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Report Title

Modeling Complex Nonlinear Optical Systems

ABSTRACT

This research dealt with the modeling of light propagation in nonlinear periodic media including bragg grating fiber arrays and periodic nonlinear
2-dimensional waveguides. The goals set were to find conditions for stable pulse propagation in the arrays and for the search of light bullets, their stability and propagation characteristics in the two dimensional waveguide. We also established conditions for optical trapping in a defect. This is a topic of great interest in the search of all optical logic systems and buffers. A second component of the project dealt with existence and stability of Bose Einstein condensates in periodic magnetic traps. There has been an extensive experimental effort on BEC trapping and our work developed a solid theoretical framework to explore such trapping mechanisms. Tools used in this research include: Dynamical systems, numerical methods for nonlinear partial differential equations, asymtotic analysis. The project had also an important educational component as it served to train 3 graduate students in Applied Mathematics and provided a seed for a new crop of students working in this field.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

T. Dohnal, A. Aceves (2005),
``Optical soliton bullets in (2+1)D nonlinear bragg resonant periodic
geometries", Studies in Applied Mathematics, {\bf 115} 209-232.

T. Dohnal and A. B. Aceves (2006) " Finite dimensional
model for defect-trapped light in planar periodic nonlinear structures", Optics Letters {\bf 31}, 3013-3015.

T. Dohnal and T. Hagstrom (2006), ``Perfectly matched layers in photonics computations: 1D and 2D Nonlinear Coupled Mode Equations," accepted to J. Comput. Phys.

T. Kapitula, B. Sandstede (2004), "Eigenvalues and resonances using the Evans function", Discrete and Continuous Dynamical Systems, Vol. 10, No. 4, 857-869.

T. Kapitula, B. Sandstede and J. Kutz (2004), "The Evans function for nonlocal equations",Indiana U. Math. J., Vol. 53, No. 4, 1095-1126.

T. Kapitula and P. Kevrekidis (2004), "Linear stability of perturbed Hamiltonian systems: theory and a case example", J. Phys. A: Math. Gen., Vol. 37, No. 30, 7509-7526.

T. Kapitula, P. Kevrekidis and B. Sandstede (2004), "Counting eigenvalues via the Krein signature in infinite-dimensional Hamiltonian systems" Physica D, Vol. 195, No. 3\&4, 263-282.
(Addendum: Counting eigenvalues via the Krein signature in infinite-dimensional Hamiltonian systems", Physica D, Vol. 201, No. 1\&2, 199-201 (2005).

T. Kapitula (2005), "Stability analysis of pulses via the Evans function: dissipative systems", Lecture Notes in Physics, Vol. 661, 407-427.

T. Kapitula and P. Kevrekidis (2005), "Bose-Einstein condensates in the presence of a magnetic trap and optical lattice: two-mode approximation", Nonlinearity, Vol. 18, No. 6, 2491-2512.

T. Kapitula and P. Kevrekidis (2005), "Bose-Einstein condensates in the presence of a magnetic trap and optical lattice", Chaos Vol. 15, No. 3, 037114.

Number of Papers published in peer-reviewed journals: 10.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

A.B. Aceves and T. Dohnal, ``Stopping and bending light in 2D photonic structures." ``Nonlinear Waves: Classical and Quantum Effects," p. 293 - 302, F. Kh. Abdullaev and V.V. Konotop (eds.), Kluwer, (2004).

A.B. Aceves and T. Dohnal, ``Stopping and bending light in 2D photonic structures." Proceedings of OSA topical meeting on Nonlinear Guided Waves and their Applications, Toronto, March (2004).

Number of Papers published in non peer-reviewed journals: 2.00

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

(d) Manuscripts

"Three is a crowd: Solitary waves in photorefractive media with three potential wells", T. Kapitula, P. Kevrekidis and Z. Chen, to appear in SIAM J. Appl. Dyn. Sys.

Number of Manuscripts: 0.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED	
Tomas Dohnal	1.00	No
Karl Frinkle	0.75	No
Bobbi Page	0.25	No
FTE Equivalent:	**2.00**	
Total Number:	**3**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Alejandro Aceves	0.50	No
Todd Kapitula	0.50	No
FTE Equivalent:	**1.00**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Personnel receiving masters degrees

NAME	
Bobbi Page	No
Total Number:	**1**

Names of personnel receiving PHDs

NAME	
Tomas Dohnal	No
Karl Frinkle	No
Total Number:	**2**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Modeling Complex Nonlinear Optical Systems

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Statement of the problem: Study the dynamics of pulses in photonic structures.

What is a photonic structure ?

- It is an "engineered" optical medium with periodic properties.
- Photonic structures are built to manipulate light (slow light, light localization, trap light)
- I'll show some examples, but this talk will concentrate on 2-dim periodic waveguides

Motivation of study

We take advantage of periodic structures and nonlinear effects to propose new stable and robust systems relevant to optical systems.

- Periodic structure - material with a periodically varying index of refraction ("grating")

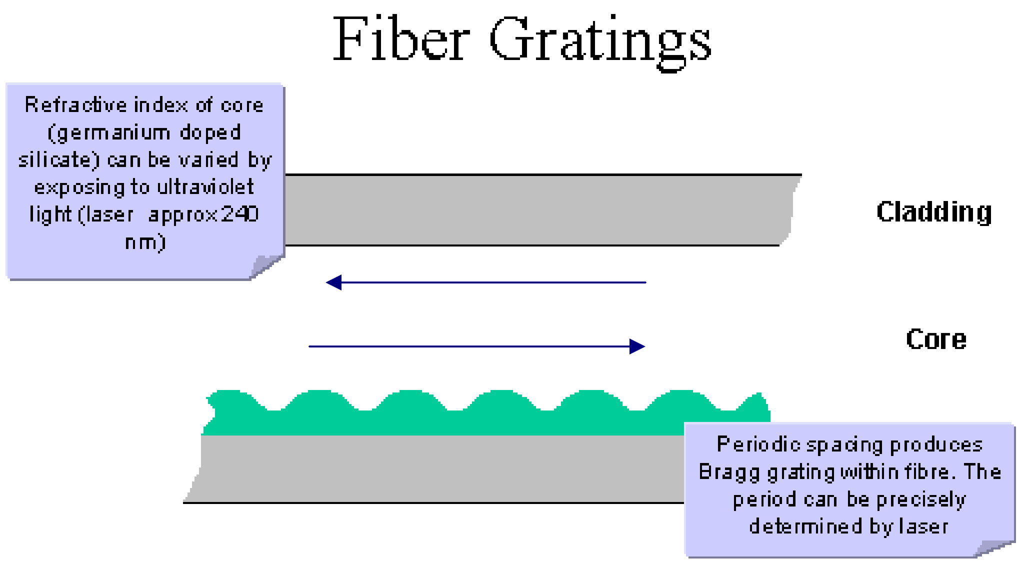
- Nonlinearity - dependence of the refractive index on the intensity of the electric field (Kerr nonlinearity)

