15629

Game Theory and Trade-Off Analysis

David Bednarz and William Jackson
Survivability Technology Area
US Army RDECOM, TARDEC
Warren, Michigan

ABSTRACT

Which is preferable: a more lethal weapon or a stealthier platform upon which to mount one's current weapon? Of course, without specifying how much more lethal the new weapon is or how much less detectable the platform is the question is unanswerable. Thus, the purpose of this paper is to formulate the question in a precise fashion so that a tradeoff between increased lethality and decreased detectability can be accomplished. Once formulated, the actual tradeoff will be accomplished using results from the area of games of timing, a sub-area of the mathematical Theory of Games.

BACKGROUND

Consider the following abstract situation which involves only lethality: Of two combatants, Red and Blue, each has a single noisy bullet, where having noisy bullets means that each combatant knows when his opponent has fired. Each combatant also has an accuracy function; that is, there are two functions aRB and aBR where $aRB(x)$ is the probability of Red killing Blue if Red fires at Blue from a distance x, and $aBR(x)$ is the probability of Blue killing Red if Blue fires at Red from a distance x.

In the above situation, when is the optimal time for Blue to fire? Here, by optimal time to fire is meant a firing time for Blue that will minimize Red's survival. Game Theory then informs us that the optimal time for Blue to fire is when the distance between Blue and Red, xf, is such that $aBR(xf) + aBR(xf) = 1$. In this situation it's clear that Blue has the advantage if at any distance his gun has a higher accuracy of killing Red than Red has of killing Blue.

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(Aside: There are various mathematical assumptions made regarding the accuracy functions above such as non-decreasing with continuous derivatives. These assumptions will generally be met by any real-life accuracy function.)

In the above situation it is assumed that Red and Blue can each always see their opponent. In order to tradeoff survivability and lethality the above situation is enhanced as follows: In addition to the accuracy functions, aRB and aBR, there are now detectability distributions, dRB and dBR, where $dRB(x)$ is the probability that Red will detect Blue by the time the distance between them is x, and similarly $dBR(x)$ is the probability that Blue will detect Red by the time the distance between them is x.

Since the interest is in the tradeoff between Blue lethality and Blue survivability and not Red lethality and Red survivability, it is assumed that Blue can always see Red, but not that Red can always see Blue.

Thus, since Blue can always see Red, $dBR(x) = 1$ for every distance x. However, since Red cannot always see Blue, $dRB(x)$ will in general be less than 1. But it is assumed that as the distance x between Red and Blue decreases, the detectability distribution dRB will increase.

INTRODUCTION TO THE EXAMPLES

For technical reasons the distance, x, between Red and Blue will always be represented by a negative value.

In all the following examples Red will have the same accuracy function, namely

$$aRB(x) = 1 + x/1000.$$

A graph will be provided in one of the examples below.

Example 1: In this example Blue will have the same accuracy function that Red possesses; that is, $a1BR = aRB$. As noted above Red will always be visible to Blue, but Blue will not always by visible to Red. Nevertheless, Red's ability to detect Blue, given by $d1RB$ will be such that Blue could not be considered a stealthy target. A graph of $d1RB$ will be given in the example.

Example 2: In this example Blue will have a more lethal accuracy function,

$$a2BR(x) = 1 + x/2000,$$

but Red's ability to detect Blue will remain the same as in Example 1.

