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14. ABSTRACT

Control of systems described by large-order models typically requires construction and use of reduced order models (ROM's) for the purpose of feedback controller design and implementation. However, controllers based on these ROM's can have deleterious interactions with un-modeled modes, and some sort of stability compensation is needed, such as Residual Mode Filters (RMF's). In this report we summarize our work on exponential closed loop stability using RMF's for Aero-Elastic Simulations. This can be achieved for systems with actuator dynamics, as well as for controllers designed only using a ROM of the actuator. Our applications include the control of fluid-structure interaction in a simple piston model and a three dimensional NACA wing to alleviate aero-elastic flutter.

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AFOSR FINAL REPORT:
REDUCED ORDER MODELING FOR AERO-ELASTIC SIMULATIONS
AFOSR award #: F49620-03-1-0020

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Abstract: Control of systems described by large-order models typically requires construction and use of reduced order models (ROM's) for the purpose of feedback controller design and implementation. However, controllers based on these ROM's can have deleterious interactions with un-modeled modes, and some sort of stability compensation is needed, such as Residual Mode Filters (RMF's). In this report we summarize our work on exponential closed loop stability using RMF's for Aero-Elastic Simulations. This can be achieved for systems with actuator dynamics, as well as for controllers designed only using a ROM of the actuator. Our applications include the control of fluid-structure interaction in a simple piston model and a three dimensional NACA wing to alleviate aero-elastic flutter.

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Significant Results Obtained in this Research

- One of the first uses of large scale CFD based aero-elastic simulations for control design (other than System ID-type approaches)

- Theoretical development of ROM/RMF for Stability in Very Large-Scale Model Reduction

- Development of closed loop stability bounds with ROM

- Development of Mixed ROM Approach for Fluid Structure Interactions and RMF Compensation for Stability

- Beginning development of residual state filtering (RSF) for Balanced Model Reduction and use of ROM/RSF Approach on Balanced Reduction(and Mixed ROM Reduction) of AGARD wing and F-16 Linearized Model

Graduate Students Supported on this Research:

C. William Hindman, PhD; completed May 2003; hired by AFRL-Kirtland Space Vehicles Directorate

Bryan Street, MS ECE; completed Spring 2004

Alex Matras, New PhD student Fall 2003; Space Scholar Summer 2004 and 2005 at AFRL-Kirtland; completed in Fall 2005

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Introduction

In the last few years, advances in computing power and algorithm development have allowed the creation of highly accurate nonlinear aero-elastic simulation codes [1,2]. These codes typically have fluid meshes with a very large number of nodes, and a lesser (but still large) number of structural nodes. System orders in the millions of degrees of freedom are not uncommon. Designing control systems using these models is not feasible with current techniques due to these large sizes. Instead, some method or combination of methods for reduced order modeling must be employed to create a low order model of the aeroelastic system.

The most obvious model reduction method to try on a large-scale system is eigen truncation. Reduced order models based on eigenreduction techniques are attractive to the control designer because of their common usage in structural dynamics control (see review in [3]). Many nonlinear solvers have built in parallelization and linear equation solvers than can be adapted to solve for the first few eigenvectors, which can then be used as a basis for a reduced order model. The current paradigm is that modal truncation is not a good technique to use in aeroelasticity because the fluid eigenvalues tend to be closely spaced, necessitating a large number of retained eigenmodes to achieve accurate results. Instead, emphasis has been placed on methods that map system inputs to system outputs directly, including approximate balanced truncation methods [4] and the Proper Orthogonal Decomposition (POD) method [5,6]. The impetus for this paper grew out of a desire to use a reduced order model of the fluid coupled to a reduced order modal truncation model of the structure.

Once a reduced order model is created, a controller can then be designed about it using conventional techniques. Problems can arise once the controller based on the reduced order model is applied to the full order system. Since the controller has no knowledge built into it about the unmodelled states in the full system, interactions can excite those states into instability. Previous work using modal truncation based ROMs [7] as well as coupled systems of modal based ROMs [8] has shown that by using a Residual Mode Filter (RMF) to filter out the unwanted interaction, stability can be restored to the system. These previous stability proofs rely on the modal nature of the systems and are not valid once the coupling terms created by balanced truncation methods are considered.

