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14. ABSTRACT
The Hybrid Projected Gradient-Evolutionary Search Algorithm (HPGES) algorithm uses a specially designed evolutionary-based global search strategy to efficiently create candidate solutions in the solution space. A local projection-based gradient search algorithm is then used to improve the candidate solutions at each generation and to construct new (potentially improved) candidate solutions for the next generation of the evolutionary search. The search terminates when a certain convergence criterion is met. The details of different components of the hybrid algorithm are described in detail in the following sections.

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Hybrid Projected Gradient-Evolutionary Search Algorithm for Mixed Integer Nonlinear Optimization Problems

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Autonomous Uninhabited (or Unmanned) Air (or Aerial) Vehicles (UAVs) have grown in significance both in space exploration missions and in military applications such as surveillance and payload delivery. The tasks that UAVs are expected to perform are also growing in complexity. One plausible scenario is that of multiple vehicles of various types, capabilities, and constraints performing a complicated task requiring coordinated decision making and execution.

UAV task scheduling can be considered a general case of the Capacitated Vehicle Routing Problem with Time Windows (VRPTW). The basic version of the vehicle routing problem is the Capacitated Vehicle Routing Problem (CVRP). The CVRP is described as follows:

> n targets must be served from a unique depot. Each target needs a quantity q_i of ammunition ($i = 1, ..., n$) and a vehicle of capacity Q is available to deliver ammunition. Since the vehicle capacity is limited, the vehicle has to periodically return to the depot for reloading. In the CVRP, it is not possible to split goal delivery. Therefore, a CVRP solution is a collection of tours where each goal is visited only once and the total tour demand is at most Q. This definition may be extended to m dissimilar vehicles that are cooperating to serve the targets.

In general, there are two different approaches to solving this problem, exact solution techniques and approximate solution techniques. Various approaches are considered in the literature for finding exact solutions. These approaches are

- Dynamic Programming
- Lagrange Relaxation-Based Methods
- Column Generation
- Branch and Bound (e.g., CPLEX)

Exact techniques are found inadequate for real-world problems of realistic size due to computational complexity, and they often are complemented with heuristics. Various approximate solution techniques have been attempted with various degrees of success. Among these techniques are

- Directed local search
- Simulated annealing and tabu search
- Evolutionary Algorithms (e.g. Genetic Algorithms)

The problem with these methods is that they require initial feasible solutions (sophisticated heuristics are often developed just to generate an initial feasible solution) and have difficulties handling constraints, especially time-windows. These methods suffer from slow rates of convergence. To treat larger instances, or to compute solutions faster, heuristic methods must be used. Among the best heuristic methods are tabu search and large neighborhood search. These approaches, however, generally suffer from slow convergence.

Another approach is mathematical programming. In this approach, the problem is transformed into a Mixed Integer Linear Programming (MILP) formulation. The resulting MILP problem poses significant challenges to MILP-based solution techniques. Although much research has been devoted to finding an effective method for solving MILP problems, a general technique capable of handling online task scheduling has been elusive due to the potential for exponential explosion in large-scale problems.

Mixed Integer Nonlinear Programming (MINLP) formulations cover a broad class of problems, including MILP, and, in general, are more difficult to solve than MILP formulations. A wide variety of optimization problems arising in engineering applications can be formulated as MINLP problems. Applications of MINLP range from control of hybrid systems [1] to task scheduling [2] and other closely related combinatorial optimization problems, such as the traveling salesman problem (TSP) and path planning [3][4]. All these problems, however, pose significant challenges to MINLP-based solution techniques. Although much research has been devoted to finding an effective method for solving MINLP problems, a general technique capable of handling real-time task scheduling as well as assignment and control problems has been elusive. This is due again to the potential for combinatorial explosion in large-scale problems [5][6].

The deficiencies in applying any stand-alone optimization technique have led researchers to attempt to combine efficient components of the different methods to build hybrid optimization solutions. Integration of different features of multiple conventional optimization and heuristic techniques is an area that has sparked considerable interest in the optimization community. The main motivation behind these endeavors is the fact that, with the increase in the dimension of the problem, the conventional methods quickly become computationally intractable. The standard techniques for solving MILP problems, e.g., branch and bound, experience an exponential rise in convergence time when the number of variables is increased [6]. This rise in complexity with the increase in dimension is evident even for a moderate increase in problem size.

