Control of Mobile Communication Systems with Time-Varying Channels via Stability Methods

Robert Buche*
Mathematics Dept.
North Carolina State University
Rayleigh, NC 27695
rtbuche@unity.ncsu.edu

Harold J. Kushner†
Applied Mathematics Department
Lefschetz Center for Dynamical Systems
Brown University, Providence, RI 02912
hjk@dam.brown.edu

December, 2002; Revised August 2003 and March 2004

Abstract

Consider the forward link of a mobile communications system with a single transmitter and connecting to K destinations via randomly varying channels. Data arrives in some random way and is queued according to the K destinations until transmitted. Time is divided into small scheduling intervals. Current systems can estimate the channel (e.g, via pilot signals) and use this information for scheduling. The issues are the allocation of transmitter power and/or time and bandwidth to the various queues in a queue and channel-state dependent way to assure stability and good operation. The decisions are made at the beginning of the scheduling intervals. Stochastic stability methods are used both to assure that the system is stable and to get appropriate allocations, under very weak conditions. The choice of Liapunov function allows a choice of the effective performance criteria. The resulting controls are readily implementable and allow a range of tradeoffs between current rates and queue lengths. The various extensions allow a large variety of schemes of current interest

*This work was started while the author was at the Applied Math. Dept. of Brown University, and partially supported by National Science Foundation Grant ECS 9979250. The author would like to thank the Dept. for the incredibly supportive research environment that it provided.

†This work was partially supported by Contract DAAD-19-02-1-0425 from the Army Research Office and National Science Foundation Grant ECS 0097447.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **MAR 2004**	2. REPORT TYPE	3. DATES COVERED **00-03-2004 to 00-03-2004**

4. TITLE AND SUBTITLE
Control of Mobile Communication Systems with Time-Varying Channels via Stability Methods

5a. CONTRACT NUMBER
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER
5e. TASK NUMBER
5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Brown University,Division of Applied Mathematics,182 George Street,Providence,RI,02912

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**		**19**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

to be covered. All essential factors are incorporated into a "mean rate" function, so that the results cover many different systems. Because of the non-Markovian nature of the problem, we use the perturbed Stochastic Liapunov function method, which is well adapted to such problems. The method is simple and effective.

1 Introduction

We consider the problem of power and time control for the forward link of a mobile communications system when the connecting channels are randomly time varying. The control can be over allocated time, power, and bandwidth, or over all simultaneously. More discussion of the possibilities appears in Section 4. There are K queues at the base station, each receiving *data* according to some random process, and each associated with a unique mobile destination. The (nonnegative) content of any queue is simply its initial content plus its arrivals minus its departures to date. Time is divided into small scheduling intervals of length Δ (typically, in the low tens of milliseconds) and the transmitter decisions concerning the possible power, time, and bandwidth allocations are made at the beginning of the intervals. With the appropriate use of pilot signals, many systems have the ability to estimate key properties of the channel (e.g., S/N ratios) and to use this information to determine the controls. Such an approach would greatly improve the performance [1]. More generally, there might be only partial knowledge of the channel. The actual physical situation that corresponds to a channel state value or to an estimate of the value is unimportant. Each value of the state or of its estimate corresponds to some set of allowed possibilities for the coding and resource allocations. For simplicity, it is assumed that the controller quantizes the state of the K channels into a finite number of values, indexed by (a K-vector) j: each value of j denotes the state of all of the individual channels.[1] The individual components of the channel state might or might not be mutually independent. The rates of transmission are measured in terms of packets. The queue state and data arrival processes need not be Markovian. The actual transmission schemes that are allowed are quite general. They can be based on TDMA, CDMA, bit interval control, or on various combinations.

To best accommodate issues of fairness as well as the particular performance criteria of interest, the power and time allocation in each scheduling interval should depend on the lengths of the queues as well as on the current knowledge of the (vector-valued) state of the channel. The resulting optimal control problem is quite difficult. The dimension (K) might be high, the queue-length processes might not be Markovian under any control scheme, and uncertainty of the channel state can lead to a complicated filtering problem. These complications suggest an approximation approach. The small duration of the scheduling interval Δ and the (usually) fast rate of change of the channel-state process suggest an approach to the control problem that is based on a "crude mean

[1] The finiteness of the number of channel states is assumed only for notational simplicity.

flow" idea. Loosely speaking, we require only that there is some allocation under which an appropriately defined "mean" system is stable (stability is defined in the next section), and then the stability approach can be carried out under quite general conditions for a great variety of physical systems.

The conditions of typical applications (e.g., Rayleigh fading) do not usually allow a Markov representation of the queue-length and channel-state processes, even under "feedback" control; hence the usual methods of stability analysis, which require a Markov model, are inappropriate. Because of the non-Markov and randomly-time-varying nature of the problem, the stability is proved using the perturbed Liapunov function methods of [8]. With this method, one starts with a basic Liapunov function that works for a simpler (e.g., a "crude mean flow") system. Then one obtains finds a perturbation to this basic Liapunov function which can be used as a Liapunov function for the actual non-Markov physical system. Analogously to the usual "stability method" procedure that is used to get controls, the controls are determined by maximizing the (conditional expectation, given the current data, of the) negative of the gradient of the basic (not the perturbed) Liapunov function. The development uses a basic Liapunov function that is a polynomial which is the sum of terms, each depending on only one component of the state of the queue. This is adequate for applications at this time. Liapunov functions of a more general form can also be used, the main requirement being that the gradient has positive components.[2]

Stability is of interest partly because it assures a robustness of behavior to "small" changes in the system. For this reason an analysis based on Markov models is inadequate. No physical model is truly Markov. If an approximating Markov model is used for analysis, then it is important to know that the stability holds for the original systems that are approximated. The perturbed Liapunov function method is a powerful tool for checking such robustness since it does not require Markovianness. Additionally, mild modifications of the proof allow the basic data (e.g., the probabilistic structure of the data arrival process) to be time varying. The Liapunov function can be chosen to reflect the relative importance of the various queues or even to accommodate constraints. For example, if it is desired to minimize the time that queue i is above a value A_i, one can let the dependence of the Liapunov function on the state of queue i be very large when it exceeds A_i. Analogously to the treatment of a related problem in [5], where a sequence of systems scaled by speed was considered, one can allow a received packet to be rejected by the receiver if it decides that it contains too many errors. Then the acceptance probability will also depend on the allocated power.

