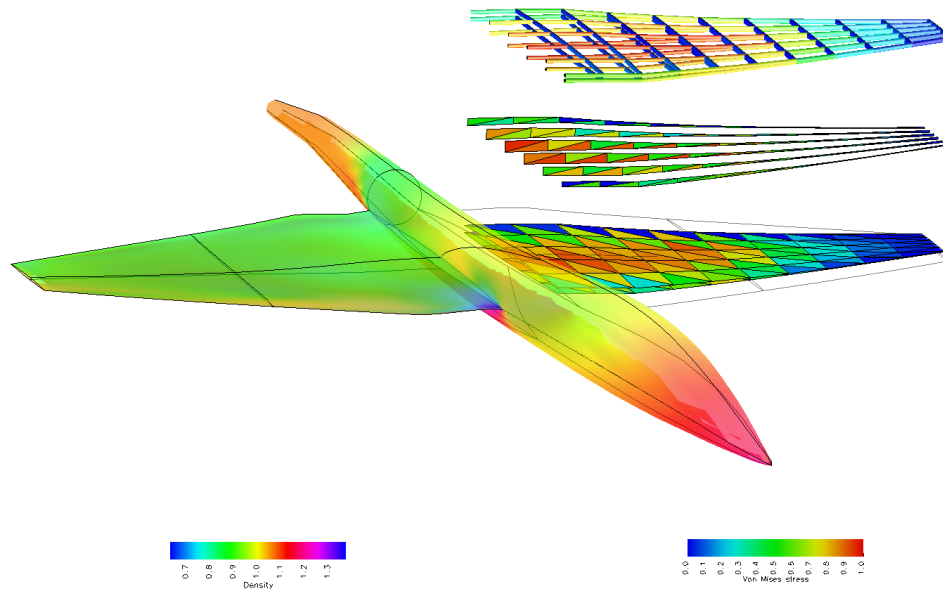# AIAA 2002–5402

# Complete Configuration Aero-Structural Optimization Using a Coupled Sensitivity Analysis Method

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# Complete Configuration Aero-Structural Optimization Using a Coupled Sensitivity Analysis Method

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**This paper focuses on the demonstration of a new integrated aero-structural design method for aerospace vehicles. The approach combines an aero-structural analysis solver, a coupled aero-structural adjoint solver, a geometry-based analysis and design integration strategy, and an efficient gradient-based optimization algorithm. The aero-structural solver ensures highly accurate solutions by using high-fidelity models for both disciplines as well as a high-fidelity coupling procedure. The Euler equations are solved for the aerodynamics and a detailed finite element model is used for the primary structure. The coupled aero-structural adjoint solution is used to calculate the needed sensitivities of aerodynamic and structural cost functions with respect to both aerodynamic shape and structural variables. The geometric outer mold line (OML) serves not only as an interface between the two disciplines for both the state and costate systems, but also as an interface between the numerical optimization algorithm and the high-fidelity analyses. Another set of design variables parameterizes a structure of fixed topology. Kreisselmeier–Steinhauser functions are used to reduce the number of structural constraints in the problem. Sample results comparing a fully coupled aero-structural design with a more traditional sequential optimization are presented.**

## Introduction

During the past decade the advancement of numerical methods for the analysis of complex engineering problems such as those found in fluid dynamics and structural mechanics has reached a mature stage: many difficult numerically intensive problems are now readily solved with modern computer facilities. In fact, the aircraft design community is increasingly using computational fluid dynamics (CFD) and computational structural mechanics (CSM) tools to replace traditional approaches based on simplified theories and wind tunnel testing. With the advancement of these numerical analysis methods well underway, the focus for engineers is shifting toward integrating these analysis tools into numerical design procedures.

