Collision Avoidance in Single-Channel Ad Hoc Networks Using Directional Antennas *

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Abstract

Three collision-avoidance protocols are analyzed that use omni-directional packet reception together with omni-directional transmissions, directional transmissions, or a combination of both. A simple model is introduced to analyze the performance of these collision avoidance protocols in multi-hop networks with arbitrary topologies. The numerical results of this analysis show that collision avoidance using a narrow antenna beamwidth for the transmission of all control and data packets achieves the highest throughput among the three collision avoidance schemes considered. Simulation experiments of the popular IEEE 802.11 MAC protocol and its variants based on directional transmissions and omni-directional packet reception validate the results predicted in the analysis. The results further show that narrow-beamwidth transmissions can also reduce the average delay experienced by nodes. It is concluded that the advantage of spatial reuse achieved by narrow-beamwidth transmissions outweighs that of conservative collision avoidance schemes featured by the omni-directional transmission of some control packets. This is due to the fact that the latter requires far more stringent coordination of nodes with their neighbors and hidden terminals, which can lead to much more channel resource wasted due to nodes' excessive waiting time.

Keywords

Collision avoidance, MAC, ad hoc networks, IEEE 802.11, directional antennas, spatial reuse

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1 Introduction

Wireless ad-hoc networks have received increasing interest in recent years due to their ease of deployment without the aid of any pre-existing network infrastructure. Designing effective medium access control (MAC) protocols to regulate access to a shared channel among geographically distributed nodes is critical to the overall performance of ad hoc networks, because of the limited channel bandwidth available to such networks.

Because the "hidden terminal" problem [8] can degrade channel throughput dramatically in multi-hop networks, many collision avoidance MAC protocols have been proposed in the recent past. The most popular collision avoidance scheme to date is such that actual data packet transmission and its acknowledgment are preceded by short request-to-send (RTS) and clear-to-send (CTS) packets between a pair of sending and receiving nodes. Other neighboring nodes that overhear the RTS or CTS packets defer their access to the shared channel to avoid collisions. In effect, this collision avoidance scheme depends on the broadcast nature of the channel and assumes that all nodes are equipped with omni-directional antennas. However, this scheme requires that all the neighbors of the sending and receiving nodes back off during the handshake which greatly reduces the possible spatial reuse in multi-hop networks.

Based on the above observations, some MAC protocols that make use of directional antennas have been proposed in the recent past. Ko et al. [4] propose two schemes. One scheme consists of nodes using directional RTS transmissions and omni-directional transmission of CTS packets in collision avoidance, and then using directional transmissions of data and acknowledgment packets after successful handshakes. The other scheme consists of nodes using both directional and omni-directional transmission of RTS packets alternately. When the location of a receiver is not well known or all the transmitting antennas are unblocked,

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an omni-directional RTS is sent. These two schemes show the tradeoff between increased possibility of simultaneous transmissions by neighboring nodes (scheme one) and reduced possibility of collisions of control packets (scheme two).

Nasipuri et al. [5] propose a MAC protocol similar to those summarized above, but use a different model. In the authors' model, each node is equipped with M antennas whose orientations can be maintained all the time irrespective of a node's movement. They also assume that nodes have directional reception capability, that is, they can activate the antenna pointing to the desired source while deactivating antennas in other directions. Therefore, the receiving node is not influenced by simultaneous transmissions from other directions. This is different from the model assumed by Ko et al. [4] where antennas are always active for receiving and thus transmissions to different antennas result in failed reception. In the proposed MAC protocol, omni-directional RTS and CTS packets are first exchanged between a pair of sending and receiving nodes and then directional transmissions of data and acknowledgment packets are used.

Simulation studies of the above proposed protocols show that they improve performance over the existing omni-directional IEEE 802.11 MAC protocol, which is the de facto standard protocol in performance studies of routing protocols for ad hoc networks. However, the studies done to date have used relatively simple network topologies and do not provide sufficient insight on the interaction between spatial reuse and collision avoidance, where the former is obtained with directional transmission while the later requires omni-directional transmission. In this paper, we investigate the interaction between these two conflicting factors by introducing a simple yet general network model from which analytical results can be derived. An important contribution of this model is that it is applicable to many other combinations of directional and omni-directional transmissions in collision avoidance protocols.

In our network model, nodes are randomly placed on a plane according to a two-dimensional Poisson distribution with density λ. Varying λ has the effect of changing the congestion level within a region, as well as the number of hidden terminals. A node is equipped with antennas such that, when directional transmission with a given beamwidth is used, nodes outside the beamwidth will not receive any signal from the node. It is also assumed that time is slotted and each node is ready to transmit independently in each time slot with probability p, where p is a protocol dependent parameter. This model was first used by Takagi and Kleinrock [7] to derive the optimal transmission range of a node in a multi-hop network, and was used subsequently by Wu and Varshney [10] to derive the throughput of non-persistent CSMA and some variants of busy tone multiple access (BTMA) protocols [8]. We used this model to derive the saturation throughput of the RTS/CTS based collision avoidance scheme in multi-hop ad hoc networks in which all transmissions are omni-directional [9]. In this paper, we advance our work to analyze MAC schemes that use directional transmissions.