The assumptions and the precise definition of stability are in Section 2, where the assumptions are discussed and seen to be unrestrictive, and a simple example given. The main theorem is stated and proved in Section 3. The scope of the approach and simplicity of application can be seen from the examples in Sections 4 and 5, which can be read without knowledge of Section 3. Some

[2]For example, if we wish to more or less equalize the lengths of the various queues, one can add terms to the Liapunov function that depend on the differences of the values of the queue components.

canonical classes of physical models (and various combinations and extensions) are discussed in Section 4. These include the standard CDMA and TDMA and show that many types of constraints can be handled. The approach and results are the same when the channel is only partly known or retransmission of poorly received packets is called for. A similar analysis can be used when there are multiple antennas [15] and frequencies. Multiple antennas and frequencies are used in the space-time coded OFDM (orthogonal frequency division multiplexing) approach [10, 13], part of the purpose of which is to provide "space-time diversity," to partially neutralize the effects of channel variations. But, via the approach of this paper and given sufficient information on the channel, one might sometimes find it preferable to use only the most appropriate combinations of antennas and frequencies for the various sources at any time. Section 5 contains a comparison of the obtained controls with optimal controls for a closely related problem.

For the special case of problems of the type of Example 4.2 where the rate of transmission is proportional to power, [1, 11] obtain rules for power allocation whose form is similar to ours and which are based on stability considerations, although the method uses large deviations estimates and the setup is Markovian. The reference [14] was perhaps the first to consider the problem of dynamic power allocation when the channels are time-varying. But, since their channel-rate and data-arrival processes are all i.i.d. sequences, the range of applications is very small. The reports [2, 3] deal with related problems. They prove interesting results concerning the limit (as $t \to \infty$) of (queue length at t)/t, and show that (under appropriate conditions) this limit is zero. This is used to show that the integral of the "rates" of transmission per unit time converges. But such a result does not imply stability of the queue length process, since it can grow sublinearly. [Consider, for example, a process modeled by a random walk in one dimension, with reflection at the origin, where the queue is only null recurrent.] Their scheduling is done continuously, rather than discretely in time. They allocate a single resource (e.g., bandwidth) and the rate is proportional to the allocation. This focus implies a more restricted set of applications. For example, it does not seem to be able to handle the "discrete selection" bit interval controls, the multiple antenna problem, or the joint bandwidth-power allocation problem, or nonlinear dependencies on the controls in general (as in Example 4.1). Their setup and proofs are much more complicated than that here. Stability of the "rate of transmission" process in our case follows from Theorem 3.1.

The work that is closest to ours is [12], where the channel-state process is Markov, the data input sequence is i.i.d., and a "complete resource pooling" condition is required. The decision rule is the same as ours for a quadratic Liapunov function. The emphasis is on stability in the heavy traffic limit, and showing how the problem simplifies there. Stability in the heavy traffic limit does not imply stability for any one of the "prelimit" processes. See also [5] for a stability analysis as the heavy traffic regime is approached.

2 Assumptions and the Control Rule

Comment on controls. For purposes of concrete visualization, one could suppose that the controls are over either the power and/or time or bandwidth allocated to each queue. Other forms are noted in the examples in Section 4. Let $u_i(j,x)$ denote the control applied to queue i in a scheduling interval, when the (vector-valued) queue state is x and the estimate of the (vector-valued) channel state is j. Some component of the control might also be a probability; e.g., as in the example at the end of the section, where the control is the probability of selecting a queue in an interval. The constraints need to be taken into account in defining admissible controls. For example, the total time scheduled in an interval cannot be greater than Δ. In the simplest model, where $u_i(j,x)$ denotes power, the total allocated power cannot be greater than that available and then we have

$$\sum_i u_i(j,x) \leq \bar{\mu}, \tag{2.1}$$

where $\bar{\mu}$ is the (real-valued) total available power. More generally, one might only require that the power " average locally in time" to that available. Other possible constraints might be a maximum length for any queue, or the control might have to take values in some discrete set. Obviously, the queue contents x_i must be nonnegative.

Assumption (A2.1) gives the general definition of admissibility and generalizes (2.1). Unless otherwise mentioned, when several users are transmitting simultaneously, it is assumed for simplicity that the mutual interference is negligible. If this is not the case, then the same method can be used, but the rates for any particular channel will depend on the vector of power allocations.

Definitions. The n-th scheduling interval is the real time interval $(n\Delta, n\Delta+\Delta]$, and the decisions for it are made at time $n\Delta$. Let L_n denote the value of the random channel state process at time $n\Delta$. Let $a_{i,n}$ (resp., $d_{i,n}$) denote the number of packets arriving to (resp., transmitted from) queue i in scheduling interval n. The number sent from queue i in any scheduling interval depends only on the channel and full queue states at the beginning of the interval, and the control allocations to queue i in the interval. Let $\mathcal{F}_n$ denote the minimal σ-algebra that measures the data (channel states, initial queue states, arrivals, control values) up to and including time $n\Delta$, and let E_n denote the associated conditional expectation.

Stability: Definition. Owing to the non-Markovianness, an appropriate definition of stability is a "uniform mean recurrence time" property, as follows. Suppose that there are $0 < q_0 < \infty$ and a real-valued function $F(\cdot) \geq 0$ such that the following holds: For any n, and $\sigma_1 = \min\{k \geq n : |x(k\Delta)| \leq q_0\}$, we have[3]

$$E_n\left[\sigma_1 - n\right] \leq F(x(n\Delta))I_{\{|x(n\Delta)|\geq q_0\}}. \tag{2.2}$$

Then the system is said to be stable.[4] The definition implies recurrence to some compact set. If the (absolute value of the state of) the process reaches a level $q_1 > q_0$, then the conditional expectation of the time required to return to a value q_0 or smaller is bounded by a function of q_1, uniformly in the past history and in n. This implies that the sequence $\{x(n\Delta)\}$ is tight or bounded in probability (see, for example, [8, Theorem 2, Chapter 6]).[5]

The basic Liapunov function. For simplicity in the computations (and because they yield controls that are readily implemented) the basic Liapunov functions will be polynomials of the form[6]

$$V(x) = \sum_i c_i(x_i + h_i)^{p_i}, \tag{2.3}$$

where $h_i \geq 0$, $c_i > 0$, and $p_i \geq 2$. The h_i serve the purpose of reducing the dependence of the controls on the queue size if the content is small. Define $p = \max_i p_i$. Let $V_x(\cdot)$ (with components $V_{x_i}(\cdot)$) denote the gradient of $V(\cdot)$.

