting signal is primarily comprised of the signal contributions from the burst envelope. The model for the low-passed signal is then the Gaussian window. A least-squares quadratic is fit to the logarithm of the filtered data with the intention of finding the position of maximum signal, where the first time derivative of the model function is zero. The time at which this occurs is estimated as being the arrival time. This time is compared with the input arrival time and their difference times the input velocity gives the position uncertainty, i.e.  $\delta x = U\delta t$ . For the simulations run, two cases were considered—one in which the signal from the measurement volume ( $200\mu m$  diameter) was used to find the arrival time and another where the signal from the narrower light sheet ( $20\mu m$  width) is used. The resulting 20:1 uncertainties are about  $1.2\mu m$  and  $0.4\mu m$  for the entire measurement volume and the narrow light sheet methods, respectively.

#### 2.3.2.1.1.1 Velocity Statistics Uncertainties

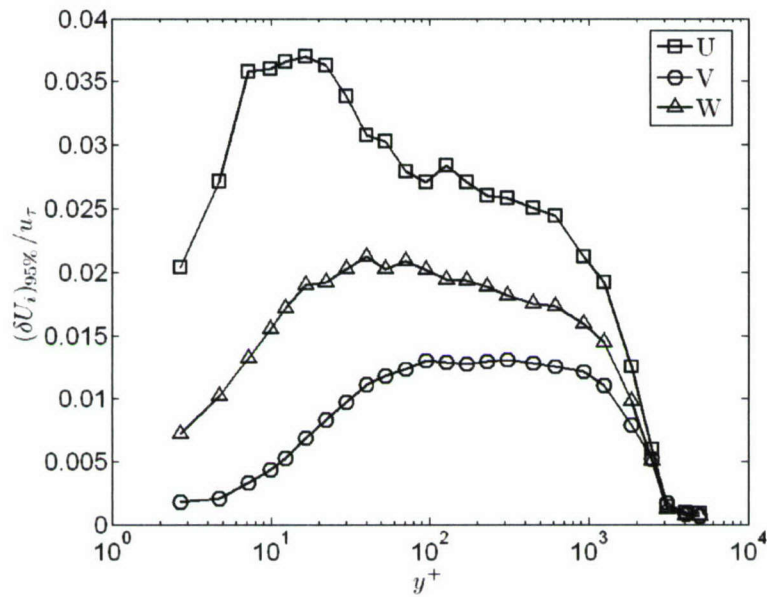
The instantaneous velocity uncertainty values were used to determine the uncertainties in the velocity statistics throughout a boundary layer profile. Data in the 2D flat-plate turbulent boundary layer (TBL) at  $Re_\theta = 7600$  were used to give the statistical variations of the velocities to allow an uncertainty assessment. Since the uncertainty of a measurement set depends upon the true distribution of the quantity sampled, this simulation takes the variation in the turbulence intensities into account. This dependency is easily seen for the estimation of the statistical mean, where the standard error of the

mean for the variable  $x$  is given as  $S_x = \frac{\sigma(x)}{\sqrt{n}}$  where  $\sigma(x)$  is the standard deviation of  $x$ ,

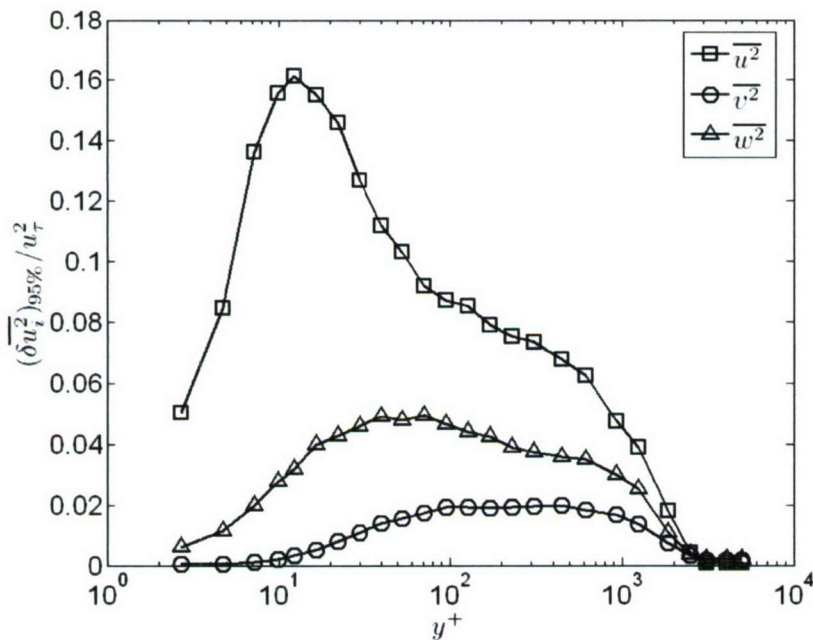
and  $n$  is the total number of samples for  $x$ . Note that this statistical error value occurs even when  $x$  is sampled without any inherent uncertainties. The addition of uncertainties in the sampling of  $x$  increases the total uncertainty in the statistical quantity.

Series of values for each velocity component were produced with mean values and standard deviations given by the profile data. Gaussian probability distribution functions were used for all the simulated velocity distributions. No cross-correlations or higher-order moment values were introduced to the simulated data sets so that any such statistics measured would indicate an uncertainty in that value. The uncertainties for the velocity statistics up to triple-products (third-moments of velocities) non-dimensionalized by the wall friction velocity in the 2D flat plate TBL are plotted in Figure 2.9-Figure 2.14. These

values may be taken as the uncertainty values for the statistical quantities estimated using the CompLDV3.



**Figure 2.9.** 20:1 uncertainties in the mean velocities throughout a 2D flat plate turbulent boundary layer profile non-dimensionalized on the wall friction velocity.



**Figure 2.10.** 20:1 uncertainties in the Reynolds normal stresses throughout a 2D flat plate turbulent boundary layer profile non-dimensionalized on the wall friction velocity.



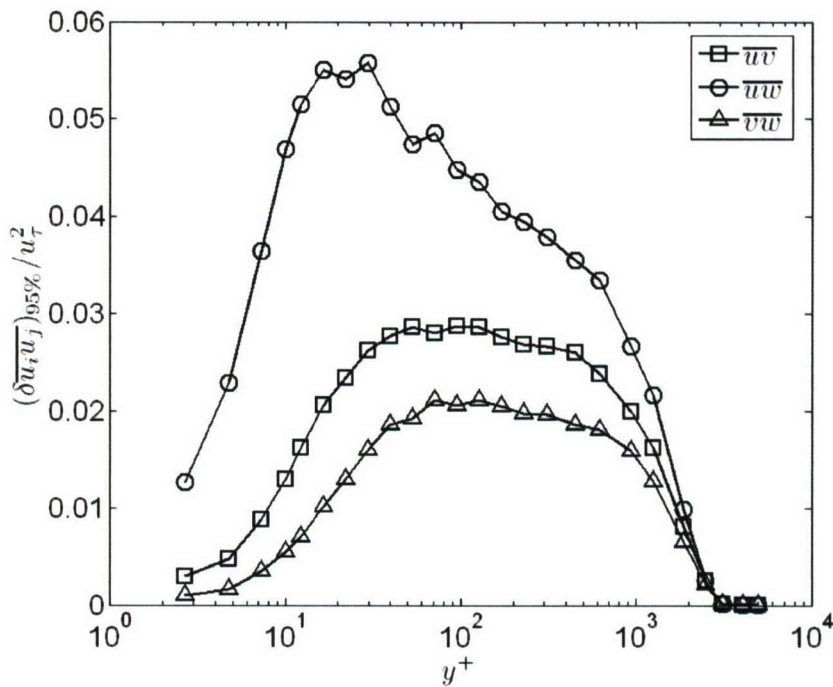


Figure 2.11. 20:1 uncertainties in the Reynolds shear stresses throughout a 2D flat plate turbulent boundary layer profile non-dimensionalized on the wall friction velocity.

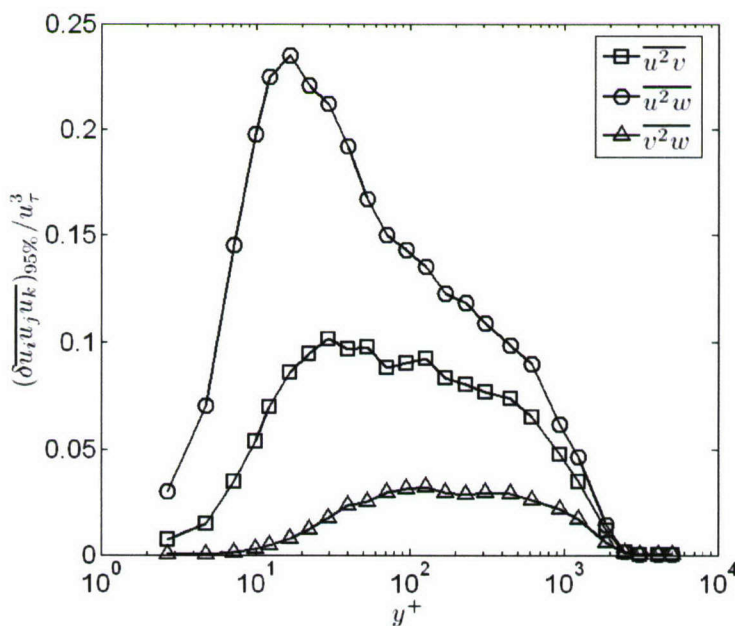


Figure 2.12. 20:1 uncertainties in velocity triple-products throughout a 2D flat plate turbulent boundary layer profile non-dimensionalized on the wall friction velocity.

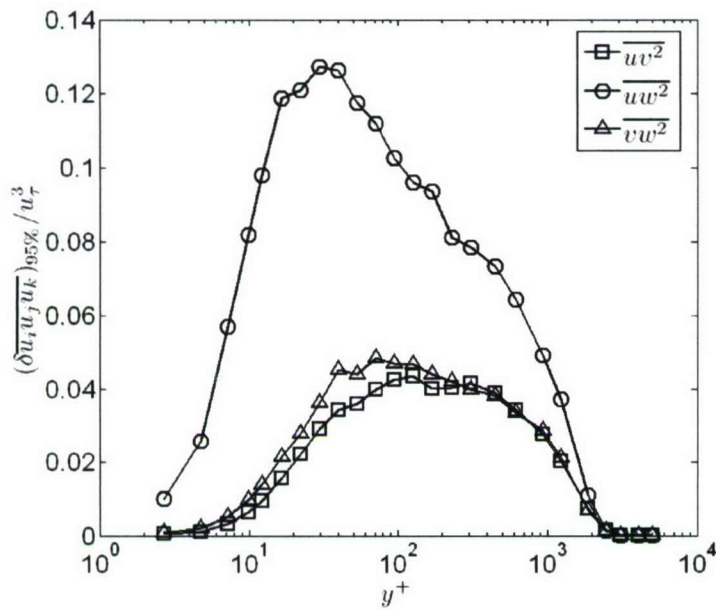


Figure 2.13. 20:1 uncertainties in velocity triple-products throughout a 2D flat plate turbulent boundary layer profile non-dimensionalized on the wall friction velocity.

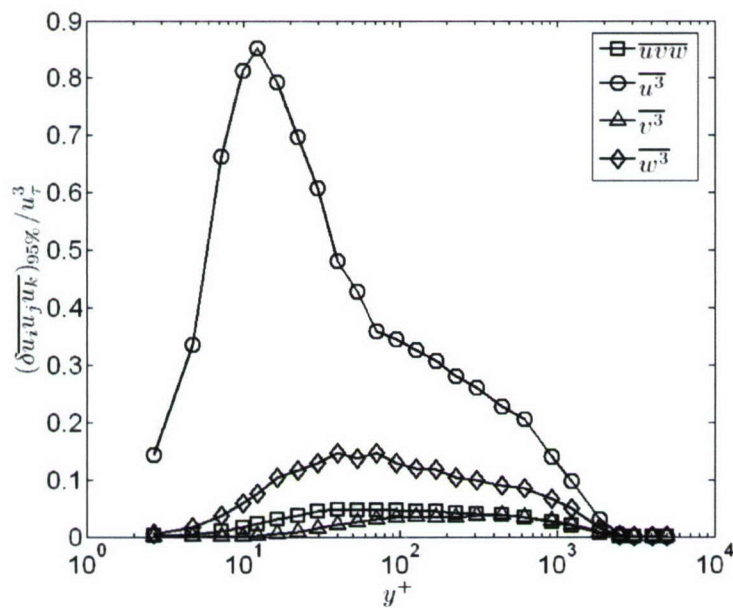


Figure 2.14. 20:1 uncertainties in velocity triple-products throughout a 2D flat plate turbulent boundary layer profile non-dimensionalized on the wall friction velocity.



### 2.3.2.1.1.2 Velocity Gradient Estimation and Uncertainties

The problem of estimating velocity gradients from CompLDV3 data is posed as follows:

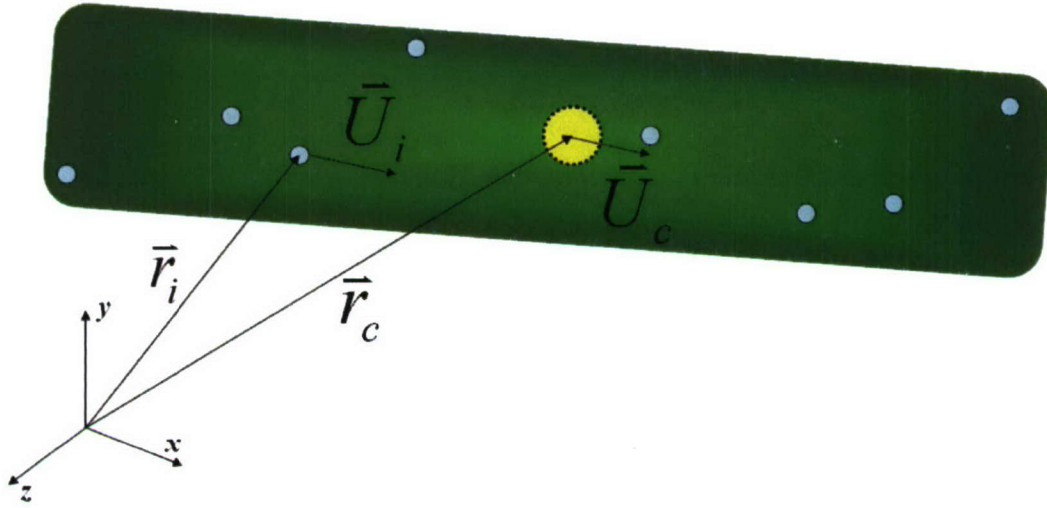
*Given the velocities and relative positions of  $N$  particles ( $N \geq 4$ ), determine the velocity gradient tensor that is consistent with the data within experimental uncertainties and the constraints imposed by coherent turbulence scales.*

The geometry of the problem is shown schematically in Figure 2.15. The particles arrive randomly in space and time. The statistics of the arrivals depend on the turbulence level as well as the velocity gradients across the volume (Albrecht et al. 2004). In the case of zero turbulence and velocity gradient, the arrival time statistics follow a Poisson (exponential) distribution while the particles are uniformly distributed in space. The probability distribution function (PDF) of the measurement-volume validation weights the arrival position so that the measured positions are not uniformly distributed. The extrapolated volume is aligned approximately with the mean flow direction in the case that turbulent flow angles are relatively small.

For a typical situation, consider a region of fluid convecting with some time- and space-local centroid velocity in the 2D flat-plate TBL at  $y^+ = 100$ . In this case, the mean-stream-wise velocity,  $U^+ \approx 16.25$ , and the mean secondary flow velocities are very small. At this height in the TBL of interest, the Reynolds normal stresses are given approximately as  $\overline{u^2}^+ \approx 5$ ,  $\overline{v^2}^+ \approx 1$ ,  $\overline{w^2}^+ \approx 3$ . Now consider the case when the region of flow moves with a centroid velocity such that each of the velocity components has a fluctuation equal to one standard deviation from the mean. In this case the centroid velocity vector is  $\bar{U}^+ = 18.5\hat{i} + 1.0\hat{j} + 1.7\hat{k}$ . The location of the leading edge of the region of fluid after it has convected over a time of  $t^+ = 10$  is a distance of  $x^+ = 185$ ,  $y^+ = 10$ ,  $z^+ = 17$ . Now superimpose the nominally spherical measurement volume with the region of convecting fluid. Some sub-region of the fluid volume will convect through this superimposed volume and the particles seeding the flow will result in scattered light signals measured by the CompLDV3. Over the time  $t^+ = 10$  several particles will pass through the measurement volume resulting in velocity and position measurements. The particle paths are extrapolated to form the description of the fluid motion that is shown schematically in Figure 2.15. Noting the scales of the problem, the length of the region of fluid for which measurements are made will be about  $x^+ = 180$ . However, since the time-scales are so small, it is not expected that significant fluctuations in the secondary velocities will exist such that the region of fluid interrogated will have dimensions in the  $y$ - and  $z$ -directions on the order of the measurement volume diameter. The result is that the region for which

flow velocities are known has a very large aspect ratio:  $AR \equiv \frac{\Delta x^+}{\sqrt{\Delta y^+ \Delta z^+}}$ . This geometry

imbalance the sensitivities of the gradient-measurement directions to the position and velocity uncertainties in the measurements. That is, since small differences are measured in the  $y$ - and  $z$ -directions, the gradients in these directions become more sensitive to the inherent uncertainties in measuring these positions and the velocities. The challenge at hand is, then, to determine an effective way to handle this geometry that results in useful gradient measurements in the  $y$ - and  $z$ -directions.



**Figure 2.15.** Schematic of an instance for the ComplDV3 extrapolated measurement volume containing several particles with estimated velocities and positions.  $\vec{r}_i$  is the position vector of the  $i^{\text{th}}$  particle,  $\vec{U}_i$  is the velocity vector of the  $i^{\text{th}}$  particle,  $\vec{r}_c$  is the position vector of the centroid of the  $N$  particles considered for velocity gradient tensor estimation, and  $\vec{U}_c$  is the centroid velocity vector for the  $N$  particles.

#### 2.3.2.1.1.2.1 Direct estimation

We begin by considering the case in which four particles are used for direct estimation of the velocity gradient tensor by solution of a fully-constrained system of equations. For the computation, we consider four particles, each with their own set of states as given by the trajectory of the  $i^{\text{th}}$  particle by Taylor expansion:

$$\vec{x}_i(t) = \vec{x}_{0i} + \vec{U}_{0i}(t - t_{0i}) + \frac{\vec{a}_{0i}}{2}(t - t_{0i})^2 + O[(t - t_{0i})^3] \quad (2.10)$$

where  $\vec{x}_i$  is the position of the  $i^{\text{th}}$  particle at time  $t$ ,  $\vec{x}_{0i}$  is the position of the  $i^{\text{th}}$  particle at time  $t_0$ ,  $\vec{U}_{0i}$  is the velocity of the  $i^{\text{th}}$  particle at time  $t_0$ , and  $\vec{a}_{0i}$  is the acceleration of the  $i^{\text{th}}$  particle at time  $t_0$ . The Latin subscripts to follow will indicate the particle in question.

Next, let us consider the expansion of the velocity component  $U$  in a Cartesian coordinate system around point  $\vec{x}_{0i}$  and time  $t_0$ :

$$U_i(t, \vec{x}_i) = U_{0i} + \nabla U_{0i} \cdot (\vec{x}_i - \vec{x}_{0i}) + \frac{\partial U}{\partial t} \Big|_{0i} (t - t_{0i}) + O\left[(t - t_{0i})^2, [(\vec{x}_i - \vec{x}_{0i}) \cdot \hat{i}]^2, [(\vec{x}_i - \vec{x}_{0i}) \cdot \hat{j}]^2, [(\vec{x}_i - \vec{x}_{0i}) \cdot \hat{k}]^2\right] \quad (2.11)$$



The quantity desired from the exercise appears directly in equation (2.11),  $\nabla U_{0i}$ , so dropping the higher-order terms yields

$$\nabla U_{0i} \cdot (\bar{x}_i - \bar{x}_{0i}) = U_i(t, \bar{x}_i) - U_{0i} + \left. \frac{\partial U}{\partial t} \right)_{0i} (t - t_{0i}) \quad (2.12)$$

The first problem that arises is that it is not possible to measure the velocity of the same particle at two instances in time and space. This indicates that both terms,  $U_i(t, \bar{x}_i)$  and  $U_{0i}$ , may not be measured for the same particle. Since, however, all the particles are passing within the same domain and sufficiently close to one-another such that truncation errors are manageable, it is possible to consider a velocity  $U_0$  and acceleration  $\bar{a}_0$  that occurs at a position  $\bar{x}_0$  at time  $t_0$  that is used as a reference for all the expansions. Further, if we assume that the velocity gradient field holds for all the particles that pass within some acceptable duration of time, we can re-write equation (2.11) as

$$U_i(t, \bar{x}_i) = U_0 + \nabla U_0 \cdot (\bar{x}_i - \bar{x}_0) - \left. \frac{\partial U}{\partial t} \right)_{0i} (t - t_{0i}) + HOT \quad (2.13)$$

and likewise equation (2.12) is written

$$\nabla U_0 \cdot (\bar{x}_i - \bar{x}_0) = U_i(t, \bar{x}_i) - U_0 - \left. \frac{\partial U}{\partial t} \right)_{0i} (t - t_{0i}) \quad (2.14)$$

One additional simplification may be made to equation (2.14) by evaluating it at the time  $t_0$  to eliminate the unsteady term:

$$\nabla U_0 \cdot [\bar{x}_i(t_0) - \bar{x}_0] = U_i(t_0, \bar{x}_i) - U_0 \quad (2.15)$$

The next issue with equation (2.15) is that it gives only a single relation for three quantities in this case (nine in the full tensor case considering other velocity components). To remedy this, three values of the particle subscript  $i$  must be used, such that four particles are needed in total. To use the particles as such, we may obtain an estimate of the velocity,  $U_i(t_0, \bar{x}_i)$ , by expanding this velocity in time around the arrival time for that particle:

$$U_i(t_0, \bar{x}_i) = U_{0i}(t_{0i}, \bar{x}_{0i}) + (\bar{a}_{0i} \cdot \hat{i})(t_0 - t_{0i}) + O[(t_0 - t_{0i})^2] \quad (2.16)$$

To clarify the subscripts in equation (2.16), the terms  $t_{0i}$ ,  $\bar{x}_{0i}$ ,  $U_{0i}$ , and  $\bar{a}_{0i}$  are the measured arrival time, position at the arrival time, velocity at the arrival time and acceleration at the arrival time, respectively. Equation (2.15) and (2.16) may next be combined to form:

$$\nabla U_0 \cdot [\bar{x}_i(t_0) - \bar{x}_0] = U_{0i}(t_{0i}, \bar{x}_{0i}) - U_0 + (\bar{a}_{0i} \cdot \hat{i})(t_0 - t_{0i}) \quad (2.17)$$

Finally, equation (2.17) may be written more explicitly by substituting for  $\bar{x}_i(t_0)$  using equation (2.10):

$$\nabla U_0 \cdot \left[ (\bar{x}_{0i} - \bar{x}_0) + \bar{U}_{0i}(t_0 - t_{0i}) + \frac{\bar{a}_{0i}}{2}(t_0 - t_{0i})^2 \right] = U_{0i}(t_{0i}, \bar{x}_{0i}) - U_0 + (\bar{a}_{0i} \cdot \hat{i})(t_0 - t_{0i}) \quad (2.18)$$

Let us now expand the dot product in the left-hand side of (2.17):

$$\begin{aligned} & \frac{\partial U}{\partial x} \left[ \Delta x_i + U_{0i}(t_0 - t_{0i}) + \frac{\bar{a}_{x0i}}{2}(t_0 - t_{0i})^2 \right] + \frac{\partial U}{\partial y} \left[ \Delta y_i + V_{0i}(t_0 - t_{0i}) + \frac{\bar{a}_{y0i}}{2}(t_0 - t_{0i})^2 \right] \\ & + \frac{\partial U}{\partial z} \left[ \Delta z_i + W_{0i}(t_0 - t_{0i}) + \frac{\bar{a}_{z0i}}{2}(t_0 - t_{0i})^2 \right] = \Delta U_i + \bar{a}_{x0i}(t_0 - t_{0i}) \end{aligned} \quad (2.19)$$

We can now obtain a system of three equations for the relationship given in equation (2.15) for four particles. The measurements for the particle with arrival time  $t_0$  are represented in equation (2.15) and to follow without a Latin subscript.

$$\begin{aligned} & \begin{bmatrix} \left[ \Delta x_1 + U_{01}(t_0 - t_{01}) + \frac{\bar{a}_{x01}}{2}(t_0 - t_{01})^2 \right] \\ \left[ \Delta x_2 + U_{02}(t_0 - t_{02}) + \frac{\bar{a}_{x02}}{2}(t_0 - t_{02})^2 \right] \\ \left[ \Delta x_3 + U_{03}(t_0 - t_{03}) + \frac{\bar{a}_{x03}}{2}(t_0 - t_{03})^2 \right] \end{bmatrix} \begin{bmatrix} \left[ \Delta y_1 + V_{01}(t_0 - t_{01}) + \frac{\bar{a}_{y01}}{2}(t_0 - t_{01})^2 \right] \\ \left[ \Delta y_2 + V_{02}(t_0 - t_{02}) + \frac{\bar{a}_{y02}}{2}(t_0 - t_{02})^2 \right] \\ \left[ \Delta y_3 + V_{03}(t_0 - t_{03}) + \frac{\bar{a}_{y03}}{2}(t_0 - t_{03})^2 \right] \end{bmatrix} \begin{bmatrix} \left[ \Delta z_1 + W_{01}(t_0 - t_{01}) + \frac{\bar{a}_{z01}}{2}(t_0 - t_{01})^2 \right] \\ \left[ \Delta z_2 + W_{02}(t_0 - t_{02}) + \frac{\bar{a}_{z02}}{2}(t_0 - t_{02})^2 \right] \\ \left[ \Delta z_3 + W_{03}(t_0 - t_{03}) + \frac{\bar{a}_{z03}}{2}(t_0 - t_{03})^2 \right] \end{bmatrix} \begin{Bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \\ \frac{\partial U}{\partial z} \end{Bmatrix} \\ & = \begin{Bmatrix} \Delta U_1 + \bar{a}_{x01}(t_0 - t_{01}) \\ \Delta U_2 + \bar{a}_{x02}(t_0 - t_{02}) \\ \Delta U_3 + \bar{a}_{x03}(t_0 - t_{03}) \end{Bmatrix} \end{aligned} \quad (2.20)$$

Or to simplify things further, we may drop the acceleration term by assuming first-order particle trajectories to obtain

$$\begin{aligned} & \begin{bmatrix} \Delta x_1 + U_{01}(t_0 - t_{01}) & \Delta y_1 + V_{01}(t_0 - t_{01}) & \Delta z_1 + W_{01}(t_0 - t_{01}) \\ \Delta x_2 + U_{02}(t_0 - t_{02}) & \Delta y_2 + V_{02}(t_0 - t_{02}) & \Delta z_2 + W_{02}(t_0 - t_{02}) \\ \Delta x_3 + U_{03}(t_0 - t_{03}) & \Delta y_3 + V_{03}(t_0 - t_{03}) & \Delta z_3 + W_{03}(t_0 - t_{03}) \end{bmatrix} \begin{Bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \\ \frac{\partial U}{\partial z} \end{Bmatrix} \\ & = \begin{Bmatrix} \Delta U_1 \\ \Delta U_2 \\ \Delta U_3 \end{Bmatrix} \end{aligned} \quad (2.21)$$

Finally, equation (2.20) or (2.21) may be solved for the velocity gradients by obtaining the inverse of the matrix on the left-hand side and doing a left-multiply of this inverse with both sides of equation (2.21).

### 2.3.2.1.1.2.1.1 Quantification of performance of the direct estimation procedure

The first step to finding the uncertainties that will result for equation (2.21) should be to validate that given ideal inputs, the system of equations will result in a stable solution when there are no uncertainties for the velocities and positions. To do this, a MATLAB simulation was written with inputs for the velocity components, mean velocity gradients, and range of y- and z-positions possible. For the simulation, a representative set of data for the 2D flat plate turbulent boundary layer was input at  $y^+ = 100$  and  $Re_\theta = 7300$  in the boundary layer research wind tunnel, as listed in Table 2.3. Particle arrivals were



staggered uniformly at times of  $[0, 25, 50, 75] \mu\text{s}$  for the four particles. The  $y$ - and  $z$ -positions were uniformly sampled within a  $200 \mu\text{m}$  diameter volume. Since no turbulence is input, any resulting variation in the velocity gradients is determined to be in error. For this simulation, a very small amount of variance in the velocity gradients was observed—on the order of the machine precision. Also, with 10,000 samples the mean velocity gradient values input were computed within  $13 \times 10^{-12}\%$  of the mean  $y$ -gradient. Some of these values are listed as Case 0 in Table 2.4. The conclusions from this exercise are that the method for estimating the velocity gradients is stable and converges to the proper mean value.

Five cases were run to explore the variance in the velocity gradients that may result from uncertainties in the velocities and particle positions. These cases are numbered 1-5 in Table 2.4. Note once again that there is no turbulence input into the simulations, all variances arise from the Gaussian uncertainties in the velocity and positions from 10,000 simulation samples. A few interesting observations are made from these data. First, the  $x$ -direction gradient variance is consistently lower than the  $y$ - and  $z$ -positions. This is an inherent property of the CompLDV3 operation geometry since the method is based upon extrapolation of the particle paths before or after the particle velocities and positions are measured while crossing the  $200 \mu\text{m}$  diameter measurement volume. The resulting extrapolated measurement volume has a typical aspect ratio of about 10 (length in  $x$ -direction divided by typical width along  $y$ - or  $z$ -directions). Such an aspect ratio is reasonable compared with, although greater than, the coherent structure length-scale ratios in the near-wall turbulent boundary layer (Simpson 1989; Jeong et al. 1997), but strongly affects the gradient estimation variance. The effect of this aspect ratio on the uncertainties for the final result is evident in a couple of ways. First, since the linear system of equations (2.21) is fully coupled among the position-direction components, the imbalance in these components results in potential for great uncertainties. A metric for this is the matrix inverse condition number that may be defined by

$$1/\kappa \equiv \left( \frac{\|A\|}{\|A^{-1}\|} \right)^{-1} \quad (2.22)$$

where  $1/\kappa$  is the metric, and the matrix  $A$  is the position matrix on the far left-hand-side of equations (2.21) for the current application. This metric is unity for a well-conditioned matrix which may be confidently inverted, and becomes decreasingly small for ill-conditioned matrices (Golub and Van Loan 1996). The average matrix inverse condition number for the simulation run with no uncertainties (Case 0 in Table 2.4) was 0.03, much less than unity. Even in the case of a structured grid, it is clear that gradients taken over smaller distances will require more resolution in the constituent values of velocity and positions used to compute these to achieve the same levels of uncertainties as those gradients taken over larger differences. Thus, one may expect to see greater uncertainties in a similar aspect ratio volume even for a structured grid in the span-wise and normal-to-wall directions compared with the stream-wise direction.

From the results for Cases 1-5, we see variances arising fully from random, incoherent Gaussian-distributed uncertainties. Larger variances in the span-wise and normal-to-wall directions compared to the stream-wise direction are consistently observed. For each case, the ratios of the span-wise or normal-to-wall direction gradient variance to the



stream-wise gradient variance are about 120-150. To make the magnitudes of these values clearer, they are compared to the 2D flat-plate turbulent boundary layer results of Honkan and Andreopoulos (1997) in viscous wall scaling. To note is that the variance for the  $x$ -direction derivative from the simulation is much smaller than the measurement variance in the 2D turbulent boundary layer indicating that this derivative may be confidently measured with the CompLDV3. In contrast, for the most realistic case for the uncertainties in the system as determined by the simulations discussed in previous sections, Case 5, the  $y$ - and  $z$ -direction derivative variances are on the order of the measurement value for both cases. This indicates that the measurement uncertainties are perhaps too great to directly estimate the velocity gradients with confidence using this technique with no corrections.

The propagation of the measurement uncertainties through to the velocity gradients is further-examined by looking at the histograms of the velocity gradients computed in Case 5. For comparison, the histograms for the simulated velocity gradient measurements for the  $x$ ,  $y$ , and  $z$ -direction gradients normalized by the measured standard deviations are plotted together in Figure 2.16. One may immediately note the similarity in these distributions. They each have high kurtosis,

$$\kappa = \frac{\overline{u^4}}{(\overline{u^2})^2} \quad (2.23)$$

with values of about 23, 22, and 16 for the stream-wise, normal-to-wall, and span-wise-direction velocity gradients respectively, indicating that the distributions are ‘peaky’ compared with the Gaussian case with a value of 3. For such distributions, the ‘tails’ of the histograms persist at higher probabilities than the comparable Gaussian distribution with the same standard deviation.

An additional simulation was done in which turbulence-like fluctuations of the velocities and velocity gradients were input to be representative of the 2D flat plate TBL at  $y^+ = 100$  and  $Re_\theta = 7500$ . The probability distributions for the turbulence were all taken to be Gaussian with statistics values given in

Table 2.6. The data of Honkan and Andreopoulos (1997) were used as estimates for the velocity gradient variances. The ranges of  $y$ - and  $z$ -positions were based upon experimentally determined ranges for the CompLDV3. Using these ranges, the uncertainties were adjusted to obtain similar results as those of the CompLDV3 measurements are listed in comparison with other benchmark measurements in Table 2.7. Again, the stream-wise direction gradient variance is considerably smaller than the normal-to-wall and span-wise variances, as expected. The estimated variance for the stream-wise gradient is about 1.85 times greater than the value input. However, the normal-to-wall direction gradient was 23.7 times greater than the input value and the span-wise direction gradient was found to be 16 times greater than that input. The three resulting histograms for the  $U$ -gradients are plotted in Figure 2.17. We do see a fair preservation of the Gaussian-distributed input gradients for the  $x$ -direction estimation, with a kurtosis value of  $\kappa = 6.4$ . But the other gradient components considered are similar to those in Figure 2.16 and exhibit large values of kurtosis of about  $\kappa = 15$ .



**Case 6 in Table 2.4 and**

Table 2.5 is the analogous case to the turbulence-like statistics simulation with the same input uncertainties and mean velocities and gradients. That case differs from Cases 1-5 in that the  $y$ - and  $z$ -positions are sampled with Gaussian statistics with a standard deviation of  $50\mu\text{m}$  as determined from empirical histograms obtained with the CompLDV3 of the  $y$ - and  $z$ -position differences among particles. The statistics are comparable to the case with turbulence-like input statistics, except the  $x$ -direction gradient exhibits a large kurtosis of  $\kappa = 23$ , which is comparable with the other gradient directions.

It is clear from table 5, that the direct estimation procedure for the velocity-gradient measurements is not sufficient for reliable measurements of the variances of velocity-gradient terms without the development of large corrections. This has led to the development of an alternative approach to estimating the velocity gradients described to follow.

**Table 2.3. Parameters for the velocity gradient uncertainty simulations at  $\text{Re}_\theta = 7500$  and  $y^+ = 100$ .**

$\text{Re}_\theta$	7500
$y^+$	100
$u_\tau$	1 m/s
$\frac{\partial U}{\partial y}$	$1478 \text{ s}^{-1}$
$\bar{U}$	16.25 m/s

**Table 2.4. Velocity gradient uncertainty simulation cases and results.**

Case	$\delta x$	$\delta y$	$\delta z$	$\delta U$	$\left(\frac{\partial U}{\partial y}\right)_{\text{Computed}}$	$\left[\left(\frac{\partial u}{\partial x}\right)^2\right]_{\text{Computed}}$	$\left[\left(\frac{\partial u}{\partial y}\right)^2\right]_{\text{Computed}}$	$\left[\left(\frac{\partial u}{\partial z}\right)^2\right]_{\text{Computed}}$
0	0	0	0	0	$1478\text{s}^{-1}$	$7.23\text{E} - 23\text{s}^{-2}$	$5.0\text{E} - 20\text{s}^{-2}$	$7.1\text{E} - 21\text{s}^{-2}$
1	0	$1\mu\text{m}$	$1\mu\text{m}$	0	$1477\text{s}^{-1}$	$62\text{s}^{-2}$	$8.9\text{E}3\text{s}^{-2}$	$7.3\text{E}3\text{s}^{-2}$
2	0	0	0	0.5%	$1470\text{s}^{-1}$	$2.1\text{E}5\text{s}^{-2}$	$2.7\text{E}7\text{s}^{-2}$	$2.4\text{E}7\text{s}^{-2}$
3	0	$30\mu\text{m}$	$30\mu\text{m}$	0	$1211\text{s}^{-1}$	$5\text{E}4\text{s}^{-2}$	$4.9\text{E}6\text{s}^{-2}$	$4.7\text{E}6\text{s}^{-2}$
4	0	0	0	0.2%	$1482\text{s}^{-1}$	$3.5\text{E}4\text{s}^{-2}$	$4.3\text{E}6\text{s}^{-2}$	$4.1\text{E}6\text{s}^{-2}$
5	$4\mu\text{m}$	$6\mu\text{m}$	$6\mu\text{m}$	0.2%	$1430\text{s}^{-1}$	$3.6\text{E}4\text{s}^{-2}$	$4.5\text{E}6\text{s}^{-2}$	$3.8\text{E}6\text{s}^{-2}$
6	$4\mu\text{m}$	$10\mu\text{m}$	$10\mu\text{m}$	0.38%	$664\text{s}^{-1}$	$2.1\text{E}5\text{s}^{-2}$	$3.1\text{E}8\text{s}^{-2}$	$2.6\text{E}8\text{s}^{-2}$

**Table 2.5. Velocity gradient uncertainty simulation results in viscous wall scaling.**

Case	$\left[ \overline{\left( \frac{\partial u}{\partial x} \right)^2} \right]_{\text{Computed}}$	$\left[ \overline{\left( \frac{\partial u}{\partial y} \right)^2} \right]_{\text{Computed}}$	$\left[ \overline{\left( \frac{\partial u}{\partial z} \right)^2} \right]_{\text{Computed}}$
1	1.7E-8	2.4E-6	2.0E-6
2	5.7E-5	7.3E-3	6.5E-3
3	1.3E-5	1.3E-3	1.3E-3
4	9.5E-6	1.2E-3	1.1E-3
5	9.8E-6	1.2E-3	1.0E-3
6	5.7E-5	8.4E-2	7.1E-2
Typical real flow at $y^+=100$ (Honkan and Andreopoulos 1997)	1.3E-3	3.8E-3	5.6E-3

**Table 2.6. Velocity gradient simulation with statistical variations given by the expected results in the 2D flat-plate TBL at  $Re_\theta = 7500$  and  $y^+=100$ .**

Input variables	Input values
$U$	16.25 m/s
$\overline{u^2}$	$5m^2 / s^2$
$\overline{v^2}$	$1.3m^2 / s^2$
$\overline{w^2}$	$2.5m^2 / s^2$
$\frac{\partial U}{\partial y}$	$1478s^{-1}$
$\overline{\left( \frac{\partial u}{\partial x} \right)^2}$	0.0013
$\overline{\left( \frac{\partial u}{\partial y} \right)^2}$	0.0038
$\overline{\left( \frac{\partial u}{\partial z} \right)^2}$	0.0056
$\delta x$	$4\mu m$
$\delta y$	$10\mu m$
$\delta z$	$10\mu m$
$\delta U$	0.38%



Standard deviation of probability distribution of y- and z-positions (a statement of measurement volume size)	50 $\mu\text{m}$
<b>Output variables</b>	<b>Output values</b>
$\overline{\left(\frac{\partial u}{\partial x}\right)^2}^+$	0.0024
$\overline{\left(\frac{\partial u}{\partial y}\right)^2}^+$	0.09
$\overline{\left(\frac{\partial u}{\partial z}\right)^2}^+$	0.08

**Table 2.7. Comparison of velocity gradient measurements at  $y^+=100$  for three bodies of work in the 2D flat plate turbulent boundary layer.**

Lowe 2006	Klebanoff 1955	Honkan and Andreopoulos 1997
$\overline{\left(\frac{\partial u}{\partial x}\right)^2}^+ = 0.0013$	$\overline{\left(\frac{\partial u}{\partial x}\right)^2}^+ = 0.0013$	$\overline{\left(\frac{\partial u}{\partial x}\right)^2}^+ = 0.0013$
$\overline{\left(\frac{\partial u}{\partial y}\right)^2}^+ = 0.0955$	$\overline{\left(\frac{\partial u}{\partial y}\right)^2}^+ = 0.0021$	$\overline{\left(\frac{\partial u}{\partial y}\right)^2}^+ = 0.0038$
$\overline{\left(\frac{\partial u}{\partial z}\right)^2}^+ = 0.0955$	$\overline{\left(\frac{\partial u}{\partial z}\right)^2}^+ = 0.0021$	$\overline{\left(\frac{\partial u}{\partial z}\right)^2}^+ = 0.0056$
$\overline{\left(\frac{\partial v}{\partial x}\right)^2}^+ = 0.0012$	$\overline{\left(\frac{\partial v}{\partial x}\right)^2}^+ = 0.00062$	$\overline{\left(\frac{\partial v}{\partial x}\right)^2}^+ = 0.0023$
$\overline{\left(\frac{\partial v}{\partial y}\right)^2}^+ = 0.0907$	$\overline{\left(\frac{\partial w}{\partial x}\right)^2}^+ = 0.001$	$\overline{\left(\frac{\partial v}{\partial y}\right)^2}^+ = 0.0032$
$\overline{\left(\frac{\partial v}{\partial z}\right)^2}^+ = 0.0893$		$\overline{\left(\frac{\partial v}{\partial z}\right)^2}^+ = 0.0055$
$\overline{\left(\frac{\partial w}{\partial x}\right)^2}^+ = 0.0017$		$\overline{\left(\frac{\partial w}{\partial x}\right)^2}^+ = 0.0034$
$\overline{\left(\frac{\partial w}{\partial y}\right)^2}^+ = 0.1310$		$\overline{\left(\frac{\partial w}{\partial y}\right)^2}^+ = 0.0063$
$\overline{\left(\frac{\partial w}{\partial z}\right)^2}^+ = 0.1208$		$\overline{\left(\frac{\partial w}{\partial z}\right)^2}^+ = 0.0064$

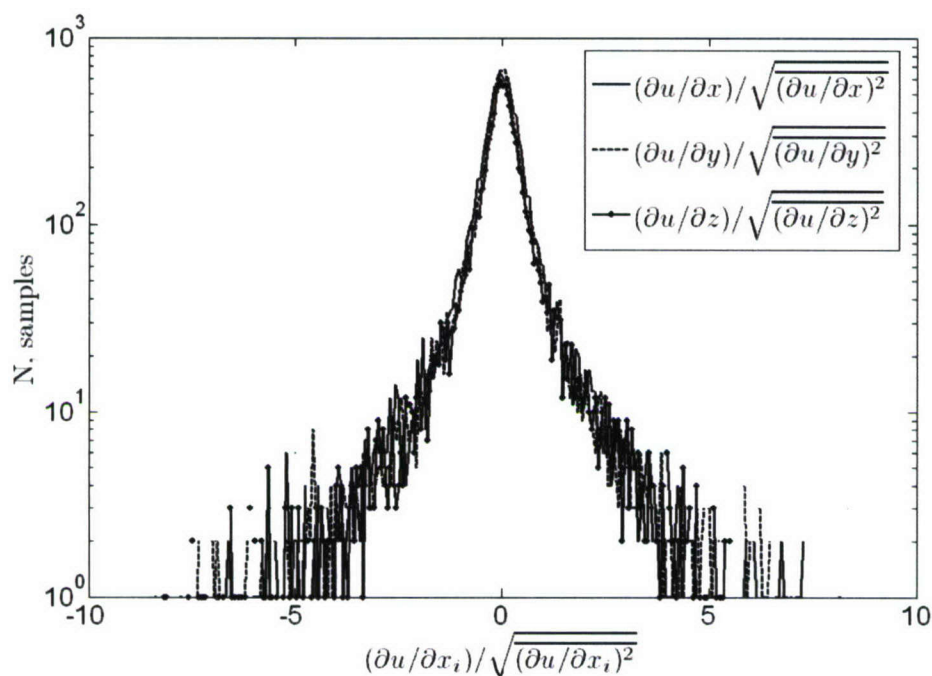


Figure 2.16. Normalized histogram illustrating the similarity of histograms for each of the three gradients considered.

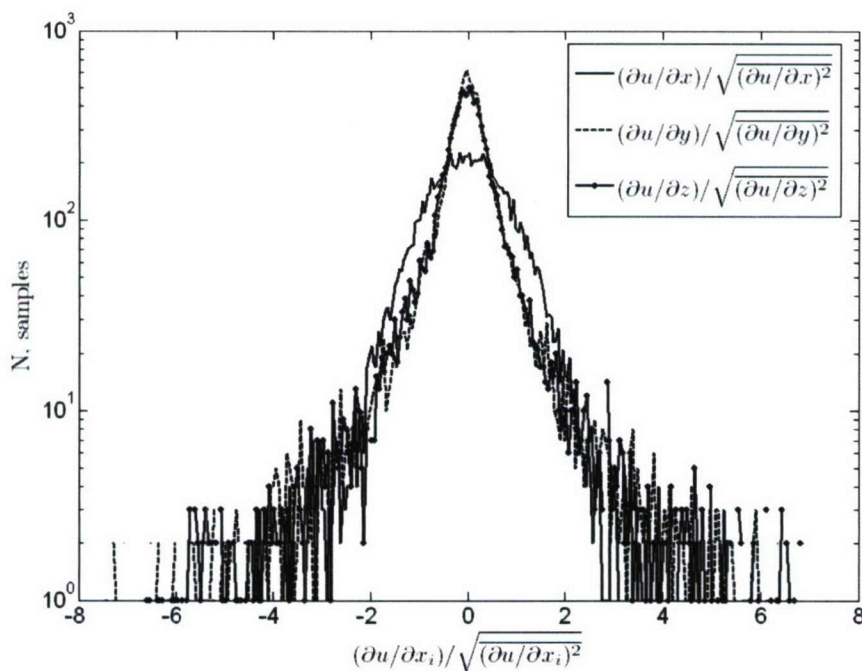


Figure 2.17. Normalized histogram for each of the three gradients considered when a Gaussian distribution of velocity gradients are simulated with variances given by Honkan and Andreopoulos (1997).



## 2.3.2.1.1.2.2 Least-squares estimation of velocity gradients

A second technique for estimating the velocity gradients is developed based upon an over-constrained system for the  $N$ -particles that cross the measurement volume within the allowable time  $t$ . To construct a least-squares cost function, some model for the distribution of the measured quantities must be assumed. In this case, we desired a coherent-structure-based model for the velocity field observed over a short record of time. The model assumed is a swirling structure aligned with the free-stream direction and with a relatively large extent in space for that direction. The near-wall coherent structures in the 2-D TBL such as the quasi-streamwise vortex of Robinson (1991) have been observed to be consistent with this model. Further from the wall, it is still observed that the stream-wise ‘legs’ of ‘crescent-shaped’ structures are really those that contribute to dissipation of the turbulence energy in the Reynolds stresses as the legs stretch and the vorticity within is intensified. The existence of these elongated dissipative structures gives credence to a technique that utilizes a long-narrow region of fluid as the basis for obtaining resolved velocity gradients. A simple structure of the velocity distribution is assumed such that the nine Cartesian-components of the velocity gradient tensor are modeled as constant within the observed region of flow.

To implement the model chosen, we refer again to Figure 2.15. The centroid velocity and position of the  $N$  particles may be readily computed from the data. In the case that the velocity field assumed is exact and the measurements are without uncertainty, then, a velocity gradient field which is consistent with the measurements will result in the following relationship:

$$\begin{aligned} U_i &= U_c + [(\bar{r}_i - \bar{r}_c) \cdot \bar{\nabla}] U_c \\ V_i &= V_c + [(\bar{r}_i - \bar{r}_c) \cdot \bar{\nabla}] V_c \\ W_i &= W_c + [(\bar{r}_i - \bar{r}_c) \cdot \bar{\nabla}] W_c \end{aligned} \quad (2.24)$$

where  $\bar{r}_i$  is the position vector of the  $i^{\text{th}}$  particle,  $\bar{U}_i = U_i \hat{i} + V_i \hat{j} + W_i \hat{k}$  is the velocity vector of the  $i^{\text{th}}$  particle,  $\bar{r}_c$  is the position vector of the centroid of the  $N$  particles considered for velocity gradient tensor estimation, and  $\bar{U}_c = U_c \hat{i} + V_c \hat{j} + W_c \hat{k}$  is the centroid velocity vector for the  $N$  particles. Equations (2.24) may also be thought of as the 3-D Taylor-series expansions for the velocity components truncated for velocity derivatives of order 2 and greater (although one may refine velocity estimates using the CompLDV3 acceleration measurements to enhance the order of the method). In the CompLDV3 measurements, uncertainty exists both for the measurements obtained as well as for the model equations (2.24). To mitigate this, redundant measurements for several particles are used along with equations (2.24) to construct objective cost functions that must be minimized by successive guesses for the velocity-gradient tensor. The cost function chosen is a least-squares error function developed from equation (2.24):

$$\begin{aligned}
\Phi_U &= \sum_{i=1}^N \left\{ U_c + [(\bar{r}_i - \bar{r}_c) \cdot \bar{\nabla}] U_c - U_i \right\}^2 \\
\Phi_V &= \sum_{i=1}^N \left\{ V_c + [(\bar{r}_i - \bar{r}_c) \cdot \bar{\nabla}] V_c - V_i \right\}^2 \\
\Phi_W &= \sum_{i=1}^N \left\{ W_c + [(\bar{r}_i - \bar{r}_c) \cdot \bar{\nabla}] W_c - W_i \right\}^2
\end{aligned} \tag{2.25}$$

where  $\Phi_U, \Phi_V, \Phi_W$  are the cost functions that are minimized to obtain the velocity gradient estimates. Note that in this implementation, the velocity components are decoupled except in the convection that is hidden in the calculation of the position vectors:

$$\bar{r}_i = U_i(t_0 - t_A)\hat{i} + [y_i + V_i(t_0 - t_A)]\hat{j} + [z_i + W_i(t_0 - t_A)]\hat{k} \tag{2.26}$$

where  $t_A$  is the arrival time for the  $i^{\text{th}}$  particle,  $t_0$  is the time at which the centroid information is computed, and  $y_i$  and  $z_i$  are the position-components directly-measured by the CompLDV3. It is taken that the arrival time measurement for the burst occurs when the particle is at the location  $x_i=0$ , which is an excellent assertion considering the overall length of the volume under consideration.

**In a similar manner as with the four-particle technique, the least-squares technique was tested to ensure that the optimization scheme would result in the proper velocity gradient tensor with no uncertainties input. A simulation technique was implemented in which particle inter-arrival times were sampled from the exponential Poisson distribution with a mean particle rate of 20,000 particle arrivals per time unit. The same representative point at  $y^+=100$  in the 2D flat-plate turbulent boundary layer was used for mean velocities and mean velocity gradients as given in Table 2.3, but no turbulence was input. Simulations with 6 and 7 particles used in the least-squares optimization were carried out. The results from these two simulations are listed in Table 2.8. It is reiterated, that no turbulence is entered either within the velocities or the gradients—all deviations from zero-variance are due to uncertainties in the technique. A comparison between these results and those in**

**Table 2.5 for the four-particle method reveals a much-reduced level of variance in the estimated velocity gradients. In particular, the variance for the  $y$ - and  $z$ -direction gradients are reduced by approximately two orders of magnitude when one compares Case 6 in table**

Table 2.5 to either case in Table 2.8 since each of these cases have the same level of input uncertainty in the velocities and positions. Based upon the promising results of this simulation, the method was applied to actual data in the 2D flat-plate TBL to obtain real-flow results discussed to follow in chapter 4.



**Table 2.8. Uncertainty simulations for the least-squares velocity gradient estimation method.**

Case	1	2
<b>Inputs</b>		
$U$	16.25 m/s	16.25 m/s
Number of particles	7	6
$\frac{\overline{\partial U}}{\partial y}$	1478.2 s <sup>-1</sup>	1478.2 s <sup>-1</sup>
$\delta x$	4 $\mu m$	4 $\mu m$
$\delta y, \delta z$	10 $\mu m$	10 $\mu m$
$\delta U, \delta V, \delta W$	0.38% of $\ \bar{U}\ $	0.38% of $\ \bar{U}\ $
Data rate	20 kHz	20 kHz
<b>Outputs</b>		
$\left. \frac{\overline{\partial U}}{\partial y} \right)_{Computed}$	1407.3 s <sup>-1</sup>	1390.2 s <sup>-1</sup>
$\left. \overline{\left( \frac{\partial u}{\partial x} \right)^2} \right)_{Computed}^+$	2E-7	3E-7
$\left. \overline{\left( \frac{\partial u}{\partial y} \right)^2} \right)_{Computed}^+$	1.8E-4	2.7E-4
$\left. \overline{\left( \frac{\partial u}{\partial z} \right)^2} \right)_{Computed}^+$	1.7E-4	2.7E-4

$\kappa \left[ \overline{\left( \frac{\partial u}{\partial x} \right)^2} \right]$	5.6	6.5
$\kappa \left[ \overline{\left( \frac{\partial u}{\partial y} \right)^2} \right]$	4.4	5.7
$\kappa \left[ \overline{\left( \frac{\partial u}{\partial z} \right)^2} \right]$	4.4	4.9

### 2.3.2.2 Calibration of third generation ComplDV

Just as with the ComplDV1, a calibration is necessary to determine the fringe pattern so that the Doppler frequencies may be interpreted properly. Following the discussions of section **Error! Reference source not found.**, the fringe field for each measurement volume has been parameterized by two constants of alignment. The two parameters are the same ones in equation (2.27)—the fringe spacing at the center of the measurement volume ( $d_0$ ) and the normalized fringe gradient,  $A \equiv \frac{1}{d_0} \frac{d[d(x')]}{dx'}$ .

#### 2.3.2.2.1 Center fringe spacing calibration

The center fringe spacing may be determined in a simple and robust manner by making laser-Doppler measurements within the inviscid core of the facility being used. Measurements of the mean velocity at this location are made using a Pitot-static probe and an electronic pressure transducer. In a manner as quickly as possible, the ComplDV3 is used for measurements at nearly the same location. The mean Doppler frequencies and the beam geometries may be used to determine the center fringe spacing,  $d_0$ . Note that due to the Gaussian nature of the beams and the fact that a small portion of the longitudinal extent of the measurement volume is ‘viewed’ by the receiving optics, the mean frequencies measured are truly indicative of the center-of-volume fringe spacing for fringe gradients that approximately follow the variations in equation (2.27) (i.e., approximately constant gradients). The issue of alignment of the ComplDV3 volume and the Pitot-static probe is eased by the configuration of the laser beams. To align, the operator wears laser-protection goggles so that the measurement volume region may be viewed and the Pitot-static probe is carefully positioned so that the beams from both transmitting heads are reflected away approximately equally. This achieves a spatial confidence about the order of the width of the Pitot-static probe itself, which is quite sufficient for the only-large-scale variations that are characteristic of an inviscid core.

#### 2.3.2.2.2 Fringe gradient calibration



The calibration of the fringe gradients is more difficult in that the spatial-extent of the measurement volume is very small and only dynamic targets are useful for fringe characterization. The wheel method for calibration for the CompLDV1 has been discussed along with its limitations. The width of the wire spanning the measurement volume simply results in too many possibilities for irresolvable systematic errors such that no number of independent calibrations will increase the fidelity of the technique. For this reason, a particle or point-like scatterer is necessary to fully resolve the fringe variations in an unbiased manner.

With the need of a point-like scatterer established, the most-obvious candidate experiment would be a particle-seeded flow. The requirements for the flow must be that it is well-understood, that it have large spatial variations at least one mean velocity component, and that those spatial variations in the velocity are repeatable. The plate and cone viscometer flow was first considered for this application after the relatively successful applications of the device as discussed Lowe (2006). The application of the CompLDV3 to this flow, however, proved difficult, as flare from the cone-surface was problematic and would not allow sufficient range of traverse within the gap so that the flow gradients could be characterized effectively. It is surmised that this problem arose due to the large intersection angles of all the beams for the CompLDV3 that results in beams striking the cone-surface within the receiving line-of-sight at relatively large distances between the cone and the measurement volume. This problem resulted in low signal-to-noise ratios that could only be remedied by increasing the parameter  $y_0$  in equation (2.20). As will be shown in section 2.3.2.3, this results in significant secondary flow in the operation of the plate and cone device and drastically reduces its effectiveness as a calibration tool.

The next ‘well-understood’ flow that is accessible to the Turbulent Boundary Layer Research Group (TBLRG) that contains velocity gradients on the orders needed is the 2D flat-plate turbulent boundary layer on the floor of the Boundary Layer Research Wind Tunnel (BLWT). This flow is attractive in a few ways. First, the BLWT facility is extremely reliable and results in very repeatable measurements from day to day. Second, the CompLDV3 is capable of very near-wall measurements, meaning that it is possible to make measurements in the region of the flow where viscous forces dominate and where lower-Reynolds number direct-numerical simulations (DNS) exist to corroborate CompLDV3 data. Third, extensive measurements exist for this flow in addition to the DNS that indicate that law-of-the-wall and the parameters defining it are well-founded. The most-obvious drawback to this approach is that the flow is turbulent, and with the highest intensities of anywhere in the boundary layer profile occurring just above the wall. This means that extensive amounts of data are needed to truly define the mean velocities at several points within the measurement volume. Nonetheless, the advantages discussed are sufficient to make the method attractive, and applications of this technique show that the flow is effective for these purposes.

The method of calibration is to find the combination of 5 values for the measurement volume fringe gradients that result in the most continuous velocity profile that is also



consistent with the viscous sublayer law-of-the-wall where the velocity gradient approaches the constant value given by the wall shear. A couple of methods may be used to determine the value of the velocity gradient at the wall for the 2D flat-plate TBL. The work of DeGraaff and Eaton (2000) has shown that the wall-friction velocity,  $u_\tau$ , may be determined effectively in this particular flow by a fit of the logarithmic region of the boundary layer profile to the law-of-the-wall in that region:

$$u/u_\tau = \frac{1}{\kappa} \ln \frac{yu_\tau}{\nu} + B \quad (2.27)$$

where  $\kappa$  and  $B$  are constants given by Coles' as  $\kappa = 0.41$  and  $B=5.0$ . Lowe and Simpson (2006) have also reported data that indicates a small difference between this fitted value and the value obtained by a fit to data in the sublayer using the rigorous equation there (Rotta 1962; Tang 2004; Durst et al. 1995)

$$U = \frac{\tau_{wx}}{\mu} y + \frac{1}{2\mu} \left. \frac{\partial P}{\partial x} \right)_{wall} y^2 + C_U y^4 \quad (2.28)$$

where  $U$  is the stream-wise mean velocity,  $\tau_{wx}$  is the stream-wise wall shear stress,  $\mu$  is the dynamic viscosity of the fluid,  $\left. \frac{\partial P}{\partial x} \right)_{wall}$  is the wall pressure gradient (zero in this case)

and  $C_U = f \left\{ \frac{\partial^3 (\overline{uv})}{\partial y^3} \right\}$ . This is observed to yield reliable shear stress estimates for  $y^+ < 9$

(Tang 2004). Many researchers in the TBLRG has used this technique successfully in low-speed smooth wall 2-D and 3-D flows (Ölçmen et al., 2001a). Since the fit to the sublayer data is done on the mean velocity, volume-averaged conventional LDV data may be used to obtain the sublayer profile in flows where viscous scales allow for direct fits to equation (2.28).

In the present work, a 2D flat-plate TBL at  $Re_\rho = 5930$  was used for a calibration case. In this case, the volume-averaged statistics for the near-wall region could be used both for a fit to equation (2.28) as well as in the logarithmic region as a fit to equation (2.27). Considering the values of wall shear for both these cases, a value of  $u_\tau \approx 0.78$  well-represents both equations (2.28) and (2.27) for the data. To obtain the velocities and positions for each CompLDV burst acquired, we must solve equations (2.21). To obtain an initial solution for this equation in terms of wind tunnel coordinates, with the  $x$ -axis aligned with the free-stream velocity, equation (2.27) is evaluated with the values for the design parameters used to align beams. Solving the system in equations (2.21) then yields an estimate for the velocities and positions. It is possible to organize the data in bins according to the position estimated and obtain statistics for the velocities measures. The mean stream-wise velocities resolved by the CompLDV3 before calibration in the 2D flat-plate TBL of interest are plotted in Figure 2.18. In this plot, there is clear discontinuity in both the values of the stream-wise mean velocities and their derivatives. The goal of the calibration is then to find a multiplicative constant for the measured-positions that results in a continuous profile and a wall velocity gradient that approaches



the predicted value. Adjustment of the calibration constant yields the calibrated data in Figure 2.19 that is considerably more continuous in both the velocity values and the gradients. Furthermore, the values of the wall velocity gradient approaches the value measured using independent methods.

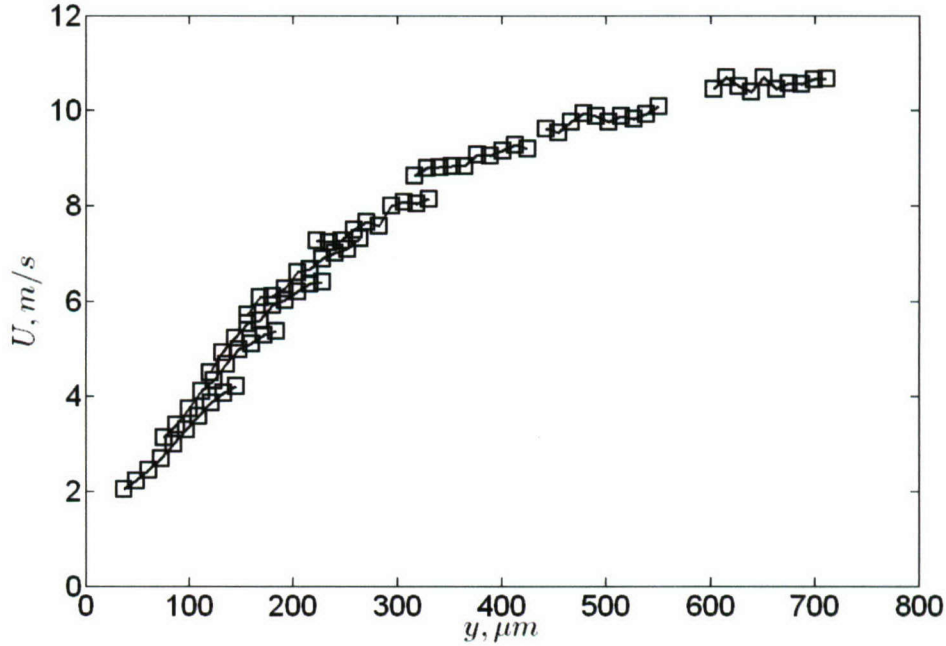
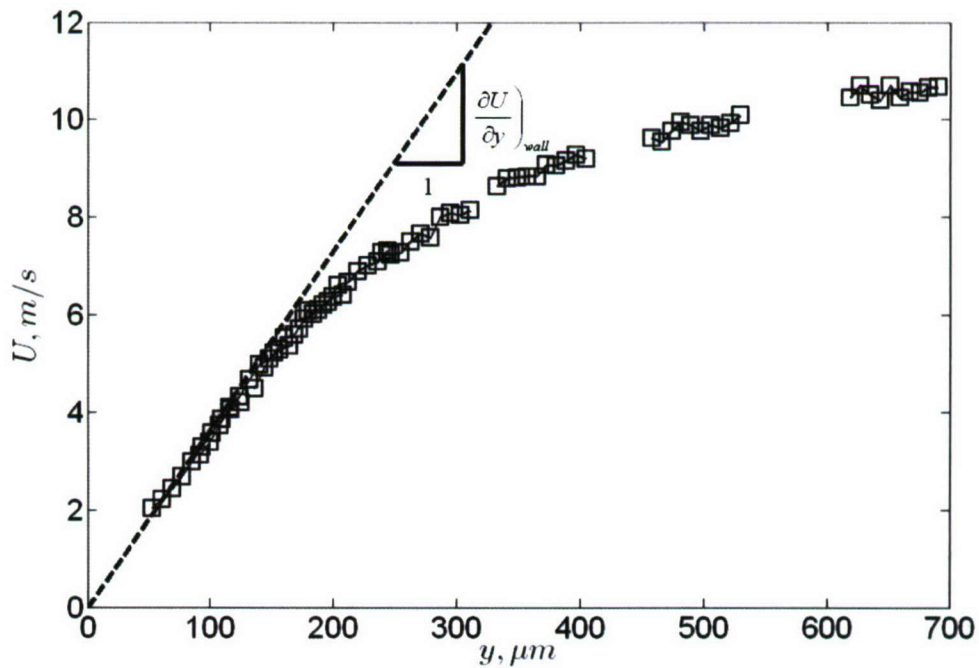


Figure 2.18. Sub-measurement volume resolution measurements in a 2D flat plate turbulent boundary layer with assumed fringe gradients.