Despite revolutionary accomplishments in single-discipline applications [23, 24], progress towards the development of high-fidelity, multidisciplinary design optimization (MDO) methods has been slow. The level of coupling between disciplines is highly problem dependent and significantly affects the choice of algorithm. Multiple difficulties also arise from the wide

variety of design problems: an approach that is applicable to one problem may not be compatible with another.

An important feature that characterizes the various solution strategies for MDO problems is the allowable level of disciplinary autonomy in the analysis and optimization components. Excellent discussions of these issues are presented by Sobieski and Haftka [28], and Alexandrov and Lewis [4]. The allowable level of disciplinary autonomy is usually inversely proportional to the bandwidth of the interdisciplinary coupling. Thus, for highly coupled problems it may be necessary to resort to fully integrated MDO, while for more weakly coupled problems, modular strategies may hold an advantage in terms of ease of implementation. With these constraints in mind, a number of ideas for solving complex MDO problems have been developed. These ideas include multilevel optimization strategies [3, 15], collaborative optimization [6, 17, 9], individual discipline feasible methods [8], as well as tightly coupled optimization procedures. The main difference between the different MDO strategies is the degree of coupling that is required between the disciplines in both the analysis and the optimization procedures.

In the particular case of high-fidelity aero-structural optimization, the coupling between disciplines has a very high bandwidth. Furthermore, the values of the

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objective functions and constraints depend on highly coupled multidisciplinary analyses (MDA). As a result, we believe that a tightly coupled MDO environment is more appropriate for aero-structural optimization.

Another important consideration when selecting a numerical design strategy is to choose between gradient- and non-gradient-based approaches. Some combination of these two approaches is likely to emerge as the leading alternative in which discontinuities in the design space, such as changes in the structural topology, are treated with a non-gradient-based procedure, while efficient convergence to a local minimum is achieved with a gradient approach. The present paper employs a gradient-based strategy that enables the use of hundreds or even thousands of design parameters to achieve near optimal shape-structure combinations. This level of design detail — which is arguably necessary at the end of the preliminary design stage — cannot be treated by non-gradient methods when the analyses involve the solution of a high-fidelity aero-structural system.

In contrast with emerging single-discipline design methodologies such as aerodynamic shape optimization or structural optimization, aero-structural design has traditionally been carried out in a cut-and-try basis. Aircraft designers have a pre-conceived idea of the shape of an "optimal" load distribution and then tailor the jig shape of the structure so that the deflected wing shape under a 1-g load gives the desired load distribution. While this approach may suffice for conventional transport aircraft, where there is considerable accumulated experience, in the case of either new planform concepts or new flight regimes, the lack of experience combined with the complexities of aero-structural interactions can lead to designs that are far from optimal.

This is certainly the case in the design of both small and large supersonic transports, where simple beam theory models of the wing cannot be used to accurately describe the behavior of the wing structure. In some cases, these aircraft must even cruise for significant portions of their flight at different Mach numbers. In addition, a variety of studies show that supersonic transports exhibit a range of undesirable aeroelastic phenomena due to the low bending and torsional stiffness that result from wings with low thickness to chord ratio. These phenomena can only be suppressed when aero-structural interactions are taken into account at the preliminary design stage [5].

Unfortunately, the modeling of the participating disciplines in most of the work that has appeared so far has remained at a relatively low level. While useful at the conceptual design stage, lower-order models cannot accurately represent a variety of nonlinear phenomena such as wave drag, which can play an important role in the search for the optimum design. An exception to low-fidelity modeling is the

recent work by Giunta [11] and by Maute et al. [22] where aero-structural sensitivities are calculated using higher-fidelity models.

The ultimate objective of our work is to develop an MDO framework for high-fidelity analysis and optimization of aircraft configurations. The framework is built upon prior work by the authors on aero-structural high-fidelity sensitivity analysis [25, 18, 19]. The objective of this paper is to present the current capability of this framework and to demonstrate it by performing a simplified aero-structural design of a supersonic business jet configuration.