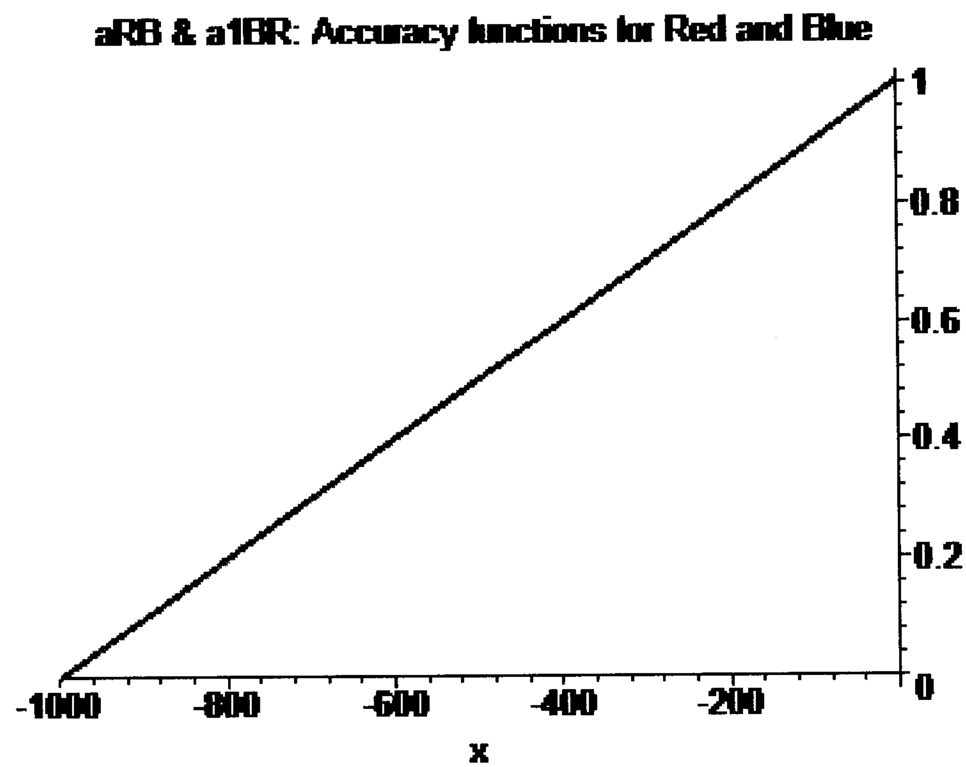
Example 3: In this example Blue reverts to a less lethal weapon, *a1BR* replaces *a2BR*, but Blue becomes more stealthy as given by *d2RB*, with again the graph given below.

Example 4: In this example Blue reverts to the more lethal weapon and retains its stealthiness.