**Figure 2.19. Sub-measurement volume resolution measurements in a 2D flat plate turbulent boundary layer with calibrated fringe gradients.**

### 2.3.2.3 Validation of the third generation CompLDV

In a similar manner as the CompLDV1, the plate and cone viscometer flow is used as a validation experiment for the CompLDV3. In applying the CompLDV3 to this flow, considerable interference from the surface of the cone was encountered that limited the region of the flow that could be interrogated. This interference precluded the use of the flow device for calibrations since the region of flow that could be measured was too small to get a high-confidence measurement of the velocity gradient. In order to obtain measurements, the housing for the rotating cone was displaced from the plate such that the cone apex was a considerable distance from the stationary glass plate. The measurements to be discussed show considerable secondary flow that further make the flow measured unsuitable for calibrations but effective as a validation case.

The flow parameters for the profile measured are given in Table 2.9. The angular speed of the cone has been verified in previous experiments already discussed. The distance between the apex and the measurement point was measured using the stepper motors of the  $x$ - $z$  probe traverse by carefully visually aligning the measurement volume on the apex of the cone. The linear-profile azimuthal velocity gradient was determined by a linear fit to the mean volume-averaged velocity statistics plotted in Figure 2.20. From this fit, the gap height is also determined, which indicates that the additional space added between the cone housing and the glass plate was about 1.46mm, which is consistent with the thickness of the two pieces of reinforced double-sided tape used to displace and secure the housing.

The three-orthogonal velocity components determined from the volume-averaged statistics for the CompLDV3 are plotted in Figure 2.21, non-dimensionalized using the gap height and the cone azimuthal velocity. As already mentioned, the extra displacement of the cone apex has caused the significant secondary flow to arise as is evident by the radial mean-velocity profile. One original hope for the current measurements was to quantify the random uncertainty in the position estimation to validate the quality of those measurements. To obtain these estimates, the flow interrogated must be free of turbulence so that there is a one-to-one correspondence between velocity and position. Unfortunately, there appears to be turbulence in the current flow most likely arising from instabilities due to the large gap between the apex of the cone and the plate. The volume-averaged Reynolds stresses corrected for velocity gradient broadening as described by Durst et al. (1995) are plotted in Figure 2.22. Since both the azimuthal and radial mean velocities exhibit significant gradients in this region of the flow, the azimuthal and radial normal stresses were corrected as

$$\overline{\Delta u_i^2} = \frac{d^2}{12} \left( \frac{\partial U_i}{\partial y} \right)^2 \quad (2.29)$$



where  $\overline{\Delta u_i^2}$  is the correction to be subtracted from the normal stress  $\overline{u_i^2}$ ,  $d$  is the effective diameter of the measurement volume (determined as  $100 \mu\text{m}$  in the CompLDV3 volume-averaged data), and  $\frac{\partial U_i}{\partial y}$  is the mean velocity gradient of the in the same velocity component as the normal-stress being corrected. Since both the radial and azimuthal velocities vary considerably, it is also necessary to correct the shear-stress value that involves those velocities. The appropriate correction in that case is

$$\overline{\Delta u_i u_j} = \frac{d^2}{12} \left( \frac{\partial U_i}{\partial y} \right) \left( \frac{\partial U_j}{\partial y} \right) \quad (2.30)$$

At maximum, the velocity gradient broadening-corrected data exhibit a turbulence intensity (defined here as  $\sqrt{\overline{u_\theta^2}} / (U_\theta)_{\text{cone}}^2 \times 100\%$ ) of about 2.5%—high enough to be considered at least transitional.

Despite that this device may not be used effectively to give an estimate for the position uncertainties, it is still useful as a flow with large velocity gradients yet relatively low turbulence intensities. This becomes useful for testing the sub-measurement volume profile-resolution capabilities of the CompLDV3 since volume-averaged data are available. The mean azimuthal velocities resolved by positioning the center of the CompLDV3 volume at five vertical locations are plotted in Figure 2.23. The profile measured is very continuous and exhibits very little scatter among the data points except near the edges of the measurement volume. The mean radial velocities for these five locations are plotted in Figure 2.24, exhibiting that the CompLDV3 has true resolution capabilities in multiple velocity-directions. This is even further demonstrated in Figure 2.25 in which the sub-measurement volume profiles of flow angle are plotted. Since the flow angle is continuously changing, no rotation of the data may found to make the flow purely one-dimensional. It is noted here that the CompLDV3 is truly sensing the rapid change of flow angle within the measurement volume at each point.

**Table 2.9. Parameters for the CompLDV3 measurements in the plate and cone flow.**

$f_{\text{cone}}$	<b>1800RPM</b>
$\omega_{\text{cone}}$	<b>188.5rad/s</b>
<b>Gap height, <math>y_{\text{max}}</math></b>	<b>1.92mm</b>
$r$	<b>26.2mm</b>
$(U_\theta(r))_{\text{cone}}$	<b>4.93m/s</b>
$\partial U_\theta / \partial y$	<b>2565s<sup>-1</sup></b>

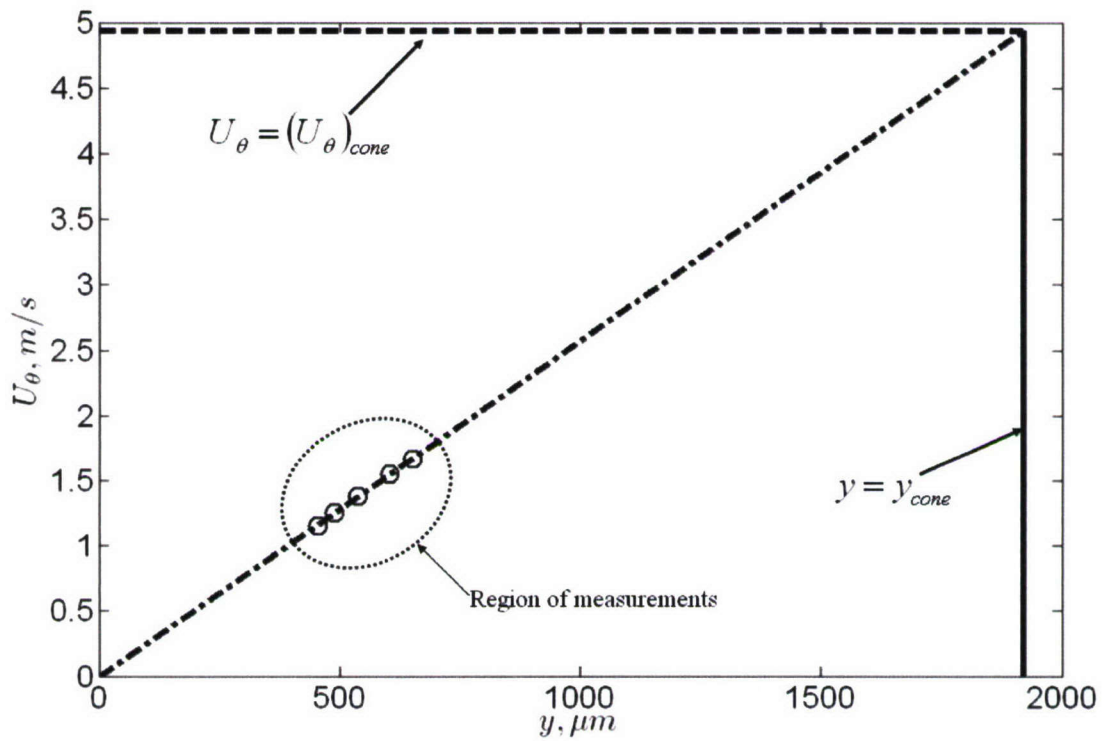


Figure 2.20. Volume-averaged mean azimuthal velocities in the plate and cone flow as measured with the ComplDV3.

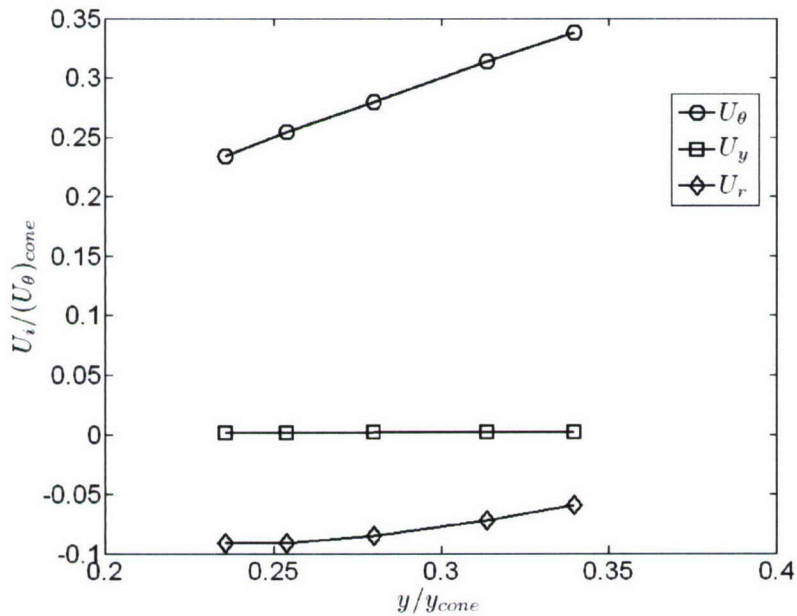


Figure 2.21. Volume-averaged mean velocities normalized by the local cone velocity and measured gap-height for the plate and cone flow with the ComplDV3.



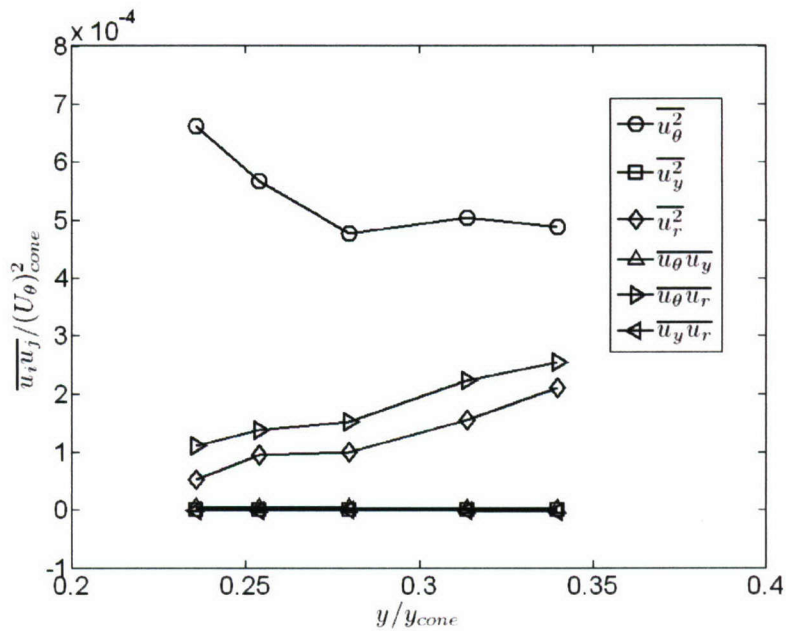


Figure 2.22. Volume-averaged Reynolds stresses normalized by the local cone velocity and measured gap-height for the plate and cone flow with the ComplDV3.

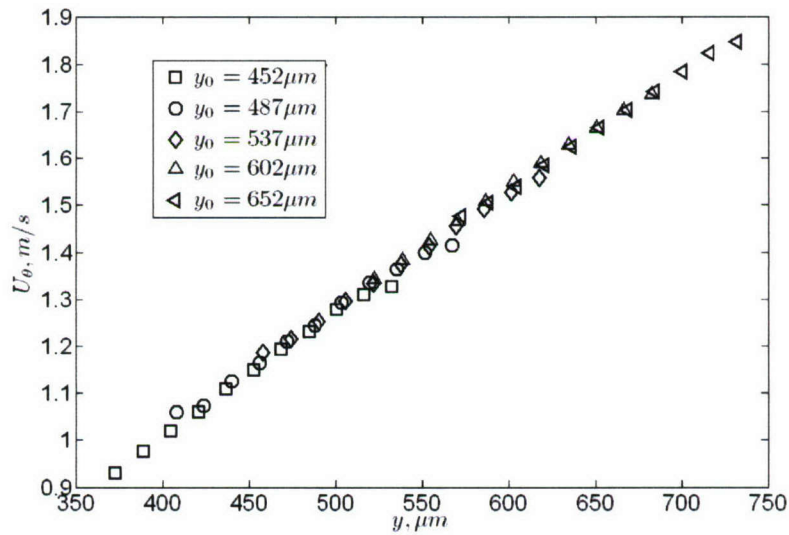
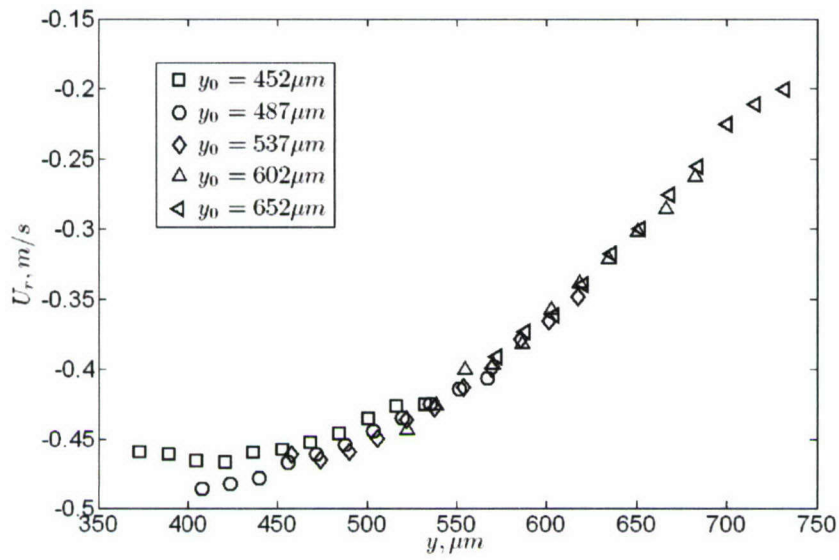
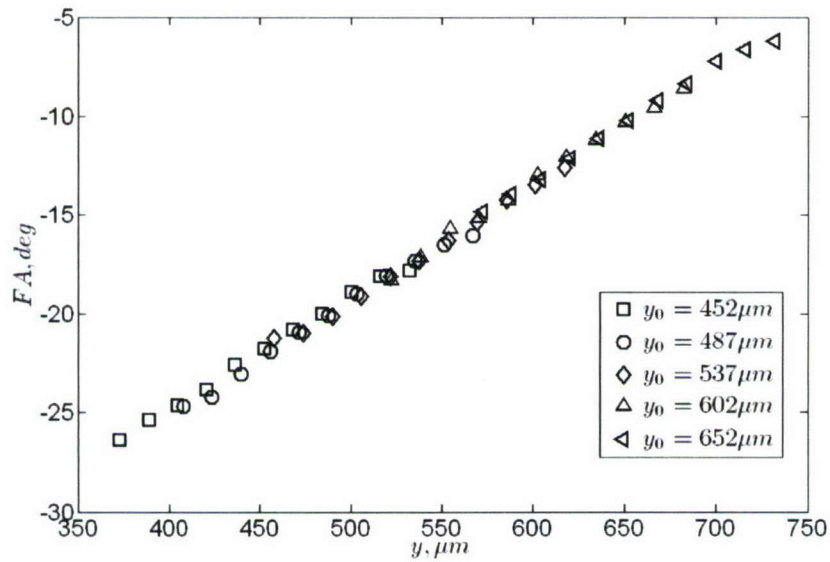


Figure 2.23. Sub-measurement-volume resolved mean azimuthal velocity measurements in the plate and cone flow using the ComplDV3.



**Figure 2.24.** Sub-measurement-volume resolved mean radial velocity measurements in the plate and cone flow using the ComplDV3.



**Figure 2.25.** Sub-measurement-volume resolved mean flow-angle measurements in the plate and cone flow using the ComplDV3.



## 2.4 Facilities and apparatus

### 2.4.1 Measurement facilities

Measurements were acquired in the Department of Aerospace and Ocean Engineering Boundary Layer Research Wind Tunnel (BLWT) and Small Boundary Layer Wind Tunnel (SBLWT). These facilities have been described by previous authors (Devenport and Simpson 1990, Ölçmen 1995, Simpson 2001, and Bennington 2003). Other information about the BLWT facility is available online at <http://www.aoe.vt.edu/research/facilities/bllab.php>. For most measurements, both tunnels were operated at 27.5m/s nominal freestream velocity. The upper walls in both tunnels were set to achieve a zero-pressure gradient flow in the vicinity of the measurement region. Once this pressure gradient was set for the smooth, bare tunnel configuration, it remained in that setting while test elements (isolated roughness elements or wing-body junction) were placed in the flow. In the case of the Reynolds number study for the smooth wall, constant pressure boundary layer, the streamwise pressure gradient was eliminated for each Reynolds number.

### 2.4.2 Flow Seeding

In order to make optical measurements in flows, particles must be introduced to serve as light scattering sites. An optimal particle scatters a large amount of light while following the flow at all the significant fluctuation frequencies. In the case of translucent particles, the theories of Rayleigh and Mie Scattering can be used to predict the amount of light scattered by a spherical particle given the wavelength of light, the diameter of the particle, and the ratio of indices of refraction from flow medium to the particle medium (Albrecht et al. 2003). As discussed in section 2.2.1, the degree to which a particle follows the flow may be estimated using a first-order lag model of equation (2.1) as discussed by Lehman, Nobach, and Tropea (2002). For the liquid DOP particles that exhibit approximately  $0.6 \mu\text{m}$  mean diameters and very little deviation from this size, we compute a time constant of  $1 \mu\text{s}$  indicating that Lagrangian fluctuations out to 160kHz are attenuated by no more than 3dB.

More sophisticated treatments of particle dynamics in two-phase flows have been considered by several authors as reviewed by McLaughlin (1994). His discussion includes the effects of particle rotation, which is driven by vorticity in the flow. For the flows studied herein, the vorticity is greatest near the wall, but as McLaughlin notes from the previous work of Goldman et al. (1967a, b), the presence of the wall reduces the particle rotation from the zeroth-order-analysis maximum of one-half the vorticity. To briefly examine the effects of particle rotation, we consider the effect that vorticity may have on the Doppler signals measured using an extreme case of particle angular velocity equals one-half the viscous sublayer vorticity in a flat-plate turbulent boundary layer (i.e., the maximum mean vorticity in a flat-plate turbulent boundary layer). Since the LDV senses the velocity of a particle as a Doppler shift in the laser light being scattered by that particle, the varying translational velocities across a particle with non-zero angular

velocity results in a broadening of the spectral distribution of Doppler frequencies. Although particle scattering theories may be used to give precise contributions of light scattered from the regions of the particle surface, we will consider for the moment that the broadening results from equal contributions from two points on opposite sides of the particle sphere that lie on the axis with the largest velocity gradient. Consider the velocities resulting in scattering to be given as

$$\begin{aligned} U_2 &= U_0 + \frac{\partial U}{\partial y} \frac{d}{2} \\ U_1 &= U_0 - \frac{\partial U}{\partial y} \frac{d}{2} \end{aligned} \quad (2.31)$$

where  $U_{1,2}$  are the velocities at the top (2) and bottom (1) of the particle,  $U_0$  is the velocity at the center and  $d$  is the diameter of the particle. Reverting back to the bounding case in the viscous sublayer of a flat-plate turbulent boundary layer, we may relate all the quantities in equations (2.31) by scaling them using the viscosity and the wall friction velocity (and denoting variables using this scaling with the superscript '+'):

$$\begin{aligned} U_2^+ &= U_0^+ + \frac{d^+}{2} \\ U_1^+ &= U_0^+ - \frac{d^+}{2} \end{aligned} \quad (2.32)$$

or

$$U_2^+ - U_1^+ = d^+ \quad (2.33)$$

Thus the amount of broadening in the velocity measured due to particle rotation is approximated by equation (2.33). Applying the viscous sublayer relationship,  $U^+ = y^+$ , to normalize equation (2.33) by the center velocity of the particle, we obtain

$$\frac{U_2^+ - U_1^+}{U_0^+} = \frac{U_2 - U_1}{U_0} = \frac{d^+}{y^+} = \frac{d}{y} \quad (2.34)$$

which gives the magnitude of the rotational velocities relative to the particle center velocity. For the particle diameters used and considering the current operational limits of the ComplDV, the maximum value of  $\left. \frac{U_2 - U_1}{U_0} \right)_{\max} = \left. \frac{d}{y} \right)_{\max} \approx 0.01$  is obtained for the

nearest-to-wall data. While this value is larger than random velocity uncertainties for the ComplDV3, it is still not significantly lower than these uncertainties and an attempt to estimate flow vorticity based upon Doppler peak broadening may result in high relative uncertainties in the vorticity measured and the estimates would become increasing poorer as the vorticity decreases. In the same way, this simple analysis bounds the possible uncertainties in the measured particle velocity due to particle rotation and exhibits that rotation does not contribute a significant amount uncertainty for all but the most-extreme cases of extremely high vorticity.

A liquid particle aerosol made with DOP has been used as flow seeding for some time by the TBLRG. The aerosol is generated with a Laskin-nozzle-style seeder that is passed through a heat-exchanger that first vaporizes and then condenses the aerosol in a controlled manner. The heat-exchanger unit known as the VAP/CON was devised and



tested by Liu et al. (1966). The original unit in use in this research group was built by Baker (1986). The VAP/CON unit (Figure 2.26) consists of a manifold that distributes the input of polydisperse DOP aerosol evenly into four stainless-steel tubes. Near the entrance of the tubes, heating tapes increase the temperature at the outer surface of the tubes to about 770°F as measured by a simple thermocouple probe. This temperature may be adjusted using Variac A/C voltage regulators. The high temperature in the first section of the tubes raises the temperature of the aerosol above the vapor point of DOP so that a DOP-laden gas results. This mixture is then cooled through the remainder of the stainless steel tubes with heat transfer from the fluid mixture to the tubes and the DOP condenses onto solid nucleation sites naturally present in the supply air. Since all the DOP/air mixture undergoes the same controlled cooling process at a constant flow-rate, the particles that result are highly uniform. The condensed DOP particles in the supply air exit the stainless steel tubes to a second manifold that recombines the streams to a single line that leads to the test facility.

To measure the performance of the VAP/CON unit, Dan Neuhart from the NASA Langley Research Center conducted tests on the particle size statistics using a TSI model 3321 Aerosol Particle Sizer Spectrometer at the facilities used by the TBLRG. While this particle sizer does not detect the presence of particles smaller than 0.3  $\mu\text{m}$  and cannot measure the size of particles smaller than 0.523  $\mu\text{m}$ , the statistics of the larger particles in the aerosol are well-gauged.

Particle distributions are best described by the log-normal distribution where the natural-log of the sample magnitude follows the Gaussian distribution. The primary statistics of interest for these measurements include the geometric mean particle diameter and the geometric standard deviation. The geometric mean is defined as

$$\overline{d}_g = \left( \prod_{i=1}^N d_i \right)^{1/N} \quad (2.35)$$

where  $d_i$  is the particle diameter measurement of the  $i^{\text{th}}$  sample. The geometric standard deviation is likewise given as

$$s_g = \exp \left[ \frac{\sum_{i=1}^N (\ln d_i - \ln \overline{d}_g)^2}{N} \right]^{1/2} \quad (2.36)$$

Several cases were considered for the particle sizing that included modulations of the supply-air pressure to the Laskin nozzle as well as varying the VAP/CON temperature. A sample normalized concentration histogram of particle diameter for the closest-case to the one used for aerodynamic measurements in the present work with a Laskin nozzle supply-air pressure of 16 psi and VAP/CON outer surface temperature of 768°F is plotted in Figure 2.27. The ordinate in this case is normalized such that the varying bin-widths in diameter do not bias the interpretation of the data. For this seeding case, an estimate for the geometric mean particle diameter is 0.61  $\mu\text{m}$  with a geometric standard deviation of 1.12. This may be interpreted to give one-standard deviation particle diameter bounds as

$\frac{\overline{d_g}}{s_g} < d_p < \overline{d_g} s_g$  or  $0.54 \mu\text{m} < d_p < 0.68 \mu\text{m}$ , a very narrowly-distributed range of particle diameters for both light-scattering and aerodynamic considerations. It is noted that the particle statistics do not change significantly with supply-air pressure when the VAP/CON is operated at around 770°F outer surface temperature. To emphasize the utility of the VAP/CON unit, Figure 2.28 is the normalized concentration histogram for the case with 15 psi supply-air pressure for the Laskin nozzle without the VAP/CON unit. Clearly, the complete histogram is even more truncated on the low-end than for Figure 2.27, and the distribution of larger-particles is more-pronounced. In this case the particle distribution statistics reflect the increase of polydisperse particle sizes, with a one-standard deviation range of  $0.57 \mu\text{m} < d_p < 0.93 \mu\text{m}$  about the geometric mean of  $\overline{d_g} = 0.728 \mu\text{m}$ . For all the CompLDV boundary layer tests, the VAP/CON unit was used to improve the signal-to-noise ratio of the signals and give confidence in the fluid dynamic results.

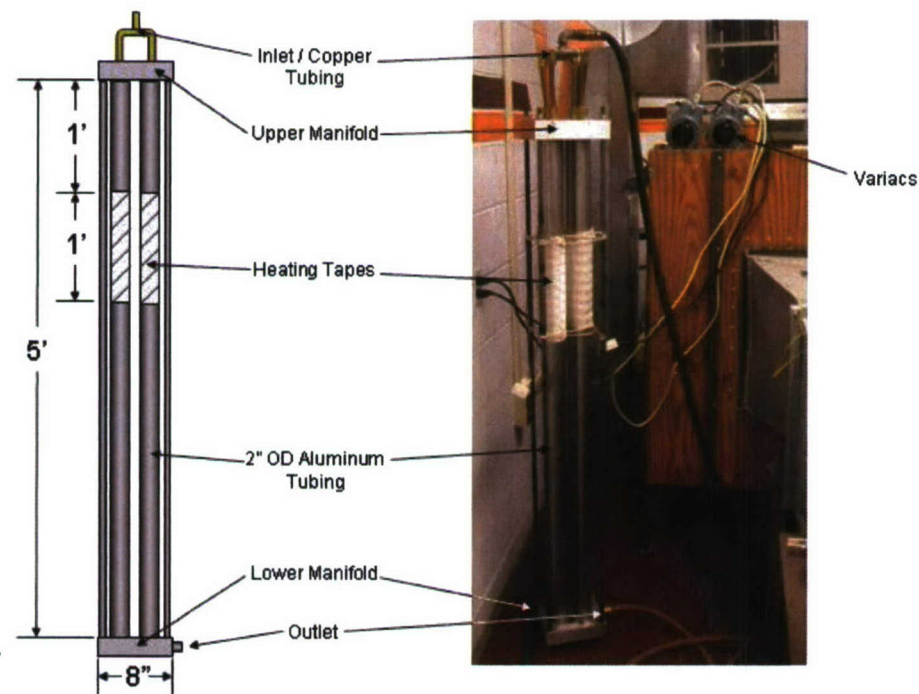


Figure 2.26. The VAP/CON monodisperse aerosol generator (figure due to Devin Stewart).



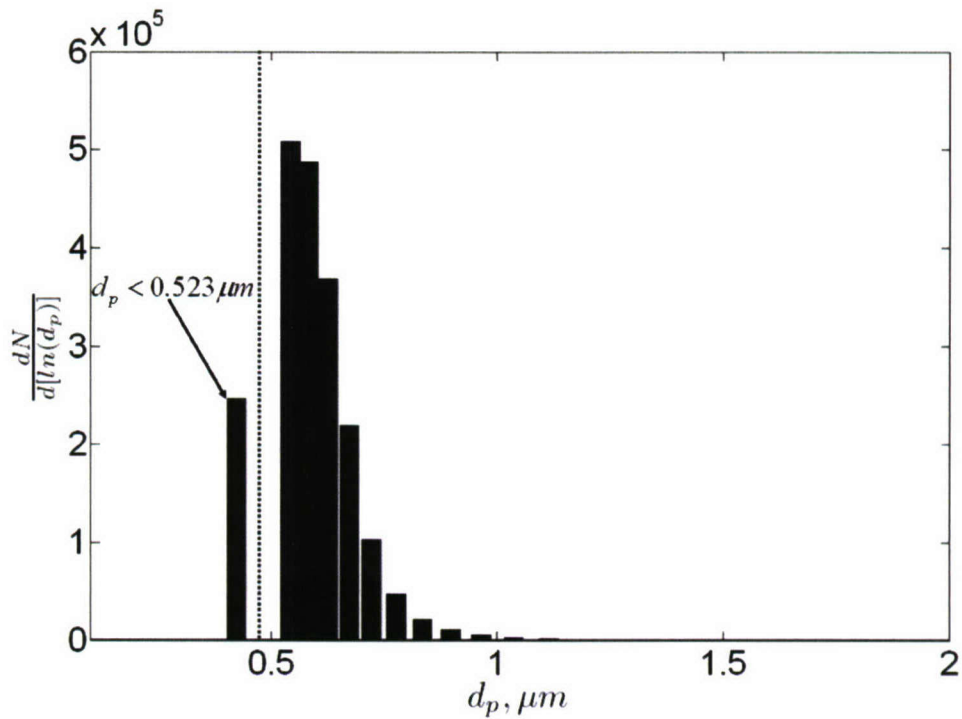


Figure 2.27. Normalized concentration histogram of the DOP particle diameter distribution for a Laskin nozzle supply air pressure of 16 psi and a VAP/CON outer surface temperature of 768°F.

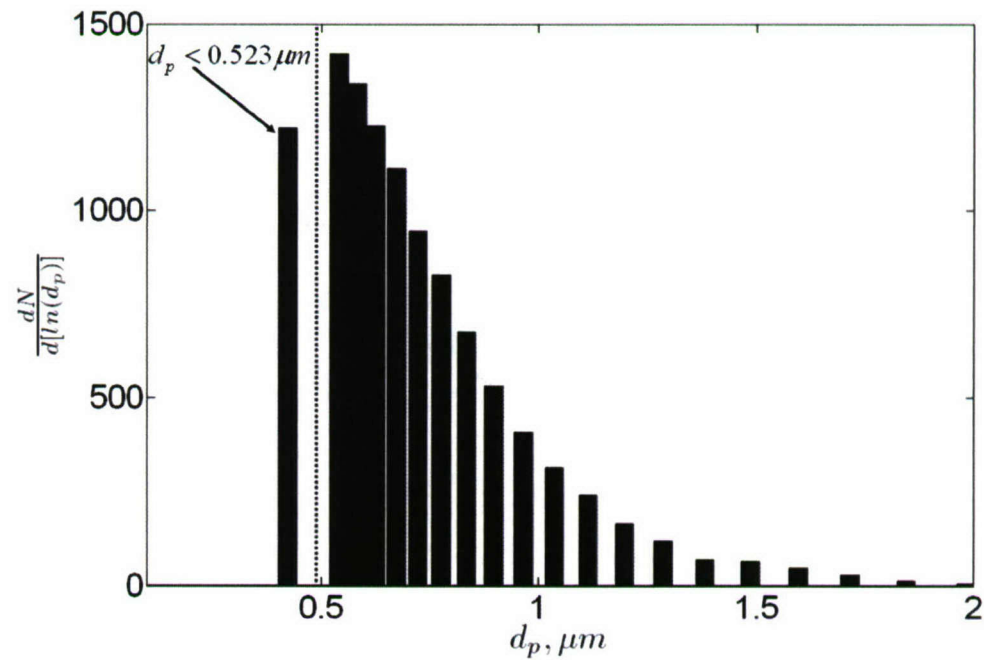


Figure 2.28. Normalized concentration histogram of the DOP particle diameter distribution for a Laskin nozzle supply air pressure of 15 psi without VAP/CON unit.

### 3 Signal processing

#### 3.1 Laser-Doppler Burst Processing

In order to take advantage of newly available digitization and PC-storage capabilities, the laser-Doppler signals which are stored must be analyzed thoroughly and efficiently. The current application is demanding in that signals are needed in rapid succession for gradient estimation and high frequency spectra. To further compound the challenge, we wish in this study to measure particle accelerations which are an order of magnitude more difficult to estimate compared to velocities. Much effort in the study has been in determining the lowest-uncertainty and most robust algorithms for obtaining the frequency, rate-of-change of frequency, and arrival time for each burst of sufficient signal-to-noise ratio (SNR) while making the minimum allowable SNR to be as low as possible so that the maximum number of bursts is accepted.

##### 3.1.1 Signal description

An adequate model for a laser-Doppler burst from a single, small particle is given as

$$s(t) = A \exp\left[-\frac{\beta}{2}(t-t_0)^2\right] \cos[\phi(t)] + n(t) \quad (3.1)$$

where  $A$  is the signal amplitude,  $\beta$  is the Gaussian window parameter analogous to the variance,  $t_0$  is the center-arrival time of the burst,  $\phi(t)$  is the phase of the signal and  $n(t)$  is the noise. For the case when the interference fringes are parallel throughout the measurement volume, the particle velocity is equal to the fringe spacing,  $d$ , times the frequency of the signal described by equation (3.1),  $U = df = \frac{d}{2\pi} \frac{d\phi(t-t_0)}{dt}$ . Likewise,

the particle acceleration is given as  $a = \frac{dU}{dt} = d \frac{df}{dt} = \frac{d}{2\pi} \frac{d^2\phi(t-t_0)}{dt^2}$ . The Taylor series expansion of the velocity of a particle as it passes the measurement volume is

$$U(t) = U_0 + \left. \frac{dU}{dt} \right|_{t=t_0} (t-t_0) + O[(t-t_0)^2] \quad (3.2)$$

For the small transit times, much smaller than the Kolmogorov time scales, the first-order approximation is sufficient. Thus, we need only estimate the velocity and acceleration at the time  $t=t_0$ . We see then that at similar Taylor expansion of the phase evolution of the signal yields dependencies only on the first two-orders,

$$\phi(t) = \phi_0 + 2\pi f_0(t-t_0) + \pi \left. \frac{df}{dt} \right|_{t=t_0} (t-t_0)^2 + O[(t-t_0)^3] \quad (3.3)$$

Thus two parameters for the way the signal changes in addition to the constant phase are necessary to describe the signal. The model signal used in this study is thus simplified from equation (3.1) as



$$s(t) = A \exp\left[-\frac{\beta}{2}(t-t_0)^2\right] \cos\left[\phi_0 + 2\pi f_0(t-t_0) + \pi \frac{df}{dt}\bigg|_{t=t_0} (t-t_0)^2\right] + n(t) \quad (3.4)$$

The time-series for an example signal is given in Figure 3.1.

The noise in a laser-Doppler signal,  $n(t)$ , arises from shot noise inherent in the photomultiplier. This noise is proportional to the square-root of the photocathode current, is broadband and follows Poisson statistics (Albrecht et al. 2003, Coates 1972). This means that the noise level is dependent upon the signal level, which is time dependent. If we consider the *noise-free* instantaneous signal level as follows:

$$\hat{s}(t) \propto A \exp\left[-\frac{\beta}{2}(t-t_0)^2\right] \quad (3.5)$$

Then the noise will be represented as

$$n(t) \propto A \exp\left[-\frac{\beta}{4}(t-t_0)^2\right] \quad (3.6)$$

Thus the signal-to-noise ratio,  $SNR(t) = \hat{s}(t) / n(t)$  is given as

$$SNR(t) \propto \exp\left[-\frac{\beta}{4}(t-t_0)^2\right] \quad (3.7)$$

This is not a new result and has been recognized in the literature (Lehmann et al. 2002). However, many simulation studies of LDV signals are done with constant Gaussian noise which theoretically results in a worst-case, but also results in a bias by over-accentuating the center of the signal. Figure 3.2 illustrates the difference in burst SNR given by the two techniques. There is significant degradation of the SNR for a burst with constant noise at  $|t\sqrt{\beta}| > 1$ , while the same drop in is not seen until about  $|t\sqrt{\beta}| > 2$  for the Gaussian model. Evidence for this model is seen in Figure 3.3, a spectrogram for a real LDV burst which contains two Doppler frequencies is given. The deepest red indicates a region of high power while cooler colors are low power. With the passage of the particle comes an increased amount of broadband noise which is not constant throughout the burst. Note that background light from scattering surfaces and grazing particles do cause a small constant amplitude noise component, but this is insignificant compared with the shot noise within the burst.

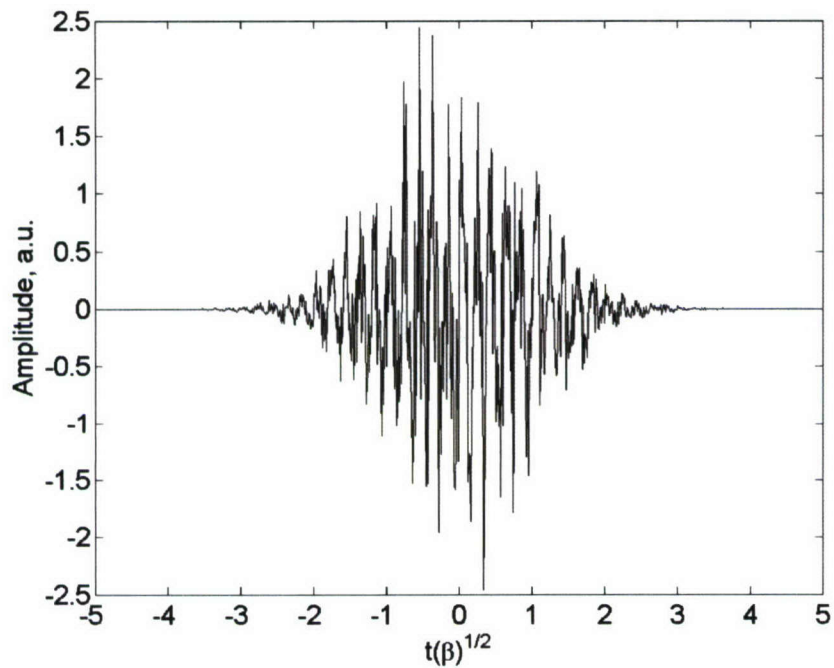


Figure 3.1. Model burst with noise.

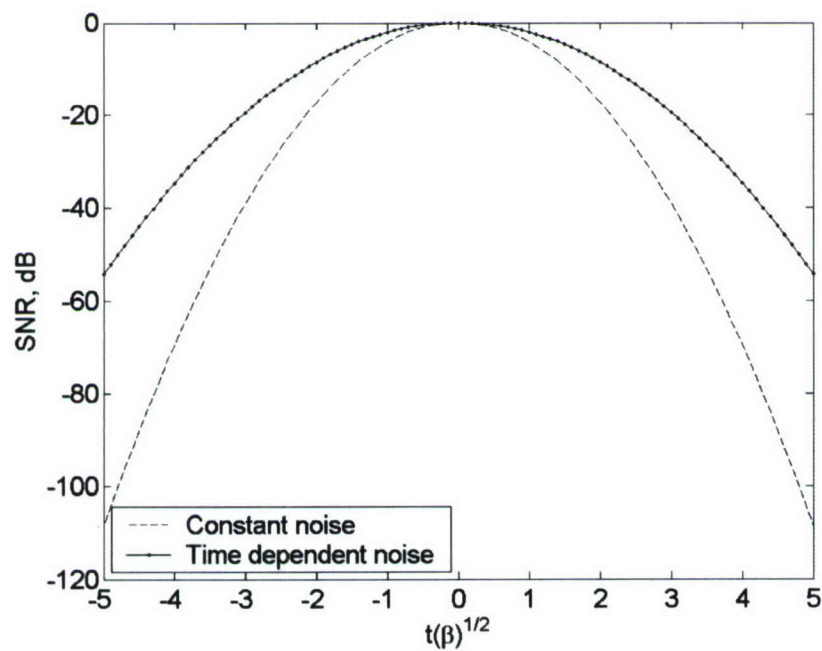
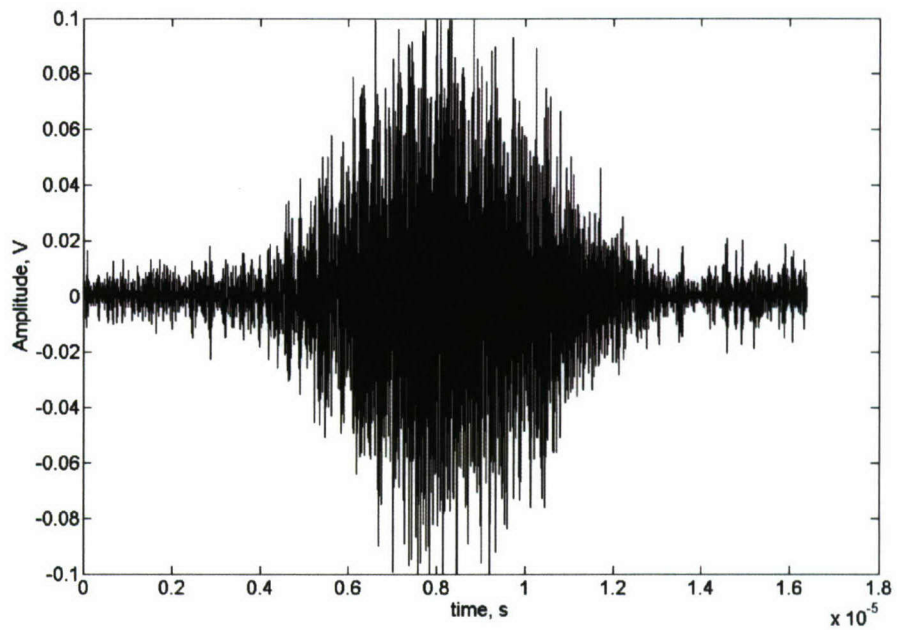
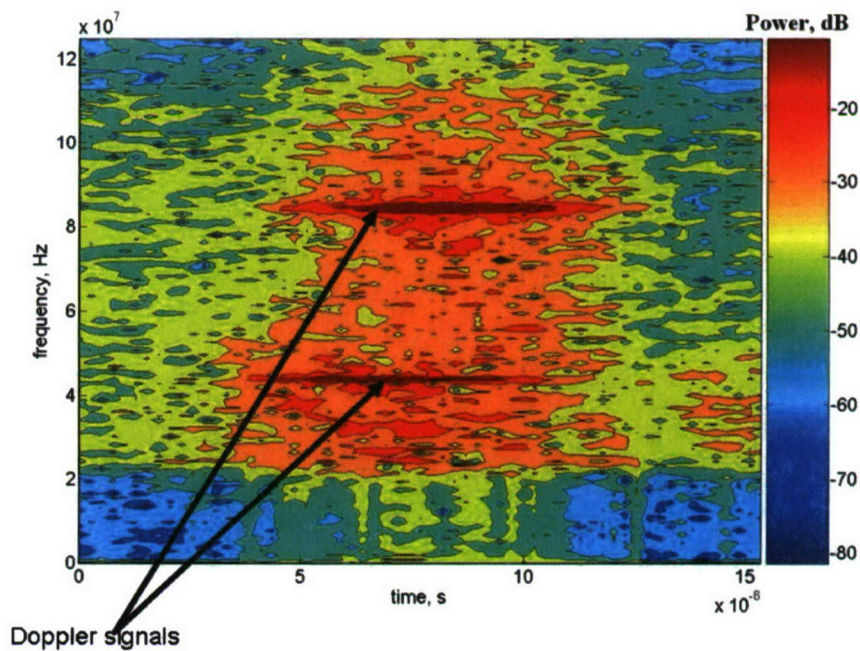


Figure 3.2. Comparison of the SNR for signals with constant noise levels versus noise levels that vary in time as a Gaussian.





(a)



(b)

**Figure 3.3. (a) Time series and (b) spectrogram of an LDV burst containing two Doppler signals. Note that the noise is not constant amplitude throughout the signal but varies with time.**

There are several fundamental challenges for processing laser-Doppler signals, the most significant of which are

- the signals are transient and brief
- the arrival time of the signal is random and unknown *a priori*
- the most probable inter-arrival time is *zero*
- the burst-to-burst SNR is fluctuating and relatively low

These issues are well-discussed throughout the literature and were recently all considered in a single work by Albrecht et al. (2002). The first two bullets underscore the need for an excellent burst recognition algorithm. The third point affects the way we must validate the bursts and indicates the necessity for separating closely spaced bursts for proper processing. The final point is the most difficult to overcome. First, it requires that the signals be validated so that the estimated SNR is likely to result in actual data. Second, the SNR is the limiting factor in the variance of signal parameter estimation due to the Cramer-Rao lower bound (Shinpaugh et al. 1992, Lehmann, Nobach, and Tropea 2002).

In the following sections the methods chosen for handling the abovementioned challenges will be discussed. These include aspects of signal recognition, signal discrimination in the presence of multiple signals, and frequency- and time-frequency domain processing.

### 3.1.2 Burst Recognition and Envelope Estimation

To achieve the lowest uncertainty and highest efficiency frequency-domain or time-frequency-domain processing of non-stationary signals such as LDV bursts, it is essential to have a good estimate of the burst envelope parameters before applying computationally-intensive processing techniques. Knowing a working estimate of the burst envelope parameters allows centering of the burst in the processing window. It also allows one to identify a time-domain cut-off for processing where the burst levels are obscured by noise; thus, creating the optimal case of a matched time-window processor (see Albrecht et al. 2002). Nobach (2002a) noted the power of burst envelope estimations for separation of dual-burst signals which are inevitable in the high-seed concentration cases of the CompLDV. Furthermore, signal characterization techniques such as the FFT or those discussed by Lehmann, Nobach, and Tropea (2002) will perform best when the burst is centered in the processing window. Finally, robust frequency- and time-frequency-domain processing is expensive, so only high-confidence burst signals should be passed to such a processor. The challenge set forth is in determining a robust and computationally inexpensive method for characterizing the burst envelope before detailed processing commences.

The Hilbert transform is often the chosen method for obtaining the window function of burst signals (e.g. Nobach 2002a and Lehmann, Nobach, and Tropea 2002). The magnitude of the so-called analytical signal obtained with the Hilbert transform yields the instantaneous window of the real signal. The problem with this technique is that it requires the same amount of time to compute as the FFT—if one already has the FFT representation of the signal *a priori*. Otherwise, the optimal computation of the Hilbert



transform requires computation of *two* FFTs. Also, the result must be low-pass filtered since instantaneous noise will distort the signal envelope from the analytical signal. It was thought that this method, though robust, was not the most efficient available for the problem.

The proposed solution to the burst recognition and envelope estimation problem involves a digital algorithm for inexpensively determining the burst envelope parameters. The digital algorithm takes advantage of the nearly Gaussian nature of the signal to robustly identify burst signals. By identifying this known pattern in the data, *a priori* knowledge is used to enhance the quality of the measurements. Details of the algorithm are discussed to follow.

### **3.1.2.1 Details of the process**

#### ***3.1.2.1.1 Pedestal Retention***

In the case that large amplitude white noise is persistent on the LDV channels, it may prove more reliable for signal detection to retain the pedestal of the signal. It is commonly taken as practice that LDV signals should be high-pass filtered to remove the pedestal of the signal prior to processing. There are several arguments for doing this, such as the high-passed signal more optimally uses digitizer bits and signal processing is typically carried out on zero-mean signals. The argument for not filtering, however, is that the burst pedestal presents the highest-fidelity method for identifying the occurrence of a particle crossing the measurement volume. This is because the power within that low-frequency signal is many orders of magnitude greater than any noise component within the signal, and usually some factor more powerful than even the Doppler signal itself.

To retain the pedestal of a burst which is Bragg shifted to RF frequencies, one must use an RF amplifier with sufficient low-frequency response. The Sonoma Instruments model 315 amplifier being used has a flat response from 10 kHz to 1 GHz, allowing even relatively long duration burst pedestals to be preserved. The other detail with retaining the pedestal information is involved with the digitizing. To use the most possible digitizer bits, it is necessary to shift the waveform so that the zero-voltage bit is at some negative bit. This is possible to do with the manual settings of the Strategic Test UF.258 digitizer card used. The automatic bit distribution is from -128 to +128 with 0 corresponding to nominally zero-differential voltage. The manual settings allow one to set the zero-differential voltage to approximately bit -80. Then if the burst ranges from, say, 0V to 0.7V, the digitizer may be set with a range of 1V and the waveform will usurp most of the bits available.

#### ***3.1.2.1.2 Digital Detection Algorithm***

As mentioned, the biggest concerns with the detection algorithm were low computational cost with high fidelity of detection. It is sought to use the burst data in a manner which is

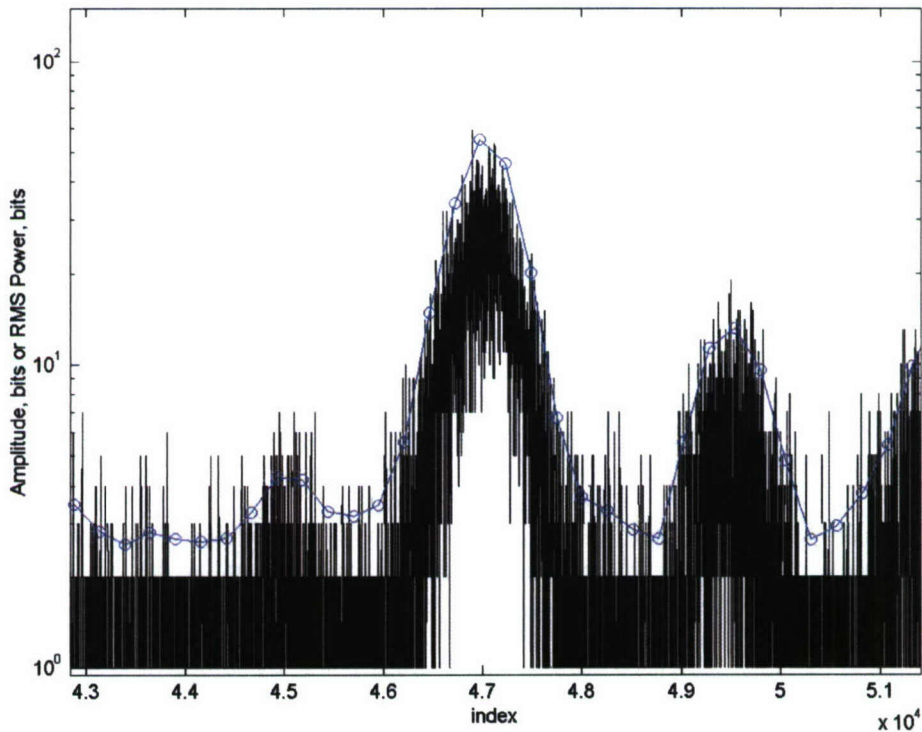
linear computationally. This precludes the use of general frequency-domain techniques like the FFT. Thus, a purely algebraic algorithm is specified, as described to follow.

### 3.1.2.1.2.1 RMS Power Characterization

The short-time RMS of a signal  $s$  is

$$RMS_T = \frac{1}{T} \int_0^T s(t)^2 dt \approx \frac{1}{N} \sum_{i=0}^N s(t_i)^2 \quad (3.8)$$

where  $T$  is the duration of the signal being averaged,  $N= Tf_s$  where  $f_s$  is the sampling rate of the digitizer. It is not a novel idea to detect burst signals based upon the RMS power of the waveform over some time (see Qui et al. 1994 for a discussion of burst detection methods). However, with a high fidelity burst envelope or pedestal, this type of metric can be quite useful. In the current algorithm, the signal is first divided into blocks and the RMS of that block computed. Figure 3.4 is a semi-logarithmic plot of a region of signal in which the burst time-series is plotted along with two-times the RMS signal power. In this signal the pedestal has been retained. The RMS values of the signal are computed every 256 burst samples in this case, though this should be adapted when flow conditions drastically change such as very near a wall. One may see that the results faithfully follow the burst envelope as detected by the eye. **This step requires  $2N$  operations where  $N$  is the number of time-series samples to be searched for bursts.**



**Figure 3.4. Burst amplitude (black) and 2x the RMS power (blue circles) for a typical LDV signal snippet.**



### 3.1.2.1.2.2 Parabolic fitting

To better identify the center of the burst and distinguish the bursts from noise spikes, parabolic fitting of the logarithm of the RMS power is used. This offers the possibility of identifying the burst envelope parameters relating to the Gaussian model case,

$$e(t) = \exp\left[-\frac{\beta}{2}(t-t_0)^2\right] \quad (3.9)$$

including the burst center time,  $t_0$ , and decay parameter,  $\beta$ , analogous to the variance in a normal distribution. A least-squares parabola is fit to every five adjacent points in the RMS power signal, stepping one RMS power data point each new parabola. If the parabola fit is concave down and occurring such that the center is within the five points, the correlation coefficient,

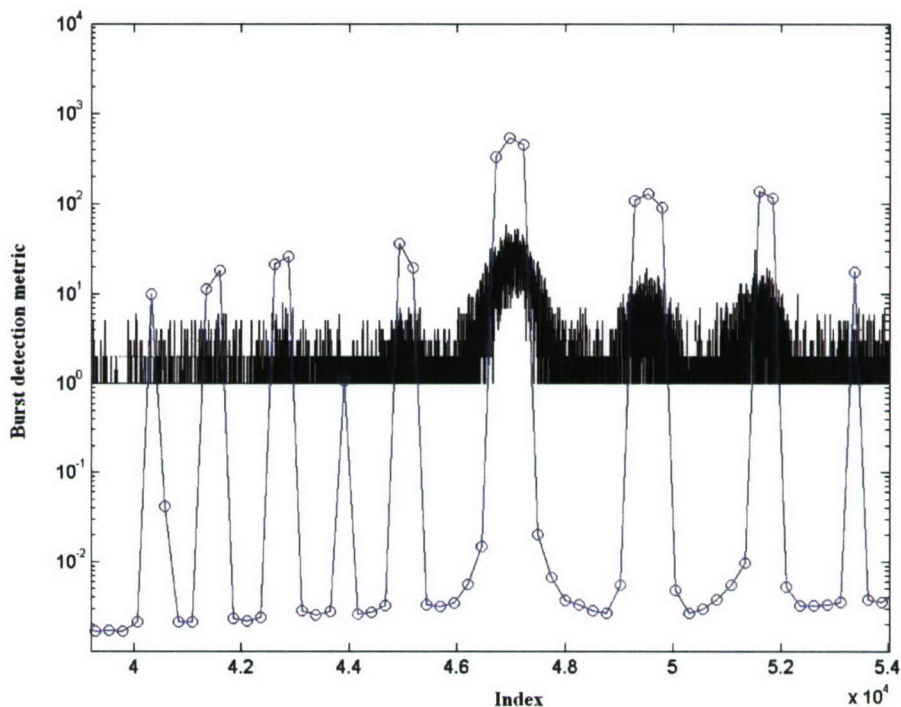
$$R_{xy} = \frac{\overline{xy}}{\sqrt{\overline{x^2}}\sqrt{\overline{y^2}}} \quad (3.10)$$

is computed. This “goodness of fit” parameter is then used as an additional metric for burst recognition by multiplying the correlation coefficient raised to the fourth-power by the RMS power of the central point of the fit. The fit parameters are further refined such that one obtains the center time of the burst and the Gaussian decay parameter; this information is saved for burst envelope identification. Note that this step may result in multiple maxima per burst if the burst duration is mismatched with the number of samples used to obtain the RMS power distribution. Ideally, no more than 10-12 RMS power points should span one burst to ensure proper parabola fitting. Figure 3.5 is a plot of the new burst recognition metric including the correlation coefficient weighting. **This step requires less than N operations.**

### 3.1.2.1.2.3 Peak detection and thresholding

The peak detection and thresholding step involves determining where the burst detection metric exhibits maxima and which of these are above a threshold. The threshold value is user-set and may be determined quickly by processing data snippets to judge the success of the threshold level. In this step, the bursts which pass the detection criteria are identified and information is stored indicating their locations and decay parameters. This step requires  $0.012N$  operations, negligibly contributing to the computational costs.

**The total cost for the burst detection is then about  $3N$ . Since this is a linear cost and the search for an event in an array is inherently linear in itself, it is much more computationally efficient than any non-linear technique such as frequency-domain detection.**



**Figure 3.5. Final burst detection metric versus sample index.**

### 3.1.2.1.3 Dual burst processing

Since the CompLDV experiences very high burst rates, the probability of closely spaced or overlapping bursts is high. To properly process these signals, we must identify those bursts and go about separating them in some way. Knowing the burst envelope parameters allows one to devise an efficient way to do just that.

Figure 3.6 is an example of the semi-logarithmic time-series of a dual burst event. The first parabola of the RMS power is noticeably distorted by the second burst, but the large amplitude will ensure that it passes validation; thus, the processor must know to identify this event as a dual burst event for proper processing. The two burst envelope parameters allow one to easily identify these events. The  $1/e^2$  definition may be used to define the ‘edge’ of the burst and the decay parameters and burst center times may be used to determine the presence of overlap. Figure 3.7 and Figure 3.8 are time-series and frequency-domain plots of the same dual burst event. In Figure 3.7, the first burst is centered in the window while the second burst is centered in Figure 3.8. The fitted burst envelope is exaggerated in duration in the case of some dual bursts, an example being made in Figure 3.7. This is acceptable because it increases the probability of the burst being processed as a dual burst.

To separate the bursts from one-another, the envelopes are used yet again. In Figure 3.7 and Figure 3.8, the blue plots are the original data while the red plots signify the



separated signals. In each processing window, the burst envelope is used as an absolute boundary such that if the signal extends beyond the instantaneous envelope value, it is set to the envelope value. Also the signal is cut so that no data occur below zero. Functionally, the following is occurring

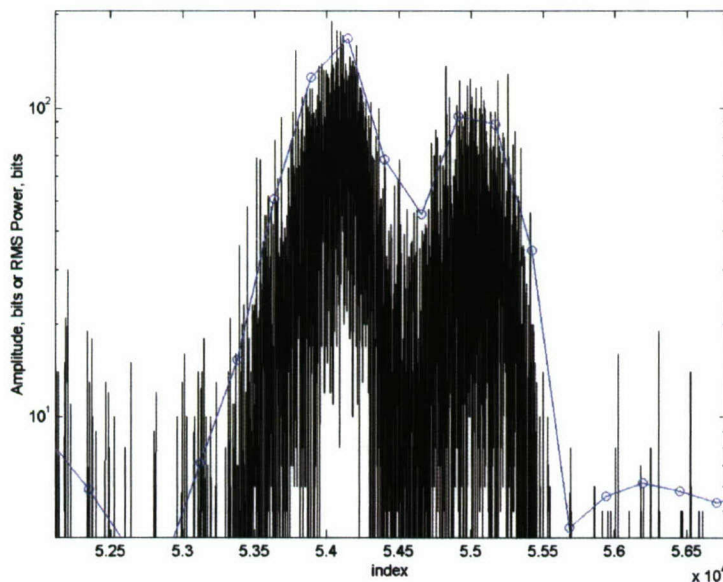
$$\tilde{s}(t) = \begin{cases} s(t), & 0 \leq s(t) \leq e(t) \\ e(t), & s(t) > e(t) \\ 0, & s(t) < 0 \end{cases} \quad (3.11)$$

Where  $s(t)$  is the measured signal,  $e(t)$  is the Gaussian envelope fit to the RMS signal, and  $\tilde{s}(t)$  is the processed signal. The result of the separation is quite dramatic, both in the time- and frequency-domains. Figure 3.8 in particular illustrates a greatly improved behavior in the semi-logarithmic burst spectrum. The red plot shows a frequency spike that is nearly parabolic, as predicted for the Gaussian-windowed sinusoid. The less dramatic results in Figure 3.7 are due to an over-prediction of the burst duration which may be corrected by adding a step in the process that notes that a burst follows that one closely in time, thus only the leading edge of the burst should be used for envelope determination.

Nobach (2002a) discusses a simple validation scheme which is derived from signal simulations. We must test to see that the bursts may truly be separated, if not, they must be discarded due to large bias in the frequency estimate. For two bursts at arrival times  $t_{01}$  and  $t_{02}$  with nominal decay parameters,  $\beta$ , the validation criteria should be

$$|t_{02} - t_{01}| \sqrt{\beta} > 2 \quad (3.12)$$

The cost of the burst separation algorithm is very modest. Compared to any frequency-domain processing done on these bursts, the linear separation schemes will be trivial.



**Figure 3.6. Semi-logarithmic amplitude of burst time-series for a dual burst, along with the RMS power metric.**

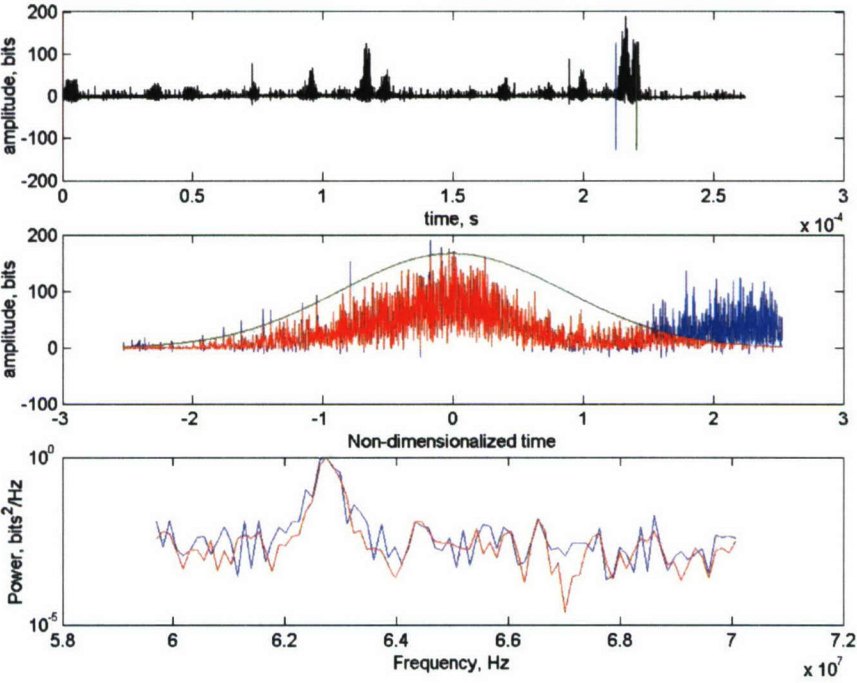


Figure 3.7. First burst in the dual burst pair example.

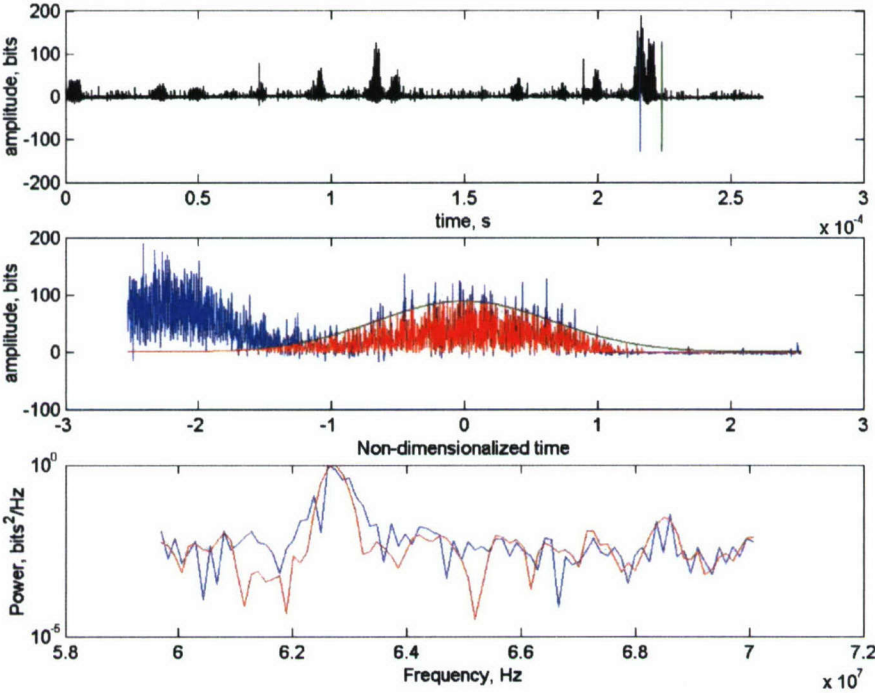


Figure 3.8. Second burst in the dual burst pair example.



### 3.1.2.1.4 Processing for Signal Phase Parameters

#### 3.1.2.1.4.1 The Cramer-Rao Lower Bound

With a confident identification of the burst using the recognition algorithms described and adequate separation of closely spaced bursts, processing for determining the burst phase parameters is possible. These parameters, as discussed, are the average frequency,  $f_0$ , and the chirp parameter,  $\gamma = \left. \frac{df}{dt} \right)_{t=t_0}$ . Many options exist for estimating the signal

phase and chirp for laser-Doppler bursts. As mentioned, however, the best unbiased estimator for any signal is given by the *Cramer-Rao Lower Bound* (CRLB), the theory of which is discussed by Whalen (1971) and Albrecht et al. (2002). Therefore, we must be aware of the limits imposed by the CRLB and seek an estimator which approaches this level of fidelity.

