REPORT DOCUMENTATION PAGE	Form Approved OMB NO. 0704-0188

Public Reporting Burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comment regarding this burden estimate or any other aspect of this collection of information, including suggesstions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington DC 20503

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE:	3. REPORT TYPE AND DATES COVERED Final Report 1-May-2001 - 31-Jul-2005

4. TITLE AND SUBTITLE Using Nuclear Magnetic Resonance to Assess and Optimize the Precision of Methods for Controlling Quantum Dynamics	5. FUNDING NUMBERS DAAD190110519
6. AUTHORS David G. Cory, Timothy F. Havel	8. PERFORMING ORGANIZATION REPORT NUMBER
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES Massachusetts Institute of Technology Office of Sponsored Programs Bldg. E19-750 Cambridge, MA 02139 -4307	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSORING / MONITORING AGENCY REPORT NUMBER 42393-PH-QC.20

11. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12. DISTRIBUTION AVAILIBILITY STATEMENT Approved for Public Release; Distribution Unlimited	12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The abstract is below since many authors do not follow the 200 word limit

14. SUBJECT TERMS Quantum Computing, Quantum Control, Nuclear Magnetic Resonance	15. NUMBER OF PAGES Unknown due to possible attachments
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev .2-89)
Prescribed by ANSI Std.
239-18 298-102

Report Title

Using Nuclear Magnetic Resonance to Assess and Optimize the Precision of Methods for Controlling Quantum Dynamics

ABSTRACT

The most significant achievements of this project were: (1) Development and validation of the hardware and software needed to implement "strongly modulating pulses", by which high precision quantum gates can be obtain in realistic systems of up to 12 qubits; (2) A detailed analysis and evaluation of a 3-qubit quantum Fourier transform via full quantum process tomography; (3) An extensive set of mathematical and computational techiques to accomplish these goals, including Hadamard products, the real density matrix, and methods of fitting superoperators to experimental data; (4) Creation of a 3-qubit "noiseless subsystem", of a Bell state on two 2-qubit decoherence-free subspaces (DFS), and the invention of "partial" pseudopure states which will enable us to demonstrate robust methods for controlling multi-DFS-qubit systems by NMR; (5) Implementation of several quantum chaotic maps, and the discovery that these provide a scalable approach to determining the magnitude and kind of errors present in complex quantum computations; (6) The invention of a "spin amplifier", by which entanglement can be used to enable single spin measurement, and a small-scale demonstration by NMR; (7) Experiments demonstrating that the foregoing advances enable implementation of complex entangling unitary and decoherent operations, culminating in creation of a 12-qubit CAT state.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

L. Viola, E. M. Fortunato, M. A. Pravia, E. Knill, R. Laflamme and D. G. Cory, “Experimental realization of noiseless subsystems for quantum information processing,” Science 293:2059–63 (2001).

G. Teklemariam, E. M. Fortunato, M. A. Pravia, T. F. Havel and D. G. Cory, “An NMR analog of the quantum disentanglement eraser,” Phys. Rev. Lett. 86:5845–9 (2001).

T. F. Havel, Y. Sharf, L. Viola and D. G. Cory, “Hadamard products of product operators and the design of gradient-diffusion experiments for simulating decohrence by NMR spectroscopy,” Phys. Lett. A 280:282–8 (2001).

E. M. Fortunato, L. Viola, J. Hodges, G. Teklemariam and D. G. Cory, “Implementation of universal control on a decoherence-free qubit,” New J. Phys. 4:5.1–20 (2002).

N. Boulant, E. M. Fortunato, M. A. Pravia, G. Teklemariam, D. G. Cory and T. F. Havel, “Entanglement transfer experiment in NMR quantum information processing,” Phys. Rev. A 65:024302 (2002).

E. M. Fortunato, M. A. Pravia, N. Boulant, G. Teklemariam, T. F. Havel and D. G. Cory, “Design of strongly modulating pulses to implement precise effective Hamiltonians for quantum information processing”, J. Chem. Phys. 116:7599-606 (2002).

N. Boulant, M. A. Pravia, E. M. Fortunato, T. F. Havel and D. G. Cory, “Experimental concatenation of quantum error correction with decoupling,” Quantum Information Processing, 1:135-44 (2002).

G. Teklemariam, E. M. Fortunato, M. A. Pravia, Y. Sharf, T. F. Havel and D. G. Cory, “Quantum erasers and probing classifications of entanglement via NMR,” Phys. Rev. A, 66:012309 (2002).

Y. S. Weinstein, S. Lloyd, J. V. Emerson and D. G. Cory, “Experimental implementation of the quantum baker's map,” Phys. Rev. Lett. 89:157902 (2002).

R. Laflamme, D. Cory, C. Negrevergne and L. Viola, “NMR quantum information processing and entanglement”, Quantum Inform. Processing and Comput. 2:166–76 (2002).