This paper presents a tightly coupled approach to high-fidelity aero-structural MDO that uses CFD and CSM. The following sections begin with the description of the aircraft optimization problem we propose to solve. We then introduce the general formulation of the sensitivity equations followed by the specific case of the adjoint equations for the aero-structural system. A detailed study of the accuracy and efficiency of the aeroelastic sensitivity information is also presented for validation purposes. Finally, we present results of the application of our sensitivity analysis method to the full aero-structural optimization of a supersonic business jet and compare the results with the more traditional approach of sequential discipline optimizations where we highlight the fact that only closely coupled optimization frameworks can arrive at the true optimum of the system.

## Aircraft Optimization Problem

For maximum lift-to-drag ratio, it is a well-known result from classical aerodynamics that a wing must exhibit an elliptic lift distribution. For aircraft design, however, it is usually not the lift-to-drag ratio we want to maximize but an objective function that reflects the overall mission of the particular aircraft. Consider, for example, the Breguet range formula for jet-powered aircraft

$$\text{Range} = \frac{V}{c} \frac{C_L}{C_D} \ln \frac{W_i}{W_f}, \tag{1}$$

where $V$ is the cruise velocity and $c$ is the thrust specific fuel consumption of the powerplant. $C_L/C_D$ is the ratio of lift to drag, and $W_i/W_f$ is the ratio of initial and final cruise weights of the aircraft.

The Breguet range equation expresses a trade-off between the drag and the empty weight of the aircraft and constitutes a reasonable objective function to use in aircraft design. If we were to parameterize a design with both aerodynamic and structural variables and then maximize the range for a fixed initial cruise weight, subject to stress constraints, we would obtain a lift distribution similar to the one shown in Figure 1.

This optimum lift distribution trades off the drag penalty associated with unloading the tip of the wing, where the loading contributes most to the maximum
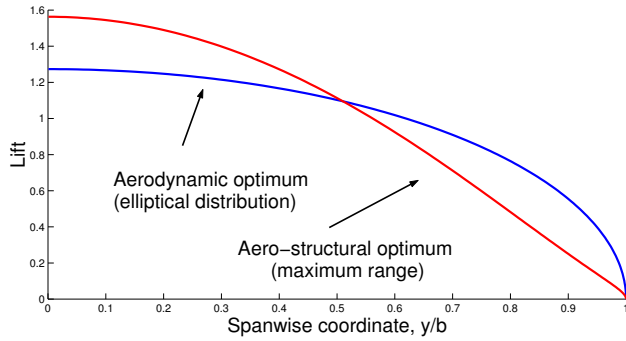
Fig. 1: Elliptic vs. aero-structural optimum lift distribution.



Fig. 2: Natural laminar flow supersonic business jet configuration.

stress at the root of the wing structure, in order to reduce the weight. The end result is an increase in range when compared to the elliptically loaded wing that results from an increased weight fraction $W_i/W_f$. The result shown in Figure 1 illustrates the need for taking into account the coupling of aerodynamics and structures when performing aircraft design.

The aircraft configuration used in this work is the supersonic business jet shown in Figure 2. This configuration is being developed by the ASSET Research Corporation and is designed to achieve a large percentage of laminar flow on the low-sweep wing, resulting in decreased friction drag [16]. The aircraft is to fly at Mach 1.5 and have a range of 5,300 nautical miles.

Detailed mission analysis for this aircraft has determined that one count of drag ($\Delta C_D = 0.0001$) is worth 310 pounds of empty weight. This means that to optimize the range of the configuration we can minimize the objective function

$$ I = \alpha C_D + \beta W, \qquad (2) $$

where $C_D$ is the drag coefficient, $W$ is the structural weight in pounds and $\alpha/\beta = 3.1 \times 10^6$.

The aircraft design is parameterized using two types of design variables. The first type of variable modifies the OML of the configuration while the second type of variable controls the sizing of underlying structure.

