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REGULATING A FORMATION OF A LARGE NUMBER OF VEHICLES

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ABSTRACT

Various control algorithms have been developed for fleets of autonomous vehicles. Many of the successful control algorithms in practice are behavior-based control or nonlinear control algorithms, which makes analyzing their stability difficult. At the same time, many system theoretic approaches for controlling a fleet of vehicles have also been developed. These approaches usually use very simple vehicle models such as particles or point-mass systems and have only one coordinate system which allows stability to be proven. Since most of the practical vehicle models are six-degree-of-freedom systems defined relative to a body-fixed coordinate system, it is difficult to apply these algorithms in practice.

In this paper, we consider a formation regulation problem as opposed to a formation control problem. In a formation control problem, convergence of a formation from random positions and orientations is considered, and it may need a scheme to integrate multiple moving coordinates. On the contrary, in a formation regulation problem, it is not necessary since small perturbations from the nominal condition, in which the vehicles are in formation, are considered. A common origin is also not necessary if the relative distance to neighbors or a leader is used for regulation. Under these circumstances, the system theoretic control algorithms are applicable to a formation regulation problem where the vehicle models have six degrees of freedom.

We will use a realistic six-degree-of-freedom model and investigate stability of a fleet using results from decentralized control theory. We will show that the leader-follower control algorithm does not have any unstable fixed modes if the followers are able to measure distance to the leader. We also show that the leader-follower control algorithm has fixed modes at the origin, indicating that the formation is marginally stable, when the relative distance measurements are not available.

Multi-vehicle simulations are performed using a hybrid leader-follower control algorithm where each vehicle is given a desired trajectory to follow and adjusts its velocity to maintain a prescribed distance to the leader. Each vehicle is modeled as a three-degree-of-freedom system to investigate the vehicle's motion in a horizontal plane. The examples show efficacy of the analysis.

Keywords: Formation, Formation Control, Multiple vehicles, Leader-Follower Algorithm, Autonomous Underwater Vehicles

I. INTRODUCTION

Various control algorithms have been developed for fleets of autonomous vehicles. The general idea is to use relatively inexpensive vehicles to cooperatively solve a difficult problem. This idea is best illustrated in a mine search problem. In a mine search problem, a large area must be searched quickly and carefully before humans can enter the area safely. The cost of each vehicle must be low because there is always some risk of losing the search vehicles.

Many of the early successful control algorithms in practice are behavior-based control. For example Balch and Arkin implemented a behavior-based controller on multiple mobile robots to move in formation [1]. In their work, several different formation shapes and algorithms are evaluated. Fredslund and Mataric used mobile robots equipped with a camera to move in formation and avoid obstacles [5].

Another common approach is a nonlinear control algorithm. In [7], an artificial potential functions were used for formation control. The algorithm was successfully implemented for the control of underwater gliders [4]. This approach has an advantage over the behavior-based control because stability can be directly proven by Lyapunov's method [7]. Ihle, Skjetne and Fossen introduced and implemented a nonlinear control algorithm for a fleet of ships [6]. In this algorithm, a leader is absent, and formation flying is accomplished by maintaining desired distance to a point called the formation reference point (FRP).

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At the same time, many system theoretic approaches for controlling fleets of vehicles have also been developed. These approaches usually use simple vehicle models such as particles or point-mass systems and have only one coordinate system. Stability of a system can be proven using the results from graph theory and decentralized control theory [9]. Although this approach gives great insight to a general multi-vehicle problem, it is difficult to apply these algorithms in practice since most of the practical vehicle models are nonlinear six-degree-of-freedom (DOF) systems defined relative to a body-fixed coordinate system.

In this paper, we consider a formation regulation problem as opposed to a formation control problem. In a formation control problem, convergence of a formation from random positions and orientations is considered, and it may need a scheme to integrate multiple moving coordinates. On the other hand, in a formation regulation problem, small perturbations from the nominal condition, in which the vehicles are in formation, are considered. Under these circumstances, the system theoretic control approaches are applicable to a multiple vehicle problem where the vehicle models have six degrees of freedom.

This paper is organized as follows. In section II, a 6DOF vehicle model and a vehicle coupling equation are developed. In section III, we will show that the leader-follower control algorithm does not have any unstable fixed modes if the followers are able to measure a distance to the leader. We also show that the leader-follower control algorithm has fixed modes at the origin, indicating that the formation is marginally stable, when the relative distance measurements are not available. In section IV, the developed design and analysis methods are applied to a hybrid leader-follower control algorithm as an example. Each vehicle is modeled as a nonlinear 3DOF system to investigate the vehicle's motion on a horizontal plane. Conclusions are given in section V.

