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LABORATORY FOR INFORMATION AND DECISION SYSTEMS
Massachusetts Institute of Technology
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Annual Technical Report for

Grant AFOSR-88-0032

on

ANALYSIS, ESTIMATION, AND CONTROL FOR PERTURBED AND SINGULAR SYSTEMS AND FOR SYSTEMS SUBJECT TO DISCRETE EVENTS

for the period

1 October 1987 through 30 September 1988

Submitted to: Major James Crowley
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October 21, 1988

I. *SUMMARY*

In this report we summarize our accomplishments in the research program presently supported by Grant AFOSR-88-0032 over the period from October 1, 1987 to September 30, 1988. The basic scope of this program is the analysis, estimation, and control of complex systems with particular emphasis on (a) the development of asymptotic methods and theories for nearly singular systems; (b) the investigation of theoretical questions related to singular systems; and (c) the analysis of complex systems subject to or characterized by sequences of discrete events. These three topics are described in the next three sections of this report. A full list of publications supported by Grant AFOSR-88-0032 is also included.

The principal investigator for this effort is Professor Alan S. Willsky, and Professor George C. Verghese is co-principal investigator. Professors Willsky and Verghese were assisted by several graduate research assistants as well as additional thesis students not requiring stipend or tuition support from this grant.

II. Asymptotic Analysis for Perturbed Systems

Our research in this area has focused on the analysis of perturbation models of linear systems and of finite-state stochastic processes. The class of models that has been the focus of much of our work is the perturbed linear system.

$$\dot{x}(t) = A(\epsilon)x(t) \tag{2.1}$$

and its counterpart with inputs

$$\dot{x}(t) = A(\epsilon)x(t) + B(\epsilon)u(t) \tag{2.2}$$

Our work during the past year has had three parts. The first has been in continuing to refine and document results on several topics, namely an efficient approach to hierarchical aggregation of finite state Markov processes (FSMP's) (corresponding to (2.1) with $A(\epsilon)$ an infinitessimally stochastic matrix) and its extension to semi-Markov processes and positive systems [1,6-8,11,15], an algebraic, ring-theoretic approach to multiple time scale decomposition for (2.1) [3,5], and a theory of asymptotic orders of reachability for (2.2) [10]. The second topic deals with both the application of the theory in [1,6-8] to the analysis of a flexible manufacturing system and the development of a new theoretical result on asymptotically accurate computation of count rates for particular events. These results are documented in [18,19], and a journal paper on this topic is now being planned.

Let us briefly describe the central idea behind the count rate result. Specifically, in a perturbed FSMP a particular event --corresponding to one or any of a set of transitions in the FSMP -- may appear to occur at different effective rates depending upon the time scale over which the process is viewed. For example, the production rate of a machine over a time period that is short with respect to the machine's mean time between failure will certainly differ from its effective rate over very long intervals in which several failures and repairs may occur. To deal with this one must make use of the multiple time-scale, hierarchically aggregated models of the process as developed in [1]. The method developed in [1] is, in a sense, optimum, as it recursively determines and keeps only those ϵ-dependent transistion rates that affect long term transition behavior. To solve the asymptotic count rate problem, however, several other issues needed to be considered. First, it turns out to be necessary to identify and account for additional terms not needed for the problem considered in [1]. For example, suppose that the transitions from state i to j and from state k to m both correspond to the same "event" in the overall system. Suppose further that the ergodic probability of state i is $O(1)$, while the i-to-j transition rate is $O(\epsilon)$, and that the ergodic probability of k is $O(\epsilon)$ and the k-to-m rate is $O(1)$. Then, the analysis in [1] would in essence neglect residency in state k. However, thanks to the $O(1)$ k-to-m rate, state k is just as important for event counting as state i. Thus, in our analysis we developed an extension of previous results to identify and retain such terms.