To perform gradient-based optimization, we need the sensitivities of the objective function (2) with respect to all the design variables. Since this objective function is a linear combination of the drag coefficient and the structural weight, its sensitivity can be written as

$$ \frac{\mathrm{d}I}{\mathrm{d}x_n} = \alpha \frac{\mathrm{d}C_D}{\mathrm{d}x_n} + \beta \frac{\mathrm{d}W}{\mathrm{d}x_n}. \qquad (3) $$

The sensitivity of the structural weight, $\mathrm{d}W/\mathrm{d}x_n$, is trivial, since the weight calculation is independent of the aero-structural solution. This gradient is calculated analytically for the structural thickness variables and by finite differences for the OML variables. The drag coefficient sensitivity, $\mathrm{d}C_D/\mathrm{d}x_n$, is not trivial since it does depend on the aero-structural solution. Details of the methodology used to compute coupled sensitivities are presented in the next section.

The OML design variables can be applied to any of the components used to define the aircraft geometry. For each wing-like component (main wing, canard, horizontal tail, etc.), the shape is modified at a number of pre-specified airfoil sections. Each of these sections may be independently modified while the spanwise resolution can be controlled by the number and position of the sections. The shape modifications to the airfoils are linearly lofted between stations. Various types of design variables may be applied to the airfoils: twist, leading and trailing edge droop, and Hicks–Henne bump functions, among others. The Hicks–Henne functions are of the form

$$ b(\zeta) = x_n \left[ \sin \left( \pi \zeta^{\frac{\log \frac{1}{2}}{\log t_1}} \right) \right]^{t_2}, \qquad (4) $$

where $t_1$ is the location of the maximum of the bump in the range $0 \leq \zeta \leq 1$ at $\zeta = t_1$, since the maximum occurs when $\zeta^\alpha = 1/2$, where $\alpha = \log(1/2)/\log t_1$. The parameter $t_2$ controls the width of the bump. The advantage of these functions is that when they are applied to a smooth airfoil, that airfoil remains smooth.

The structural design variables are the thicknesses of the structural finite elements. The topology of the structure remains unchanged, i.e., the number of spars and ribs and their position are fixed throughout the optimization. However, because the OML determines the location of the nodes of the structural model, variations of the OML have an effect on the depth of the spars and ribs of the wing box.

Among the constraints to be imposed, the most obvious one is that during cruise the lift must always equal the weight of the aircraft. In our optimization problem we constrain the $C_L$ by periodically adjusting the angle-of-attack of the aircraft within the aero-structural solution until the desired lift is obtained within a pre-specified tolerance. Otherwise, OML design changes would quickly result in lower drag coefficients simply because of reduced lift. Some design problems may require that the objective function be

minimized over a range of operating conditions (multipoint design). In these situations, the appropriate lift constraint would be imposed at each design point using the same procedure.

In addition to maintaining the $C_L$, the stresses are also constrained so that the yield stress of the material is not exceeded at a number of load conditions. There are typically thousands of finite elements describing the structure of the aircraft, and it can become computationally very costly to treat these constraints separately. The difficulty of the problem is that even though there are efficient ways of computing sensitivities of a few functions with respect to many design variables, and of computing sensitivities of many functions with respect to a few design variables, there is no known efficient method for computing sensitivities of many functions with respect to many design variables. Thus, we are left to choose between treating a large number of design variables or being able to handle multiple cost functions and constraints.

In the case of most structural design problems, the preferred approach is the direct method, where one calculates the sensitivities of the stress in each element independently, while limiting the number of design parameters to lower the total cost of computing sensitivities. The number of sensitivity analyses needed to compute the complete sensitivity matrix scales independently of the number of stress constraints but linearly with respect to the number of design parameters. Unfortunately, the most efficient approach for aerodynamics problems is quite different: the number of aerodynamic cost/constraint functions is relatively small, while the number of necessary shape design parameters is very large. It is then more efficient to compute gradients via the adjoint method, whose cost scales linearly with the number of cost/constraint functions but is independent of the number of design parameters. The coupled aero-structural problem requires a compromise between these two approaches. However, without a viable strategy to reduce the need for a large number of aerodynamic design variables we are left with the option of trying to reduce the number of independent structural constraints.