1 Residual Mode Filters and Actuated Systems

The Residual Mode Filter (RMF) is a technique that can be used to regain stability in a high order system driven by a reduced order controller when unmodelled interactions have caused instability. In general, RMF's work by splitting the state vector x into components x_N, x_R, and x_Q where x_N contains those states in the reduced order model (including all initially unstable states), x_r contains the remaining stable modes, and x_Q contains those modes that have been driven unstable through feedback control. The RMF is then applied to cancel the x_Q modes from interacting with the controller, thereby causing the system to regain stability.

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Now if instead of a single dynamical system there is a coupled problem, things become more complicated. Utilizing the method described in [9], the system can be split into a coupled aeroelastic response and an aerodynamic actuation dynamic. In this formulation, one system can be seen acting as an actuator on the other, and the combined system can be denoted as:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du; \quad x(0) = x_0 \\ \dot{z} = Fz + Hu_c \\ u = Lz \end{cases} \tag{1}$$

where z denotes actuator states (aerodynamic actuation dynamic) and x system states (coupled aeroelastic reponse). The system for the case we are considering can then be decomposed into modal form:

$$\begin{cases} \dot{x}_N = A_N x_N + B_N u \\ \dot{x}_R = A_R x_R + B_R u \\ \dot{x}_Q = A_Q x_Q + B_Q u \\ y = C_N x_N + C_R x_R + C_Q x_Q + Du \end{cases} \tag{2}$$

where x_N, x_R, and x_Q are defined as before. Note that all the inherently unstable modes are collected in x_N, therefore A_R and A_Q will be stable matrices (in the sense that all the eigenvalues will lie in the left half plane). An output feedback state estimator controller designed for this system will have the following form:

$$\begin{cases} \dot{\hat{x}}_N = A_N \hat{x}_N + B_N L\hat{z} + K_N(y - \hat{y}_N) \\ \hat{y}_N = C_N \hat{x}_N + DL\hat{z} \\ \dot{\hat{z}} = F\hat{z} + Hu_c \\ u_c = G_N \hat{x}_N \end{cases} \tag{3}$$

Because of how we have defined the states, this controller applied to the full order system will be stable if the x_Q terms are deleted. If the following error terms are then defined:

$$\begin{cases} e_N = \hat{x}_N - x_N \\ e_z = \hat{z} - z \end{cases} \tag{4}$$

Then the complete set of equations with the x_Q modes deleted is:

$$\begin{bmatrix} \dot{x}_N \\ \dot{e}_N \\ \dot{x}_R \\ \dot{z} \\ \dot{e}_z \end{bmatrix} = \begin{bmatrix} A_N & 0 & 0 & B_N L & 0 \\ 0 & (A_N - K_N C_N) & K_N C_R & 0 & (B_N L - K_N DL) \\ 0 & 0 & A_R & B_R L & 0 \\ HG_N & HG_N & 0 & F & 0 \\ 0 & 0 & 0 & 0 & F \end{bmatrix} \begin{bmatrix} x_N \\ e_N \\ x_R \\ z \\ e_z \end{bmatrix} \tag{5}$$

This system is by definition stable. Note in particular the stable sub-block consisting of the first 4 rows and columns of eqn. (5), we will be using this sub-block later. If we next introduce the unstable modes x_Q back into our system, some method to compensate for their instability is required. A RMF can be used to filter the output signal y being fed into the state estimator in this manner:

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$$\begin{cases}
\dot{\hat{x}}_N = A_N \hat{x}_N + B_N L \hat{z} + K_N \left(y - \hat{y}_N - \hat{y}_Q \right) \\
\hat{y}_N = C_N \hat{x}_N + DL\hat{z} \\
\dot{\hat{z}} = F\hat{z} + Hu_c \\
\dot{\hat{x}}_Q = A_Q \hat{x}_Q + B_Q L\hat{z} \\
\hat{y}_Q = C_Q \hat{x}_Q \\
u_c = G_N \hat{x}_N
\end{cases} \qquad (6)$$