A second issue that needed to be considered is the changing nature of events as we look at aggregated versions of the FSMP corresponding to long time-scale behavior. In particular, while an event corresponds to a set of transitions between states of the original process, these states may be aggregated together at some time-scale, and at that scale the event becomes a random variable taking on a value corresponding to each aggregate state (e.g. while an initial model of a machine would have transitions corresponding to completion of a part, an aggregated version capturing only transitions between "failed" and "operational" states would not; in this case one would associate a variable representing effective production rate while in each of these aggregate states). Combining these ideas with the results of [1] and results from renewal theory leads to an effective procedure for computing asymptotic event rates.

The final topic that has been considered during the year is the asymptotic analysis of estimation problems for perturbed FSMP's. We have been successful in obtaining some significant new theoretical results and some simulation results that support a number of additional conjectures. This research is still in progress, and a paper is being planned for the coming year. We limit ourselves here to a brief illustration of some of the ideas. Consider the 4-state process depicted in Figure 2.1. Because of the weak left-right coupling, this process has two time scales: fast transitions between top and bottom states and much less frequent right-left transitions. Using the method of [1], the long-term, aggregate behavior of this process can be described as in Figure 2.2, where L = {1,2}, R = {3,4}, and γ_1, γ_2 can be computed from the λ's and μ's.

Suppose that we are interested in estimating only the slow, long-term L-R behavior from measurements of the original process:

$$dy(t) = g(\epsilon)h(\rho(t))dt + dv(t) \qquad (2.3)$$

where $\rho(t)$ is the 4-state process, $v(t)$ is a standard Wiener process, and $g(\epsilon)$ is a gain, controlling measurement signal-to-noise ratio. There are two interesting possibilities for the measurement function $h(\rho)$:

$$h(1) = h(2) = \alpha \quad , \quad h(3) = h(4) = \beta \qquad (2.4)$$

or

$$h(1) = h(3) = \alpha \quad , \quad h(2) = h(4) = \beta \qquad (2.5)$$

In the first case, we only receive information on L-R status from the measurements, and thus one would expect that a suboptimal estimator based on the aggregate model of Fig. 2.2 would perform well. This is indeed the case, and the precise result will be reported in the forthcoming paper.

In the case of (2.5) we directly measure only top-bottom behavior. However, we do obtain indirect information about L-R status thanks to differences in the λ's. We have several results on methods for extracting this information. For example, if $g(\epsilon) \longrightarrow 0$ as $\epsilon \longrightarrow 0$, i.e. if the instantaneous SNR is small, then information is accrued only by the integrated effect of $y(t)$ over a substantial time period. What would seem plausible in this case is to replace $h(\rho)$ by its average, depending upon whether $\rho(t)$ is on the left or right:

$$h_a(L) = p_1\alpha + p_2\beta \quad , \quad h_a(R) = p_3\alpha + p_4\beta \qquad (2.6)$$

where the p_i are the ergodic probabilities (for the corresponding ergodic classes) original process when ϵ=0. We have precise theoretical results for the asymptotic optimality of the estimator based on the aggregate model of Figure 2.2 and (2.6).

The work described previously has largely been restricted to linear, time invariant (LTI) systems so far. It is of interest to consider extensions to the case of perturbed periodically varying linear systems. Opportunities for studying problems of this sort arise in the context of electrical machines and power electronics. Deferring a discussion of modeling and analysis of switched power electronic circuits to Section IV, we mention here some recent work of ours in stability studies for electrical machines.

The most commonly considered nominal operating condition for the nonlinear model of an electrical machine system is constant speed operation. The linearized model at this operating condition is usually periodically varying, because of the periodic variation of the machine's inductance matrix with rotor position. However, for some machines a time invariant nonlinear transformation of the original nonlinear model leads to a time invariant linearized model, provided the inputs (voltages and torques) satisfy certain ideal properties. This "Blondel-Park" transformation radically simplifies the stability analysis. Most analytical stability studies in the machines literature therefore make (sometimes drastic!) modeling assumptions that lead to a transformable model subjected to ideal inputs.