II. VEHICLE MODEL FORMULATION

We will model our vehicle as a rigid body. Its dynamics are expressed by 6DOF equations of motion.
Consider a general nonlinear 6DOF model,

$$\dot{\xi} = f(\xi, \mu) \tag{1}$$

where ξ is the state vector, and μ is the control input vector. Since it has six degrees of freedom, the state vector is usually given as $\xi = [u\ v\ w\ p\ q\ r]^T$. The right hand side of Eq. (1) usually consists of damping terms, Corioli's forces, control forces, etc.
It will be convenient to define a translational velocity vector, x_v and a rotational velocity vector, x_r as

$$x_v = [u \quad v \quad w]^T \text{ and } x_r = [p \quad q \quad r]^T. \tag{2}$$

Equation (1) is usually defined in body-fixed coordinates, which move with the vehicle. Inertial position and orientation of the vehicle are found through coordinate transformations. The inertial position is found by integrating Eq. (3),

$$\dot{\eta}_1 = J_1 x_v \tag{3}$$

where

$$J_1 = \begin{bmatrix} c\psi c\theta & -s\psi c\varphi + c\psi s\theta s\varphi & s\psi s\varphi + c\psi c\varphi s\theta \\ s\psi c\theta & c\psi c\varphi + s\psi s\theta s\varphi & -c\psi s\varphi + s\psi s\theta c\varphi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix}$$

where $c(.) = cos(.)$ and $s(.) = sin(.)$.
and the angles of the vehicle are obtained from

$$\dot{\eta}_2 = J_2 x_r \tag{4}$$

where

$$J_2 = \begin{bmatrix} 1 & s\varphi t\theta & c\varphi t\theta \\ 0 & c\varphi & -s\varphi \\ 0 & s\varphi / c\theta & c\varphi / c\theta \end{bmatrix}$$

where $t(.) = tan(.)$.

Stabilization of a fleet of vehicles is studied by Roy, Saberi, and Herlungston [9]. They give necessary and sufficient conditions for formation stabilization of a fleet for a given observation graph. For stability of the fleet, existence of unstable fixed modes is important. A fleet can be formation stabilized with a decentralized dynamic linear time-invariant (LTI) controller if it does not have any unstable fixed modes, and if there exists a linear dynamic LTI controller such that all eigenvalues are placed in the open left half plane (OLHP). Unfortunately, this analysis method was developed using a simple model (i.e., a particle) for each vehicle. Although it provides a great insight to the formation control problem, it does not mention some practical issues. For example, vehicles are usually modeled as a six-degree-of-freedom system defined in a body-fixed coordinate system, and a position of a vehicle is determined only through coordinate transformations. Moreover, if there are multiple vehicles, there would be multiple body-fixed coordinates and inertial coordinates. Difficulties arise when we have to consider these coordinates together.

The main objective of formation flying is that all vehicles keep a prescribed distance to their neighbors (or a leader) and move in a common direction. This condition can be realized as an operating condition of the fleet, and this can be viewed as a regulation problem. In this problem setting, the system theoretic approaches for a fleet may be applicable. In this paper, we consider a leader-follower system assuming that the followers measure a distance to the leader continuously.

First we derive linear equations for the leader. The nonlinear 6 DOF model is linearized for a straight line trajectory to obtain the following linear model

$$\begin{aligned} \dot{\xi}_0 &= A_0 \xi_0 + B_0 \mu_0 \\ y_0 &= C_0 \xi_0 \end{aligned} \tag{5}$$

Note that the vehicle equations might have more than six equations to perform specific tasks (such as trajectory following).

A follower has more states than the leader because followers regulate formation errors as well. The nonlinear 6 DOF model of the ith follower is also linearized for a straight line trajectory to obtain the following linear model

$$\dot{\xi}_i = A_i \xi_i + B_i \mu_i \tag{6}$$

Now define a vehicle separation vector as d_{0i}.

$$d_{0i} = d_0 - d_i \tag{7}$$

where

d_0 is a position of the leader.

d_i is a position of the follower.