Grossman and Jain [7] have presented a hybrid MILP and constraint programming (CP) techniques to solve problems that are intractable if solved using either of the two techniques alone. The class of problems they tested was formulated in terms of a hybrid MILP/CP model that involves some of the MILP constraints, a reduced set of CP constraints and an equivalence relation between the MILP and CP variables. Fletcher [8] has presented another approach by outer approximation. The outer approximation scheme solves MINLP problems by a finite sequence of Nonlinear Programming (NLP) sub-problems and MINLP master problems. Other interesting attempts at hybrid solutions combine some conventional methods with heuristic or metahueristic approaches in the hope of having their strengths complement one another. Salomon [9] has combined gradient search methods with evolutionary techniques to achieve faster convergence on continuous-parameter optimization problems.

In our work, we present a new hybrid method for solving MINLPs for a class of combinatorial optimization problems formulated as follows:

$$\left\{\begin{array}{c} \min_x \ \Im(x) \\ \text{subject to}: \underline{B} \le Cx \le \overline{B} \\ \text{Where } x = \begin{pmatrix} x_c \\ x_d \end{pmatrix} \\ x_c \in \Re^{n-k} \quad x_d \in \{-1, 1\}^k \end{array}\right\} \tag{1}$$

In the following sections, the Hybrid Projected Gradient-Evolutionary Search (HPGES) algorithm that was developed to solve this problem is described in detail.

1 Hybrid Projected Gradient-Evolutionary Search Algorithm

The HPGES algorithm uses a specially designed evolutionary-based global search strategy to efficiently create candidate solutions in the solution space. A local projection-based gradient search algorithm is then used to improve the candidate solutions at each generation and to construct new (potentially improved) candidate solutions for the next generation of the evolutionary search. The search terminates when a certain convergence criterion is met. The details of different components of the hybrid algorithm are described in detail in the following sections.

1.1 Discrete Variables Relaxation

The generalized MINLP can be stated as:

$$\left\{\begin{array}{c} \min_x \ \Im(x) \\ \text{subject to}: \underline{B} \le Cx \le \overline{B} \end{array}\right\}, \quad x = \begin{pmatrix} x_c \\ x_d \end{pmatrix} \tag{2}$$

where x is the decision vector with x_c and x_d indicating continuous and discrete decision variables, respectively. $\underline{B}$ and $\overline{B}$ are, respectively, the lower and upper bounds for the constraint set C_x where C is a matrix of appropriate dimensions. We assume that $x \in \Omega$, where Ω is a bounded space.

A class of algorithms for solving Eq. (1) is based on penalty function methods. Among the earliest such methods is the one proposed in [10], where the authors observed that a binary variable $x_{d,i}$ could be replaced by a continuous unbounded variable if an appropriate constraint is added. Even though the reformulation in [10] appears attractive, it suffers from a number of serious drawbacks. Introducing a nonlinear equality constraint transforms a convex MINLP problem into a non-convex NLP problem, which in general is not easier to solve. Another approach aimed at relaxing the MINLP problem into a NLP problem is suggested in [1], where use of a penalty function and a constraint was advocated:

$$
\begin{aligned}
&\min_x \ \Im(x) + \sigma_1 \left(x_d^T x_d - \sum_{i=1}^{m} x_{d,\mathrm{i}} \right)^2 \\
&\text{subject to}: \underline{B} \le Cx \le \overline{B} \\
&\left(x_d^T x_d - \sum_{i=1}^{m} x_{d,\mathrm{i}} \right)^2 \le \sigma_2 \\
&0 \le x_{d,\mathrm{i}} \le 1, i = 1,\ldots,m
\end{aligned}
\tag{3}
$$

In Eq. (3), x is the same as in Eq. (2), where $x_d = [x_{d,1},\ldots,x_{d,\mathrm{m}}]^T$ is the relaxed binary decision variable, where the $x_{d,i}$, $1 \le i \le m$, are expected to converge to 0 or 1 (the positive penalty term $\left(x_d^T x_d - \sum_{i=1}^{m} x_{d,\mathrm{i}} \right)^2$ will have the minimum value of zero when $x_{d,i}$, $1 \le i \le m$, are either 0 or 1) and non-negative σ_1 and σ_2 are appropriately selected at each iteration. The algorithm repeatedly solves the optimization problem in Eq. (3) with an increasing penalty factor σ_1 and tighter tolerance level σ_2 until the difference between two successive iterations becomes sufficiently small and the elements of the solution vector approach 0 or 1. If the algorithm converges to a solution whose components are not 0 or 1, then the procedure is restarted from an alternative initial point. Other forms of penalty function are also suggested in the literature.