In [25], we have obtained succinct and easily tested necessary and sufficient conditions for Blondel-Park transformability of a machine with affine magnetic properties. The conditions are constructive in the sense that a suitable transformation is yielded if one exists. This then leads to the possibility, elaborated on in [25] and [26], of studying machines in <u>nearly ideal</u> operation, i.e. nearly transformable and/or operating with nearly ideal inputs. For such machines, nominal operation corresponds to periodic, nearly constant speed.

There are two tasks for a perturbation approach here. The first is to approximately compute the perturbed periodic steady state. The second is to approximately assess the stability of this periodic steady state.

Perturbation methods for both tasks are described in [25], [26], in each case exploiting the fact that ideal operation involves the analysis of an LTI model. In particular, the second task involves stability analysis of the nth-order system

$$\dot{x}(t) = (A + \epsilon B(t))x(t),\ B(t) = B(t + T),\ \text{periodic} \tag{2.7}$$

The Floquet theory shows that asymptotic stability is equivalent to the eigenvalues of the monodromy matrix (the state transistion matrix over a period) of the above system all having magnitude less than 1. In the case where A has distinct eigenvalues, we use eigenvalue sensitivity results to reduce the computation involved in approximate stability assessment to evaluation of the n scalar integrals $\int_0^T \omega_i^T B(t)v_i dt$, where ω_i and v_i are the left and right eigenvectors associated with the ith eigenvalue of A, $i = 1,\ldots,n$. Note that the perturbation matrix B(t) enters these integrals linearly, which can be important for design and physical interpretation of the results.

Future work in this direction will examine the potential and limitation of such first-order sensitivity methods. For example, if the average value of B(t) is the zero matrix, then the first order results above are not helpful, and one has to look further. Another interesting problem that we intend to study in more detail is that of periodic singular perturbations, corresponding to A in (2.7) being singular. Finally, the above problems and methods may be extended to more general systems. In particular, the idea of using a time invariant nonlinear transformation of a nonlinear model to obtain a time invariant linearization instead of a periodically varying one does not seem to have been discussed outside the machines literature, though related ideas such as feedback linearization and pseudo-linearization have been discussed.

III. Singular Systems

During the past year we have made substantial progress on the analysis of two-point boundary-value descriptor systems (TPBVDS's):

$$Ex(k+1) = Ax(k) + B\mu(k) \tag{3.1}$$

with boundary conditions

$$v = V_i x(0) + V_f x(N) \tag{3.2}$$

and output

$$y(k) = Cx(k) \tag{3.3}$$

Such models are a natural choice for the description of non-causal (e.g. spatial) phenomena and signal processing tasks.

Our earlier work dealt with some of the basic system-theoretic properties of these systems. Specifically, we investigated well-posedness for TPBVDS's and in the process discovered a very useful normalized form. We also investigated the two natural "state" processes for these systems, namely the inward process, corresponding to propagating the boundary condition inward, and the outward process, which involves propagating the effect of the input outward toward the boundary. With these notions in hand we investigated corresponding pairs of notions of reachability and observability.

Our work this year has built on these earlier results, yielding several papers [2,14,20,21,24] and a recently completed Ph.D. thesis [13], with several additional papers planned for the coming year. One part of our work focused on the class of stationary TPBVDS's -- i.e. systems for which the weighting pattern from u(k) to y(k) is shift-invariant (this is not always true for (3.1)-(3.3), even if E,A,B, and C are constant matrices). For such processes, explicit and comparatively simple expressions can be obtained for reachability and observability conditions, and we have also developed an associated theory characterizing minimal TPBVDS's [20]. When u(k) is a white noise process, one can also analyze the statistics of x(k) and in particular can develop the concept of stochastic stationarity -- i.e. when the correlation of x(k) and x(m) depends only on k-m. This has led us to discover a new class of generalized Lyapunov equations which are also related to a novel notion of stability for TPBVDS's related to asymptotic behavior as the size of the interval of definition grows without bound [21].