##### 3.1.2.1.4.1.1 Formulation for CRLB

The computation of the CRLB is based upon the model signal to be processed. The measured signal is taken as equation (3.1) with the model signal,

$$\hat{s}(t) = A \exp\left[-\frac{\beta}{2}(t-t_0)^2\right] \cos[\phi(t)] \quad (3.13)$$

Assuming a Gaussian noise distribution, the so-called Fisher information matrix is computed for a discrete signal at times  $[t_1, t_2, \dots, t_k]$  as

$$J_{ij} = \frac{1}{\sigma_n^2} \sum_{k=1}^N \frac{\partial \hat{s}(t_k)}{\partial \theta_i} \frac{\partial \hat{s}(t_k)}{\partial \theta_j} \quad (3.14)$$

where  $\sigma_n^2$  is the variance of the noise and  $\theta$  is a vector of the signal parameters, in our case,  $\theta = [A \ \beta \ \gamma \ f_0 \ \phi_0]^T$ . Note that it is assumed that  $t_0$  is known and is zero. The pertinent expressions for the partial derivatives are

$$\begin{aligned} \frac{\partial \hat{s}}{\partial A} &= \Gamma \sin \Phi \\ \frac{\partial \hat{s}}{\partial \beta} &= -\frac{A}{2} t^2 \Gamma \sin \Phi \\ \frac{\partial \hat{s}}{\partial \gamma} &= A \pi t^2 \Gamma \cos \Phi \\ \frac{\partial \hat{s}}{\partial f_0} &= 2A \pi t \Gamma \cos \Phi \\ \frac{\partial \hat{s}}{\partial \phi_0} &= A \Gamma \cos \Phi \end{aligned} \quad (3.15)$$

Where  $\Gamma = \exp\left(-\frac{\beta}{2}t^2\right)$  and  $\Phi = \pi(\gamma t^2 + 2f_0 t) + \phi_0$ . The CRLB is directly related to the Fisher information matrix such that the diagonal of the inverse of equation (3.14) gives

the CRLB for each parameter. It is then possible, to compute the CRLB for representative signal parameters.

To generalize the signal variance estimates, Shinpaugh et al. (1992) illustrated the usefulness of the proper normalization for the variance and a more pertinent definition of the SNR. It is shown that the CRLB for the simpler case of the constant frequency burst will collapse for all signal parameters when the frequency and chirp variance is normalized using the spectral line width of the Fourier transformed signal,  $\delta f = 1/T$  where  $T$  is the duration of the sampling period, and the SNR is modified by multiplying with the number of signal samples,  $N$ . The expression for the SNR used by Shinpaugh et al. (1992) is

$$SNR_1 = N(SNR) = N \frac{\sigma_{signal}^2}{\sigma_n^2} \quad (3.16)$$

or in decibel form

$$SNR_1)_{dB} = 10 \log_{10} \left( N \frac{\sigma_{signal}^2}{\sigma_n^2} \right) \quad (3.17)$$

where  $\sigma_{signal}^2$  is the mean-square of the model signal given by equation (3.13) and  $\sigma_n^2 / N$  is one-half the noise variance per spectral line in the spectral vicinity of the signal. The  $SNR_1$  formulation is particularly meaningful because the noise metric is representative of the narrowband noise that directly affects signal parameter estimation. Heuristically speaking, it is only the narrowband noise that degrades spectral estimates because broadband noise may be filtered before processing.

#### 3.1.2.1.4.1.2 Results for CRLB

Although equations (3.14) and (3.15) indicate that the CRLB is dependent upon absolute values of the signal parameters, it was found that it is very insensitive to *phase* parameters and the signal amplitude when the results were normalized and indexed with  $SNR_1$ . The Gaussian window parameter,  $\beta$ , was chosen such that the burst decayed to the  $e^{-2}$  value at the edge of the window. This may be stated in a relationship between the period,  $T$ , and  $\beta$  as  $\beta = 8/(T/2)^2$ . For reference, the exact signal parameters used for the CRLB reported are given in Table 3.1, although it is reiterated that the CRLB obtained is generally applicable for a signal following the model of equation (3.14).

The parameters of interest to our measurements are the minimum variance (or similarly, the standard deviation) for the frequency and the chirp. Figure 3.9 is a plot of the CRLB for the frequency standard deviation normalized by the spectral line width,  $\delta f = 1/T$ . Figure 3.10 is a plot of the CRLB for the standard deviation of the chirp parameter normalized by  $(\delta f)^2$ . Table 3.2 lists the values of the CRLB at various levels of  $SNR_1$ . For reference on the  $SNR_1$  scale, a practical lower limit for the value of  $SNR_1$  allowing estimates to approach the CRLB variance was shown by Shinpaugh et al. (1992) to be about 20dB. Below this value, the signal parameters could not be estimated. More will be



said to follow concerning the minimum  $SNR_1$  for unbiased estimates of the chirp parameter.

**Table 3.1. Signal parameters for the reported Cramer-Rao Lower Bound.**

Parameter	Value
Sampling frequency, $f_s$	250 MS/s
$N$	8192
$T$	32.77 $\mu$ s
$\delta_{SL}$	30.5 kHz
$A$	1 arbitrary unit
$f_0$	3.06 MHz
$\gamma$	7.45E8 Hz <sup>2</sup>
$\phi_0$	0
$\beta$	1.49E10 Hz <sup>2</sup>

**Table 3.2. The Cramer-Rao Lower Bound for the signal parameters in Table 3.1.**

SNR <sub>1</sub>	$\sigma_\gamma / \delta_{SL}^2$	$\sigma_{f_0} / \delta_{SL}$	SNR <sub>1</sub>	$\sigma_\gamma / \delta_{SL}^2$	$\sigma_{f_0} / \delta_{SL}$
0	7.990821	0.919593	36	0.126646	0.014575
2	6.347335	0.730459	38	0.100598	0.011577
4	5.041867	0.580224	40	0.079908	0.009196
6	4.004897	0.460888	42	0.063473	0.007305
8	3.181203	0.366097	44	0.050419	0.005802
10	2.526919	0.290801	46	0.040049	0.004609
12	2.007203	0.230991	48	0.031812	0.003661
14	1.594378	0.183483	50	0.025269	0.002908
16	1.26646	0.145746	52	0.020072	0.00231
18	1.005985	0.11577	54	0.015944	0.001835
20	0.799082	0.091959	56	0.012665	0.001457
22	0.634733	0.073046	58	0.01006	0.001158
24	0.504187	0.058022	60	0.007991	0.00092
26	0.40049	0.046089	62	0.006347	0.00073
28	0.31812	0.03661	64	0.005042	0.00058
30	0.252692	0.02908	66	0.004005	0.000461
32	0.20072	0.023099	68	0.003181	0.000366
34	0.159438	0.018348	70	0.002527	0.000291

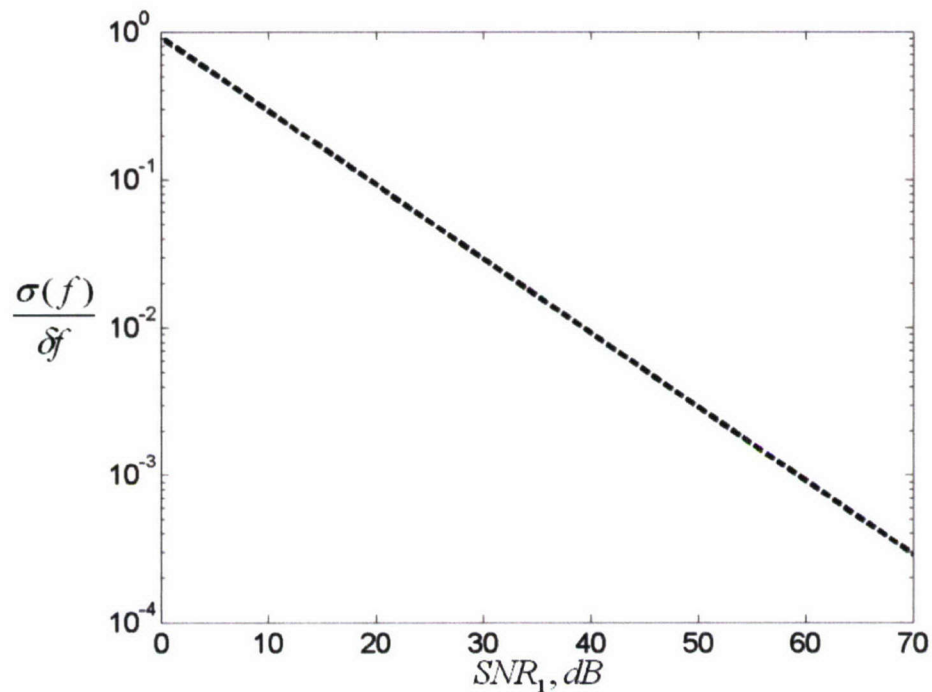


Figure 3.9. The Cramer-Rao Lower Bound for the standard deviation in frequency for the signal described by equation (3.14).

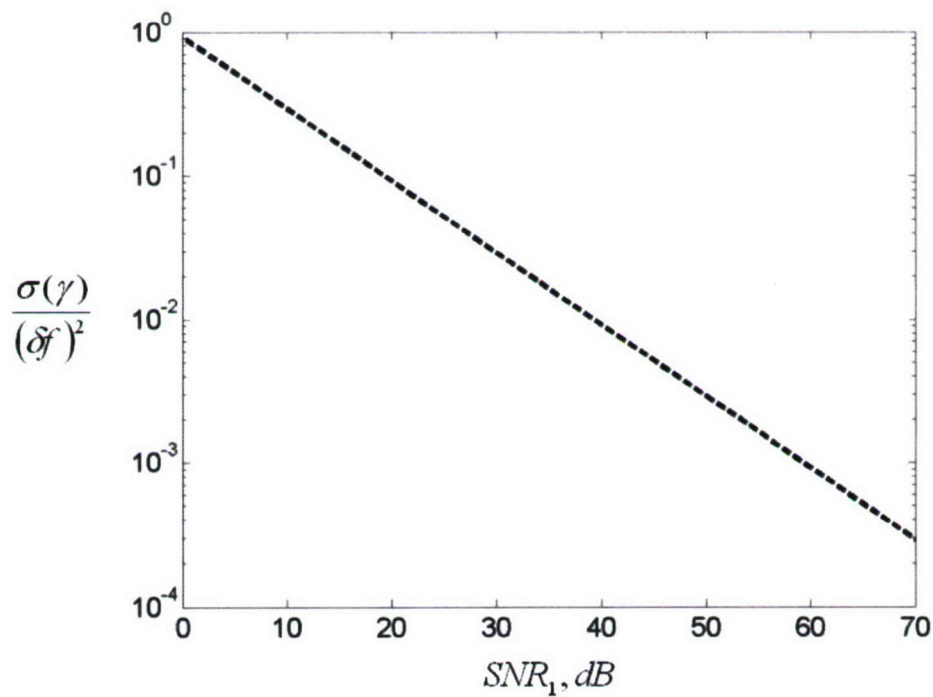


Figure 3.10. The Cramer-Rao Lower Bound for the standard deviation in chirp for the signal described by equation (3.14).



### 3.1.2.1.4.2 Processing options

The options for signal processing for laser-Doppler bursts have been discussed extensively in the literature by Shinpaugh et al. (1992) and more recently by Albrecht et al. (2002). Many techniques used for general signal processing applications have been considered for obtaining the burst frequency, chirp, and time-frequency information. These include spectral analysis via Fourier transforms; correlation-domain analysis; time-frequency analysis by quadrature demodulation, wavelet transforms, spectrograms (or short-time Fourier transforms), and Wigner-Ville transforms; the discrete chirp Fourier transform and many model-based parametric techniques which optimize the model parameters to best fit the signal. In the following section the literature will be briefly reviewed for these techniques and considerations for this specific application will be discussed.

#### 3.1.2.1.4.2.1 Spectral Analysis

The work by Shinpaugh et al. (1992) extensively explores the use of the fast Fourier transform (FFT) for estimating the spectra of Doppler bursts. In this method, the presence of a burst is first identified through signal recognition and then is centered into a processing window. The signal is transformed using the discrete Fourier transform (DFT),

$$X_k = \frac{1}{N} \sum_{j=1}^N x_j \exp(-ik(2\pi j)/N) \quad (3.18)$$

where  $X_k$  is the DFT of the discrete signal,  $x_j$  at sampled at times  $t_j$ , and  $k$  is the spectral line number. The FFT is an efficient implementation of the DFT which results in  $N \log_2 N$  number of floating point operations, instead of the  $N^2$  operations implied by direct implementation of (3.18). The discrete frequency domain power spectrum of a burst is estimated as

$$S_k = X_k X_k^* \quad (3.19)$$

Note that when the expected value of (3.19) is obtained for a stationary signal, the autospectrum will result.

To determine the Doppler frequency, the spectrum is considered in the expected frequency range. The signal will result in a large amount of power being concentrated around the Doppler frequency, as is shown in slices of constant time in the spectrogram in Figure 3.3. The final frequency estimate is determined by interpolation using the spectral lines adjacent to the maximum power spectral line. There are two effective ways of interpolating the spectra that are worth mentioning. First, since the Gaussian burst transforms as a Gaussian curve in the frequency domain when the chirp is small (see Albrecht et al. 2002 and Lehmann et al. 2002), a three-point Gaussian interpolation is appropriate. This is equivalent to a parabolic fit to the logarithm of the power spectral peak near the Doppler frequency such that,

$$f_D = \delta_{SL} \left( k + \frac{\ln\left(\frac{S_{k-1}}{S_{k+1}}\right)}{2 \ln\left(\frac{S_{k+1}S_{k-1}}{S_k^2}\right)} \right) \quad (3.20)$$

where  $k$  is the spectral line number of the maximum value in the power spectrum. A second interpolation scheme which is very effective is the centroid fit which is given as

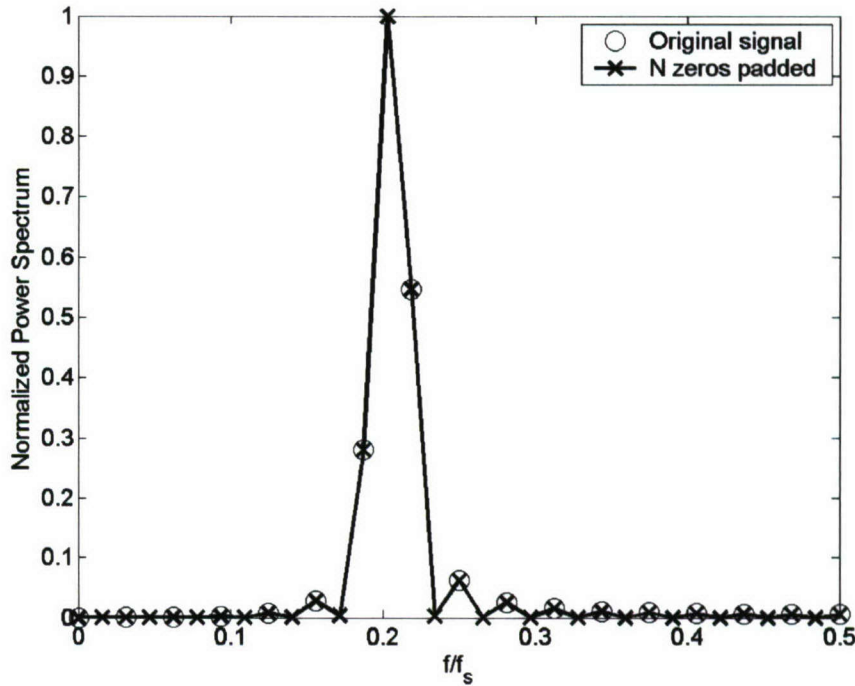
$$f_D = \delta_{SL} \left( \frac{\sum_{p=k-j}^{k+j} p S_p}{\sum_{p=k-j}^{k+j} S_p} \right) \quad (3.21)$$

where  $j$  is an odd number (typically  $j=1$ ) which will allow sufficient interpolation of the Doppler peak.

An additional option for enhancing the capabilities of the DFT in estimating the signal is via zero-padding. Since the resolution of the spectral lines is limited by the period over which the signal was sampled, one may reduce the spectral line width by artificially extending the sampling period by appending zeros to the signal. Of course, no additional information is added to the signal, so the theoretical bandwidth of the signal (or the amount of information the signal is capable of conveying) is unaltered. However, the technique is effective at interpolating the spectrum which may allow for more refined estimates of the Doppler frequency. An example of a power spectrum obtained using the DFT for a signal without zero-padding is compared with the DFT for the same signal but zero-padded with  $N$  zeros is given in Figure 3.11.

Shinpaugh et al. (1992) tested the performance of the spectral interpolation schemes and zero-padding using signal simulations of bursts with constant Doppler frequencies. The results indicate that the FFT technique does *approach* the CRLB for values of  $SNR_1$  greater than about 20 dB. Below this value the signal and noise peaks become inseparable and unbiased estimation is prohibited. In the case of the centroid interpolation, the zero-padding technique is ineffective in improving the signal estimation variance. However for the Gaussian interpolation scheme, zero-padding is very effective for  $SNR_1 > 30$  dB. For low  $SNR_1$  the Gaussian and centroid interpolations performed very similarly, but the centroid interpolation out-performs the Gaussian interpolation at higher values of  $SNR_1$  when no zero-padding is used. If signals are high fidelity and processing time allows, the Gaussian interpolation with zero-padding is preferred.





**Figure 3.11.** Spectrum of a signal with and without zero-padding.  $f_s$  is the sampling frequency of the signal.

#### 3.1.2.1.4.2.1.1 DFT Spectral analysis for time-varying frequency

The implementation of the FFT-based spectral analysis for signals with non-constant frequencies was considered by Lehmann et al. (2002). The scheme is based upon finite difference of the Doppler frequency as estimated in two parts of the signal. The concept is shown graphically in Figure 3.12. For a chirp signal with a constant rate-of-change of frequency, the chirp is then given by

$$\gamma = \frac{f_{D2} - f_{D1}}{t_2 - t_1} \quad (3.22)$$

where the subscripts represent the appropriate portion of the burst.

Since the burst amplitude is modulated, it is necessary to amplitude equalize the signal to avoid biasing the center of the burst. The signal envelope is removed using the empirical value of the window function as estimated using the Hilbert transform. The amplitude equalized signal is related as follows:

$$\hat{x}_k = \frac{x_k}{|x_k + iH\{x_k\}|} \quad (3.23)$$

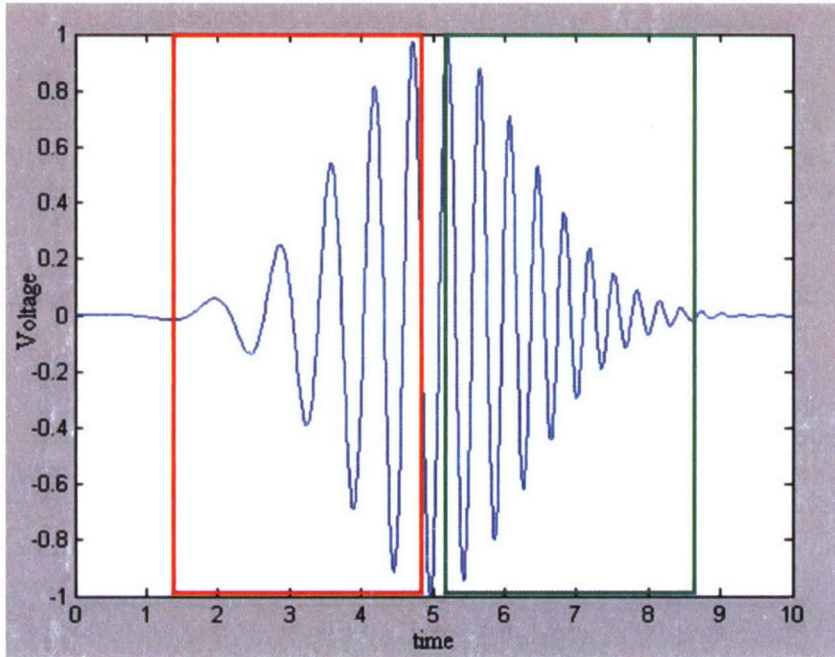
where  $H\{x_k\}$  is the discrete Hilbert transform of the signal and  $x_k + iH\{x_k\}$  is the so-called analytical signal for  $x_k$ . Note that the Hilbert transform may be computed using the inverse FFT given the DFT of  $x_k$  by the following modifications to  $X_k$  (Whalen 1971),

$$\hat{X}_k = \begin{cases} X_k, & k = 0 \\ 2X_k, & 1 \leq k < N/2 \\ 0, & N/2 \leq k < N \end{cases} \quad (3.24)$$

The Hilbert transform is then

$$H\{x_k\} = \text{Im}\left[F^{-1}\{\hat{X}_k\}\right] \quad (3.25)$$

where  $F^{-1}\{\hat{X}_k\}$  is the inverse Fourier transform of  $\hat{X}_k$ .



**Figure 3.12. Illustration of a chirped burst and two nominal processing zones.**

Given the amplitude-equalized signal split into two parts, the average Doppler frequency for each part is determined. Since any contribution from constant amplitude noise will decrease the SNR near the signal edges, a window function is applied to the signal. Lehmann et al. (2002) considered both the Hanning window,

$$w_k = \frac{1}{2} \left[ 1 + \cos\left(\frac{2\pi k}{N}\right) \right] \quad (3.26)$$

as well as the Gaussian window,

$$w_k = \exp\left[-\xi \left(\frac{k}{N-1} - \frac{1}{2}\right)^2\right] \quad (3.27)$$

where the authors used  $\xi = 40$ . For further reference to the characteristics of window functions, the definitive work by Harris (1978) is very informative.

In the study by Lehmann et al. (2002), the authors considered the effects of the processing window width and window choice for varying amounts of noise and signal chirp. The simulations indicated that separating the burst signal into the first 2/3rds and



last 2/3rds resulted in approximately the lowest combination of variance in the midpoint frequency estimation as well as the chirp estimation. Further, they indicated that a reduced bias and measurement variance could be achieved using the Gaussian window compared to the trigonometric Hanning window. The most likely explanation for this is that the Gaussian window behaves favorably under Fourier transformation such that the spectral peak of the signal is also Gaussian. Knowing a decent model for the spectral peak shape allows one to more-effectively interpolate the spectral peak. Of course, in the case of large chirp, the Gaussian spectral approximation fails; this magnitude of frequency change is, in general however, out of the realm of expected values for these measurements.

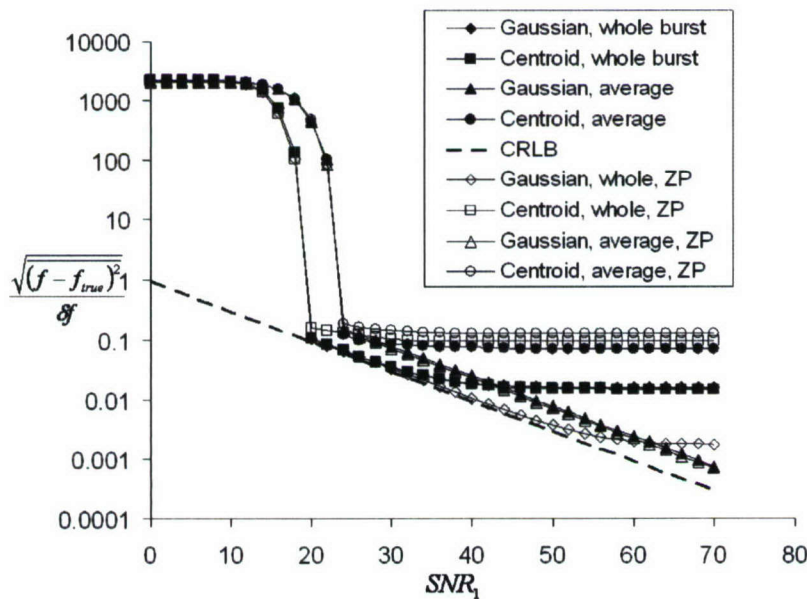
To benchmark the performance of similar algorithms which were implemented in the current system, signal simulations of chirped Doppler bursts with noise were performed. The signal parameters for the simulation were the same as the ones used by Lehmann et al. (2002) and are given in Table 3.3. Eight cases were considered as outlined in Table 3.4. It was sought to examine the impact of zero-padding, interpolation function, and mid-point frequency estimation scheme used. For each noise level the signal was processed 5000 times, each time with an independently realized Gaussian noise time series. The results for the study are shown in Figure 3.13-Figure 3.16. Note that in the cases in which zero-padding was employed, the spectral line width was still calculated based upon the sampling period of the actual signal before zero-padding. There are a few interesting features from the simulations that deserve mention. First, the zone of non-biased estimation occurs at  $SNR_1 \approx 20dB$  for the midpoint frequency when the entire burst is used. However, for the cases in which the burst is separated into parts in order to compute the parameters, the break occurs at  $SNR_1 \approx 24dB$ . It is also seen from these simulations that the chirp rate estimator RMS error fails to approach the CRLB. This failure is attributed to the loss of amplitude information when the envelope of the signal is removed to avoid biases (Lehmann et al. 2002). However, for the techniques tested, for both the frequency and the chirp rate it is found that the Gaussian spectral peak interpolation scheme is the superior technique. For expected noise levels in the 20-40 dB range, case 2 with no zero-padding is seen to perform as well case 3 utilizing zero-padding. Since zero-padding requires more processing power, this knowledge is quite useful for reducing processing time.

**Table 3.3. Signal parameters for the chirped burst simulation.**

Parameter	Value
Sampling frequency, $f_s$	1 S/s
$N$	256
$T$	255 s
$\delta_{sl}$	0.0078 Hz
$A$	1.3 arbitrary units
$f_0$	0.04 Hz
$\gamma$	0.00015 Hz <sup>2</sup>
$\phi_0$	1.2 rad

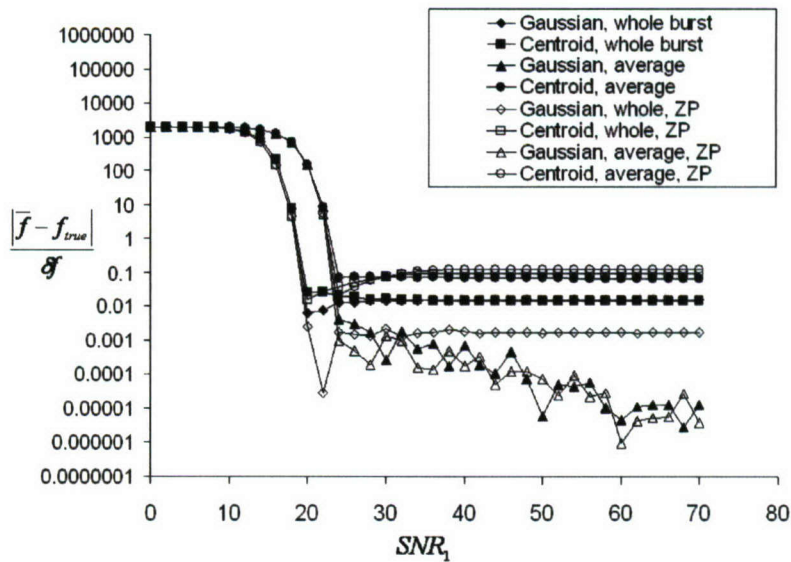
**Table 3.4. Processing cases for the chirped burst simulation study. Note: “Whole burst” means that the mid-point frequency was estimated from power spectrum of entire burst; “Average” means mid-point frequency was estimated by averaging the frequency determined from the two portions of the burst.**

Case	Description
1 Whole burst	Signal length N, centroid interpolation
2 Whole burst	Signal length N, Gaussian interpolation
3 Whole burst	Signal length 2N, Gaussian interpolation
4 Whole burst	Signal length 2N, centroid interpolation
1 Average	Signal length N, centroid interpolation
2 Average	Signal length N, Gaussian interpolation
3 Average	Signal length 2N, Gaussian interpolation
4 Average	Signal length 2N, Centroid interpolation

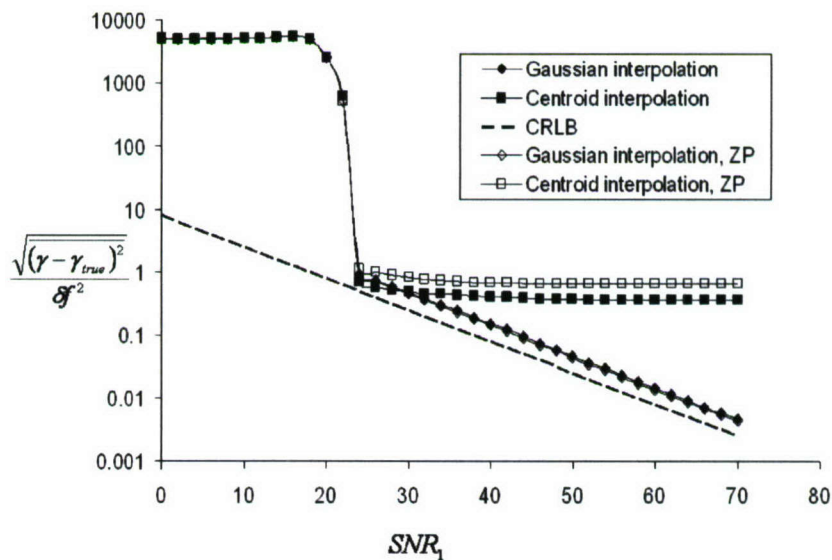


**Figure 3.13. RMS error in midpoint frequency calculations for signal simulations of a chirped burst with noise. Gaussian, Gaussian frequency interpolation; Centroid, Centroid frequency interpolation; whole, entire burst used for computing single DFT; average, burst broken into two parts to form two power spectral densities and frequency result determined by average; ZP, N zeros padded.**

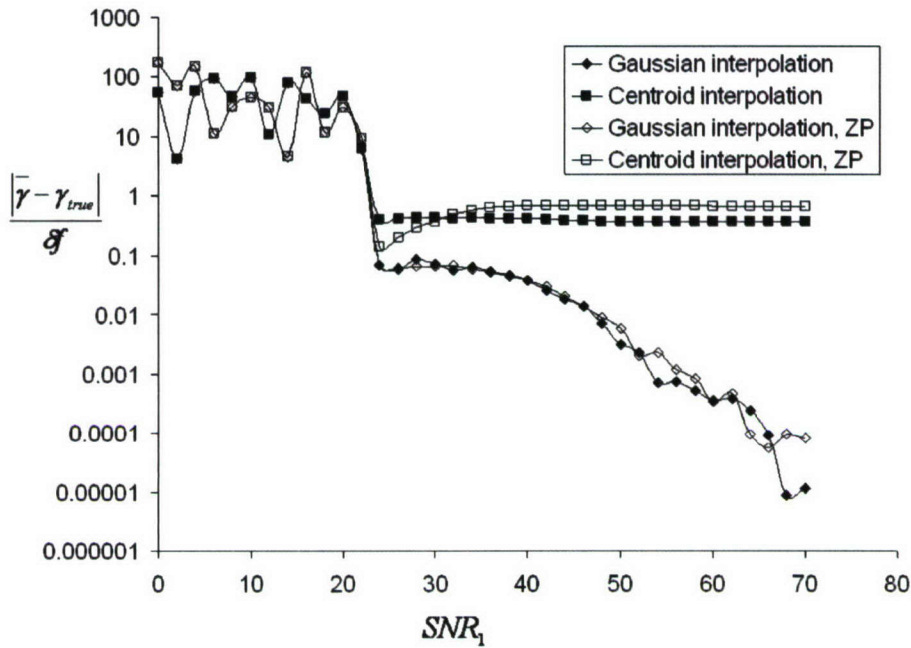




**Figure 3.14. Bias of the frequency measurement for the parameters of signal 1. Gaussian, Gaussian frequency interpolation; Centroid, Centroid frequency interpolation; whole, entire burst used for computing single DFT; average, burst broken into two parts to form two power spectral densities and frequency result determined by average; ZP, N zeros padded.**



**Figure 3.15. RMS chirp rate error from signal simulations of a chirped burst with noise for . Gaussian, Gaussian frequency interpolation; Centroid, Centroid frequency interpolation; ZP, N zeros padded.**



**Figure 3.16. Bias in the chirp rate. Gaussian, Gaussian frequency interpolation; Centroid, Centroid frequency interpolation; ZP, N zeros padded.**

#### 3.1.2.1.4.2.2 Correlation Domain Processing

The correlation domain processing is a technique for determining the Doppler frequency of a burst signal by analysis the time-delay correlation of the measured signal. This technique is most-often used by commercial processors to leverage the favorable noise response of the time-delay correlation functions. The correlation between signals  $a(t)$  and  $b(t)$  is written as

$$R_{ab}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} a(t-\tau)b(t)dt \quad (3.28)$$

where  $R_{ab}$  is the time-delay correlation of the signals. When  $a=b$  the correlation is known as the autocorrelation, otherwise it is the cross-correlation of the signal. An important note here is in the analysis of the noise when the expected value of  $R_{ab}$  is obtained. In this case we can represent the signals  $a$  and  $b$  as the sum of the desired signals  $s_a$  and  $s_b$  and the unwanted noise  $n_a$  and  $n_b$ . Making the substitution into (3.28) we obtain

$$R_{ab}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} [s_a(t-\tau)s_b(t) + n_a(t-\tau)s_b(t) + s_a(t-\tau)n_b(t) + n_a(t-\tau)n_b(t)]dt \quad (3.29)$$

In the case that  $a \neq b$ , then presumably the noise for each signal has resulted from separate stochastic processes and are thus unrelated. The resulting correlation will then only contain components from the first time within the integrand such that



$$E[R_{ab}(\tau)] = E\left[\frac{1}{T} \int_{-T/2}^{T/2} [s_a(t-\tau)s_b(t)] dt\right] \quad (3.30)$$

In the more common case as in typical laser-Doppler velocimetry signals when only a single measurement of a Doppler signal exists, the autocorrelation of the signal will be

$$E[R_{aa}(\tau)] = E\left[\frac{1}{T} \int_{-T/2}^{T/2} [s_a(t-\tau)s_a(t) + n_a(t)^2] dt\right] \quad (3.31)$$

Thus the autocorrelation will only contain noise contributions at zero-delay. The frequency of the signal may then be determined by any of several methods, including counting signal periods or fit to model signals.

Implementing this technique has similar costs as the FFT, but is not the preferred method for this study. As discussed by vanMaanen (1999), while the correlation technique will effectively separate signal from noise in a stationary signal, Doppler signals are transient, finite, and non-stationary. In this way, it is not possible to find the expected value of the autocorrelation and so noise will still be present throughout the correlation estimate. While it is possible to eliminate broadband noise to improve correlation estimates, ultimately it is the narrowband noise which may not be filtered that contaminates the correlation (vanMaanen 1999). Since the algorithms for frequency extraction from correlation signals are less general and robust than the FFT-based algorithms and since the chirp rate was also desired, the correlation technique was not considered for this study.

#### 3.1.2.1.4.2.3 Quadrature Demodulation

The quadrature demodulation technique (QDT) is a very power method for obtaining efficient estimates of the phase evolution of signals. The method utilizes the analytical signal, defined as

$$S(t) = r(t) + ic(t) = A(t)e^{i\phi(t)} \quad (3.32)$$

where  $r$  and  $c$  form an orthogonal quadrature pair such that

$$\int_{-T/2}^{T/2} r(t)c(t) dt = 0 \quad (3.33)$$

If both  $r$  and  $c$  are known, then the signal the time-varying amplitude,  $A(t)$ , and phase,  $\phi(t)$  may be determined as the magnitude and argument, respectively, of  $S(t)$ . In many cases, the quadrature pair of a signal is not measurable, so the Hilbert transform is used to determine  $c(t)$ .

As discussed by Lehmann, Nobach, and Tropea (2002), this technique offers great promise for determining the desired parameters of the signal phase. A succinct description of the performance of this estimation technique for signals in the presence of noise is given by Czarske (1999). The drawback of the QDT, however, is the requirement for high signal-to-noise ratio signals for performance that approaches the CRLB. Using the data from the work by Lehmann, Nobach, and Tropea (2002), the limiting noise level for which the CRLB-level variance is achieved is about  $SNR_l = 41 dB$ . In the LDV systems

utilized for this work, such signal-to-noise ratios could not be achieved on a regular basis and thus the power of the QDT may not be tapped.

#### 3.1.2.1.4.2.4 Discrete chirp Fourier transform

The discrete chirp Fourier transform (DCFT) is a method for examining signals with quadratic phase evolutions. The transform is directly analogous to the Fourier transform, but with an additional phase term for the chirp rate,

$$CF[s(t)](l, m) = \sum_{k=0}^{N-1} s_k e^{-i\left(\pi m \left(\frac{k-N/2}{N}\right)^2 + 2\pi l \frac{k-N/2}{N}\right)} \quad (3.34)$$

where  $l$  is the frequency spectral line number and  $m$  is the chirp spectral line number. Just as the discrete Fourier transform is limited in its frequency range by the Nyquist criterion, the chirp rate range is limited by a similar parameter,  $\gamma_{\max} = \frac{1}{4} f_s^2$ . Another observation

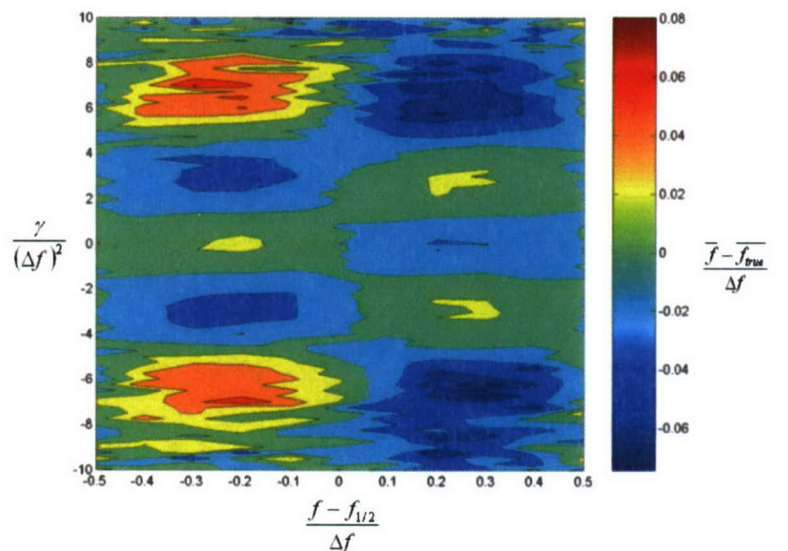
for the technique is the relation of the chirp line width,  $\delta\gamma = \left(\frac{f_s}{N}\right)^2$ . This parameter has

been shown in an above section to scale the uncertainties for the chirp estimation. It should be noted at this point that the chirp line width is a large value for typical Doppler signals in turbulent flows. A typical set of values for  $f_s$  and  $N$  is 250 MS/s and 4096 samples respectively. This results in a chirp line width of 3.7 GHz/s, with typical resolutions being some fraction of this number.

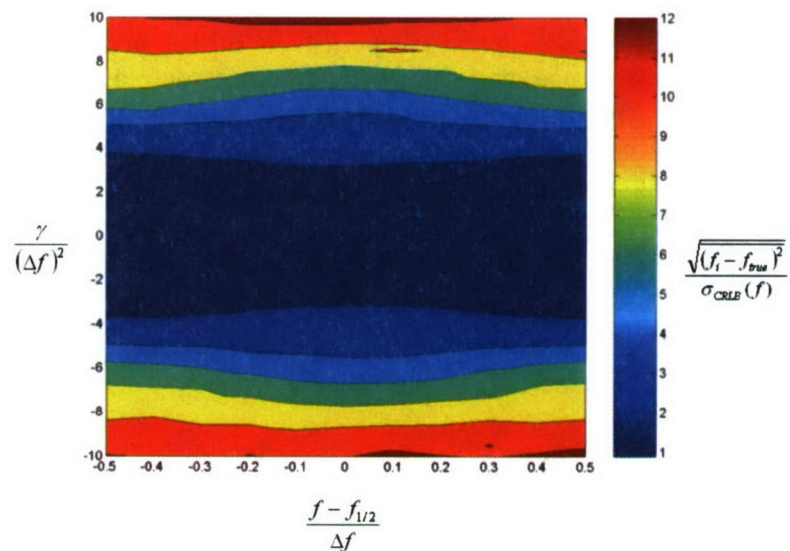
The primary interest in the DCFT is to extend the application of the acceleration estimation to lower values of signal-to-noise ratio. This presumption is due to the very robust operation of the DFT techniques for estimation of Doppler frequencies from low SNR signals, and the similarity between the DCFT and DFT techniques. An algorithm for achieving this was developed to implement the DCFT for LDV burst signals. In this algorithm, a combination of the DFT and DCFT is used to bound the value of the frequency and then resolve both the frequency and chirp rate to nearly the lowest-possible uncertainties. First the DFT is computed for the burst centered in the processing window. Gaussian interpolation is used around the Doppler frequency peak, giving an estimate of the frequency for the case when the chirp rate is zero. Next the DCFT is computed at the three spectral lines adjacent to Doppler frequency peak estimate (i.e., along the  $l$  index in equation 3.34) and along the range of realizable chirp rates (i.e., the  $m$  index in equation 3.34). Next, the maximum power spectral density value in the two-dimensional map is determined. Finally, two interpolations around this value of  $(l, m)$  are done. Since at significant chirp rates the frequency estimate from the DFT can be highly uncertain as seen in Figure 3.17, a new estimate for the Doppler frequency is determined by interpolating along the spectral lines using Gaussian interpolation. In Figure 3.17 and similar figures to follow, on the abscissa  $f_{1/2}$  refers to the frequency equally between adjacent spectral lines. Referring to Figure 3.17, note that at chirp rates above 3 chirp line widths, significantly higher RMS errors than the Cramer-Rao lower bound exist and bias errors become more predominate. To refine the chirp rate estimate, two interpolation



methods are considered, a parabolic interpolation and a centroid interpolation. These methods were chosen because it was observed that the power spectral densities are much more diffuse in the chirp domain than in the frequency domain where the Gaussian interpolation is perfectly suited for the Gaussian-windowed signals.



(a)

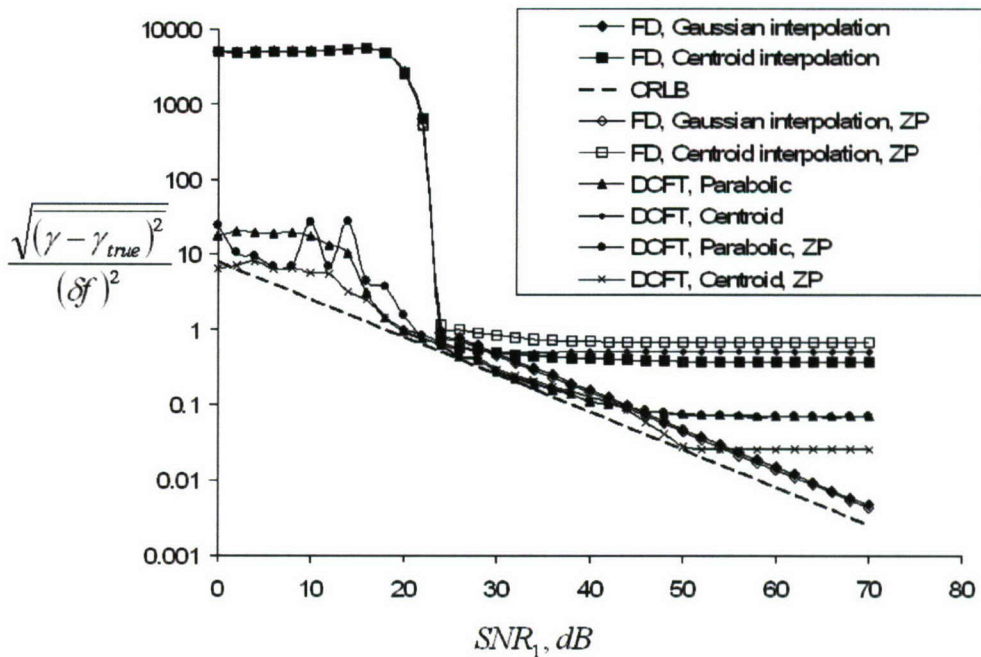


(b)

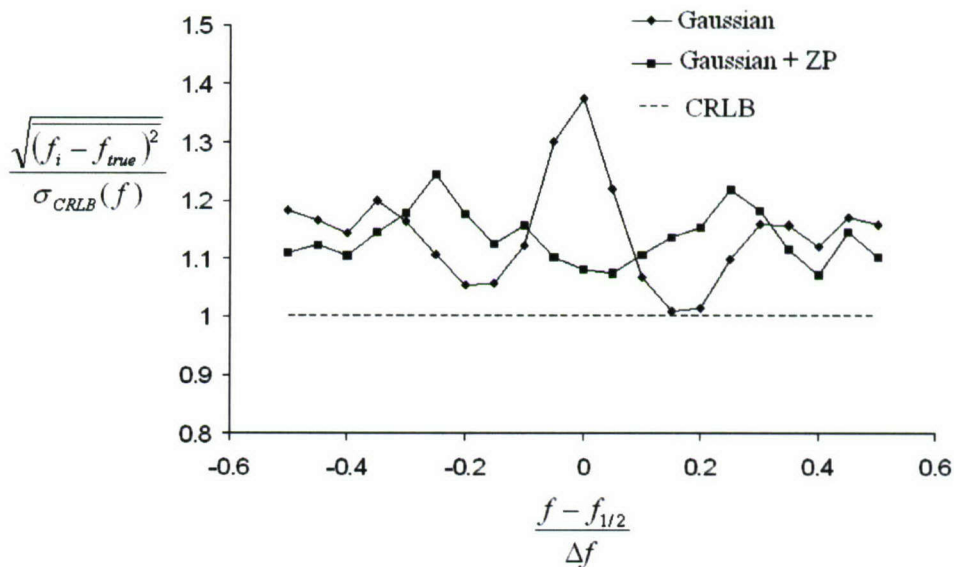
**Figure 3.17. Simulation results for the DFT with Gaussian interpolation to determine the frequency of a Gaussian windowed LDV burst over a cycle of one frequency spectral line and several chirp lines. (a) Contours of frequency bias errors normalized by the spectral line width. (b) Contours of RMS errors for the estimated frequency normalized by the Cramer-Rao Lower Bound.**

To examine the behavior of the DCFT/DFT algorithm in response to burst signals containing noise, signal simulations similar to the ones discussed in the spectral analysis section were executed. Both methods of chirp domain interpolation discussed above were considered to determine which may be better suited for the application. As with the previous simulations, the desired results were the estimator biases and variances for the frequency and chirp rate for given noise levels. Figure 3.18 is a plot of the chirp rate variance with  $SNR_1$  for the DCFT along with the finite difference spectral techniques discussed previously. Two important aspects are immediately evident. First, as expected the DCFT converges to give unbiased estimations at the same value of  $SNR_1$  as for the FFT for frequency estimation. Second, the DCFT results in RMS errors approaching the CRLB more closely than the finite difference method. To further study the response of the DCFT to signals with different frequency and chirp rates, a range of both those parameters were considered for the representative level of  $SNR_1=24dB$ . Signals with a chirp-rate of 1.47 in chirp spectral lines and a sweep of frequencies across a single spectral line were investigated at the constant value of  $SNR_1$ . The results are presented in Figure 3.19-Figure 3.22 with frequency spectral line units on the abscissa and normalized units on the ordinate. The important note from these plots is that the signal estimator approaches the Cramer-Rao lower bound to within about 5-40% for all cases, indicating a high fidelity estimator. Particularly useful in visualizing the effectiveness of this algorithm over the unrefined DFT-based techniques is comparison of Figure 3.17 to Figure 3.23. These two figures are exactly analogous; however Figure 3.23 utilizes the DCFT for determining both the frequency and chirp rate jointly. Since the modulation of the Doppler frequency peak due to a variable frequency is accounted in the model signals in the DCFT, the quality of the midpoint frequency estimation is not affected by the chirp rate.

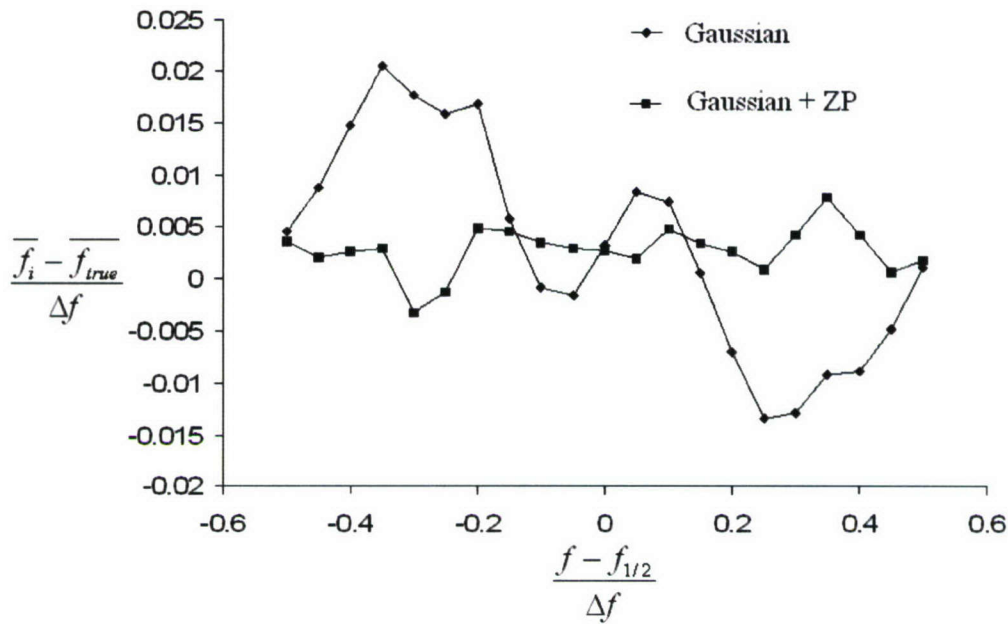




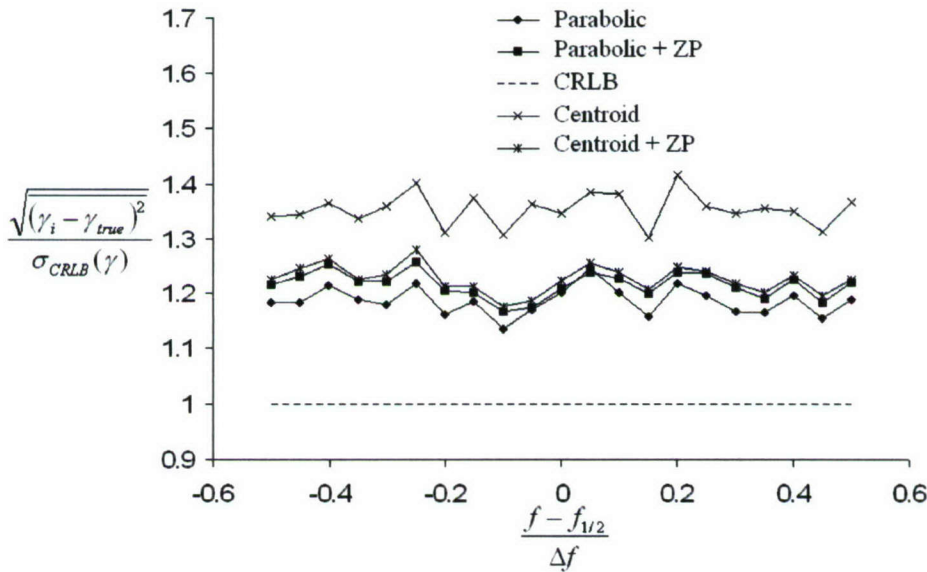
**Figure 3.18.** Uncertainty of estimated Doppler frequency chirp ( $\gamma$ ) as a function of normalized signal-to-noise ratio,  $SNR_1$  for finite difference methods (FD) and discrete chirp Fourier transform methods (DCFT). CRLB: Cramer-Rao lower bound, ZP: 2-times zero-padding, Parabolic: Parabolic interpolation of chirp power distribution, Centroid: centroid interpolation of chirp power distribution.



**Figure 3.19.** Relative frequency estimator RMS error over a frequency sweep with  $SNR_1=24\text{dB}$ . Gaussian, Gaussian interpolation of spectral lines in frequency domain; Gaussian + ZP, Gaussian interpolation of spectral lines in frequency domain with  $N$  zeros padded; CRLB, Cramer-Rao Lower Bound.

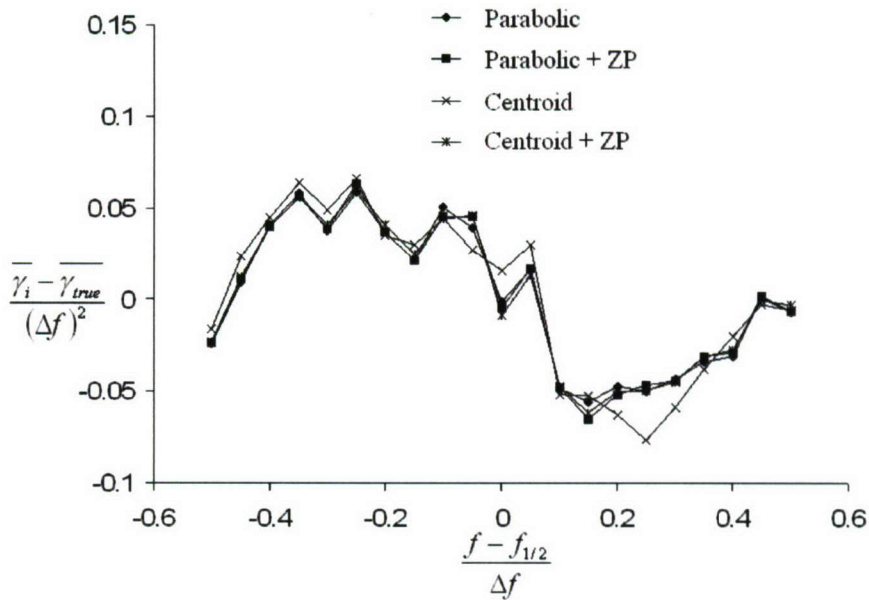


**Figure 3.20. Relative frequency estimator bias error over a frequency sweep with  $SNR_f=24\text{dB}$ . Gaussian, Gaussian interpolation of spectral lines in frequency domain; Gaussian + ZP, Gaussian interpolation of spectral lines in frequency domain with  $N$  zeros padded.**

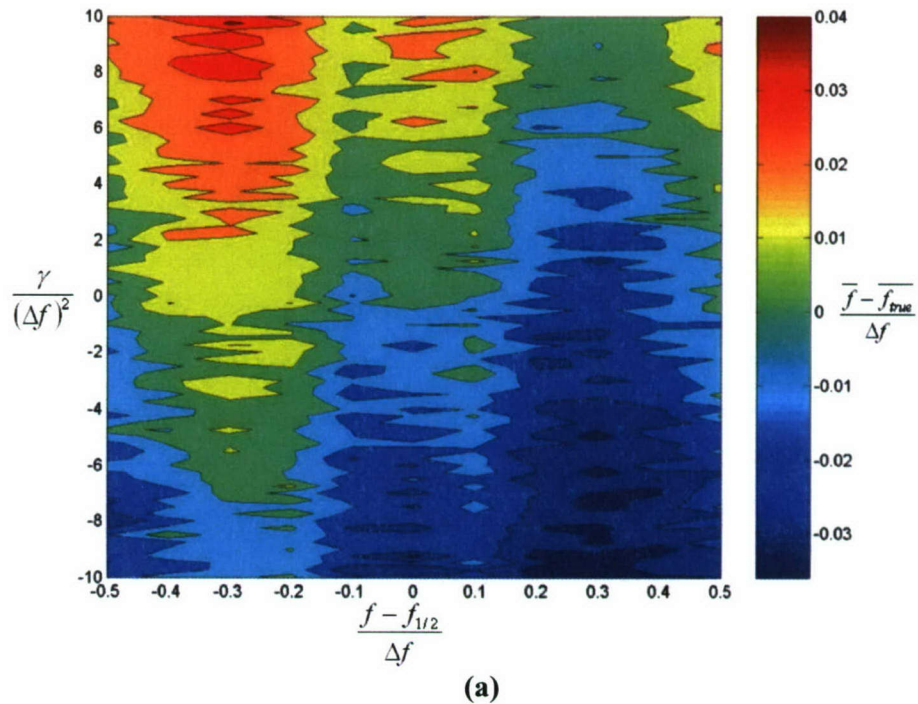


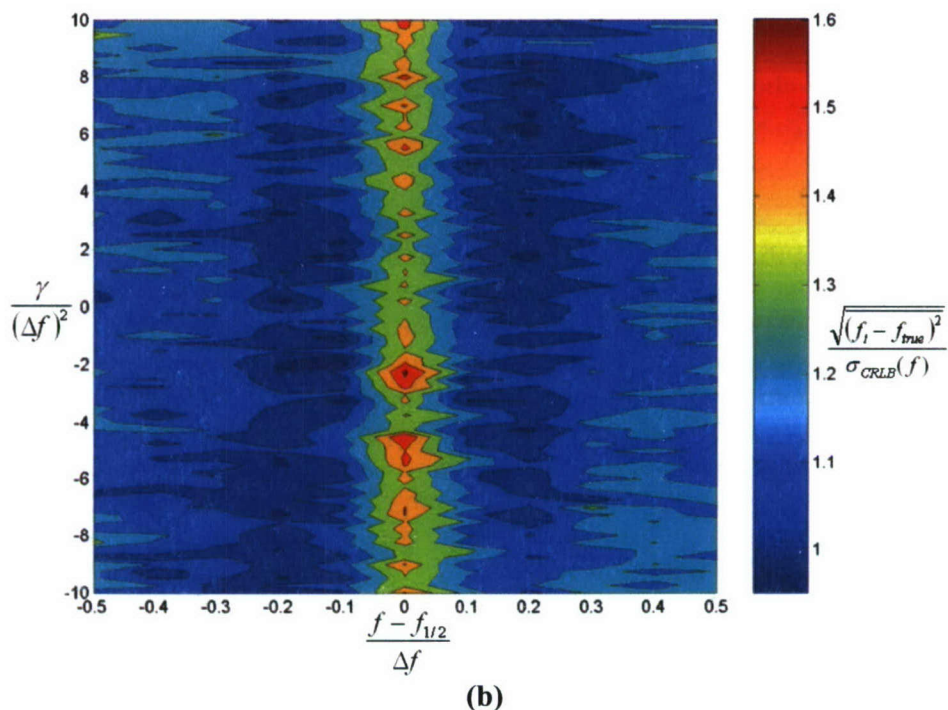
**Figure 3.21. Relative chirp rate estimator RMS error over a frequency sweep with  $SNR_f=24\text{dB}$ . Parabolic, Parabolic interpolation of chirp lines; Parabolic + ZP, Parabolic interpolation of chirp lines with  $N$  zeros padded. Centroid, Centroid interpolation of chirp lines; Centroid + ZP, Parabolic interpolation of chirp lines with  $N$  zeros padded; CRLB, Cramer-Rao Lower Bound.**





**Figure 3.22. Relative chirp rate estimator bias error over a frequency sweep with  $SNR_I=24\text{dB}$ . Parabolic, Parabolic interpolation of chirp lines; Parabolic + ZP, Parabolic interpolation of chirp lines with  $N$  zeros padded. Centroid, Centroid interpolation of chirp lines; Centroid + ZP, Parabolic interpolation of chirp lines with  $N$  zeros padded; CRLB, Cramer-Rao Lower Bound.**





**Figure 3.23. Simulation results for the DFT/DCFT combination technique with Gaussian interpolation to determine the frequency of a Gaussian windowed LDV burst over a cycle of one frequency spectral line and several chirp lines. (a) Contours of frequency bias errors normalized by the spectral line width. (b) Contours of RMS errors for the estimated frequency normalized by the Cramer-Rao Lower Bound.**

#### 3.1.2.1.4.2.5 General time-frequency analysis

The time-frequency analysis is a challenging and well-researched field. Both these points are evident when examining a review of the available techniques by Boashash 1992. The challenge of the time-frequency analysis is inherent in the nature of problem: time and frequency are conjugate variables and thus subject to Heisenberg's uncertainty principle. This principle states that the uncertainty in time and frequency are related such that  $\delta f \delta t = C$  where  $C$  is a constant (Døssing 1998). In this way, we may either know the frequency with low uncertainty by sacrificing the time uncertainty or vice-versa. This makes time-frequency analysis very useful for applications where signal bandwidth is very large such as voice analysis. To utilize the analysis in an effective way for low-bandwidth LDV signals, we must choose the technique which optimizes the combination of frequency and time uncertainties to achieve the desired measurement for the signal.

The general time-frequency analysis tools available are many (see Boashash 1992 for a review). A few popular time-frequency techniques include the spectrogram (or short-time Fourier transform), the wavelet analysis, and the Wigner-Ville transform. Each of these is a linear transform that attempts to capture the non-stationary events through adapting either the kernel of the transform integral (Wigner-Ville), the width of the processing



window (spectrogram), or both (wavelet). Researchers have implemented each of these techniques for laser-Doppler signals to obtain time-frequency information from the signals.

Lehmann (1998) implemented both the spectrogram and the Wigner-Ville transform to examine LDV signals from oscillating droplets. His research indicated that the cross-product terms inherent in the Wigner-Ville transform made the method unusable even for moderate amounts of noise in the signal. The spectrogram method was used successfully in the study by Lehmann (1998) to make estimates of the time-frequency variation of the signals due to the robustness of the discrete Fourier transform in the presence of noise. Unfortunately, the relative bandwidths encountered in the signals for the study by Lehmann (1998) were very large and not typical for those encountered in turbulence measurements. The broadening of the spectral peak which occurs when the spectrogram is used severely limits the quality of the frequency estimates. This is seen using Figure 3.13 as an example. Even if a high level of  $\text{SNR}_1$  is achieved for the short-duration processing window, the scaling of the error with the spectral line width makes 'instantaneous' frequency estimates highly uncertain. Small variations in signal frequency are undetectable, being lost in the estimator variance.

The wavelet transform has also been examined for implementation in Doppler signal analysis (Nobach and vanMaanen 2001). The wavelet transform has a varying resolution of time and frequency such that low frequencies are measured precisely but with high uncertainty in the time at which they occurred and high frequency events are measured precisely in time but with high frequency uncertainty. The result is that the relative frequency uncertainty is constant for all frequencies or conversely the relative time uncertainty is constant for all signal periods. The motivation for developing this technique for LDV signal processing was improved estimations of the burst arrival time. Since the burst duration itself is smaller than the smallest time scales in the flows in interest in the current work, this additional refinement beyond the envelope estimates from the burst recognition algorithms was unnecessary. Presently, no researchers have implemented the wavelet transform for analysis of the performance of wavelets for Doppler frequency time-variation. It is expected that due to the small bandwidth of the signals of interest, this technique will yield unsatisfactory results much like the spectrogram.

### 3.1.3 Validation of the complete burst processor

The individual algorithms discussed above for determining the location and duration of burst, estimating frequency and chirp rate and validating the burst were coded into a comprehensive processor. In order to validate the operation of these algorithms as implemented together in the processing code, sample burst signals with arrival time-values from an LDV dataset at  $y^+ = 181$  in a 2D constant pressure boundary layer were used. Furthermore, the mean-square velocity fluctuations were matched with the velocity data at the same point such that,  $\overline{u^2} = 4.7 \text{ m}^2 / \text{s}^2$ ,  $\overline{v^2} = 1.4 \text{ m}^2 / \text{s}^2$ ,  $\overline{w^2} = 2.4 \text{ m}^2 / \text{s}^2$ . The velocity signals generated were single-tone sinusoids, with a frequency of 100Hz to allow simple comparison in the time series between input and estimated velocities. No

accelerations were input. The velocities were transformed to the non-orthogonal optical coordinates so realistic Doppler frequencies were used to generate the bursts at the appropriate time-indices. For each burst, the phase was randomly varied to eliminate any processor preference for such a characteristic. To simulate the effects of noise, the bursts were introduced with constant-amplitude Gaussian noise such that  $SNR_I=34dB$ —this value was chosen due to its proximity to the values encountered in flow situations. A time-series snippet comparing the input to the output velocities for the  $U$  channel is given in Figure 3.24. The mean-square estimates obtained with the processed results were 0.18%, -0.21%, and -0.18% in error for  $\overline{u^2}$ ,  $\overline{v^2}$ ,  $\overline{w^2}$ , respectively. In addition, the velocity-acceleration correlations found were each on the order of  $100m^2/s^3$ , or about 1-2% of the maximum value in the 2D flat plate boundary layer flow for which these data were simulated.