For this reason, we lump the individual element stresses using Kreisselmeier–Steinhauser (KS) functions. In the limit, all element stress constraints can be lumped into a single KS function, thus minimizing the cost of a large-scale aero-structural design cycle. Suppose that we have the following constraint for each structural finite element,

$$g_m = 1 - \frac{\sigma_m}{\sigma_y} \geq 0, \tag{5}$$

where $\sigma_m$ is the element von Mises stress and $\sigma_y$ is the yield stress of the material. The corresponding KS function is defined as

$$\text{KS}(g_m) = -\frac{1}{\rho} \ln \left( \sum_m e^{-\rho g_m} \right). \tag{6}$$

This function represents a lower bound envelope of all the constraint inequalities and $\rho$ is a positive parameter that expresses how close this bound is to the actual minimum of the constraints. This constraint lumping method is conservative and may not achieve the same optimum that a problem treating the constraints separately would. However, the use of KS functions has been demonstrated and it constitutes a viable alternative, being effective in optimization problems with thousands of constraints [2].

Having defined our objective function, design variables and constraints, we can now summarize the aircraft design optimization problem as follows:

$$
\begin{aligned}
&\text{minimize} &&I = \alpha C_D + \beta W \\
&x_n \in \mathbb{R}^n && \\
&\text{subject to} &&C_L = C_{L_T} \\
& && \text{KS} \geq 0 \\
& && x_n \geq x_{n_{\min}}.
\end{aligned}
$$

The stress constraints in the form of KS functions must be enforced by the optimizer for aerodynamic loads corresponding to a number of flight and dynamic load conditions. Finally, a minimum gage is specified for each structural element thickness.

## Analytic Sensitivity Analysis

Our main objective is to calculate the sensitivity of a multidisciplinary function of interest with respect to a number of design variables. The function of interest can be either the objective function or any of the constraints specified in the optimization problem. In general, such functions depend not only on the design variables, but also on the physical state of the multidisciplinary problem. Thus we can write the function as

$$I = I(x_n, y_i), \tag{7}$$

where $x_n$ represents the vector of design variables and $y_i$ is the state variable vector.

For a given vector $x_n$, the solution of the governing equations of the multidisciplinary system yields a vector $y_i$, thus establishing the dependence of the state of the system on the design variables. We denote these governing equations by

$$\mathcal{R}_k(x_n, y_i(x_n)) = 0. \tag{8}$$

The first instance of $x_n$ in the above equation indicates the fact that the residual of the governing equations may depend *explicitly* on $x_n$. In the case of a CFD solver, for example, changing the surface shape results in different values of the residual for at least the mesh

points closest to the surface, even if the solution is not recomputed. By solving the governing equations we determine the state, $y_i$, which depends *implicitly* on the design variables through the solution of the system. These equations may be nonlinear, in which case the usual procedure is to drive residuals, $\mathcal{R}_k$, to zero using an iterative method.

Since the number of equations must equal the number of state variables, the ranges of the indices $i$ and $k$ are the same, i.e., $i, k = 1, \ldots, N_{\mathcal{R}}$. In the case of a structural solver, for example, $N_{\mathcal{R}}$ is the number of unconstrained degrees of freedom, while for a CFD solver, $N_{\mathcal{R}}$ is the number of mesh points multiplied by the number of state variables stored at each point. In the more general case of a multidisciplinary system, $\mathcal{R}_k$ represents *all* the governing equations of the different disciplines, including their coupling.
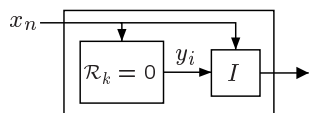


Fig. 3: Schematic representation of the governing equations ($\mathcal{R}_k = 0$), design variables ($x_n$), state variables ($y_i$), and objective function ($I$), for an arbitrary system.