Section 2 presents the approximate throughput analysis of three MAC schemes that rely on omni-directional packet reception. In the first MAC scheme, all packet transmissions are omni-directional. This scheme corresponds to the usual sender-initiated collision-avoidance scheme commonly in use, which emphasizes making nodes around the receiver back off until the sender terminates its transmission. In the second scheme, all packet transmissions are directional. Obviously, this scheme emphasizes spatial reuse. In the third scheme, RTS packets are transmitted directionally, CTS packets are transmitted omni-directionally, and then data packets and acknowledgment packets are transmitted directionally. This scheme tries to strike a balance between collision avoidance and spatial reuse.

Section 3 presents numerical results derived from the analytical model, and clearly shows that, contrary to the conclusions that can be derived from limited scenarios with regular topologies [4, 5], the benefits of antenna directionality outweigh the advantages derived from trying to eliminate the interference from all hidden terminals by using omni-directional CTS transmissions.

Section 4 presents the simulation results of the popular IEEE 802.11 MAC protocol and its variants obtained by using directional transmissions. Both analytical and simulation results show that the MAC scheme in which all transmissions are directional achieves the best performance among the three schemes analyzed in terms of throughput and delay. Section 5 concludes that aggressive spatial reuse achieved by directional transmissions is much more advantageous than conservative collision avoidance typified by omni-directional transmissions in multi-hop ad hoc networks and proposes some directions for future work.

2 Approximate Analysis

In this section, we analyze the following three MAC schemes:

1. All packet transmissions are omni-directional. We assume that correct collision avoidance is enforced, i.e., once a node starts sending a CTS packet in reply to an RTS destined to it, the following handshake can go on unobstructed.[1] We call this scheme "ORTS-OCTS."

2. All packet transmissions are directional, including

[1]The reader is referred to [3] for a discussion of issues involved in achieving correct collision avoidance.

RTS and CTS packets. This scheme tries to maximize spatial reuse. We call this scheme "DRTS-DCTS."

3. Directional transmissions of RTS packets are used together with omni-directional transmissions of CTS packets. Data packets and acknowledgments are transmitted directionally. We call this scheme "DRTS-OCTS."

As we have stated, we adopt the model used by Takagi and Kleinrock [7], which makes the analysis of a multi-hop network tractable. According to our network model, the nodes are two-dimensionally Poisson distributed with density λ, i.e., the probability $p(i, S)$ of finding i nodes in an area of S is given by:

$$p(i, S) = \frac{(\lambda S)^i}{i!} e^{-\lambda S}.$$

Each node has the same transmission and receiving range of R, and N is the average number of nodes within a circular region of radius R; therefore, we have $N = \lambda \pi R^2$. To simplify the analysis, we assume that both omni-directional and directional transmissions have equal gains. Without this assumption, it is much more difficult to model the interference from omni-directional and directional transmissions by other nodes when two nodes are engaged in a handshake. In practice, it is possible to achieve equal gain of omni-directional and directional transmissions by means of power control.

We also assume that nodes operate in time-slotted mode, with time slots of length τ much smaller than the length of any packet. Given that τ is very small, the performance of the time-slotted protocol is very close to the performance of the asynchronous version of the protocol. The transmission times of RTS, CTS, data, and ACK packets are normalized with regard to τ and are denoted by l_{rts}, l_{cts}, l_{data}, and l_{ack}, respectively. For the sake of simplicity, we also assume that all packet lengths are multiples of the length of a time-slot.

We derive the throughput of these protocols based on a heavy-traffic assumption, i.e., a node always has a packet to be sent. We also assume that a silent node begins transmission with probability p at each time slot. Obviously, p cannot be 1 due to the workings of collision avoidance, such as deferring, backing off, etc. Thus, p is a protocol-specific parameter but is slot independent. The above assumptions have been widely used in performance evaluation of MAC protocols (e.g., [2, 7, 10]) to make the analysis tractable.

We assume that a node becomes ready independently with probability p_0 at each time slot, and that a node initiates a successful handshake with any other node with probability p_s. Obviously, $p_s < p < p_0$. To be specific, $p = p_0 \cdot$ Prob.$\{$Channel is sensed idle in a slot$\}$. With this model, Wu and Varshney [10] and we [9] used two Markov chains to analyze the performance of CSMA, BTMA and collision

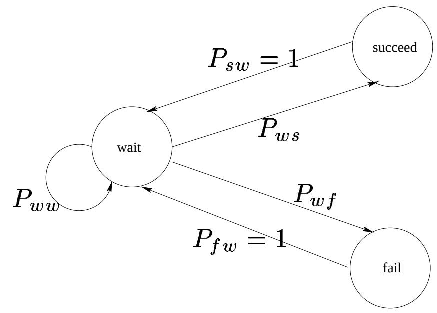


Figure 1. Markov chain model for a node

avoidance with omni-directional antennas. A Markov chain of the shared channel is used to derive the rough relationship between p and p_0, and a Markov chain is used to model the state of a node to derive the throughput. It is shown that the throughput is largely decided by p.

Due to the workings of collision avoidance and resolution, p must be kept very small, which means that p is likely to assume values that do not exceed 0.1. Here we do not analyze the relationship between p and p_0, as has been done before [9, 10]. This is because of the difficulty in modeling the channel when the directional transmission mode is used, especially in the third scheme we investigate, in which nodes can switch between directional and omni-directional transmission modes. Instead, we just assume that p takes on a range of values and then derive the throughput using the node model only. Given that the key objective of the model is to provide a comparative analysis of collision-avoidance strategies, and that the probability of successful handshakes by any one node in an ad hoc network cannot be very large, our approximation is very sensible.