In the follower's view point, the separation vector changes as

$$\dot{d}_{0i} = R_{0i} x'_{v0} - x_{vi} - x_r \times d_{0i} \tag{8}$$

where

x_{vi} and x_{v0} are the follower and the leader's translational velocity in the follower's body-fixed coordinates.

x_{v0}' is the leader's translational velocity in the leader's body-fixed coordinates.

x_r is the follower's rotational velocity.

R_{0i} is a coordinate transformation matrix from the leader to the follower's coordinates.

Note that the transformation matrix is necessary since the leader's velocity is defined with respect to its body-fixed coordinates.

Now we convert d_{0i} to a deviation variable and define it as a formation error, e_{0i}.

$$e_{0i} = d_{0i} - \bar{d}_{0i} \tag{9}$$

where $\bar{d}_{0i}$ is a desired nominal distance between the vehicles. Taking a time derivative and substituting it to Eq. (8) yields

$$\dot{e}_{0i} = R_{0i} x'_{v0} - x_{vi} - x_r \times \left(e_{0i} + \bar{d}_{0i}\right) \tag{10}$$

Relationship between vehicles can be described in a similar way. However, if the vehicles move in formation, the cross product goes away, and the rotational matrix becomes the identity matrix. This is because the local frames are parallel. Hence the equation is simplified to the following simple linear relationship of their translational velocities

$$\dot{e}_{0i} = x_{v0} - x_{vi} \tag{11}$$

As long as the vehicles' orientation is approximately the same, this simplification may be valid. Then the state equation for the ith follower is

$$\begin{aligned} \dot{\xi}_i &= A_i \xi_i + B_i \mu_i \\ \dot{e}_{0i} &= -x_{vi} + x_{v0} \\ y_i &= \begin{bmatrix} C_{i1} & 0 \\ 0 & C_{i2} \end{bmatrix} \begin{bmatrix} \xi_i \\ e_{0i} \end{bmatrix} \end{aligned} \tag{12}$$

which can be written in the following compact form

$$\begin{aligned} \dot{\xi}_i &= A_i \xi_i + B_i \mu_i \\ \dot{e}_{0i} &= D_i \xi_i + E \xi_0 \\ y_i &= \begin{bmatrix} C_{i1} & 0 \\ 0 & C_{i2} \end{bmatrix} \begin{bmatrix} \xi_i \\ e_{0i} \end{bmatrix} \end{aligned} \tag{13}$$

where

$$D = \begin{bmatrix} -I & 0 \end{bmatrix} \text{ and } E = \begin{bmatrix} I & 0 \end{bmatrix}$$

III. STABILITY OF A LEADER-FOLLOWER SYSTEM

Using the models developed in the previous section, we investigate the stability of an *n*-vehicle leader-follower system. The *n*-vehicle leader-follower system can be written as

$$\begin{aligned} \dot{\tilde{\xi}} &= F\tilde{\xi} + G\tilde{\mu} \\ \tilde{y} &= H\tilde{\xi} \end{aligned} \tag{14}$$

where $\tilde{\xi} = [x_0^T\ x_1^T\ e_{01}^T\ x_2^T\ e_{02}^T\ \ldots\ x_n^T\ e_{0n}^T]^T$, $\tilde{\mu} = [u_0^T\ u_1^T\ u_2^T\ \ldots\ u_n^T\ ]^T$, $\tilde{y} = [y_0^T\ y_1^T\ y_2^T \ldots\ y_n^T\ ]^T$ and

$$F = \begin{bmatrix} A_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 & 0 & 0 & 0 \\ E & D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_2 & 0 & 0 & \vdots \\ E & 0 & 0 & D_2 & 0 & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & \ddots & 0 \\ E & 0 & 0 & \cdots & 0 & D_n & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} B_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & B_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots \\ 0 & 0 & B_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & \ddots & B_n \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}, \text{ and}$$

$$
H = \begin{bmatrix} C_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & C_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{12} & 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & C_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & C_{n1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & C_{n2} \end{bmatrix}
$$

Notice that F matrix is in lower block triangular form, and G and H matrices are in block diagonal form. In other words, the plant is in input-output LBT (lower block triangular) form [11]. For a plant in this form, stability of the overall system is determined by stability of subsystems. Also, a decentralized controller may be designed one by one by neglecting the off diagonal elements. For instance, a decentralized linear quadratic Gaussian (LQG) regulator, which is able to tolerate nonlinear coupling and other disturbances, may be designed [11]. This property makes the leader-follower system a preferred system in practice especially if an advanced controller is required for controlling each vehicle.