G. Teklermariam, E. M. Fortunato, M. A. Pravia, T. F. Havel and D. G. Cory, “Experimental investigations of decoherence on a quantum information processor”, Chaos, Solitons & Fractals 16:457-65 (2003).

T. F. Havel, “Robust procedures for converting among Kraus, Lindblad and matrix representations of quantum dynamical semigroups,” J. Math. Phys. 44:534-57 (2003).

N. Boulant, T. F. Havel, M. A. Pravia and D. G. Cory, “A robust method for estimating the Lindblad operators of a dissipative quantum process from measurements of the density operator at multiple time points”, Phys. Rev. A 67:042322 (2003).

E. M. Fortunato, L. Viola, M. A. Pravia, E. Knill, R. Laflamme, T. F. Havel and D. G. Cory, “Exploring noiseless subsystems via nuclear magnetic resonance”, Phys. Rev. A 67:062303 (2003).

G. Teklemariam, E. M. Fortunato, C. C. Lopez, J. Emerson, J.-P. Paz, T. F. Havel and D. G. Cory, “A method for modeling decoherence on a quantum information processor”, Phys. Rev. A 67:062316 (2003).

N. Boulant, K. Edmonds, J. Yang, M.A. Pravia and D.G. Cory, “Experimental demonstration of an entanglement swapping operation and improved control in NMR quantum-information processing”, Phys. Rev. A 68:032305 (2003).

M. A. Pravia, N. Boulant, J. Emerson, A. Farid, E. M. Fortunato, T. F. Havel and D. G. Cory, “Robust control of quantum information”, J. Chem. Phys. 119:9993 (2003).

J. Emerson, Y. S. Weinstein, M. Saraceno, S. Lloyd, and D. G. Cory, “Pseudo-random unitary operators for quantum information science”, Science, 302:2098-100 (2003).

S. Sinha, J. Emerson, N. Boulant, E. M. Fortunato, T. F. Havel and D. G. Cory, “Experimental simulation of spin squeezing by nuclear magnetic resonance”, Quantum Information Processing 2:433-48 (2003).

T. F. Havel, “The real density matrix”, Quantum Information Processing 1:511-38 (2003).

N. Boulant, J. Emerson, T. F. Havel, D. G. Cory and S. Furuta, “Incoherent noise in quantum information processing”, J. Chem. Phys. 121:2955-61 (2004).

Y. S. Weinstein, T. F. Havel, J. Emerson, N. Boulant, M. Saraceno, S. Lloyd and D. G. Cory, “Quantum process tomography of the quantum Fourier transform”, J. Chem. Phys. 121:6117-33 (2004).

N. Boulant, L. Viola, E. M. Fortunato and D. G. Cory, “Experimental implementation of a concatenated quantum error-correcting code”, Phys. Rev. Lett. 94:130501 (2005).

P. Cappellaro, J. Emerson, N. Boulant, C. Ramanathan, S. Lloyd and D. G. Cory, “Entanglement assisted metrology”, Phys. Rev. Lett. 94:020502 (2005).

C. Negrevergne, T. S. Mahesh, C. A. Ryan, M. Ditty, F. Cyr-Racine, W. Power, N. Boulant, T. F. Havel, D. G. Cory and R. Laflamme, “Benchmarking quantum control methods on a 12-qubit system”, Phys. Rev. Lett. 96:170501 (2006).

Number of Papers published in peer-reviewed journals: 25.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

M. D. Price, E. M. Fortunato, M. A. Pravia, C. Breen, Swami Kumaresean, G. Rosenberg and D. G. Cory, “Information transfer on an NMR quantum information processor”, Concepts in Magn. Reson. 13:151-8 (2001).

T. F. Havel and C. J. L. Doran, “Interaction and entanglement in the multiparticle spacetime algebra”, in Applications of Geometric Algebra in Computer Science and Engineering, (L. Dorst, C. J. L. Doran and J. Lasenby, eds.), Birkhauser (2002).

R. Laflamme, E. Knill, D.G. Cory, E. M. Fortunato, T. F. Havel, C. Miquel, R. Martinez, C. Negrevergne, G. Ortiz, M. A. Pravia, Y. Sharf, S. Sinha, R. Somma and L. Viola, “Introduction to NMR quantum information processing”, Los Alamos Science 27:2-37 (2002).

T. F. Havel and C. J. L. Doran, “Geometric algebra in quantum information processing”, in Quantum Information and Computation (S.J. Lomonaco, Jr. and H.E. Brandt, eds.) Contemporary Math. 305:81-100 (Am. Math. Soc., Providence RI, 2002).

T. F. Havel, D. G. Cory, S. Lloyd, N. Boulant, E. M. Fortunato, M. A. Pravia, G. Teklemariam, Y. S. Weinstein, A. Bhattacharyya and J. Hou, “Quantum information processing by nuclear magnetic resonance spectroscopy”, Am. J. Phys. 70(3):345–61 (2002).