The node model is a three-state Markov chain shown in Fig. 1, where the *wait* is the state where the node defers for other nodes or backs off, *succeed* is the state where the node can complete a successful four-way handshake with other nodes, and *fail* is the state where the node initiates a handshake that is unsuccessful or cannot be completed due to collisions and interference.

With some simplifications, all the three schemes we are investigating share the same node model, while they differ in the duration of certain states and the transition probabilities among these states.

2.1 ORTS-OCTS

We have analyzed the ORTS-OCTS scheme before [9] and here we present a simplified analysis for completeness.

For the sake of simplicity, we regard *succeed* and *fail* as the states in which two different kinds of *virtual* packets are

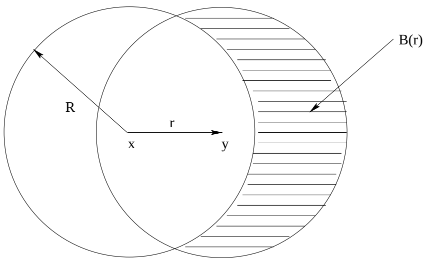


Figure 2. ORTS-OCTS scheme

transmitted and their lengths in time slots are:

$$\begin{aligned} T_{succeed} &= (l_{rts}+1)+(l_{cts}+1)+(l_{data}+1)+(l_{ack}+1) \\ &= l_{rts}+l_{cts}+l_{data}+l_{ack}+4 \end{aligned}$$

and

$$\begin{aligned} T_{fail} &= (l_{rts}+1)+(l_{cts}+1) \\ &= l_{rts}+l_{cts}+2 \end{aligned}$$

respectively as they are the durations that a node should stay in these two states. Obviously, the duration of a node in *wait* state T_{wait} is 1.

Because by assumption collision avoidance is enforced in each node, no node is allowed to transmit data packets continuously; therefore, the transition probabilities from *succeed* to *wait* and from *fail* to *wait* are both one.

To derive the transition probability P_{ws} from *wait* to *succeed*, we need to calculate the probability $P_{ws}(r)$ that node x successfully initiates a four-way handshake with node y at a given time slot when they are at a distance r apart. Before calculating $P_{ws}(r)$, we define $B(r)$ to be the area that is in the hearing region of node y but outside the hearing region of node x, i.e., the interfering region "hidden" from node x shown as the shaded area in Fig. 2. Takagi and Kleinrock [7] have shown that $B(r)$ equals

$$B(r) = \pi R^2 - 2R^2 q\left(\frac{r}{2R}\right)$$

where $q(t) = \arccos(t) - t\sqrt{1-t^2}$.

Then $P_{ws}(r)$ can be calculated as:

$$P_{ws}(r) = P_1 \cdot P_2 \cdot P_3 \cdot P_4$$

where

$$\begin{aligned} P_1 &= \text{Prob.}\{x \text{ transmits in the time slot}\}, \\ P_2 &= \text{Prob.}\{y \text{ does not transmit in the time slot}\}, \\ P_3 &= \text{Prob.}\{\text{none of the nodes within } R \text{ of } x \\ &\qquad \text{transmits in the same slot}\}, \\ P_4(r) &= \text{Prob.}\{\text{none of the nodes in } B(r) \\ &\qquad \text{transmits for } (2l_{rts}+1) \text{ slots} \mid r\}. \end{aligned}$$

The reason for the last term is that the vulnerable period for an RTS is only $2l_{rts}+1$, and once the RTS is received successfully by the receiving node (which can then start sending the CTS), the probability of further collisions is assumed to be negligibly small.

Given a planar area of size S where nodes are randomly placed according to a two-dimensional Poisson distribution, the probability that none of them transmits in a time slot is given by:

$$\begin{aligned} P &= \sum_{i=0}^{\infty}(1-p)^i \frac{(\lambda S)^i}{i!}e^{-\lambda S} \\ &= \sum_{i=0}^{\infty}\frac{[(1-p)\lambda S]^i}{i!}e^{-(1-p)\lambda S}\cdot e^{-p\lambda S} \\ &= e^{-p\lambda S} = e^{-pS\lambda\pi R^2/(\pi R^2)} \\ &= e^{-pNS/(\pi R^2)}. \end{aligned} \tag{1}$$

Obviously, $P_1 = p$ and $P_2 = (1-p)$. P_3 can be obtained by

$$\begin{aligned} P_3 &= \sum_{i=0}^{\infty}(1-p)^i \frac{(\lambda\pi R^2)^i}{i!}e^{-\lambda\pi R^2} \\ &= \sum_{i=0}^{\infty}(1-p)^i \frac{N^i}{i!}e^{-N} \\ &= e^{-pN}. \end{aligned}$$

Similarly, the probability that none of the nodes in $B(r)$ transmits in a time slot is given by

$$\begin{aligned} p_4(r) &= \sum_{i=0}^{\infty}(1-p)^i \frac{(\lambda B(r))^i}{i!}e^{-\lambda B(r)} \\ &= e^{-p\lambda B(r)}. \end{aligned}$$

Hence, $P_4(r)$ can be expressed as

$$\begin{aligned} P_4(r) &= (p_4(r))^{2l_{rts}+1} \\ &= e^{-p\lambda B(r)(2l_{rts}+1)}. \end{aligned}$$

Since each sending node chooses any one of its neighbors with equal probability and the average number of nodes within a region of radius r is proportional to r^2, thus the probability density function of the distance r between node x and y is

$$f(r) = 2r, \quad 0 < r < 1.$$

where we have normalized r with regard to R by setting $R = 1$.