We assume that the leader measures all of its states (i.e., $C_{01} = I$), and the followers measure all of its states and distance to the leader (i.e., $C_{i1} = C_{i2} = I$). The following controller is used for the leader

$$
\mu_0 = K_0 y_0 \tag{15}
$$

and the following controller is used for the i[th] follower.

$$
\mu_i = K_i y_i \tag{16}
$$

where

$$
K_i = \begin{bmatrix} K_{i1} & K_{i2} \end{bmatrix}
$$

Closed-loop equation for this system is

$$
\dot{\tilde{\xi}} = F_c \tilde{\xi} \tag{17}
$$

where

$$
F_c = \begin{bmatrix} A_0 + B_0K_{01}C_{01} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & A_1 + B_1K_{11}C_{11} & B_1K_{12}C_{12} & 0 & 0 & 0 & 0 \\ E & D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_2 + B_2K_{21}C_{21} & B_2K_{22}C_{22} & 0 & \vdots \\ E & 0 & 0 & D_2 & 0 & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & \ddots & B_nK_{n2}C_{n2} \\ E & 0 & 0 & \cdots & 0 & D_n & 0 \end{bmatrix}
$$

Because F_c matrix of the closed loop equation is in block lower triangular form, eigenvalues of overall system are a union of eigenvalues of subsystems. Hence stability of the fleet can be established if each vehicle can be stabilized. Moreover, since we assumed that each vehicle was controllable and observable, the fleet can be stabilized with proper feedback controllers. This result is not surprising because some leader-follower controllers have been designed successfully without formal analysis (for example [3, 8]).

This result can also be concluded in terms of fixed modes ([9], [10]). The n-vehicle leader-follower system has a fixed mode, λ_i, if the following condition is satisfied.

$$
det\left(\lambda_i I - F_c\right) = 0 \tag{18}
$$

The fleet can be formation stabilized if all fixed modes are in OLHP. Because the closed loop system matrix is in block lower triangular form, Eq. (18) can be written as

$$
det\left(\lambda_i I - A_0 - B_0K_{01}C_{01}\right) \cdot det\left(\lambda_i I - \begin{bmatrix} A_1 + B_1K_{11}C_{11} & B_2K_{12}C_{12} \\ D_1 & 0 \end{bmatrix}\right) \cdots
$$
$$
\cdot\, det\left(\lambda_i I - \begin{bmatrix} A_n + B_nK_{n1}C_{n1} & B_nK_{n2}C_{n2} \\ D_n & 0 \end{bmatrix}\right) = 0
$$

which indicates that fixed modes of the system are a union of uncontrollable and unobservable modes of each vehicle. Therefore, this system obviously has no fixed modes, and the fleet can be formation stabilized. Another observation is that when the followers become incapable of measuring the distance to the leader, the fleet would have multiple eigenvalues at the origin. In other words, the formation becomes marginally stable. This can easily be seen by substituting C_{i2} matrix with the zero matrices.

IV. EXAMPLE: A HYBRID LEADER-FOLLOWER ALGORITHM

We consider a hybrid leader-follower algorithm introduced in [3]. This algorithm is developed for a fleet of autonomous underwater vehicles (AUVs) searching for underwater mines. In this algorithm it is assumed that all vehicles know their inertial position, and the leader broadcasts its inertial position to the followers. Only communication required for formation flying is the position of the leader. Each vehicle follows a given search trajectory (or path), and the followers adjust their velocity to maintain a prescribed distance to the leader.

In this example, a 3 DOF model of a REMUS given in [8] is adapted. A linear state space model of the vehicle is given in Eq. (19).

$$
\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -0.37 & 0 & 0 & 0 \\ 0 & -2.78 & -0.63 & 0 \\ 0 & -5.00 & -1.97 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} 0.03 & 0 \\ 0 & 0.28 \\ 0 & -1.60 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{prop} \\ \delta_r \end{bmatrix} \tag{19}
$$

where

- u and v are forward and lateral velocities of the vehicle in m/sec defined in body fixed coordinates.
- r is yaw rate in rad/sec.
- ψ is yaw angle in rad.
- u_{prop} is a propeller thrust force in N
- δ_r is a rudder angle in rad

The operating condition, x_o, is x_o = [1.5 0 0 0].