T. F. Havel, “Metric matrix embedding in protein structure calculations, NMR data analysis, and relaxation theory”, Magn. Reson. Chem. 41:S37 (2003).

T. F. Havel and C. J. L. Doran, “A Bloch-sphere-type model for two qubits in the geometric algebra of a 6-D Euclidean vector space”, Proc. SPIE Symp. Defense & Security (vol. 5436, Quantum Information and Computation II, E. Donkor, A. R. Pirich and H. E. Brandt, eds.; 2004).

T.F. Havel, S.S. Somaroo, W. Zhang and D.G. Cory, “Geometric algebra and transition-selective implementations of the controlled-NOT gate”, Concepts in Magn. Reson. 23A:49-62 (2004).

C. Ramanathan, N. Boulant, Zhiying Chen, D. G. Cory, I. L. Chuang and M. Steffen, “NMR quantum information processing”, in Experimental Aspects of Quantum Computing (H. O. Everitt, ed.; Springer, 2004).

D. G. Cory and T. F. Havel, “Physics - Ion entanglement in quantum information processing”, Science 304:1456-1457 (2004).

Number of Papers published in non peer-reviewed journals: 10.00

(c) Papers presented at meetings, but not published in conference proceedings (N/A for none)

Posters and talks presented at meetings with T. F. Havel as (co)author:

"Quantum information science and technology, talk presented at the Advanced Materials Research Institute Symposium, Univ. of New Orleans, LA (Feb., `01).

"Exploring the quantum/classical interface by NMR", poster presented at the International Conference on Spintronics and Quantum Information Technology, Hawaii (May, `01).

"What's hot in NMR computing" talk presented at the 16th Waterloo Summer School, Univ. of Waterloo, Ontario, Canada (Jun., `01).

"Hadamard products of product operators" poster presented at the Gordon Research Conference on Magnetic Resonance, William Rogers Univ., Bristol, RI (Jun., `01).

ibid, talk presented to the Semiconductor Physics Group at the Cavendish Labs, Cambridge Univ., U.K. (Jul., `01).

"Interaction and entanglement in the multiparticle space-time algebra", talk given at the Workshop on Applied Geometrical Algebras in Science and Engineering, Cambridge, U.K. (Jul., `01).

"NMR approaches to quantum information processing," talk presented at the Intnl. Soc. for Magn. Reson. Ann. Meeting, Rhodes Greece (Aug., `01).

ibid, talk given at the Intnl. Conf. on Magn. Reson. Microscopy, Nottingham UK (Sep., `01).

"Large scale quantum computation by NMR," poster presented with D.G. Cory at the DARPA Spintronics Review, Long Beach, CA (Sep. `01).

"Quantum information science and technology," talk presented at the opening conference of the Workshop on Information Theory and Combinatorics, organized by R. Alswede, Zentrum fuer Interdisziplinaer Forschung (ZIF) der Univ. Bielefeld, FRG (Feb., `02).

"The real density matrix," poster presented at the Experimental NMR Conference (ENC), Asilomar CA (Apr., `02).

ibid, talk presented at the First Feynman Festival, Univ. of Maryland, College Park MD (Aug. `02).

ibid, talk presented at the 33rd Snowbird Conference on Progress in Quantum Electronics, Snowbird, UT (Jan. `03).

"A quantum cookbook: A set of laboratory exercises, based on NMR spectroscopy, for a first course in quantum mechanics," talk presented at the Gordon Conference on Physics Education Research: Quantum Mechanics, Mt. Holyoke College, MA (June, `02).

"Quantum dynamical semigroup tomography," talk presented at the AMS Eastern Regional Meeting, Special Session on Quantum Information (Aug., `02).

ibid, AMS/MAA/SIAM Joint Meeting, Baltimore, MD (Jan. `03).

ibid, ARO Workshop on Quantum Control (D.G. Cory & T.F. Havel, organizers), MIT, Cambridge, MA (Oct. `02).

"Density operators in the multiparticle space-time algebra," talk presented at the Applied Geometrical Algebras Meeting, organized by the Inst. for Math. & Appl., Trinity College, Cambridge UK (Sep., `02).

"Information dynamics in open quantum systems," talk presented at the conference on Information Transfer and Combinatorics, organized by R. Alswede, Zentrum fuer Interdisziplinaer Forschung (ZIF) der Univ. Bielefeld, FRG (Nov., `02).

"A robust method for estimating the Lindblad operators of a dissipative quantum process from measurements of the density operator at multiple time points," poster presented at the ESF Conference on Advances in Quantum Information Processing: From Theory to Experiment, Ettore Majorana Center, Erice, Italy (Mar. `03).

"A hierarchical model of decoherence and its NMR implementation," Pacific Inst. for the Math. Sciences, Quantum Mechanics on the

Large Scale (organized by P.C.E. Stamp, G.A. Sawatzky, A.J. Leggett, T.F. Havel, S. Popescu & R. Gill), Banf, Canada (Apr. `03).