Now we can calculate P_{ws} as follows:

$$\begin{aligned} P_{ws} &= \int_0^1 2rP_{ws}(r)dr \\ &= 2p(1-p)e^{-pN}\int_0^1 re^{-p\lambda B(r)(2l_{rts}+1)}dr \\ &= 2p(1-p)e^{-pN}\int_0^1 re^{-pN[1-2q(r/2)/\pi](2l_{rts}+1)}dr. \end{aligned}$$

From the Markov chain shown in Fig. 1, the transition probability P_{ww} that node x continues to stay in *wait* state in a slot is just $(1-p)e^{-pN}$, i.e., it does not initiate any transmission and there is no node around it initiating a transmission. Let π_s, π_w and π_f denote the steady-state probability of state *succeed*, *wait* and *fail* respectively. From Fig. 1, we have

$$\begin{aligned}\pi_w &= \pi_w P_{ww} + \pi_s + \pi_f \\ \pi_w &= \pi_w P_{ww} + 1 - \pi_w \\ \pi_w &= \frac{1}{2-P_{ww}} \\ &= \frac{1}{2-(1-p)e^{-pN}}.\end{aligned}$$

and then the steady-state probability of state *succeed* π_s can be calculated as:

$$\pi_s = \pi_w P_{ws} = \frac{P_{ws}}{2-(1-p)e^{-pN}}.$$

Accordingly, the throughput Th is:

$$\begin{aligned}Th &= \frac{\pi_s \cdot l_{data}}{\pi_w T_w + \pi_s T_s + \pi_f T_f} \\ &= l_{data}\pi_s[\pi_w + (l_{rts}+l_{cts}+2)(1-\pi_w-\pi_s) \\ &\quad + (l_{rts}+l_{cts}+l_{data}+l_{ack}+4)\pi_s]^{-1}.\end{aligned}$$

2.2 DRTS-DCTS

Following an approach similar to what we have presented for ORTS-OCTS, it is straightforward to show that the values of many quantities for DRTS-DCTS remain the same as those in the ORTS-OCTS scheme. They are summarized as follows:

$$\begin{aligned}T_{succeed} &= l_{rts}+l_{cts}+l_{data}+l_{ack}+4 \\ T_{wait} &= 1 \\ P_{sw} &= P_{fw} = 1 \\ P_{ww} &= (1-p)e^{-p'N} \quad (p' = p\theta/2\pi) \\ \pi_w &= 1/(2-P_{ww}) \\ \pi_s &= P_{ws}/(2-P_{ww}) \\ \pi_f &= 1-\pi_w-\pi_s. \end{aligned} \tag{2}$$

The only unknown quantities are T_{fail} and P_{ws}. Because the DRTS-DCTS scheme cannot prevent interference from neighboring nodes, the handshake between any pair of sending and receiving nodes may be interrupted at almost any time and the failed period can last from $T_1 = l_{rts}+1$ to $T_2 = l_{rts}+l_{cts}+l_{data}+l_{ack}+4$.

We assume that the length of the failed period follows a truncated geometric distribution with parameter p, lower

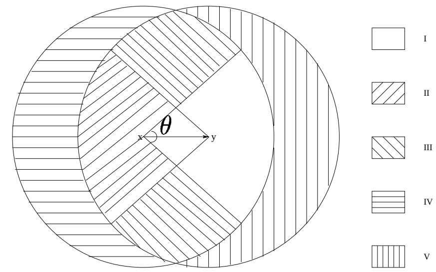


Figure 3. DRTS-DCTS scheme

bound T_1 and upper bound T_2. Then we take the mean value of the truncated geometric distribution as T_{fail}, which is shown below:

$$T_{fail} = \frac{1-p}{1-p^{T_2-T_1+1}} \sum_{i=0}^{T_2-T_1} p^i (T_1+i). \tag{3}$$

As discussed above, to calculate P_{ws}, we need to first calculate $P_{ws}(r)$, the probability of successful transmission in a time slot initiated by node x to node y which is at a distance r. $P_{ws}(r)$ equals the product of the probability that x transmits, the probability y listens and the probability that no node that can interfere x's transmission to y actually transmits which is denoted by $P_I(r)$, i.e., $P_{ws}(r) = p(1-p)P_I(r)$. The region around nodes x and y is shown in Fig. 3, where θ is the beamwidth of transmissions. We assume complete attenuation of the transmitted signal outside the range of the beamwidth.

From Fig. 3, we can see that the region around nodes x and y can be divided into five areas. Denote by S_i the size of Area i, they are:

$$\begin{aligned}S_I &= \theta/(2\pi) \\ S_{II} &= \theta/(2\pi) - r^2\tan(\theta/2)/(2\pi) \\ S_{III} &= 2q(r/2)/\pi - \theta/\pi + r^2\tan(\theta/2)/(2\pi) \\ S_{IV} &= 1-2q(r/2)/\pi \\ S_V &= 1-2q(r/2)/\pi \end{aligned} \tag{4}$$

where we have normalized r with regard to R by setting $R=1$ and S_i with regard to πR^2. Calculation of these areas is straightforward and omitted here.

Before proceeding, we emphasize key assumptions that we are making regarding the use of directional antennas. First, when a node is transmitting with one of its antennas, it appears "blind" to other directions. This is also the case when each node is equipped with only one steerable antenna. When a node is transmitting, it cannot sense any channel activity at all. Second, we assume omni-directional reception.