Assumptions. The assumptions are discussed after being stated. Controls satisfying (2.4) are called admissible.

(A2.1) There is a real (resp., vector[7]) valued function $f(\cdot)$ and sets U_i such that such that the controls satisfy

$$f(j, u(x)) \leq \bar{\mu}, \quad , u_i(j,x) \in U_i, \quad \text{all } j, x, \tag{2.4}$$

where $\bar{\mu}$ is a real (resp., vector) valued constant. The set of controls satisfying (2.4) is compact.

(A2.2) The maximum number that can be sent from queue i over a scheduling interval is bounded by the minimum of (a constant times Δ) and x_i. [I.e., the actual number of packets sent can be no greater than x_i.] There are functions $g_i(\cdot)$, with values $g_i(j, x, u_i(j,x))$, and upper-semicontinuous[8] in $u_i \in U_i$ for each value of the other variables, such that if $x(n\Delta) = x$ and $L_n = j$ then the

[3] The stability condition in [14] requires only that return times be finite w.p.1, but the mean return time might still be infinite. The form (2.2), based on mean return times, seems more appropriate in applications.

[4] Restricting attention to the sampling times $n\Delta$ is unrestrictive in view of (A2.6).

[5] A sequence $\{X_n\}$ is bounded in probability if $\lim_{\kappa\to\infty} \sup_n P\{|X_n| \geq \kappa\} = 0$.

[6] Other polynomial forms can be used in the development, provided that $V(x) \to \infty$ as $x \to \infty$, $V_{x_i}(x) \to 0$ as $x_i \to 0$, and $V_{x_i}(x) \to \infty$ as $x_i \to \infty$, $V_{x_i}(x)/V(x) \to 0$ as $x \to \infty$, and $E_n e_n / |V_x(x)| \to 0$ as $x \to \infty$, where e_n is defined in (3.5).

[7] We have the vector case if, for example, both power and time are being constrained or in the multiple antenna case.

[8] All that is needed is that there be a maximizer in (2.6) that satisfies (2.4).

conditional (given the data to $n\Delta$) mean number of packets sent from queue i on $(n\Delta, n\Delta + \Delta]$ can be represented as

$$E_n d_{i,n} = \Delta g_i(j, x, u_i(j, x)). \tag{2.5}$$

(A2.3) There are positive constants $\bar{\lambda}_i^a, i \leq K$, such that the sums

$$C_{i,n}^a = \sum_{l=n}^{\infty} E_n \left[a_{i,l} - \Delta\bar{\lambda}_i^a\right], \quad i \leq K,$$

are well defined and bounded, uniformly in n.

(A2.4) There are $\Pi_j \geq 0$ such that $\sum_j \Pi_j = 1$ and $E_n \left[I_{\{L_l=j\}} - \Pi_j\right] \to 0$ fast enough as $l - n \to \infty$ so that $\sum_{l=n}^{\infty} \left|E_n I_{\{L_l=j\}} - \Pi_j\right|$ is bounded uniformly in n.

(A2.5) Define the average $\bar{g}_i(x, u(x)) = \sum_j \Pi_j g_i(j, x, u_i(j, x))$. There are $c_0 > 0$, $K_0 > 0$, and an admissible control $\tilde{u}(x) = \{\tilde{u}_i(j, x); i, j\}$, such that if $x_i \geq K_0$ then

$$\bar{\lambda}_i^a - \bar{g}_i(x, \tilde{u}(x)) \leq -c_0$$

and $g_i(j, x, \tilde{u}_i(j, x)))$ does not depend on x. [9]

(A2.6) There is a constant C such that for each i

$$E_n \left[a_{i,n}\right]^l \leq C\Delta, \quad l \leq p_i,$$

where p_i is defined by (2.3).

The control rule. Controls satisfying (A2.1) are called admissible. The actual control is any admissible control that maximizes in

$$\max_u \left\{ \sum_i V_{x_i}(x) g_i(j, x, u_i(j, x)) \right\}. \tag{2.6}$$

It is assumed that there is at least one maximizing admissible control $\bar{u}_i(j, x)$.

Discussion of the conditions. Condition (A2.1) is intended to cover constraints such as (2.1) or where the controls take values in some finite set. The $f(\cdot)$ and U_i are always defined by the problem at hand, and there is no need to

[9] The K_0 is introduced in (A2.5) only because if the queue content is smaller than the maximum of what can be transmitted on a scheduling interval, then the mean (weighing with the Π_j) output might be too small to assure the $-c_0$ value. For example, if a queue is empty, then the arrivals will exceed the departures. K_0 will be of the order of Δ.

specify them any further. The only additional conditions on the set of admissible controls is that there exist an admissible control satisfying (A2.5) and one maximizing in (2.6). For cases where the control is unique and readily found, see the examples in Sections 4 and 5.

If control is over power only and the total instantaneous power must be no greater than $\bar{\mu}$, and can take only some finite set of values, then (2.4) reduces to (2.1) and U_i is the set of allowed discrete values for queue i. Condition (A2.4) would hold if the channel state process were a function of a finite-state ergodic Markov process. It need not be Markovian itself. Loosely speaking, it would hold if the future becomes less and less predictable (given the distant past) and the conditional likelihood of any channel state approaches an average value. Condition (A2.5) is likely very close to being necessary. It concerns an averaged system, where the input is just a fluid flow at the mean rate and the output is also a fluid flow, whose value is the mean over all channel states (i.e., there is no randomness). If there was no control under which the condition held, then even this mean system would be unstable and the physical system might not be stable.

Condition (A2.6) will hold if the inputs occur continuously in time, but at a bounded maximum rate (the usual case). Then the maximum input to any queue over a scheduling interval is proportional to Δ. Alternatively, suppose that the input rate is unbounded and a batch of packets can arrive all at once. Suppose (without loss of generality) that they arrive at instants $k\Delta, k = 1, 2, \ldots,$ and have bounded pth moments. Let the values over the scheduling intervals be mutually independent, and let the probability of an input to queue i during any interval be $c_i\Delta$ (a discrete time Poisson process). Then (A2.6) holds.

Comments on (A2.3). Condition (A2.3) says that the expectation of the number of packets arriving on any distant future interval given the data to the remote past converges to the average number of arrivals per unit time as the difference between the times goes to infinity. The following simple examples illustrate the condition. First suppose that the $\{a_{i,n}; i, n\}$ are mutually independent with the inputs to each queue being identically distributed, with $\bar{\lambda}_i^a$ defined so that $Ea_{i,n} = \Delta\bar{\lambda}_i^a$. Then the summands in $C_{i,n}^a$ are all zero and $C_{i,n}^a = 0$. If the input sequence is only M-dependent, then all but at most the first M summands in $C_{i,n}^a$ are zero.