Relaxing binary variables through a combination of augmented penalty functions and additional constraints has been found to suffer from two major setbacks. First, in general, extraneous local optima will be introduced to the optimization surface within the feasible region, and hence the true optimal solution could be compromised. Second, the solution for the relaxed binary variable will converge to 0 or 1 only when the penalty factor, σ_1 in Eq. (3) for instance, approaches infinity. For the quadratic cost function in Eq. (3), however, the use of $\sigma(x_d - \mathbf{0.5})^T (x_d - \mathbf{0.5})$ (where **0.5** is the vector all of whose elements are 0.5) as the augmented penalty function, and the addition of $0 \le x_{d,i} \le 1$, $1 \le i \le m$, as the additional constraint, is found to be adequate even with moderate values of σ. This is the relaxation strategy adopted in the solution to the generalized MINLP with the graph partitioning application that is studied in this work, transforming a MINLP problem into a constrained NLP problem to be solved with the HPGES algorithm.

1.2 Projected Variable Metric Method

Unconstrained NLP problems have been studied extensively in the past, and a host of well-known techniques, including conjugate direction methods, restricted step methods, Newton-like methods, and secant methods, is reported in the literature (see [8], [11] and [12] for instance). The convergence rate of these methods is super-linear, and global convergence is guaranteed for convex and smooth functions. The solution to constrained NLP problems, however, becomes prohibitively expensive for large-scale problems. This work advocates a projection technique that has demonstrated favorable numerical properties in the solution to the generalized MINLPs. [13].

1.3 Hybrid Evolutionary-Gradient Algorithm

In this section, a hybrid evolutionary-gradient based method for MINLP problems with inequality constraints is introduced. In this approach, the evolutionary algorithm is used as an engine for global search, while the projected variable metric method is used for efficient and swift local search.

A local search algorithm starts with an initial solution and seeks better solutions in the existing candidate solution's neighborhood. The quality of the solutions obtained by a search method is directly influenced by the definition of the neighborhood space in the search process. Efficient generation of superior neighborhoods is important in order to have an effective search. Being trapped in local optima is the other factor affecting the quality of solutions generated using local search methods. Combinations of local search techniques with global search strategies such as genetic algorithms were devised to overcome such problems. These search strategies are classified as metahueristics since they act as guidance strategies for their respective local search procedures.

Genetic algorithms (GAs) are a class of guided search heuristics for the optimization process based on the gradual evolution of genes in a population of solutions represented as chromosomes. Candidate solutions, or chromosomes, are usually fixed-length integer-, real- or binary-valued strings. A selection mechanism chooses parent chromosomes to go through recombination and mutation procedures possibly to produce better candidate solutions. Improved solutions may replace some unfit members of the old population.

The steps involved in combining the two aforementioned methods to form the Hybrid Projected Gradient-Evolutionary Search (HPGES) algorithm are as follows (see Figure 1):

1. Create a random initial population. Use a Linear-programming solver (e.g., CPLEX) to ensure that at least some of the individuals in the initial population are feasible. (This would not be the case for a totally random initialization.)
2. Create new individuals by using the local improvement scheme (gradient projection) on the population of the candidate solutions. Infeasible individuals are also made feasible through the projection process.
3. Select individuals from the pool of improved feasible candidate solutions for crossover.
4. Apply convex crossover, create new candidate solutions, and update the corresponding search direction matrix for each newly created individual.
5. Select the individuals in the population pool for the next iteration.
6. Stop if convergence criterion is met. Otherwise, go to step 2.

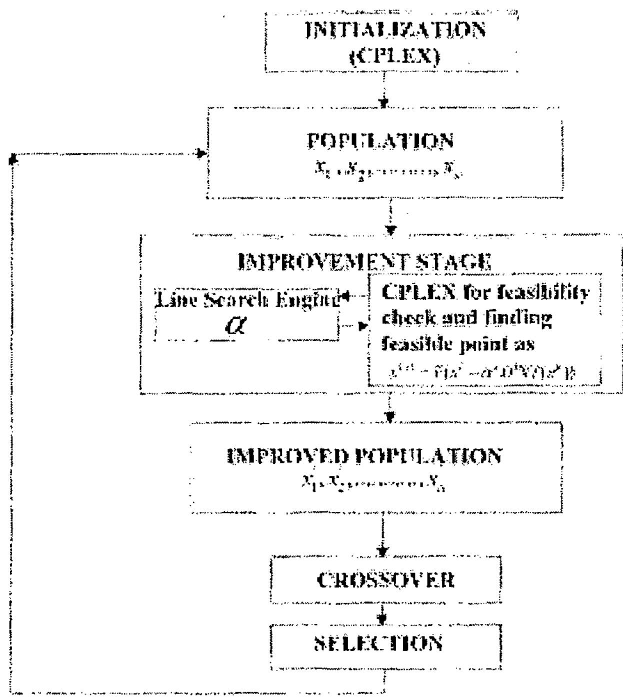

Figure 1: Hybrid Projected Gradient-Evolutionary Search Algorithms for MINLPs

To create the initial population with random feasible solutions, we use the LP solver of CPLEXR and solve the following LP problem repeatedly with a random vector L_i:

$$\left\{ \begin{array}{c} \min_x \quad L_i x \\ \text{subject to}: \underline{B} \leq Cx \leq \overline{B} \end{array} \right\} \tag{4}$$