Accordingly, for the handshake between nodes x and y to be successful, all of the following conditions should be met:

1. In Area I, no node transmits in one slot and the probability is:

$$p_1 = e^{-pS_I N}$$

 This is because the nodes in area I do not know that node x is transmitting and can interfere with the handshake between nodes x and y, if they do transmit.

2. In Area II, it must be true that no node transmits in $2l_{rts}$ slots in the direction of node y and does not transmit in the slot when the transmission of node y arrives at them. Thus the probability is:

$$p_2 = e^{-p' S_{II} N(2l_{rts})} \cdot e^{-pS_{II}N}$$

 where $p' = p\theta/(2\pi)$.

3. In Area III, no node transmits in the direction to nodes x and y during the whole handshake and the span angle of the direction θ' is $\theta + \phi$, where ϕ is the angle formed by the two lines joining a node in Area III with nodes x and y if ϕ is less than θ; otherwise, θ' is just 2θ. When nodes x and y are very close to each other, $\theta' \approx \theta$. Though the range of θ' is between θ and 2θ, for simplicity, we just choose $\theta' = \theta$. Therefore,

$$\begin{aligned} p_3 &= e^{-p'' S_{III} N(l_{rts}+l_{rts}+1+l_{cts}+1+l_{data}+1+l_{ack}+1)} \\ &= e^{-p'' S_{III} N(2l_{rts}+l_{cts}+l_{data}+l_{ack}+4)} \end{aligned}$$

 where $p'' = p\theta'/(2\pi) = p\theta/(2\pi)$.

4. In Area IV, no node transmits in node x's direction when node y is transmitting; therefore, there are two such periods. One is the time when node y transmits a CTS packet to node x and the other is the time when node y transmits an acknowledgment packet to node x. The durations of these two periods in the number of time slots are approximately $l_{rts} + l_{cts} + 1$ and $l_{rts} + l_{ack} + 1$ respectively which follows the assumption that nodes transmit in each time slot independently with probability p. Accordingly, the probability p_4 that no interference from nodes in Area IV is:

$$\begin{aligned} p_4 &= e^{-p' S_{IV} N(l_{rts}+l_{cts}+1)} \cdot e^{-p' S_{IV} N(l_{rts}+l_{ack}+1)} \\ &= e^{-p' S_{IV} N(l_{rts}+l_{cts}+1+l_{rts}+l_{ack}+1)} \\ &= e^{-p' S_{IV} N(2l_{rts}+l_{cts}+l_{ack}+2)} \end{aligned}$$

5. In Area V, no node transmits in node y's direction when node x is transmitting. Similar to the case discussed above, there are two such periods. One is the time when node x transmits an RTS packet to node y and the other is the time when node x transmits a data packet to node y. The durations of these two periods in the number of time slots are approximately $l_{rts} + l_{rts} + 1$ and $l_{rts} + l_{data} + 1$ respectively. Therefore, the probability p_5 that no interference from nodes in Area V is:

$$\begin{aligned} p_5 &= e^{-p' S_V N(l_{rts}+l_{rts}+1)} \cdot e^{-p' S_V N(l_{rts}+l_{data}+1)} \\ &= e^{-p' S_V N(l_{rts}+l_{rts}+1+l_{rts}+l_{data}+1)} \\ &= e^{-p' S_V N(3l_{rts}+l_{data}+2)} \end{aligned}$$

Having that $P_I(r) = p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5$, P_{ws} can be obtained by considering all posssible values of r, i.e.,

$$P_{ws} = \int_0^1 2rP_{ws}(r)dr.$$

Then throughput Th can be calculated as follows:

$$\begin{aligned} Th &= \frac{\pi_s \cdot l_{data}}{\pi_w T_w + \pi_s T_s + \pi_f T_{fail}} \\ &= l_{data}\pi_s[\pi_w + T_{fail}(1 - \pi_w - \pi_s) \\ &\quad + (l_{rts} + l_{cts} + l_{data} + l_{ack} + 4)\pi_s]^{-1}. \end{aligned}$$

2.3 DRTS-OCTS

Once again, following an approach similar to what was used for ORTS-OCTS, many of the variables needed to compute the throughput of DRTS-OCTS can be calculated and equal the values shown in (2), with the exception that $P_{ww} = (1-p)e^{-pN}$, which is product of the probability that the node does not transmit and the probability that no node around it transmits omni-directionally. Though nodes may sometimes transmit directionally, considering that almost each handshake, either failed or successful, consists of a CTS packet that is transmitted omni-directionally, and that the CTS packet can virtually silence all the hidden terminals, this is a reasonable approximation.

To calculate P_{ws}, we first need to calculate $P_{ws}(r)$. In the DRTS-OCTS scheme, the interfering region of nodes x and y can be divided into three areas. Similarly, denote by S_i the size of Area i and they are:

$$S_I = \theta/(2\pi)$$

$$S_{II} = 1 - \theta/(2\pi)$$

$$S_{III} = 1 - 2q(r/2)/\pi.$$

For the handshake between nodes x and y to be successful, the following conditions should be met:

1. In Area I, no node transmits in one slot and the probability is:

$$p_1 = e^{-pS_I N}$$

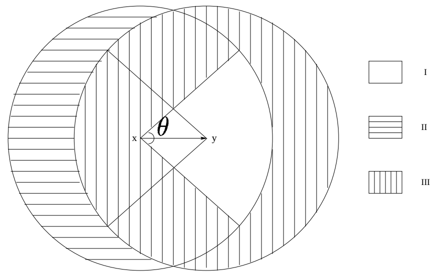


Figure 4. DRTS-OCTS

2. In Area II, it should be that no node transmits in $2l_{rts}$ slots in the direction to node y and no node transmits in the slot when the transmission of node y arrives at them. Thus the probability is:

$$p_2 = e^{-p' S_{II} N(2l_{rts})} \cdot e^{-p S_{II} N}$$

where $p' = p\theta/(2\pi)$.