We have also investigated estimation problems for TPBVDS's [2,14] and a class of 2-D (i.e. spatial) TPBVDS's [24]. In particular, in [2,14] we introduce a new class of generalized Riccati equations, the solutions to which provide the basis for a generalization of the Mayne-Fraser two-filter algorithm for optimal smoothing.

Our most recent and just completed work [13] contains significant extensions to all of these results. In particular

- We now have an explicit representation of reachability, observability, and minimality for TPBVDS's that need not be stationary.

- We have now solved the deterministic realization problem by introducing a new type of transform generalizing the z-transform in a way that treats zero and infinite eigenmodes in a similar way. This work has led to a new factorization problem, a generalized notion of McMillan degree for stationary TPBVDS's, an algorithm for constructing minimal realizations, and a clearer picture of the more complex nature of minimality conditions for TPBVDS's.

- We have made some progress on the stochastic realization problem. Specifically we have a theory for TPBVDS realizations of processes with rational spectra.

- We have completed a thorough study of the smoothing problem for TPBVDS's. This includes a complete theory for our generalized Riccati equations, including existence and uniqueness conditions and their relation to reachability and observability. In addition, we have been able to provide a precise probabilistic interpretation for the solutions to these equations (which is <u>not</u> obvious in this case), and this in turn has led to a generalization of the Rauch-Tung-Striebel smoothing algorithm.

One major reason for studying singular systems is that descriptions of interconnected systems typically combine dynamic and algebraic (or static) constraints, and therefore appear naturally in singular form. Reduction to regular state-space form is often the simplest way to proceed in the case of small or moderate size singular descriptions. Singular descriptions of large interconnected systems, on the other hand, will typically have sparse structure that needs to be respected and exploited; reduction to regular state-space form destroys sparsity. We have developed some new results in this direction, as outlined next.

The specific task that we address is the computation of selected modes of a continuous time singular system of the form

$$E\dot{x}(t) = Ax(t) \tag{3.4}$$

where E and A are square $N \times N$ matrices. An important application occurs in the context of small-signal stability analysis for power systems. If there are g generators and b buses in the network, then N may be on the order of $b + 12g$ if the generators are represented by 12th-order dynamic models and the loads are taken to be static, yielding an N that could be a few thousands!

The corresponding model (3.4) is very sparse (and in fact was a major stimulus for the original development of sparse matrix methods). The system (3.4) in this case will have 12g, or several hundred, finite eigenvalues. Typically only g oscillatory modes need to be accurately computed, however. These modes are the least damped ones, rather than the slowest or the fastest ones considered in singular perturbations. Application of a full generalized eigenanalysis algorithm such as the standard QZ algorithm (even preceded by Van Dooren's preparatory "pencil deflation") would be prohibitively expensive.

An approach to this problem, termed Selective Modal Analysis (SMA), has been developed for regular state-space descriptions in work of Perez-Arriaga, Verghese and co-workers[1], and applied with considerable success to power systems and other settings. One application that is currently being considered is that of large Markov models, where the dominant and subdominant modes are of interest. We have made significant progress on extending SMA to singular systems in the past few months,[27].

A key quantity in SMA for regular state-space descriptions is what we have termed the participation factor of the kth state variable in a given mode. To see the form of the extension to singular systems, let λ denote the frequency of a mode of (3.4), and let $v \neq 0$ and $w \neq 0$ denote the associated right and left (generalized) eigenvectors, so

$$(\lambda E - A)v = 0, \quad w^T(\lambda E - A) = 0 \qquad (3.5)$$

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[1] I.J. Perez-Arriaga, G.C. Verghese, F.L. Pagola, J.L. Sancha and F.C. Schweppe, "Developments in Selective Modal Analysis of small-signal stability in electric power systems," to appear in *Automatica*.

We shall assume that the eigenvectors are normalized so that $w^T E v = 1$. Now the participation factor p_k of the kth component of x in the given mode is defined to be $p_k = w^T E_k v_k$, where E_k is the kth column of E and v_k is the kth component of v.