By introducing a new error term e_Q defined as:

$$e_Q = \hat{x}_Q - x_Q \qquad (7)$$

a new matrix equation can be written that now incorporates the entire system:

$$\begin{bmatrix} \dot{x}_N \\ \dot{e}_N \\ \dot{x}_R \\ \dot{z} \\ \dot{x}_Q \\ \dot{e}_Q \\ \dot{e}_z \end{bmatrix} = \left[\begin{array}{cccc|c|cc} A_N & 0 & 0 & B_N L & 0 & 0 & 0 \\ 0 & (A_N - K_N C_N) & K_N C_R & 0 & 0 & -K_N C_Q & (B_N L - K_N DL) \\ 0 & 0 & A_R & B_R L & 0 & 0 & 0 \\ HG_N & HG_N & 0 & F & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & B_Q L & A_Q & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & A_Q & B_Q L \\ 0 & 0 & 0 & 0 & 0 & 0 & F \end{array} \right] \begin{bmatrix} x_N \\ e_N \\ x_R \\ z \\ x_Q \\ e_Q \\ e_z \end{bmatrix} \qquad (8)$$

Note the sub-block formed by the first four rows and columns; this is the same sub-block as in eqn. (5), and was by definition stable. Because of this, the stability of the entire matrix is seen by successively including an additional row and column of eqn. (8) and noting the diagonal nature of subsequent subsystems. Since all the inherently unstable modes were collected into A_N, A_Q is stable, and F is also stable. Therefore the entire matrix given in eqn. (8) is stable.

2 Residual Mode Filters with Reduced Order Balanced Actuators

For the actual problems of interest in this work, which consist of fluid systems acting as actuators and coupled to structural systems, it is not practical to include the entire model of the fluid system within the controller. If a ROM of the fluid is used that is based on a balanced reduction method, the actuator can be represented by:

$$\begin{cases}
\dot{z}_N = F_N z_N + F_{NR} z_R + H_N u_c \\
\dot{z}_R = F_R z_R + F_{RN} z_N + H_R u_c \\
u = L_N z_N + L_R z_R
\end{cases} \qquad (9)$$

where z_N represents those states that are retained in the ROM and z_R those states that are discarded. Note that the key difference to the work described in [8] is the inclusion of the cross-coupling terms F_{NR} and F_{RN}. Because of the nature of a balancing reduction, the influence the z_R states have on the output u and the retained states z_N is small (and can be made smaller by including more states in z_N). Because of this, it is natural to introduce a singular perturbation term in front of the z_R equation, and analyze what happens as this term is varied from zero to some small positive value:

$$\begin{cases} \dot{z}_N = F_N z_N + F_{NR} z_R + H_N u_c \\ \varepsilon \dot{z}_R = F_R z_R + F_{RN} z_N + H_R u_c \\ u = L_N z_N + L_R z_R \end{cases} \qquad (10)$$

When $\varepsilon = 0$, z_R can be solved for in terms of z_N and u_c:

$$\begin{cases} z_R = -F_R^{-1} F_{RN} z_N - F_R^{-1} H_R u_c \\ \dot{z}_N = F_N z_N + F_{NR}\left(-F_R^{-1} F_{RN} z_N - F_R^{-1} H_R u_c\right) + H_N u_c \\ u = L_N z_N + L_R\left(-F_R^{-1} F_{RN} z_N - F_R^{-1} H_R u_c\right) \end{cases} \qquad (11)$$

If the following substitutions are then made:

$$\begin{cases} \widetilde{F}_R = -L_R F_R^{-1} H_R G_N \\ \widetilde{F}'_R = -L_R F_R^{-1} F_{RN} \end{cases} \qquad (12)$$

Then the actuation equations can be written as:

$$\begin{cases} \dot{z}_N = \left(F_N - F_{NR} F_R^{-1} F_{RN}\right) z_N + \left(H_N - F_{NR} F_R^{-1} H_R\right) G_N \hat{x}_N \\ \varepsilon \dot{z}_R = F_R z_R + F_{RN} z_N + H_R u_c \\ u = \left(L_N + \widetilde{F}'_R\right) z_N + \widetilde{F}_R \hat{x}_N \end{cases} \qquad (13)$$