The effects we seek:
• existence of stable localized solutions – solitary waves, solitons
• short formation lengths of these stable pulses
• possibility to control the pulses – speed, direction (2D, 3D)

Prospective applications:
• rerouting of pulses
• optical memory
• low-loss bending of light

The University of New Mexico

1d strucutres: Optical fiber gratings



$$E_f(Z,T) \quad \text{(forwardmovingenvelope)}$$

$$E_b(Z,T) \quad \text{(backwardmovingenvelope)}$$

Equations studied

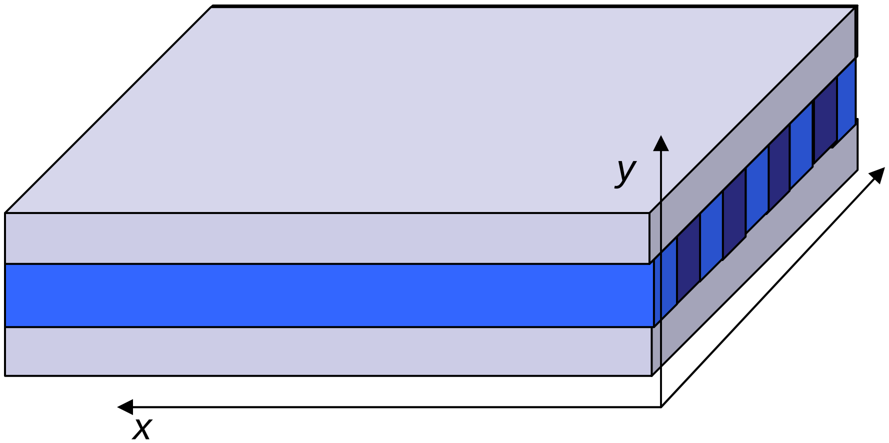
1-dim Coupled Mode Equations

$$\partial_t E_+ = -c_g \partial_z E_+ + i\kappa E_- + i\Gamma(|E_+|^2 + 2|E_-|^2)E_+$$

$$\partial_t E_- = c_g \partial_z E_- + i\kappa E_+ + i\Gamma(|E_-|^2 + 2|E_+|^2)E_-$$

Gap solitons exist. Velocity proportional to amplitude mismatch
Between forward and backward envelopes.

II. 2D structures:0 Waveguide gratings



Assumptions:

- dynamics in *y* arrested by a fixed *n*(*y*) profile
- *xy*-normal incidence of pulses
- characteristic length scales of coupling, nonlinearity and diffraction are in balance

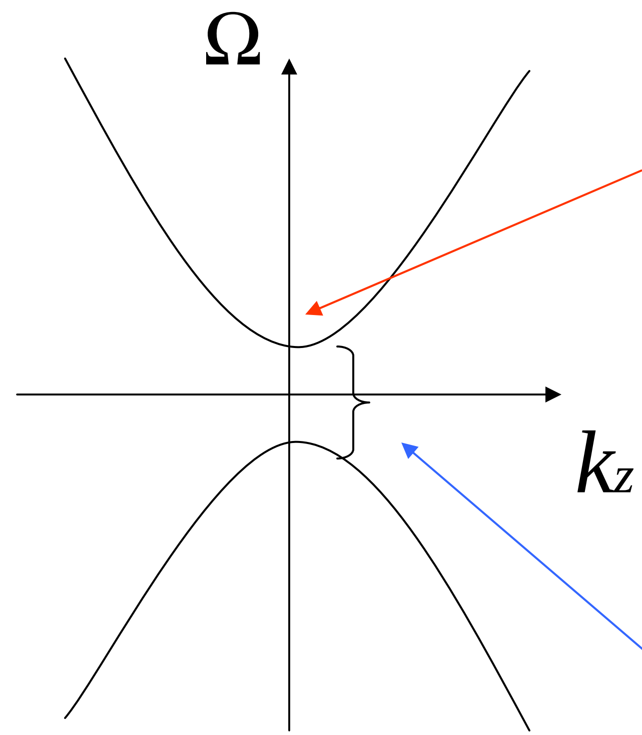
BARE 2D WAVEGUIDE

- 2D NL Schrödinger equation
- collapse phenomena: point blow-up

WAVEGUIDE GRATING

- 2D CME
- no collapse
- possibility of localization

Dispersion relation for coupled mode equations



Close, but outside the gap
Well approximated by the
2D NLSE + higher order
corrections.
Collapse arrest shown by
Fibich, Ilan, A.A (2002).
But dynamics is unstable

Frequency gap region.
We will study dynamics in this
regime.