Let us now examine $C_{i,n}^a$ more closely to understand why our requirement on its convergence and boundedness is reasonable. Henceforth let us lump together all arrivals in the nth interval and suppose (w.l.o.g) that it occurs at the end. Consider queue i. First suppose that arrivals occur in batches at (real time) moments $\tau_{i,n}^a, i = 1, 2, \ldots$, which is assumed to be a renewal process with values in $\{n\Delta, n = 1, 2, \ldots\}$. Define the mean interval $m_i^a = E[\tau_{i,n+1}^a - \tau_{i,n}^a]$, and let the number of arrivals at the renewal time $\tau_{i,n}^a$ be denoted by $v_{i,n}^a$. For each queue i, let the $v_{i,n}^a, n < \infty$, be mutually independent and identically distributed with mean $\bar{v}_i^a$, and suppose that the sequences for the different queues are mutually independent. Define $\bar{\lambda}_i^a = \bar{v}_i^a / m_i^a$ and let $\Delta\mu_{i,n,l}^a, l = 1, 2, \ldots,$ denote the sequence of renewal times that occur after time $n\Delta$. By the independence and renewal property, for any integer $v \geq 1$, we have $E_n[\mu^a_{i,n,v+1} - \mu^a_{i,n,v}] = m^a_i/\Delta$ and, hence,

$$E_n \sum_{l=\mu^a_{i,n,v}+1}^{\mu^a_{i,n,v+1}} \left[a_{i,l} - \frac{\Delta \bar{v}^a_i}{m^a_i} \right] = \left[\bar{v}^a_i - \frac{m^a_i}{\Delta} \frac{\Delta \bar{v}^a_i}{m^a_i} \right] = 0.$$

Thus, $C^a_{i,n}$ reduces to

$$C^a_{i,n} = E_n \sum_{l=n}^{\mu^a_{i,n,1}} \left[a_{i,l} - \frac{\Delta \bar{v}^a_i}{m^a_i} \right],$$

which is just a scaled residual time term

$$\bar{v}^a_i E_n \left[1 - \frac{\Delta \mu^a_{i,n,1} - n\Delta}{m^a_i} \right],$$

where the ratio is the time to the next arrival divided by the mean interarrival time. Thus (A2.3) holds. If the amounts remain independent, but the interarrival times are correlated, then $C^a_{i,n}$ is just the residual time term plus

$$\bar{v}^a_i \sum_{l:\tau^a_{i,l} > n\Delta} E_n \left[1 - \frac{\tau^a_{i,l+1} - \tau^a_{i,l}}{m^a_i} \right]$$

and the condition holds under suitable mixing-type conditions on the interarrival times. Analogously, if the independence of the amounts is dropped, then the condition will hold under suitable mixing-type conditions on the interarrival times and amounts.

Example. The simplest example is where the $a_{i,n}, i \leq K$, sequences are i.i.d., L_n is an ergodic Markov chain and only one queue can be selected in any interval. Let $\Delta \bar{\lambda}^a_i$ denote the mean number of arrivals to queue i in an interval. Then $C^a_{i,n} = 0$ in (A2.3). In this example, let $u_i(j,x)$ denote the probability of selecting queue i under channel state j and queue state x. Let $\bar{\lambda}^d_i(j)$ denote the maximum rate of transmission per unit time if i is selected. Then we have (rate per unit time times Δ times the probability of selecting i) $\Delta g_i(j,x,u_i(j,x)) = \min\{\Delta \bar{\lambda}^d_i(j), x_i\} u_i(j,x)$. Suppose that there are $\alpha_i(j)$ with $\sum_j \alpha_i(j) = 1$ for all i, such that $\bar{\lambda}^a_i - \sum_j \Pi_j \bar{\lambda}^d_i(j) \alpha_i(j) < 0$. all i. Then (A2.5) holds. This is just a requirement on a mean flow and is analogous to the mean flow condition in [14].

3 Proof of Stability

The key is to show that if a certain "mean flow" system is stable, then the actual physical system is. Proofs of stability most often work with Markov models, where the Liapunov functions are supermartingales for large values of the state. In our case, the sequence of interest is $\{x(n\Delta)\}$, which will not be Markov unless the members of the sequences of channel states and arrival numbers are all mutually independent. In the present case, these latter sequences are not necessarily even Markov. The queue might grow "on the average" when in certain channel states and decrease "on the average" when in others. All that we care about is that in the long term these effects (and the effects of the bursty non Markov arrival processes) are averaged in some way due to the random variations of the channel and arrival processes so that the behavior of the physical system is approximated by the "mean flow " system. The idea of a stochastic Liapunov function that has the supermartingale property for large queue states is still crucial, but it is obtained a little indirectly. One starts with a Liapunov function, which is a function only of the queue state, and that can be used for the mean flow system. This cannot be applied to the true physical system to get the desired result. But it turns out that that this Liapunov function can be modified by the addition of certain bounded terms that allow averaging of the random effects of the input and channel processes, and the modification will serve our needs. The basic Liapunov function is (2.3). The perturbations will be defined first, and then the modified or perturbed Liapunov function will be defined. As usual in stability proofs using Liapunov functions, the work consists essentially of a sequence of bounds on conditional expectations of differences between of various functions.

The basic idea behind the perturbed Liapunov function method can be loosely summarized as follows. Suppose that $E_n V(x(n\Delta + \delta)) - V(x(n\Delta)) = c_n$, a random quantity. Suppose that there is a constant $\bar{c} < 0$ such that $\delta V_n = \sum_{i=n} E_n[c_i - \bar{c}]$ is well defined. Define $V_n = V(x(n\Delta)) + \delta V_n$. Then $E_n \delta V_{n+1} - \delta V_n = -(c_n - \bar{c})$ and $E_n V_{n+1} - V_n = c_n - [c_n - \bar{c}] = \bar{c} < 0$. Thus, the use of the perturbation allows the replacement of the random c_n by a "mean." This idea will be used in the proof.