Now we define a deviation of heading from a search path, θ, as

$$
\theta = \psi - \beta \tag{20}
$$

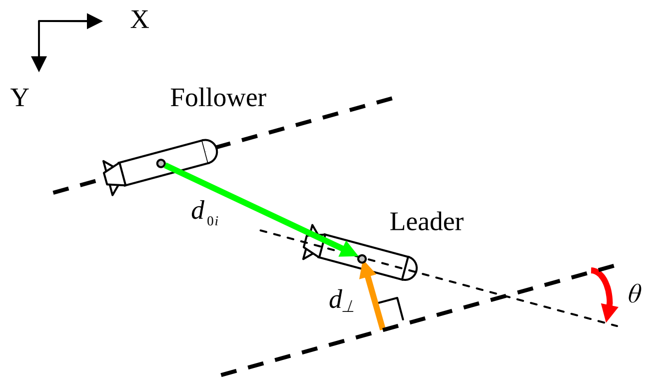

Figure 1: A Hybrid Leader-Follower Algorithm

where β is an angle of the path with respect to the inertial coordinates (see Fig. 1).
We take a time derivative of Eq. (20) and obtain

$$\dot{\theta} = r \tag{21}$$

where β was assumed to be constant.
A kinematic equation for the perpendicular distance from the path, $d_\perp$, is defined as

$$\dot{d}_\perp = u \sin\theta + v \cos\theta \tag{22}$$

Linearizing Eq. (22) at the previously defined operating point yields

$$\dot{d}_\perp = v + 1.5\theta \tag{23}$$

Augmenting Eq. (21) and Eq. (23) with Eq. (19) yields a state space model for the leader.

$$\begin{bmatrix} \dot{u}_0 \\ \dot{v}_0 \\ \dot{r}_0 \\ \dot{\theta}_0 \\ \dot{d}_{\perp,0} \end{bmatrix} = \begin{bmatrix} -0.37 & 0 & 0 & 0 & 0 \\ 0 & -2.78 & -0.63 & 0 & 0 \\ 0 & -5.00 & -1.97 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1.5 & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ v_0 \\ r_0 \\ \theta_0 \\ d_{\perp,0} \end{bmatrix} + \begin{bmatrix} 0.03 & 0 \\ 0 & 0.28 \\ 0 & -1.60 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{prop,0} \\ \delta_{r,0} \end{bmatrix} \tag{24}$$

We denote Eq. (24) compactly as

$$\dot{\xi}_0 = A\xi_0 + B\mu_0 \tag{25}$$

Equation for the formation error is added to Eq. (24) to obtain a follower model given in Eq. (26).

$$\begin{bmatrix} \dot{u}_i \\ \dot{v}_i \\ \dot{r}_i \\ \dot{\theta}_i \\ \dot{d}_{\perp,i} \\ \dot{e}_{x0i} \end{bmatrix} = \begin{bmatrix} -0.37 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.78 & -0.63 & 0 & 0 & 0 \\ 0 & -5.00 & -1.97 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1.5 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ r_i \\ \theta_i \\ d_{\perp,i} \\ e_{x0i} \end{bmatrix} + \begin{bmatrix} 0.03 & 0 \\ 0 & 0.28 \\ 0 & -1.60 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{prop,i} \\ \delta_{r,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_0 \tag{26}$$

Note that only the x-component of the formation error was augmented.
Equation (26) is written compactly as

$$\begin{aligned} \dot{\xi}_i &= A\xi_i + B\mu_i \\ \dot{e}_{x0i} &= D\xi_i + E\xi_0 \end{aligned} \tag{27}$$

where $\xi_i = [u_i\ v_i\ r_i\ \theta_i\ d_{\perp i}]^T$, $\mu_i = [u_{prop,i}\ \delta_{r,i}]^T$, D = [-1 0 0 0 0], and E = [1 0 0 0 0].

Decentralized LQR Design
Assuming that each vehicle is able to measure all local states, a linear quadratic regulator (LQR) is designed for each vehicle separately.
The LQ control law for the leader is

$$\mu_0 = K_0 \xi_0 \tag{28}$$

where

$$K_0 = -R_0^{-1} B^T P_0$$

where P_0 is a positive definite solution of the algebraic control Riccati equation

$$0 = P_0 A + A^T P_0 - P_0 B R_0^{-1} B^T P_0 + Q_0$$

The control law given in Eq. (28) minimizes a quadratic cost function

$$J_0 = \int_{t_0}^{\infty} \left(\xi_0^T Q_0 \xi_0 + \mu_0^T R_0 \mu_0 \right) dt$$