"Representations of quantum operations in geometric algebra," invited lectures given at the NATO Advanced Study Institute on Computational Noncommutative Algebra and Applications, Il Ciocco Resort, Barga, Italy (Jul. `03).

"A Bloch-sphere-type model for two qubits in the geometric algebra of a 6-D Euclidean vector space", SPIE Symp. on Defense & Security, session on Quantum Information and Computation (Apr. `04).

"Simple but useful mathematical tools for the description and analysis of decoherence processes", Entanglement, Information and Noise, Krzyzowa, Poland (Jun. `04).

"Experimental methods for quantum control in nuclear spin systems", Quantum Information and Quantum Control, Fields Institute, Toronto, Canada (Jul. `04).

"Quantum simulation with macroscopic lattices of nuclear spins", AFOSR Workshop on Quantum Computation for Physical Modelling, Martha's Vineyard, USA (Sep. `04).

"Some connections between protein NMR spectroscopy and NMR quantum computing", Progress in Quantum Electronics, Snowbird, UT (Jan. `05).

"Reflection symmetries for multi-qubit densities", Quantum Information Processing, MIT (Jan. `05).

"Quantum information processing with nuclear spin based devices", NSTI Nanotechnology Conference and Trade Show, Anaheim (May `05).

"Quantum control of nuclear spins", CMI / EPSRC Quantum information theory and technology summer school, Belfast, Northern Ireland (Sep. `05).

Number of Papers not Published: 29.00

(d) Manuscripts

P. Cappellaro, J. S. Hodges, T. F. Havel and D. G. Cory, "Principles of control for decoherence-free subsystems", J. Chem. Phys. (in press); reprint available from http://arxiv.org/abs/quant-ph/0604203.

Number of Manuscripts: 1.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED	
Tatjana Atanasijevic	0.10	No
Nicolas Boulant	0.75	No
Paola Cappellaro	0.15	No
Hyung Joon Cho	0.12	No
Evan Fortunato	0.25	No
Marco Pravia	0.10	No
FTE Equivalent:	**1.47**	
Total Number:	**6**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED	
Anatoly Dementyev	0.50	No
FTE Equivalent:	**0.50**	
Total Number:	**1**	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
David G. Cory	0.05	No
FTE Equivalent:	**0.05**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Personnel receiving masters degrees

NAME	
Karen Ka Yan Lee	No
Total Number:	**1**

Names of personnel receiving PHDs

NAME	
Nicolas Boulant	No
Suddhasattwa Sinha	No
Paola Cappellaro	No
Evan Fortunato	No
Marco Pravia	No
Grum Teklemariam	No
Hyung Joon Cho	No
Total Number:	**7**

Names of other research staff

NAME	PERCENT_SUPPORTED	
Timothy F. Havel	0.05	No
Chandrasekhar Ramanathan	0.05	No
FTE Equivalent:	**0.10**	
Total Number:	**2**	

Sub Contractors (DD882)

Inventions (DD882)

DAAD19-01-1-0519: FINAL REPORT

Technical Summary

(1) Development and validation of the hardware and software needed to implement "strongly modulating pulses", by which high precision quantum gates can be obtain in realistic systems of up to 12 qubits:

- E. M. Fortunato, M. A. Pravia, N. Boulant, G. Teklemariam, T. F. Havel and D. G. Cory, "Design of strongly modulating pulses to implement precise effective Hamiltonians for quantum information processing", *J. Chem. Phys.* **116:7599-606** (2002).

  Abstract. We describe a method for improving coherent control through the use of detailed knowledge of the system's Hamiltonian. Precise unitary transformations were obtained by strongly modulating the system's dynamics to average out unwanted evolution. With the aid of numerical search methods, pulsed irradiation schemes are obtained that perform accurate, arbitrary, selective gates on multiqubit systems. Compared to low power selective pulses, which cannot average out all unwanted evolution, these pulses are substantially shorter in time, thereby reducing the effects of relaxation. Liquid-state nuclear magnetic resonance techniques on homonuclear spin systems are used to demonstrate the accuracy of these gates both in simulation and experiment. Simulations of the coherent evolution of a three-qubit system show that the control sequences faithfully implement the unitary operations, typically yielding gate fidelities on the order of 0.999 and, for some sequences, up to 0.9997. The experimentally determined density matrices resulting from the application of different control sequences on a three-spin system have overlaps of up to 0.99 with the expected states, confirming the quality of the experimental implementation.

- M. A. Pravia, N. Boulant, J. Emerson, A. Farid, E. M. Fortunato, T. F. Havel and D. G. Cory, "Robust control of quantum information", *J. Chem. Phys.* **119:9993** (2003).