Governing equations : 2D Coupled Mode Equations

$$\partial_t E_+ = -c_g \partial_z E_+ + id\partial_{x^2} E_+ + i\kappa E_- + i\Gamma(|E_+|^2 + 2|E_-|^2)E_+$$

$$\partial_t E_- = c_g \partial_z E_- + id\partial_{x^2} E_- + i\kappa E_+ + i\Gamma(|E_-|^2 + 2|E_+|^2)E_-$$

advection | diffraction | coupling | non-linearity

$$c_g, d, \kappa, \Gamma \geq 0, \quad E_\pm : [-L_x, L_x] \times [-L_z, L_z] \times [0, \infty) \to C.$$

Summary of results (details to follow on next slides)

- **Found stationary solutions of the governing equations of the 2d structure (see next slide)**
- **Obtained conditions for bullet propagation in such structures**
- **Obtained conditions for light trapping at a defect by a resonance mechanism between the incident optical bullets and defect modes**
- **Derived a finite dimensional dynamical system to study the dynamics inside the trap**

Stationary solutions via Newton's iteration

If $E_\pm(x,z,t) = \mathcal{E}_\pm(x,z)\,\mathrm{e}^{-i\omega t}$ then

$$\begin{aligned} \omega\mathcal{E}_+ + ic_g\partial_z\mathcal{E}_+ + \partial_x^2\mathcal{E}_+ + \kappa\mathcal{E}_- + \Gamma(|\mathcal{E}_+|^2 + 2|\mathcal{E}_-|^2)\mathcal{E}_+ = 0, \\ \omega\mathcal{E}_- - ic_g\partial_z\mathcal{E}_- + \partial_x^2\mathcal{E}_- + \kappa\mathcal{E}_+ + \Gamma(|\mathcal{E}_-|^2 + 2|\mathcal{E}_+|^2)\mathcal{E}_- = 0. \end{aligned} \tag{1}$$

Solve (1) as a **NL eigenvalue problem** for $\left(\omega, \begin{pmatrix}\mathcal{E}_+ \\ \mathcal{E}_-\end{pmatrix}\right)$ via Newton's iteration.

Need one more equation:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\mathcal{E}_+|^2 + |\mathcal{E}_-|^2 dz dx = N.$$

Initial guess: $\left(\omega^{(0)}, \mathcal{E}_\pm^{(0)}(x,z)\right)$

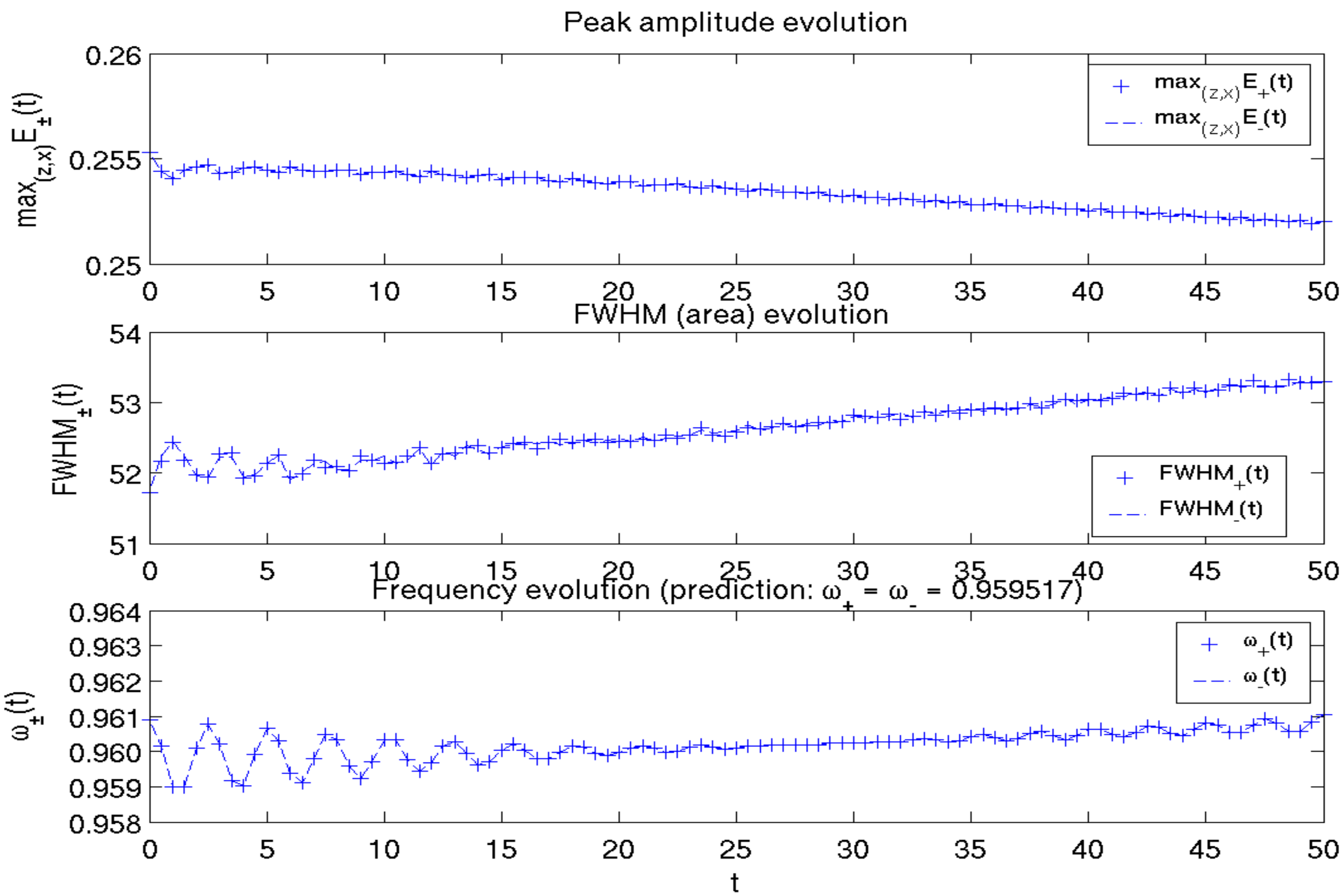
separable waveform $\mathcal{E}_\pm^{(0)}(x,z) = \mathcal{F}_\pm(z)G(x)$, $\omega^{(0)} = \kappa\cos(\delta)$
where $\mathcal{F}_\pm(z)\,\mathrm{e}^{-i\omega^{(0)}t}$ is the 1D gap soliton with $v = 0$ (free parameter $\delta \in (0,\pi)$)