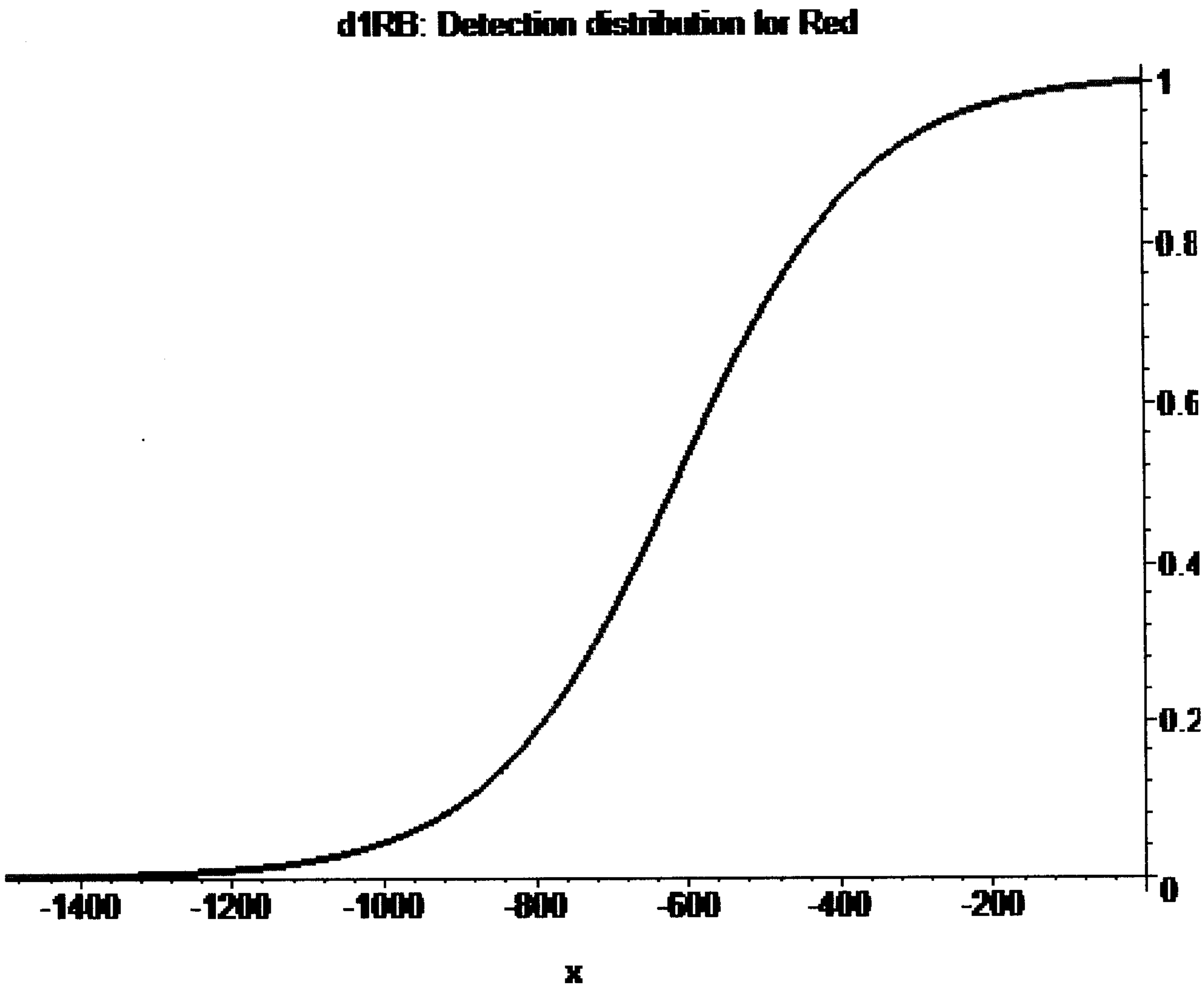
EXAMPLES

Example 1:

Red accuracy = $aRB(x)$ = Blue accuracy = $a1BR(x) = 1 + x/1000$.



Red's ability to detect Blue, $d1RB(x)$, is given by



In this example Red's survivability is .498.