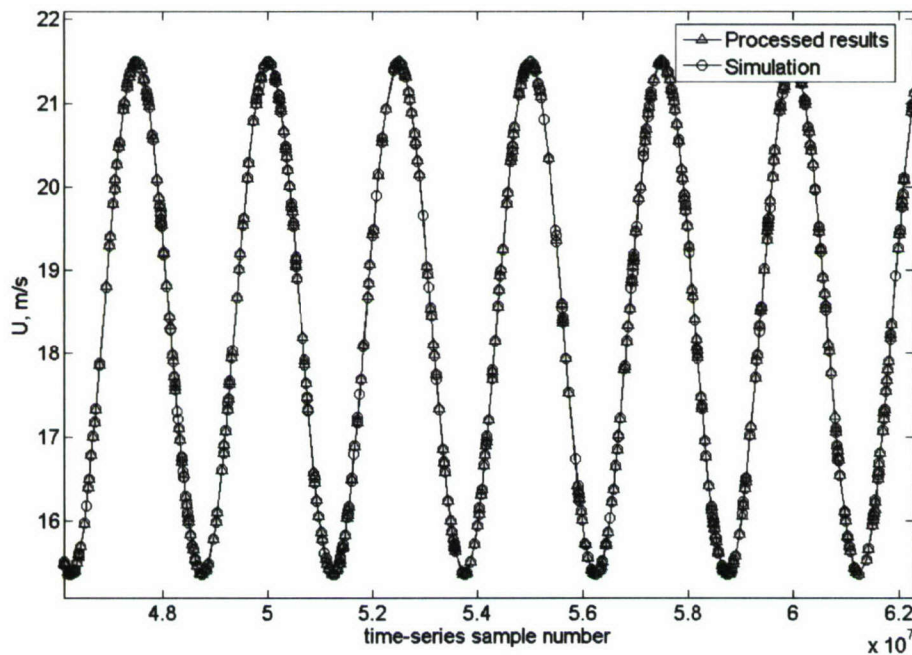


Figure 3.24. Comparison of the simulated  $U$  signal with the one processed using the ComplDV processing algorithms.

### 3.2 Spectral Analysis for Non-equidistantly Sampled Data

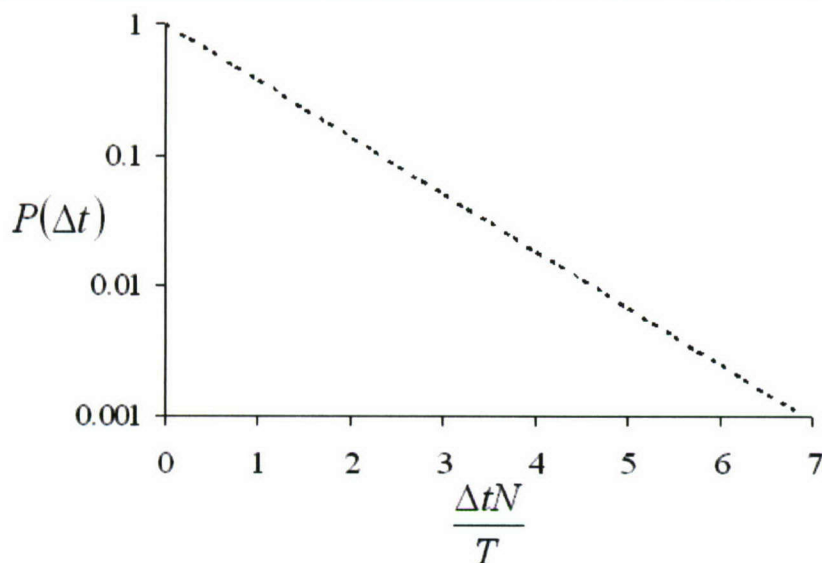
The subject of time-delay correlation and spectral estimation for signals arising from laser-Doppler measurements has been visited by numerous researchers since the advent of single-burst frequency estimation. An excellent review of this work up to the current state-of-the-art is given by Benedict, Nobach and Tropea (2000). The problem at hand is to efficiently and robustly estimate the time-delay correlation of a signal which is



sampled at a random time distribution. This time distribution follows Poisson statistics such that the probability of encountering a given arrival time,  $\Delta t$ , is given as

$$P(\Delta t) = \frac{N}{T} \exp\left(-\frac{N\Delta t}{T}\right) \quad (3.35)$$

where  $N$  is the number of Doppler measurements and  $T$  is the period of time for acquisition (see Benedict, Nobach, and Tropea 2000 or vanMaanen 1999 for a more detailed discussion of the arrival time phenomena). An example of such a distribution is given in Figure 3.25. The result of the Poisson arrival time distribution is that there are many data points with instantaneous ‘sampling rates’ greater than the mean rate,  $N/T$ , but also some instantaneous sampling rates much lower than the mean rate.



**Figure 3.25.** Graphical depiction of the Poisson arrival time statistics for particle-laden flows.

### 3.2.1 Methods for estimating time-delay and frequency-domain information from LDV data

Three classifications of methods exist for computing the frequency content of LDV signals. These are

1. Direct transform methods
2. Evenly-sampled time-series reconstruction via signal interpolation
3. Slot correlation

Of these methods, only 2. and 3. offer viable estimates as Benedict, Nobach, and Tropea (2000) show by comparison. The direct transform methods suffer from significant bias and high estimation variance which worsens with increasing noise levels. Signal reconstruction may be done by any interpolation scheme desired. However, due to the random nature of the arrival time, higher-order schemes fail since adjacent data-points that are used to interpolate intermediate values have unknown relationships. The result is that high-order interpolation results in progressively greater low-pass filtering and bias of the spectral results. The most successful of these interpolation schemes has been the

zeroth-order sample-and-hold. This method has the benefit of being very quick and simple to program and process data, but still results in low-pass filtering of the data. The frequency content of a signal which has been sample-and-hold processed will exhibit first-order system-type attenuation with the cut-off frequency being  $\frac{N}{2\pi T}$  (Adrian and Yao 1987). Corrections for the auto- and cross-correlations resulting from the sample-and-hold signals have been derived by Nobach, Müller, and Tropea (1998) and Müller, Nobach, and Tropea (1998) respectively. These methods significantly increase the viability of the sample-and-hold estimators for frequencies nearing the mean particle rate.

The third estimation scheme is a very robust statistical technique for obtaining the time-delay correlation functions for laser-Doppler data. The basic slot correlation of signals  $u(t_i)$  and  $v(t_j)$  is given by

$$R_{uv}(n\Delta\tau) = \frac{\sum_{i=1}^N \sum_{j=1}^N u_i v_j b_n(t_i - t_j)}{\sum_{i=1}^N \sum_{j=1}^N b_n(t_i - t_j)} \quad (3.36)$$

where

$$b_n(t_i - t_j) = \begin{cases} 1 & \text{for } \left| \frac{t_i - t_j}{\Delta\tau} - n \right| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

This is the basic algorithm which Benedict, Nobach, and Tropea (2000) credit to Mayo et al. (1974). Three important improvements have been made for this algorithm, known as the methods of fuzzy slotting, local normalization, and local time estimation. The fuzzy slotting technique was first proposed by Nobach, Müller, and Tropea (1998) as an improvement over the slotting technique of Mayo et al. (1974). The algorithm given by equation (3.36) was modified such that the weighting parameter,  $b_n(t_i - t_j)$  was no longer digital but could be distributed according to the proximity of the sample to the two nearest adjacent bins. The result is the fuzzy slotting condition,

$$b_n(t_i - t_j) = \begin{cases} 1 - \left| \frac{t_i - t_j}{\Delta\tau} - n \right| & \text{for } < 1 \\ 0 & \text{otherwise} \end{cases} \quad (3.37)$$

This significantly reduces the variance for the slotting methods and makes it much more attractive for general use. vanMaanen, Nobach, and Benedict (1999) published a short communication wherein they exhibit the usefulness of the fuzzy slotting scheme along with that of the local normalization scheme. The merging of these two techniques resulted in an even lower estimator variance. The local normalization is given by

$$\hat{R}_{uv}(n\Delta\tau) = \frac{\sum_{i=1}^N \sum_{j=1}^N u_i v_j b_n(t_i - t_j)}{\sqrt{\sum_{i=1}^N \sum_{j=1}^N u_i^2 b_n(t_i - t_j)} \sqrt{\sum_{i=1}^N \sum_{j=1}^N v_j^2 b_n(t_i - t_j)}} \quad (3.31)$$



where  $\hat{R}_{uv}(n\Delta\tau)$  is now the time-delay correlation coefficient with  $\hat{R}_{uv}(0) = 1$ . Finally, the local time estimation technique was developed by Nobach (2002b) to obtain the proper location of time-delay bins given sample sets that may be non-homogeneously distributed within the bins. In this case, the integer bin number,  $n$ , is replaced such that the correlation coefficient,  $\hat{R}_{uv}(\tau_n)$ , is defined at bin delay  $\tau_n$  as given by

$$\tau_n = \frac{\sum_{i=1}^N \sum_{j=1}^N (t_i - t_j) b_n(t_i - t_j)}{\sum_{i=1}^N \sum_{j=1}^N b_n(t_i - t_j)} \quad (3.32)$$

The local time estimation is very useful particularly at the smallest time delays where finite transit time limits the minimum inter-arrival time. Note that in each of these statistical slotting techniques, the general form of the equations allows a non-unity weighting of the  $i^{\text{th}}$  sample by some  $w_i$  and/or analogous weighting for the  $j^{\text{th}}$  sample. Since it has been shown by work in the past that the commonly-corrected velocity-data rate bias is very small for three-component LDV systems, no necessity for non-unity weighting exists for the present study.

A caveat of the local time estimation is that the resulting correlation function is still unevenly sampled, albeit to a much lesser degree than the original signal. Nobach (2002) suggests that interpolation of the correlation values to evenly sampled delays gives good results, especially since the shift of the bin location is very small. With this newly re-sampled correlation function, the spectral method of choice is the discrete transform with variable windowing developed by Tummers and Passchier (1996)

$$\hat{S}(f_i) = 2\Delta\tau \sum_{n=1}^L W_n \hat{R}_{uv}(n\Delta t) (\cos(2\pi f_i n\Delta t) + i \sin(2\pi f_i n\Delta t)) \quad (3.33)$$

where  $W_n$  is the variable window chosen to be a Tukey-Hanning window such that

$$W_n = \begin{cases} 1/2 + 1/2 \cos(\pi f_i n\Delta t / \kappa) & \text{for } |f_i n\Delta t| < \kappa \\ 0 & \text{otherwise} \end{cases} \quad (3.34)$$

where Benedict, Nobach, and Tropea (2000) suggest a value of  $\kappa = 6$ .

For a continuous spectral distribution as with turbulence, adjacent bin smoothing may be used to reduce the scatter in the frequency-domain data. In this technique, the power-spectral values for two adjacent estimates are averaged and replaced with a single estimate at their average frequency. This technique was used in the results section to obtain smooth estimates of the power spectra for all three-velocity components and corresponding cross-spectra.

### 3.2.2 Slot correlation validation

It was desired to validate the implementation of the slot correlation algorithms just discussed so that the flow data to be processed could be analyzed confidently. To this end, the slot correlation with fuzzy slotting, local normalization, and local time estimation was used to compute the cross-correlation between known sample sets. The

sets were constructed using an even sample set and then interpolating for time-indices taken from an actual LDV file at  $y^+=181$  in a 2D flat plate turbulent boundary layer flow. The signals entered include a sine-cosine pair, the auto-correlation of a sine function, and the cross and auto-correlations of Gaussian noise with unit standard deviations. The sampling frequency for the original signal was taken as the mean burst frequency from the LDV data, 9563.3 Hz. The frequency for the sine-cosine pair was 100 Hz.

The time-delay (cross) correlation is defined as

$$R_{uv}(\tau) = \frac{1}{2T} \int_{-T}^T u(t+\tau)v(t)dt \quad (3.38)$$

In the case of the cross-correlation between the sine and cosine functions with frequency,  $2\pi f = \omega$ ,

$$R_{\sin(\omega t)\cos(\omega t)}(\tau) = \frac{1}{2T} \int_{-T}^T \sin[\omega(t+\tau)]\cos(\omega t)dt = \frac{1}{2} \sin(\omega\tau) \quad (3.39)$$

The autocorrelation is given as

$$R_{\sin(\omega t)\sin(\omega t)}(\tau) = R_{\cos(\omega t)\cos(\omega t)}(\tau) = \frac{1}{2} \cos(\omega\tau) \quad (3.40)$$

In the other case of the random noise content the auto- and cross- correlations become, respectively

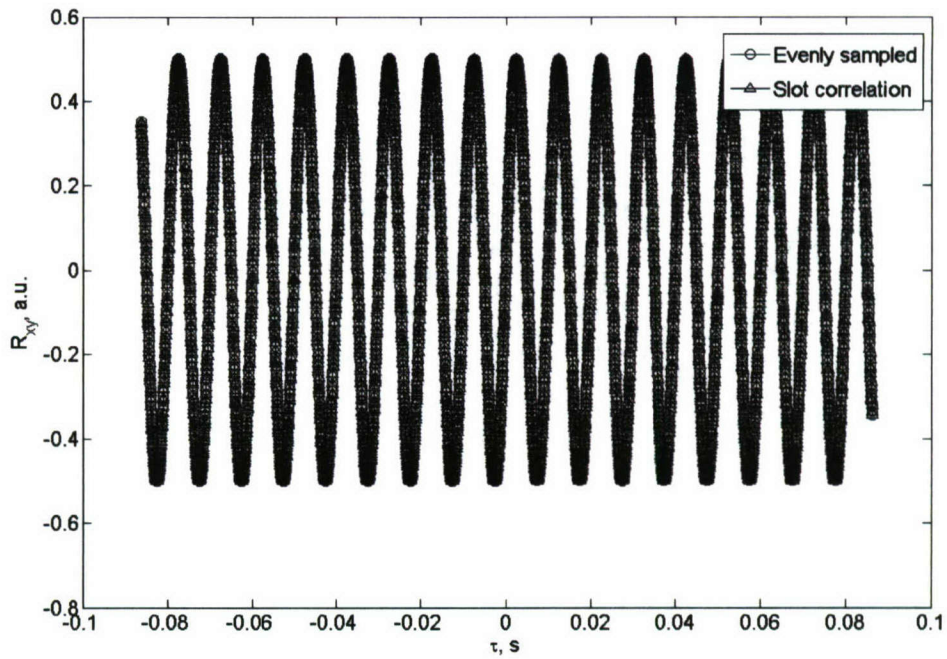
$$\begin{aligned} R_{uu}(\tau) &= \delta(\tau = 0) \\ R_{uv}(\tau) &= 0 \end{aligned} \quad (3.41)$$

The delay-space results for the first case described analytically by equation (3.39) are shown graphically in Figure 3.26 and Figure 3.27. The agreement between the cross-correlation derived from the evenly-sampled data and the LDV-time-series data is very good. To quantify the agreement, the RMS error and correlation coefficient were computed. The RMS error divided by the amplitude of the correlation was found to be 0.0076%. The correlation coefficient was computed as 0.9998. The sinusoid autocorrelation case is presented in Figure 3.28 and Figure 3.29 with equal levels of correlation between the input and computed correlations.

To obtain the Gaussian noise, two methods were employed. In the first method, a regularly sampled set was first obtained and then linearly interpolated to obtain the noise at the times from the LDV time-series list. Unfortunately, this method results in filtering and introduces very short time correlations evident in Figure 3.30 and Figure 3.31 in which similar data were used to obtain the time-delay correlations using the slotting technique as well as direct estimation from the evenly sampled data (Figure 3.32). Note that to obtain the evenly sampled data, the LDV-like data were evenly sample-and-hold interpolated. To remedy the filtering, a second method was used in which a Gaussian random number was generated for each point in the LDV time-series list. It is impossible to rigorously compare this plot to the regularly-sampled case since it will contain aliased values for regular samples. However, from the results of the slot correlation (Figure 3.33 and Figure 3.34), we see much reduced correlations near zero delay indicating that the



noise was found to be uncorrelated at non-zero values of the autocorrelation. Figure 3.35 also exhibits favorable behavior for the cross-correlation of two independently realized noise time-series with the same LDV arrival time distribution. Thanks to these simulation data, much confidence in the slot correlation algorithms used was obtained.



**Figure 3.26. Cross-correlation of sine-cosine pair.**

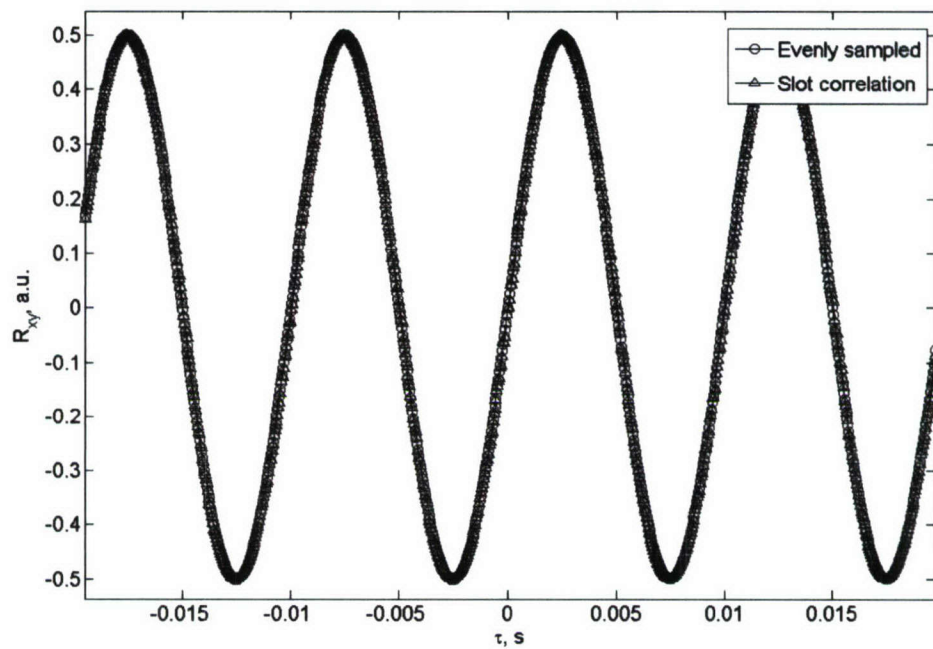


Figure 3.27. Cross-correlation of sine-cosine pair near zero delay.

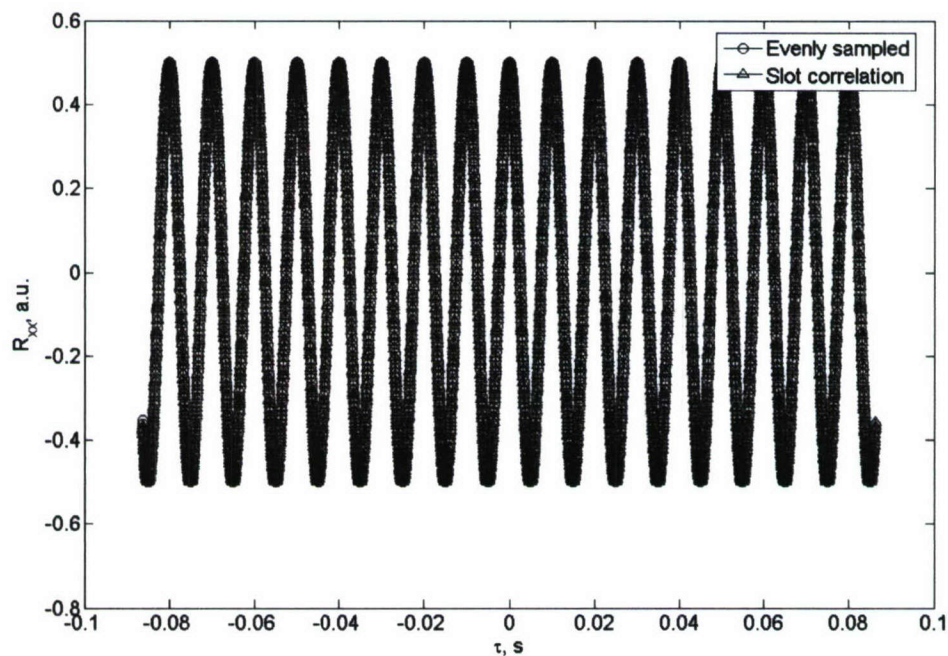


Figure 3.28. Autocorrelation of sine.



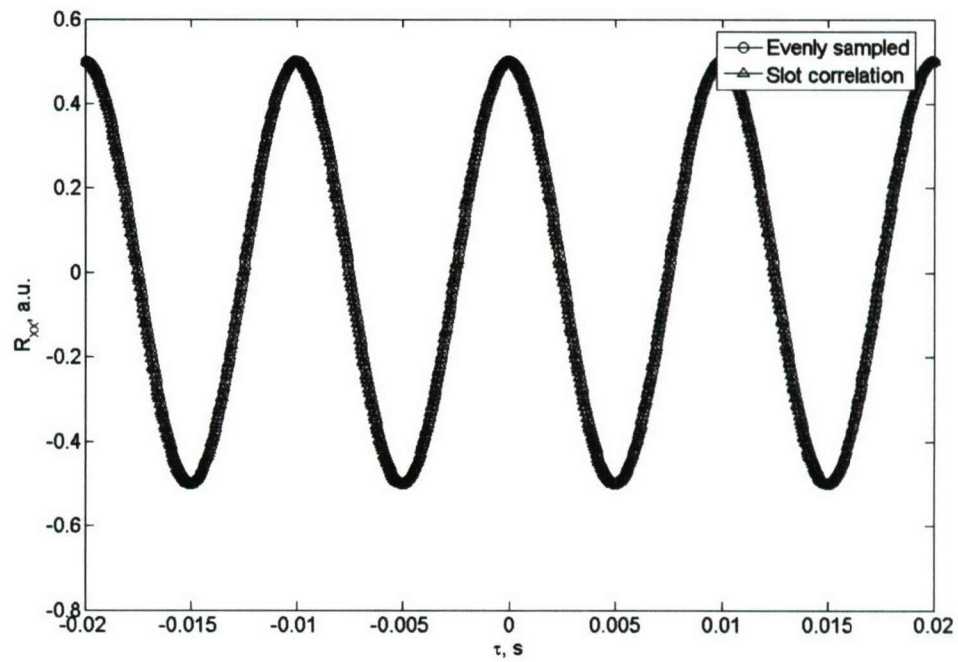


Figure 3.29. Autocorrelation of sine near zero delay.

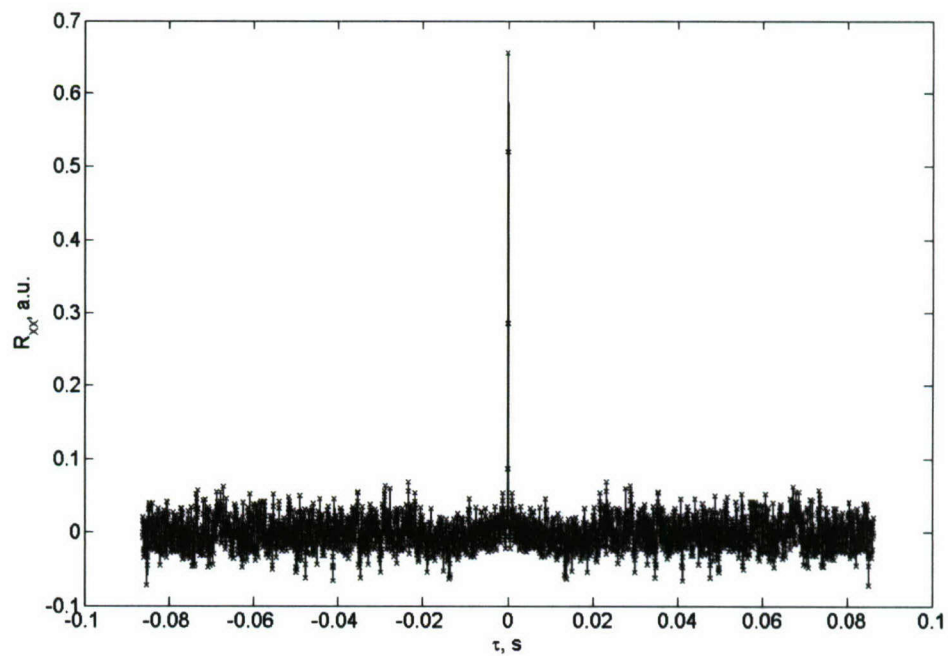
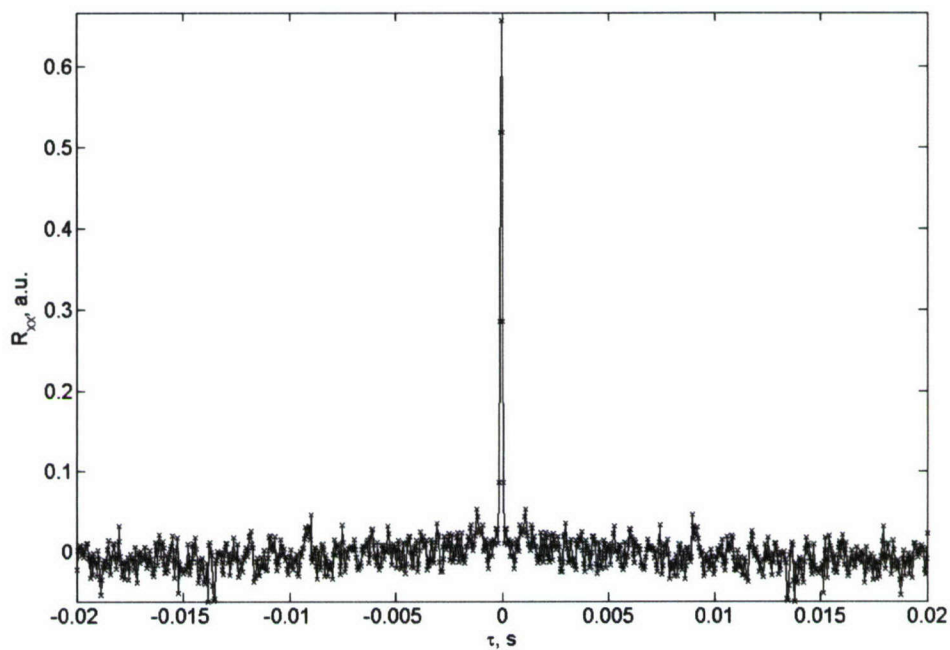
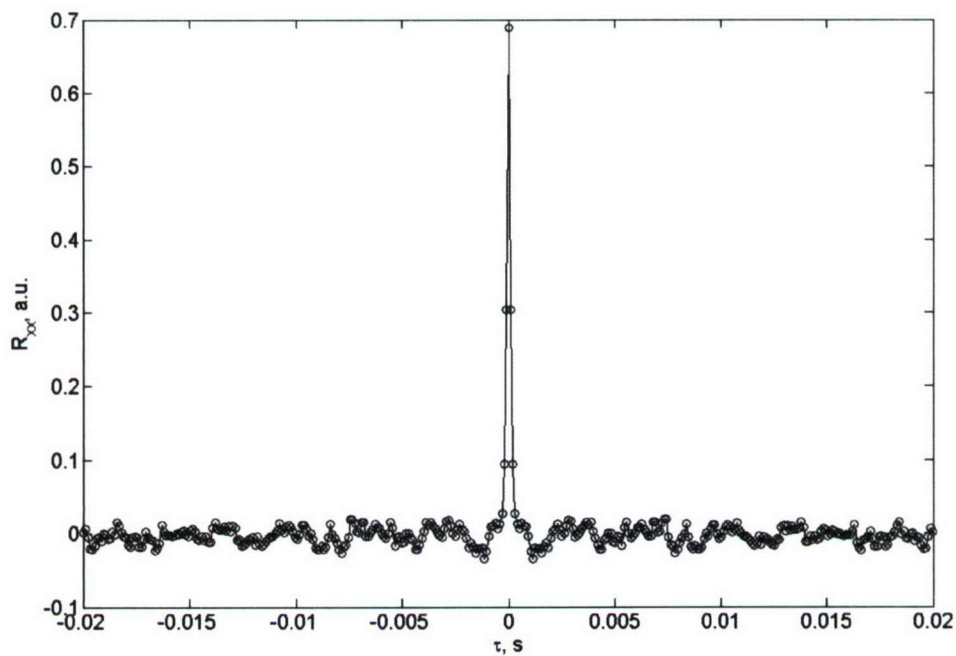


Figure 3.30. Autocorrelation of Gaussian noise as computed with the slot correlation with linear interpolation.

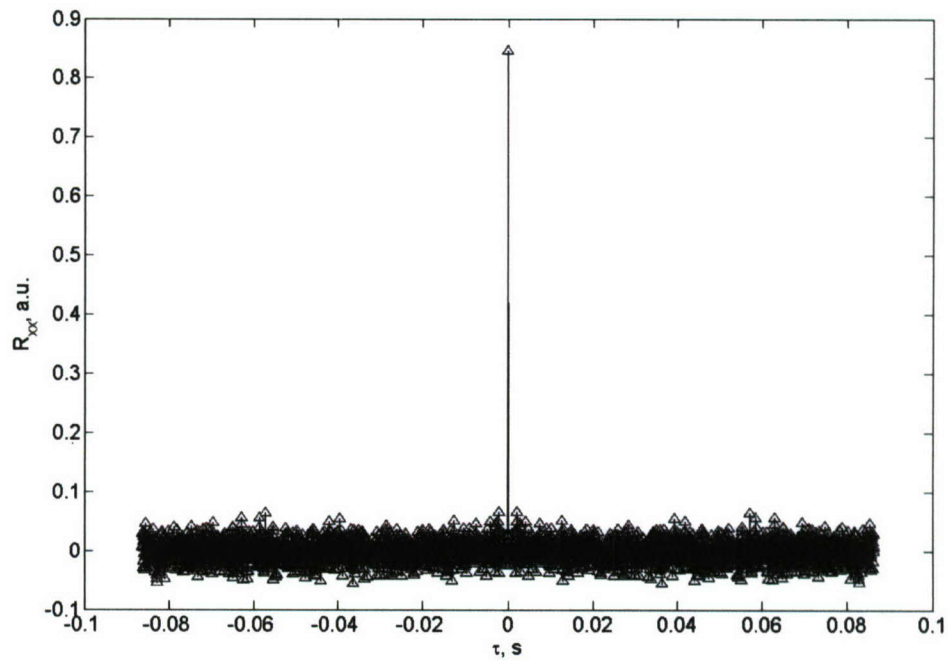


**Figure 3.31.** Autocorrelation of linearly interpolated Gaussian noise near zero delay as computed with the slot correlation.

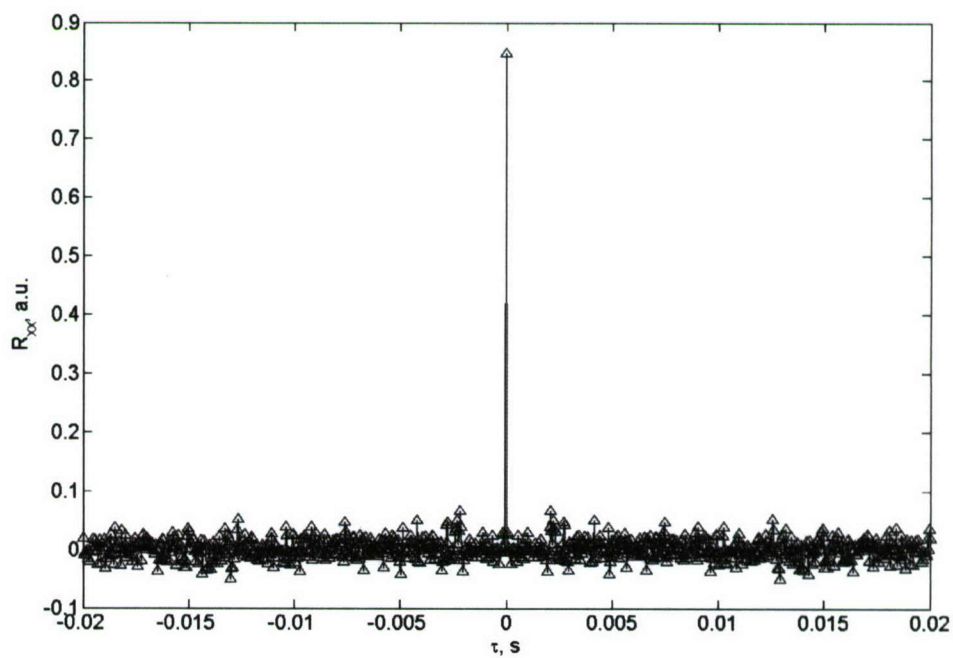


**Figure 3.32.** Autocorrelation of regularly sampled data obtained from linearly interpolated Gaussian noise.

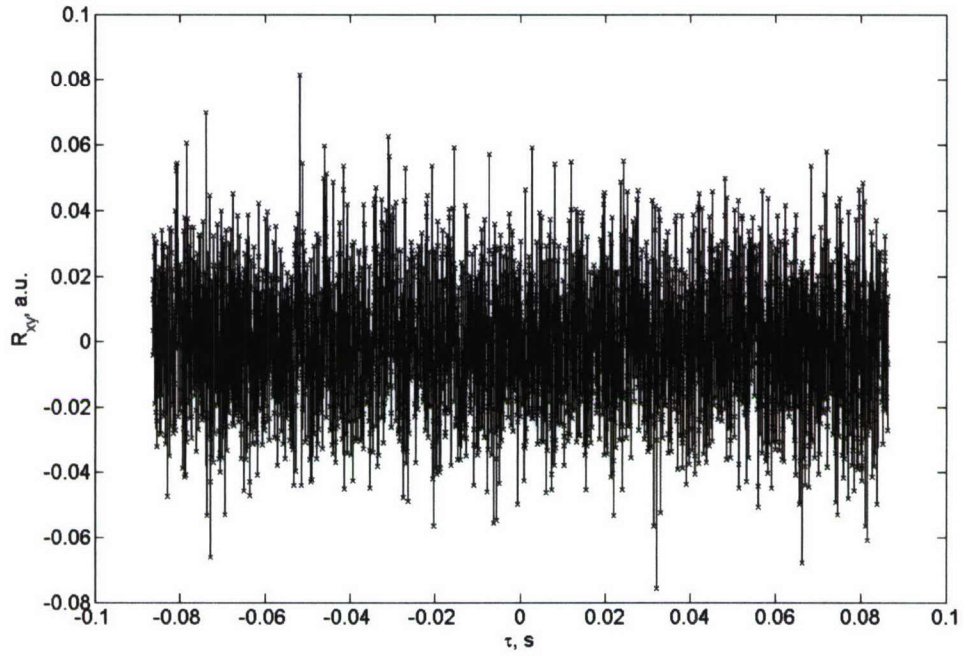




**Figure 3.33. Autocorrelation of Gaussian noise with no interpolation as computed with the slot correlation.**



**Figure 3.34. Autocorrelation of Gaussian noise with no interpolation near zero delay as computed with the slot correlation.**



**Figure 3.35. Cross-correlation of two Gaussian noise signals with no interpolation as computed with the slot correlation.**



## 4 Results

In this chapter, we examine results obtained using the CompLDV instrumentation in turbulent boundary layers. Several flows of fundamental interest have been examined and the measurements have yielded important results for better understanding these flows. The goals of this chapter are two-fold. First it is hoped that the reader gains an appreciation for the depth of information that is available when applying the CompLDV to turbulent flows. Second, the results give insight into flow phenomena that have only been fully-examined by DNS or in some cases not at all, and we wish to comment on these insights herein.

The information in this chapter is broken down into the different flows examined using the CompLDV. Within each of the flow cases, many different measurement quantities are considered, including measurement-volume-averaged velocity statistics, near-wall spatially-resolved velocity statistics, spectral results, velocity-acceleration statistics, Reynolds stress transport, and velocity gradient statistics. For each of these cases, previous work may be referenced that give representative results for similar flow cases, although many of the results to be presented are completely novel at the Reynolds numbers examined.

### 4.1 2D flat-plate turbulent boundary layers

The 2D flat-plate turbulent boundary layer (2DFPTBL) is the most-basic of TBL flows. In this flow a boundary layer is formed on the surface of a flat plate and the flow undergoes no mean pressure gradients in any direction. Analyses are possible in the near-wall region of the flow that lead to a few relationships that are thought to hold for all flat-plate TBLs. First, we may consider the momentum equation in the nearest-to-wall region.

Since all the flow velocities are zero directly on the wall, the convective terms are also zero there and in a flow without pressure gradients as in this one, the Navier-Stokes equations reduce to

$$\frac{1}{\rho} \left( \frac{\partial \tau_x}{\partial y} \right)_{y=0} = 0 \quad (4.1)$$

where  $\tau_x$  is the shear stress in the flow. The mean shear stress for this particular flow consists of the Newtonian viscous shear stress from the mean velocity gradient and the turbulent Reynolds shear stress from the Reynolds-averaged Navier-Stokes (RANS) equations:

$$\overline{\tau_x} = \mu \frac{\partial \overline{U}}{\partial y} - \overline{\rho uv} \quad (4.2)$$

Thanks to the no-slip condition, the turbulent stresses are necessarily zero at the wall, and the shear stress at the wall is exactly

$$\overline{(\tau_x)}_{wall} = \mu \left( \frac{\partial \overline{U}}{\partial y} \right)_{wall} \quad (4.3)$$

While this equation is valid directly on the wall, it is unclear how the contributions from the Reynolds shear stress will vary just above the wall. To further consider this, we may obtain the Taylor series expansions for the fluctuating velocities in the very near-wall region following the discussion by Simpson (2003) or Pope (2000):

$$\begin{aligned} u &= a_1 + b_1 y + c_1 y^2 + O(y^3) \\ v &= a_2 + b_2 y + c_2 y^2 + O(y^3) \\ w &= a_3 + b_3 y + c_3 y^2 + O(y^3) \end{aligned} \quad (4.4)$$

The constants  $a_1$ ,  $a_2$ , and  $a_3$  are all necessarily zero due to the no-slip condition and the non-penetration condition. Thanks to the no-slip condition also, the  $u$  and  $w$  velocities do not change anywhere on the surface so that

$$\left( \frac{\partial u}{\partial x} \right)_{wall} = \left( \frac{\partial u}{\partial z} \right)_{wall} = \left( \frac{\partial w}{\partial x} \right)_{wall} = \left( \frac{\partial w}{\partial z} \right)_{wall} = 0 \quad (4.5)$$

Now considering the continuity equation at the wall,

$$\left( \frac{\partial u}{\partial x} \right)_{wall} + \left( \frac{\partial v}{\partial y} \right)_{wall} + \left( \frac{\partial w}{\partial z} \right)_{wall} = 0 \quad (4.6)$$

we obtain the important result that

$$\left( \frac{\partial v}{\partial y} \right)_{wall} = 0 \quad (4.7)$$

This indicates that the term  $b_2$  in equations (4.4) is also zero to satisfy continuity at the wall. Then equation (4.4) to leading order becomes

$$\begin{aligned} u &\sim b_1 y \\ v &\sim c_2 y^2 \\ w &\sim b_3 y \end{aligned} \quad (4.8)$$

These results may now be used to estimate the near-wall variation of the Reynolds stresses as

$$\begin{aligned} \overline{u^2} &\sim \overline{b_1^2} y^2 \\ \overline{v^2} &\sim \overline{c_2^2} y^4 \\ \overline{w^2} &\sim \overline{b_3^2} y^2 \\ \overline{uv} &\sim \overline{b_1 c_2} y^3 \end{aligned} \quad (4.9)$$

We now see that  $\frac{\partial \overline{uv}}{\partial y} \sim y^2$  and thus changes slowly in the vicinity of the wall. This

important result indicates that there is a significant region of flow near the wall where only viscous shear stress exists. This region is known as the viscous sublayer and is characterized by a constant velocity gradient and thus a linear profile in the mean velocity.



It is now convenient to obtain the velocity and length scales that are indicative of this near-wall flow. As the discussion has indicated, this region of the flow is dominated by the wall shear and the action of the velocity gradient through the viscosity. An appropriate velocity scale based upon the wall shear may be proposed based upon dimensional analysis of the wall-shear and fluid density,

$$\tau_w \equiv \rho u_\tau^2 \quad (4.10)$$

Thus the wall friction velocity,  $u_\tau = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\nu \left( \frac{\partial U}{\partial y} \right)_{wall}}$ . The other parameter needed

for scaling is the viscosity, for which the kinematic viscosity is chosen to account for fluid density. The length scale for this region of the flow then becomes

$$\delta_\nu = \frac{\nu}{u_\tau} \quad (4.11)$$

The linear portion of the sublayer in dimensional terms is given as

$$U = y \left( \frac{\partial U}{\partial y} \right)_{wall} \quad (4.12)$$

We now apply the wall friction velocity and viscous length scale to obtain

$$U^+ = y^+ \quad (4.13)$$

where  $U^+ \equiv U/u_\tau$  and  $y^+ \equiv y/\delta_\nu$ .

The linear portion of the sublayer has been shown by Spalart (1988) as well as DeGraaff and Eaton (2000) to extend to approximately  $y^+=4$ , although it remains a good approximation for even larger values of  $y^+$ .

In another region where  $y^+$  is significantly greater than for the upper limits of the viscous sublayer, there exists a region in which a scaling transition must transpire, such that both viscous wall-scaling and outer flow boundary layer thickness scaling holds. Two prominent arguments exist for obtaining the correct functional relationships for this region due to von Kármán (1930) and Millikan (1938), but both are based upon dimensional analysis and the existence of a region where two length-scales are simultaneously valid. The analytical result for this region is a logarithmic variation in the velocity profile, given as

$$U^+ = \frac{1}{\kappa} \ln y^+ + B \quad (4.14)$$

where  $\kappa$  and  $B$  are empirical constants. Coles' values for these constants are most-often used for 2DFPTBLs,  $\kappa = 0.41$ ,  $B = 5.0$  (Coles 1956).

Beyond these fundamental developments for the mean velocity profile in the 2DFPTBL, significant work has been conducted to better-understand the turbulence structure in this flow. The pioneering work of Klebanoff (1956) in which he used hot-wires and all-analog equipment to obtain velocity time derivatives and two-point correlations that lead to robust estimates of velocity gradients throughout most of the 2DFPTBL at  $Re_\theta \approx 7300$  has stood the test of time as a standard reference for the turbulent kinetic energy (TKE) dissipation rate in this flow. A key conclusion from Klebanoff's work was that significant

anisotropies in the dissipation rate exist near the wall and must be modeled for proper computations. The low-Reynolds number 2DFPTBL direct numerical simulation (DNS) of Spalart (1988) revealed much about the behavior of this flow in the very-near-wall region, where the viscous-dominated flow is very similar for a wide-range of Reynolds numbers. Since the work of Spalart was a full simulation, direct estimates of the complete Reynolds stress transport budgets were obtained and the role of anisotropies in the turbulence structure was examined. In particular, valuable data for the velocity/pressure gradient correlation, a key term in the Reynolds stress transport that requires much effort from the modeling community (Pope 2000), was obtained. This term is responsible for distributing turbulence energy from the Reynolds stresses that are being produced by the mean velocity gradients to those normal-stress components that have no means of production. The results of Spalart indicate that in the log-layer, the velocity/pressure gradient correlations draw energy from the stream-wise normal-stress at twice the rate of the dissipation.

Similar studies to Klebanoff's were repeated with elaborate hot-wire arrangements by Balint et al. (1991) and Honkan and Andreopoulos (1997) among others. These studies measured all-nine components of the velocity gradient tensor in low Reynolds number turbulent boundary layer flows, but with probe-sizes that were several times the Kolmogorov length scales in the low Reynolds number flows. The data showed good agreement with the data of Spalart (1988) for a relatively small portion of the boundary layer that was reported, in the region of  $20 < y^+ < 80$ . The spectral distributions of the vorticity (enstrophy) were also examined and revealed a persistence of high-energy modes to higher wave-numbers than in the case of the velocity fluctuations.

#### 4.1.1 CompLDV Measurements

Four different Reynolds numbers for the 2DFPTBL have been measured using the CompLDV technologies. The parameters for the flows measured are given in Table 4.1.

**Table 4.1. Flow parameters for the 2DFPTBLs measured for the current study.**

$Re_\theta$	$U_\infty$ , m/s	$u_r$ , m/s	$\delta$ , mm	$\theta$ , mm	$\delta^*$	$H \equiv \delta^* / \theta$	$\delta_v$ , $\mu m$	Instrument
4113	13.78	0.537	39.9	4.96	6.82	1.375	30.7	C3
5929	21.00	0.777	39.5	4.70	6.44	1.370	21.2	C3
7497	27.95	1.022	39.0	4.46	6.00	1.345	16.1	C3
6350	26.86	1.032	38.0	3.9	5.1	1.31	16.0	C1

##### 4.1.1.1 The near-wall region

As discussed in Chapter 2, the CompLDV3 has powerful position-resolution capabilities that enable a detailed examination of phenomena that change rapidly in space. One possibility for utilizing these capabilities is in obtaining sub-measurement-volume profiles of velocity statistics in the 2DFPTBLs being studied. In the plots to follow for the Reynolds-averaged velocity statistics, dimensional plotting has been used so that the reader may gain an appreciation for the spatial scales that are being resolved. In these plots, five vertical positions of the measurement volume are used to obtain all the results.



Each symbol on the plot represents about 10 microns of vertical distance in the flow, so that the spatial resolution of these statistics is on that order.

The near-wall position-resolved CompLDV3 data are very valuable for determining the skin friction. As discussed briefly in Chapter 2, the skin friction for smooth walls can be obtained by using the nearest-wall data for the parallel-to-wall velocities. Fitting these data to the rigorous viscous sublayer equations (Rotta 1962; Tang 2004; Durst et al. 1995)

$$\begin{aligned} U &= \frac{\tau_{wx}}{\mu} y + \frac{1}{2\mu} \left. \frac{\partial P}{\partial x} \right)_{wall} y^2 + C_U y^4 \\ W &= \frac{\tau_{wz}}{\mu} y + \frac{1}{2\mu} \left. \frac{\partial P}{\partial z} \right)_{wall} y^2 + C_W y^4 \end{aligned} \quad (4.15)$$

where  $U$  and  $W$  are mean velocities in the plane parallel to the wall,  $\mu$  is the dynamic viscosity of the fluid,  $\left. \frac{\partial P}{\partial x} \right)_{wall}$  and  $\left. \frac{\partial P}{\partial z} \right)_{wall}$  are the mean wall pressure gradients,  $C_U = f \left\{ \frac{\partial^3 (\overline{uv})}{\partial y^3} \right\}$ , and  $C_W = f \left\{ \frac{\partial^3 (\overline{vw})}{\partial y^3} \right\}$ . This is observed to yield reliable shear stress estimates for  $y^+ < 9$  (Tang 2004).

The fit of equation (4.15) for the nearest-wall data were used to obtain the skin-friction velocities in Table 4.1, and to obtain the wall velocity gradients plotted in Figure 4.1-Figure 4.3 for the three Reynolds numbers measured with the CompLDV3. As may be noted, there is excellent agreement between these nearest-wall data and the linear sublayer relationship equivalent to equation (4.13).

In addition to the mean-velocity position-resolved profiles, the Reynolds stresses are also resolved within the measurement volume. These results are given in Figure 4.4-Figure 4.9 for the normal and shear stresses for each of the Reynolds numbers measured with the CompLDV3. These measurements are the most-highly-spatially-resolved measurements for these quantities at these flow scales that exist anywhere to the author's knowledge.

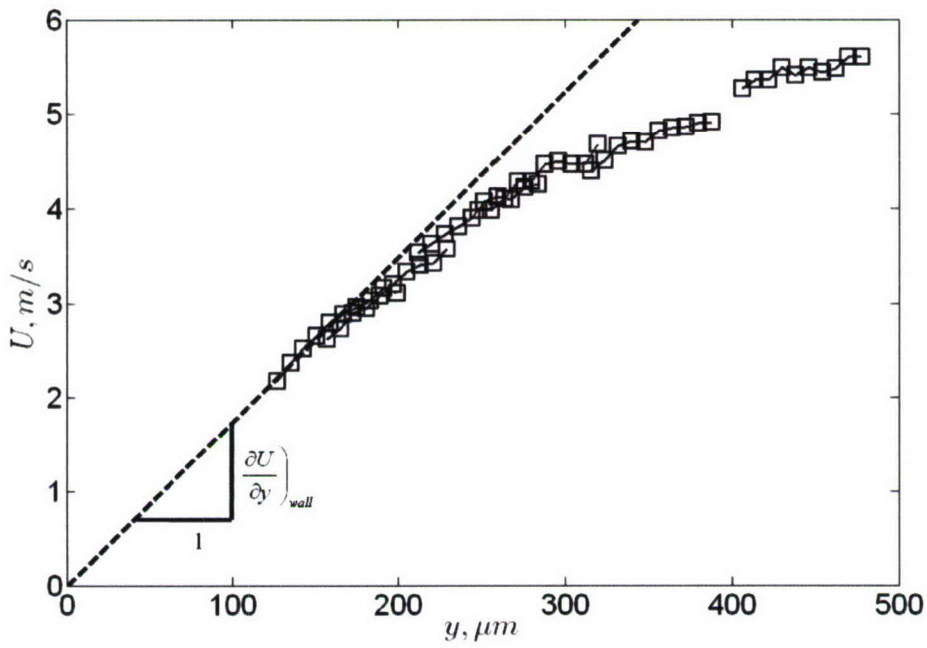


Figure 4.1. Resolved measurements of the stream-wise mean velocity in the 2DFPTBL at  $\text{Re}_\theta = 4113$ .

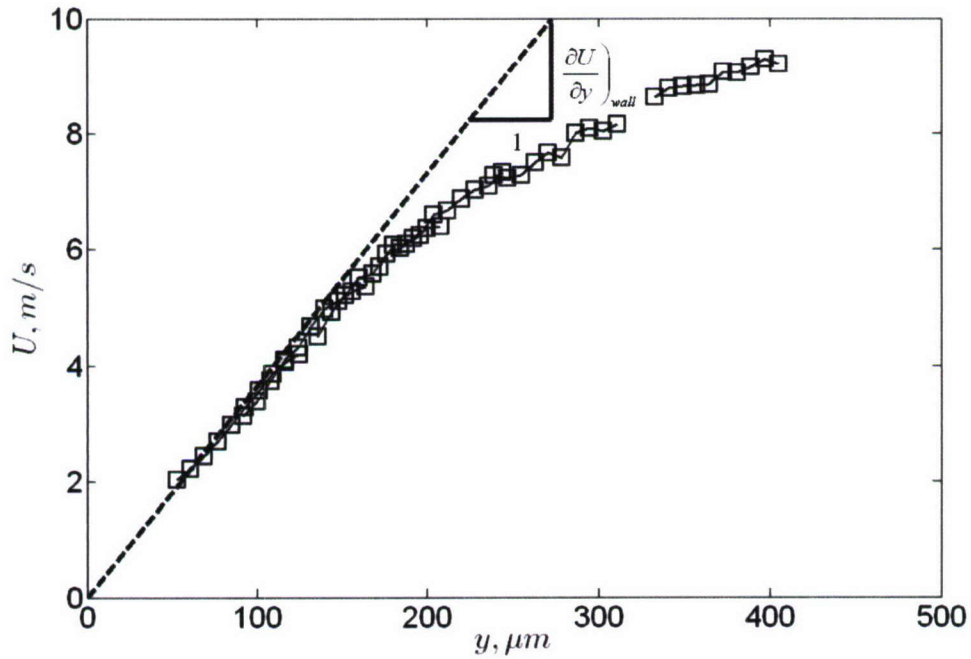


Figure 4.2. Resolved measurements of the stream-wise mean velocity in the 2DFPTBL at  $\text{Re}_\theta = 5929$ .



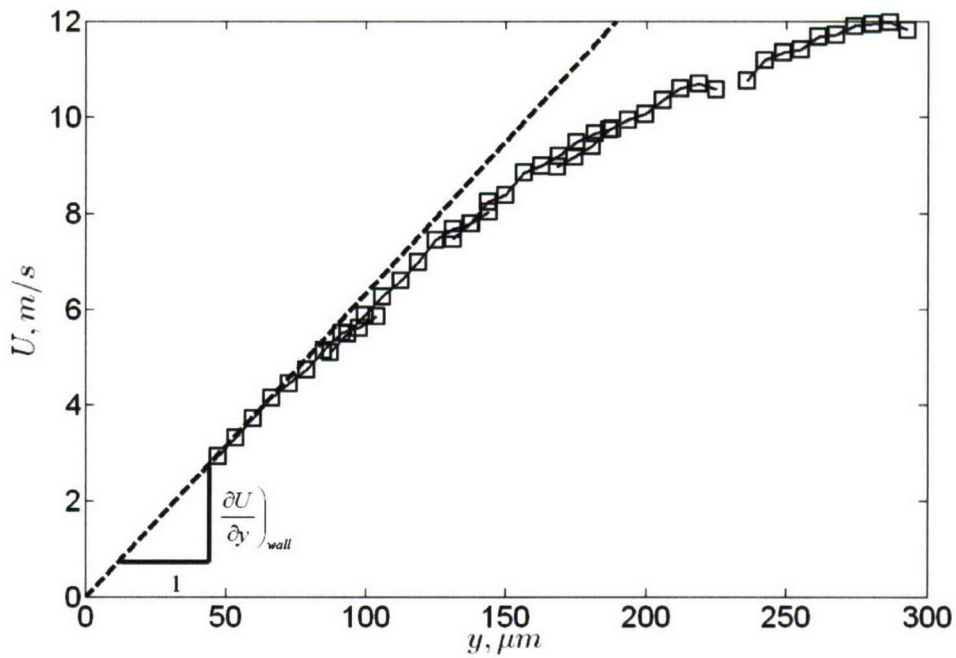


Figure 4.3. Resolved measurements of the stream-wise mean velocity in the 2DFPTBL at  $Re_\theta = 7497$ .

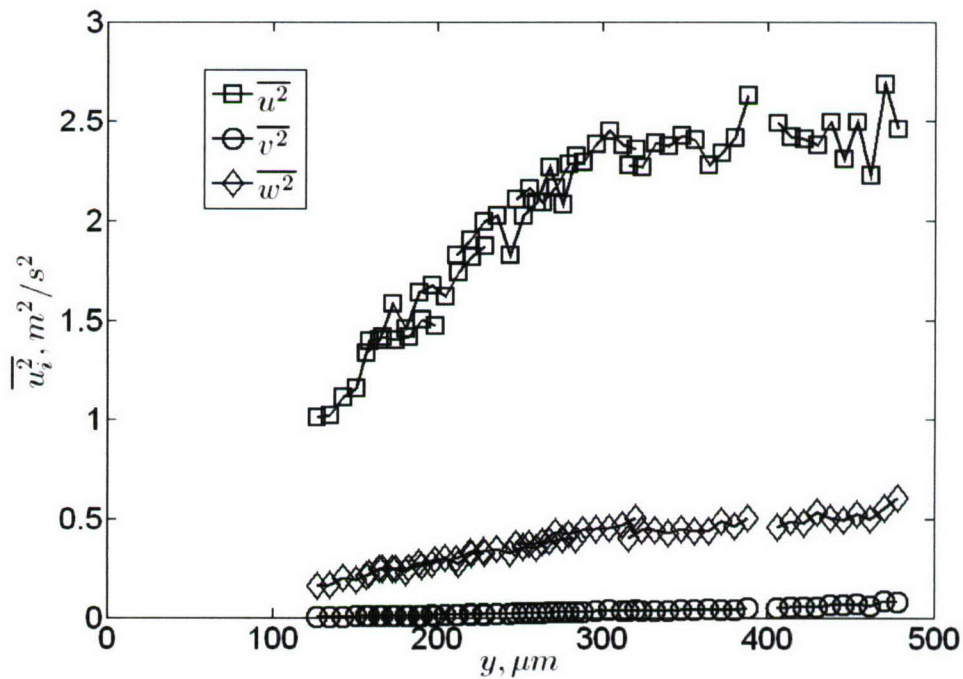


Figure 4.4. Resolved measurements of the Reynolds normal stresses in the 2DFPTBL at  $Re_\theta = 4113$ .

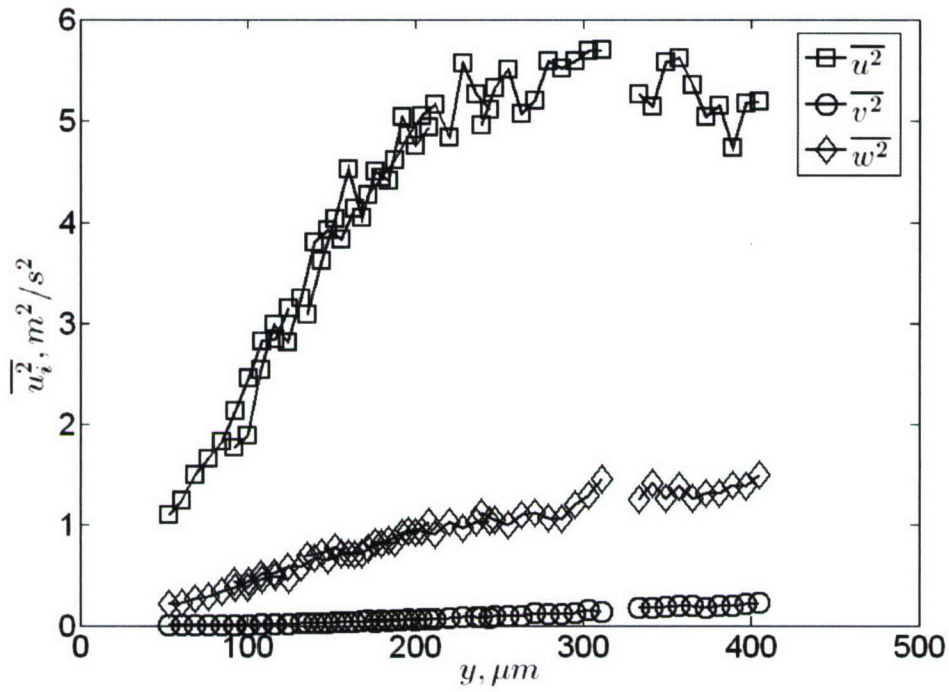


Figure 4.5. Resolved measurements of the Reynolds normal stresses in the 2DFPTBL at  $Re_\theta = 5929$ .

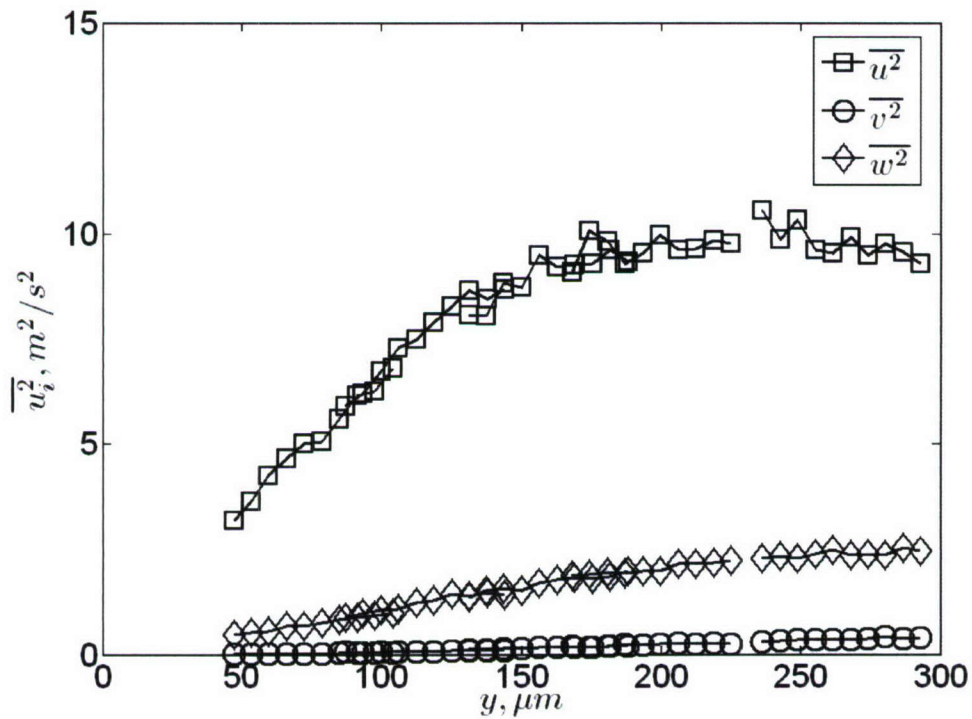


Figure 4.6. Resolved measurements of the Reynolds normal stresses in the 2DFPTBL at  $Re_\theta = 7497$ .



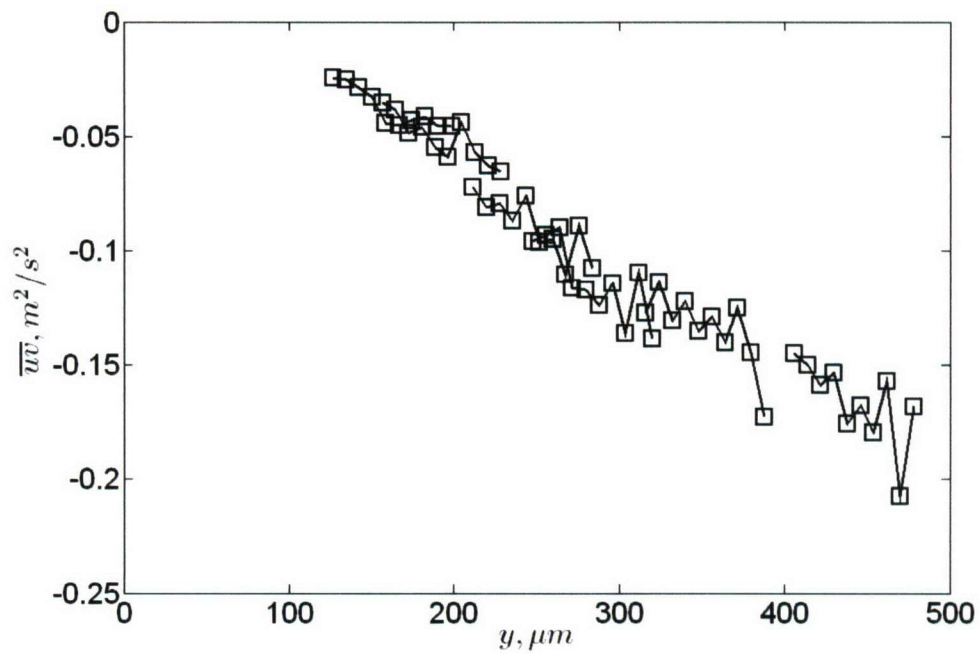


Figure 4.7. Resolved measurements of the Reynolds normal stresses in the 2DFPTBL at  $Re_\theta = 4113$ .

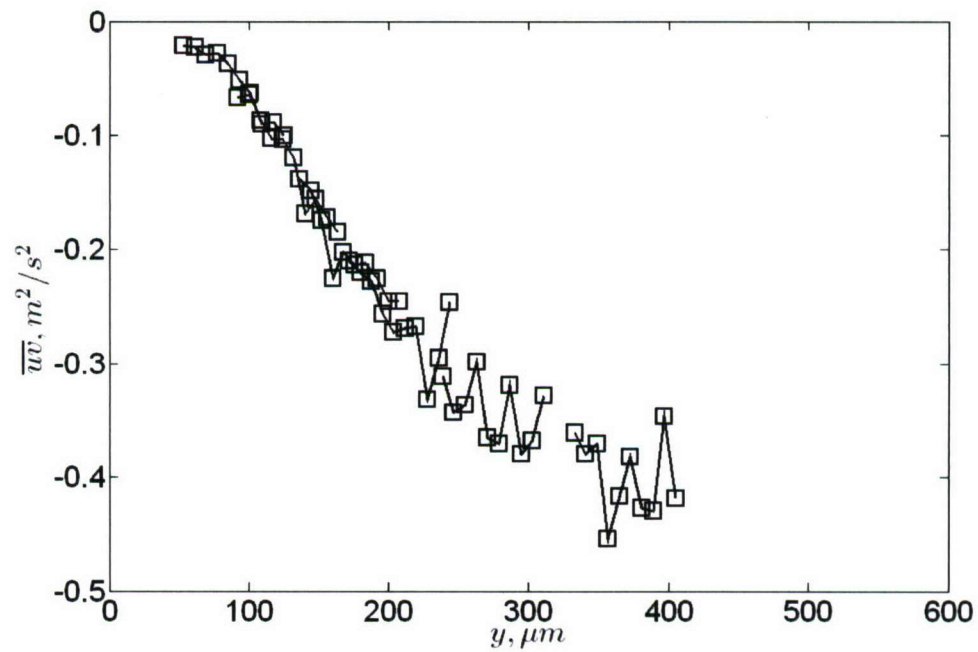
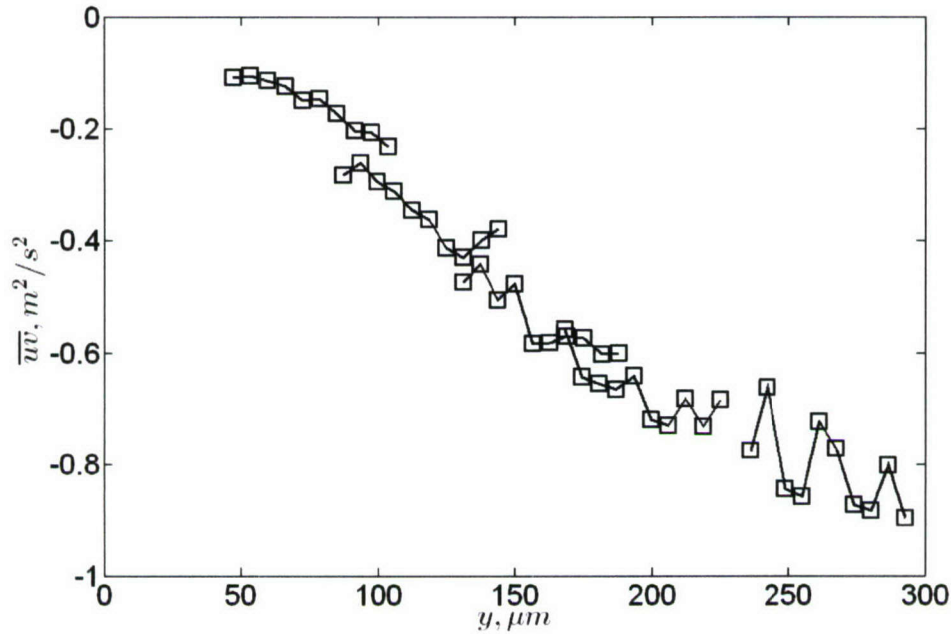


Figure 4.8. Resolved measurements of the Reynolds normal stresses in the 2DFPTBL at  $Re_\theta = 5929$ .