Substitute and integrate in z:

$$G'' + b(G^3 - G) = 0, \qquad b = 2\frac{\kappa}{\delta}(\sin(\delta) - \delta\cos(\delta)), \qquad \delta \in (0, \pi/2)$$

$$\Rightarrow G(x) = \sqrt{2}\,\mathrm{sech}(\sqrt{b}x)$$

Stationary case I



Nonexistence of minima of the Hamiltonian

Theorem 1. *The Hamiltonian functional of 2D CME has no minima constrained to a fixed total power.*

Proof: Suppose $\exists(\mathcal{E}_+(x,z), \mathcal{E}_-(x,z))$ s.t. $H(\mathcal{E}_+, \mathcal{E}_-) = \min_S H$, where

$$S = \{(f_1(x,z), f_2(x,z)) \text{ s.t. } f_{1,2} : \mathbb{R}^2 \to \mathbb{C},\ \sum_{k=1}^{2} \|f_k\|_2^2 = \|\mathcal{E}_+\|_2^2 + \|\mathcal{E}_-\|_2^2\}$$

Consider

$$S_1 = \{(\tilde{\mathcal{E}}_+, \tilde{\mathcal{E}}_-) : \tilde{\mathcal{E}}_\pm = \alpha\mathcal{E}_\pm(x/\mu, z/\nu) \text{ and } \alpha, \mu, \nu > 0, \alpha^2\mu\nu = 1\}.$$

Clearly $S_1 \subset S$ and $(\mathcal{E}_+, \mathcal{E}_-) \in S_1$.

Within S_1 the Hamiltonian $H = H_r = A_1\frac{1}{\nu} + A_3\alpha^4\nu^2 - A_4\alpha^2 - A_2$ with

$$A_1 = ic_g \int_{\mathbb{R}^2} \mathcal{E}_-^* \partial_z \mathcal{E}_- - \mathcal{E}_+^* \partial_z \mathcal{E}_+ dxdz,\ \ A_2 = \kappa \int_{\mathbb{R}^2} \mathcal{E}_- \mathcal{E}_+^* + \mathcal{E}_-^* \mathcal{E}_+ dxdz,$$
$$A_3 = \int_{\mathbb{R}^2} |\partial_x \mathcal{E}_+|^2 + |\partial_x \mathcal{E}_-|^2 dxdz,\ \ A_4 = \frac{\Gamma}{2} \int_{\mathbb{R}^2} |\mathcal{E}_+|^4 + 4|\mathcal{E}_+|^2|\mathcal{E}_-|^2 + |\mathcal{E}_-|^4 dxdz$$

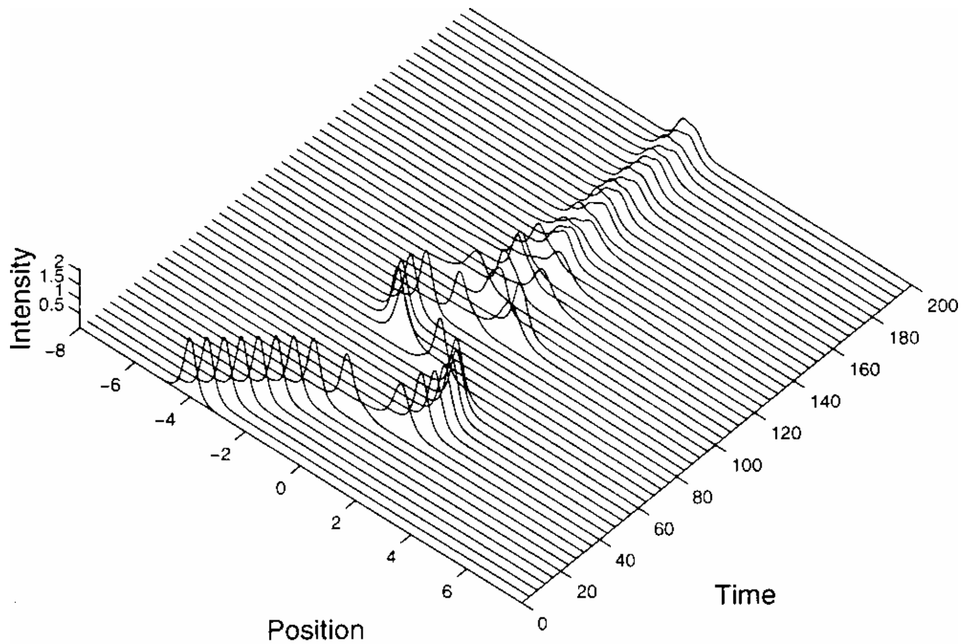
$A_1, A_2 \in \mathbb{R}$ and $A_3, A_4 > 0$. The only C.P. of H_r is

$$(\alpha^*, \nu^*) = \left(\frac{A_1\sqrt{2A_3}}{A_4^{3/2}}, \frac{A_4^2}{2A_1A_3}\right)$$

and by the 2nd derivative test (α^*, ν^*) is a **saddle**! $\square$

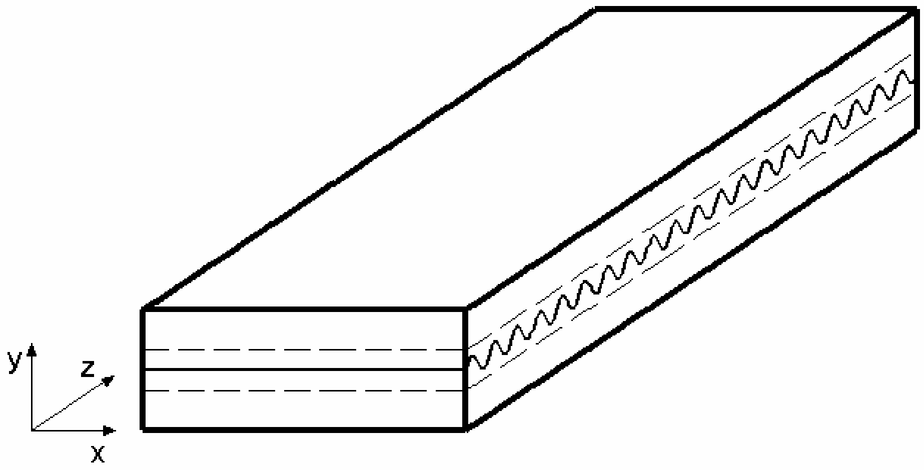
Defects in 1d gratings

- Trapping of a gap soliton bullet in a defect. (this concept has been theoretically demonstrated by Goodman, Weinstein, Slusher for the 1-dim (fiber Bragg grating with a defect) case).