For the follower, we will ignore the coupling term, $E\xi_0$, and design an LQR for Eq. (27) to minimize a cost function,

$$J_i = \int_{t_0}^{\infty} \left(\begin{bmatrix} \xi_i^T & e_{x0i}^T \end{bmatrix}^T Q_i \begin{bmatrix} \xi_i \\ e_{x0i} \end{bmatrix} + \mu_i^T R_i \mu_i \right) dt$$

The solution to this LQR problem is

$$\mu_i = \begin{bmatrix} K_{i1} & K_{i2} \end{bmatrix} \begin{bmatrix} \xi_i \\ e_{x0i} \end{bmatrix}$$

where

$$[K_{i1} \quad K_{i2}] = -R_i^{-1}[B^T \quad 0]P_i$$

where P_i is a positive definite solution of the algebraic control Riccati equation

$$0 = P_i\begin{bmatrix} A & 0 \\ D & 0 \end{bmatrix} + \begin{bmatrix} A^T & D^T \\ 0 & 0 \end{bmatrix}P_i - P_i\begin{bmatrix} B \\ 0 \end{bmatrix}R_i^{-1}[B^T \quad 0]P_i + Q_i$$

Simulation Result

In this section, we will perform a simulation with three vehicles: a leader and two followers. In order to see usefulness and effectiveness of the design methodology, a nonlinear 3 DOF model was used for each vehicle. Because each vehicle has a trajectory to follow, formation flying is accomplished if the followers maintain a prescribed forward distance to the leader. The desired distance was set to 10m in x_i-direction for this example while the trajectories were separated by 40m. Figure 2 shows the vehicle trajectories (Triangles) and search paths (Dashed lines). Although the vehicles were initially placed at random position and orientation, they converged to their assigned search trajectory and maintained prescribed distance to the leader. Figure 3 shows formation errors. (Positive error means a follower is behind the desired position and vise versa.) The simulation result indicates that the assumptions are valid, and the controller design methodology is simple, robust, and efficient. Even if the followers lose communication with the leader, the followers would still follow the trajectory since the coupling only causes their velocity to change.

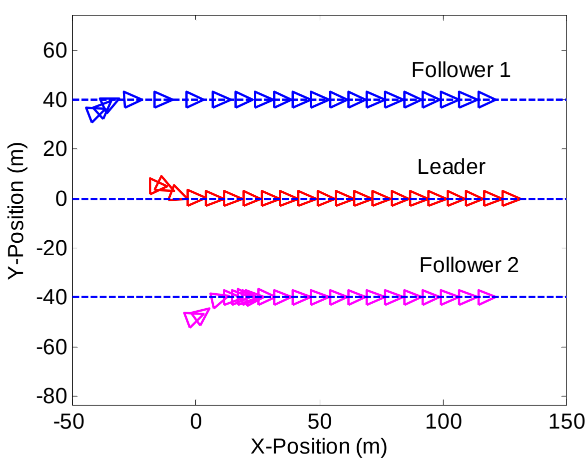

Figure 2: Vehicle Trajectories

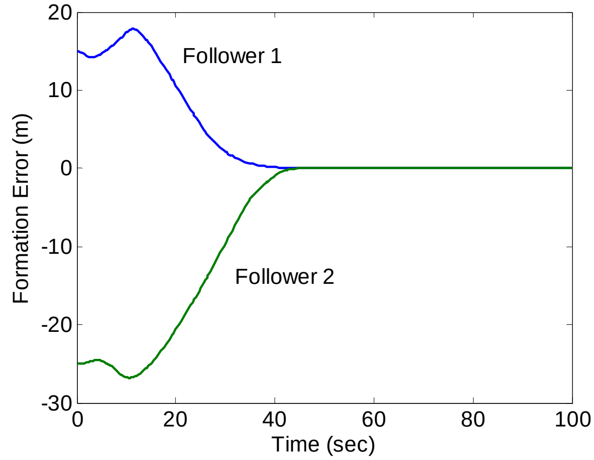

Figure 3: Formation Errors

V. CONCLUSION

In this paper, a formation control problem was formulated into a formation regulation problem, and a design methodology of a simple and robust decentralized control for a fleet of vehicles was introduced. Stability of the system was shown in terms of existence of unstable fixed modes. It was shown that the stability of the *n*-vehicle leader-follower system was achieved by stabilizing each vehicle separately. This property would allow control engineers to design an advanced controller for higher performances. As an example, a hybrid leader-follower system was designed. Simulation results showed that the design and analysis methods are valid and useful.

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