A graphical representation of the system of governing equations is shown in Figure 3, with the design variables $x_n$ as the inputs and $I$ as the output. The two arrows leading to $I$ illustrate the fact that the objective function typically depends on the state variables and may also be an explicit function of the design variables.

As a first step toward obtaining the derivatives that we ultimately want to compute, we use the chain rule to write the total sensitivity of $I$ as

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} + \frac{\partial I}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}x_n}, \qquad (9)$$

for $i = 1, \ldots, N_{\mathcal{R}}$, $n = 1, \ldots, N_x$. Index notation is used to denote the vector dot products. It is important to distinguish the total and partial derivatives in this equation. The partial derivatives can be directly evaluated by simply varying the denominator and re-evaluating the function in the numerator while keeping everything else constant. The total derivatives, however, require the solution of the multidisciplinary problem. Thus, all the terms in the total sensitivity equation (9) are inexpensively computed except for $\mathrm{d}y_i/\mathrm{d}x_n$.

Since the governing equations must always be satisfied, the total derivative of the residuals (8) with respect to any design variable must also be zero. Expanding the total derivative of the governing equations with respect to the design variables we can write,

$$\frac{\mathrm{d}\mathcal{R}_k}{\mathrm{d}x_n} = \frac{\partial \mathcal{R}_k}{\partial x_n} + \frac{\partial \mathcal{R}_k}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}x_n} = 0, \qquad (10)$$

for all $i, k = 1, \ldots, N_{\mathcal{R}}$ and $n = 1, \ldots, N_x$. This expression provides us with the means to compute the total sensitivity of the state variables with respect to the design variables. By rewriting equation (10) as

$$\frac{\partial \mathcal{R}_k}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}x_n} = -\frac{\partial \mathcal{R}_k}{\partial x_n}, \qquad (11)$$

we can solve for $\mathrm{d}y_i/\mathrm{d}x_n$ and substitute this result into the total derivative equation (9), to obtain

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} - \underbrace{\frac{\partial I}{\partial y_i}\left[\frac{\partial \mathcal{R}_k}{\partial y_i}\right]^{-1}}_{-\Psi_k}\overbrace{\frac{\partial \mathcal{R}_k}{\partial x_n}}^{-\,\mathrm{d}y_i/\,\mathrm{d}x_n}. \qquad (12)$$

The inversion of the Jacobian $\partial \mathcal{R}_k/\partial y_i$ is not necessarily calculated explicitly. In the case of large iterative problems neither this matrix nor its factorization are usually stored due to their prohibitive size.

The approach where we first calculate $\mathrm{d}y_i/\mathrm{d}x_n$ using equation (11) and then use the result in the expression for the total sensitivity (12) is called the *direct* method. Note that solving for $\mathrm{d}y_i/\mathrm{d}x_n$ requires the solution of the matrix equation (11) *for each design variable $x_n$*. A change in the design variable affects only the right-hand side of the equation, so for problems where the matrix $\partial \mathcal{R}_k/\partial y_i$ can be explicitly factorized and stored, solving for multiple right-hand-side vectors by back substitution would be relatively inexpensive. However, for very large systems — such as the ones encountered in CFD — the matrix $\partial \mathcal{R}_k/\partial y_i$ is never factorized explicitly and the system of equations requires an iterative solution which is usually as costly as solving the governing equations. When we multiply this cost by the number of design variables, the total cost for calculating the sensitivity vector may become unacceptable.