Participation factors are independent of the units chosen for the components of x, and sum to 1 for a given mode. There are several reasons for considering the kth participation factor to be a measure of how important it is to retain x_k in any reduced order model for computing the selected mode. The definition above specializes to the one used in the regular case, where $E = I$.

A generalization of the prototype SMA algorithm is then as follows. First re-order the components of x into two vectors, an n-vector r containing those components of x that are deemed "significant" for computing the mode λ, and an (N - n)-vector z containing the remaining, less significant components. (For the power system models that we have experience with, n is only of the order of 2g to 3g, where g is as defined earlier.) The participation factors defined above turn out to be exactly the measures of significance needed for the algorithm below, as will be noted, so approximate knowledge of participation factors provides a good guide for any required re-ordering. Now re-order and partition the description (3.4) to conform with the partitioning of x:

$$(E_r \quad E) \begin{matrix} \dot{r} \\ \dot{z} \end{matrix} = (A_r \quad A_z) \begin{matrix} r \\ z \end{matrix} \qquad (3.6)$$

To avoid excessive notation, we shall assume that no ordering was actually needed, so that r comprises the first n components of x.

The basic philosophy of SMA is to make approximations in the less significant part of the model in order to reduce the computational burden. Suppose at the jth iteration of our algorithm we have an estimate λ^j of λ, and an estimate

$$\begin{matrix} v_r^j \\ v_z^j \end{matrix} \quad \text{to} \quad \begin{matrix} v_r \\ v_z \end{matrix} \tag{3.7}$$

We then find our next (and presumably improved) estimates λ^{j+1} and v_r^{j+1} by solving the following reduced-order N × (n + 1) generalized eigenvlaue problem from (3.6):

$$((\lambda^{j+1}E_r - A_r)\ ,\ (\lambda^j E_z - A_z)v_z^j) \begin{matrix} v_r^{j+1} \\ 1 \end{matrix} = 0 \tag{3.8}$$

It is expected that sparse matrix methods can be used to advantage here, but this remains to be studied. To complete the iteration cycle, we find v_z^{j+1} by solving the following simple N × (N − n) linear system of equations derived again from (3.6):

$$(\lambda^{j+1}E_z - A_z)v_z^{j+1} = -(\lambda^{j+1}E_r - A_r)v_r^{j+1} \tag{3.9}$$

We expect once more that sparse matrix methods can be used here.

Our preliminary analysis shows that the above iteration converges locally to λ with a linear convergence rate of $-(w^T E_r v_r)/(w^T E_z v_z)$. This is just the (negative of the) ratio of the sum of participation factors of the r variables to the sum of participation factors of the z variables. In other words, for the above reduced-order iteration to converge locally, the variables with large participation in the selected mode must be retained in the r variables. Once again, all these results specialize as expected in the regular case.

The existing SMA algorithms for the regular state-space case can iterate on several selected modes simultaneously. The corresponding extension to the singlular system case remains to be worked out. We also expect to pursue the application to singular perturbations for singular systems. If λ^j in (3.8) is set to 0 at each iteration, we obtain an iteration in the spirit of singular perturbations to compute the slow modes of the system. Further study of this, and comparison with SMA, is needed.

IV. Systems Subject to Discrete Events

Our work in this area has had two parts. The first part was concerned with the completion of our study of stochastic Petri net models for cardiac arrhythmias [12]. The second part, on which we focus here, consisted of the initiation of an investigation of a class of discrete-event dynamic systems. The thesis proposal [17] outlines a number of research problems on which we are working and on which we now have some results.