Ref: R. Goodman et.al., JOSA B 19, 1635 (July 2002)

Governing equations : 2D CME



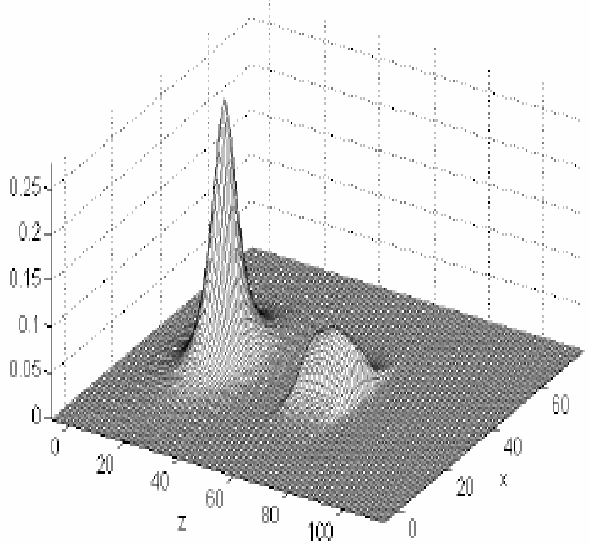
$$\partial_t E_+ = -\partial_z E_+ + i\partial_{x^2} E_+ + i\kappa(x,z)E_- + V(x,z)E_+ + i\Gamma(|E_+|^2 + 2|E_-|^2)E_+$$

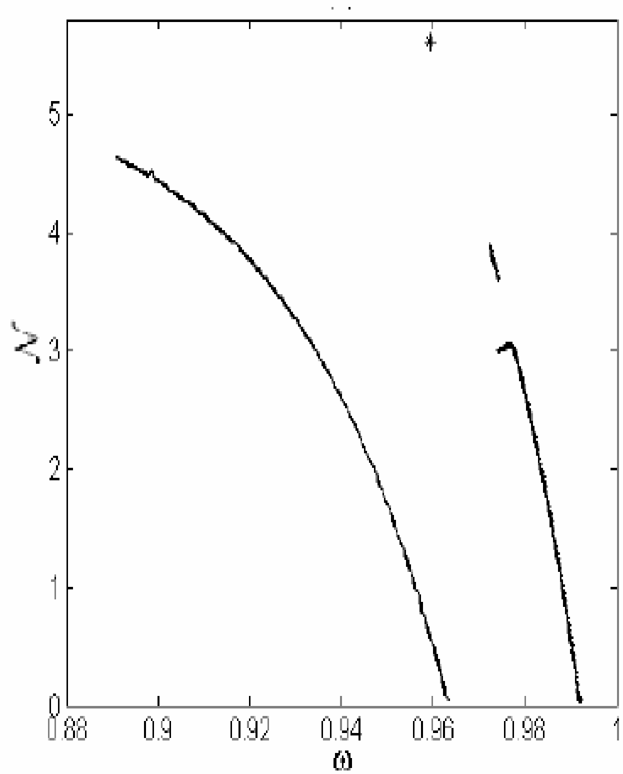
$$\partial_t E_- = \partial_z E_- + i\partial_{x^2} E_- + i\kappa(x,z)E_+ + V(x,z)E_- + i\Gamma(|E_-|^2 + 2|E_+|^2)E_-$$

advection | diffraction | coupling | defect | non-linearity

$$\kappa, \Gamma \geq 0, \quad E_\pm : [-L_x, L_x] \times [-L_z, L_z] \times [0, \infty) \to C.$$

2-D version of resonant trapping





left: GS modulus

right: defect potential $V(x, z)$

$(-)$ Bifurcation curves for the NL defect mod

$(*)$ stationary GS with $\omega_0 \approx 0.96$

3 trapping cases (GS: ω(v=0) ≈ 0.96)

1. Trapping into two defect modes

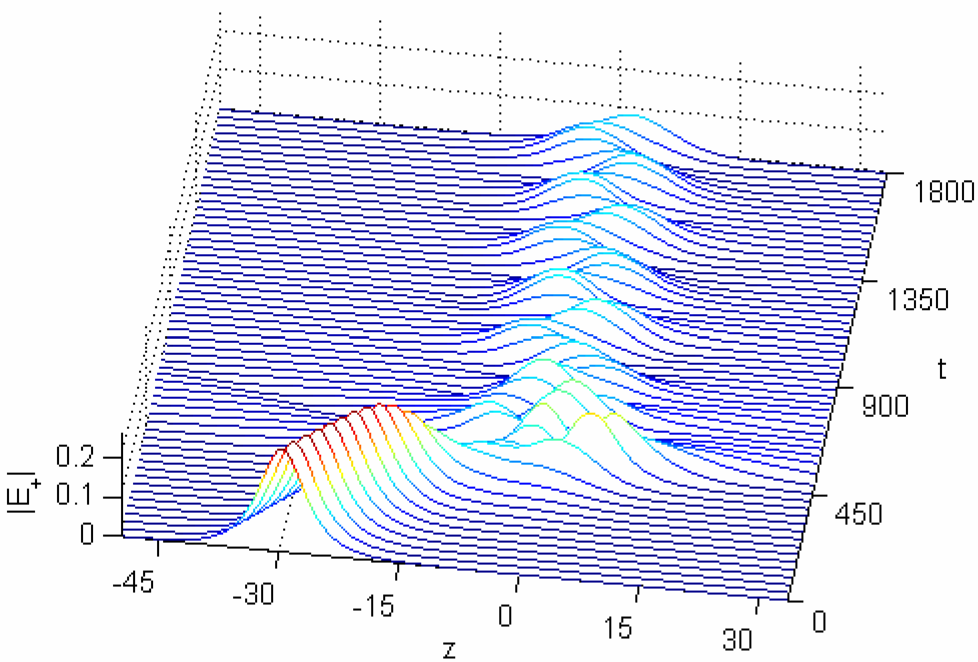
$$V = V_1(x)T(z;9) + V_2(z)T(x;7), \quad \kappa = \sqrt{1+k^2(\tanh^2(kz)-1)},$$