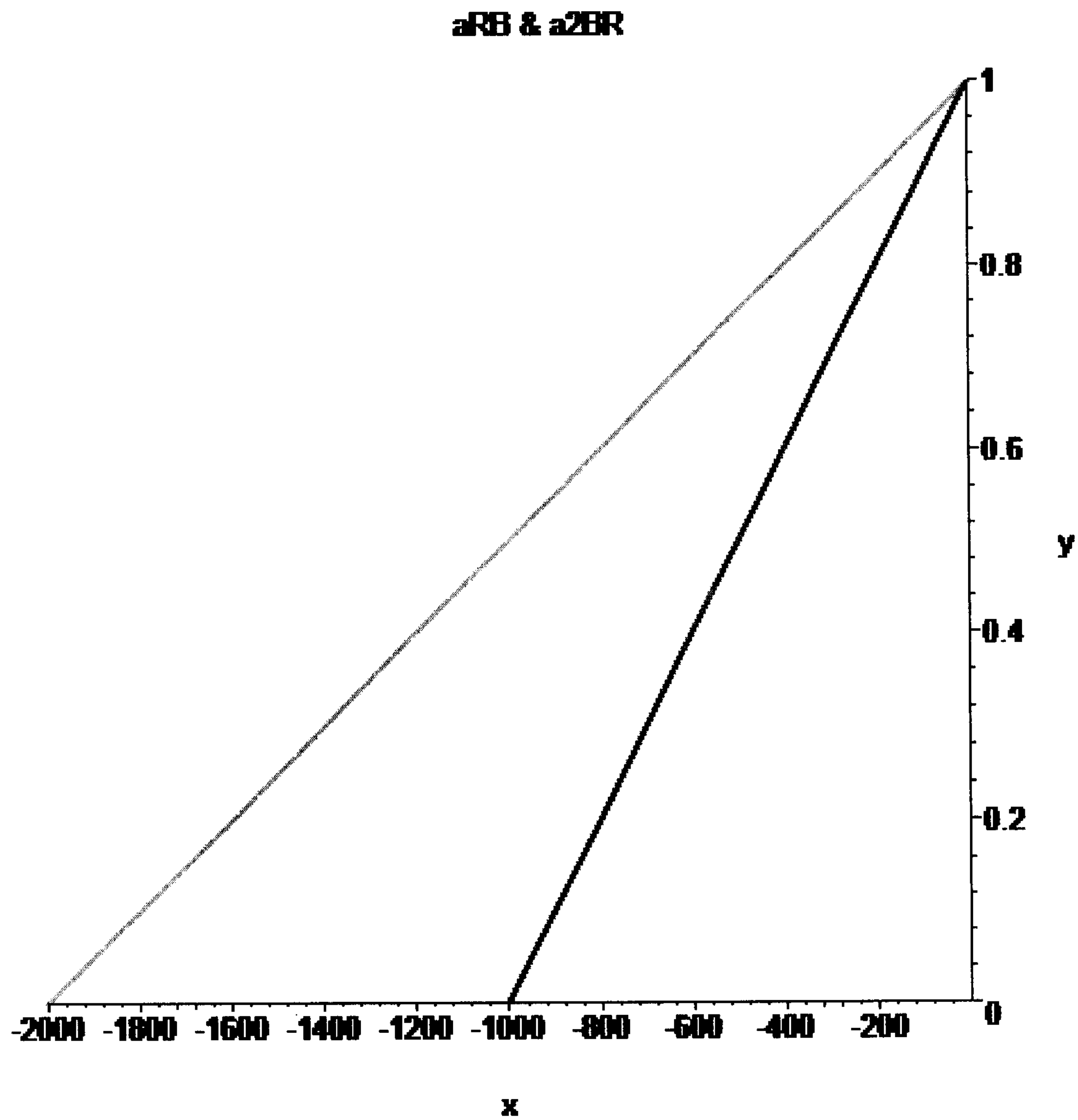
Recall that in the case where Red could always see Blue, Red's survivability was .500. Thus, in this example, given the accuracies of Red's and Blue's weapons, Blue possesses essentially no stealthiness.

Example 2:

In this example Red's accuracy and Red's ability to detect Blue remain the same as they were in Example 1. However, Blue has improved accuracy over Example 1. Blue's accuracy is now given by

$$A2BR(x) = 1 + x/2000.$$

Red's and Blue's accuracies are given in the following graph.