**Figure 4.9. Resolved measurements of the Reynolds normal stresses in the 2DFPTBL at  $Re_\rho = 7497$ .**

#### 4.1.2 Volume-averaged velocity statistics

To obtain full-profiles of data for comparisons to previous results, volume-averaged statistics were compiled. In the case of the CompLDV3 results to be presented, the option of limiting the spatial extent of the measurement volume existed due to the position resolution of the measurements. In order to get fully-converged statistics at all points, it was decided to limit the measurement volume diameter to about 100 microns, since this spatial range still contains a sufficient number of velocity estimations for converged statistics. By allowing volume-averaging, the aspect of velocity-gradient broadening must be considered and applied. In general, the velocity gradient broadening for a Reynolds stress term may be extended from the work of Durst et al. (1995) to obtain

$$\overline{\Delta u_i u_j} = \frac{d^2}{12} \frac{\partial U_i}{\partial y} \frac{\partial U_j}{\partial y} \quad (4.16)$$

where  $\overline{\Delta u_i u_j}$  is the apparent turbulence measured due to the velocity gradients. For all the volume-averaged measurements presented, this correction has been subtracted from the raw data to obtain the best-estimate for the velocity statistics.

The stream-wise velocity data are plotted on viscous wall-scaling in Figure 4.10. The data exhibit excellent agreement with this scaling out to very high values of  $y^+$ , as expected for these Reynolds numbers. A comparison to an independent data set of DeGraaff and Eaton (2000) for the stream-wise velocities is given in Figure 4.11—again with excellent agreement among the data sets. The Reynolds normal stresses are

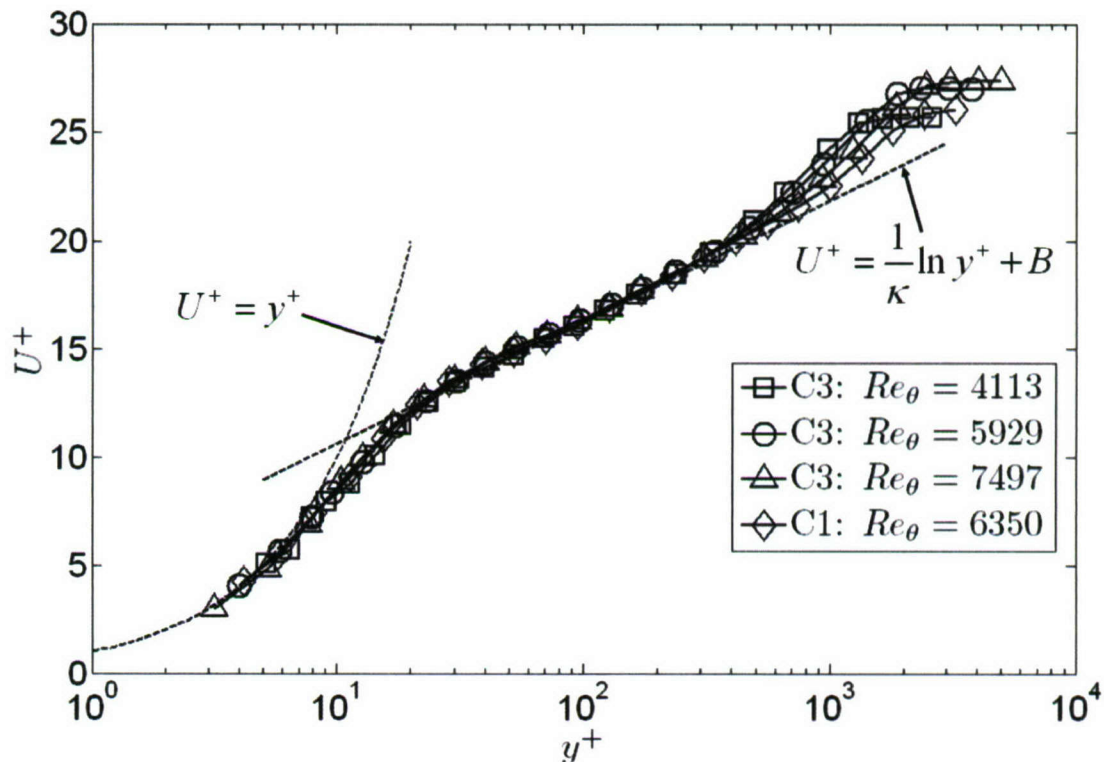


compared in Figure 4.12-Figure 4.14. The Reynolds shear stress scales relatively well on wall variables as seen in Figure 4.15.

As many other researchers have noted, a failure of collapse among the Reynolds numbers for all except the nearest-wall data is evident in the peak-region of the stream-wise normal stress. In the present work, this is also seen to be the case in the span-wise normal stress which obeys similar near-wall functional relationships as discussed in section 4.1. To improve this situation, DeGraaff and Eaton have proposed the use of a mixed scaling

parameter,  $u_\tau U_\infty = \frac{\sqrt{C_f/2}}{u_\tau^2}$  for the stream-wise normal stress. The present data is plotted

with those of DeGraaff and Eaton in Figure 4.16 using the mixed scaling. Further, the span-wise normal stress is plotted with the same scaling in Figure 4.17. Both these plots exhibit excellent collapse of the data for  $y^+ < 50$ . This improvement has been achieved by recognizing the role of the Reynolds number in the peak-value of the Reynolds normal stresses parallel to the wall.



**Figure 4.10. Stream-wise mean velocities in the 2DFPTBL at four Reynolds numbers. C1: Measurements acquired with the ComplDV1; C3: Measurements acquired with the ComplDV3.**

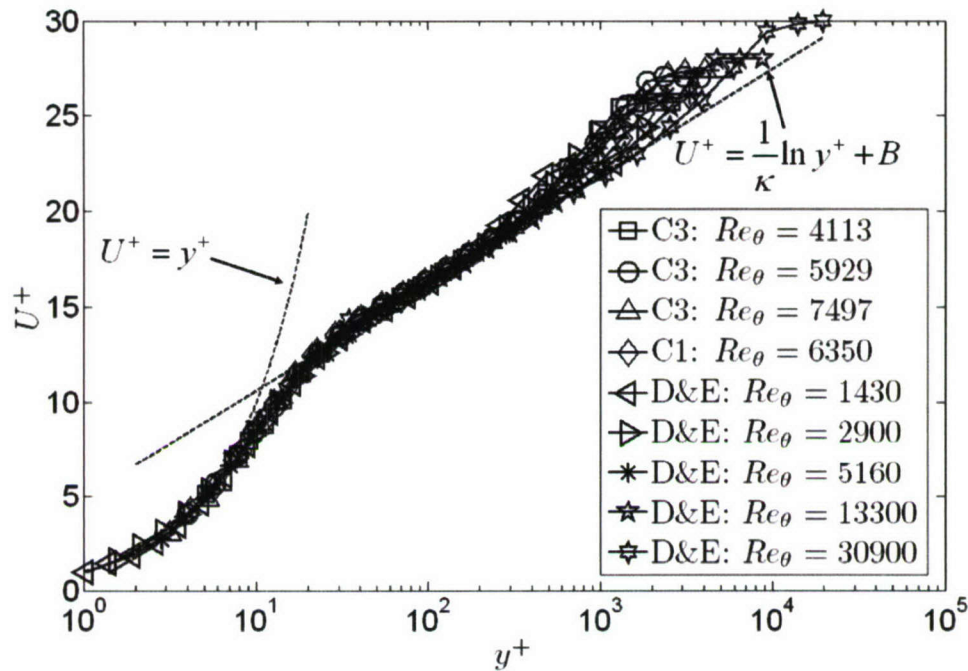


Figure 4.11. Comparison of the stream-wise mean velocities in the 2DFPTBL compared to the results of DeGraaff and Eaton (2000). C1: Measurements acquired with the CompLDV1; C3: Measurements acquired with the CompLDV3; D&E: measurements reported by DeGraaff and Eaton (2000).

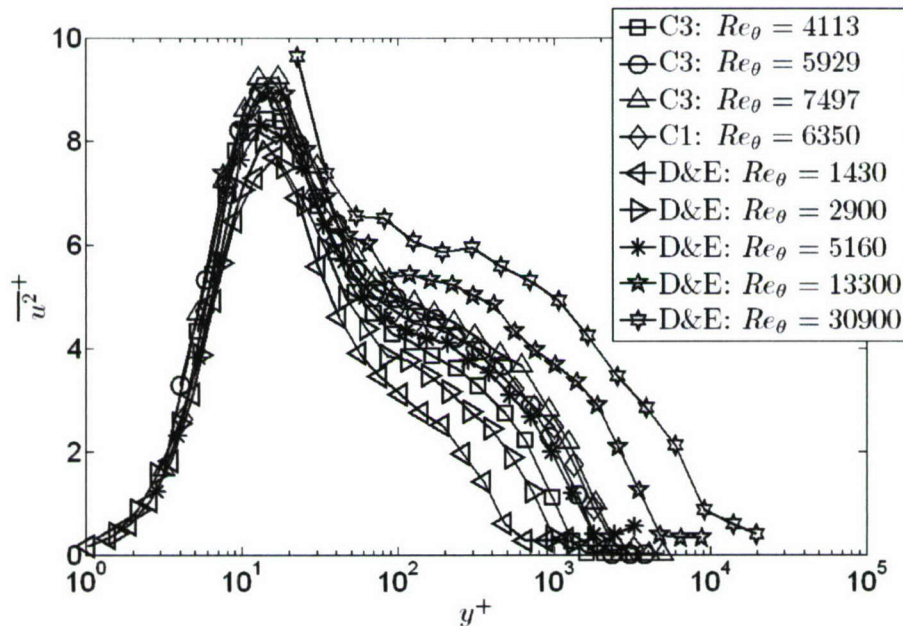


Figure 4.12. Comparison of the stream-wise Reynolds stress in the 2DFPTBL compared to the results of DeGraaff and Eaton (2000). C1: Measurements acquired with the CompLDV1; C3: Measurements acquired with the CompLDV3; D&E: measurements reported by DeGraaff and Eaton (2000).



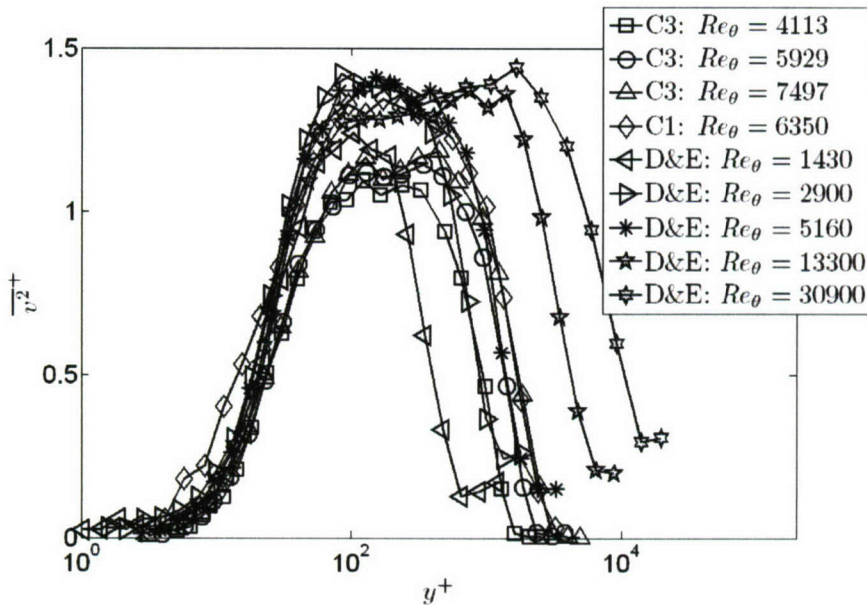


Figure 4.13. Comparison of the normal-to-wall Reynolds stress in the 2DFPTBL compared to the results of DeGraaff and Eaton (2000). C1: Measurements acquired with the ComplDV1; C3: Measurements acquired with the ComplDV3; D&E: measurements reported by DeGraaff and Eaton (2000).

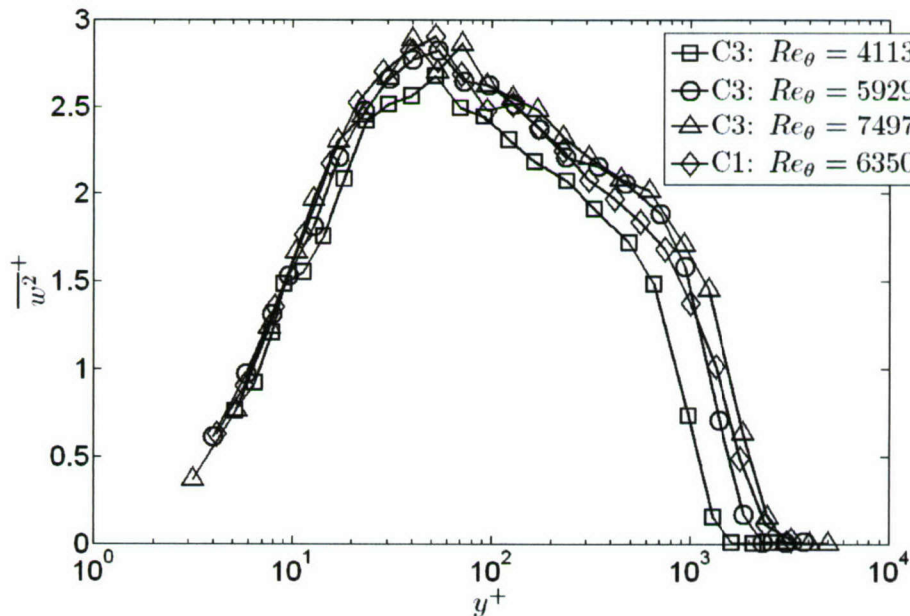


Figure 4.14. Span-wise Reynolds stress in the 2DFPTBL at four Reynolds numbers. C1: Measurements acquired with the ComplDV1; C3: Measurements acquired with the ComplDV3.

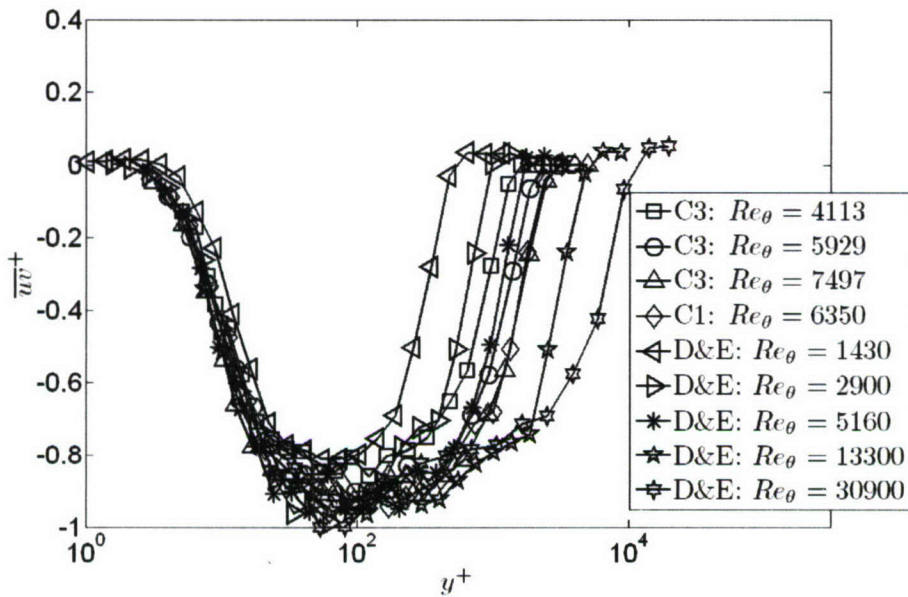


Figure 4.15. Comparison of the Reynolds shear stress in the 2DFPTBL compared to the results of DeGraaff and Eaton (2000). C1: Measurements acquired with the ComplDV1; C3: Measurements acquired with the ComplDV3; D&E: measurements reported by DeGraaff and Eaton (2000).

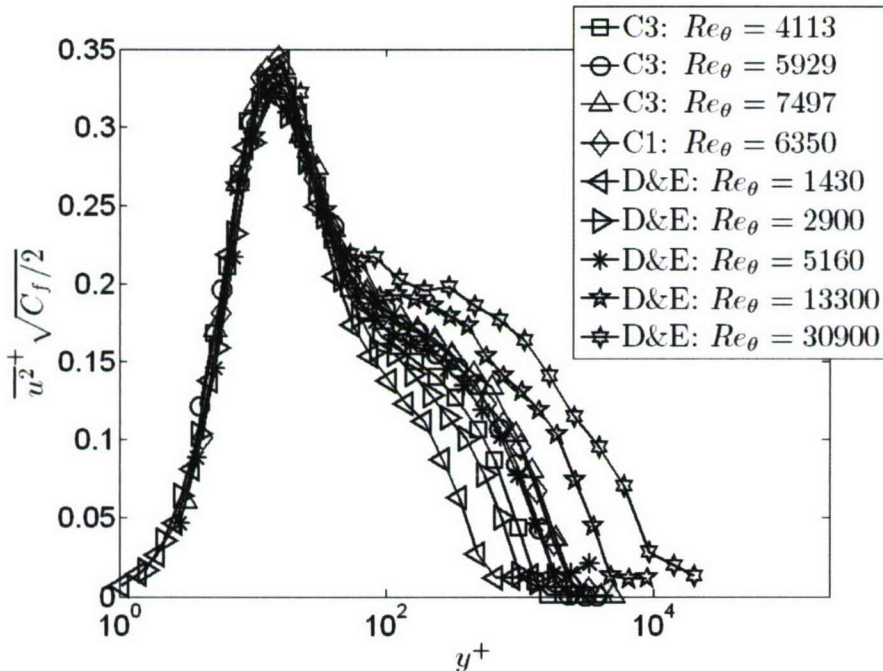
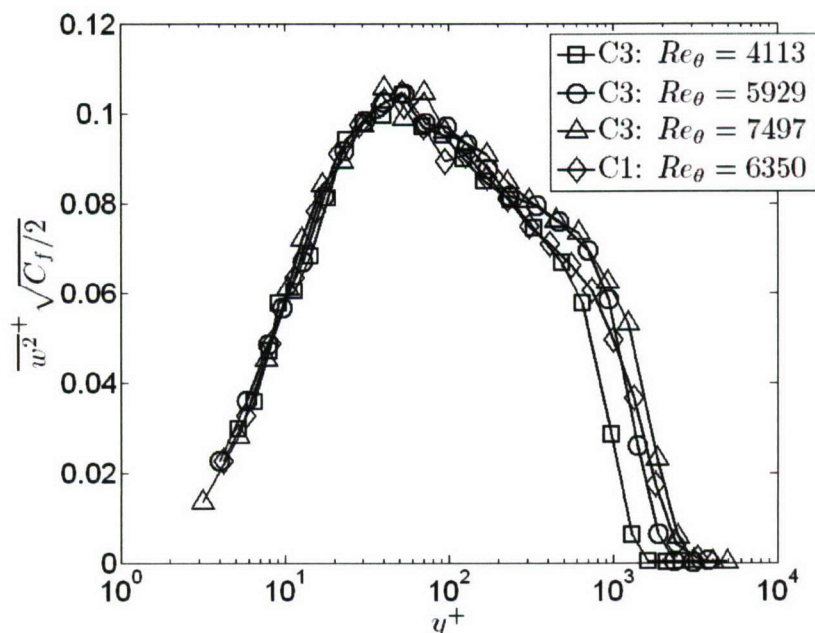


Figure 4.16. Comparison of the stream-wise Reynolds normal stress in the 2DFPTBL compared to the results of DeGraaff and Eaton (2000) with mixed scaling to account for Reynolds number effects. C1: Measurements acquired with the ComplDV1; C3: Measurements acquired with the ComplDV3; D&E: measurements reported by DeGraaff and Eaton (2000).





**Figure 4.17. Span-wise Reynolds normal stress in the 2DFPTBL with mixed scaling to account for Reynolds number effects. C1: Measurements acquired with the CompLDV1; C3: Measurements acquired with the CompLDV3; D&E: measurements reported by DeGraaff and Eaton (2000).**

### 4.1.3 Spectral analysis

Typical all-channel-coincident average data rates for the CompLDV systems in the log-layer and above range from 20-50kHz in the low-speed facilities used. These data give information content on a wide range of time-scales since contiguous sampling is done in lots of 0.54s and these lots of data contain information at frequencies even higher than the average data rate, as discussed in section 3.2. To analyze these data, spectral analysis is considered in this section by obtaining the auto- and cross-spectra of velocity fluctuations in the 2DFPTBL.

The velocity frequency spectrum may be defined as

$$E_{ij}(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} R_{ij}(\tau) \exp(-i\omega\tau) d\tau \quad (4.17)$$

where  $R_{ij}(\tau)$  is the time-delay correlation of variables  $u_i$  and  $u_j$ ,  $\omega$  is the angular frequency, and  $\tau$  is the time-delay. The spectrum has the property that

$$\overline{u_i u_j} = \int_0^{\infty} |E_{ij}(\omega)| d\omega \quad (4.18)$$

so that the total energy in the spectrum equals the covariance of the fluctuations forming the spectrum.

The frequency content of velocity fluctuations may be interpreted to give several results. The most fundamental of these is the universal-equilibrium results of Kolmogorov (1991) that states that all fully-turbulent flows of sufficient Reynolds number contain a universal ‘inertial subrange’ for which the cascade of turbulence energy through the scales of motion is dominated entirely by the flow dissipation rate. Further, Kolmogorov asserted that in the same circumstances there must also exist a universal function for the cascade of turbulence energy in extremely high wave-numbers that is determined entirely by the viscosity and the dissipation rate. These postulates may be described as the condition of ‘local isotropy’ and form a universal similarity in the velocity spectra cascade for all turbulent flows of ‘sufficient Reynolds number’ and ‘sufficiently far from flow boundaries’. The result of these assertions is the famous Kolmogorov  $-5/3$  spectra:

$$E_{11}(\kappa_1) = C_1 \varepsilon^{2/3} \kappa_1^{-5/3} \quad (4.19)$$

where  $\kappa_1$  is the wave-number in the  $x_1$  direction and  $C_1$  is a ‘universal’ constant. For comparison to the frequency spectrum already discussed, Taylor’s hypothesis may be invoked as an approximation to convert the wave-number to a frequency by the following relationship

$$\kappa_1 \approx \frac{\omega}{2\pi U_1} \quad (4.20)$$

While the Kolmogorov relationship in equation (4.19) is one of the firmest foundations in all of turbulence research and is evoked often for the purpose of modeling and analysis, two key aspects limit its use in the turbulent boundary layer, particularly near-walls. First and foremost, the requirement of being ‘sufficiently far from boundaries’ is inherently violated through some region of the *boundary layer*. Second, the universality of equation (4.19) is in question as it has been discovered that it is likely no-flows are fully-turbulent by the strictest definition, as intermittency of dissipation occurs (see for example Pope 2000, pp. 255-262). While the later shortcoming results in small deviations from the predictions of local-isotropy and may be generally compensated by additional hypotheses, it nonetheless reveals the limitations of a ‘universal’ description in the field of turbulence research.

Despite the discrepancies resulting from internal intermittency just discussed, the concept of local-isotropy is still effective away from boundaries. The working-applicability of this concept was assessed in the outer-regions of turbulent boundary layers by Saddoughi and Veeravalli (1994). They explored the spectral distributions of all three simultaneously-measured velocity components in the outer-layer of very high Reynolds number 2DFPTBLs and found substantial evidence for local-isotropy over significant wave-number ranges, even in the presence of the inherently anisotropic mean shear stress.

Lumley (1967) has derived a power-law similar to the Kolmogorov spectrum but for cross-spectra for Reynolds shear stresses. The universal functional form follows when considering the dimensional scaling of the cross-spectrum with the mean strain rate and the dissipation rate in a wave-number range of overlap between the Kolmogorov scales and the strain rate scales. The result is

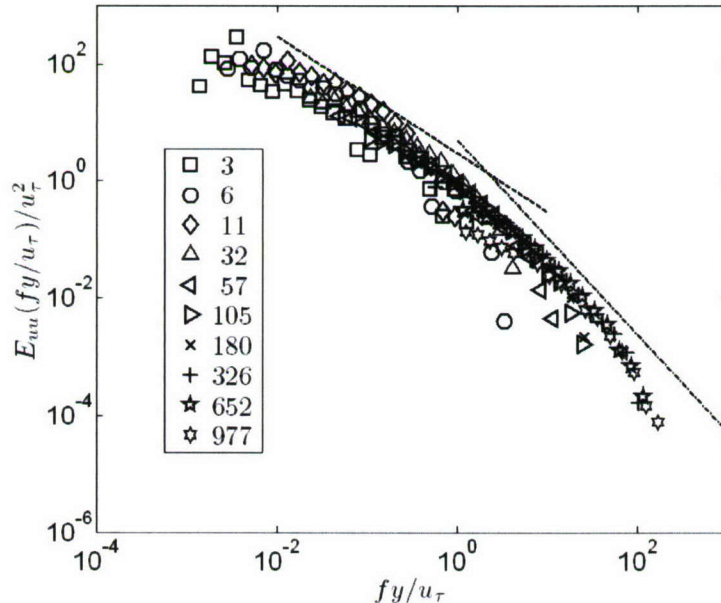
$$E_{12}(\kappa_1) = -C_{12} S \varepsilon^{1/3} \kappa_1^{-7/3} \quad (4.21)$$

where  $C_{12}$  is a ‘universal’ constant and  $S$  is the mean strain rate in the flow.



A frequency-based analog to the wave-number scaling of Perry et al. (1985) has been used to present the velocity spectral data for the first three cases in Table 4.1. This scaling is based upon a conceptual model for the coherent structures within the boundary layer. The idea presented is that of an attached-eddy concept where it is surmised that some region of the velocity spectrum is dominated by eddies with a size-scale on the order of the distance from the wall. Extensive data at three-different Reynolds numbers using this scaling is presented in Figure 4.18-Figure 4.29. Significant regions of the stream-wise Reynolds normal stress collapse for all three Reynolds numbers for heights above  $y^+ = 20$ . Regions of the Kolmogorov ( $-5/3$ ) power law are present more apparently in the two higher-Reynolds-number cases than in the lowest Reynolds number case where the slope for the subrange is greater than  $(-5/3)$ . The  $(-1)$  slope in the plots is derived based upon the turbulence production due to the presence of a mean velocity gradient (Tchen 1953). A region within each of the stream-wise normal stress spectra exhibits a  $(-1)$  slope for the lower-frequencies. The span-wise normal stress spectra scale similarly as the stream-wise. The normal-to-wall spectra do not scale as well using these parameters, particularly nearer the wall.

The shear stress cross-spectra exhibit the  $(-7/3)$  slope for high frequencies, and the collapse of the data using the wall-scaling is excellent, which may be expected in a coherent-structure-based model since it is known that coherent structures are responsible for the shear stresses in this flow (Robinson 1991).



**Figure 4.18.** Energy spectra on viscous wall scaling for the stream-wise Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 4113$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .

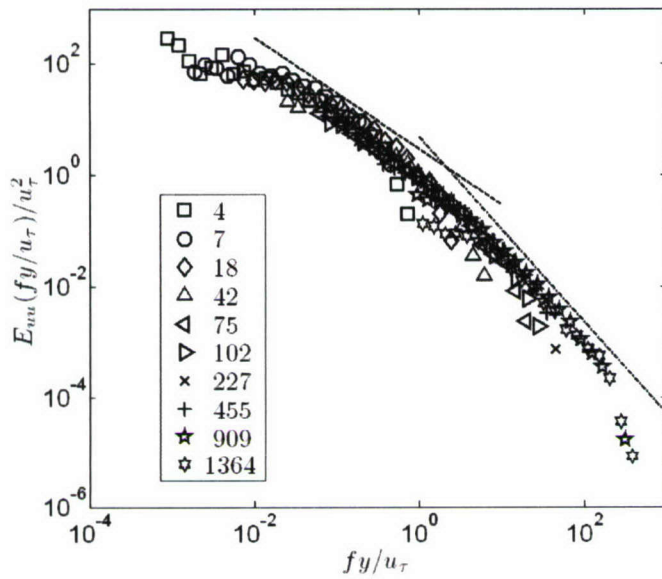


Figure 4.19. Energy spectra on viscous wall scaling for the stream-wise Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 5929$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^1$  and  $\sim f^{5/3}$ .

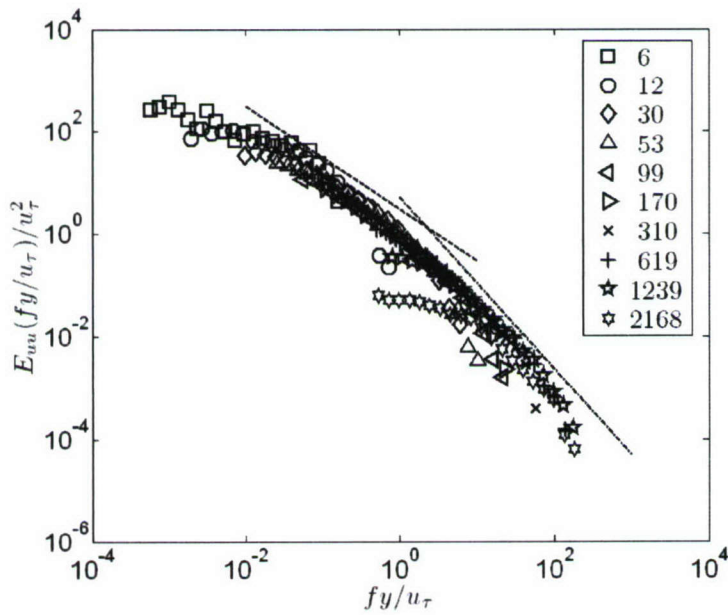
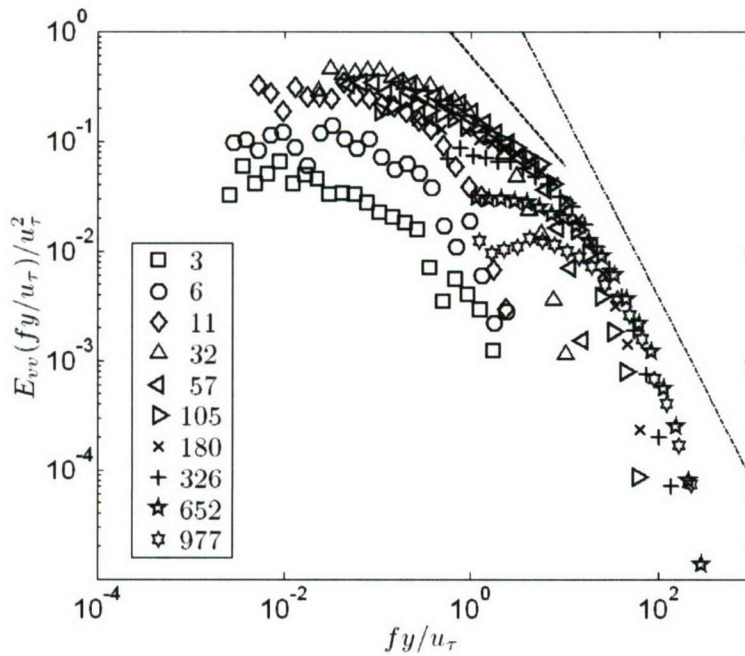
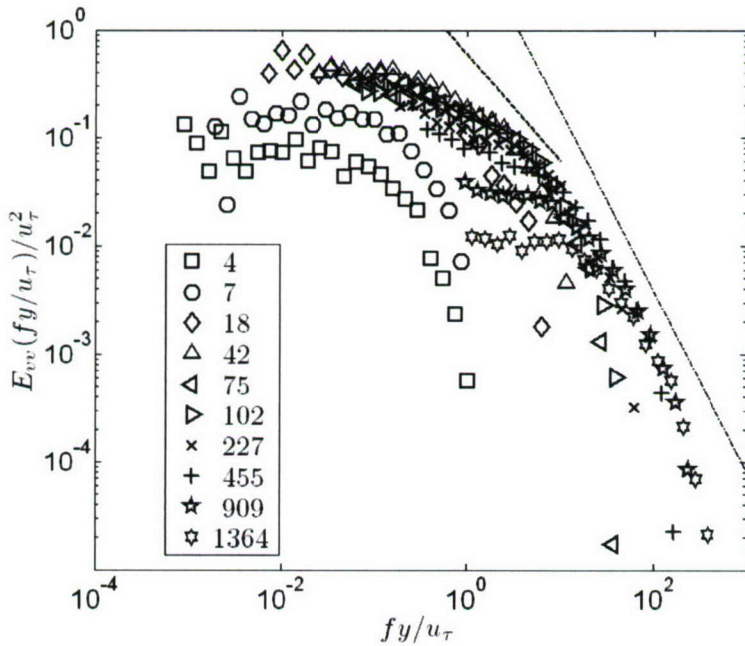


Figure 4.20. Energy spectra on viscous wall scaling for the stream-wise Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 7497$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^1$  and  $\sim f^{5/3}$ .

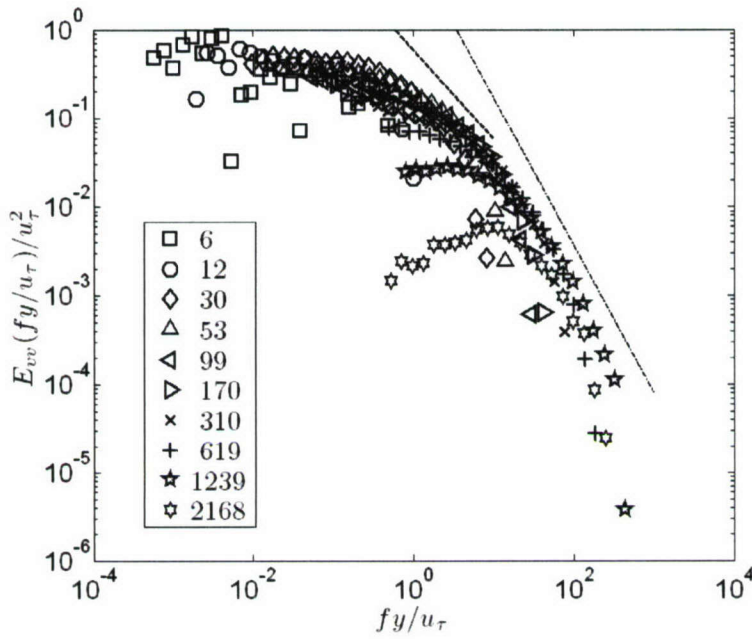




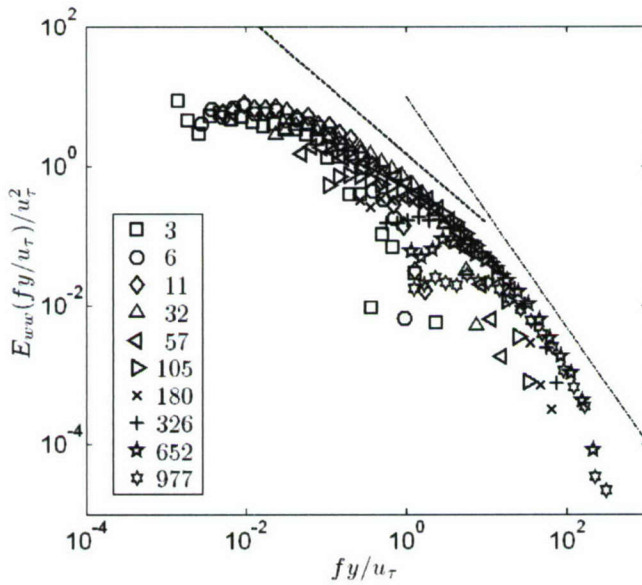
**Figure 4.21.** Energy spectra on viscous wall scaling for the normal-to-wall Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 4113$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .



**Figure 4.22.** Energy spectra on viscous wall scaling for the normal-to-wall Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 5929$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .

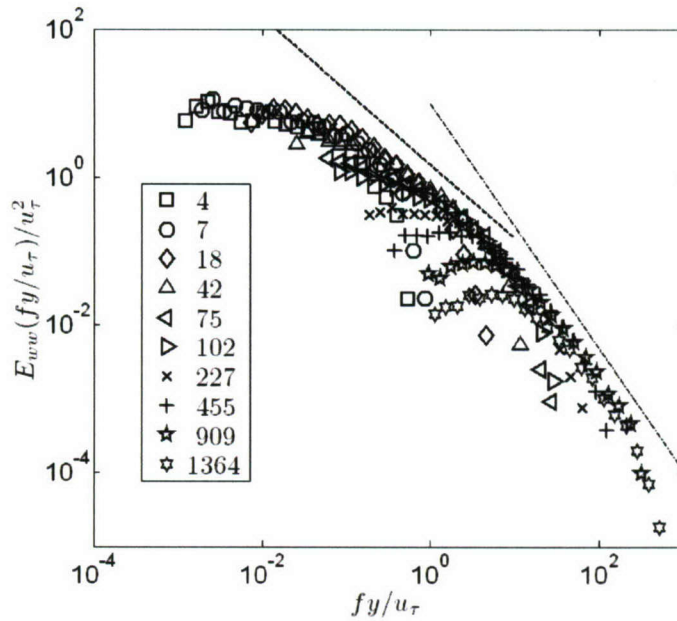


**Figure 4.23.** Energy spectra on viscous wall scaling for the normal-to-wall Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 7497$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .

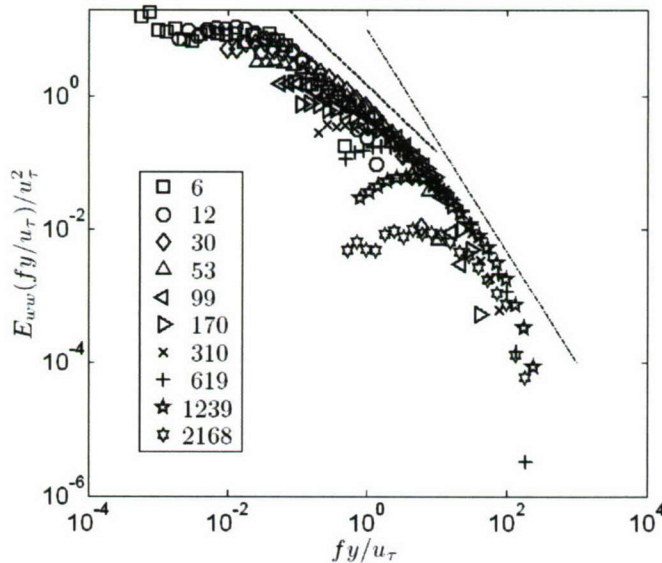


**Figure 4.24.** Energy spectra on viscous wall scaling for the span-wise Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 4113$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .





**Figure 4.25.** Energy spectra on viscous wall scaling for the span-wise Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 5929$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .



**Figure 4.26.** Energy spectra on viscous wall scaling for the span-wise Reynolds normal stress in the 2DFPTBL at  $Re_\theta = 7497$ . The legend gives vertical locations in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .

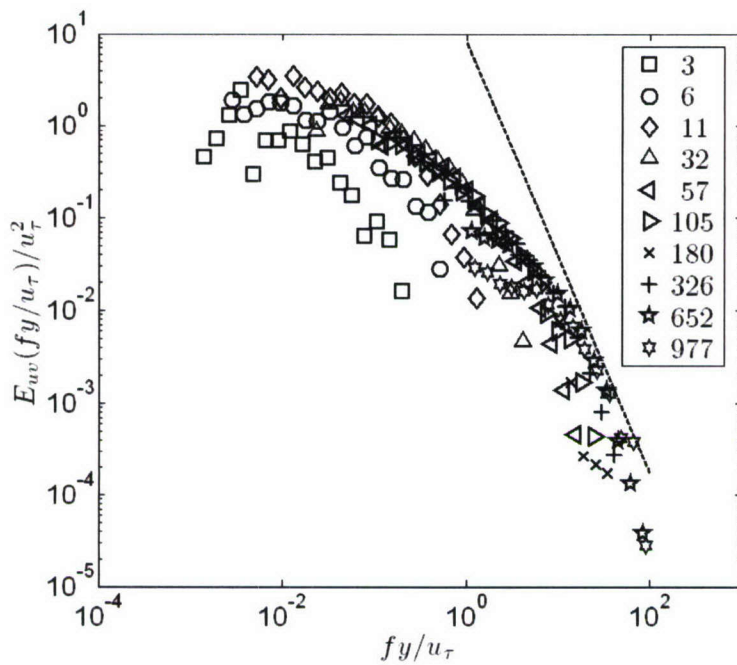


Figure 4.27. Energy spectra on viscous wall scaling for the Reynolds shear stress in the 2DFPTBL at  $Re_\theta = 4113$ . The legend gives vertical locations in  $y^+$ . The line in the plot is at a slope of  $\sim f^{7/3}$ .

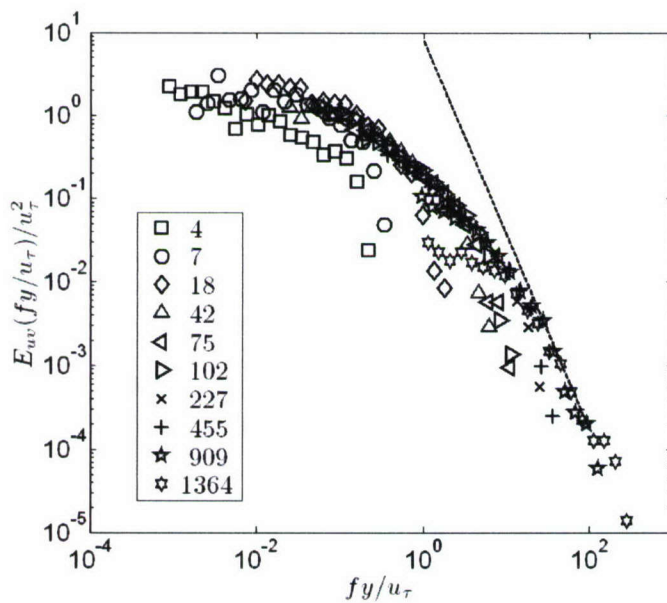
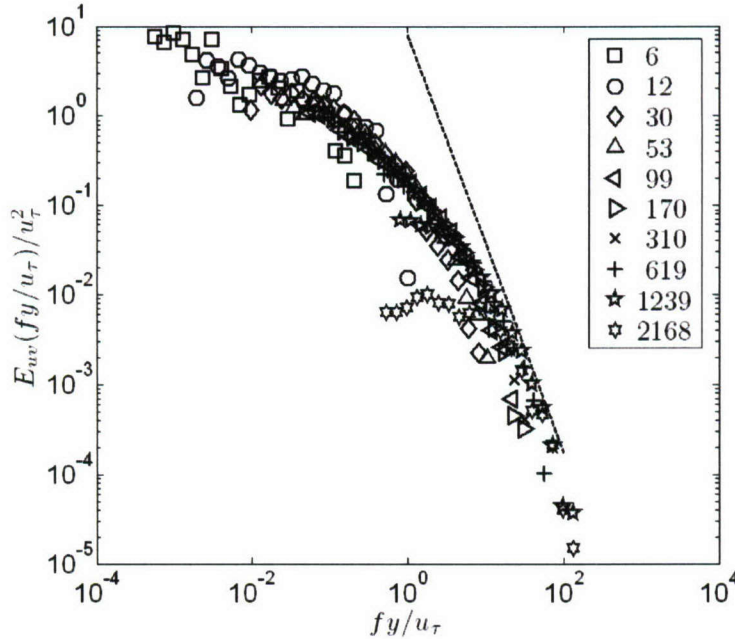


Figure 4.28. Energy spectra on viscous wall scaling for the Reynolds shear stress in the 2DFPTBL at  $Re_\theta = 5929$ . The legend gives vertical locations in  $y^+$ . The line in the plot is at a slope of  $\sim f^{7/3}$ .





**Figure 4.29.** Energy spectra on viscous wall scaling for the Reynolds shear stress in the 2DFPTBL at  $Re_\theta = 7497$ . The legend gives vertical locations in  $y^+$ . The line in the plot is at a slope of  $\sim f^{7/3}$ .

#### 4.1.3.1 Reynolds shear stress coherency

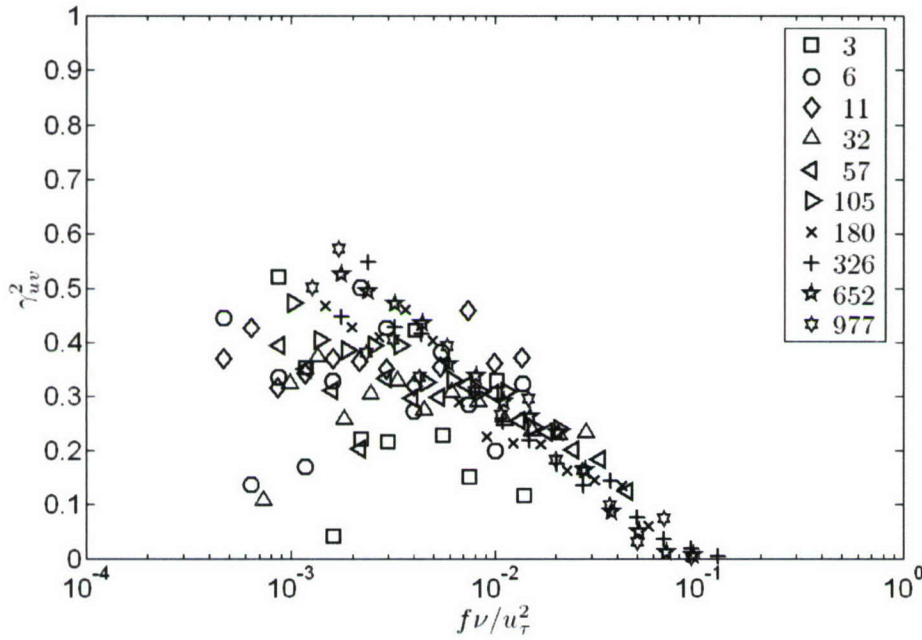
The velocity spectral data may be used to obtain spectral coherency information. The coherency is a sort of correlation coefficient in the frequency domain and may be defined as

$$\gamma_{ij}^2 \equiv \frac{|E_{ij}|^2}{E_{ii}E_{jj}} \quad (4.22)$$

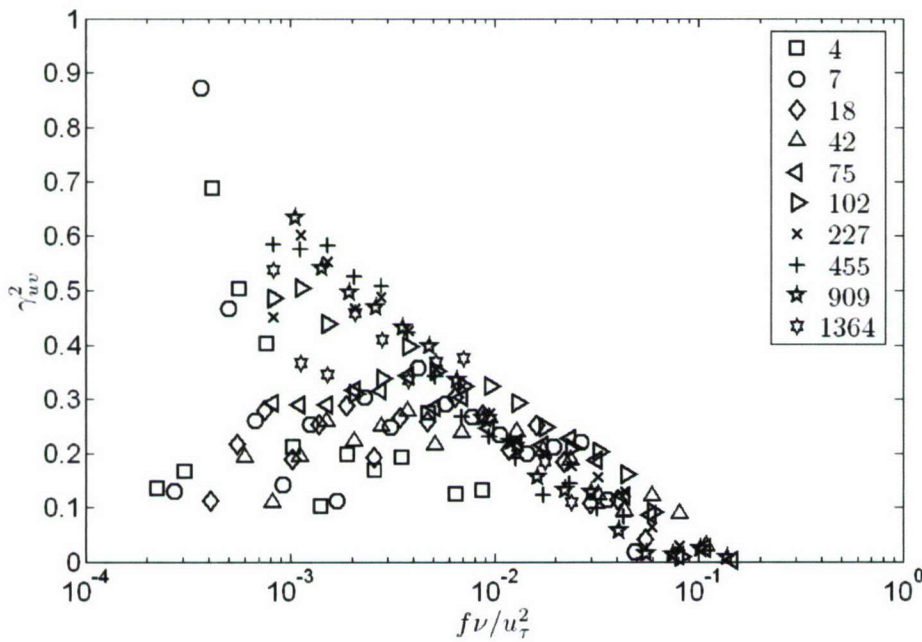
where no summing is employed. This information is quite rich, as Ha and Simpson (1993) and Saddoughi and Veeravali (1994) have discussed. The coherency function has a maximum range of  $[0 \ 1]$  and indicates what fraction of turbulence motion is being correlated to produce shear stresses in the frequency ranges measured.

The coherency data for the Reynolds shear stress,  $\overline{uv}$ , in the 2DFPTBL are given in Figure 4.30-Figure 4.32 for three Reynolds numbers. In frequency regions where the coherency is very low, it may be regarded that the turbulence energy in those regions is uncorrelated and simple modeling may be used to account for the energy there. This is particularly useful in large-eddy simulation (LES) studies, where a decision must be made as to what range of scales to simulate directly while leaving the remaining high wave-numbers to be modeled. The coherency data presented herein indicate that a significant amount of the shear-stress-producing motions happen at frequencies lower

than  $f\nu/u_\tau^2 \approx 0.1$  throughout the boundary layer. Unfortunately, this still indicates a large range of frequencies (wave-numbers) that exhibit significant coherency.

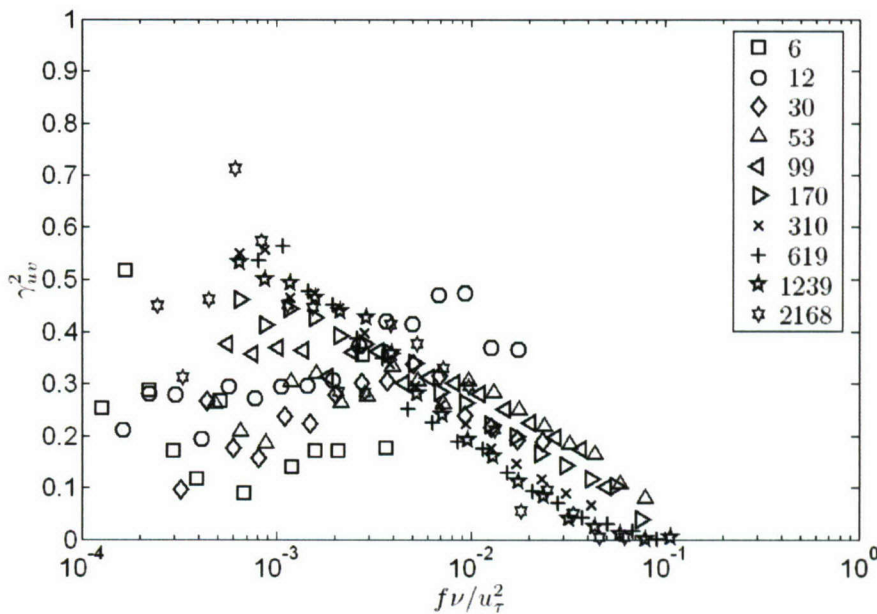


**Figure 4.30.** Coherency measurements for the Reynolds shear stress for the 2DFPTBL at  $Re_\theta = 4113$ . The legend gives vertical locations in  $y^+$ .



**Figure 4.31.** Coherency measurements for the Reynolds shear stress for the 2DFPTBL at  $Re_\theta = 5929$ . The legend gives vertical locations in  $y^+$ .





**Figure 4.32.** Coherency measurements for the Reynolds shear stress for the 2DFPTBL at  $Re_\theta = 7497$ . The legend gives vertical locations in  $y^+$ .

#### 4.1.4 Turbulence transport

Understanding the turbulence transport in shear flows is a key topic in fundamental research due to the immediate implications that these equations have on modeling for the Reynolds stresses. Although the most-obvious uses for the models of the turbulence structural (transport) terms is in Reynolds-averaged Navier-Stokes (RANS) solutions, other solution techniques benefit from the information obtained by studying Reynolds-averaged turbulence structure. For instance, wall functions that should obey RANS transport may be employed in LES to model the near-wall region of the filtered grid (Pope 2000 gives a brief description). Further, hybrid LES/RANS approaches have been developed and shown to be much more computationally efficient than LES and produce reasonable results (Labourasse and Sagaut 2002; Davidson and Dahlström 2004; Tessicini et al. 2005). Still further, RANS approaches remain the standard scheme employed by industry for computing complex engineering flows and design optimization. Improved knowledge for the behavior of Reynolds stress transport is the best hope in improving this efficient computation method.

With the present technique, two possibilities exist for obtaining Reynolds stress transport balances from the data sets. Since it is still impossible to obtain a direct measurement of the velocity/pressure gradient correlation within a flow, both methods rely upon the balance of information contained in transport equations. The Reynolds stress transport equation is repeated from chapter 0:

$$\frac{D\overline{u_i u_j}}{Dt} = P_{ij} + \Pi_{ij} + \nu \nabla^2 \overline{u_i u_j} - \varepsilon_{ij} - \frac{\partial \overline{u_i u_j u_k}}{\partial x_k} \quad (4.23)$$

where the production rate of the Reynolds stress tensor  $\overline{u_i u_j}$  is  $P_{ij} = -\overline{u_i u_k} \frac{\partial \overline{U_j}}{\partial x_k} - \overline{u_j u_k} \frac{\partial \overline{U_i}}{\partial x_k}$ , the velocity-pressure-gradient tensor is

$$\Pi_{ij} = -\frac{1}{\rho} \overline{u_i \frac{\partial p}{\partial x_j}} + \overline{u_j \frac{\partial p}{\partial x_i}}$$

with  $p$  being the fluctuating static pressure, and the dissipation-

rate tensor is  $\varepsilon_{ij} = 2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}}$ . Also from chapter 0, an equivalent equation may be

derived that replaces some terms in equation (4.23) with velocity/acceleration fluctuation correlations:

$$\frac{1}{\rho} \overline{u_j \frac{\partial P}{\partial x_i}} + \overline{u_i \frac{\partial P}{\partial x_j}} = -(\overline{a_i u_j} + \overline{a_j u_i}) - 2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} + \nu \nabla^2 \overline{u_i u_j} \quad (4.24)$$

To follow, measurements from the CompLDV1 and CompLDV3 will be discussed that utilize both of these techniques for obtaining near-wall estimates of the turbulence structural parameters of equation (4.23).

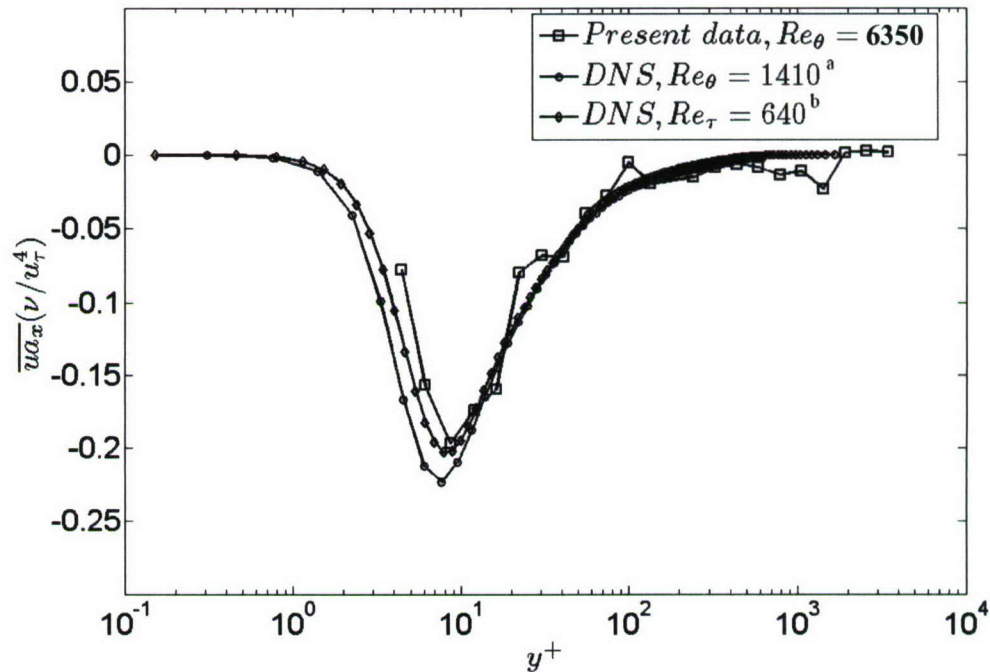
#### 4.1.4.1 Velocity/acceleration correlations

In Chapter 1, the role of the fluctuating velocity-acceleration correlation has been examined in terms of Reynolds stress transport. The immediate power of such measurements is that combinations of terms in the Reynolds stress transport equations may be characterized by a single statistical measurement at one location in the flow—no gradients need be computed. The CompLDV technique allows through optimized optics design and signal processing measurement of the acceleration of particles as they pass through the measurement volume.

The measurements for the streamwise velocity–acceleration correlation profile in wall units are given in Figure 4.33. For comparison, the DNS data of Spalart (1988) for the same flow at  $Re_\theta = 1410$  and that of Abe et al. (2001) in a turbulent channel at  $Re_\tau = 640$  are plotted along with the current data. The measurements show agreement with the low Reynolds number DNS within experimental uncertainties, indicating little Reynolds number effects for this quantity when wall scaling is used. The near-wall differences that are seen may be attributed to combinations of uncertainties in the  $y$ -distance from the wall, the friction velocity, and the velocity–acceleration correlation. The relationship between coherent motions and the velocity–acceleration correlations is considered by decomposing the contributions from the in-plane quadrants. The major contributors to the Reynolds shear stress are the correlated motions known as sweeps which occur for  $u > 0$ ,  $v < 0$  and ejections occurring when  $u < 0$ ,  $v > 0$ . It is desired to relate the sweep and ejection motions in  $\overline{uv}$  of  $\overline{ua_x}$ . The results from the quadrant analysis are plotted in Figure 4.34. These results indicate that it is the sweep motions that dominate the velocity–acceleration correlation very near wall for  $y^+ < 10$ . For heights



above  $y^+ \approx 14$ , the contributions switch such that the ejections become dominant in producing the correlation, though the difference is approaching experimental uncertainties. An explanation of the mechanisms for these phenomena is proposed by considering the shape of the probability density functions for the streamwise velocities very near the wall. Figure 4.35 gives the skewness of the streamwise velocity histograms throughout the profile. It is seen that in the very near-wall region the histograms are positively skewed, indicating that the range of positive  $u'$  fluctuations is larger than the negative ones. This makes sense intuitively because there is a limit on the lowest velocity since the flow is always downstream, but the greatest possible velocities are related to the higher-momentum large-scale eddies sweeping toward the wall. Note also that the skewness of  $u'$  changes sign at the same location that the contributions from sweeps and ejections switch dominance. Heuristically, then, the large difference between the mean velocity very near the wall and the relatively infrequent high-speed sweeps leads to high local viscous shear that acts to limit the convection of momentum that has reached the wall and thus stabilize the mean shear rate in a Lagrangian sense.



**Figure 4.33. Stream-wise velocity acceleration correlation for the 2D FPTBL at  $Re_\theta = 6350$  compared to the data of a) Spalart (1988) and b) Abe et al. (2001).**

**Adapted from Lowe and Simpson (2006).**

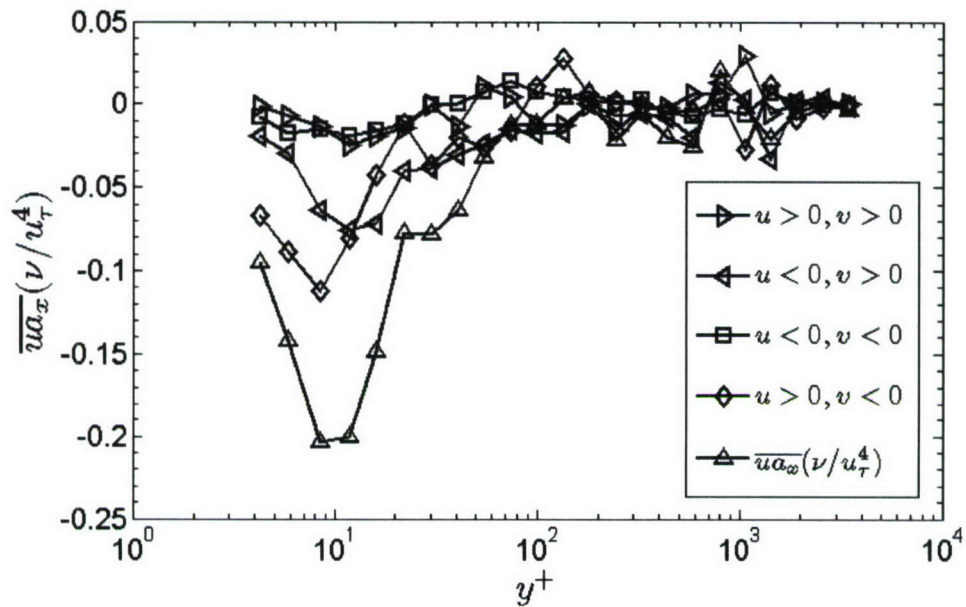


Figure 4.34. Velocity quadrant analysis of the streamwise velocity–acceleration correlation for the 2D FPTBL at  $Re_\theta = 6350$ . Adapted from Lowe and Simpson (2006).

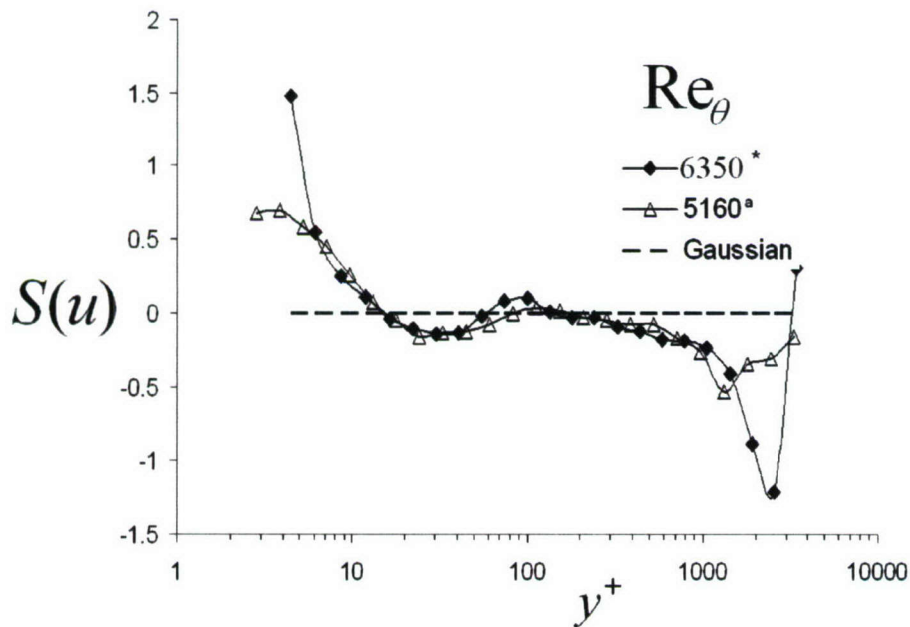


Figure 4.35. Skewness of the streamwise velocity fluctuation. (\*) Current data, (a) data of DeGraaff and Eaton (2000). The dashed line indicates the value for a Gaussian distribution. Adapted from Lowe and Simpson (2006).