Based on this definition of the actuator, the control equations become:

$$\begin{cases} \dot{\hat{x}}_N = A_N \hat{x}_N + B_N L_N \hat{z}_N + K_N\left(y - \hat{y}_N - \hat{y}_Q\right) \\ \hat{y}_N = C_N \hat{x}_N \\ \dot{\hat{z}}_N = \left(F_N - F_{NR} F_R^{-1} F_{RN}\right) \hat{z}_N + \left(H_N - F_{NR} F_R^{-1} H_R\right) G_N \hat{x}_N \\ \dot{\hat{x}}_Q = A_Q \hat{x}_Q + B_Q\left(L_N + \widetilde{F}'_R\right) \hat{z}_N + B_Q \widetilde{F}_R \hat{x}_N \\ \hat{y}_Q = C_Q \hat{x}_Q \\ u_c = G_N \hat{x}_N \end{cases} \qquad (14)$$

The system (defined to be stable) with the x_Q modes deleted and $\varepsilon = 0$ is given by:

$$\dot{\omega} = A_{11} \omega \qquad (15)$$

Where:

$$\omega \equiv \begin{bmatrix} x_N \\ e_N \\ x_R \\ z_N \\ e_z \end{bmatrix} \qquad (16)$$

And:

$$A_{11} = \left[\begin{array}{cccc|c} \left(A_N + B_N \widetilde{F}_R\right) & B_N \widetilde{F}_R & 0 & B_N\left(L_N + \widetilde{F}'_R\right) & 0 \\ \left(-B_N \widetilde{F}_R\right) & \widetilde{E}_N & K_N C_R & -B_N \widetilde{F}'_R & \left(B_N L_N\right) \\ B_R \widetilde{F}_R & B_R \widetilde{F}_R & A_R & B_R\left(L_N + \widetilde{F}'_R\right) & 0 \\ Z_1 & Z_1 & 0 & Z_2 & 0 \\ \hline 0 & 0 & 0 & 0 & Z_2 \end{array}\right] \qquad (17)$$

With:

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$$\begin{cases} \tilde{E}_N = A_N - K_N C_N - B_N \tilde{F}_R \\ Z_1 = \left(H_N - F_{NR} F_R^{-1} H_R\right) G_N \\ Z_2 = F_N - F_{NR} F_R^{-1} F_{RN} \end{cases} \tag{18}$$

Proving that the system is stable with the x_Q modes added back in and an epsilon not equal to zero requires the use of the Klimushchev-Krasovskii theorem [10], formulated for the following equation:

$$\begin{cases} \dot{\omega} = A_{11} \omega + A_{12} z_R \\ \varepsilon \dot{z}_R = A_{21} \omega + A_{22} z_R \end{cases} \tag{19}$$

Given a linear singularly perturbed system of the form in eqn. (19) with A_{22} and A_{11}-$A_{12}A_{22}^{-1}A_{21}$ stable, then there exists some $\varepsilon_o>0$ such that for each $0<\varepsilon<\varepsilon_o$, eqn. (19) is stable.

We will use this lemma by writing the complete system in the form of eqn. (19), with:

$$\omega = \begin{bmatrix} x_N \\ e_N \\ x_R \\ z_N \\ x_Q \\ e_Q \\ e_z \end{bmatrix} \tag{20}$$

The matrices A_{11}, A_{12}, A_{21}, and A_{22} are then:

$$A_{11} = \left[\begin{array}{cccc|c|c|c} \left(A_N + B_N \tilde{F}_R\right) & B_N \tilde{F}_R & 0 & B_N\left(L_N + \tilde{F}_R'\right) & 0 & 0 & 0 \\ \left(-B_N \tilde{F}_R\right) & \tilde{E}_N & K_N C_R & -B_N \tilde{F}_R' & 0 & K_N C_Q & \left(B_N \tilde{F}_R'\right) \\ B_R \tilde{F}_R & B_R \tilde{F}_R & A_R & B_R\left(L_N + \tilde{F}_R'\right) & 0 & 0 & 0 \\ Z_1 & Z_1 & 0 & Z_2 & 0 & 0 & 0 \\ \hline B_Q \tilde{F}_R & B_Q \tilde{F}_R & 0 & B_Q\left(L_N + \tilde{F}_R'\right) & A_Q & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & A_Q & B_Q L_N \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & Z_2 \end{array}\right] \tag{21}$$

$$A_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad A_{21} = \begin{bmatrix} H_R G_N & H_R G_N & 0 & F_{RN} & 0 & 0 & 0 \end{bmatrix} \qquad A_{22} = F_R$$