3. In Area III, which is in effect Area IV in Fig. 3, the probability p_3 that no interference from nodes in the area is:

$$\begin{aligned} p_3 &= e^{-p' S_{III} N(l_{rts}+l_{cts}+1)} \cdot e^{-p' S_{III} N(l_{rts}+l_{ack}+1)} \\ &= e^{-p' S_{III} N(l_{rts}+l_{cts}+1+l_{rts}+l_{ack}+1)} \\ &= e^{-p' S_{III} N(2l_{rts}+l_{cts}+l_{ack}+2)} \end{aligned}$$

From the above, we can derive $P_{ws}(r)$:

$$P_{ws}(r) = p \cdot (1-p) \cdot p_1 \cdot p_2 \cdot p_3$$

Therefore, P_{ws} equals

$$P_{ws} = \int_0^1 2r P_{ws}(r) dr.$$

From the above analysis, we also know that the handshake between nodes x and y may fail due to the interference from the nodes in Area III. Thus, the failed period can range from $l_{rts} + 1$ to $l_{rts} + l_{cts} + l_{data} + l_{ack} + 4$. We also assume that it follows a truncated geometric distribution and use its mean length as T_{fail}. To take into account the greater effect of omni-directional transmissions of CTS packets that may well collide with ongoing transmissions, we use $l_{rts} + l_{cts} + 2$ as the lower bound of the distribution instead. Then the throughput can be calculated accordingly.

3 Numerical Results

In this section, we investigate only the case in which data packets are much longer than control packets. This warrants an RTS/CTS based collision avoidance handshake before actual data packet transmissions. In addition, only the results of one typical configuration are shown here as similar results can be readily obtained for other configurations. In this configuration, τ denotes the duration of one slot; the lengths of RTS, CTS and ACK packets are 5τ; and data packets last 100τ.

The results for the maximum achievable throughput when antenna beamwidth changes from $\theta = 15°(\pi/12)$ to $\theta = 180°(\pi)$ in increment of $\theta = 15°(\pi/12)$ are shown in Fig. 5.

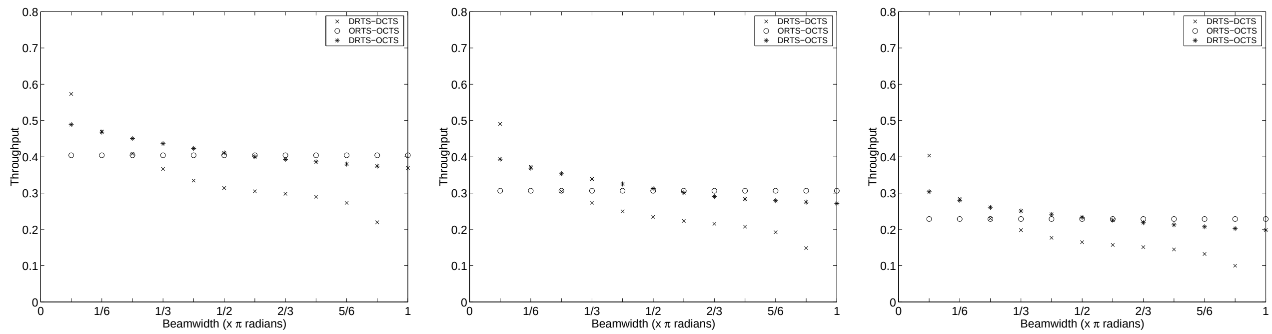
Fig. 5 clearly shows that the DRTS-DCTS scheme is the best among the three when the antenna beamwidth is narrow, while DRTS-OCTS only outperforms ORTS-OCTS marginally. This result contrasts with results from prior studies of collision avoidance schemes based on simulation [4]. The intuition behind this result is that, with a narrow transmission beamwidth, although DRTS-DCTS scheme risks a higher collision probability, the advantage of possible spatial reuse and less required co-ordination (thus nodes spend less time in deferring) outweighs a more conservative collision avoidance scheme like ORTS-OCTS.

When the antenna beamwidth is wider, the performance of DRTS-DCTS drops significantly, because the advantage of directional transmissions diminishes accordingly.

4 Simulation Results

The numerical results in the previous section show that the DRTS-DCTS scheme performs the best among the three MAC schemes. In this section, we investigate the performance of the popular IEEE 802.11 DFWMAC protocol and its variants using directional transmissions.