Returning to the total sensitivity equation (12), we observe that there is an alternative option when computing the total sensitivity $\mathrm{d}I/\mathrm{d}x_n$. The auxiliary vector $\Psi_k$ can be obtained by solving the *adjoint equations*

$$\frac{\partial \mathcal{R}_k}{\partial y_i}\Psi_k = -\frac{\partial I}{\partial y_i}. \qquad (13)$$

The vector $\Psi_k$ is usually called the *adjoint vector* and is substituted into equation (12) to find the total sensitivity. In contrast with the direct method, the adjoint vector does not depend on the design variables, $x_n$, but instead depends on the function of interest, $I$.

We can now see that the choice of solution procedure (direct vs. adjoint) to obtain the total sensitivity (12) has a substantial impact on the cost of sensitivity analysis. Although all the partial derivative terms are the same for both the direct and adjoint methods, the order of the operations is not. Notice that for any number of functions, $I$, we can compute $\mathrm{d}y_i/\mathrm{d}x_n$ once for

each design variable (direct method). Alternatively, for an arbitrary number of design variables, we can compute $\Psi_k$ once for each function (adjoint method).

The cost involved in calculating sensitivities using the adjoint method is therefore practically independent of the number of design variables. After having solved the governing equations, the adjoint equations are solved only once for each $I$. Moreover, the cost of solution of the adjoint equations is similar to that of the solution of the governing equations since they are of similar complexity and the partial derivative terms are easily computed.

Therefore, if the number of design variables is greater than the number of functions for which we seek sensitivity information, the adjoint method is computationally more efficient. Otherwise, if the number of functions to be differentiated is greater than the number of design variables, the direct method would be a better choice.

The adjoint method has been widely used for both structural sensitivity analysis [1] and aerodynamic shape optimization [13, 14].

## Aero-Structural Adjoint Equations

Although the theory we have just presented is applicable to multidisciplinary systems, provided that the governing equations for all disciplines are included in $\mathcal{R}_k$, we now explicitly discuss the sensitivity analysis of multidisciplinary systems, using aero-structural optimization as an example. This example illustrates the fundamental computational cost issues that motivate our choice of strategy for sensitivity analysis. The following equations and discussion can easily be generalized for cases with additional disciplines.

In the aero-structural case we have coupled aerodynamic ($\mathcal{A}_k$) and structural ($\mathcal{S}_l$) governing equations, and two sets of state variables: the flow state vector, $w_i$, and the vector of structural displacements, $u_j$. In the following expressions, we split the vectors of residuals, states and adjoints into two smaller vectors corresponding to the aerodynamic and structural systems

$$\mathcal{R}_{k'} = \left[ \begin{array}{c} \mathcal{A}_k \\ \mathcal{S}_l \end{array} \right], \quad y_{i'} = \left[ \begin{array}{c} w_i \\ u_j \end{array} \right], \quad \Psi_{k'} = \left[ \begin{array}{c} \psi_k \\ \phi_l \end{array} \right]. \quad (14)$$

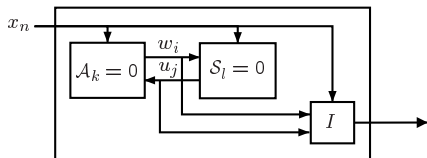Figure 4 shows a diagram representing the coupling in this system.



Fig. 4: Schematic representation of the aero-structural system.

Using this new notation, the adjoint equations (13) for the case of the aero-structural system can be written as

$$\left[ \begin{array}{cc} \dfrac{\partial \mathcal{A}_k}{\partial w_i} & \dfrac{\partial \mathcal{A}_k}{\partial u_j} \\ \dfrac{\partial \mathcal{S}_l}{\partial w_i} & \dfrac{\partial \mathcal{S}_l}{\partial u_j} \end{array} \right]^T \left[ \begin{array}{c} \psi_k \\ \phi_l \end{array} \right] = - \left[ \begin{array}{c} \dfrac{\partial I}{\partial w_i} \\ \dfrac{\partial I}{\partial u_j} \end{array} \right]. \quad (15)$$

Note that the residual sensitivity matrix in this equation is identical to that of the Global Sensitivity Equations (GSE) first presented by Sobieski [27]. This matrix, in addition to containing the diagonal terms that appear when we solve the single discipline adjoint equations, also has off-diagonal terms expressing the sensitivity of one discipline to the state variables of the other. The details of the partial derivative terms in this matrix and the right-hand side (for cases when $I = C_D$ and $I = \text{KS}$) of this equation (15) have been previously published and will not be repeated here [18, 19].