The data for the 2DFPTBL at  $Re_\theta = 6350$  gives good estimates for the stream-wise velocity/acceleration correlation, as shown in Figure 4.33. For this flow and this component of the Reynolds stress,  $i = j = 1$ , the viscous diffusion term,  $D_{v11} = \nu \nabla^2 \overline{u^2}$ , reduces to  $D_{v11} = \nu (\partial^2 \overline{u^2} / \partial y^2)$  since derivatives in the stream-wise and span-wise directions are negligible compared with derivatives in the normal-to-wall direction. While in sections to follow, direct estimates of the dissipation rate from velocity gradient measurements will be considered, to estimate the dissipation rate for the current purposes, first the turbulent kinetic energy (TKE) dissipation rate is estimated by balance of the boundary layer approximation for the TKE transport equation:

$$U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} = P - \varepsilon + \nu \frac{\partial^2 k}{\partial y^2} - \frac{1}{2} \frac{\partial (\overline{u^2 v} + \overline{v^3} + \overline{w^2 v})}{\partial y} - \frac{1}{\rho} \frac{\partial \overline{vp}}{\partial y} \quad (4.25)$$

where  $k \equiv \frac{1}{2} (\overline{u^2} + \overline{v^2} + \overline{w^2})$  is the TKE and  $\varepsilon$  is the TKE dissipation rate. The ComplDV1 measurements afford direct estimation of all the terms in equations (4.25) except for the pressure diffusion,  $-\frac{1}{\rho} \frac{\partial \overline{vp}}{\partial y}$ . While spectral estimates of the TKE

dissipation rate are possible with the measurements made with the ComplDV1, DNS of Spalart (1988) indicate that the model of Lumley (1978),  $-\frac{1}{\rho} \overline{vp} \approx \frac{1}{5} (\overline{u^2 v} + \overline{v^3} + \overline{w^2 v})$  is

well-suited for the balance of equation (4.25) results in high-confidence estimates of the dissipation rate. The TKE transport budget for the current flow is given in Figure 4.36. All gradients were computed using central differencing. Given the TKE dissipation rate, one may estimate the dissipation rate of  $\overline{u^2}$ , by either assuming isotropy of the dissipation rate or else some model for accounting for the anisotropic dissipation. It was found that common models predict isotropic dissipation rates throughout the boundary layer above  $y^+ = 10$ , so the isotropic estimate,  $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33}$  was used. The balance of equation (4.24), yielding estimates for the streamwise velocity/pressure gradient correlation is given in Figure 4.37. Note that the velocity–acceleration correlation data of Figure 4.33 have been spatially smoothed to result in more realistic estimates for the velocity/pressure gradient correlations reported in Figure 4.37. Comparing these results with those of Spalart (1988) reveals that the velocity–pressure gradient is greater near the wall compared with the lower Reynolds number DNS.

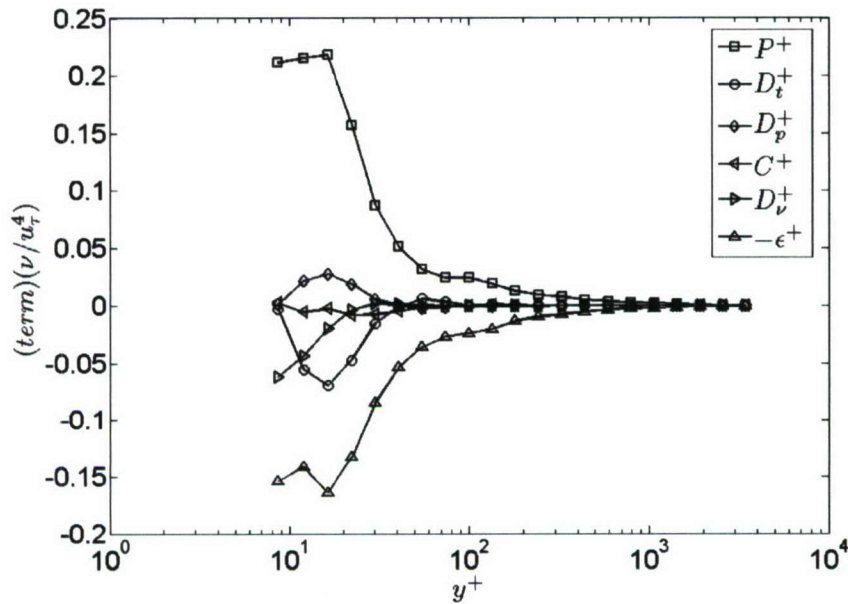


Figure 4.36. Transport budget for the turbulent kinetic energy in the 2D FPTBL at  $Re_\theta = 6350$ . Adapted from Lowe and Simpson (2006).

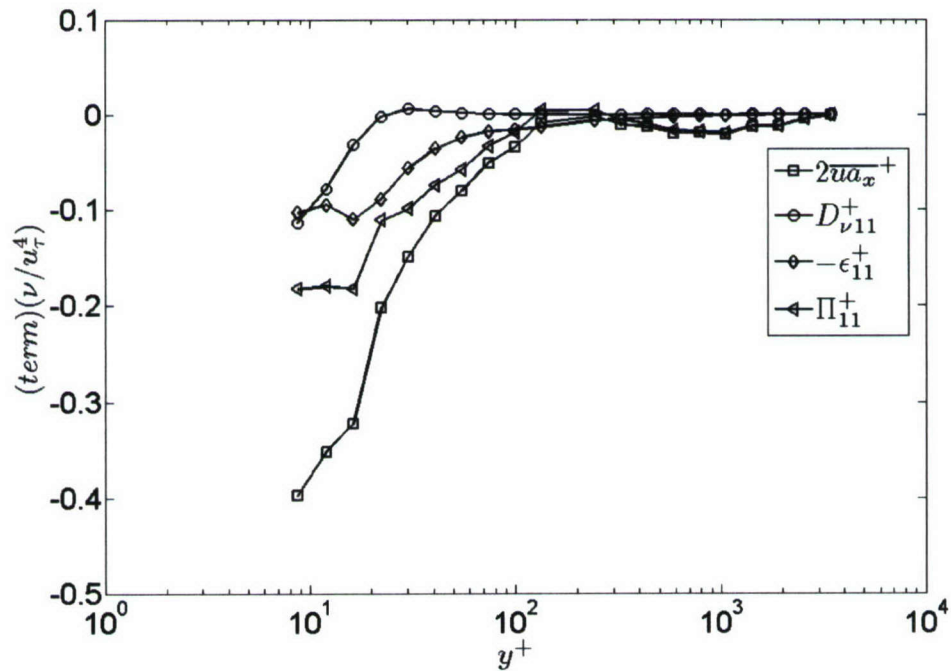


Figure 4.37. The balance of the stream-wise velocity-acceleration equation using isotropic dissipation estimates and measurements of the viscous diffusion and velocity-acceleration correlation in the 2D FPTBL at  $Re_\theta = 6350$ . Adapted from Lowe and Simpson (2006).



#### 4.1.4.2 Reynolds stress transport using dissipation rate estimates

It is possible using the capabilities of the CompLDV to obtain dissipation rate estimates within turbulent boundary layers. Two methods exist for obtaining these estimates—TKE dissipation rate estimates from velocity spectra and direct estimation using the velocity gradient tensor measurements. These methods are discussed in more detail to follow.

##### 4.1.4.2.1 Spectral dissipation rate estimates

Several methods exist for estimating the TKE dissipation rate from velocity spectral data. For spectral measurements that contain a sufficient range of scales to resolve the dissipative range such that the wave-numbers as high as  $k_1 = 1/\eta$  are measured where the

Kolmogorov length scale  $\eta \equiv (\nu^3 / \varepsilon)^{1/4}$ , isotropic relations give

$$\varepsilon = 15\nu \int_0^\infty k_1^2 E_{11}(k_1) dk_1 = \frac{15}{2}\nu \int_0^\infty k_1^2 E_{22}(k_1) dk_1 = \frac{15}{2}\nu \int_0^\infty k_1^2 E_{33}(k_1) dk_1 \quad (4.26)$$

Since most spectral data are acquired with single-point probes (as in this study), it is necessary to convert the frequency/time conjugate relationship to the wave-number/distance conjugate pair. This may be accomplished for high wave-number data sufficiently far from boundaries using Taylor's Hypothesis for frozen turbulence, as already discussed. Since relatively small scales of  $\eta$  are present throughout most of the 2DFPTBL flows measured, there was not a sufficient range of  $k_1$  obtained to directly apply equation (4.26).

Methods of fitting theoretical or model functions to spectral estimates may also lead to dissipation rate estimates. One possibility for this is using equation (4.19) for fits. One must identify the region of the spectrum in which the roll-off is approximately constant at  $k_1^{-5/3}$  and then the value of the dissipation is directly estimated from the scaling of the measurements given the value of the universal constant. In the present study, a method of fitting a model spectrum function that is consistent with the Kolmogorov (-5/3) spectrum is used. The Taylor's Hypothesis is employed to obtain wave-number distributions from the frequency spectra and the Pope (2000) model spectrum that consists of three-blended functions for the low-wave-number energy-containing range, the inertial subrange (-5/3 spectrum), and the dissipative range. Pope specifies his model in terms of the wave-number magnitude and the total-energy spectrum function:

$$E(\kappa) = C\varepsilon^{2/3}\kappa^{-5/3} f_L(\kappa L) f_\eta(\kappa\eta) \quad (4.27)$$

where  $\kappa = \sqrt{\kappa_1^2 + \kappa_2^2 + \kappa_3^2}$ ,  $C=1.5$ ,  $L \equiv k^{3/2} / \varepsilon$  and the blending functions are given as

$$f_L(\kappa L) = \left( \frac{\kappa L}{\sqrt{(\kappa L)^2 + c_L}} \right)^{5/3+p_0} \quad (4.28)$$

where  $c_L=6.78$  for  $p_0=2$  and

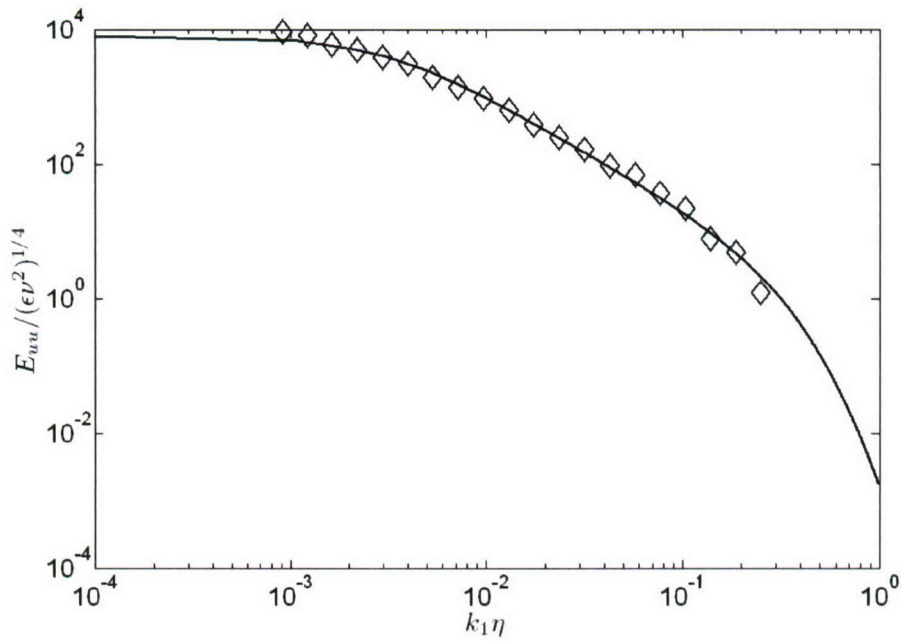
$$f_{\eta}(\kappa\eta) = \exp\left\{-\beta\left[(\kappa\eta)^4 + c_{\eta}^4\right]^{1/4} + c_{\eta}\right\} \quad (4.29)$$

where  $\beta = 5.2$  and  $c_{\eta} = 0.40$ . Each of these constants is determined using the extensive spectral data that exist in the field. To obtain the one-dimensional spectrum which may be of use in the current work, we may use isotropic assumptions inter-relating the wave-number directions to obtain

$$E_{11}(\kappa_1) = \int_{\kappa_1}^{\infty} \kappa^{-1} E(\kappa) \left(\frac{\kappa_1^2}{\kappa}\right) d\kappa \quad (4.30)$$

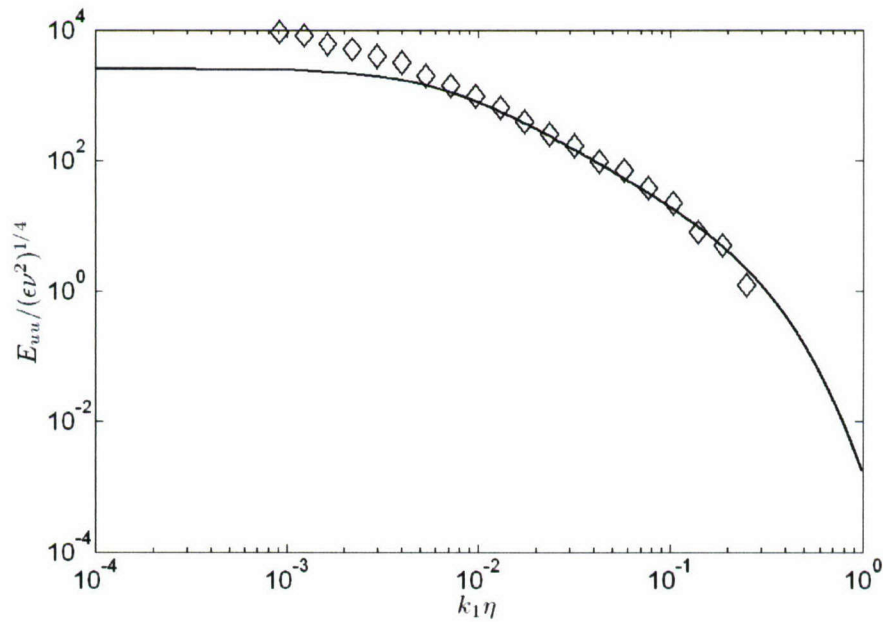
Of course this model may not perform well when the spectral distribution is significantly different from the model case, such as in the case of the very near wall data, but within the boundary layer it is a good approximation. This assertion is brought to point with an example of CompLDV3 spectral data fitted to Pope's spectra in Figure 4.38. These data were obtained in the 2DFPTBL at  $Re_{\theta} = 7497$  at the height  $y^+ = 619$  where the Kolmogorov length scale was found to be  $\eta = 61 \mu m$ . This fit was accomplished by varying the Kolmogorov length scale to obtain the best correlation coefficient in the fit. The data points at the lowest and highest ends were disregarded for this fit since the low-end wave-numbers suffer uncertainties due to variable wavespeeds (Ahn and Simpson 1987) while the high-end wave-numbers are more uncertain due to large numbers of samples that are needed to obtain high confidence levels at such dynamic ranges. In implementing this fitting technique, it is found for the 2DFPTBL, which is known to have highly non-isotropic Reynolds stresses, that the dissipation length-scale parameter,  $L$ , is best modified to  $L = \varepsilon^{-1} \left(\frac{3}{2} u^2\right)^{3/2}$  to provide favorable fits to the energy-containing range of  $E_{11}$ . This is exemplified in Figure 4.39 where the TKE has been used to compute  $L$ , instead of simply the stream-wise normal stress as in Figure 4.38.





**Figure 4.38.** Fit to Pope's model for the stream-wise velocity auto-spectrum of the CompLDV3 from the 2DFPTBL at  $Re_\theta = 7497$  at  $y^+ = 600$  where  $\eta = 61 \mu m$  and

$$L = \left( \frac{3 \overline{u^2}}{2} \right)^{3/2} / \epsilon.$$

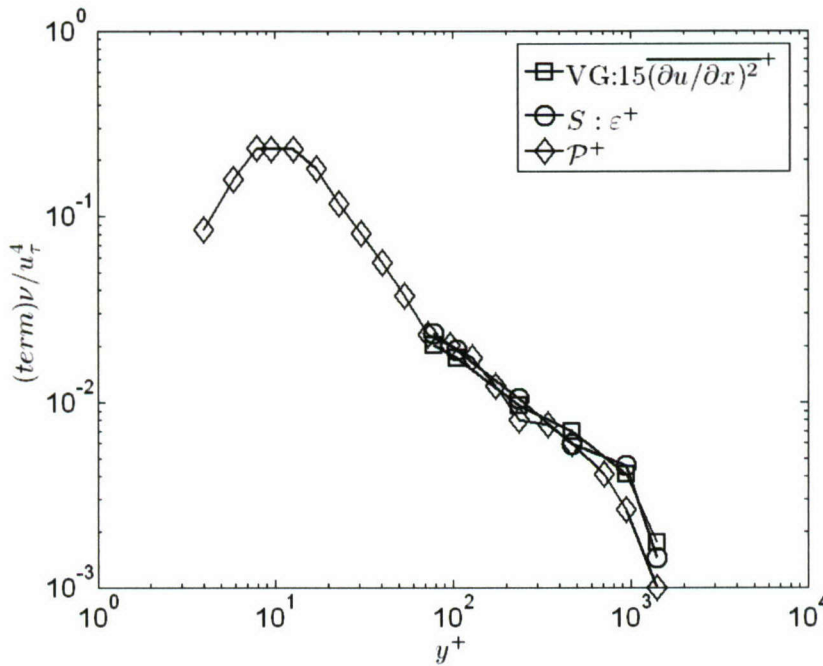


**Figure 4.39.** Same as Figure 4.38, but with  $L \equiv k^{3/2} / \epsilon$ .

#### 4.1.4.2.2 *Direct dissipation rate estimates from the velocity gradient tensor*

The second method available from the CompLDV3 for estimating the dissipation rate is by direct evaluation of the dissipation rate from velocity gradient measurements. The technique for estimating the velocity gradient using particle velocity and position measurements was discussed in section 2.3.2.1.1.2.2. In that section, a least-squares method of extraction of velocity gradients was considered based upon the notion of a stream-wise-oriented turbulent motion with generally constant velocity gradients that convect frozen through the measurement volume. With the levels of uncertainty predicted for the CompLDV3, this technique was shown to work sufficiently-well using simulated data given at least 6-7 particles for the least-squares fit for estimating the velocity gradients. To determine a reasonable time-window that may be used, previous experimental information was used to obtain an estimate for the size and convection speed of the near-wall eddies. For the coherent structures in the near-wall region, it has been observed that the convective speed of eddies is about  $U_c / u_\tau \approx 14$  (Kline et al. 1967; Ahn and Simpson 1987) and typical active near-wall structures exist in very long dimensions of  $x^+ = 500$  or greater (Robinson 1991). In practice, this model for obtaining the velocity gradients requires some adaptation. First, it has been found that the stream-wise gradients (those that are obtained using the arrival time technique for determining the positions) give results much too small when such a large time-window is allowed. The effect at work there is filtering of the small-scales that contribute to the velocity gradients in this direction. The stream-wise and normal-to-wall gradients are estimated over small-scales in those directions, so the extended stream-wise window does not appear to attenuate those data. To remedy this problem of filtering, a shorter time-window of  $t^+ = 6$  is used and the results are much-improved. An example of these results is given in Figure 4.40 where spectral dissipation rate estimates are compared to the directly-estimated mean-square of the stream-wise gradient of the stream-wise velocity and to the TKE production computed from the Reynolds-averaged profile measurements. The comparison of the two dissipation rate measurements to the production measurements rests on the assumption of the isotropic relation,  $\varepsilon = 15\nu \overline{\left(\frac{\partial u}{\partial x}\right)^2}$ , but the direct comparison of the two estimates of dissipation rate to one-another is more rigorous since the spectral fitting discussed above seeks to directly estimate the term  $\overline{\left(\frac{\partial u}{\partial x}\right)^2}$ . The quality of the comparison for these two measurements of  $\overline{\left(\frac{\partial u}{\partial x}\right)^2}$  gives confidence in the gradient estimation scheme for stream-wise gradients.





**Figure 4.40.** Comparison of the dissipation estimate for the directly-estimated stream-wise velocity gradient in the stream-wise direction to the spectral dissipation rate estimates and the production for the 2DFPTBL at  $Re_\rho = 5929$ . VG, Directly-estimated dissipation rate from velocity gradients; S, Spectral-estimates of dissipation rate; Diamonds are TKE production computed from Reynolds-averaged statistics.

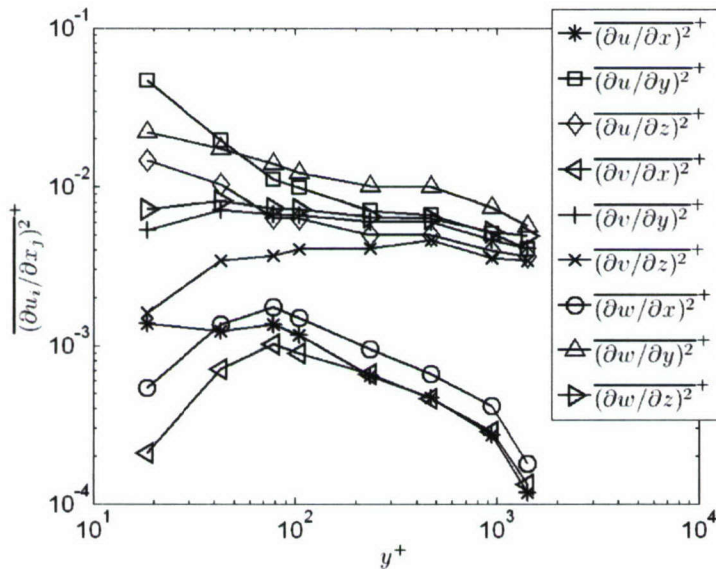
In contrast to the stream-wise gradients, span-wise and normal-to-wall gradients exhibited values that level-off above the expected values by comparison to the production or DNS values for the non-isotropic dissipation rate. The raw data for the mean-square velocity gradients in a 2DFPTBL is given in Figure 4.41. It is clear from these data that the span-wise and normal-to-wall gradients of all velocity components are in significant error since the stream-wise gradient values have been verified by comparison to the expected results in Figure 4.40. The approach for correcting the situation is to apply a constant additive correction that is appropriate for each term. The premise for applying such a correction is that it is a white-noise floor that limits the minimum variance for these velocity gradients. Unfortunately, such a noise floor must be considered to vary from point-to-point since the flow parameters such as turbulence level are varying thusly, and it is evident by the very low values that are measured for the term  $\overline{\left(\frac{\partial v}{\partial z}\right)^2+}$  in the near-wall region that a constant correction is not perfect.

Future research will be conducted to predict this noise floor for each point based upon time-delay correlations and power spectral densities of the velocity gradients. The values of the time-delay correlation near zero-delay may be used with some model for the

variation to obtain new estimates for the auto-covariance at zero time-delay. As has been discussed already in section 3.2, the random noise content of the signal will be concentrated in the zero-delay sample (i.e., the auto-covariance), and with some assumption of the functional form for the time-delay correlation near zero-delay, the auto-covariance may be corrected. The power spectral density may be used in a similar manner to correct the value of the gradient variance, as it forms a Fourier transform pair with time-delay auto-correlation. In the case of the power spectral density, the random noise content will be assumed white since we have no information to indicate otherwise. It is known that the power spectral density of a turbulence quantity is continuous and as it approaches the high wave-numbers, it necessary extends to zero due to the action of viscosity on the small scales. Then, the power spectral density may be used to indicate the portion of the high frequency spectrum where the spectral values cease to decrease and level-off. This noise-floor may be considered constant across the computed frequencies, integrated, and subtracted from the gradient variance.

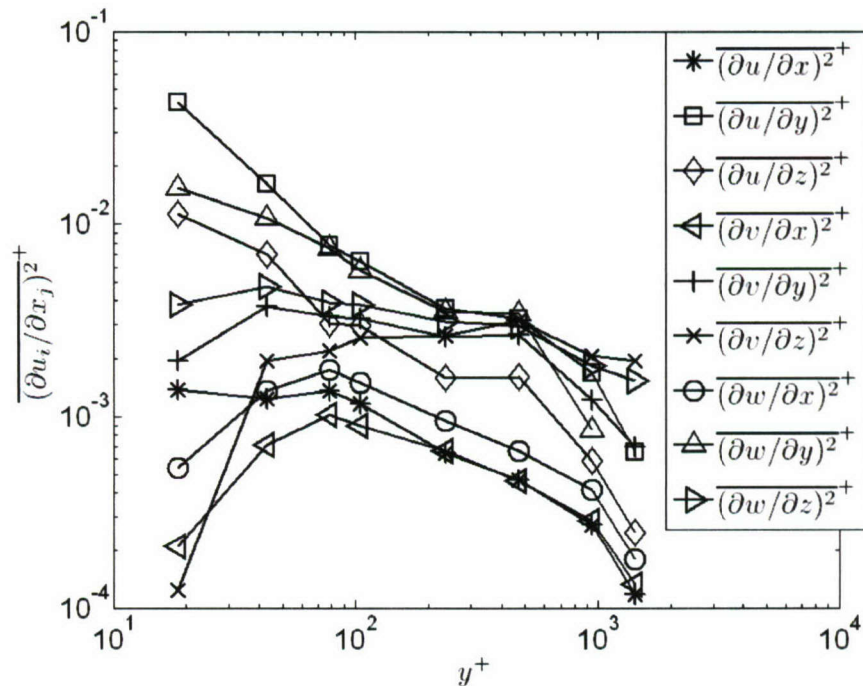
For the current purposes, the assumption of a constant noise-floor for each gradient variance will be regarded to make the necessary corrections. The data from Figure 4.41 is corrected by subtracting constant noise-floor values from each of the terms at each point and the result is plotted in Figure 4.42. The goal in selecting these noise-floor values was to obtain mean-square velocity gradient estimates of the same order of magnitude as the stream-wise gradients for the normal-to-wall and span-wise values in the outer-portion of the boundary layer where locally-isotropic conditions exist. The amount that may be

subtracted is limited in some cases, most-notably for  $\overline{\left(\frac{\partial v}{\partial z}\right)^2}$ , by the values of these terms at the lower  $y^+$  region.



**Figure 4.41. Raw mean-square velocity gradient information for the 2DFPTBL at  $Re_\theta = 5929$ .**



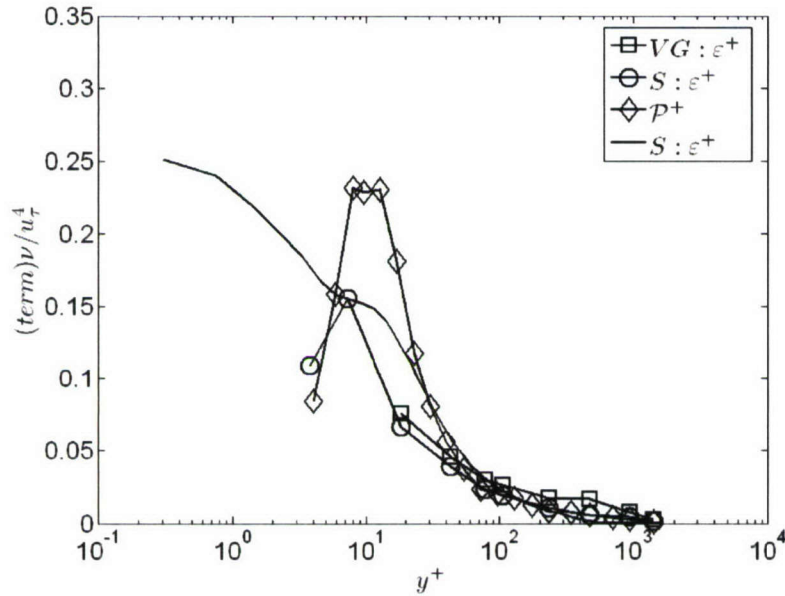


**Figure 4.42.** Corrected mean-square velocity gradient information for the 2DFPTBL at  $Re_\theta = 5929$ .

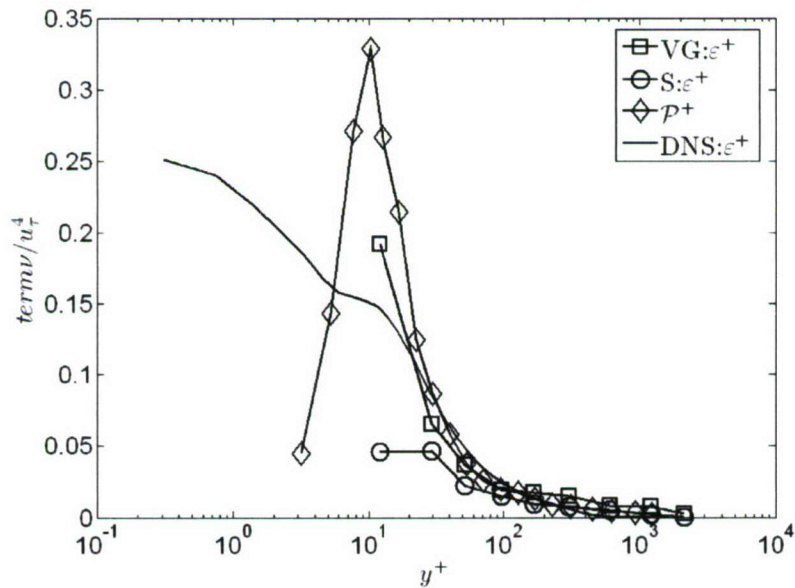
#### 4.1.4.2.3 Results for dissipation rate measurements

Both the methods just methods just discussed for estimating the dissipation rate throughout the 2DFPTBL have been applied to the CompLDV3 measurements. In this section, we compare the results for the TKE as well as Reynolds normal stress dissipation rates to the DNS results of Spalart (1988) for the low Reynolds number 2DFPTBL. In Figure 4.43 the TKE dissipation rate as measured by the spectral and velocity gradient methods are compared for the 2DFPTBL at  $Re_\theta = 5929$  and compared with the production of TKE for that profile as well as the dissipation rate data of Spalart. Analogous data are plotted for the higher Reynolds number 2DFPTBL in Figure 4.44. The underestimation of TKE dissipation rate in near-wall TBLs using single-point spectral data has been documented a number of times (Sandborn and Braun 1956; Balint et al. 1991). Surprisingly, the lower-Reynolds number case follows the spectral value for the lowest  $y^+$  value reported. In the higher-Reynolds number case, the velocity gradient dissipation rate data follows the production values of the TKE reasonably throughout the range of measurements. The distributions of the Reynolds normal stress dissipation rates measured with the velocity gradient technique are plotted in Figure 4.45 and Figure 4.46 for the lower-Reynolds number and higher-Reynolds number cases, respectively. To the author's knowledge, these measurements are the first of their kind at Reynolds numbers of this magnitude in turbulent boundary layers. To note is the persistence of anisotropy of the dissipation rate to values of  $y^+$  of about 100, since models such as the wall/isotropic

dissipation blending function of Lai and So (2000) give more-or-less isotropic predictions at  $y^+ = 10$ .

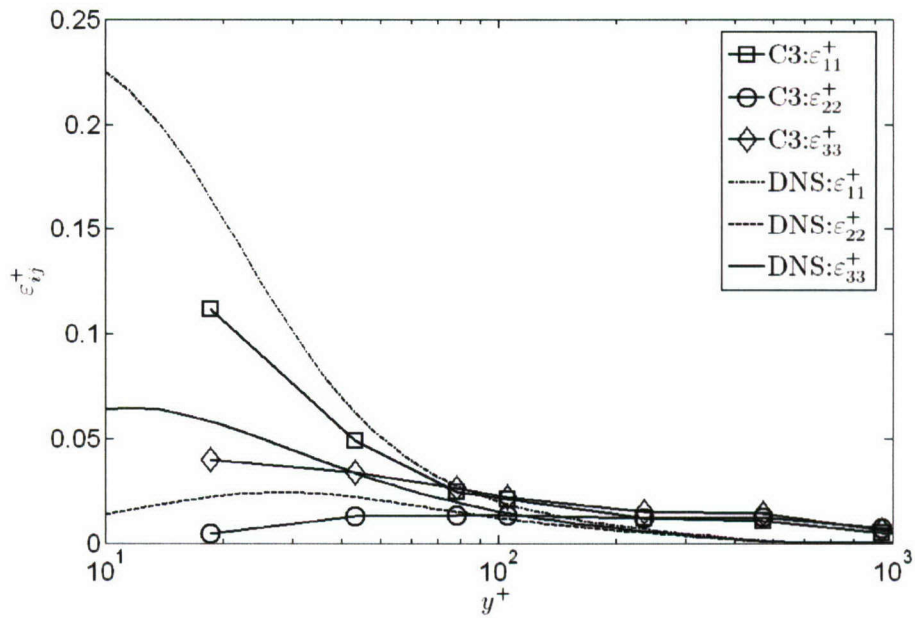


**Figure 4.43.** Dissipation rate measurements for the 2DFPTBL at  $Re_\theta = 5929$ . VG, Direct estimation of velocity gradients; S, Spectral estimation with Taylor's Hypothesis; DNS, Low Reynolds number simulation data of Spalart (1988).

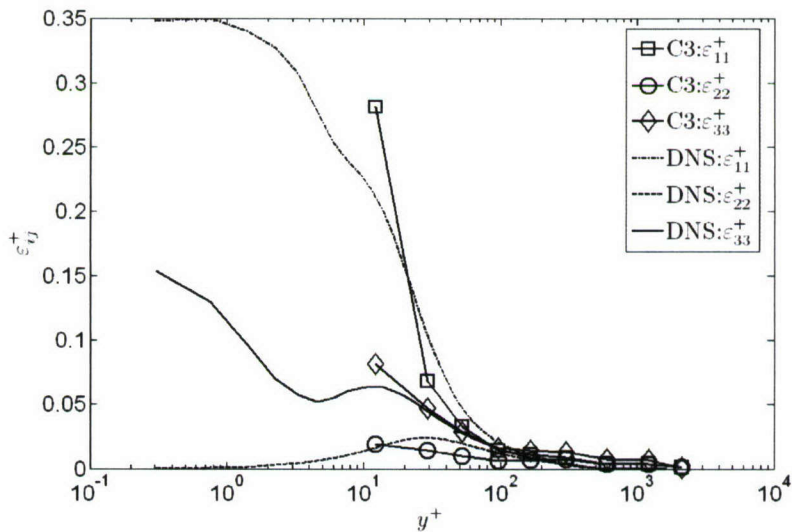


**Figure 4.44.** Dissipation rate measurements for the 2DFPTBL at  $Re_\theta = 7497$ . VG, Direct estimation of velocity gradients; S, Spectral estimation with Taylor's Hypothesis; DNS, Low Reynolds number simulation data of Spalart (1988).





**Figure 4.45.** Non-isotropic Reynolds stress dissipation rates measured in the 2DFPTBL at  $Re_\theta = 5929$ . C3, ComplDV3; DNS, Simulation data of Spalart (1988).

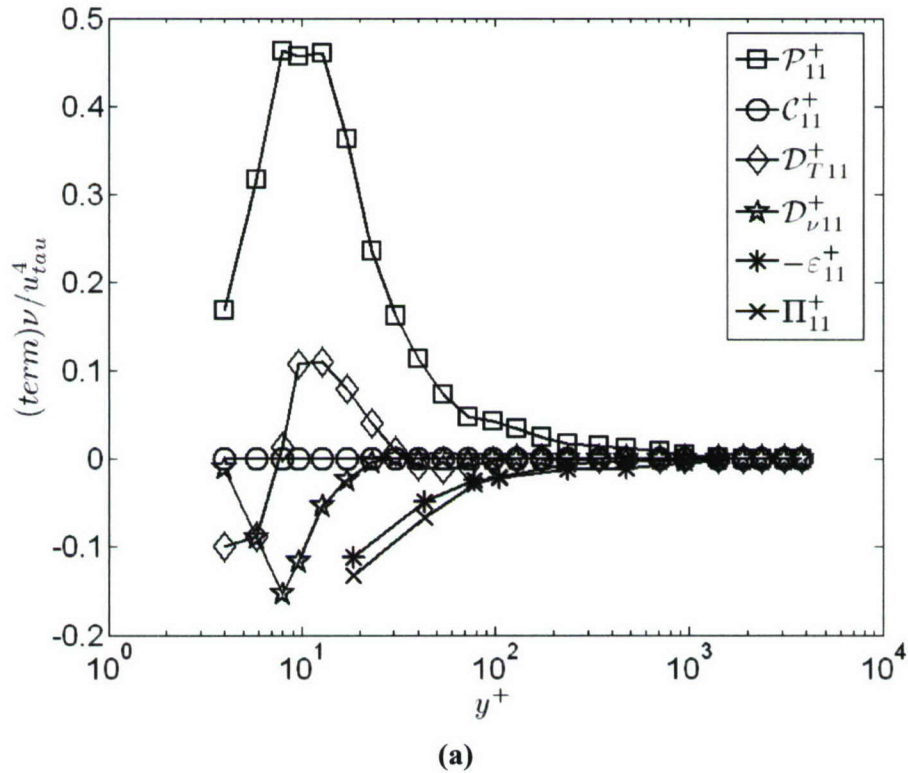


**Figure 4.46.** Non-isotropic Reynolds stress dissipation rate measured in the 2DFPTBL at  $Re_\theta = 7497$ . C3, ComplDV3; DNS, Simulation data of Spalart (1988).

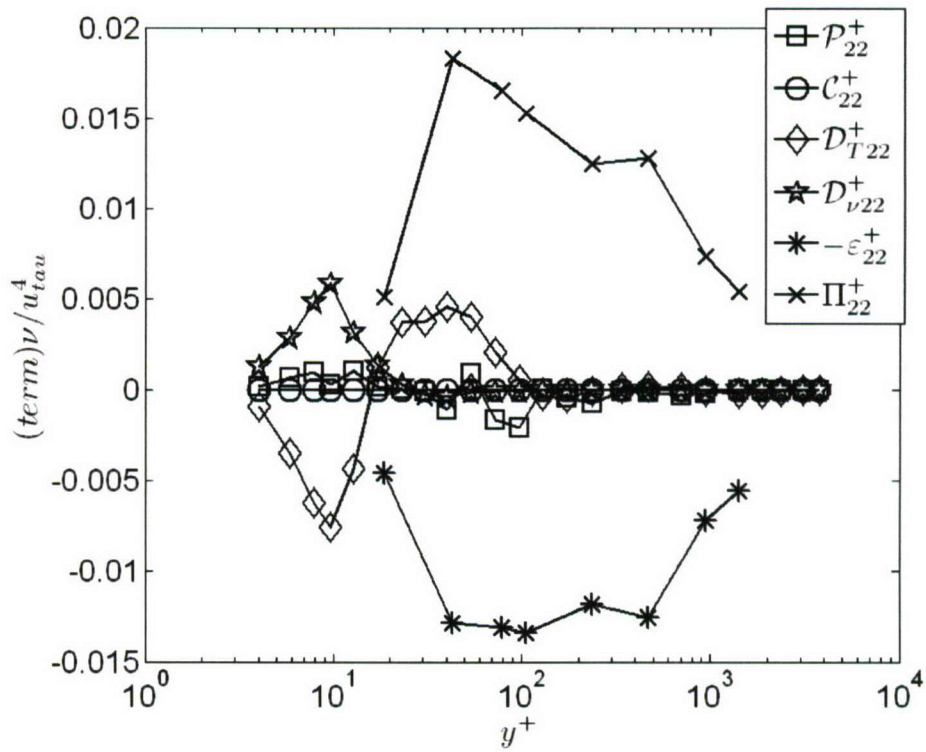
#### 4.1.4.2.4 Reynolds stress transport balances

The Reynolds stress transport balances have been estimated using the CompLDV3 data for the 2DFPTBL at two Reynolds numbers. For all terms involving gradients of Reynolds averaged quantities, central differencing was employed. To obtain the velocity/pressure gradient correlation the balance of equation (4.23) was obtained using the non-isotropic dissipation rates from direct velocity gradient measurements. In the case of the Reynolds shear stress, the statistics of the cross-products of velocity gradients have not yet been computed and the dissipation rate for the shear stress was taken as zero.

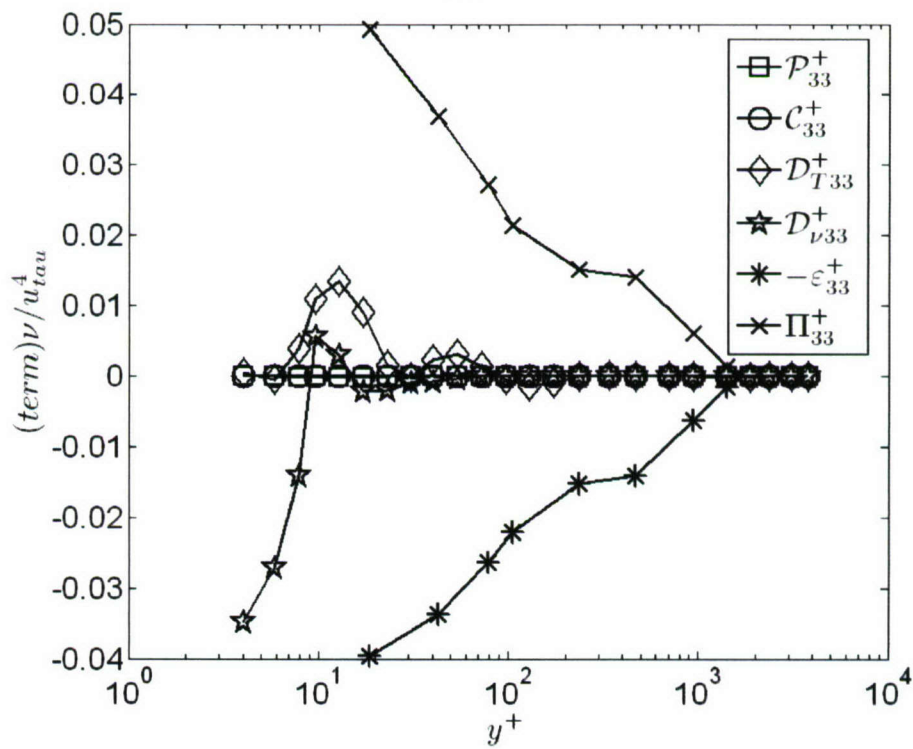
The transport rate budgets for each of the non-zero stresses in the flow and the TKE are plotted in Figure 4.47 and Figure 4.48 for the  $Re_\theta = 5929$  and  $Re_\theta = 7497$ , respectively. The importance of the velocity pressure gradient is again re-iterated based upon these data. The trends therein resemble the near-wall budgets of the DNS by Spalart (1988), although similar to the results in Figure 4.37, it appears that the velocity/pressure gradient correlation in the stream-wise normal stress plays an increasingly larger role with Reynolds number.



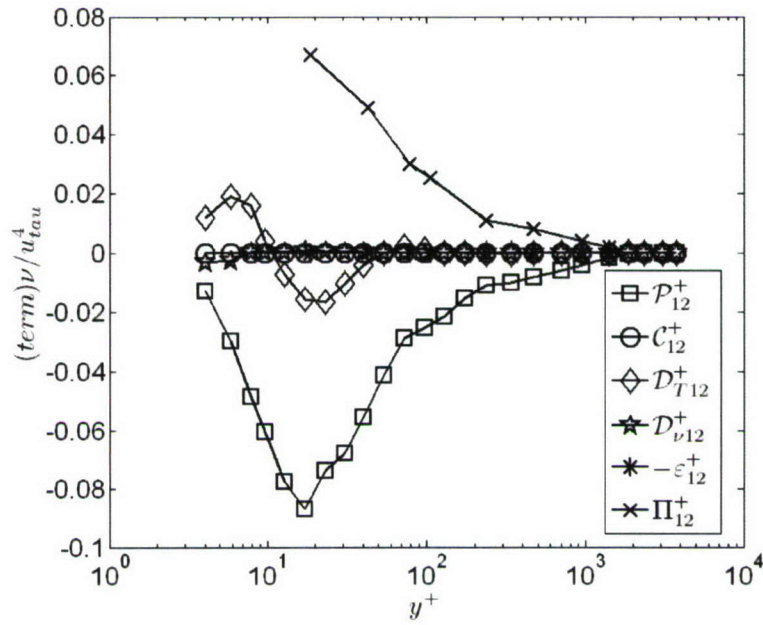




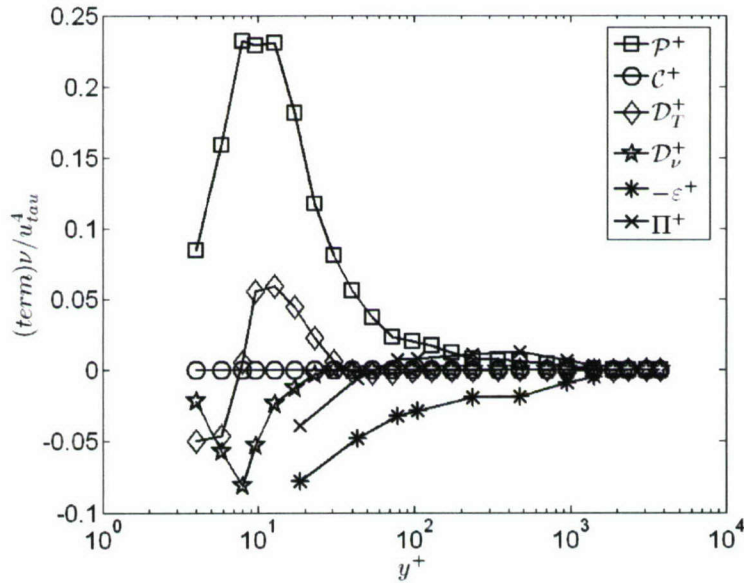
(b)



(c)



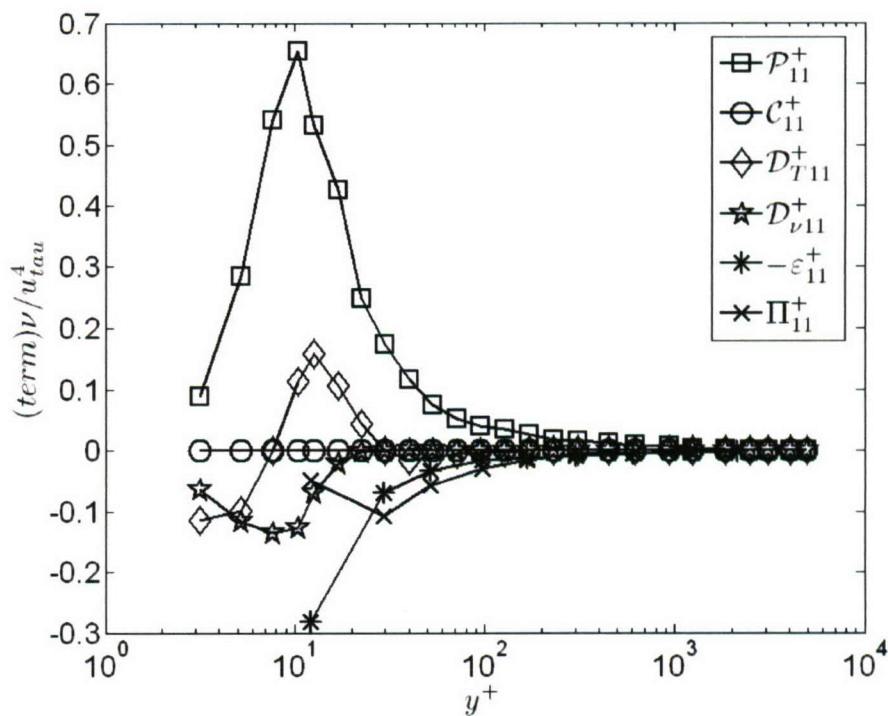
(d)



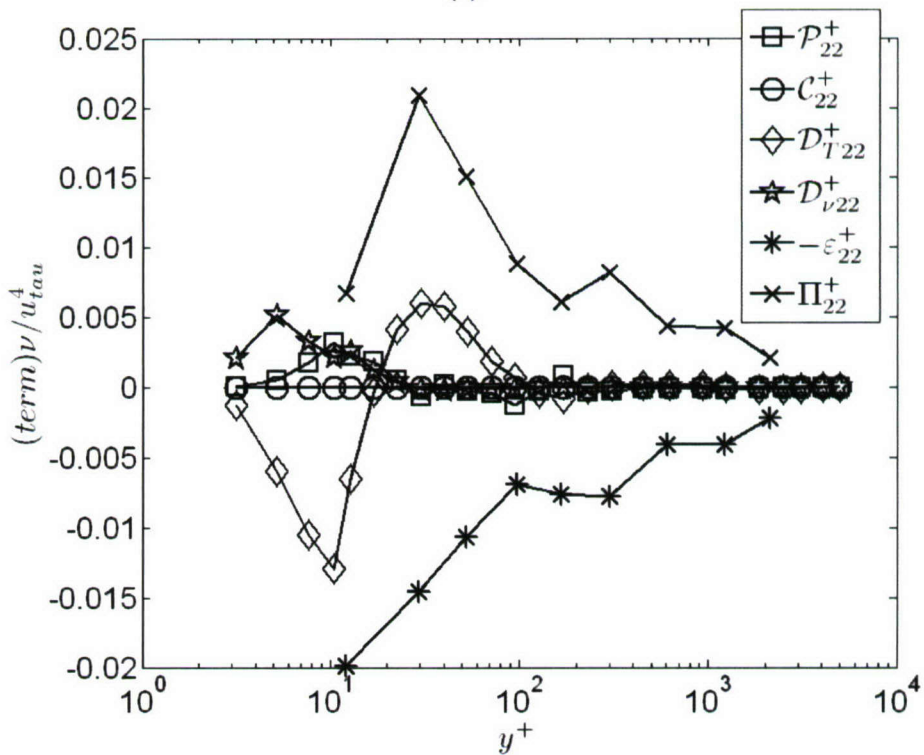
(e)

**Figure 4.47. Reynolds stress transport balances using the non-isotropic dissipation rates for the 2DFPTBL at  $Re_\theta = 5929$ . (a)  $\overline{u^2}^+$ , (b)  $\overline{v^2}^+$ , (c)  $\overline{w^2}^+$ , (d)  $\overline{uv}^+$ , (e)  $\frac{1}{2}(\overline{u^2}^+ + \overline{v^2}^+ + \overline{w^2}^+)$ .  $\mathcal{P}_{ij}$ =Production;  $\mathcal{C}_{ij}$ =Convection;  $\mathcal{D}_{vij}$ =Viscous diffusion;  $\mathcal{D}_{Tij}$ =Turbulent diffusion;  $\epsilon_{ij}$ =Dissipation rate;  $\Pi_{ij}$ =Velocity/pressure gradient correlation.**

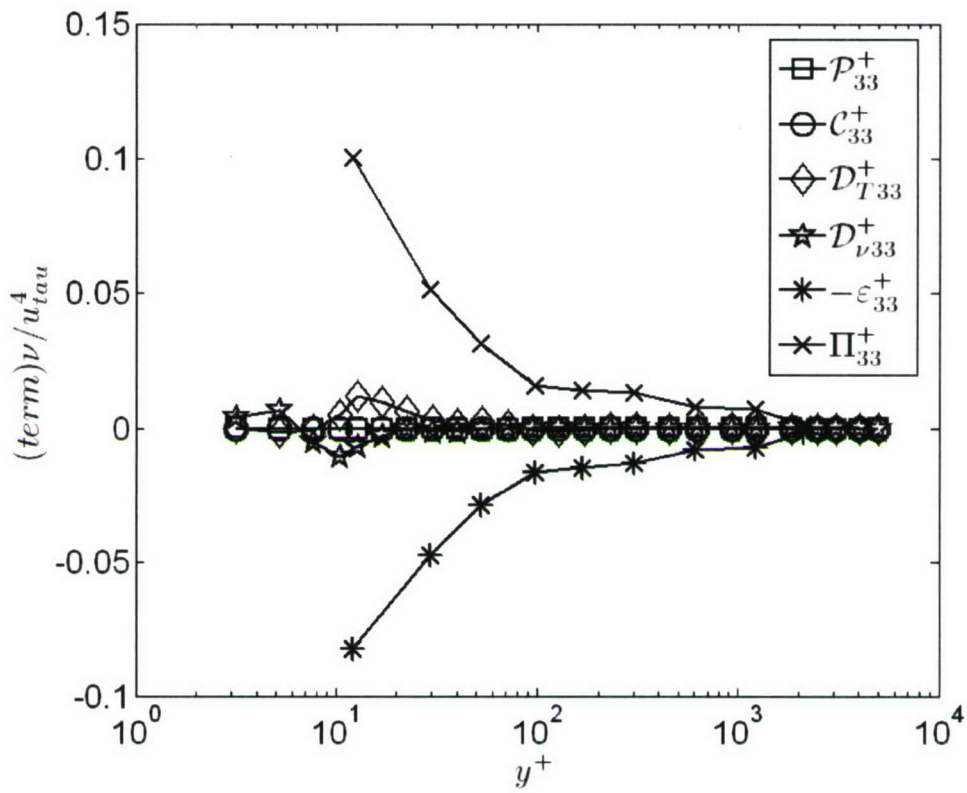




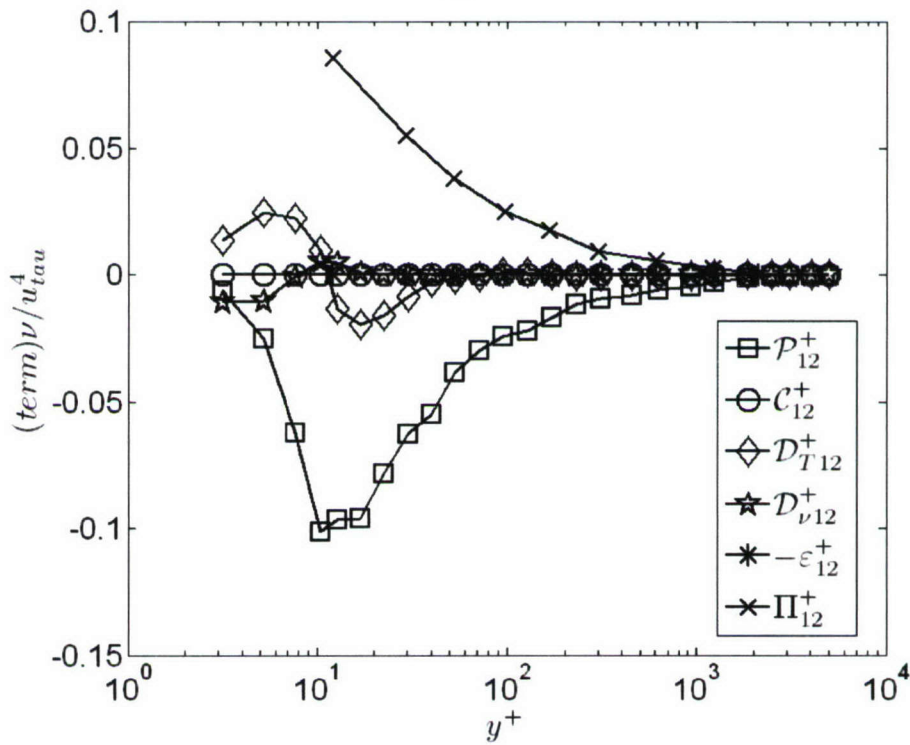
(a)



(b)



(c)



(d)



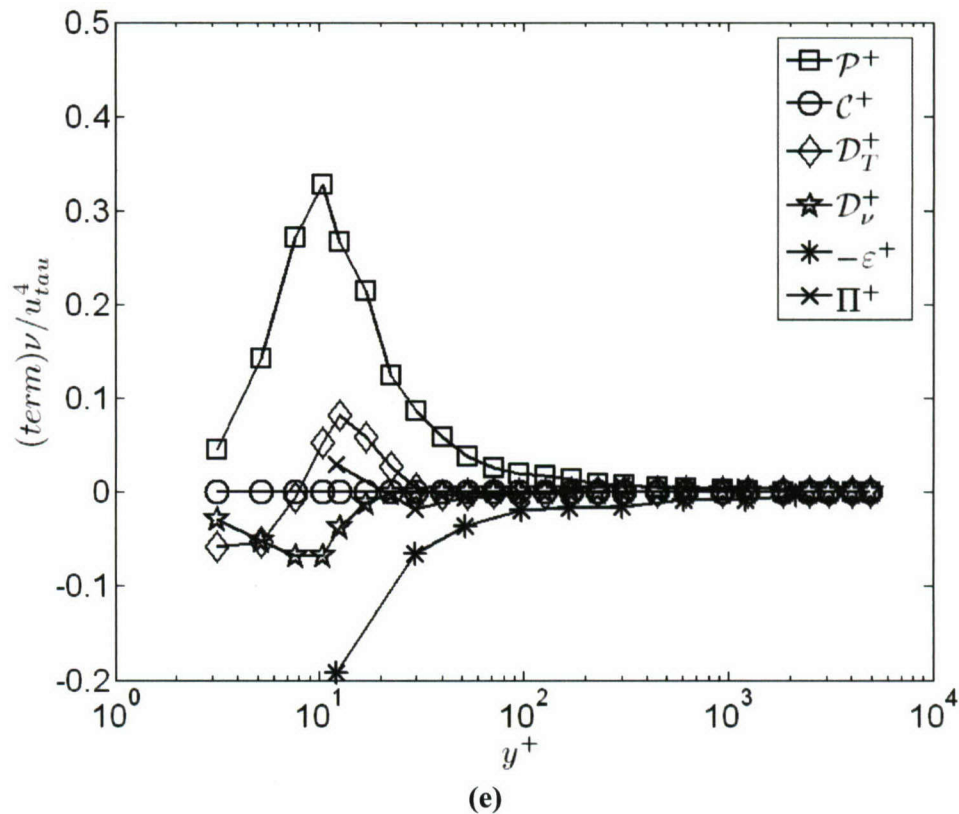


Figure 4.48. Same as Figure 4.47 for  $Re_\theta = 7497$ .

## 4.2 Isolated Protuberances

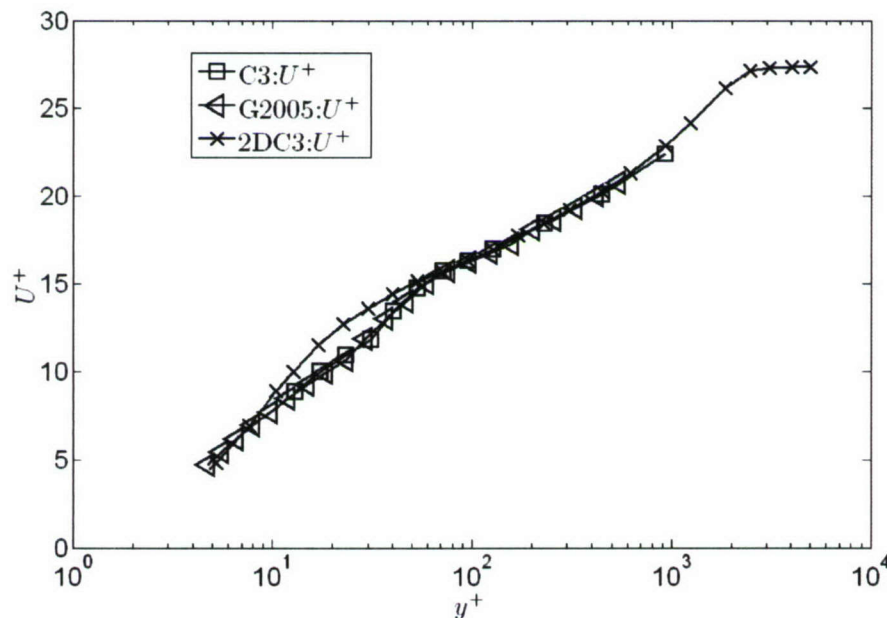
George (2005) has extensively studied the velocity statistics in the wakes of isolated cylindrical and axisymmetric Gaussian elements in hopes of a build-up approach for modeling distributed roughness made up of the same elements. These measurements have revealed intense action in both coherent and inactive motions shedding from the top of the cylindrical elements in addition to large downwash that leads to rapid reattachment of the flow. Bennington (2004) continued this study by expanding the portfolio of isolated elements for which detailed velocity statistics exist. This work exposed the importance of sharp edges on these elements in producing large TKE due to the abrupt separations that occur there.

The current study is an extension of the work done by George, further characterizing the turbulence structure in the wake of three isolated cylinders submerged in a 2DFPTBL at  $Re_\theta = 7497$ . The diameter of these cylinders were each 1.98mm with heights of  $k=0.38\text{mm}$ ,  $0.76\text{mm}$ , and  $1.52\text{mm}$ , corresponding to viscous roughness scales,  $k^+ \equiv \frac{ku_\tau}{\nu}$ , of  $k^+=23.5$ ,  $47.1$ , and  $94.1$ , respectively. In the discussions to follow, the Reynolds-averaged velocity statistics for profiles taken at locations of  $x/d=2.75$  on the

centerline of the cylinder ( $z/d=0$ ) will be compared with those of George (2005) to verify the consistency in the flows. To augment the current understanding of this flow, measurements of the velocity auto- and cross-spectra will be considered for the additional information about the frequency of motions that occur in the wakes of these elements.

### 4.2.1 Volume-averaged statistics

Just as discussed in section 4.1.2, the CompLDV3 probe-volume was limited to 100 microns in the vertical direction and all quantities were corrected for velocity gradient broadening to obtain the final statistics estimates. For normalization purposes, the 2DFPTBL value for the wall-friction velocity was used to compare with the data of George. This flow-field is characterized as being very-rapidly changing, in each direction due to the small sizes of the elements in each dimension. Considering this fact and the uncertainty inherent in locating the measurement positions relative to the cylindrical test element, the agreement in all the data is excellent. The velocity statistics data (mean velocities and Reynolds stresses) for the small cylinder,  $k^+=23.5$ , are plotted in Figure 4.49-Figure 4.51. These data agree well with the exception of the magnitude of the peak values for the span-wise normal stress,  $\overline{w^2}^+$ . This trend follows for the data for the medium-height cylinder,  $k^+=47.1$  (Figure 4.52-Figure 4.55) and the large cylinder,  $k^+=94.1$  (Figure 4.56-Figure 4.60). Despite the differences, the data from the two experiments are remarkably similar and give confidence in the further interpretation of CompLDV3 data.



**Figure 4.49** Stream-wise mean velocity profile for the small isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, CompLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the CompLDV3.



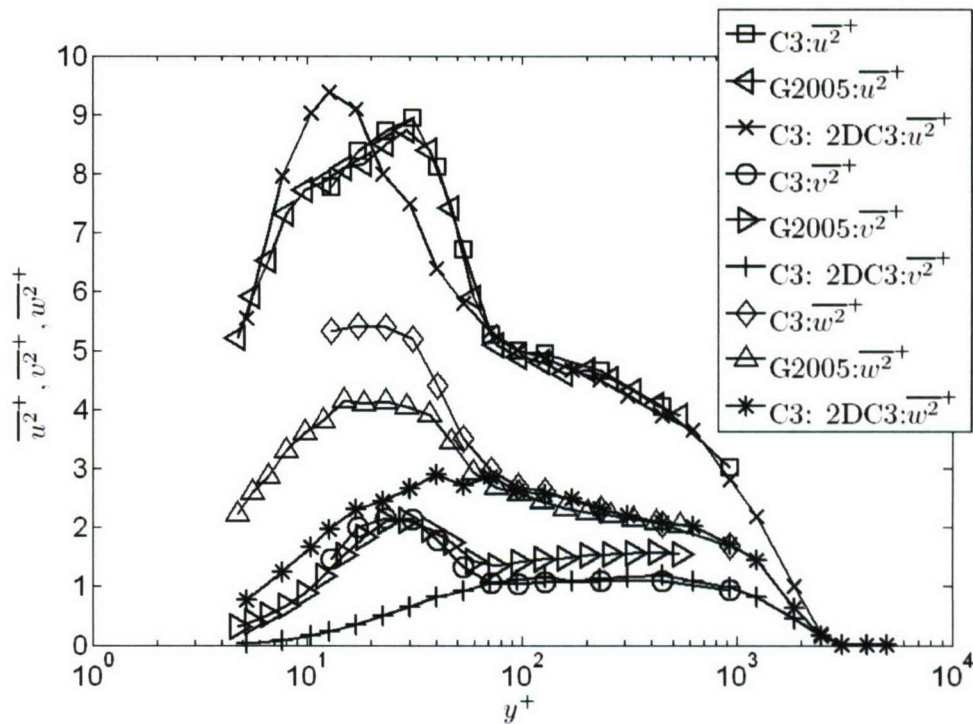


Figure 4.50. Reynolds normal stress profiles for the small isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplLDV3.