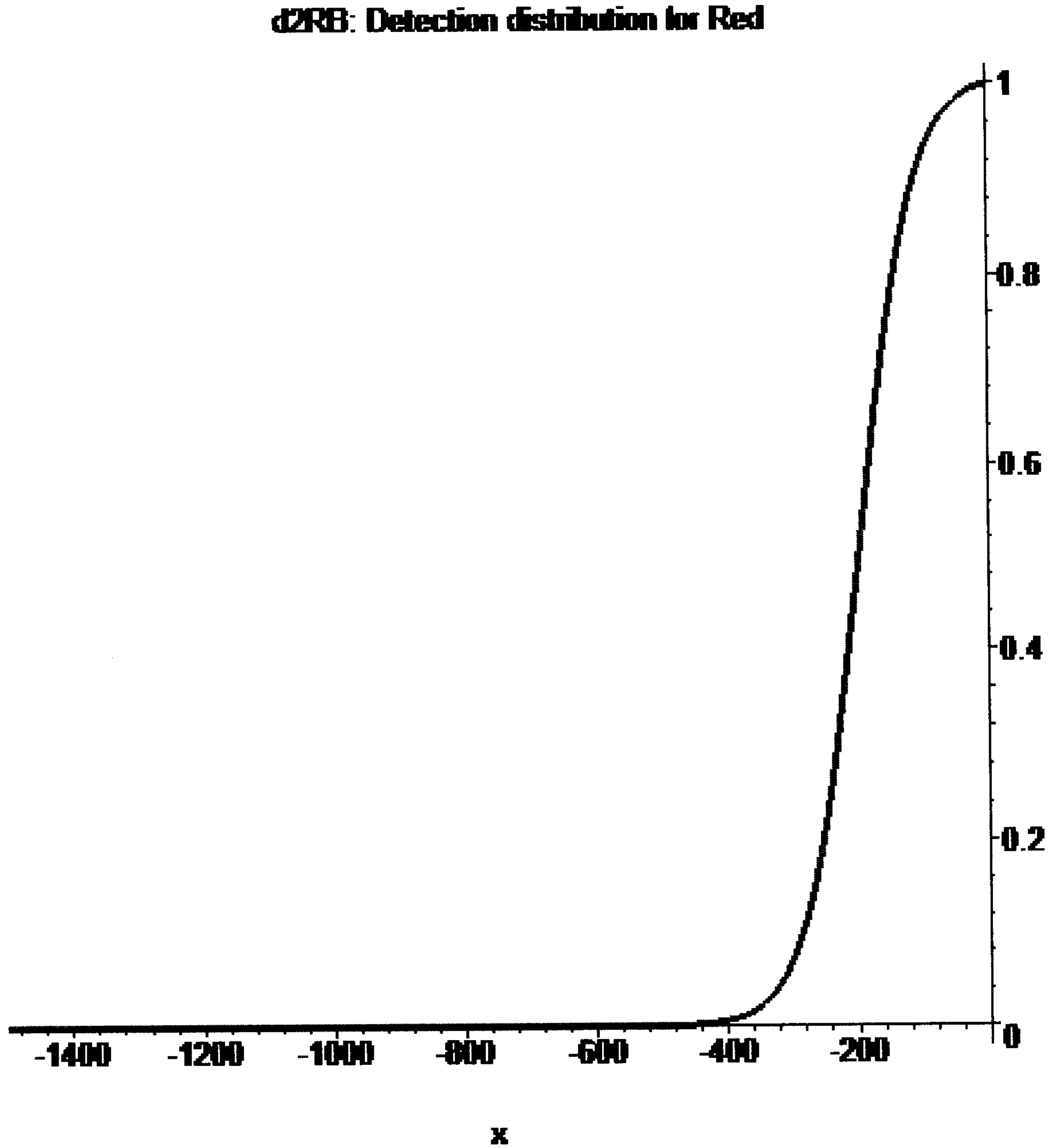


In this example Red's survivability is .330, down from the .498 of Example 1.
Example 3:

In this example Red and Blue have the same accuracy, just as they had in Example 1,

$$aRB = a1BR = 1 + x/1000,$$

but Blue is much stealthier than in Example 1. In this example Red's ability to detect Blue is given by *d2RB*, whose graph is given below.