  Abstract. Errors in the control of quantum systems may be classified as unitary, decoherent, and incoherent. Unitary errors are systematic, and result in a density matrix that differs from the desired one by a unitary operation. Decoherent errors correspond to general completely positive superoperators, and can only be corrected using methods such as quantum error correction. Incoherent errors can also be described, on average, by completely positive superoperators, but can nevertheless be corrected by the application of a locally unitary operation that "refocuses" them. They are due to reproducible spatial or temporal variations in the system's Hamiltonian, so that information on the variations is encoded in the system's spatiotemporal state and can be used to correct them. In this paper liquid-state nuclear magnetic resonance is used to demonstrate that such refocusing effects can be built directly into the control fields, where the incoherence arises from spatial inhomogeneities in the quan-

tizing static magnetic field as well as the radio-frequency control fields themselves. Using perturbation theory, it is further shown that the eigenvalue spectrum of the completely positive superoperator exhibits a characteristic spread that contains information on the Hamiltonians' underlying distribution.

(2) A detailed analysis and evaluation of a 3-qubit quantum Fourier transform via full quantum process tomography:

- Y. S. Weinstein, T. F. Havel, J. Emerson, N. Boulant, M. Saraceno, S. Lloyd and D. G. Cory, "Quantum process tomography of the quantum Fourier transform", *J. Chem. Phys.* **121:6117-33** (2004).

Abstract. The results of quantum process tomography on a three-qubit nuclear magnetic resonance quantum information processor are presented and shown to be consistent with a detailed model of the system-plus-apparatus used for the experiments. The quantum operation studied was the quantum Fourier transform, which is important in several quantum algorithms and poses a rigorous test for the precision of our recently developed strongly modulating control fields. The results were analyzed in an attempt to decompose the implementation errors into coherent (overall systematic), incoherent (microscopically deterministic), and decoherent (microscopically random) components. This analysis yielded a superoperator consisting of a unitary part that was strongly correlated with the theoretically expected unitary superoperator of the quantum Fourier transform, an overall attenuation consistent with decoherence, and a residual portion that was not completely positive although complete positivity is required for any quantum operation. By comparison with the results of computer simulations, the lack of complete positivity was shown to be largely a consequence of the incoherent errors which occurred over the full quantum process tomography procedure. These simulations further showed that coherent, incoherent, and decoherent errors can often be identified by their distinctive effects on the spectrum of the overall superoperator. The gate fidelity of the experimentally determined superoperator was 0.64, while the correlation coefficient between experimentally determined superoperator and the simulated superoperator was 0.79; most of the discrepancies with the simulations could be explained by the cumulative effect of small errors in the single qubit gates.

(3) An extensive set of mathematical and computational techniques to accomplish these goals, including Hadamard products, the real density matrix, and methods of fitting superoperators to experimental data:

- T. F. Havel, Y. Sharf, L. Viola and D. G. Cory, "Hadamard products of product operators and the design of gradient-diffusion experiments for simulating decohrence by NMR spectroscopy," *Phys. Lett. A* **280:282–8** (2001).

Abstract. An extension of the product operator formalism of NMR is introduced, which uses the Hadamard matrix product to describe many simple spin 1/2 relaxation processes. The utility of this formalism is illustrated by deriving NMR gradient-diffusion experiments to simulate several decoherence models

of interest in quantum information processing, along with their Lindblad and Kraus representations.

- T. F. Havel, “Robust procedures for converting among Kraus, Lindblad and matrix representations of quantum dynamical semigroups,” *J. Math. Phys.* **44:534-57** (2003).

  Abstract. Given a quantum dynamical semigroup expressed as an exponential superoperator acting on a space of *N*-dimensional density operators, eigenvalue methods are presented by which canonical Kraus and Lindblad operator sum representations can be computed. These methods provide a mathematical basis on which to develop novel algorithms for *quantum process tomography* –the statistical estimation of superoperators and their generators – from a wide variety of experimental data. Theoretical arguments and numerical simulations are presented which imply that these algorithms will be quite robust in the presence of random errors in the data.

- N. Boulant, T. F. Havel, M. A. Pravia and D. G. Cory, “A robust method for estimating the Lindblad operators of a dissipative quantum process from measurements of the density operator at multiple time points”, Phys. Rev. A 67:042322 (2003).

  Abstract. We present a robust method for quantum process tomography, which yields a set of Lindblad operators that optimally fit the density operators measured at a sequence of time points. The benefits of this method are illustrated using a set of liquid-state nuclear magnetic resonance measurements on a molecule containing two coupled hydrogen nuclei which are sufficient to fully determine its relaxation superoperator. It was found that the complete positivity constraint, which is necessary for the existence of the Lindblad operators, was also essential for obtaining a robust fit to the measurements. The general approach taken here promises to be broadly useful in studying dissipative quantum processes in many of the diverse experimental systems currently being developed for quantum-information processing purposes.

- T. F. Havel, “The real density matrix”, *Quantum Information Processing* **1:511-38** (2003).