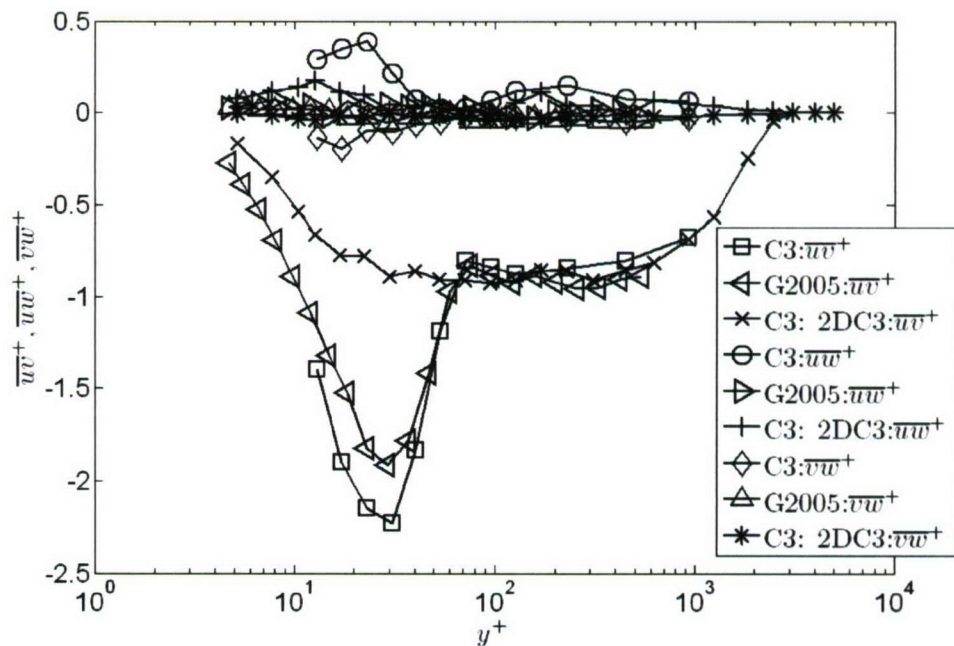
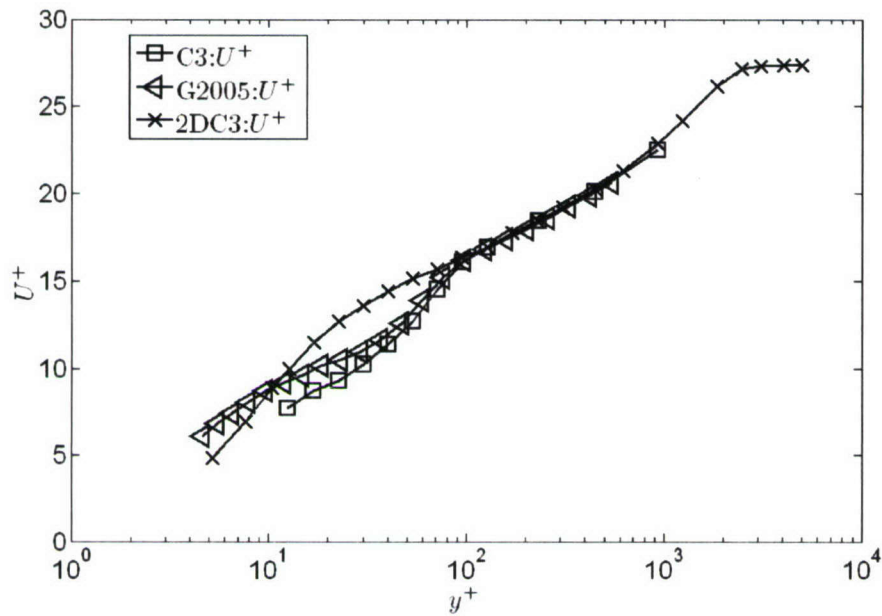
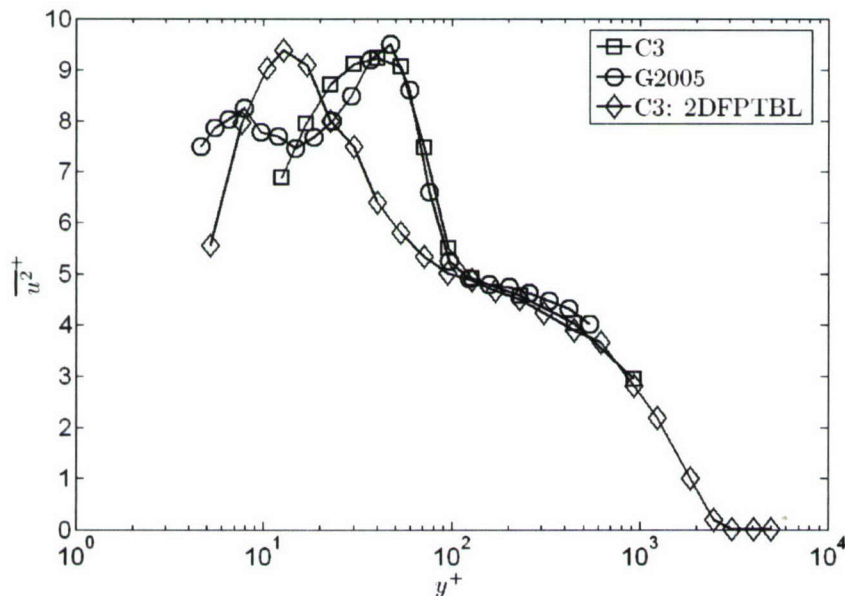


Figure 4.51. Reynolds shear stress profiles for the small isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplLDV3.



**Figure 4.52.** Stream-wise mean velocity profile for the medium-height isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, CompLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the CompLDV3.



**Figure 4.53.** Stream-wise Reynolds normal stress profiles for the medium-height isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, CompLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the CompLDV3.



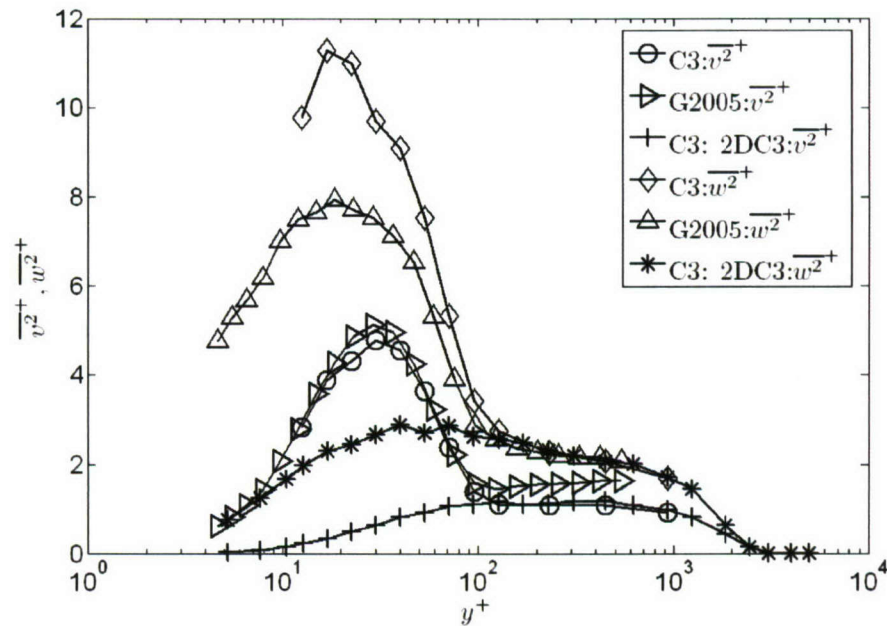


Figure 4.54. Span-wise and normal-to-wall Reynolds normal stress profiles for the medium-height isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplLDV3.

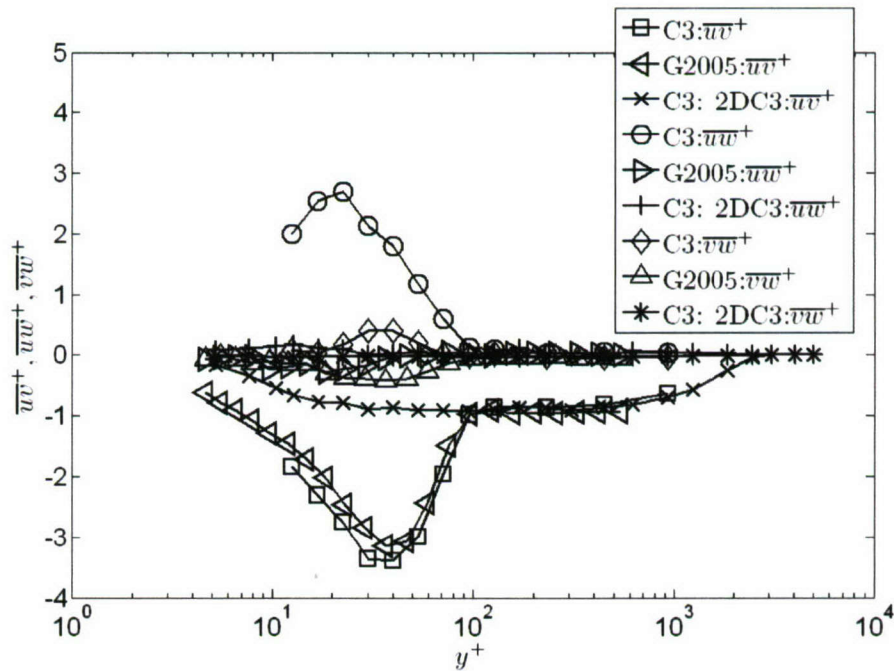
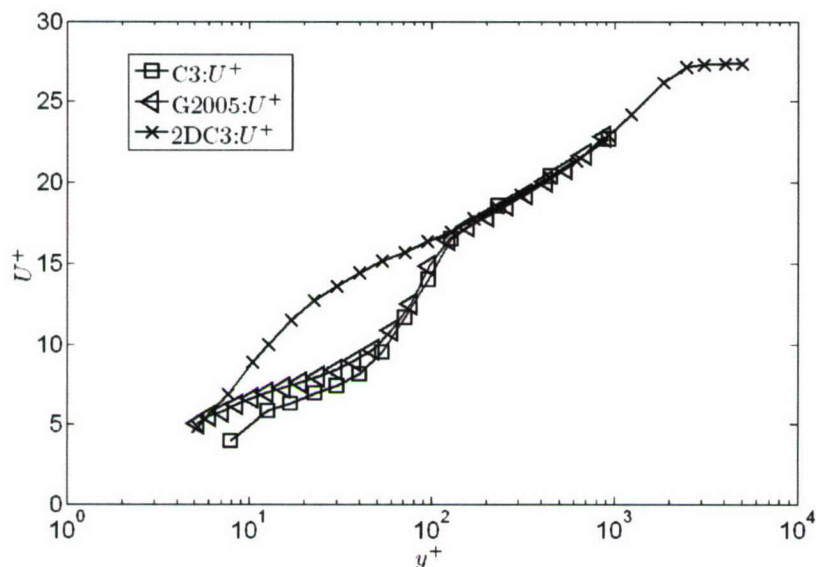
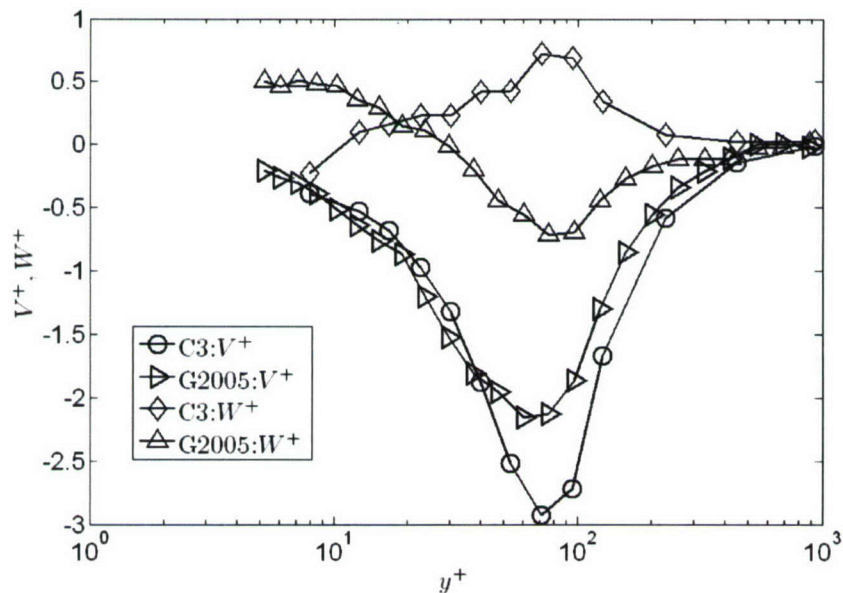


Figure 4.55. Reynolds shear stress profiles for the medium-height isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplLDV3.

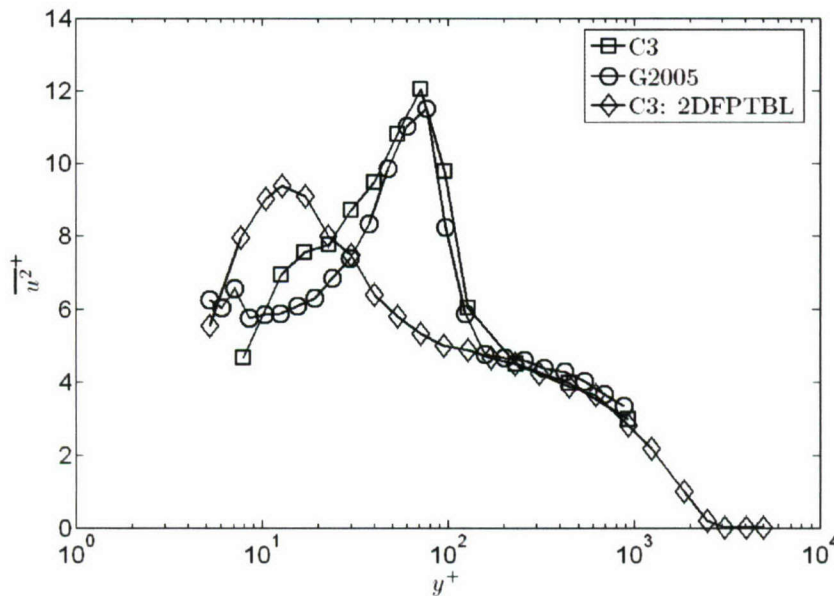


**Figure 4.56.** Stream-wise mean velocity profile for the large isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplDV3.

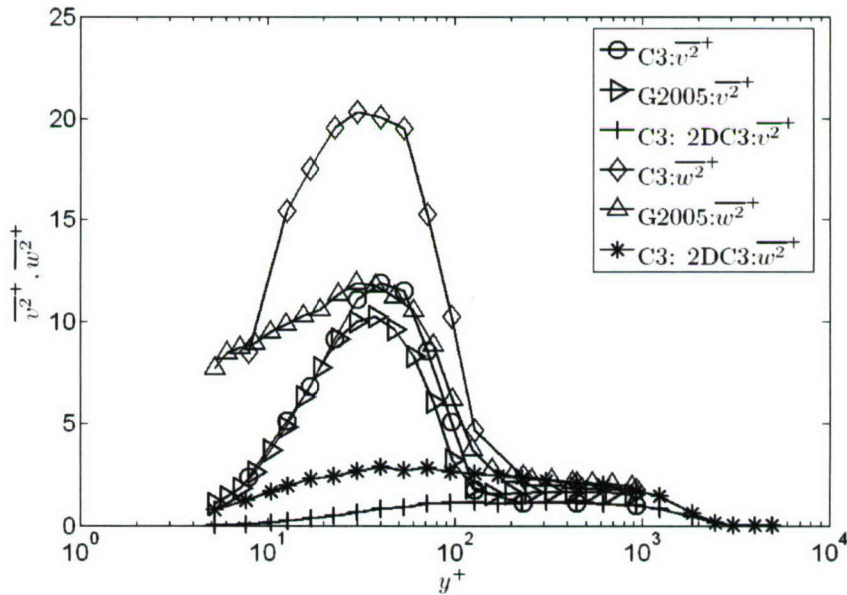


**Figure 4.57.** Normal-to-wall and span-wise mean velocity profiles for the large isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplDV3.

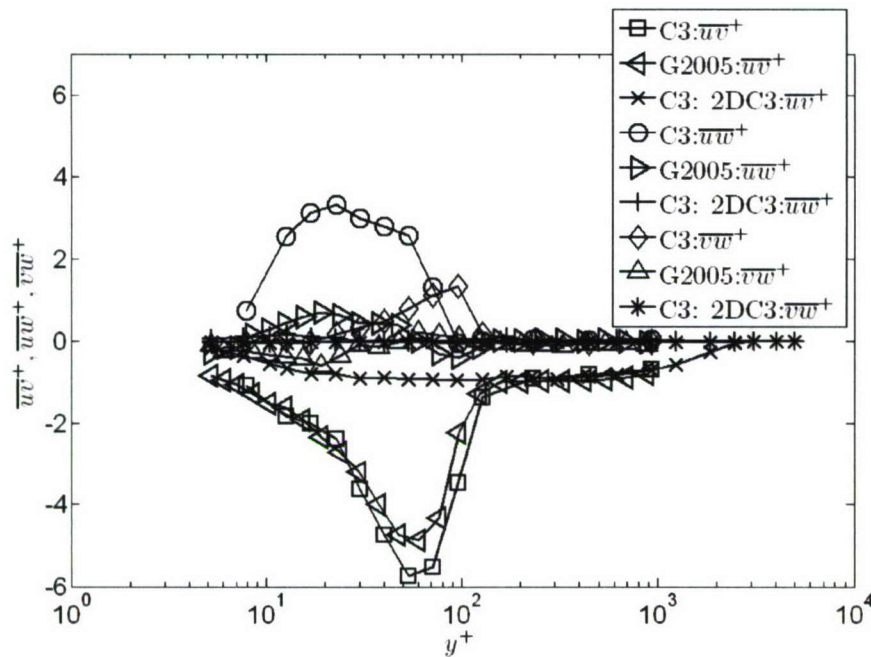




**Figure 4.58.** Stream-wise Reynolds normal stress profiles for the large isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplLDV3.



**Figure 4.59.** Normal-to-wall and span-wise Reynolds normal stress profiles for the large isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, ComplLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the ComplLDV3.



**Figure 4.60. Reynolds shear stress profiles for the medium-height isolated cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . C3, CompLDV3 measurements; G2005, George (2005) measurements; 2DC3, 2DFPTBL measured with the CompLDV3.**

## 4.2.2 Spectral analysis

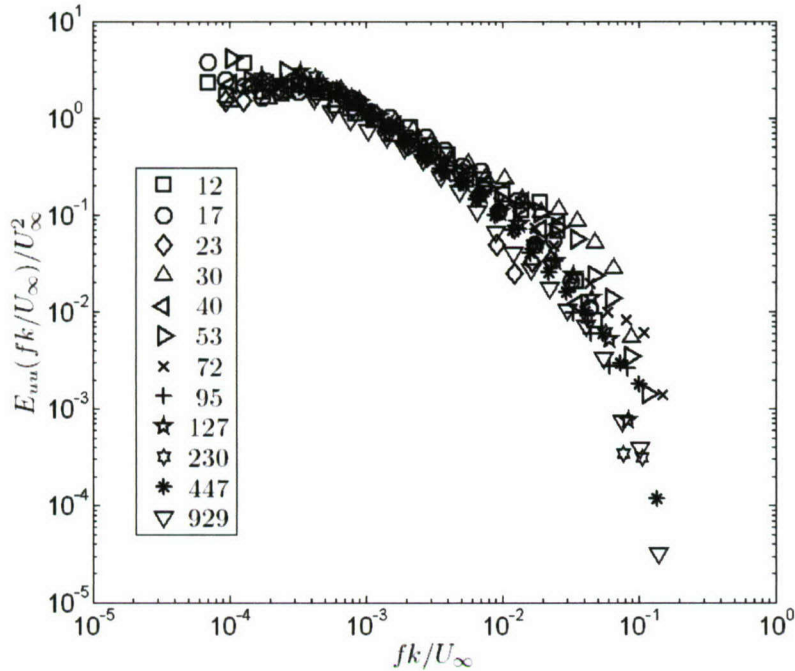
### 4.2.2.1 Small cylinder

The spectral distributions of the Reynolds stresses in the wake of the small isolated cylinder have been computed for the stream-wise location of  $x/d=2.75$  downstream from the center of the element. In order to achieve maximum data rates, the spatial-extent of the CompLDV3 volume was not significantly limited during post-processing and resulted in a diameter of about 200 microns. For this small cylinder, with a height of 381 microns, the relative extent of the measurement volume is significant. However, information about the scales of motion that occur downstream of this element are still useful for interpretation. Furthermore, it may be possible in future work to restrict the measurement volume to obtain spectral estimates at higher spatial resolutions, although at much-reduced data rates.

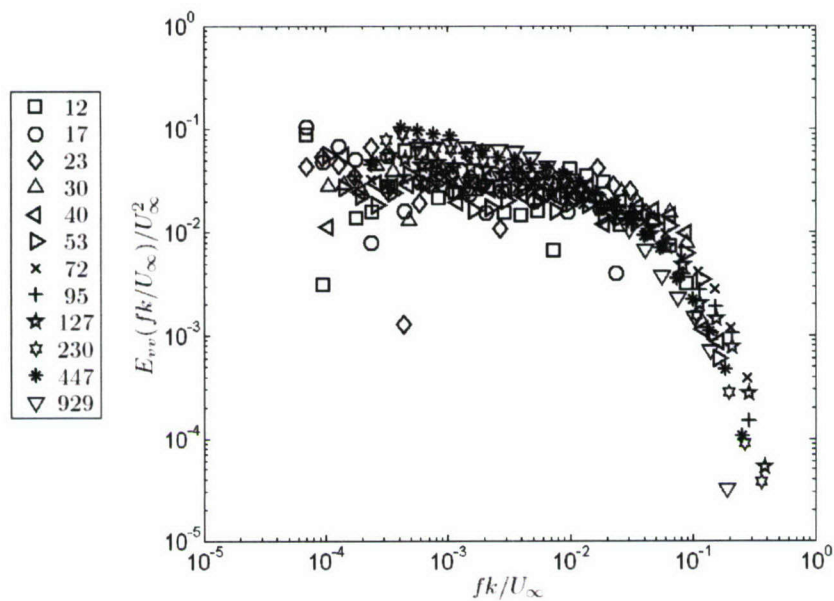
The velocity spectra have been normalized using the free-stream velocity and the height of the isolated cylinder. The normal-stress spectra are given in Figure 4.61-Figure 4.63. Shear stress cross-spectra and coherency are plotted in Figure 4.64 and Figure 4.65, respectively. Around the element height there is an enhancement of the high-frequency motions for the stream-wise normal stress and Reynolds shear stress that is most-likely linked to the large shear rate that occurs over the top of the element. As the flow abruptly encounters the cylinder, large velocity gradients occur perpendicular to the element height. Since a velocity gradient is in itself an inverse time-scale, the inverse of a large



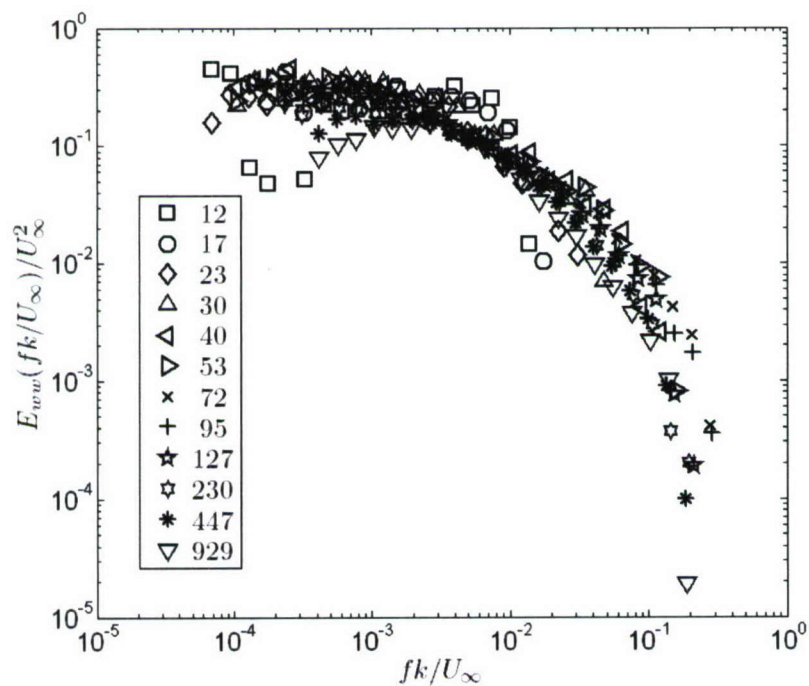
gradient value is a very small time scale. Since the mean approach flow is in the streamwise direction, it is the streamwise velocity that reflects the enhancement of the high-frequency motion in the normal stresses. The instabilities in the shear layer that occurs after separation off the top of the cylinder result in coordinated motions that lead to the high shear stresses measured. The enhancement of the Reynolds shear stress in the high frequencies is also evident in Figure 4.64.



**Figure 4.61. Auto-spectra of the stream-wise velocity fluctuations behind the small cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .**

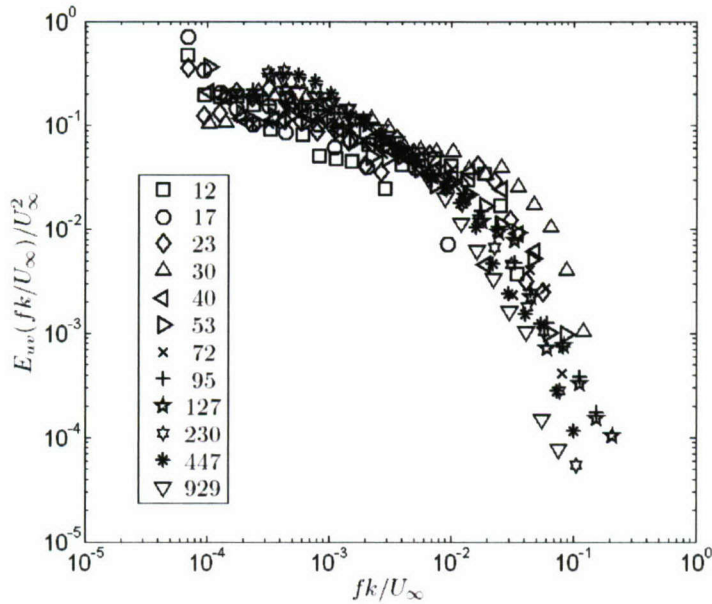


**Figure 4.62.** Auto-spectra of the normal-to-wall velocity fluctuations behind the small cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

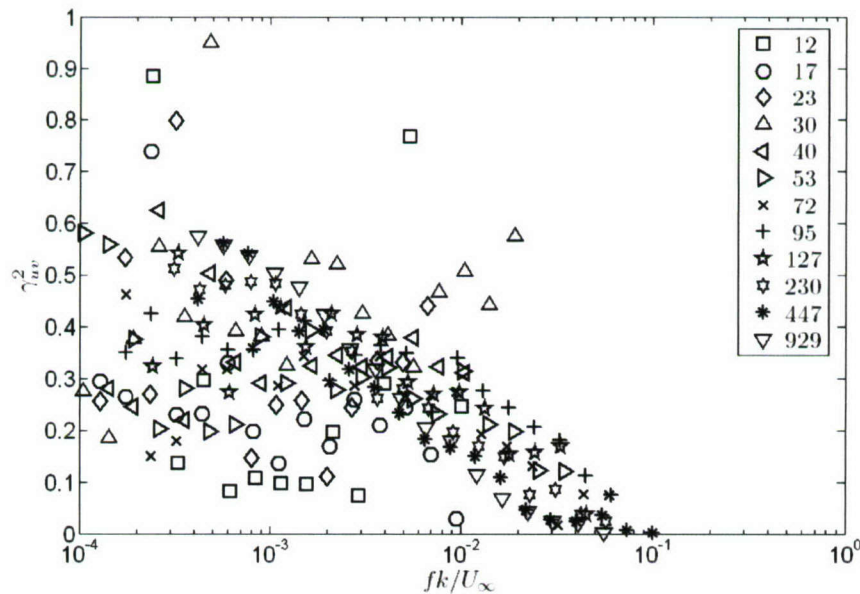


**Figure 4.63.** Auto-spectra of the span-wise velocity fluctuations behind the small cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .





**Figure 4.64.** Cross-spectra of the Reynolds shear stress components,  $u'$  and  $v'$ , behind the small cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

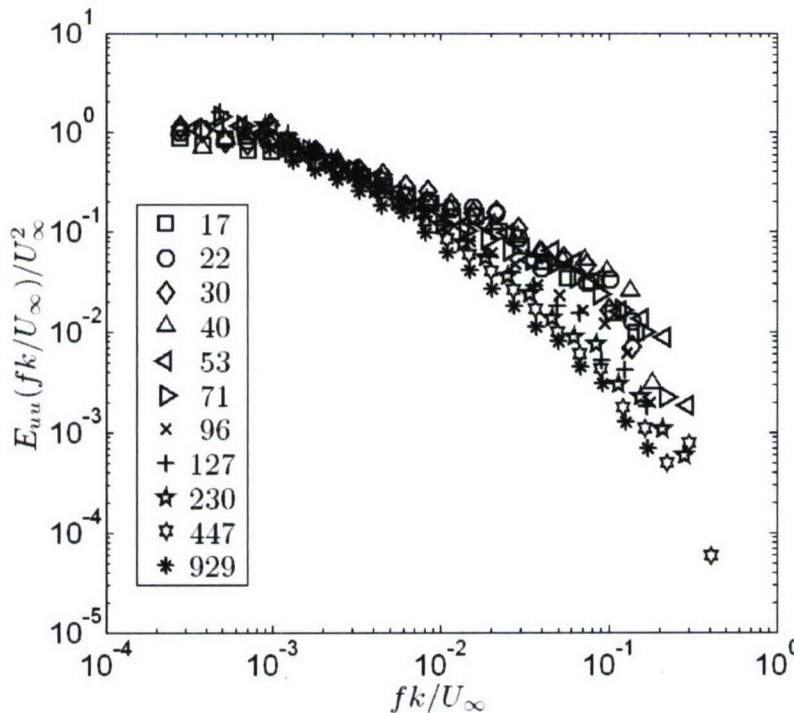


**Figure 4.65.** Spectral coherency between the Reynolds shear stress components,  $u'$  and  $v'$ , behind the small cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

#### 4.2.2.2 Medium-height cylinder

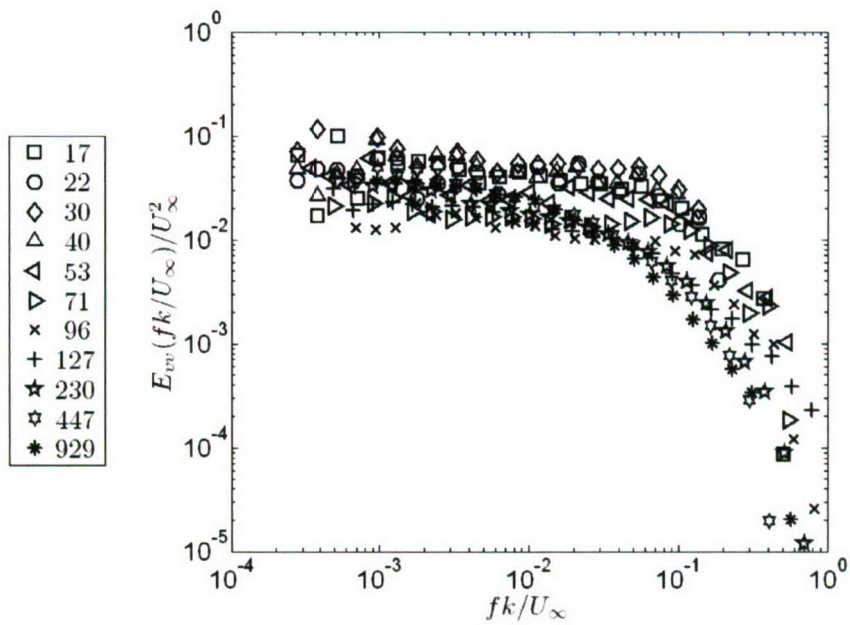
The spectral distributions of the Reynolds stresses in the wake of the medium-height isolated cylinder for the stream-wise location of  $x/d=2.75$  downstream from the center of the element are considered in this section. The concerns in the previous section about probe-volume dimensions still hold but are not as severe considering the cylinder height of 762 microns.

Again, the velocity spectra have been normalized using the free-stream velocity and the height of the isolated cylinder. The normal-stress spectra are given in Figure 4.66-Figure 4.68, while the shear-stress cross-spectra are given in Figure 4.69 with coherency of those fluctuations plotted in Figure 4.70. Again, evidence exists for the argument of the previous section concerning the behavior of the intense shear-layer just after separation from the top of the element. It is clearer for the present case that around the element height there is a definite enhancement of the high frequency motions in the stream-wise normal stresses as compared with the data much above the element. The vertical and span-wise normal stresses exhibit significant augmentation of the values of the velocity spectra in the same region, but for the most part these appear as a uniform scaling increase compared to the outer-flow data. The cross-spectra of the Reynolds shear stress exhibit substantial enhancement in the high-frequency ranges that again supports the notion of instabilities in the shear layer downstream of the element.

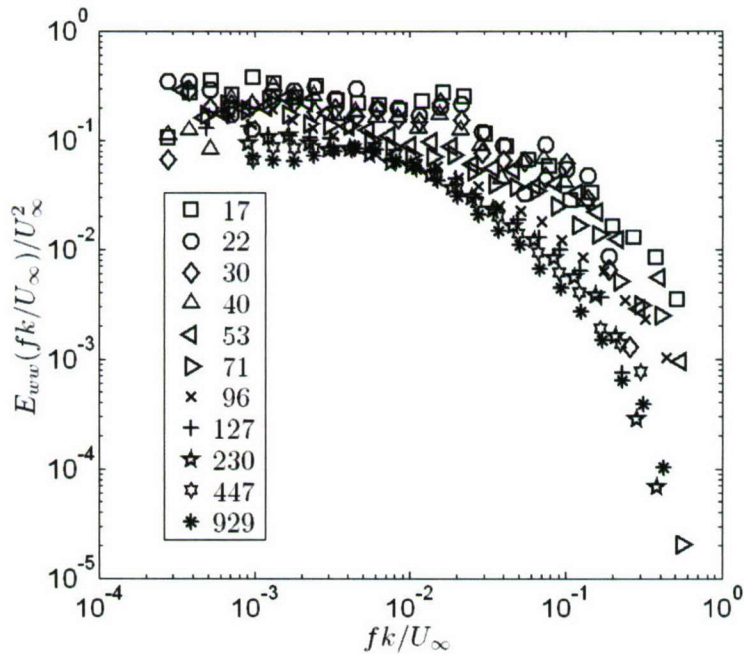


**Figure 4.66.** Auto-spectra of the stream-wise velocity fluctuations behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .





**Figure 4.67.** Auto-spectra of the normal-to-wall velocity fluctuations behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .



**Figure 4.68.** Auto-spectra of the span-wise velocity fluctuations behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

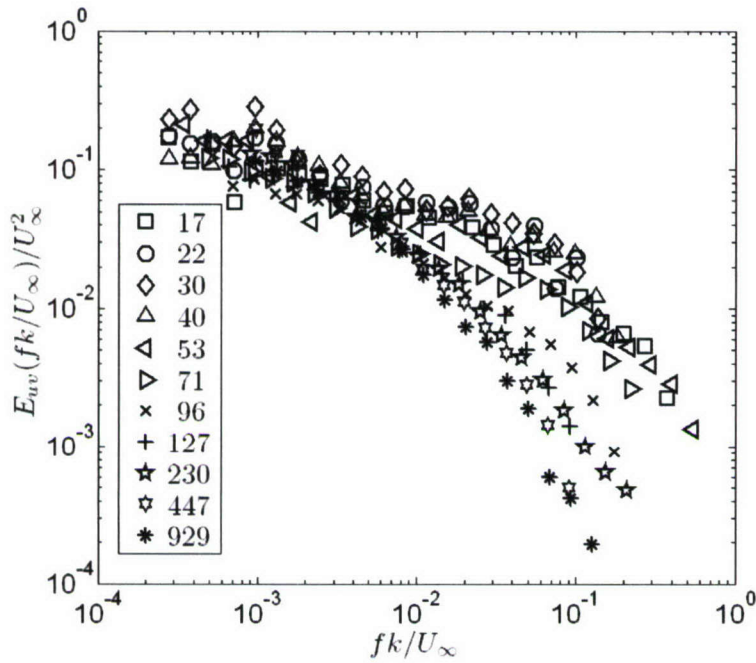


Figure 4.69. Cross-spectra of the Reynolds shear stress components,  $u'$  and  $v'$ , behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

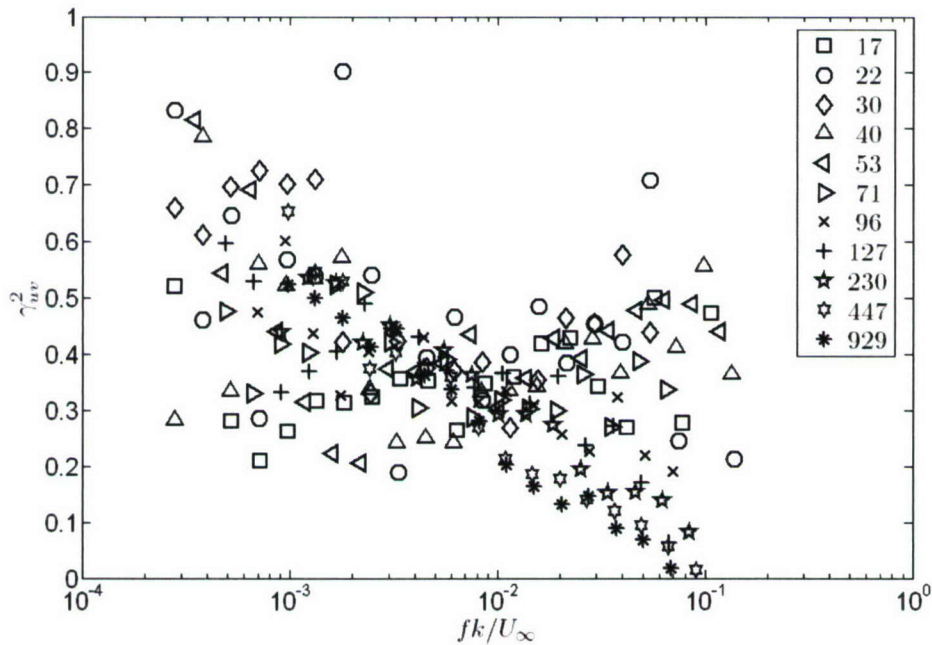


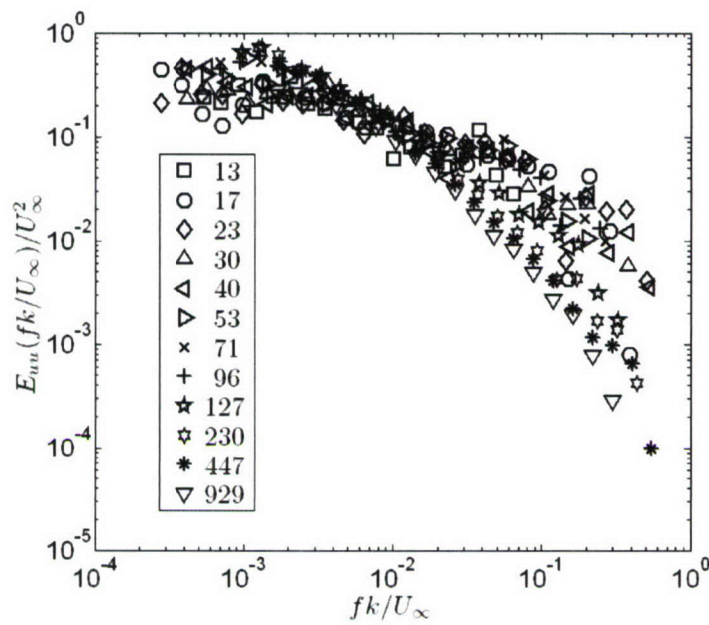
Figure 4.70. Spectral coherency between the Reynolds shear stress components,  $u'$  and  $v'$ , behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .



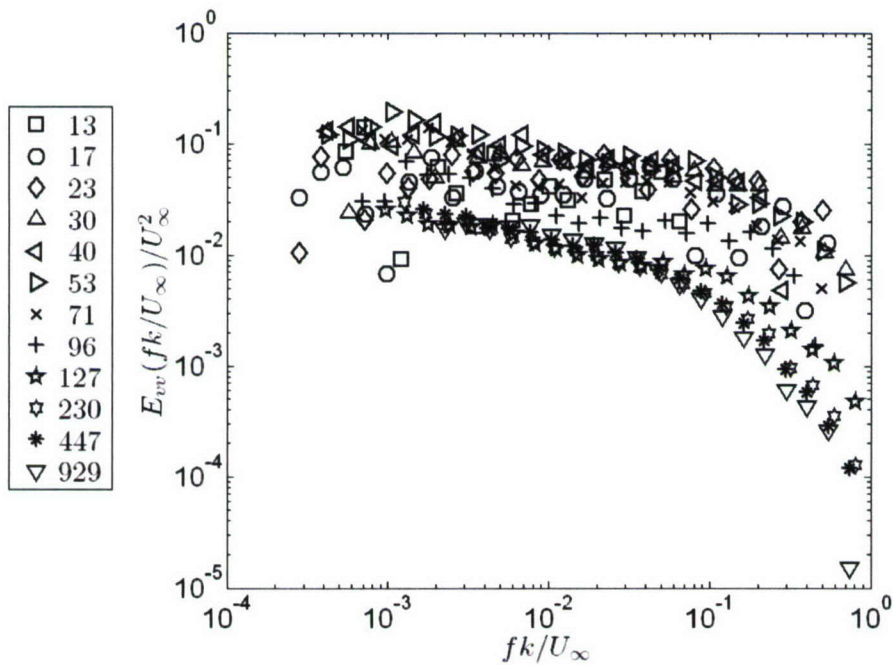
### 4.2.2.3 Large cylinder

The normal-stress spectra for the large cylinder are given in Figure 4.71-Figure 4.73, while the shear-stress cross-spectra are given in Figure 4.74 with coherency of those fluctuations plotted in Figure 4.75. Very similar phenomena appear in the high frequencies of the stream-wise normal stress and Reynolds shear stress as discussed for the small and medium cylinders. Effects of the element on the Reynolds shear stress spectra appear out to  $y^+=230$ , more than twice the height of the cylinder.

An additional interesting feature appears in these data for the span-wise normal stress spectra. Energy is concentrated in the spectra around the element height at  $fk/U_\infty \approx 0.05$ . This is somewhat unexpected since the approach flow is fully-turbulent, and that may lead one to the conclusion that a deterministic occurrence in frequency is unlikely. The extensive velocity statistics data of George (2005) give some insight into this phenomenon. The large cylinder has a relatively-intense mean vortex, denoted by George as the roughness top vortex structure (RTVS) with its center of rotation at  $y^+=30$ . In the same region as the vortex center, there is an intense region of the shear stress  $\overline{uw}$  that is also aligned with the span-wise edge of the cylinder. This seems to indicate a vertical fluctuation of the vortex center location that delivers with it either positive stream-wise-momentum fluid from areas away from the element when it fluctuates toward the wall or negative-momentum fluid from the separated region behind the cylinder when the center of the vortex moves upward. It seems likely that the mechanism for this phenomenon is the interaction of the two legs of the RTVS on either side of the element. Since the frequency of this interaction is relatively high, it may be presumed that coherent structures of large stream-wise extent may establish a sufficiently stable flow for periods long enough to induce this unsteady motion.



**Figure 4.71.** Auto-spectra of the stream-wise velocity fluctuations behind the large cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .



**Figure 4.72.** Auto-spectra of the normal-to-wall velocity fluctuations behind the large cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .



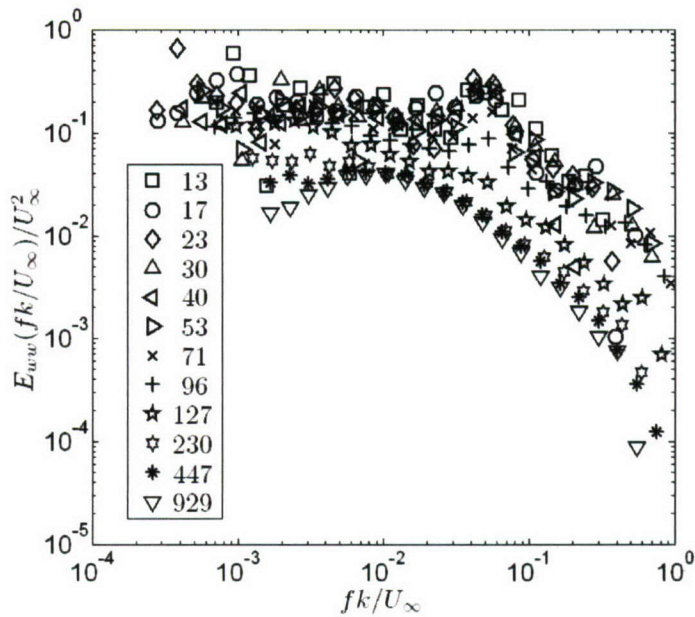


Figure 4.73. Auto-spectra of the span-wise velocity fluctuations behind the large cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

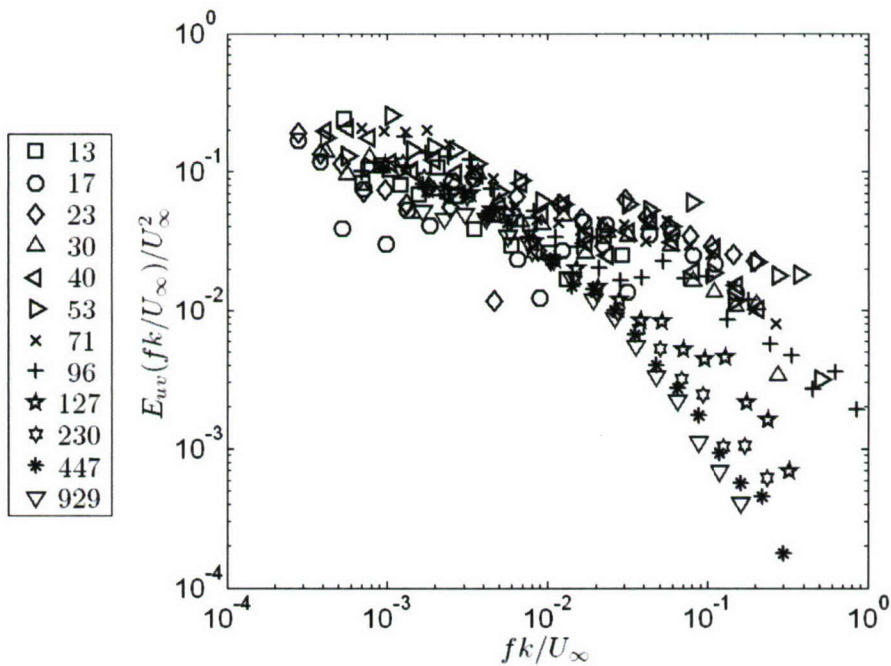
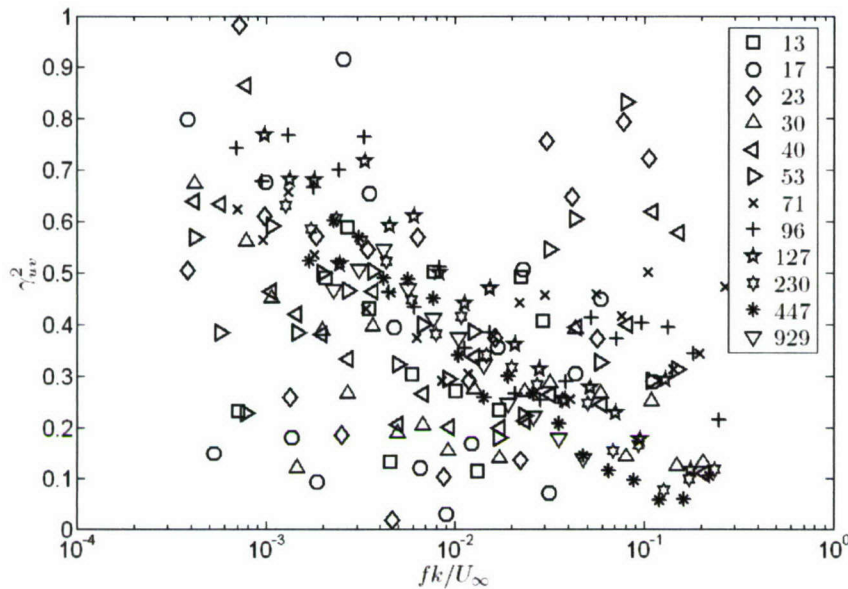


Figure 4.74. Cross-spectra of the Reynolds shear stress components,  $u'$  and  $v'$ , behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .



**Figure 4.75.** Spectral coherency between the Reynolds shear stress components,  $u'$  and  $v'$ , behind the medium-height cylinder at the location  $x/d=2.75$ ,  $z/d=0$ . The legend gives the measurement height in  $y^+$ .

### 4.3 3D Turbulent Boundary Layers

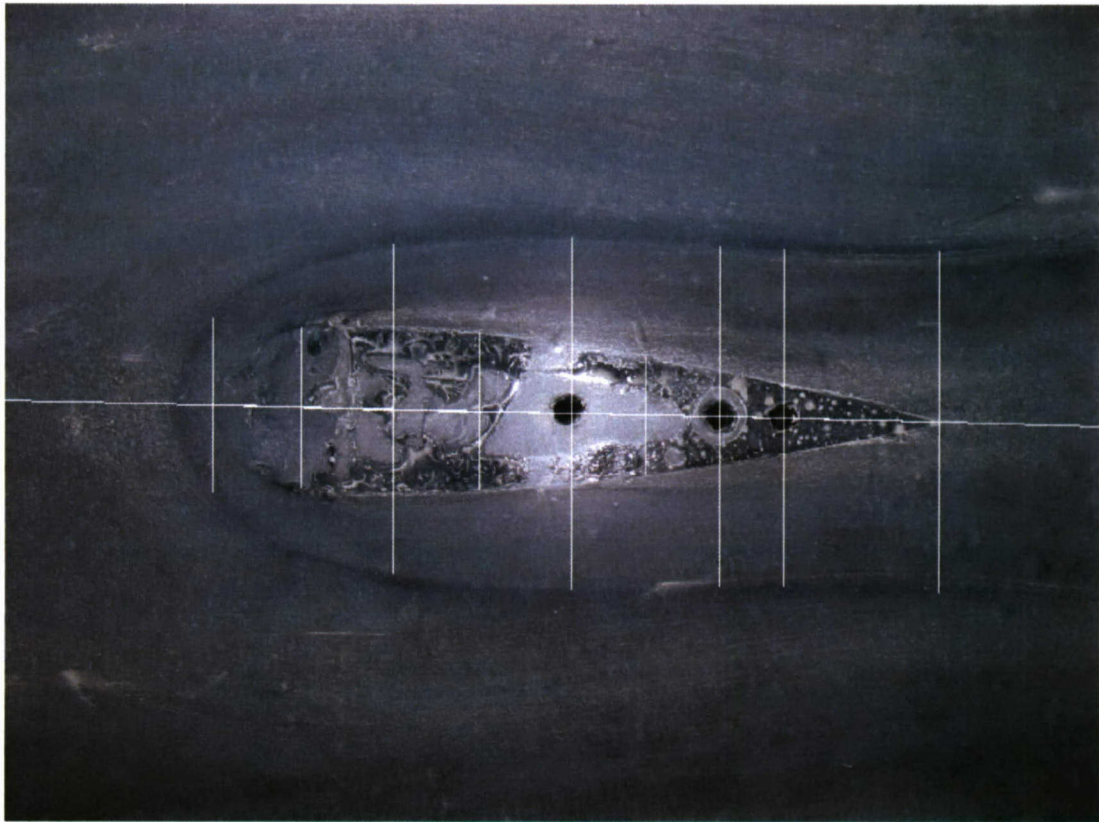
The attached turbulent boundary layer in the vicinity of a wing/body junction has been measured using the CompLDV3 technologies. The geometry of the wing is a 3:2 elliptical nose joined at the maximum thickness to a NACA 0020 airfoil with a maximum thickness of 7.17cm. This particular flow is well-studied by the TBLRG (Devenport and Simpson 1990; Ölçmen and Simpson 1995; Simpson 1996; Ölçmen et al. 2001a; 2001b; Simpson 2001; George 2005). The complex flow includes a meandering separated region very near the junction region of the wing, a highly-unsteady horseshoe vortex that is formed at the leading edge of the wing/body junction, and span-wise pressure gradients that generate stream-wise vorticity and strong three-dimensionality even outside of the attached vortex region.

#### 4.3.1 Setup and oil-flow

The wing/body junction was installed into the Department of Aerospace and Ocean Engineering Boundary Layer Research Wind Tunnel in as nearly the same manner as possible to that done by previous researchers. To continue the same study as Ölçmen and Simpson (1995), the tunnel was set to a nominal freestream velocity of 27.5m/s such that the 2DFPTBL in the absence of the wing/body junction was at a Reynolds number of  $Re_\theta = 7497$  as documented in section 4.1.1.



The setup of the flow was initially tested for symmetry and consistency with previous experiments using oil-flow visualization. The procedures and oil-pigment formula described by Tian et al. (2004) were used. In Figure 4.76, a digital photograph of the oil-flow has been examined using a mechanical drawing program that allows precise gauging of the 2D positions of points in the photograph. To assess the symmetry of the flow, the perpendicular distance from the chord-line to the horseshoe vortex washout identified by the dark line absent of pigment from the oil-flow was measured at several stream-wise locations. An excellent symmetry was achieved within the spatial uncertainties of the oil-flow technique.



**Figure 4.76. Oil-flow visualization for the wind-tunnel floor in the vicinity of the wing junction. The lines on the plot were drawn using mechanical drawing software and are equal length from the chord-line of the wing. Credit goes to Joshua DeMoss for this figure.**

### 4.3.2 CompLDV3 data

The CompLDV3 data were acquired at two stations that exhibit attached 3D TBL flows. The locations of the stations relative to the wing/body junction are shown in Figure 4.77, where stations 3 and 5 have been measured for the current study. At each of the stations, detailed, low-uncertainty conventional 3-component LDV data exist and stand as the

benchmark for the current results—all CompLDV3 velocity statistics results are compared to those of Ölçmen and Simpson (1995) for consistency.

As discussed in section 2.3.2, the CompLDV3 operates best when rotated to the approximate flow angle to be measured (see the geometry of  $\beta_{\text{Probe}}$  in Figure 4.77). With the previous data for these flows, it was possible to obtain these probe rotations *a priori*, although when measuring flows for the first time, a CompLDV3 profile may be done to obtain the mean velocities and flow angles for more subsequent, rotated measurements. The probe rotations used for measurements at station 3 were  $33.2^\circ$  for the near-wall data and  $20^\circ$  for the data further from the wall. For station 5 the near-wall data were taken with a probe rotation of  $25.2^\circ$  while an  $8^\circ$  rotation was used for the data further from the wall.

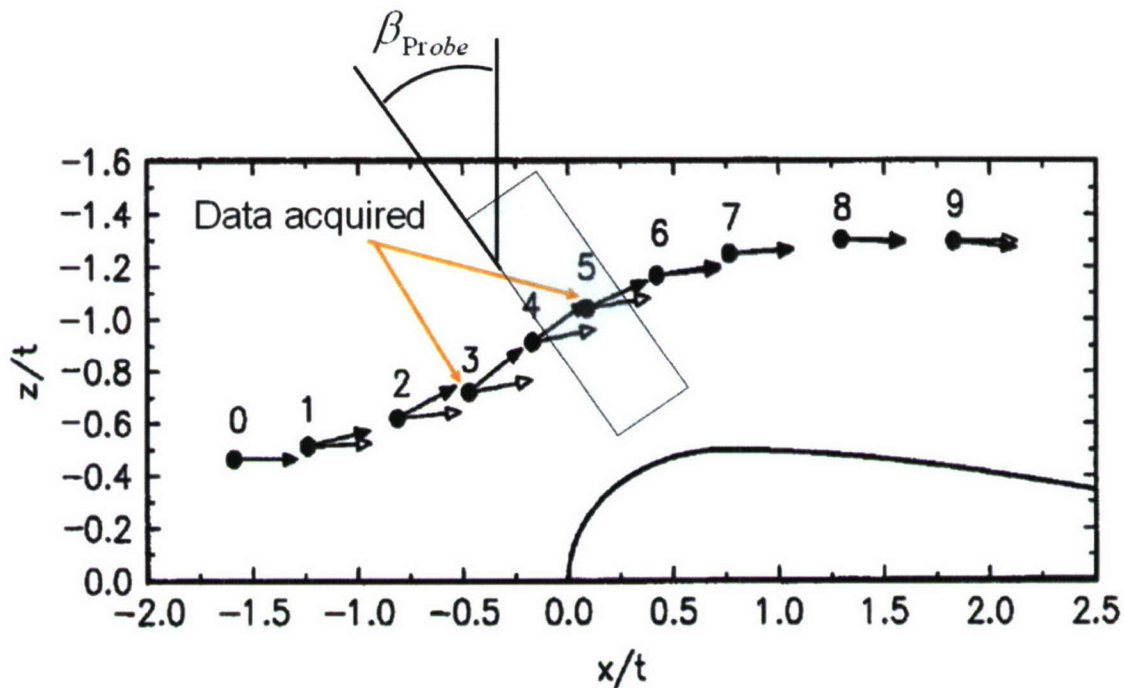


Figure 4.77. Plot of the 3D boundary layer measurements in the vicinity of the wind-tunnel floor near a wing junction. The filled arrow heads indicate the direction of the wall shear while the open arrows indicate the free-stream velocity direction. The box surrounding station 5 represents the CompLDV3 probe orientation with the length aligned with the projection of the beam bisectors of the two heads. This figure adapted from figure 1 in Ölçmen et al. (2001a) with kind permission of Springer Science and Business Media.

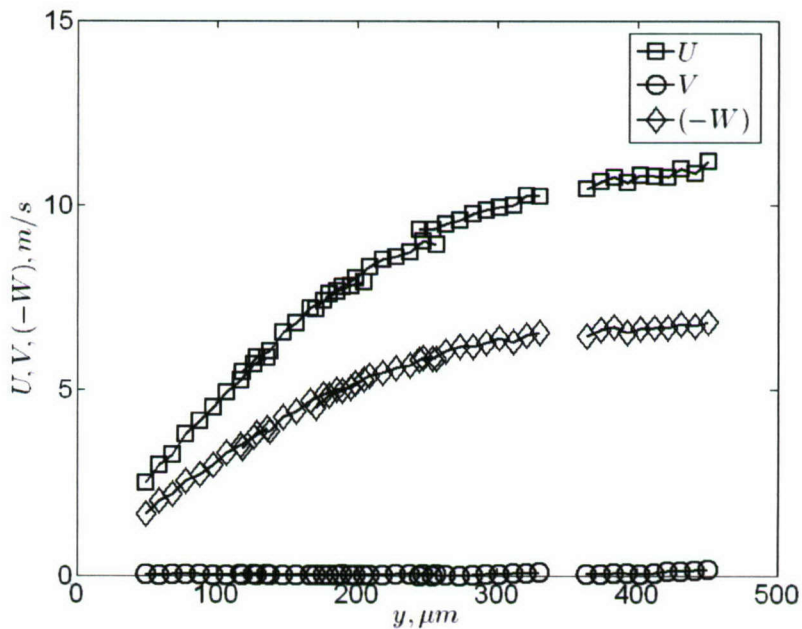
#### 4.3.2.1 Reynolds-averaged statistics

##### 4.3.2.1.1 The near-wall region

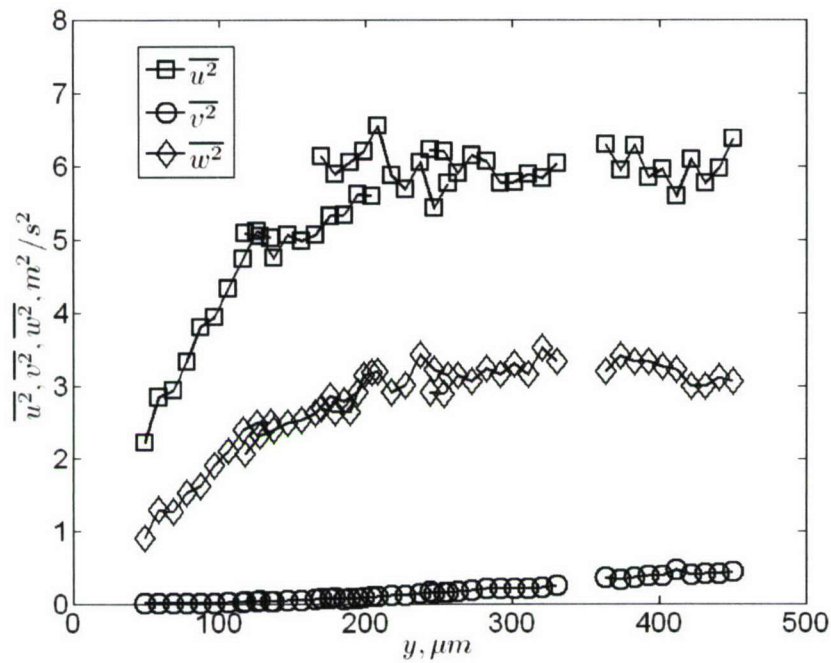


The CompLDV3 position resolution capabilities were used to obtain information in the near-wall regions of stations 3 and 5. The resolved measurements for station 3 are plotted in Figure 4.78-Figure 4.80 while station 5 data are given in Figure 4.81-Figure 4.84 . Again, as was mentioned in section 4.1.1.1, these are the most-spatially-resolved measurements ever obtained in these flows, with a resolution between each point of about 10 microns. As mentioned in section 2.3.2.3, the CompLDV3 is truly operating in a multi-dimensional manner as is evident in the plot of the flow angle for station 5, Figure 4.82. The CompLDV3 actually detects changes of flow angle occurring within the measurement volume.

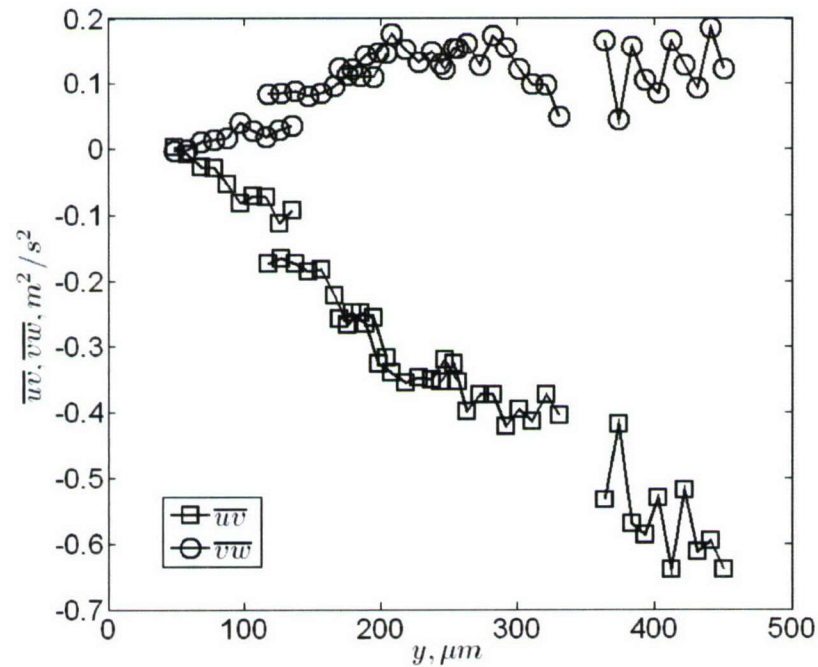
These data were used to obtain estimates of the wall-shear stress by considering the velocity gradient of the resultant co-planar wall-velocity:  $Q \equiv \sqrt{U^2 + W^2}$  . The values obtained from these fits of the near-wall data gave wall-friction velocities of  $u_\tau = 1.05\text{m/s}$  and  $u_\tau = 1.20\text{m/s}$  for stations 3 and 5 respectively. In comparison, Ölçmen and Simpson (1995) report values of  $u_\tau = 0.957\text{m/s}$  and  $u_\tau = 1.15\text{m/s}$  for the same respective values. The volume-averaged data for the CompLDV3 to follow are normalized using the wall-friction values obtained in this study and are compared to the data of Ölçmen and Simpson (1995) using the wall friction values found in their study.



**Figure 4.78. Sub-measurement volume resolution mean velocities for the wing/body junction flow at station 3.**

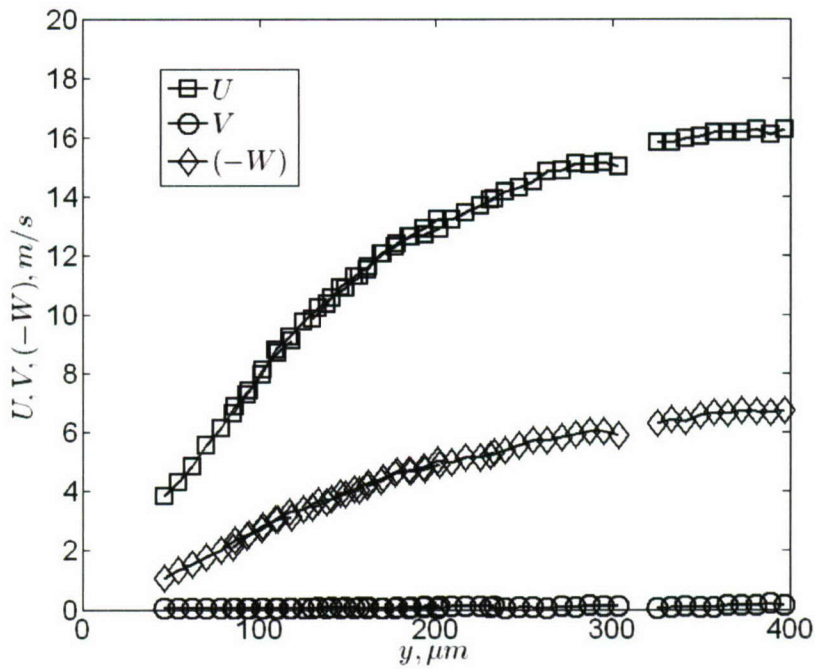


**Figure 4.79.** Sub-measurement volume resolution Reynolds normal stresses for the wing/body junction flow at station 3.