The main idea in the improvement stage (see Figure 1) is to use the local search algorithm in order to swiftly improve the candidate solutions in the population pool. The improved feasible solutions are then presented to a selection and diversification operator (in this case, the convex crossover of the selected individuals in the population) in order to explore the solution space. A variety of GA operators can be used at each stage (see [14] for details).

1.4 Numerical Implementation Results

Although our algorithm provides a general framework for a wide range of problems formulated as MINLPs, the initial tests were done on sample graph portioning problems, and the results were compared to best reported results from the literature. Many real-world problems of interest can be formulated as graph portioning problems, which in turn can be cast into a formulation suitable for the proposed HPGES algorithm. The results discussed in the following section outline the utility of the algorithm. These results can probably be improved for particular applications by incorporating heuristic schemes suitable for each problem.

1.5 The Graph Partitioning Problem

The proposed algorithm for MINLP is tested on the multiset min-cut graph partitioning problem. In this problem, the vertices of a graph are partitioned into sets of given sizes while minimizing the weighted sum of the cut edges, that is, the edges connecting vertices in different sets. Let A be an n-by-n weight matrix associated with a directed graph with vertex set $V = \{1, 2, \ldots n\}$. Multiset min-cut graph partitioning can be represented as follows:

Given m_i, $1 \le i \le k$, partition V into k disjoint subsets so that $V = V_1 \cup V_2 \cup V_3 \ldots \cup V_k$, $|V_i| = m_i$, $1 \le i \le k$, in a way that the sum of the weights of the cut edges is minimum. For implementing the algorithm, we used random graphs with n vertices, where an edge with a random weight between any two vertices is created with probability $p = 1/\sqrt{n}$. In partitioning, we assumed that all m_i, $1 \le i \le k$, are equal. We also tested the algorithms on graphs from standard test datasets. The simulation software has been written in MATLAB, and the coded algorithm has been tested on a Pentium 4, 2.2 GHz CPU with 1 GB RAM. (For results refer to Table 1.)

2 Simulation Results and Discussion

The simulation results for randomly generated and standard benchmark problem instances are presented in Table 1. In all the instances, the HPGES algorithm was able to converge to optimal or near optimal solutions within the iteration count specified. The time taken in convergence for the main hybrid routine is also tabulated in Table 1. The results indicate that the growth in CPU time is not exponential in the growth in problem size. The results are also encouraging in the sense that the randomly generated graphs are generally ill conditioned and do not possess a structure particularly amenable to numerical implementation. Some heuristic-based methods for graph partitioning could also be included to improve the results and reduce CPU time. However, the objective of these examples is to evaluate the utility of the proposed algorithm.

Problem size	Problem type	Maximum Iterations	Number of cuts	Elapsed Time (*sec*)
20	Random	20	100	17.468
40	Random	20	400	43.672
60	Random	20	900	90.077
80	Random	20	1600	166.888
136	Benchmark	20	11	284.965
548	Benchmark	20	47	546.236

Table 1: Simulation Results with Running Times for the Graph Partitioning Problem

3 Conclusion and Future Work

To solve the online scheduling problem of cooperating UAVs, we developed a new Hybrid Evolutionary Gradient Projection algorithm for mixed integer nonlinear optimization problems with linear inequality constraints. The core engine of the algorithm is a novel local improvement scheme based on projected-gradient search. The evolutionary mechanism, on the other hand, guides the solution search out of local optima, towards regions closer to global optima. The projection algorithm enables simultaneous addition or removal of multiple constraints to or from the active constraint set, increasing the speed of convergence. The penalty function used to relax the integer variables is shown not to introduce any additional local optima inside the feasible region, hence avoiding one of the main deficiencies of the similar relaxation attempts in the past. The modified cost function and the appropriately constructed projection mechanism guarantee a feasible solution. The evolutionary component of the hybrid algorithm maintains diversity of the search regions, hence improving the possibility of finding a solution at or near a global optimum in a reasonable time. The gradient search component, on the other hand, significantly speeds up the convergence of the hybrid algorithm, compensating for the slow convergence of the evolutionary search. The hybrid algorithm also requires a smaller population as compared to conventional, pure evolutionary algorithms. Efforts to compare thoroughly the numerical properties of this new algorithm with those of other available MINLP solvers are currently underway.

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