$$\text{where } V_1 = 2\beta^2 \text{sech}^2(\beta x), \quad V_2 = \frac{1}{2}\frac{k^2\sqrt{1-k^2}\,\text{sech}^2(kz)}{1+k^2(\tanh^2(kz)-1)} \quad \text{and} \quad T(y;c) = \frac{1}{2}(\tanh(y+c)-\tanh(y-c))$$

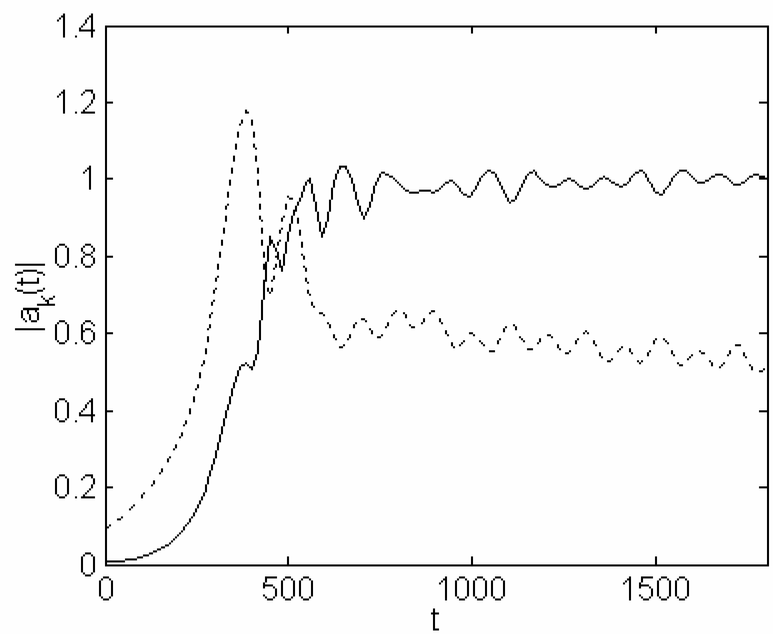
$$\text{with} \quad k = 0.18 \text{ and } \beta = 0.16$$

$$\Rightarrow \quad 2 \text{ linear defect modes}: \omega_L \approx 0.963,\ 0.992$$

$$|E_+(x=0,z,t)|$$



$$|a_k(t)|$$



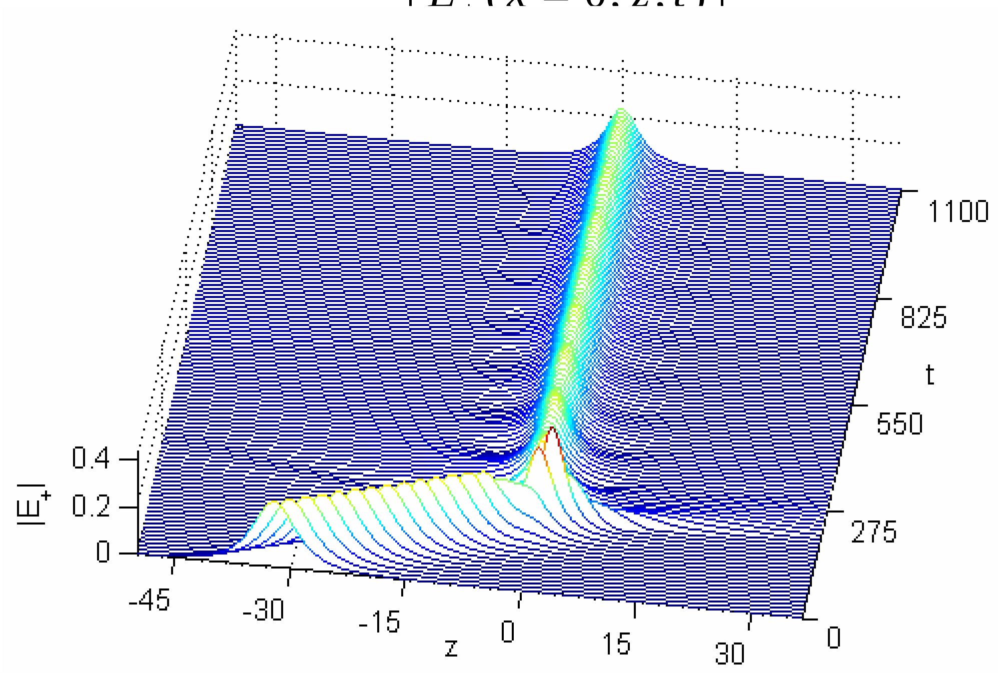
3 trapping cases (GS: $\omega(v=0) \approx 0.96$)

2. Trapping into one defect mode

$$V = 0.3e^{-(ax^2+bz^2)}, \quad \kappa = 1 + 0.1e^{-(ax^2+bz^2)}, a = 0.25, b = 0.3$$

$$\Rightarrow \quad 1 \text{ linear defect mode : } \omega_L \approx 0.995$$

$$|E_+(x=0,z,t)|$$



3 trapping cases (GS: ω(v=0) ≈ 0.96)