The specific class of models we are considereing is essentially the same as that introduced and investigated by Wonham, Ramadge, Varaiya, and others. Specifically our model is a finite-state automaton

$$A = (X, \Sigma, U, Y, f, g)$$

Here X is the state-set, Σ is the input alphabet, and the state transition function f maps a subset of $X \times \Sigma$ into X -- the fact that it is a <u>subset</u> of $X \times \Sigma$ corresponds to the fact that we allow the possibility that not all input values can be applied at every state $x \in X$. The set $U \subset \Sigma$ corresponds to the set of <u>controllable</u> events. In the original theory and in most of our work control corresponds to disabling certain of the controllable events -- i.e. preventing them from occurring. One can also imagine control in which an event in U is forced to occur. We have also considered two types of outputs. One is state output, i.e. $g: X \longrightarrow Y$, while the other is partial event observations, i.e. $g: \Sigma^* \longrightarrow Y^*$, where S^* denotes the set of strings of elements of S. We assume in our work that g is defined in a memoryless fashion -- i.e. g maps individual symbols to individual symbols -- but that some events in Σ may be unobserved, i.e. for some $\sigma \in \Sigma$, $g(\sigma)$ may be the empty string.

Our work has had as its central motivation the development of a regulator theory for discrete - event systems. This ultimate goal has led us to consider a number of more basic problems:

- Stability and Stabilization. The notion of a system recovering from an error is essential in defining any meaningful concept of regulation. We have developed such a notion and have also developed a procedure for designing stabilizing controllers using state feedback.

- Observability and Observers. In this case we have focused on the partial event sequence observational model. This leads to a weaker notion of observability. Since some events are not observed, it is only possible to determine the state every once in a while.

- Tracking. Here we consider the problem of tracking a specified event sequence or reconstructing an event sequence based on the output sequence. We have focused our attention so far on the latter problem. This has led us to a theory of invertible languages and a concept of error recovery that we refer to as resiliency.

- Computational Complexity and Composite Systems. Most discrete-event systems are made up of interconnections of several or many component systems. Thus it is of interest to develop both a theory of composition of such systems and the specialization of our other results and concepts (such as the test for stability) to such composites. In particular, it is important to determine algorithms that take advantage of this structure in minimizing the required computations.

- Higher-Level Aggregation. We have begun to develop a theory of "task-level descriptions" for discrete event systems. Here a task consists of a sequence of events. A problem we have considered is the problem of designing a controller so that ony tasks -- i.e. only certain sets of event sequences -- are executed. Once such a controller is implemented, one can consider defining a higher-level description in which an entire sequence (corresponding to a task) is mapped into a single event in the aggregate model.

At this point we have some results in each of these areas and expect considerable progress during the coming year.

A rather different context for studying systems subject to discrete events is provided by switched power electronic circuits. These sytems involve both discrete events (switch transitions) and continuously evolving quantities (voltages and currents in the circuit). There is considerable interest in obtaining approximate continuous-time models that average out the effects of the discrete events. Such averaged models yield far more insight than the detailed switched models can, and also provide a more fruitful starting point for control design. One of the obstacles to obtaining such averaged models, however, is the fact that the switching is often state dependent.

Circuits in the most popular class for which an averaging procedure exists, so-called switched converters, switch between linear networks at frequencies much higher than the natural frequencies of any of the individual networks. In the limit of infinite switching frequency, it is well known in control theory that the average behavior is obtained by directly averaging the dynamics in the individual switch configurations. A particularly interesting case is when the behavior of the converter corresponds to a sliding mode.

A more challenging modeling problem is posed by what are termed resonant converters. These comprise resonant circuits that are periodically driven by a switched waveform at an off-resonance frequency. The response is controlled by adjusting the switching frequency, which determines how close to resonance the system operates. In many practically important circuits, the resonant circuit is actually nonlinear, because of the presence of a diode bridge: The bridge reverses the polarity of the load seen by the circuit as a function of the polarity of the current through the circuit. A typical model is the following (where x may be thought of as the voltage across the capacitor of a tuned series LC pair, and $\dot{x}$ as the current through the series inductor):

$$x(t) + \omega_0^2 x(t) = \omega_0^2[V_1 . \mathrm{sign}(\sin \omega t) - V_2 . \mathrm{sign}(\dot{x}(t))] \qquad (4.1)$$

The control input here is the switching frequency ω, and we are interested in such quantitiies as the amplititude of $\dot{x}(t)$ and the average value of $|\dot{x}(t)|$.