To use the K-K theorem, we need to show that A_{11} is stable and that A_{11}- $A_{12}A_{22}^{-1}A_{21}$ is stable. The stability of A_{11} can be seen from examining A_{11} in eqn. (17) and then applying the successive partitions as shown in eqn. (21), since A_Q is defined to be stable and Z_2 is shown to be stable from eqn. (17). $A_{11} - A_{12}A_{22}^{-1}A_{21}$ is also stable since it equals A_{11}. Therefore, the entire system is stable due to the K-K theorem for some epsilon $0 < \varepsilon < \varepsilon_o$.

3 Results

This proof followed the outline for a modal based actuator given in [8], where an example using an Aeroelastic Piston model was given, which is based on the ideas developed in [11]. The same model was used, but now with a reduced order balanced model of the actuator. An output feedback state estimator controller was designed using the reduced order system and actuator that gave good results when applied to the ROM. When the same controller was applied to the FOM, the interaction with an unmodeled mode drove the response unstable. Including a RMF as in eqn. (14) causes the full order response to be stabilized, as seen in Figure 1.
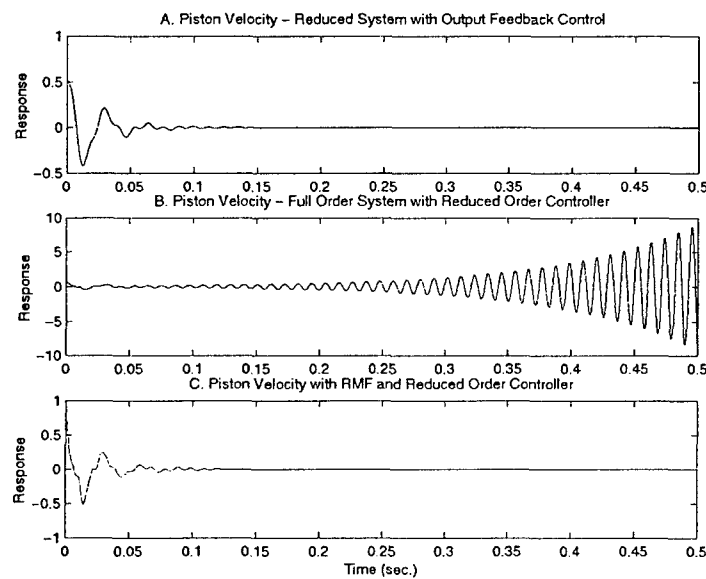

Figure 1:   Residual Mode Filter Example

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Publications Related to This Research

C. Hindman, M. Balas, and M. Lesoinne,"Exponential Stability of Controllers for Fluid-Structure Interaction Using Reduced Order System Models with Residual Mode Filters", Proceedings of OPTI 2005, Skiathos, Greece, June 2005. This paper uses the ROM/RMF structure on an actuated piston. We prove that exponential stability can be restored using RMFs to augment the ROM-based controller with actuator dynamics.

M. Balas, C. Hindman and P. Waters,"Control-Based Reduced Order Structures Models for Use in AeroServoElastic Simulations", AIAA DTE Summit, Woodland Hills, CA, Nov 2004.This summarizes our research on the ROM/RMF approach but without actuator dynamics.

Hindman, C., Balas, M. and Lesoinne, M., Exponential Stability of Controllers Using Reduced Order Actuator and System Models with Residual Mode Filters, 2000 American Control Conference, Chicago, IL, 2000.This develops a ROM/RMF approach for aero-servoelastic problems but without stability proofs. The NACA 3-d wing is used as an application here.

Lesoinne, M. and Hindman, C., A Mixed Model Reduction Technique for Aeroservoelasticity, AIAA Paper # 99-1266, 40th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, St. Louis, MO., Apr. 12-15, 1999. Some of the ideas in this work were applied to our current work in the aero-elastic problems above.