Theorem 3.1. *Under* (A2.1)–(A2.6) *and rule* (2.6)*, the system is stable.*

Proof. For each i, j, n, define

$$C^d_{i,j,n} = -\sum_{l=n}^{\infty} E_n \left[I_{\{L_l = j\}} - \Pi_j \right],$$

$$\delta V^a_{i,n} = V_{x_i}(x(n\Delta)) C^a_{i,n},$$

$$\delta V^d_{i,j,n} = \Delta V_{x_i}(x(n\Delta)) g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta)) C^d_{i,j,n},$$

where $\tilde{u}(\cdot)$ is defined in (A2.5), and $C^d_{i,j,n}$ is well defined by (A2.4). We will use the "perturbed" (and time dependent) Liapunov function V_n whose value at time $n\Delta$ is

$$V_n = V(x(n\Delta)) + \sum_i \delta V^a_{i,n} + \sum_{i,j} \delta V^d_{i,j,n}, \tag{3.1}$$

where $V(\cdot)$ is defined by (2.3). Let $\bar{u}(\cdot)$ be an admissible control that maximizes in (2.6). Since $x_i(n\Delta + \Delta) - x_i(n\Delta) = a_{i,n} - d_{i,n}$ and $E_n d_{i,n} = \Delta \sum_j g_i(j, x(n\Delta), u_i(j, x(n\Delta))) I_{\{L_n = j\}}$, a first order expansion under $\bar{u}(\cdot)$ yields

$$
\begin{aligned}
&E_n V(x(n\Delta + \Delta)) - V(x(n\Delta)) = \sum_i V_{x_i}(x(n\Delta)) \left[E_n \left(a_{i,n} - d_{i,n}\right)\right] + e_n^1 \\
&= \sum_i V_{x_i}(x(n\Delta)) \left[E_n a_{i,n} - \Delta \sum_j g_i(j, x(n\Delta), \bar{u}_i(j, x(n\Delta))) I_{\{L_n = j\}}\right] + e_n^1,
\end{aligned} \tag{3.2}
$$

where the "error" term e_n^1 is

$$
e_n^1 = E_n V(x(n\Delta + \Delta)) - V(x(n\Delta)) - \sum_i V_{x_i}(x(n\Delta)) \left[E_n a_{i,n} - E_n d_{i,n}\right].
$$

By the fact that $\bar{u}(\cdot)$ maximizes in (2.6), for each j,

$$
\begin{aligned}
&- \sum_i V_{x_i}(x(n\Delta)) g_i(j, x(n\Delta), \bar{u}_i(j, x(n\Delta))) \\
&\qquad \leq - \sum_i V_{x_i}(x(n\Delta)) g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta)).
\end{aligned} \tag{3.3}
$$

Analogously, we can write

$$
E_n \delta V_{i,n+1}^a - \delta V_{i,n}^a = -V_{x_i}(x(n\Delta)) E_n \left[a_{i,n} - \Delta \bar{\lambda}_i^a\right] + e_{i,n}^a, \tag{3.4}
$$

where

$$
e_{i,n}^a = E_n \left[V_{x_i}(x(n\Delta + \Delta)) - V_{x_i}(x(n\Delta))\right] C_{i,n+1}^a.
$$

Note that the main term on the right of (3.4), added to the component $V_{x_i}(x(n\Delta)) E_n a_{i,n}$ in (3.2), yields the "average" $V_{x_i}(x(n\Delta)) \Delta \bar{\lambda}_i^a$. Thus by adding the perturbation $\delta V_{i,n}^a$ to $V(x)$, we effectively replace the $E_n a_{i,n}$ in (3.2) by the average $\Delta \bar{\lambda}_i^a$. This is the motivation behind the form chosen for the perturbation. A similar result will be seen to hold for the service variables.

If $x_i(n\Delta) \geq K_0 + K_1$, where K_1 is the maximum that can be transmitted on an interval and K_0 is defined in (A2.5), then, $x_i(n\Delta + \Delta) \geq K_0$ and, by (A2.5), $g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta))) = g_i(j, x(n\Delta + \Delta), \tilde{u}_i(j, x(n\Delta + \Delta)))$. Using this, we have

$$
\begin{aligned}
&E_n \delta V_{i,j,n+1}^d - \delta V_{i,j,n}^d \\
&\qquad = \Delta V_{x_i}(x(n\Delta)) g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta))) \left[I_{\{L_n = j\}} - \Pi_j\right] + e_{i,j,n}^d,
\end{aligned} \tag{3.5}
$$

where

$$
e_{i,j,n}^d = \Delta E_n \left[V_{x_i}(x(n\Delta + \Delta)) - V_{x_i}(x(\Delta))\right] g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta))) C_{i,j,n+1}^d.
$$

Note that the jth summand in the brackets in the right hand term of (3.2), with $\bar{u}(\cdot)$ replaced by $\tilde{u}(\cdot)$ and added to the first term on the right of (3.5), is $-\Delta V_{x_i}(x(n\Delta)) g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta))) \Pi_j$.

Define $\delta_{i,l} = a_{i,l} - d_{i,l}$. By the definition of $V(\cdot)$ and the binomial expansion,

$$e_n^1 = \sum_i c_i \sum_{l=2}^{p_i} \binom{p_i}{l} (x_i(n\Delta) + h_i)^{p_i - l} E_n \delta_{i,n}^l,$$

where the terms with the large parentheses are the binomial coefficients. The terms $e_{i,n}^a$ and $e_{i,j,n}^d$ are bounded by a constant times the absolute value of

$$\sum_i E_n \left[(x_i(n\Delta) + h_i + \delta_{i,n})^{p_i - 1} - (x_i(n\Delta) + h_i)^{p_i - 1} \right].$$

Substitute the inequality (3.3) into (3.2). Then combining the above computations and canceling terms where possible yields

$$\begin{aligned} &E_n V_{n+1} - V_n \leq O(\Delta) + e_n + \\ &\sum_{i: x_i(n\Delta) \geq K_0 + K_1} \Delta V_{x_i}(x(n\Delta)) \left[\bar{\lambda}_i^a - \sum_j g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta))) \Pi_j \right], \end{aligned} \tag{3.6}$$

where the $O(\Delta)$ term is due to the components i for which $x_i(n\Delta) < K_0 + K_1$, and is bounded in x, and

$$e_n = e_n^1 + \sum_i e_{i,n}^a + \sum_{i,j} e_{i,j,n}^d = O(1)\Delta \sum_i (x_i(n\Delta) + h_i)^{p_i - 2}. \tag{3.7}$$