We use GloMoSim 2.0 [11] as the network simulator and implement the directional schemes (DRTS-DCTS and DRTS-OCTS) with the assumption that there is a neighbor protocol that can actively maintain a list of neighbors as well as their locations. Direct sequence spread spectrum (DSSS) parameters are used throughout the simulations, which are shown in Table 1. The raw channel bit rate is 2Mbps. We use a uniform distribution to approximate the Poisson distribution used in our model, which is mainly used to facilitate our derivation of analytical results. To be specific, we place nodes in concentric circles or rings. That is, given that a node's transmitting and receiving range is R and that there are on average N nodes within this circular region, we place N nodes in a circle of radius R, subject to a uniform distribution. Because there are on average 2^2N nodes within a circle of radius $2R$, we place $2^2N - N = 3N$ nodes outside the previous circle of radius R but inside the concentric circle of radius $2R$, i.e., the ring with radii R and $2R$, subject to the same uniform distribution. Then $3^2N - 2^2N = 5N$ nodes can be placed in an outer ring with radii $2R$ and $3R$. Because we cannot gen-



(a) Throughput (N=3) (b) Throughput (N=5) (c) Throughput (N=8)

Figure 5. $l_{rts} = l_{cts} = l_{ack} = 5\tau, l_{data} = 100\tau$.

Table 1. IEEE 802.11 protocol configuration parameters

RTS	CTS	data	ACK	DIFS	SIFS
20B	14B	1460B	14B	50μsec	10μsec

contention window	slot time	sync. time	prop. delay
31–1023	20μsec	192μsec	1μsec

erate an infinite network model, we just focus our attention on the performance of the innermost N nodes. According to our experiments, conclusions drawn from a circular network of radius of more than $3R$ will not affect the conclusion to be drawn in this section, i.e., boundary effects can be safely ignored when the circular network's radius is $3R$. Therefore, we present only the results for a circular network of radius $3R$. To avoid some extreme cases, we only use network topologies that satisfy the following conditions:

- For the inner N nodes, each node should have at least 2 neighbors and at most $2N - 2$ neighbors.
- For the intermediate outer $3N$ nodes, each node should have at least 1 neighbor and at most $2N - 1$ neighbors.

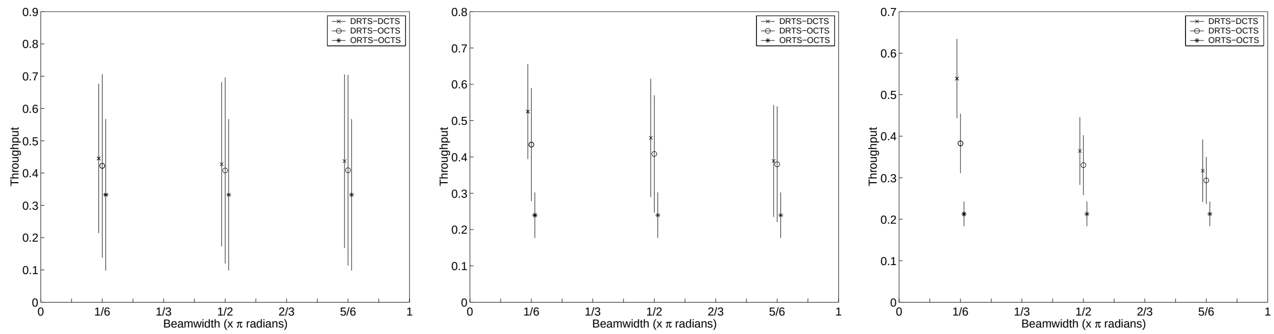
In our simulation, each node has a constant-bit-rate (CBR) traffic generator with data packet size of 1460 bytes, and one of its neighbors is randomly chosen as the destination for each packet generated. All nodes are always backloged. We run simulation programs with $N = 3, 5, 8$ with beamwidth $\theta = 30°, 90°, 150°$ respectively. We generate 50 random topologies that satisfy the uniform distribution and then get averaged throughput and delay for the N nodes in the innermost circle of radius R for each configuration. The results are shown in Figs. 6 and 7.

In these figures, the vertical lines show the range of throughput achieved by each scheme. The lines are shifted a bit so that they look clearer. At first, it can be seen that the throughput of the DRTS-DCTS scheme does not degrade much in a relatively large range of transmission beamwidths when N is small and the throughput of the DRTS-OCTS scheme degrades very little regardless of transmission beamwidth. This is a bit different from the prediction in the analysis where the throughput of both schemes degrades gradually with the increase of beamwidth. This can be explained as follows: The analysis does not take into account the fact that the number of nodes in a certain region can only be an integer. Instead, an infinite division of neighbors is implicitly assumed.

In the analytical model, it seems that the effects of a node vary in regions, which is clear from the derivation of the probability that one node succeeds in completing a four-way handshake with the other node. In the derivation, transmissions of the sending and receiving nodes are vulnerable to nodes from different regions for different periods. On the contrary, in simulations (and in reality) nodes have a definitive influence on the handshake of a pair of sending and receiving nodes. Referring to Fig. 3, for example, a node is either in Area III or V, but not in both and nor can it exert separate effects in these two areas. Thus, suppose that the neighbors of a node are distributed evenly around it, say three, it does not make any difference if this node transmits with beamwidth of 30°or 90°.[2] Despite the inaccuracies of the analytical model, the simulation results still largely agree with what is predicted in the analytical model. That is, that the DRTS-DCTS scheme outperforms the other two MAC schemes when the beamwidth of transmissions is narrow.

An interesting result is that the DRTS-DCTS scheme

[2] In reality, it is more desirable to transmit with narrower beamwidth, because signal energy is more concentrated and a higher signal-to-noise ratio can be achieved, though physical layer impairment other than Gaussian white noise is not modeled in the simulations.