3. No trapping

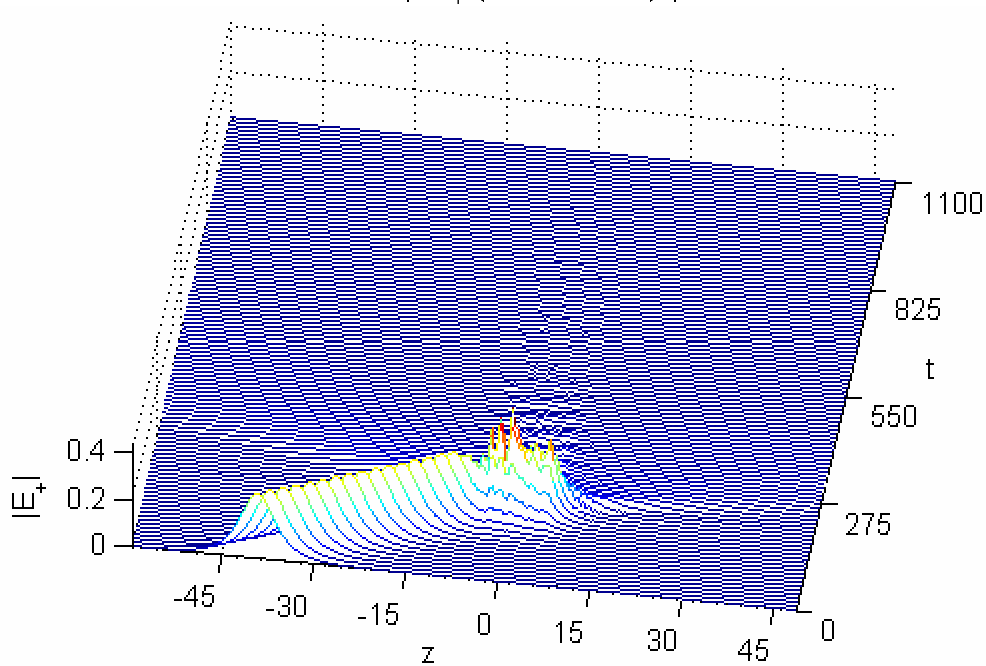
$$V = V_1(x)T(z;5) + V_2(z)T(x;5), \quad \kappa = \sqrt{1 + k^2(\tanh^2(kz) - 1)},$$

where $V_1 = 2\beta^2 \mathrm{sech}^2(\beta x)$, $V_2 = \dfrac{1}{2}\dfrac{k^2\sqrt{1-k^2}\,\mathrm{sech}^2(kz)}{1+k^2(\tanh^2(kz)-1)}$ and $T(y;c) = \dfrac{1}{2}(\tanh(y+c) - \tanh(y-c))$

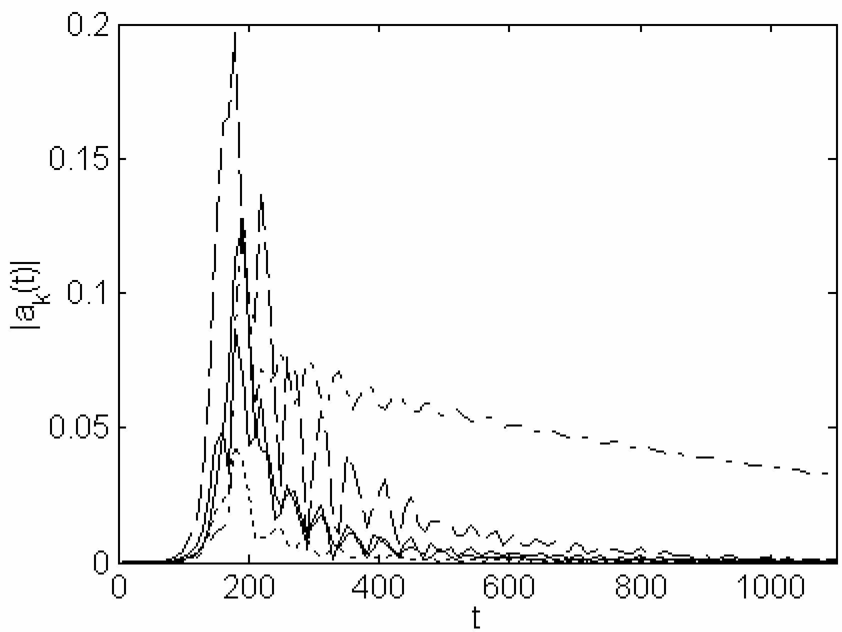
with $k = 0.85$ and $\beta = 1$

$\Rightarrow$ 5 linear defect modes: $\omega_L \approx$ -0.47, 0.26, 0.47, 0.68 and 0.87 ← ALL FAR FROM RESONANCE

$|E_+(x=0,z,t)|$



$|a_k(t)|$



Finite dimensional approximation of the trapped dynamics

For trapped solutions with small amplitude:

$$\begin{pmatrix} E_+(x,z,t) \\ E_-(x,z,t) \end{pmatrix} \approx \sum_{k=1}^{N} a_k(t) e^{-i\omega_{L_k} t} \begin{pmatrix} \psi_{+_k}(x,z) \\ \psi_{-_k}(x,z) \end{pmatrix},$$

where $(\psi_{+_k}, \psi_{-_k})^T, k = 1, \ldots N$ are the defect modes.