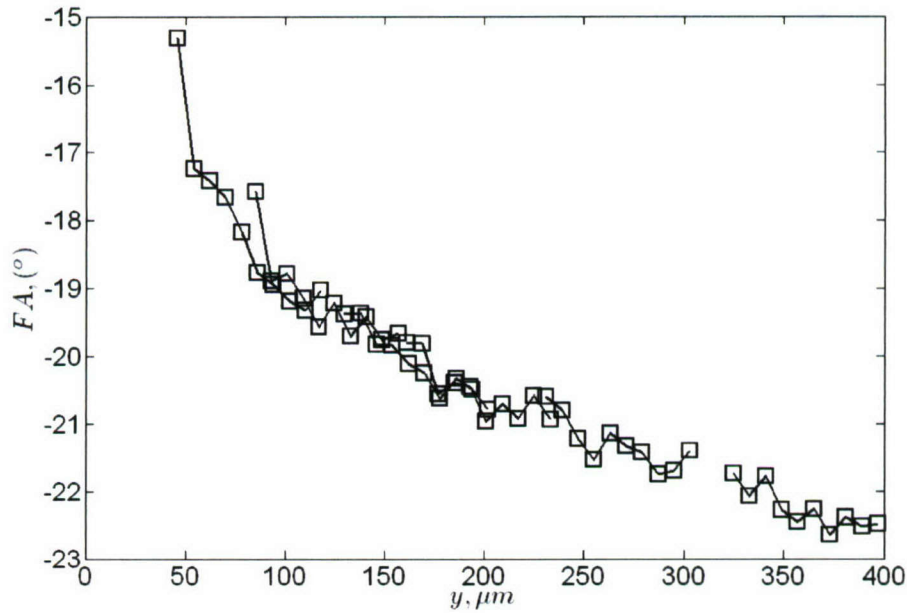


**Figure 4.80.** Sub-measurement volume resolution Reynolds shear stresses for the wing/body junction flow at station 3.





**Figure 4.81.** Sub-measurement volume resolution mean velocities for the wing/body junction flow at station 5.



**Figure 4.82.** Sub-measurement volume resolution mean flow angle for the wing/body junction flow at station 5.

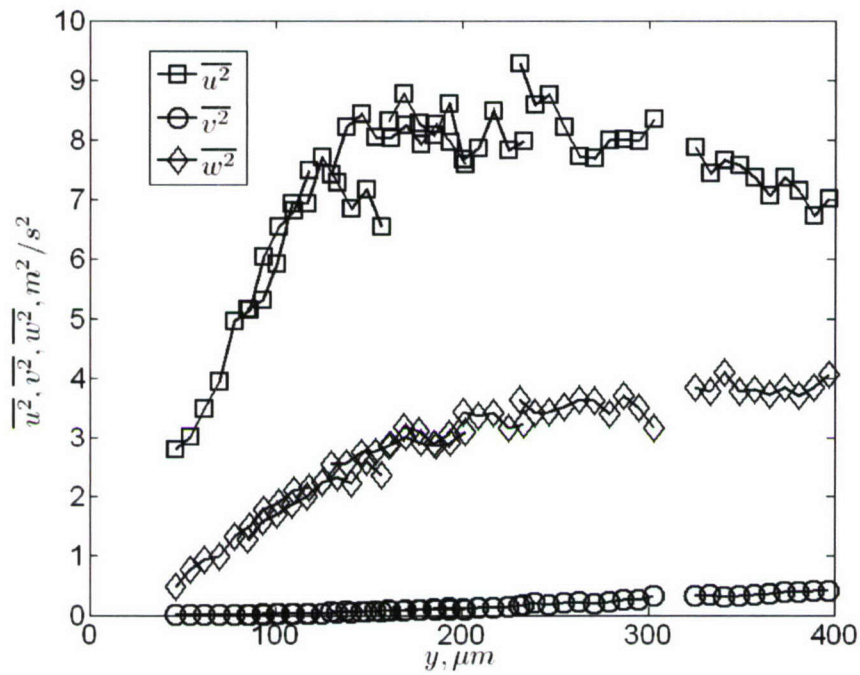


Figure 4.83. Sub-measurement volume resolution Reynolds normal stresses for the wing/body junction flow at station 5.

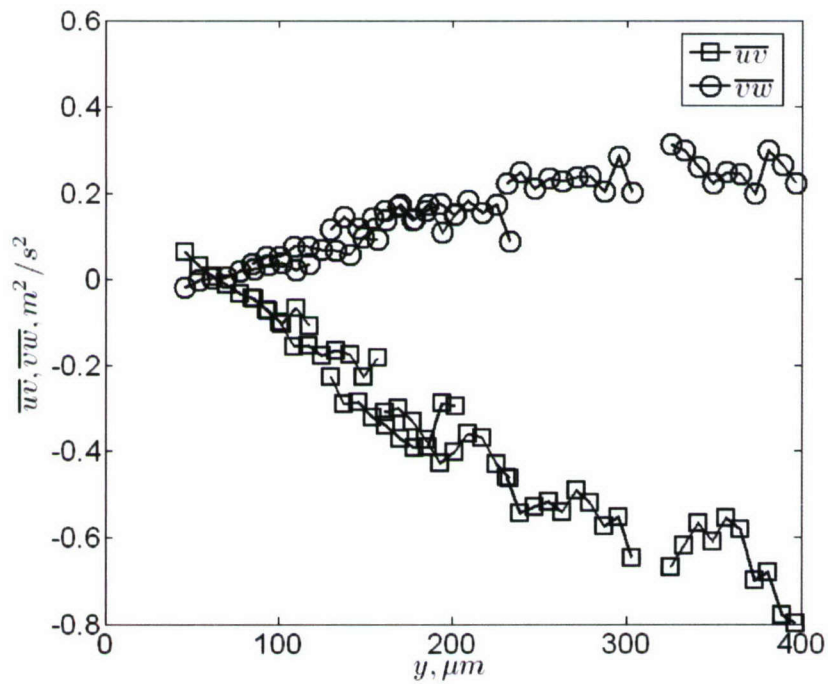
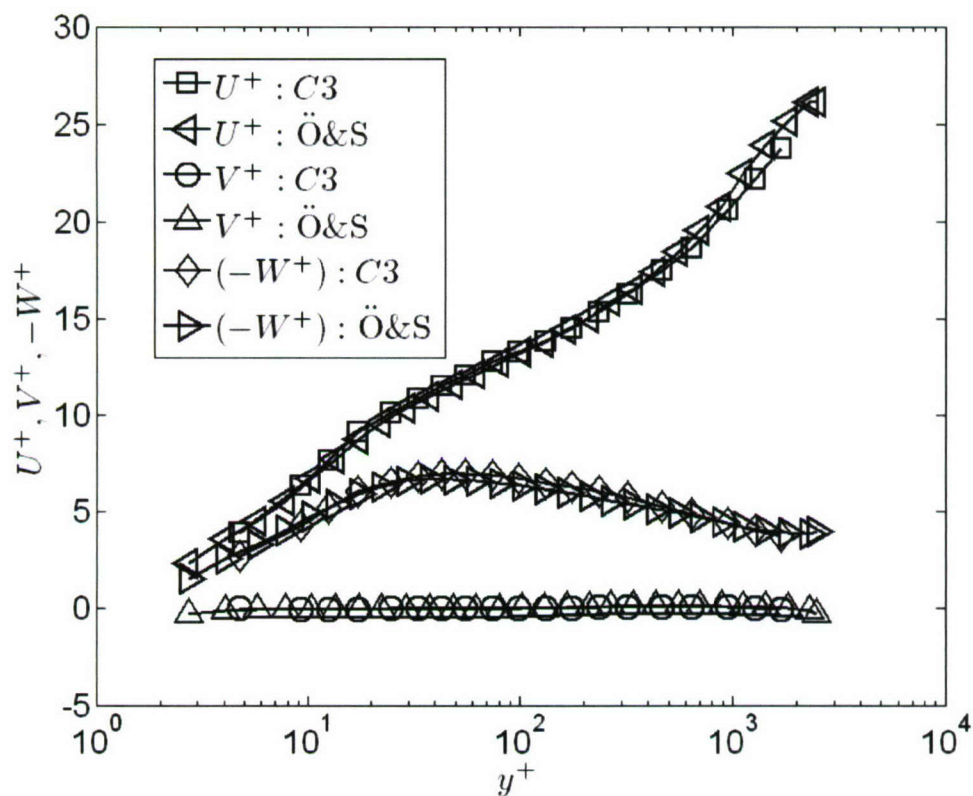


Figure 4.84. Sub-measurement volume resolution Reynolds shear stresses for the wing/body junction flow at station 5.



### 4.3.2.1.2 Volume-averaged statistics

The volume-averaged velocity statistics obtained with the CompLDV3 are compared to the results of Ölçmen and Simpson (1995) in Figure 4.85-Figure 4.88 for the data at station 3 and Figure 4.89-Figure 4.92 for station 5. Just as discussed in section 4.1.2, the CompLDV3 probe-volume was limited to 100 microns in the vertical direction and all quantities were corrected for velocity gradient broadening. Throughout the datasets, the agreement in the statistics is excellent between the current data and those of Ölçmen and Simpson (1995), despite the small discrepancy in the friction velocities.



**Figure 4.85. Mean velocities for the wing/body junction flow at station 3. C3: CompLDV3; Ö&S: Ölçmen and Simpson (1995).**

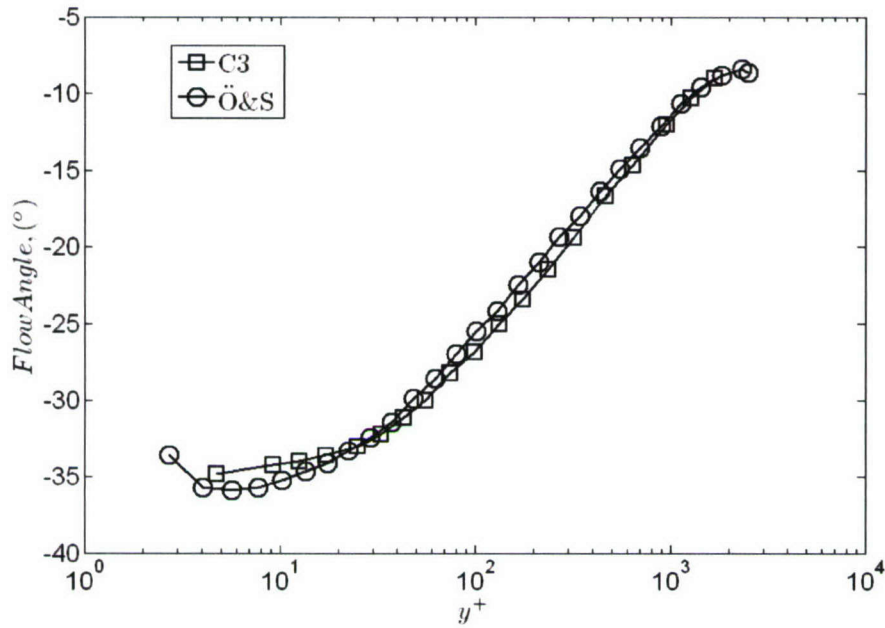


Figure 4.86. Mean flow angle for the wing/body junction flow at station 3. C3: CompLDV3; Ö&S: Ölçmen and Simpson (1995).

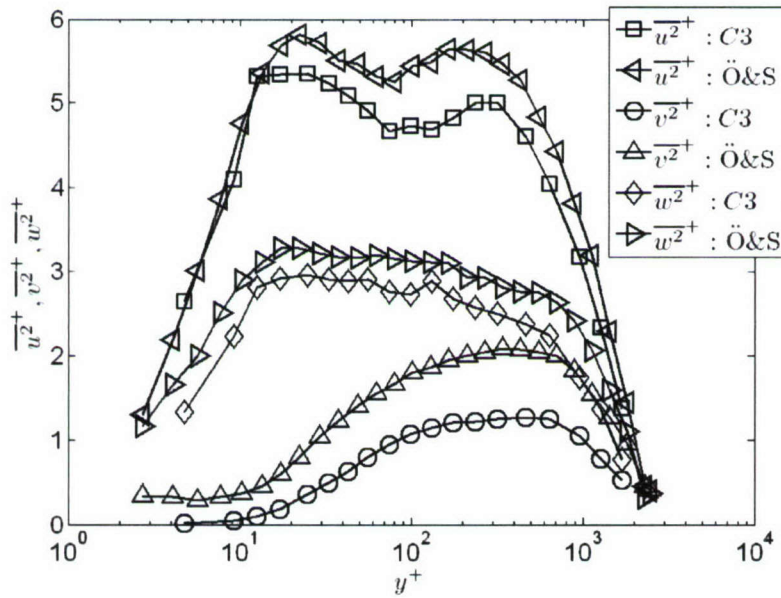


Figure 4.87. Reynolds normal stresses for the wing/body junction flow at station 3. C3: CompLDV3; Ö&S: Ölçmen and Simpson (1995).

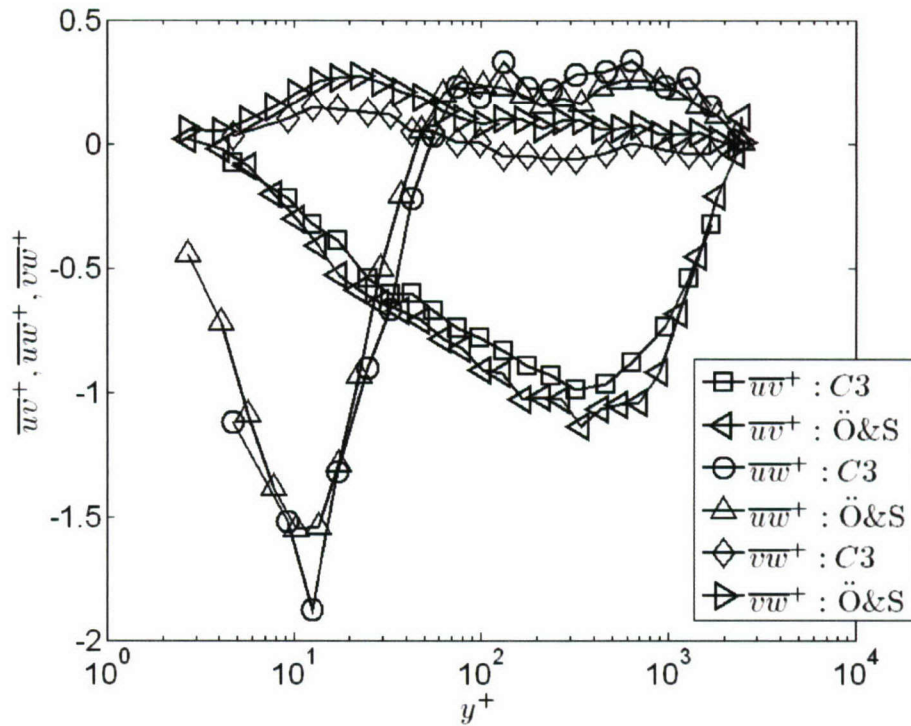


Figure 4.88. Reynolds shear stresses for the wing/body junction flow at station 3. C3: ComplDV3; Ö&S: Ölçmen and Simpson (1995).

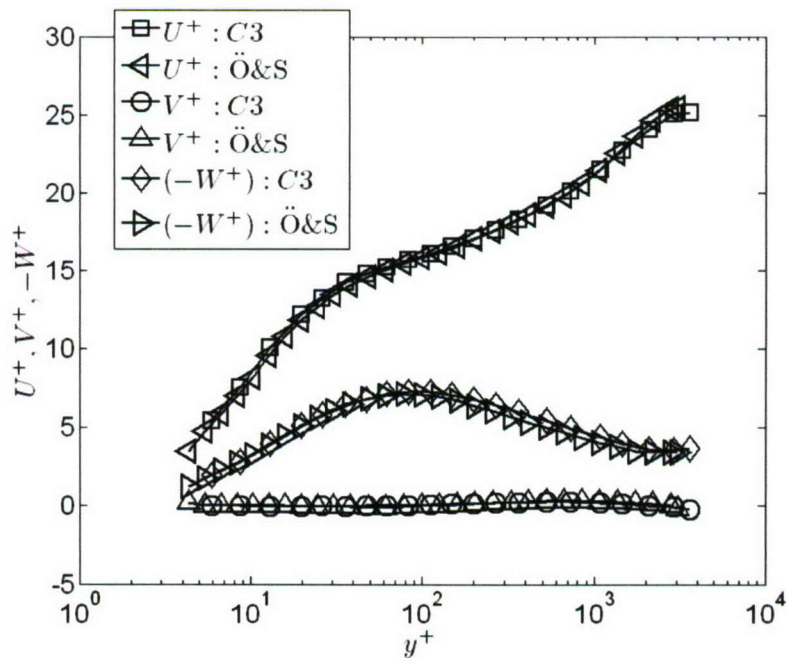


Figure 4.89. Mean velocities for the wing/body junction flow at station 5. C3: ComplDV3; Ö&S: Ölçmen and Simpson (1995).



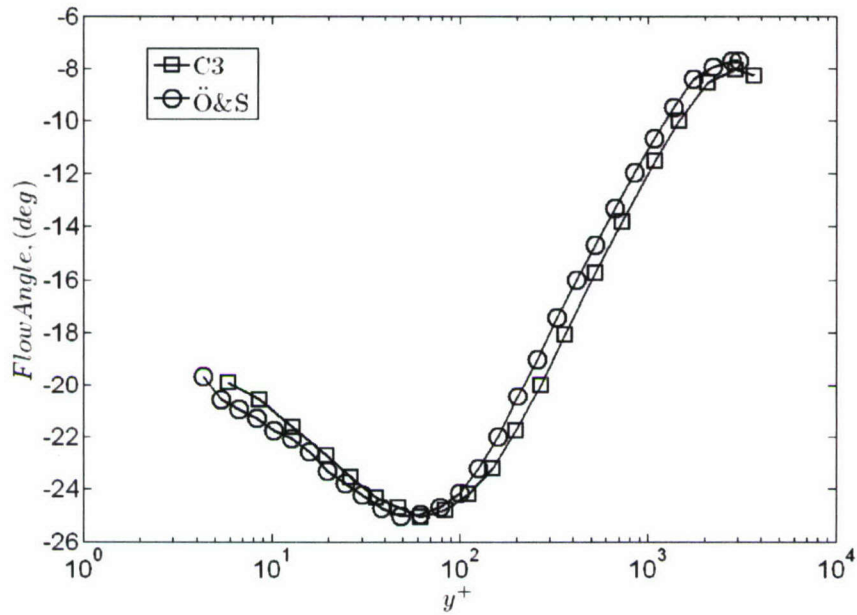


Figure 4.90. Mean flow angle for the wing/body junction flow at station 5. C3: ComplDV3; Ö&S: Ölçmen and Simpson (1995).

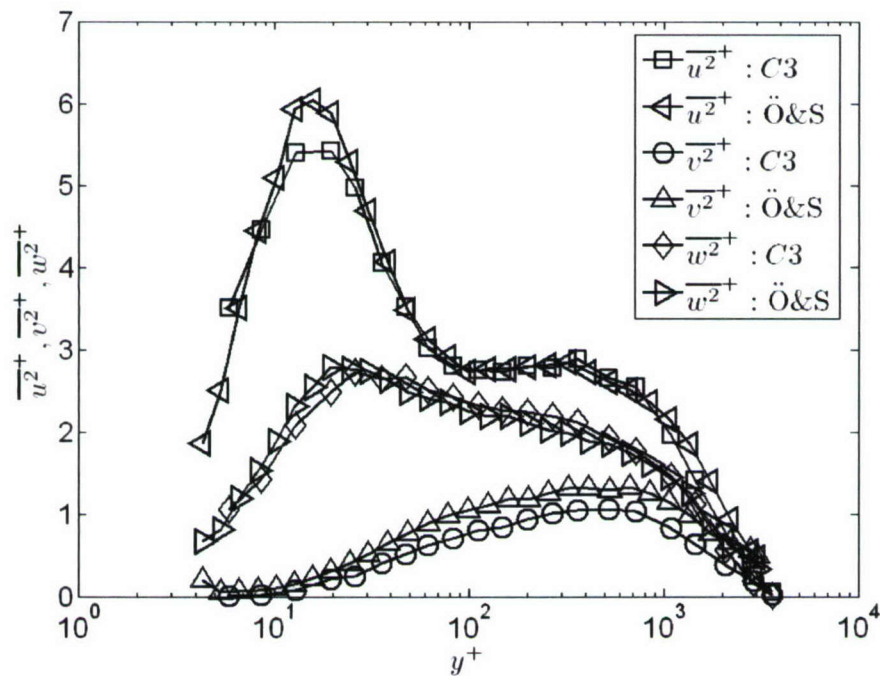
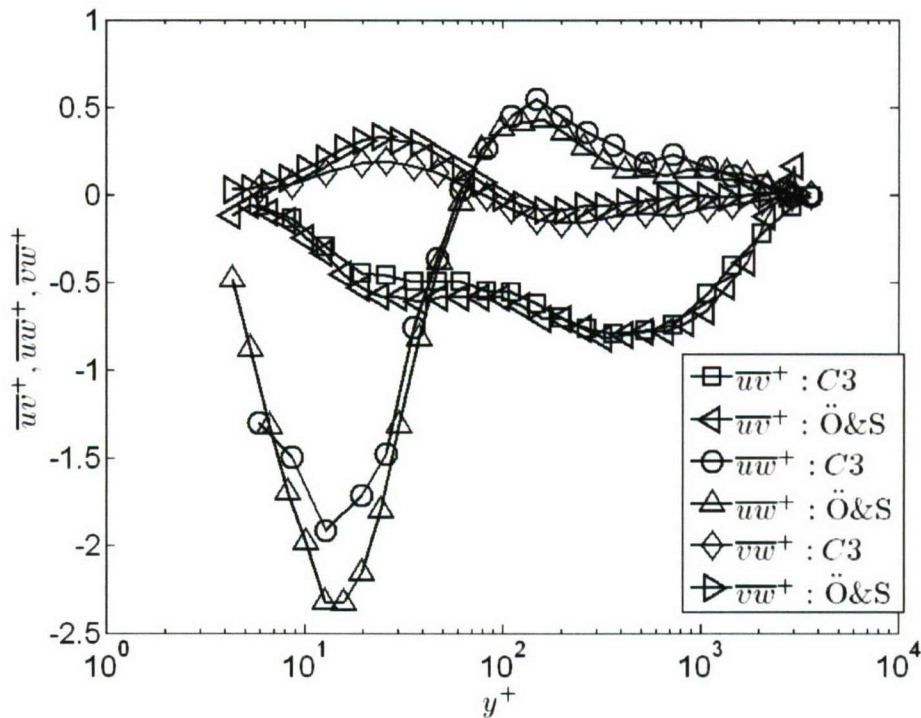


Figure 4.91. Reynolds normal stresses for the wing/body junction flow at station 5. C3: ComplDV3; Ö&S: Ölçmen and Simpson (1995).

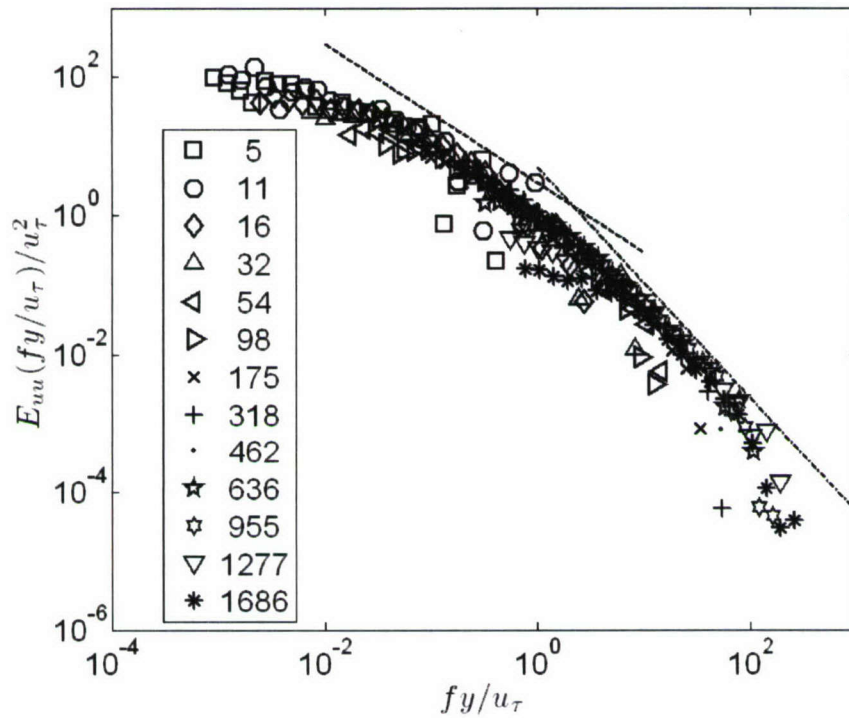


**Figure 4.92. Reynolds shear stresses for the wing/body junction flow at station 5. C3: CompLDV3; Ö&S: Ölçmen and Simpson (1995).**

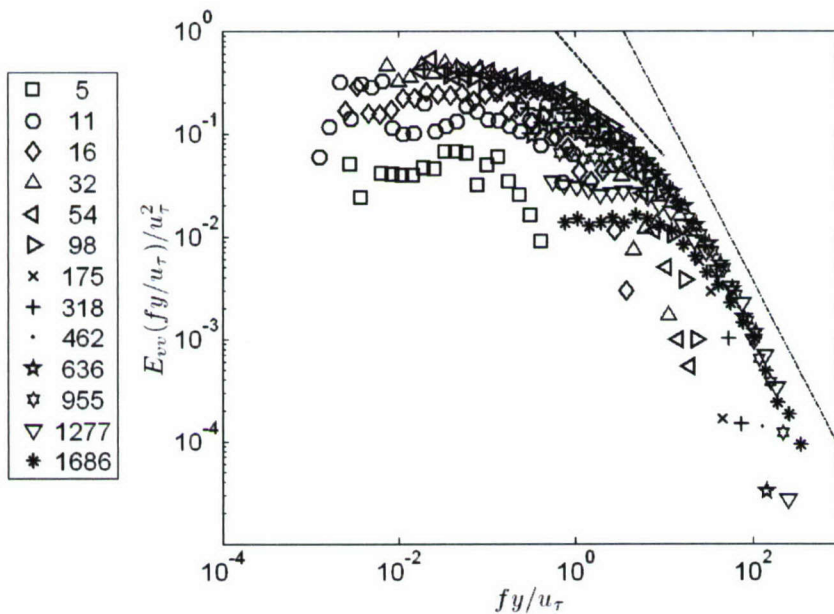
#### 4.3.2.2 Spectral Analysis

The spectral analysis techniques of the CompLDV3 were also applied to the wing/body junction flow. The scaling of Perry et al. (1985) was again used to plot the data and proves to be a good scaling for both the profiles considered. The frequency spectra of the normal-stresses for station 3 are given in Figure 4.93-Figure 4.95 while the magnitude of the cross-spectra of the Reynolds shear stress  $\overline{uv}$  is plotted in Figure 4.96. Analogous plots for station 5 are given in Figure 4.98-Figure 4.101. The spectral distributions are extremely similar to those measured for the 2DFPTBL, and all show the expected ranges of power-laws including the Reynolds shear stress cross-spectra that show evidence of the  $(-7/3)$  power-law.

The coherency of  $\overline{uv}$  is plotted in Figure 4.97 for station 3 and in Figure 4.102 for station 5. The data for station 3 indicates a loss of coherency at non-dimensional frequencies even lower than in the 2DFPTBL and little scatter among the different coherency estimates in the higher frequencies. In contrast, station 5 exhibits a broader spread in these data at higher frequencies, which most-likely indicates the role of multiple scales of motion that determine the Reynolds stresses throughout the profile. In such a 3D flow where flow angles vary so dramatically throughout the profile, the existence of multiple scales is not unexpected due the different histories of eddies that pass that location as the instantaneous flow angle changes.



**Figure 4.93.** Auto-spectra of the stream-wise velocity fluctuations at station 3 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .



**Figure 4.94.** Auto-spectra of the normal-to-wall velocity fluctuations at station 3 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .



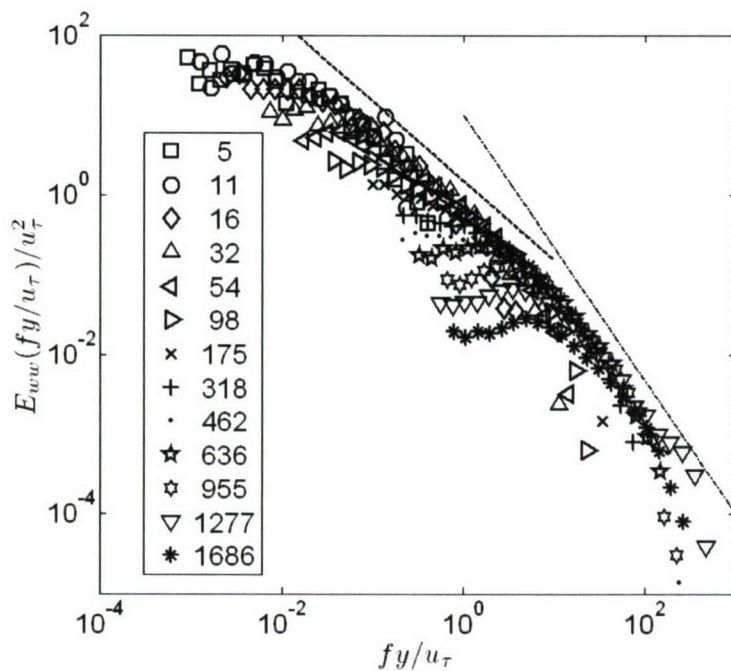


Figure 4.95. Auto-spectra of the span-wise velocity fluctuations at station 3 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The lines in the plot are at slopes  $\sim f^1$  and  $\sim f^{5/3}$ .

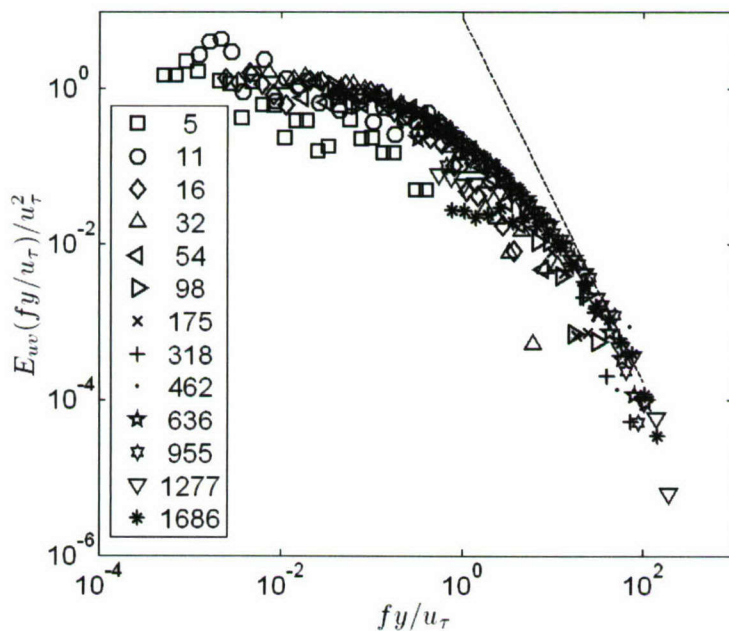


Figure 4.96. Cross-spectra of the Reynolds shear stress components,  $u'$  and  $v'$ , at station 3 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The line in the plot is at a slope of  $\sim f^{7/3}$ .

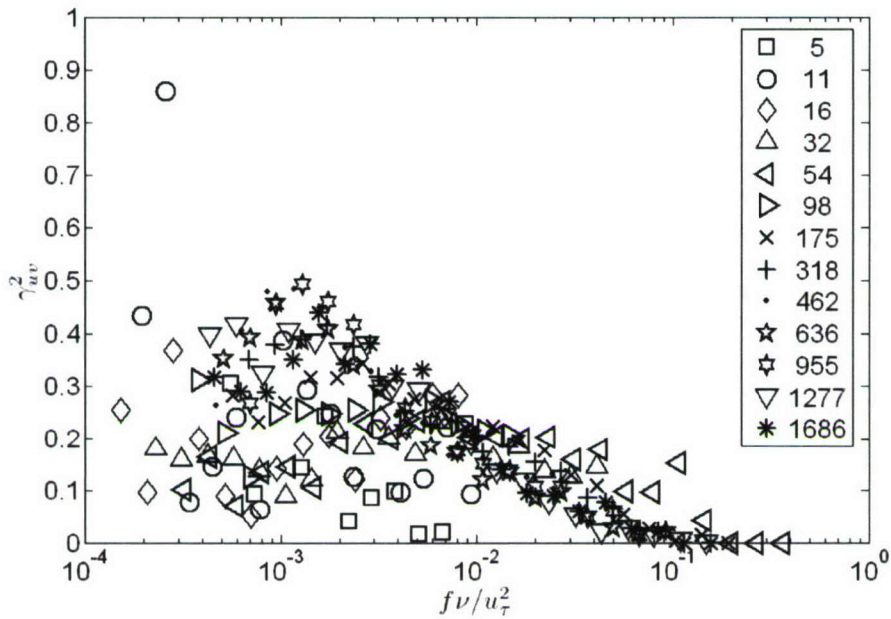


Figure 4.97. Coherency measurements for the Reynolds shear stress,  $\overline{uv}$ , at station 3. The legend gives vertical locations in  $y^+$ .

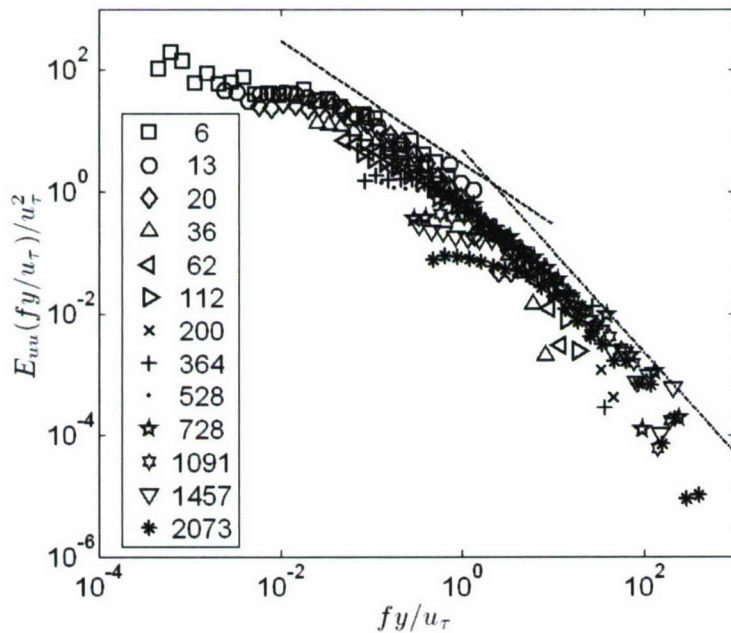


Figure 4.98. Auto-spectra of the stream-wise velocity fluctuations at station 5 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .

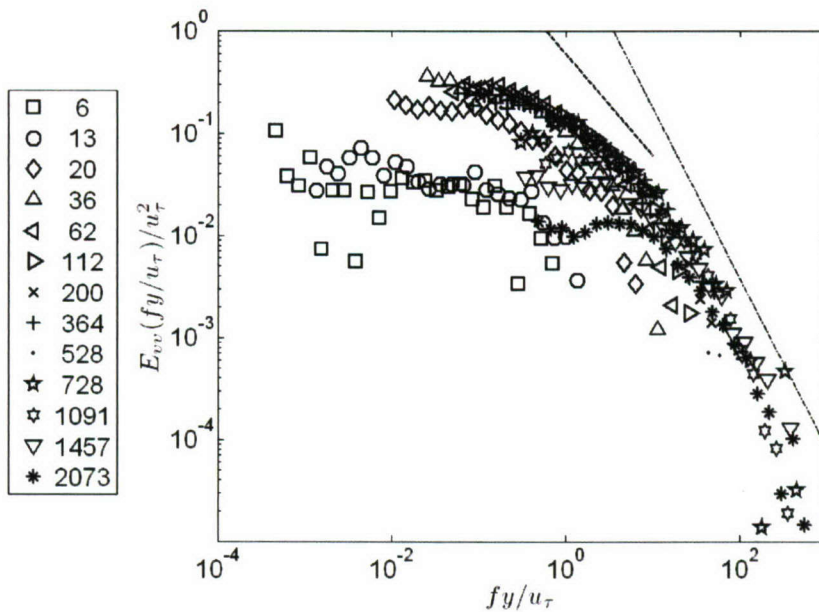


Figure 4.99. Auto-spectra of the normal-to-wall velocity fluctuations at station 5 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .

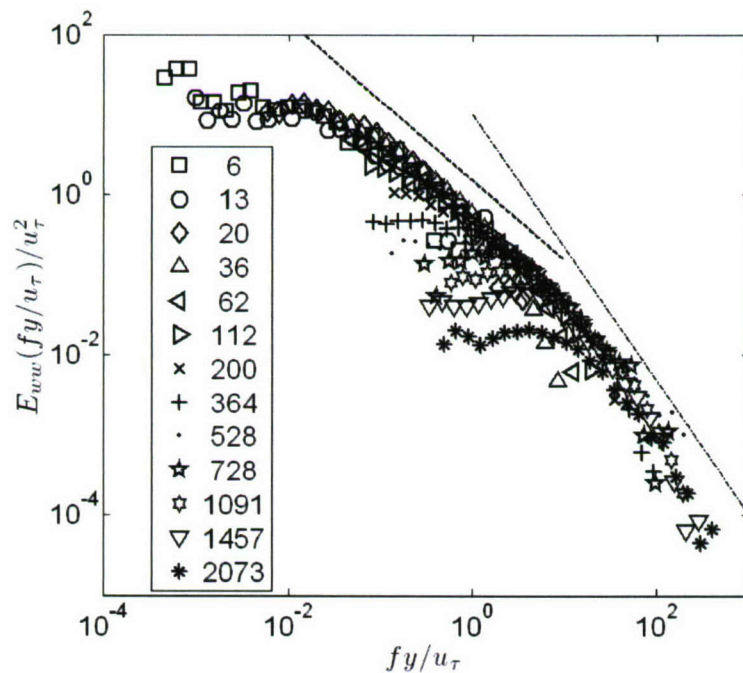


Figure 4.100. Auto-spectra of the span-wise velocity fluctuations at station 5 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The lines in the plot are at slopes  $\sim f^{-1}$  and  $\sim f^{-5/3}$ .



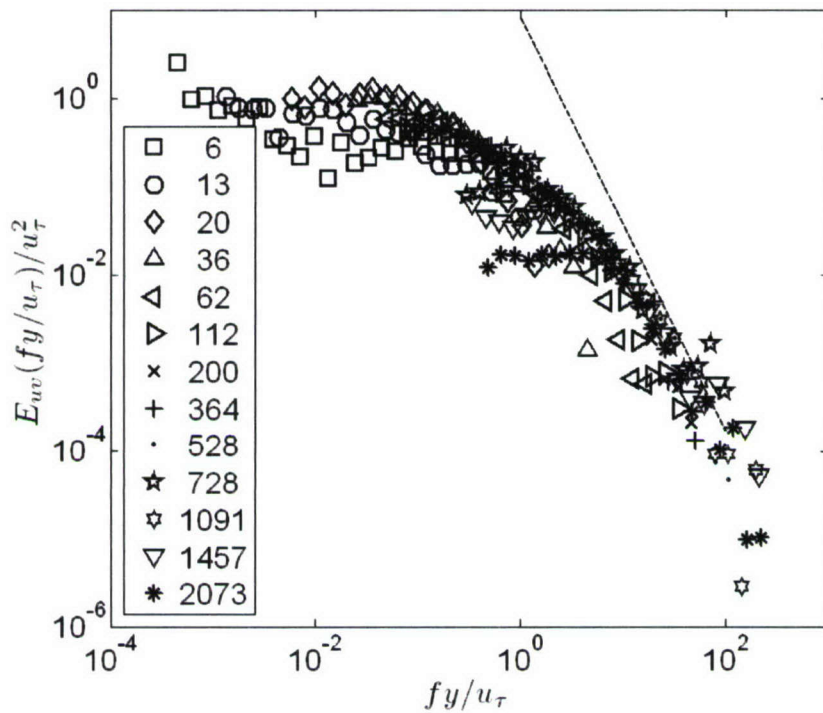


Figure 4.101. Cross-spectra of the Reynolds shear stress components,  $u'$  and  $v'$ , at station 5 in the WBJ flow with viscous wall scaling. The legend gives measurement positions in  $y^+$ . The line in the plot is at a slope of  $\sim f^{7/3}$ .

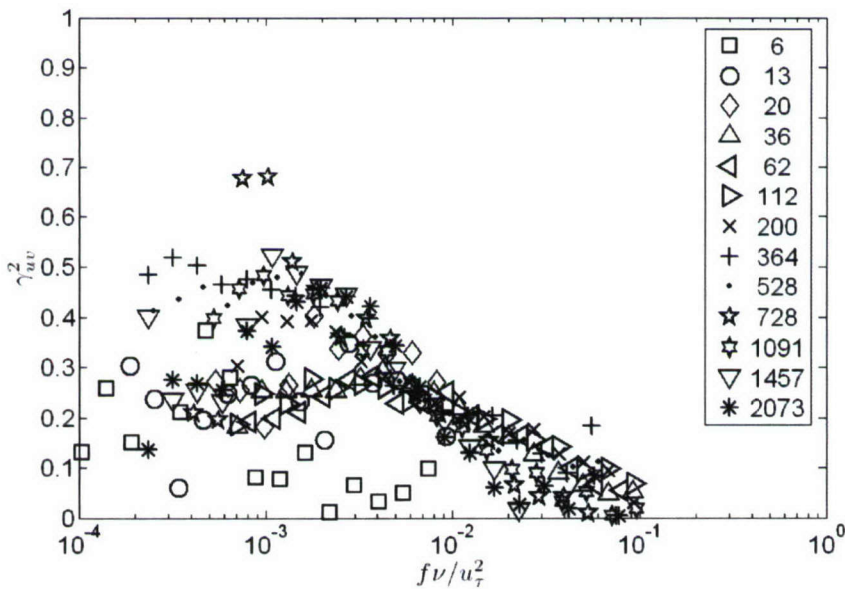


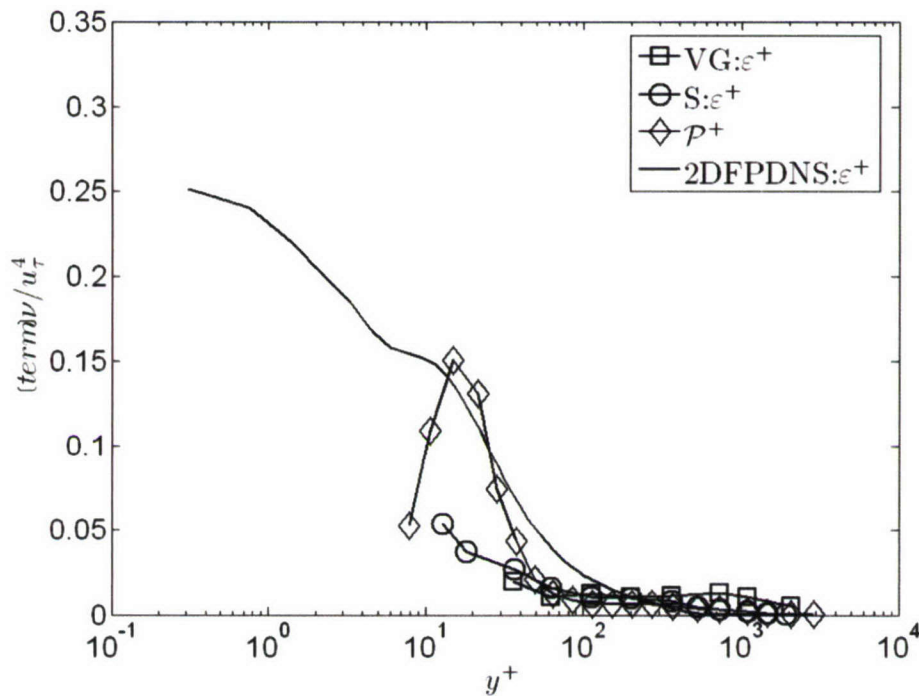
Figure 4.102. Coherency measurements for the Reynolds shear stress,  $\overline{uv}$ , at station 5. The legend gives vertical locations in  $y^+$ .

### 4.3.3 Turbulence transport

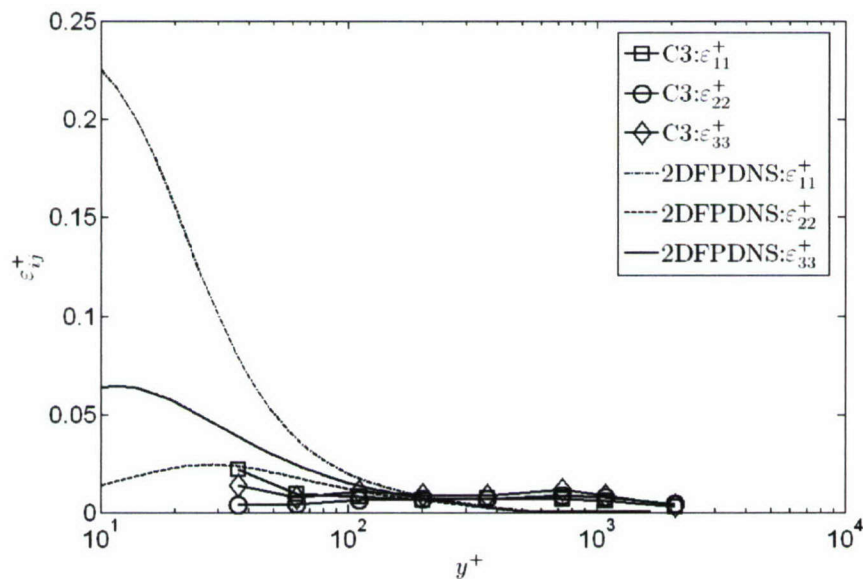
The velocity gradient dissipation estimation technique has been applied to the data at station 5 of the wing/body junction flow. The same parameters were used to obtain the velocity gradients as discussed in section 4.1.4. Tensor coordinate system rotations were applied to the gradient tensors measured so that the data could be examined in tunnel coordinates. The spectral techniques also discussed for the 2DFPTBL were applied to obtain TKE dissipation rate estimates. The estimates for the TKE dissipation rate are given in Figure 4.103. The spectral dissipation rate estimates closely follow the production values above  $y^+=60$ . The velocity-gradient estimation of the TKE dissipation rate seems to overshoot the value in the outer portion of the boundary layer, but this will benefit from the continuing research discussed concerning point-by-point random noise reduction. The non-isotropic dissipation rates obtained from the velocity gradient tensor estimates are plotted in Figure 4.104. These data are the first such ever obtained in a 3D turbulent boundary layer according to the author's knowledge. These values are seen to be low compared with the 2DFPTBL DNS of Spalart (1988) in the region of  $y^+ < 100$ , indicating the importance of non-equilibrium effects wherein several of the transport terms play significant roles.

The transport rate budgets were computed for station 5 and are presented in Figure 4.105. For these data, the boundary layer form of the Reynolds stress transport equations was again considered such that only vertical gradients of Reynolds-averaged terms were computed. To obtain the velocity/pressure gradient correlation, the balance of equation (4.23) was obtained using the non-isotropic dissipation rates from direct velocity gradient measurements. In the case of the Reynolds shear stress, the statistics of the cross-products of velocity gradients have not yet been computed and the dissipation rate for the shear stress was taken as zero.

Using the novel dissipation rate measurements, these transport budgets are also groundbreaking. Again, the importance of the velocity/pressure gradient term is clear. In the case of the stream-wise Reynolds stress, the velocity/pressure gradient correlation is the major sink of the energy up to at least  $y^+=100$ . For the span-wise normal stress, the sum of the production term and the velocity pressure gradient term result in a function very similar to the production of the stream-wise Reynolds normal stress. The trade-off between these terms sustains the value of the span-wise stress throughout the profile, although through distinctly different mechanisms.

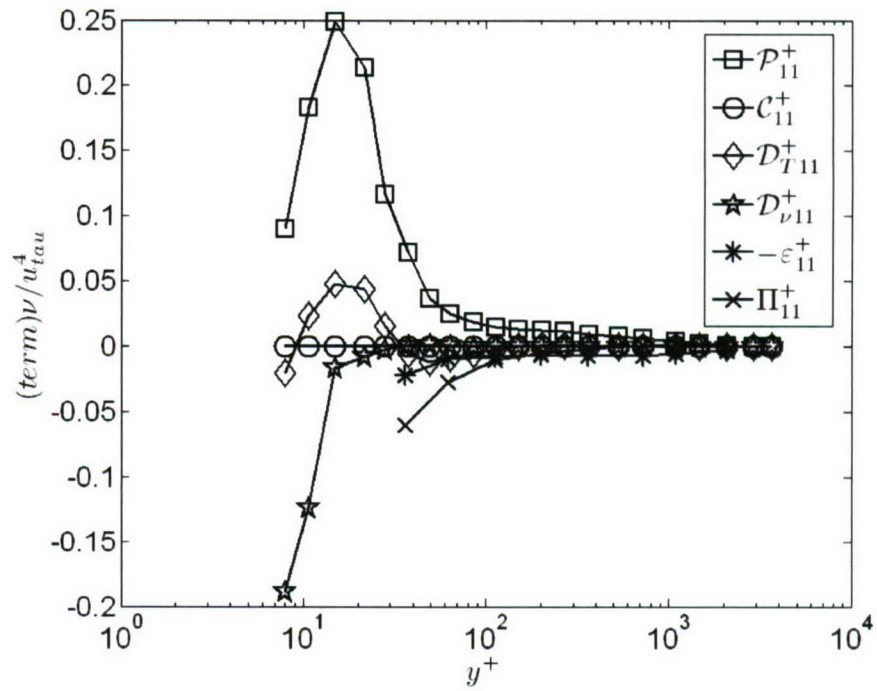


**Figure 4.103.** TKE dissipation rate and production for station 5 in the wing/body junction flow. VG, direct estimation of the dissipation rate using the velocity gradient tensor; S, estimation of the dissipation rate using spectral fits; 2DFPDNS, 2D flat plate data of Spalart.

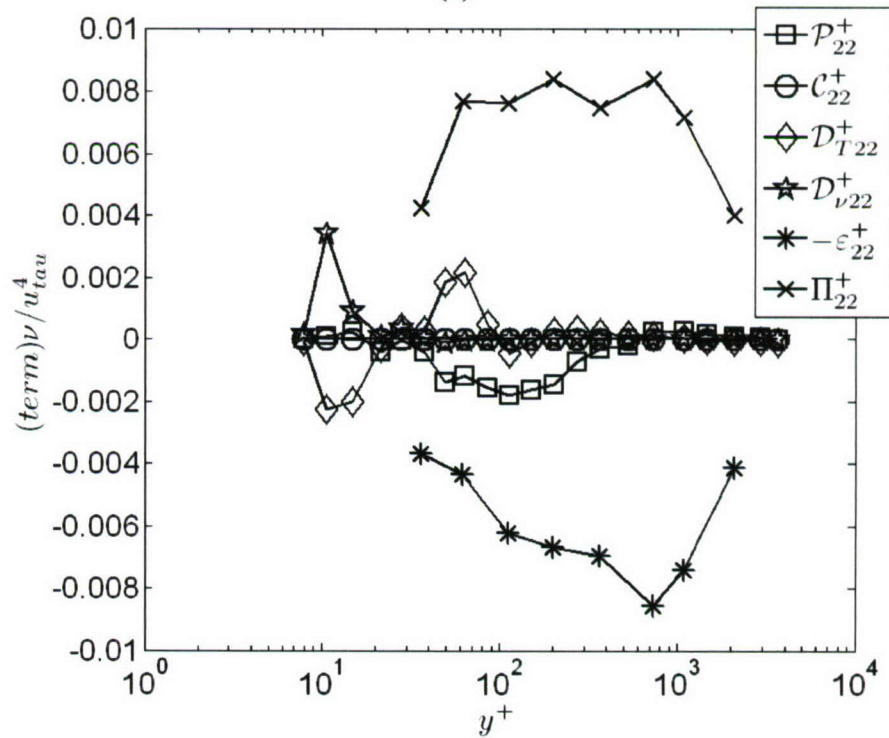


**Figure 4.104.** Non-isotropic dissipation rate measurements at station 5 in the wing/body junction flow. C3, ComplDV3 measurement; 2DFPDNS, 2D flat plate data of Spalart.

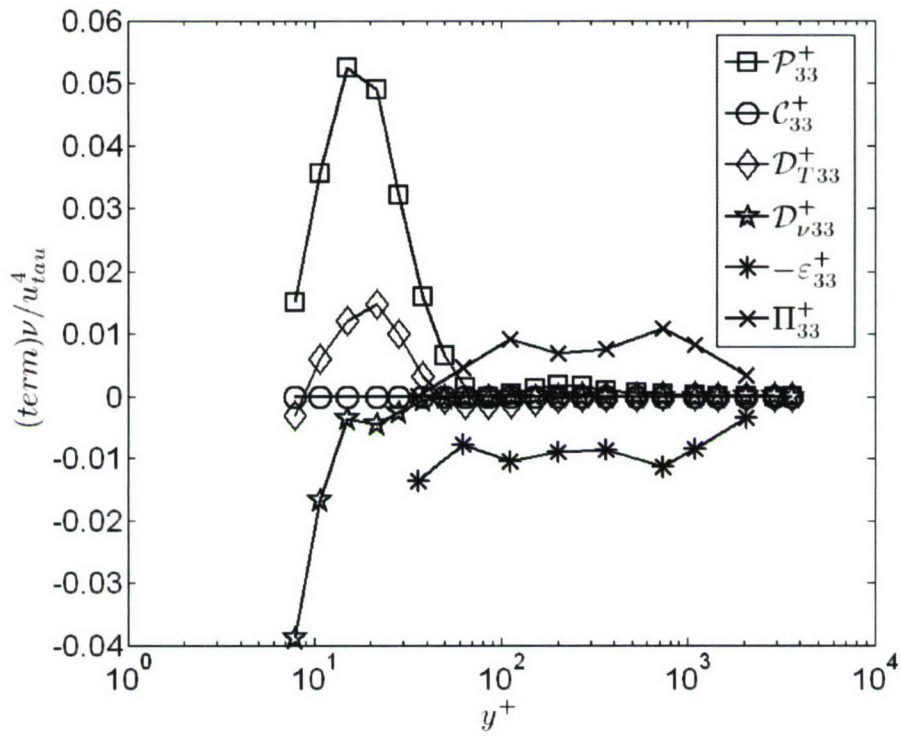




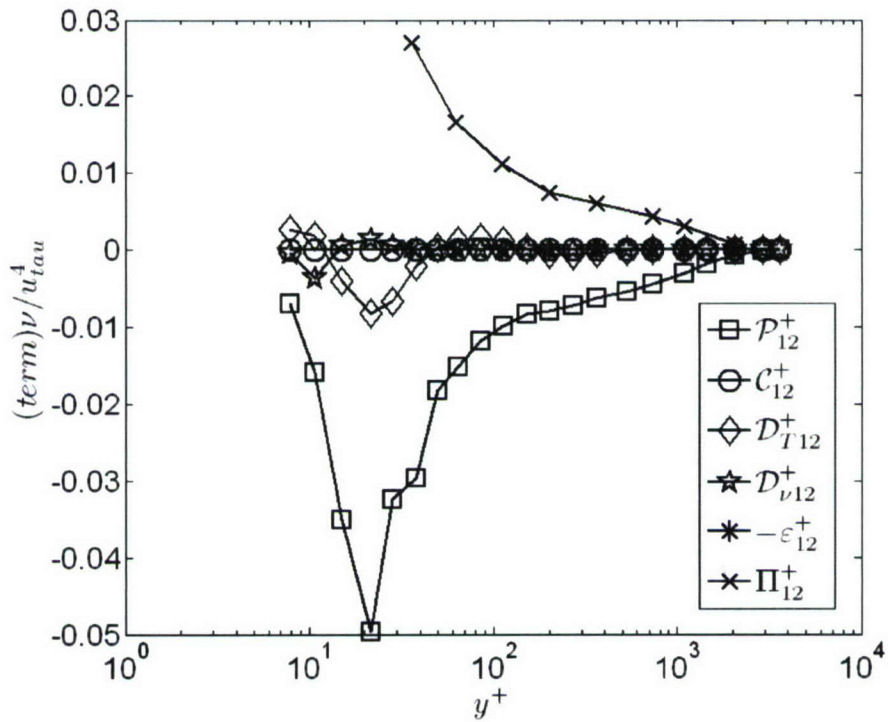
(a)



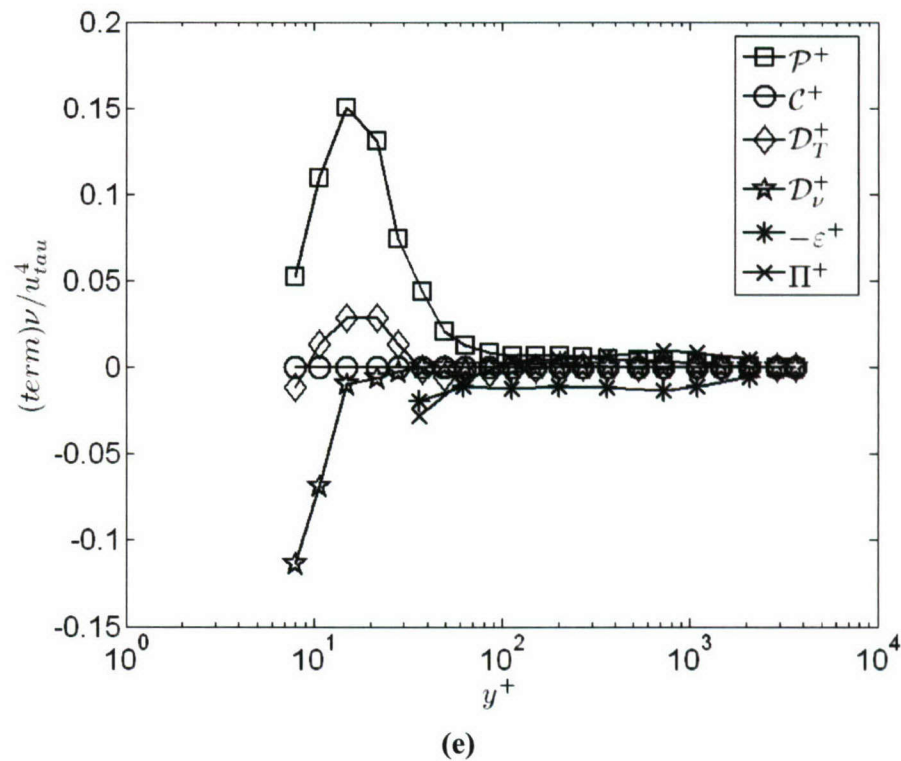
(b)



(c)



(d)



**Figure 4.105. Reynolds stress transport balances using the non-isotropic dissipation rates for the wing/body junction flow at station 5. (a)  $\overline{u^2}^+$ , (b)  $\overline{v^2}^+$ , (c)  $\overline{w^2}^+$ , (d)  $\overline{uv}^+$ , (e)  $\frac{1}{2}(\overline{u^2}^+ + \overline{v^2}^+ + \overline{w^2}^+)$ .  $\mathcal{P}_{ij}$ =Production;  $\mathcal{C}_{ij}$ =Convection;  $\mathcal{D}_{vij}$ =Viscous diffusion;  $\mathcal{D}_{Tij}$ =Turbulent diffusion;  $\varepsilon_{ij}$ =Dissipation rate;  $\Pi_{ij}$ =Velocity/pressure gradient correlation.**



## 5 Concluding remarks

In this study, a unique non-intrusive flow instrument has been used to obtain measurements in high Reynolds number turbulent boundary layers of fundamental interest. Several important leaps in measurements capabilities are enabled by the unique CompLDV technique, including direct-measurement of the complete velocity gradient tensor, sub-measurement-volume velocity statistics profiles, novel velocity-acceleration correlation measurements, and large-dynamic-range velocity spectra measurements very near the wall in turbulent boundary layer flows. These measurements are highly-resolved and allow the application of the technique to nearly any flow scale—there are no fundamental limitations on the technique for a significant range of Reynolds numbers.

New techniques have been developed and validated for extracting Doppler burst signal information, including an algorithm that approaches the theoretically lowest-possible uncertainties in both frequency and rate-of-change-of-frequency estimates for Gaussian-windowed signals in the presence of noise. Complete software suites have been prepared for every aspect of the operation of the CompLDV—from data acquisition, to advanced signal processing, to non-equidistantly-sampled data correlation analysis. Efficient software solutions have been devised for implementing the signal processing and data correlations for the extensive data sets that utilize the SGI-3800—a 512 processor supercomputer that was donated to the Department of Aerospace and Ocean Engineering in part to satisfy the needs of the CompLDV program.

The CompLDV equipment has been used to give novel results in real flows of fundamental interest to the turbulence research community. Several important ‘firsts’ stem from this study of two- and three-dimensional turbulent boundary layers, listed in Table 5.1. The first-ever direct measurement of the velocity-acceleration fluctuation correlations in any flow have been measured in this work. These data give a single-point statistical measure of turbulence structural terms in the Reynolds stress transport (RST) equations, with no assumptions. While multi-point measurement schemes are common and give excellent results for some terms in the RST equations such as the production, turbulent and viscous diffusion, and convection, the author knows of no-other direct, single-point measurement technique for obtaining such measurements. A novel scheme has been devised for estimating the velocity gradient tensor using the first-order particle trajectories with the CompLDV. This technique has given the first measurements of non-isotropic dissipation rate in turbulent boundary layers at high Reynolds numbers. Also, the first-ever non-isotropic dissipation rate measurements in a three-dimensional turbulent boundary layer have been reported herein. These non-isotropic dissipation rates have been used to obtain RST balances in two-dimensional and three-dimensional turbulent boundary layers. The quality of the measurements results in excellent estimates of the velocity/pressure gradient correlations throughout much of the boundary layer. The information for this difficult-to-model (and measure) term will be used in future studies to evaluate, improve and develop models for its behavior. The most spatially-resolved near-wall measurements of velocity-statistics ever acquired in high Reynolds number turbulent boundary layers have been reported in this study using the position-resolution



capabilities of the CompLDV. These measurements show operation of the probe to within 50 microns of the test window. The implications of this to the application of very-high Reynolds number flows are clear—the best estimates of wall-velocity gradients and Reynolds stresses will be possible and fundamental scaling of the high-Reynolds number turbulent boundary layer may truly be assessed. In the current report, extensive spectral measurements through turbulent boundary layers are presented, including near-wall data for these measurements. The non-intrusiveness of the technique makes the instrument extremely capable for these measurements—with no wall-interference phenomena as may be obtained with hot-wire techniques.

Several future applications exist for the CompLDV. First, only a fraction of the data that has already been acquired using the technique is presented in the present report. The richness of the information contained within the measurements is astounding, and each profile acquired yields extensive depths of new data. Additional data have already been acquired in the two-dimensional and three-dimensional turbulent boundary layers presented herein subjected to elevated-freestream turbulence levels. The CompLDV2 and CompLDV3 techniques were synchronized to obtain two-point measurements in those flows, and future work will utilize these data for answering fundamental questions about the role of freestream turbulence in boundary layer flows. Colleagues in the TBLRG are continuing the development of the CompLDV technique to apply it for understanding noise generation due to distributed roughness elements. The velocity gradient measurements available may allow for direct evaluation of the total derivative of the density fluctuations in the vicinity of the roughness elements. The correlations between the turbulence structure and the density-derivative will reveal important information about the mechanisms of roughness noise generation. The possibilities of applications of this novel, one-of-a-kind technique are endless. As the instrument continues to mature and is used by more and more researchers, it is firmly believed that absolutely new understandings of the fluid dynamics of high Reynolds number turbulent boundary layers will be revealed.

**Table 5.1 List of first-ever-reported measurements obtained in the CompLDV technologies**

- First-ever direct measurements of velocity-acceleration statistics in any turbulent flow
- First-ever non-isotropic dissipation rate measurements in high Reynolds number turbulent boundary layers
- First-ever non-isotropic dissipation rate measurements in a 3D turbulent boundary layer
- Highest-spatial-resolution measurements of velocity statistics in the near wall region of 2D and 3D turbulent boundary layers

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