Note that, by (A2.5), $[\bar{\lambda}_i^a - \sum_j g_i(j, x(n\Delta), \tilde{u}_i(j, x(n\Delta)))\Pi_j] \leq -c_0$ for all i such that $x_i(n\Delta) \geq K_0$. Thus under any control $\bar{u}(\cdot)$ determined by the maximization rule (2.6), we must have

$$E_n V_{n+1} - V_n \leq - \sum_{i: x_i(n\Delta) \geq K_0 + K_1} \Delta V_{x_i}(x(n\Delta)) c_0 + O(\Delta) + e_n. \tag{3.8}$$

The dependence of e_n on $x_i(n\Delta)$ is dominated by a constant times $\Delta x_i^{p-2}(n\Delta)$. The main term on the right of (3.8) is of the order of $\sum_{i: x_i(n\Delta) \geq K_0 + K_1} x_i^{p-1}(n\Delta)$. Hence, there is a constant K_2 such that if $|x(n\Delta)| \geq K_2$, then we can write

$$\begin{aligned} E_n V_{n+1} - V_n &\leq -\Delta \sum_{i: x_i(n\Delta) \geq K_0 + K_1} V_{x_i}(x(n\Delta)) c_0 / 2 \\ &= -\Delta \sum_{i: x_i(n\Delta) \geq K_0 + K_1} p_i \left(x_i(n\Delta) + h_i \right)^{p_i - 1} c_0 / 2. \end{aligned} \tag{3.9}$$

In particular,

$$E_n V_{n+1} - V_n \to -\infty, \text{ uniformly in } n \text{ as } x(n\Delta) \to \infty. \tag{3.10}$$

Before continuing, let us note the following. Suppose that $x(n\Delta)$ were Markov, and there were constants $c_1 > 0, K_3 > 0$, such that $E_n V(x(n\Delta +$

$\Delta) - V(x(n\Delta)) \le -c_1$ if $V(x(n\Delta)) \ge K_3$. Then the segments of the process $V(x(n\Delta))$ on the time intervals on which the $V(x(n\Delta))$ are no less than K_3 are supermartingales and classical stability theorems [7] imply that $E_x\tau \le V(x)/c_1$, where τ is the return time to the set where $V(x) \le K_3$ and E_x is the expectation given initial condition x. This is (2.2) in this special case.

In the general case of interest in the theorem, $x(n\Delta)$ is not necessarily Markov and the Liapunov function is (3.1). The result (3.10) implies that the segments of the process V_n on the time intervals on which the $|x(n\Delta)|$ are no less than K_2 are supermartingales. Obtaining the bounds on the conditional mean return time from (3.10) is similar to what is done in the Markov case. One just has to ensure that the effects of the perturbations can be appropriately "dominated." Using (A2.3) and (A2.4), we have the bound

$$\left|\sum_i \delta V^a_{i,n} + \sum_{i,j} \delta V^d_{i,j,n}\right| = O(1)\sum_i (x_i(n\Delta) + h_i)^{p_i - 1}, \tag{3.11}$$

which is of smaller order than $V(\cdot)$. Hence, V_n is bounded below and goes to infinity (uniformly in n) as $x(n\Delta) \to \infty$. This and (3.10) imply the conditions of [8, Theorem 2, Chapter 6] which yields (2.2), and we fill in the details next.

By (3.10), there are $c_1 > 0, q_0 > 0$, such that, for $|x(n\Delta)| \ge q_0$, $E_n V_{n+1} - V_n \le -c_1$. Given small $\delta > 0$, (3.11) implies that for q_0 sufficiently large, $|V(x(n\Delta)) - V_n| \le \delta(1 + V(x(n\Delta))$. Let σ_0 be a stopping time for which $|x(\sigma_0)| = c_2 > q_0$, and define the stopping time $\sigma_1 = \min\{n\Delta > \sigma_0 : |x(n\Delta)| \le q_0\}$. Then we have

$$E_{\sigma_0} V_{\sigma_1} - V_{\sigma_0} \le -c_1 E_{\sigma_0}[\sigma_1 - \sigma_0]. \tag{3.12}$$

Using (3.12) and the bound (3.11) on $V_n - V(x(n\Delta))$ to bound $V_{\sigma_i} - V(x(\sigma_i))$, $i = 0, 1$, yields

$$\begin{aligned} &-\delta E_{\sigma_0}[1 + V(x(\sigma_1))] + E_{\sigma_0} V(x(\sigma_1)) \\ &\qquad \le E_{\sigma_0} V_{\sigma_1} \le -c_1 E_{\sigma_0}(\sigma_1 - \sigma_0) + [\delta + V(x(\sigma_0))(1 + \delta)] \end{aligned}$$

or

$$E_{\sigma_0}(\sigma_1 - \sigma_0) \le \frac{2\delta + V(x(\sigma_0))(1 + \delta) + \delta E_{\sigma_0} V(x(\sigma_1))}{c_1}$$

which is equivalent to (2.2), since $V(x(\sigma_1)) \le \sup_{|x| \le q_0} V(x)$. ■

4 Examples

The examples are only idealized outlines of a sampling of the possibilities. In all cases, we write $x(n\Delta) = x$. Let the bit interval for queue i be be Δ_i^b. The data arrival process need not be specified in the following discussions.

Example 4.0. The simplest example is where only one queue can be powered in any interval, as follows. The constraint is (2.1) and in (A2.1), $U_i = \{0, \bar{u}\}$. There are $\lambda_i(j) \geq 0$ such that $\min\{\lambda_i(j), x_i\}$ is the number of packets transmitted in the interval if queue i is selected. Let $I_i(j,x)$ denote the indicator function of the event that queue i is selected. Then $g_i(\cdot)$ is defined by $\Delta g_i(j,x,u_i(j,x)) = \min\{\lambda_i(j), x_i\} I_i(j,x)$. To maximize the expression (2.6), we choose the i that gives $\arg\max\{V_{x_i}(x)\min\{\lambda_i(j), x_i\}\}$.