Substituting into 2D CME with the defect potentials

$$i \sum_{k=1}^{N} a'_k \begin{pmatrix} \psi_{+_k} \\ \psi_{-_k} \end{pmatrix} + \Gamma \begin{pmatrix} (NL)_+ \\ (NL)_- \end{pmatrix} = 0,$$

where $(NL)_\pm = (|E_\pm|^2 + 2|E_\mp|^2)E_\pm$.
Due to orthogonality

$$i a'_k(t) + \Gamma \int (NL)_+ \psi^*_{+_k} + (NL)_- \psi^*_{-_k} dx dz = 0 , \qquad k = 1, \ldots, N.$$

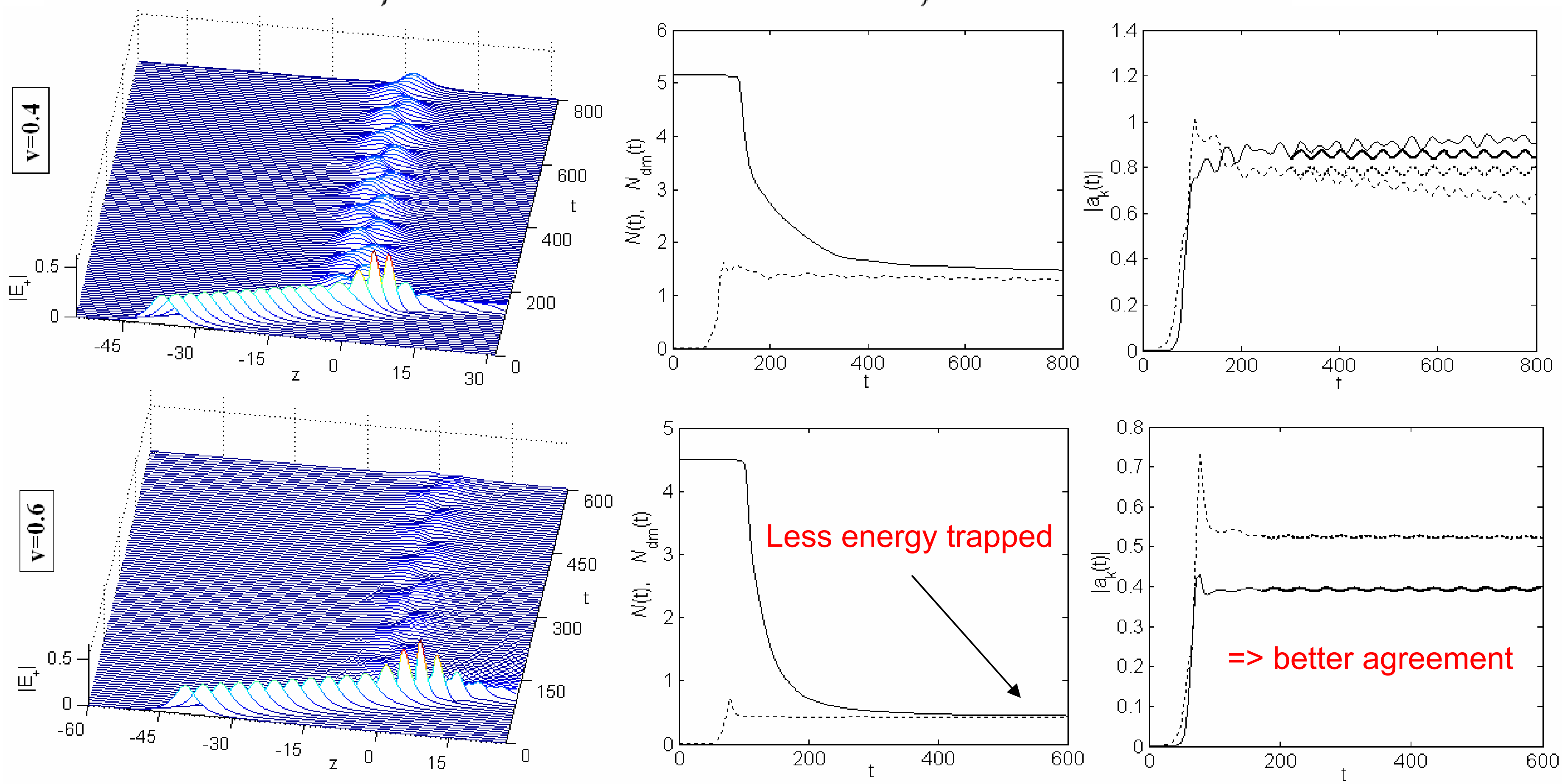
Finite dimensional approximation of the trapped dynamics

For $N = 2$:

after transformation $\tilde{a}_{1,2}(t) = a_{1,2}(t)e^{\mp\frac{i\Delta\omega}{2}t}$, where $\Delta\omega = \omega_{L_1} - \omega_{L_2}$

$$i\tilde{a}_1' - \frac{\Delta\omega}{2}\tilde{a}_1 + \alpha_1|\tilde{a}_1|^2\tilde{a}_1 + \beta|\tilde{a}_2|^2\tilde{a}_1 + \frac{\beta}{2}\tilde{a}_2^2\tilde{a}_1^* = 0$$

$$i\tilde{a}_2' + \frac{\Delta\omega}{2}\tilde{a}_2 + \alpha_2|\tilde{a}_2|^2\tilde{a}_2 + \beta|\tilde{a}_1|^2\tilde{a}_2 + \frac{\beta}{2}\tilde{a}_1^2\tilde{a}_2^* = 0.$$



Conclusions (Part I)

- 2d nonlinear photonic structures show promising properties for quasi-stable propagation of “slow” light-bullets
- With the addition of defects, we presented examples of resonant trapping
- Research in line with other interesting schemes to slow down light for eventually having all optical logic devices (eg. buffers)

Part II: Dynamics in Bose-Einstein Condensates

Statement of the problem:

In the study of the dynamics of matter waves for Bose-Einstein condensates, it is of great importance to understand the scenarios under which solutions such as necklaces, multi-poles, and vortices will exist and persist.

Part 2: Bose Einstein Condensates (summary of most important results)

- Shown that, under suitable assumptions, that N-solitons are stable for a large class of integrable partial differential equations including the equation that govern BEC
- Shown how the presence of an optical lattice along with the magnetic trap can influence the dynamics of matter waves in Bose-Einstein condensates
- Illustrated the manner in which increasing the number of potential wells in photorefractive media effects the dynamics associated with solitary waves
- Begun to develop some ``rules-of-thumb" regarding the existence and stability of waves in two-dimensional Bose-Einstein condensates

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- “Finite-dimensional model for defect-trapped light in planar periodic nonlinear structures”, Alejandro B. Aceves and Tomáš Dohnal Optics Letters, Vol. 31, Issue 20, pp. 3013-3015 (2006).