  Abstract. We introduce a nonsymmetric real matrix which contains all the information that the usual Hermitian density matrix does, and which has exactly the same tensor product structure. The properties of this matrix are analyzed in detail in the case of multi-qubit (e.g., spin = 1/2) systems, where the transformation between the real and Hermitian density matrices is given explicitly as an operator sum, and used to convert the essential equations of the density matrix formalism into the real domain.

(4) **Creation of a 3-qubit "noiseless subsystem", of a Bell state on two 2-qubit decoherence-free subspaces (DFS), and the invention of "partial" pseudopure states which will enable us to demonstrate robust methods for controlling multi-DFS-qubit systems by NMR:**

- L. Viola, E. M. Fortunato, M. A. Pravia, E. Knill, R. Laflamme and D. G. Cory, "Experimental realization of noiseless subsystems for quantum information processing," *Science* **293:2059–63** (2001).

  Abstract. We realize the basic noiseless subsystem that preserves one bit of quantum information against general collective errors. Liquid-state nuclear magnetic resonance techniques were used to encode the information into the noiseless subsystem of three carbon spins and to expose them to a complete set of engineered collective noise processes. The achieved fidelities demonstrate both the robustness of the protected information and an actual improvement in error-correcting an entire class of error models. Our results open novel practical prospects for reliable quantum information processing.

- E. M. Fortunato, L. Viola, J. Hodges, G. Teklemariam and D. G. Cory, "Implementation of universal control on a decoherence-free qubit," *New J. Phys.* **4:5.1–20** (2002).

  Abstract. We demonstrate storage and manipulation of one qubit encoded into a decoherence-free subspace (DFS) of two nuclear spins using liquid state nuclear magnetic resonance techniques. The DFS is spanned by states that are unaffected by arbitrary collective phase noise. Encoding and decoding procedures reversibly map an arbitrary qubit state from a single data spin to the DFS and back. The implementation demonstrates the robustness of the DFS memory against engineered dephasing with arbitrary strength as well as a substantial increase in the amount of quantum information retained, relative to an un-encoded qubit, under both engineered and natural noise processes. In addition, a universal set of logical manipulations over the encoded qubit is also realized. Although intrinsic limitations prevent maintenance of full noise tolerance during quantum gates, we show how the use of dynamical control methods at the encoded level can ensure that computation is protected with finite distance. We demonstrate noise-tolerant control over a DFS qubit in the presence of engineered phase noise significantly stronger than observed from natural noise sources.

- N. Boulant, M. A. Pravia, E. M. Fortunato, T. F. Havel and D. G. Cory, "Experimental concatenation of quantum error correction with decoupling," *Quantum Information Processing*, **1:135-44** (2002).

  Abstract. We experimentally explore the reduction of decoherence via concatenating quantum error correction (QEC) with decoupling in liquid-state NMR quantum information processing. Decoupling provides an efficient means of suppressing decoherence from noise sources with long correlation times, and then QEC can be used more profitably for the remaining noise sources.

- E. M. Fortunato, L. Viola, M. A. Pravia, E. Knill, R. Laflamme, T. F. Havel and D. G. Cory, "Exploring noiseless subsystems via nuclear magnetic resonance", *Phys. Rev. A* **67:062303** (2003).

  Abstract. Noiseless subsystems offer a general and efficient method for protecting quantum information in the presence of noise that has symmetry properties. A paradigmatic class of error models displaying nontrivial symmetries emerges under collective noise behavior, which implies a permutationally invariant interaction between the system and the environment. We expand our previous investigation of the noiseless subsystem idea (L. Viola et al., *Science* **293**, 2059, 2001) by reporting and analyzing NMR experiments that demonstrate the preservation of a qubit encoded in a three-qubit noiseless subsystem for general collective noise. A complete set of input states is used to determine the superoperator for the implemented one-qubit process and to confirm that the fidelity of entanglement is improved for a large, noncommutative set of engineered errors. To date, this is the largest set of error operators that has been successfully corrected for by any quantum code.

- N. Boulant, L. Viola, E. M. Fortunato and D. G. Cory, "Experimental implementation of a concatenated quantum error-correcting code", *Phys. Rev. Lett.* **94:130501** (2005).

  Abstract. Concatenated coding provides a general strategy to achieve the desired level of noise protection in quantum information processing. We report the implementation of a concatenated quantum error-correcting code able to correct phase errors with a strong correlated component. The experiment was performed using liquid-state nuclear magnetic resonance techniques on a four spin subsystem of labeled crotonic acid. Our results show that concatenation between active and passive quantum error correction is a practical tool to handle realistic noise involving both independent and correlated errors.

- P. Cappellaro, J. S. Hodges, T. F. Havel, D. G. Cory, "Principles of Control for Decoherence-Free Subsystems", J. Chem. Phys. (in press); reprint available from http://arxiv.org/abs/quant-ph/0604203.