Example 4.1. [Power only control.] Suppose that the queued data is optimally recoded before transmission and that the queues and outputs are measured in terms in bits. The Shannon capacity of a white-Gaussian channel with unrestricted alphabet and fixed symbol intervals is $\log(1 + S/N)/2$ per symbol, where S and N are the signal and noise powers, resp., at the receiver [6, Eqn. 10.17]. [For complex signals multiply by 2.] Suppose that this form models all the channels and there is negligible mutual interference. Let K_i denote the number of symbols that can be transmitted per second for queue i. Since the symbol intervals are assumed to be very short relative to the scheduling interval, with appropriate coding the total number of bits transmitted over a scheduling interval is well approximated by ΔK_i times the capacity. The power constraint is $\sum_i u_i(j,x) \leq \bar{\mu}$ for all j, x, where $\bar{\mu}$ is the total available power and $u_i(j,x)$ is the actual power applied to transmission from queue i in the interval. This constraint is (A2.1) in this case, where the U_i can be taken to be the nonnegative real numbers $[0, \infty)$. Then, there are constants α_{ij}, N_{ij} such that if the channel state is fixed at j, the channel capacity $C_i(j, u_i(j,x))$ for queue i under $u_i(j,x)$ is

$$C_i(j, u_i(j,x)) = \frac{1}{2}\log\left[1 + \alpha_{ij} u_i(j,x)/N_{ij}\right]. \tag{4.1}$$

The $\alpha_{i,j}$ scales for channel attenuation. Then we can suppose that, when the channel state is j, $d_{i,n} = \Delta g_i(j,x,u_i(j,x)) = \min\{\Delta C_i(j, u_i(j,x)), x_i\}$. This defines the functions $g_i(\cdot)$. Let $V(x) = \sum_i x_i^p$, for some $p > 1$. Then, when the channel state is j, the control maximizes

$$\sum_i x_i^{p-1} \min\left\{\Delta \frac{K_i}{2} \log\left(1 + \alpha_{ij} u_i(j,x)/N_{ij}\right), x_i\right\}. \tag{4.2}$$

Due to the strict concavity of the function $\log(1 + cu)$, several queues might be powered simultaneously. If desired, one can add the constraint that only one queue is powered during any scheduling interval. In this case one looks for the summand in (4.2) that is largest when $u_i(j,x) \equiv \bar{u}$, and all power goes to that user. If the maximizer is not unique, then select any one, or randomize among them.

Example 4.2. [Power only control.] Now suppose that the number transmitted is proportional to the power, and one might transmit from several queues simultaneously with negligible mutual interference. In particular, let there be constants $\bar{\lambda}_i^d(j)$ such that the number of packets per unit time that can be sent from queue i under channel state j is $\bar{\lambda}_i^d(j) u_i(j,x)$ for $u_i(j,x) \in U_i$, a finite set for each i. We still require the constraint (2.1). The function $g_i(\cdot)$ is defined by $\Delta g_i(j,x,u_i(j,x)) = \min\left\{\Delta\bar{\lambda}_i^d(j)u_i(j,x), x_i\right\}$. Given $V(x)$ of the form in Example 4.1, the maximization is trivial.

There are many ways that one can realize the number transmitted being proportional to power. If bandwidth is the single resource to be allocated and power is proportional to the bandwidth, then it is often the case that the rate is proportional to power. With CDMA, one possibility is to allow several "spreading sequences" per queue [15]. Another possibility is to allow the bit interval (duration) to be controllable. In particular suppose that (for queue i) there are basic time intervals δ_i^b, of which the actual bit interval Δ_i^b is one of the multiples $m\delta_i^b, m = 1, \ldots, m_i$, for some given $m_i < \infty$. Suppose that the channels are white-Gaussian, that there is perfect synchronization at the receiver, and that the system design is such that if the power allocation to any queue is not zero then it must be such that the SNR per bit at the receiver is at least some given positive number $c_i(j)$. This minimal amount $c_i(j)$, together with the bit interval, determines the capacity. Thus, cutting the bit interval in half would require twice the power for the same SNR. The chosen bit interval is constant during a scheduling interval.

Example 4.3. [Power and time control.] In Examples 4.1 and 4.2, the *total* power was constrained to $\bar{\mu}$ at each time and the transmissions from all selected queues in an interval were done simultaneously. Alternatively, suppose that we schedule by dividing time as well as power in the following generalized form of TDMA. For each queue i, channel state j, and queue state vector x, partition the nth scheduling interval into K subintervals (some subintervals might have length zero) of total lengths $\Delta_i(j,x), i \le K$, where $\sum_i \Delta_i(j,x) = \Delta$. Summarizing, in the nth scheduling interval, we transmit from queue i on a total time $\Delta_{i,n}$, which can depend on i and on the current queue and channel states x, j.

Suppose that we can vary the power within an interval provided that it averages to $\bar{\mu}$ over the interval. In particular, let $p_i(j,x)$ denote the power applied to queue i during the part of the interval that queue i is being worked on. Then the full *vector-valued* control $u_i(j,x)$ is $(p_i(j,x), \Delta_i(j,x))$. Then, in lieu of the pointwise constraint (2.1), we now constrain *average total power* over the interval and the constraint (2.1) is replaced by

$$\sum_i p_i(j,x)[\Delta_i(j,x)/\Delta] \le \bar{\mu}, \quad \text{all } j. \tag{4.3}$$

Now consider the setup in Example 4.1, but where only one queue can be powered at a time and the scheduling interval is divided among the queues, as above. Define $g_i(\cdot)$ by

$$\Delta g_i(j,x,u_i(j,x)) = \min\left\{\Delta_i(j,x)\frac{K_i}{2}\log\left[1+\alpha_{ij}p_i(j,x)/N_{ij}\right], x_i\right\}.$$

Then, the rule (2.6) is the maximizing (power and time) value in

$$\max \left\{ \sum_i V_{x_i}(x) g_i(j, x, u_i(j, x)) \right\},$$

with constraints (4.3) and $\sum_i \Delta_i(j, x) = \Delta$.

Example 4.4. The general scheme is not restricted to systems with only one antenna. This will be briefly illustrated when there are two antennas, and we have power only control, with one queue being scheduled at a time, analogously to the situation of Example 4.0. For notational simplicity, suppose that there are two antennas. Let the channel state for antenna α be denoted by j^α, write the full channel state as $j = (j^1, j^2)$, and let $u_i^\alpha(j, x)$ denote the power that antenna α applies to queue i. The total power constraints are $\sum_i u_i^\alpha(j, x) = \bar{u}^\alpha, \alpha = 1, 2$. The U_i in (A2.1) becomes a product set $U_i^1 \times U_i^2$, where $U_i^\alpha = \{0, \bar{u}^\alpha\}$.