(a) N=3 (b) N=5 (c) N=8

Figure 6. Throughput Comparison

still outperforms the other two schemes with relatively wide transmission beamwidths (say 150°). This is in fact due to the peculiarities of the binary exponential backoff (BEB) scheme used in the IEEE 802.11 MAC protocol, which is not reflected in the analytical model. The BEB scheme always favors the node that succeeds last in transmitting a data packet. Then the node may monopolize the channel for a very long time during which there is no contention loss and throughput can be very high for a particular node, while other nodes suffer starvation. The fairness problem is especially conspicuous when very few nodes are competing in a region and is investigated elsewhere, e.g. [1,6]. We find that it is much more unfair when transmission beamwidth is wider, as it is more probable that the few competing nodes are more easily affected by one "greedy" node and prevented from transmitting. However, when N is larger, the fairness problem is less severe, as it is much more difficult for a node to win exclusive access to the shared channel from all the other competing nodes at the same time. Due to space limitation, the results are not shown here.

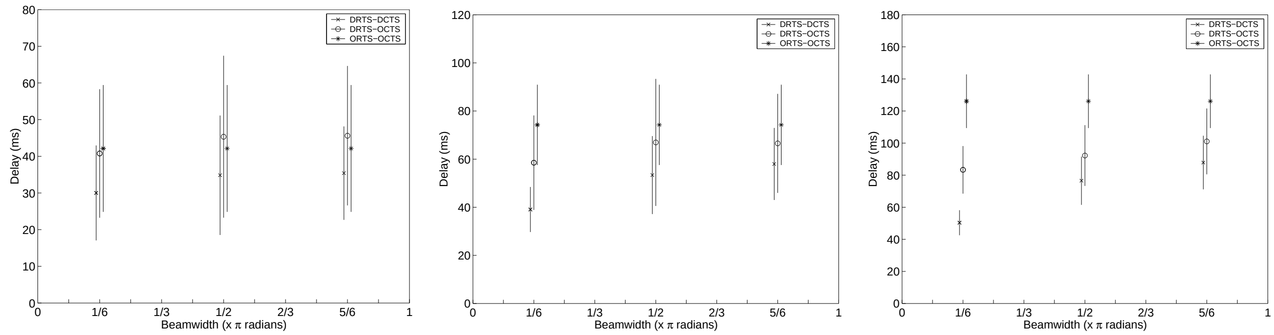
We also collected statistics about the number of transmitted RTS packets that lead to ACK timeouts due to collisions of data packets as well as the total number of transmitted RTS packets that can lead to either an incomplete RTS-CTS-data handshake or a successful four-way handshake. Then we calculate the ratio of these two numbers which in fact models imperfectness of collision avoidance. The larger the ratio is, the poorer is collision avoidance. The results, which are omitted here due to space limitation, show that the DRTS-DCTS and DRTS-OCTS schemes have higher collision occurrences than ORTS-OCTS. This is to be expected, because both schemes are more aggressive in achieving spatial reuse and do not force all the neighbors around the sending and receiving nodes to defer access to the shared channel. It is also clear that, due to the contention-based nature of IEEE 802.11 MAC protocol as well as its failure to ensure that data packets are transmitted free from collisions, it is very difficult to make all competing nodes to coordinate well when N is large and hence the collision ratio is still rather high.

It is also clear from Fig. 7 that, with a more aggressive way of channel access to achieve spatial reuse, the DRTS-DCTS scheme also enjoys on average less delay than the other two schemes, especially when N is large. In addition, it is also desirable to use narrower transmission beamwidths in the DRTS-DCTS scheme even when it does not affect the throughput much for small values of N, as nodes will be less affected by surrounding sending and receiving nodes and thus can spend less time in waiting.

5 Conclusion

In this paper, we have presented the first analytical model for collision avoidance protocols with omni-directional transmission mode, all-directional transmission mode, and a combination of both. This work extends the analytical model first introduced by Takagi and Kleinrock for the analysis of CSMA in multi-hop networks with omni-directional transmissions to the case of collision-avoidance protocols with omni-directional and directional transmissions. This basic model can be applied to many other variants of collision avoidance protocols and uses of directional transmissions.

The results from our analytical model reveal that, in multi-hop ad hoc networks infested with hidden terminals, the cost to coordinate neighboring nodes well to avoid collisions can be much more than the gain in silencing potentially interfering neighboring nodes, as many more nodes are forced to wait and spatial reuse is greatly reduced. Though directional transmission of control packets may lead to more collisions, the gain in more spatial reuse and less time in waiting still outweighs the disadvantage



(a) N=3 (b) N=5 (c) N=8

Figure 7. Delay Comparison

of higher collision probability. In addition, the consistent use of directional transmissions in the DRTS-DCTS scheme also avoids the disruptive effects of the other conservative collision avoidance schemes like the DRTS-OCTS scheme, because the omni-directional transmission of CTS packets can interfere with other on-going handshake in the neighborhood given the imperfectness of collision avoidance. In fact, the scheme can almost nullify the benefit of spatial reuse enabled by directional transmissions in sufficiently random networks. Simulation experiments of the popular IEEE 802.11 MAC protocol as well as its variants that make use of directional transmissions validate the above arguments and show that the DRTS-DCTS scheme indeed achieves much better spatial reuse (thus higher throughput) and less time in waiting (thus less delay) at the expense of higher collision rate.

Our findings suggest that it is worthwhile to consider adopting the all-directional transmission scheme to enhance throughput and reduce channel access delay in the framework stipulated by the IEEE 802.11 protocol stack. More generally, our results indicate that further research is still needed on new collision avoidance schemes tailored to directional antennas that permit more aggressive channel reuse.

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