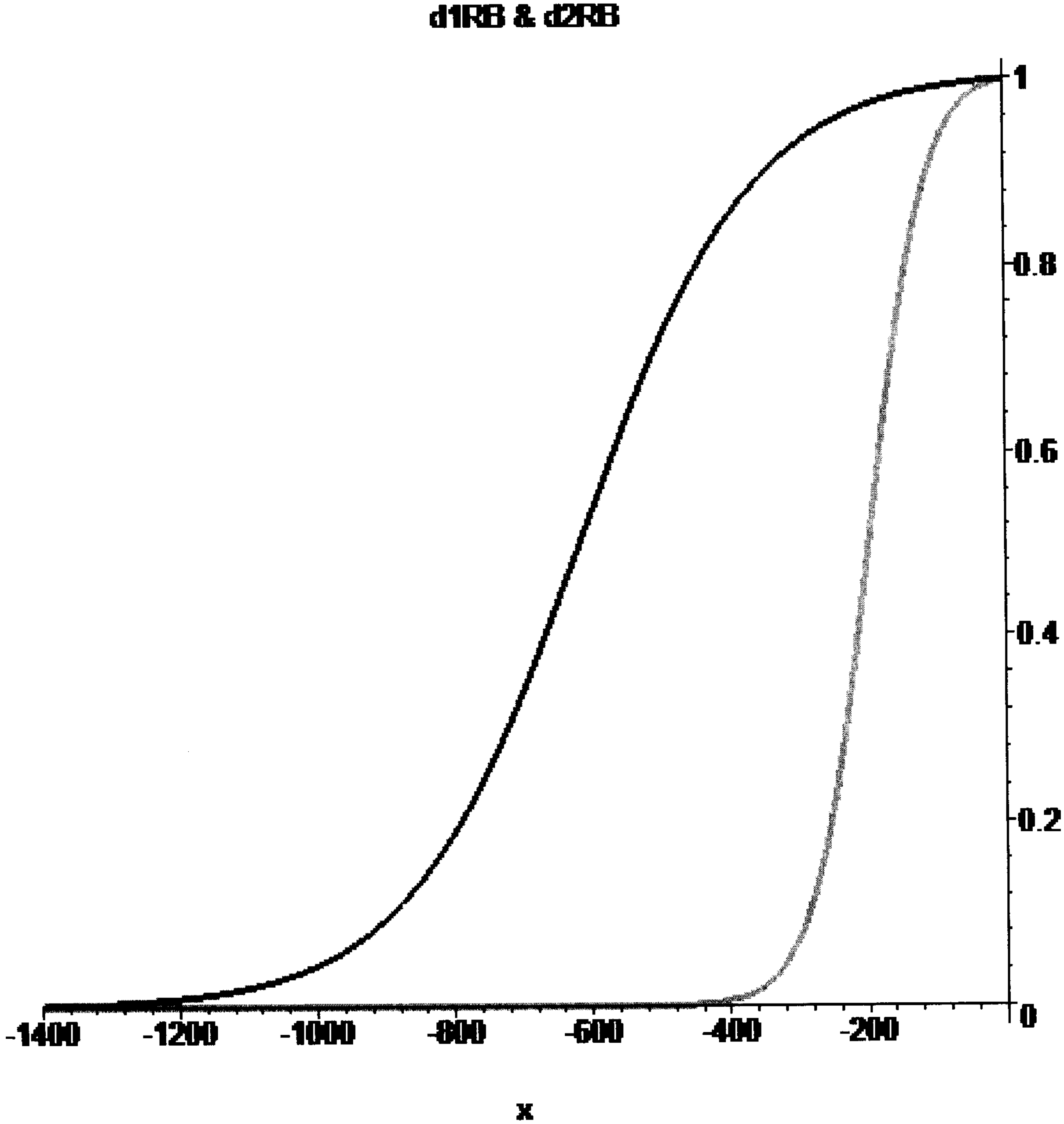
Red's survivability in this example is .326. Thus, the increase in Blue's stealthiness from *d1RB* to *d2RB* is just slightly more advantageous than the increase in Blue's lethality from *a1BR* to *a2BR*, as evidenced by this Example and Example 2.

In the next and last example Blue will have both increased stealthiness and a more lethal weapon.

Example 4:

In this example Blue has the stealthiness of Example 3, that is, Red's ability to detect Blue is given by *d2RB*, and Blue's ability to kill Red is given by *a2BR*.

A graph comparing *a2BR* with *a1BR* = *aRB* was given above, and a graph comparing *d2RB* with *d1RB* is given below.



In this example Red's survivability is .182.

TRADING OFF LETHALITY AND SURVIVABILITY

Since Blue's increased stealthiness reduces Red's survivability approximately the same amount as Blue's increased lethality reduced it, the choice between which direction to pursue in the development of the Blue platform will depend upon the costs, integration factors, and perhaps other parameters, involved in the development of Blue's stealthiness as opposed to the development of a more lethal weapon for Blue.

FINAL REMARKS AND REFERENCES

The methodology used in obtaining the results in this paper is based upon techniques developed in the following.

Dresher, Melvin, *The Mathematics of Games of Strategy*, Dover Publications, 1981.

Sweat, Calvin, "A Single-Shot Noisy Duel with Detection Uncertainty," *Operations Research*, Volume 19, 1971.