The scheduling can be done independently between the antennas, or there could be coordination. We will consider all possibilities. First allow the two antennas to be scheduled independently. One simply uses (2.6) for each antenna with $u_i^\alpha(j, x)$ depending only on (i, j^α, x). Let $\min\{\lambda_i^\alpha(j), x_i\}$ denote the number of packets transmitted in an interval on antenna α when queue i is selected for it and the (channel, queue) state is (j, x). When the antennas are scheduled independently, there are two functions $g_i^\alpha(j^\alpha, x, u_i^\alpha(j^\alpha, x))$, $\alpha = 1, 2$, each obtained analogously to the construction in Example 4.0. Now let us coordinate the decisions. The simplest form of coordination is to modify the foregoing by allowing only one queue to be powered at a time at each antenna. Let $I_i^\alpha(j, x)$ denote the indicator of the event that queue i is selected by antenna α. Then the constraints can be considered to be

$$\sum_i I_i^\alpha(j, x) = 1, \alpha = 1, 2, \quad \sum_\alpha I_i^\alpha(j, x) \leq 1. \tag{4.4}$$

The expression (2.6) becomes

$$\max \sum_i V_{x_i}(x) \left\{ \min\left\{\lambda_i^1(j), x_i\right\} I_i^1(j, x) + \min\left\{\lambda_i^2(j), x_i\right\} I_i^2(j, x) \right\}. \tag{4.5}$$

Another option allows the possibility of using a more sophisticated coding scheme, say that of space time coding [13], still under the constraints (4.4). In this case, both antennas are used for the selected queue, with the coding accounting for the fact that there are two antennas. This gives a different function $g(\cdot)$. There might be some channel and queue states for which the value of (4.5) is larger than the value of (2.6) under space-time coding and conversely. One would take the best choice.

5 Examples of Controls and Relation to Optimal Controls

Consider Example 4.2, where the rate of transmission is proportional to power (the model used in [1]) and let $V(x) = \sum_i x_i^{p+1}, p \geq 1$. Then the control given by (2.6) is the maximizer in

$$\max \left\{ \Delta \sum_i x_i^p \bar{\lambda}_i(j) u_i(j,x) \right\}$$

over the allowed discrete values. In the rest of the discussion of the controls, we ignore the small queues, which are readily accounted for.

A two-dimensional case. Consider a two-queue and two-channel state example with no constraints on the power allocation. Let $\bar{\lambda}_1^d(1) > \bar{\lambda}_2^d(1)$, and $\bar{\lambda}_1^d(2) < \bar{\lambda}_2^d(2)$. For channel state 1, apply all power to queue 1 when

$$x_1/x_2 \geq \left[\bar{\lambda}_2^d(1)/\bar{\lambda}_1^d(1)\right]^{1/p}. \tag{5.1}$$

For channel state 2, apply all power to queue 2 when

$$x_2/x_1 \geq \left[\bar{\lambda}_1^d(2)/\bar{\lambda}_2^d(2)\right]^{1/p}. \tag{5.2}$$

As $p \to \infty$, the switching lines move to the diagonal.

An alternative Liapunov function. Now consider the Liapunov function $V(x) = \sum_i [x_i + h_i]^{p+1}$. With equal h_i, this gives results that are closer to the numerically computed optimal controls for an approximating diffusion with cost rate $\max\{x_1, x_2\}$ (see below). The inequalities (5.1) and (5.2) are replaced by. respectively,

$$x_1 + h_1/x_2 + h_2 \geq \left[\bar{\lambda}_2^d(1)/\bar{\lambda}_1^d(1)\right]^{1/p}, \tag{5.3}$$

$$x_2 + h_2/x_1 + h_1 \geq \left[\bar{\lambda}_1^d(2)/\bar{\lambda}_2^d(2)\right]^{1/p}. \tag{5.4}$$

Here the cost rate is less sensitive to differences in the x_i unless they are large.

Comparison with optimal controls. It is interesting to relate the control forms determined by ((5.1), (5.2)) and by ((5.3), (5.4)) to optimal controls for diffusion processes with similar dynamics. Figure 5.1 gives the optimal switching surfaces for control problem for the diffusion process $dx = \bar{g}(x, u(x))dt + \sigma dw$. The qualitative form of the controls did not depend on σ. We let $\bar{\lambda}_1^d(1) > \bar{\lambda}_2^d(1), \bar{\lambda}_1^d(2) < \bar{\lambda}_2^d(2)$, and the optimal controls are obtained by numerical solution using the methods of [9]. The cost rate is $k(x) = \max\{x_1, x_2\}$ for the left-hand figure and $\sum_i x_i$ for the right-hand figure. The total cost was either of the ergodic or discounted form with small discount factor, with little difference in the controls between them. Similar results hold for discrete time problems. The slopes for the linear cost rate tend to be larger than those for the $\max\{x_1, x_2\}$ criterion. Equivalently, for the linear cost rate we put more emphasis on using the most efficient channel and less on attaining closeness of the queues. Note the similarity of the controls to the forms given by ((5.1), (5.2)) and ((5.3), (5.4)). The controls in the left hand figure have the form of ((5.3), (5.4)), and those in the right hand figure have the form of ((5.1), (5.2)). Thus, with appropriate choices of the parameters, the controls are close to those for optimal processes for closely related problems.

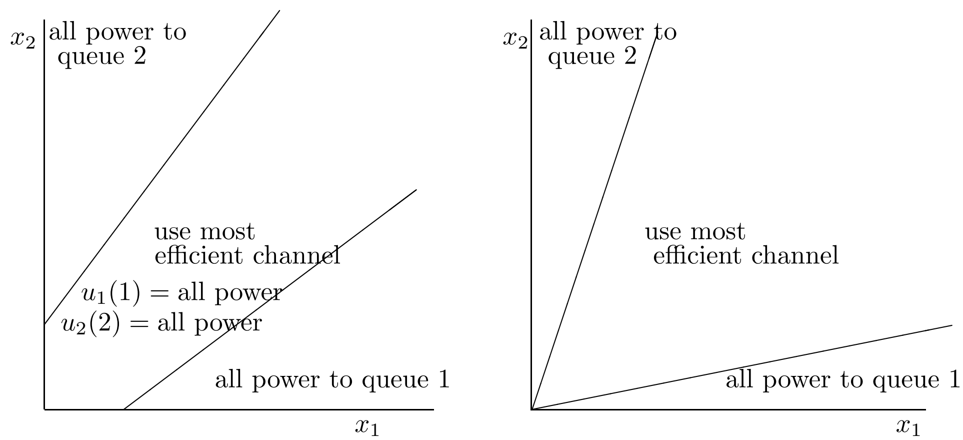

Figure 5.1. Switching curves for the optimal control:
$k(x) = \max\{x_1, x_2\}$ and $\sum_i x_i$.

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