  Abstract. Decoherence-Free Subsystems (DFS) are a powerful means of protecting quantum information against noise with known symmetry properties. Although Hamiltonians theoretically exist that can implement a universal set of logic gates on DFS encoded qubits without ever leaving the protected subsystem, the natural Hamiltonians that are available in specific implementations do not necessarily have this property. Here we describe some of the principles that can be used in such cases to operate on encoded qubits without losing the protection offered by the DFS. In particular, we show how dynamical decoupling can be used to control decoherence during the unavoidable excursions outside of the DFS. By means of cumulant expansions, we show how the fidelity of quantum gates implemented by this method on a simple two-physical-qubit DFS depends on the correlation time of the noise responsible for decoherence. We further show by means of numerical simulations how our previously introduced "strongly modulating pulses" for NMR quantum information processing can permit high-fidelity operations on multiple DFS encoded qubits in practice, provided that the rate at which the system can be modulated is fast compared to the correlation time of the noise. The principles thereby illustrated are expected to be broadly applicable to many implementations of quantum information processors based on DFS encoded qubits.

(5) Implementation of several quantum chaotic maps, and the discovery that these provide a scalable approach to determining the magnitude and kind of errors present in complex quantum computations:

- Y. S. Weinstein, S. Lloyd, J. V. Emerson and D. G. Cory, "Experimental implementation of the quantum baker's map," *Phys. Rev. Lett.* **89:157902** (2002).

  Abstract. This Letter reports on the experimental implementation of the quantum baker's map via a three bit nuclear magnetic resonance quantum information processor. The experiments tested the sensitivity of the quantum chaotic map to perturbations. In the first experiment, the map was iterated forward and then backward to provide benchmarks for intrinsic errors and decoherence. In the second set of experiments, the least significant qubit was perturbed in between the iterations to test the sensitivity of the quantum chaotic map to controlled perturbations. These experiments can be used to investigate existing theoretical predictions for quantum chaotic dynamics.

- J. Emerson, Y. S. Weinstein, M. Saraceno, S. Lloyd, and D. G. Cory, "Pseudo-random unitary operators for quantum information science", *Science*, **302:2098-100** (2003).

  Abstract. In close analogy to the fundamental role of random numbers in classical information theory, random operators are a basic component of quantum information theory. Unfortunately, the implementation of random unitary operators on a quantum processor is exponentially hard. Here we introduce a method for generating pseudo-random unitary operators that can reproduce those statistical properties of random unitary operators most relevant to quantum information tasks. This method requires exponentially fewer resources, and hence enables the practical application of random unitary operators in quantum communication and information processing protocols. Using a nuclear magnetic resonance quantum processor, we were able to realize pseudo-random unitary operators that reproduce the expected random distribution of matrix elements.

(6) The invention of a "spin amplifier", by which entanglement can be used to enable single spin measurement, and a small-scale demonstration by NMR:

- P. Cappellaro, J. Emerson, N. Boulant, C. Ramanathan, S. Lloyd and D. G. Cory, "Entanglement assisted metrology", *Phys. Rev. Lett.* **94:020502** (2005).

  Abstract. We propose a new approach to the measurement of a single spin state, based on nuclear magnetic resonance (NMR) techniques and inspired by the coherent control over many-body systems envisaged by quantum information processing. A single target spin is coupled via the magnetic dipolar inter-

action to a large ensemble of spins. Applying radio frequency pulses, we can control the evolution so that the spin ensemble reaches one of two orthogonal states whose collective properties differ depending on the state of the target spin and are easily measured. We first describe this measurement process using quantum gates; then we show how equivalent schemes can be defined in terms of the Hamiltonian and thus implemented under conditions of real control, using well established NMR techniques. We demonstrate this method with a proof of principle experiment in ensemble liquid state NMR and simulations for small spin systems.

(7) Experiments demonstrating that the foregoing advances enable implementation of complex entangling unitary and decoherent operations, culminating in creation of a 12-qubit CAT state:

- G. Teklemariam, E. M. Fortunato, M. A. Pravia, T. F. Havel and D. G. Cory, "An NMR analog of the quantum disentanglement eraser," *Phys. Rev. Lett.* **86:5845–9** (2001).

  Abstract. We report the implementation of a three-spin quantum disentanglement eraser on a liquid-state NMR quantum information processor. A key feature of this experiment was its use of pulsed magnetic field gradients to mimic projective measurements. This ability is an important step towards the development of an experimentally controllable system which can simulate any quantum dynamics, both coherent and decoherent.

- N. Boulant, E. M. Fortunato, M. A. Pravia, G. Teklemariam, D. G. Cory and T. F. Havel, "Entanglement transfer experiment in NMR quantum information processing," *Phys. Rev. A* **65:024302** (2002).