One obstacle to averaging (4.1) using established results[2] is the $\mathrm{sign}(\dot{x}(t))$ term, which does not satisfy a Lipschitz condition, so a routine application of existing results is ruled out. However, this particular discontinuity is actually not troublesome because the system trajectories pass through the line $\dot{x}(t) = 0$ in the phase plane. We are presently examining methods for distinguishing between "troublesome" discontinuities (which may nevertheless have well defined sliding solutions, in the Fillipov sense) and harmless discontinuities.

The standard approach to studying (4.1) involves transforming variables to $r(t)$ and $\psi(t)$, defined by $x(t) = r \cos(\omega_0 t + \psi)$ and $\dot{x}(t) = -r\omega_0 \sin(\omega_0 t + \psi)$, and then averaging the resulting system under the assumption that V_1 and V_2 are small. We are studying an alternate and related route, [28], but one that seems more direct. For this, we define the running average

$$x(t)_T = (1/T) \int_{t-T}^{t} x(\sigma) e^{2\pi j\sigma/T} d\sigma \qquad (4.2)$$

[2] J.A. Sanders and F. Verhulst, *Averaging Methods in Nonlinear Dynamical Systems*, Springer-Verlag, New York 1985.

where $T = 2\pi/\omega$. It is now easy to see that if ω is constant then

$$dx(t)/dt_{\,T} = d\,x(t)_{\,T}/dt - j\omega\, x(t)_{\,T} \tag{4.3}$$

Averaging (4.1) and using (4.3) under the assumption of constant ω then yields

$$[p^2 - 2j\omega p + (\omega_0^2 - \omega^2)]\; x(t)_{\,T} = c \tag{4.4}$$

where $p = d/dt$, and c is constant if $V_2 = 0$. Furthermore, c is nearly constant if $\dot{x}$ does not differ significantly from its steady-state waveform and/or if V_2 is small.

It follows that the dynamics of the resonant converter described by (4.1), after a small step change in ω, can be deduced from the characteristic polynomial associated with the left hand side of (4.4). The roots of this polynomial are at $j(\omega \pm \omega_0)$, which correlates well with the observed ringing of the envelope of $\dot{x}$ at the difference frequency $\omega - \omega_0$ in the resonant converter. Prior to this analysis, only a more complicated and unilluminating sampled-data analysis had been presented in the literature.

There are several questions that remain about the analysis of even this simple example. We intend to further compare the approach via $x_{\,T}$ to the more traditional approach, and to extend our results all the way to the small-signal transfer functions that are needed for analytical design of a feedback controller for such resonant converters. It is expected that this will yield valuable insight for more general averaging problems involving switched systems.

PUBLICATIONS

The publications listed below represent papers, reports, and theses supported in whole or in part by the Air Force Office of Scientific Research under Grant AFOSR-88-0032:

(1) J.R. Rohlicek and A.S. Willsky, "The Reduction of Perturbed Markov Generators: an Algorithm Exposing the Role of Transient States", Journal of the ACM, Vol. 35, No. 3, July 1988, pp. 675-696.

(2) R. Nikoukhah, M.B. Adams, A.S. Willsky, and B.C. Levy, "Estimation for Boundary-Value Descriptor Systems", to appear in Circuits, Systems, and Signal Processing.

(3) X.-C. Lou, G.C. Verghese, and A.S. Willsky, and P.G. Coxson, "Conditions for Scale-Based Decompositions in Singularly Perturbed Systems," Linear Algebra and Its Applications.

(4) H.J. Chizeck, A.S. Willsky, and D.A. Castanon, "Optimal Control of Systems Subject to Failures with State-Dependent Rates", in preparation for submission to International J. of Control.

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