  Abstract. We report the implementation of an entanglement transfer on a four-qubit liquid-state nuclear-magnetic resonance quantum information processor. This consists of creating an (pseudopure) entangled state among two directly coupled spins, and then transferring this two-spin state to another pair of spins whose direct interactions are negligible. Such transfers are expected to be an essential operation in scalable quantum computer architectures, and provide a useful benchmark for the coherent control available in specific implementations.

- G. Teklemariam, E. M. Fortunato, M. A. Pravia, Y. Sharf, T. F. Havel and D. G. Cory, "Quantum erasers and probing classifications of entanglement via NMR," *Phys. Rev. A* **66:012309** (2002).

  Abstract. We report the implementation of two- and three-spin quantum erasers using nuclear magnetic resonance (NMR). Quantum erasers provide a means of manipulating quantum entanglement, an important resource for quantum information processing. Here, we first use a two-spin system to illustrate the essential features of quantum erasers. The extension to a three-spin "disentanglement eraser" shows that entanglement in a subensemble can be recovered if a proper measurement of the ancillary system is carried out. Finally, we use the same pair of orthogonal decoherent operations used in quantum erasers to probe the two classes of entanglement in tripartite quantum systems: the Greenberger-Horne-Zeilinger state and the W state. A detailed

presentation is given of the experimental decoherent control methods that emulate the loss of phase information in strong measurements, and the use of NMR decoupling techniques to implement partial trace operations.

- G. Teklemariam, E. M. Fortunato, C. C. Lopez, J. Emerson, J.-P. Paz, T. F. Havel and D. G. Cory, "A method for modeling decoherence on a quantum information processor", *Phys. Rev. A* **67:062316** (2003).

Abstract. We develop and implement a method for modeling decoherence processes on an N-dimensional quantum system that requires only an N2-dimensional quantum environment and random classical fields. This model offers the advantage that it may be implemented on small quantum-information processors in order to explore the intermediate regime between semiclassical and fully quantum models. We consider in particular $\sigma_z\sigma_z$ system-environment couplings which induce coherence (phase) damping, although the model is directly extendable to other coupling Hamiltonians. Effective, irreversible phase damping of the system is obtained by applying an additional stochastic Hamiltonian on the environment alone, periodically redressing it and thereby irreversibly randomizing the system phase information that has leaked into the environment as a result of the coupling. This model is exactly solvable in the case of phase damping, and we use this solution to describe the model's behavior in some limiting cases. In the limit of small stochastic phase kicks the system's coherence decays exponentially at a rate that increases linearly with the kick frequency. In the case of strong kicks we observe an effective decoupling of the system from the environment. We present a detailed implementation of the method on a nuclear magnetic resonance quantum-information processor.

- N. Boulant, K. Edmonds, J. Yang, M.A. Pravia and D.G. Cory, "Experimental demonstration of an entanglement swapping operation and improved control in NMR quantum information processing", *Phys. Rev. A* **68:032305** (2003).

Abstract. We demonstrate the implementation of an entanglement swapping operation on a four-qubit liquid-state nuclear-magnetic-resonance (NMR) quantum-information processor. We use this experiment as a benchmark to illustrate the progress made in the field of quantum control using strongly modulating pulses and a correction scheme for removing distortions introduced by the nonlinearities in the transmitter and probe circuits. The advances include compensating for incoherent errors caused by the spatial variation of the system Hamiltonian in the NMR sample. The goal of these control sequences is to cause the collapse of the Kraus operator-sum representation of the super-operator into one unitary operator so that the ensemble appears to evolve as one coherent whole.

- S. Sinha, J. Emerson, N. Boulant, E. M. Fortunato, T. F. Havel and D. G. Cory, "Experimental simulation of spin squeezing by nuclear magnetic resonance", *Quantum Information Processing* **2:433-48** (2003).

Abstract. We report on the experimental simulation of spin squeezing using a liquid-state NMR quantum information processor. This was done by identifying

the energy levels within the symmetric subspace of a system of n spin-1/2 nuclei with the energy levels of the simulated spin-(n/2) system. The results obtained for our simulations of spin-1 and spin-3/2 systems are consistent with earlier theoretical studies of spin squeezing, and illustrate interesting relations between the degree of squeezing and the strength of the correlations among the underlying spin--1/2 particles.

- C. Negrevergne, T. S. Mahesh, C. A. Ryan, M. Ditty, F. Cyr-Racine, W. Power, N. Boulant, T. F. Havel, D. G. Cory and R. Laflamme, "Benchmarking quantum control methods on a 12-qubit system", *Phys. Rev. Lett.* **96:170501** (2006).

  Abstract. In this Letter, we present an experimental benchmark of operational control methods in quantum information processors extended up to 12 qubits. We implement universal control of this large Hilbert space using two complementary approaches and discuss their accuracy and scalability. Despite decoherence, we were able to reach a 12-coherence state (or a 12-qubit pseudopure cat state) and decode it into an 11 qubit plus one qutrit pseudopure state using liquid state nuclear magnetic resonance quantum information processors.