Since the factorization of the full matrix in the coupled-adjoint equations (15) would be extremely costly, our approach uses an iterative solver, much like the one used for the aero-structural solution, where the adjoint vectors are *lagged* and the two different sets of equations are solved separately. For the calculation of the adjoint vector of one discipline, we use the adjoint vector of the other discipline from the previous iteration, i.e., we solve

$$\underbrace{\frac{\partial \mathcal{A}_k}{\partial w_i} \psi_k = -\frac{\partial I}{\partial w_i}}_{\text{Aerodynamic adjoint}} - \frac{\partial \mathcal{S}_l}{\partial w_i} \tilde{\phi}_l, \quad (16)$$

$$\underbrace{\frac{\partial \mathcal{S}_l}{\partial u_j} \phi_l = -\frac{\partial I}{\partial u_j}}_{\text{Structural adjoint}} - \frac{\partial \mathcal{A}_k}{\partial u_j} \tilde{\psi}_k, \quad (17)$$

where $\tilde{\psi}_k$ and $\tilde{\phi}_l$ are the lagged aerodynamic and structural adjoint vectors respectively. Upon convergence, the final result given by this system is the same as that of the original coupled-adjoint equations (15). We call this the *lagged-coupled adjoint* (LCA) method for computing sensitivities of multidisciplinary systems. Note that these equations look like the single discipline adjoint equations for the aerodynamic and structural solvers, with the addition of forcing terms on the right-hand side that contain the off-diagonal terms of the residual sensitivity matrix. This allows us to use existing single-discipline adjoint sensitivity analysis methods with only small modifications. Note also that, even for more than two disciplines, this iterative solution procedure is nothing more than the well-known block-Jacobi method.

Once both adjoint vectors have converged, we can compute the final sensitivities of the objective function by using the following expression

$$\frac{dI}{dx_n} = \frac{\partial I}{\partial x_n} + \psi_k \frac{\partial \mathcal{A}_k}{\partial x_n} + \phi_l \frac{\partial \mathcal{S}_l}{\partial x_n}, \quad (18)$$

which is the coupled version of the total sensitivity equation (12).

Note again, that the details of the partial derivative terms in the LCA equations (16) and (17) and the total sensitivity equation (18) can be found in previous publications [18, 19].

For the aero-structural optimization problem at hand the aerodynamic portion is usually characterized by a single objective function and at most a few aerodynamic constraints, but a large number of design variables. On the other hand, the structural portion of the optimization problem is characterized by a large number of constraints: the stress in each element of the finite-element model cannot exceed the material yield stress for a number of load conditions. Constrained gradient optimization methods generally require that the user provide the gradient of both the cost function and each nonlinear constraint with respect to all of the design variables in the problem. Using the adjoint approach, the evaluation of the gradient of each constraint would require an independent coupled solution of a large adjoint system. Since the number of structural constraints is similar to the number of design variables in the problem ($\mathcal{O}(10^3)$ or larger), the usefulness of the adjoint approach could be put in question.

Both of the remaining alternatives, the direct and finite-difference methods, prove overly costly since they both require a number of solutions that is comparable to the number of design variables. In the absence of other choices that can efficiently evaluate the gradient of a large number of constraints with respect to a large number of design variables, it is necessary to reduce the size of the problem either through a reduction in the number of design variables or through a reduction in the number of nonlinear constraints.

The reason for the choice of the KS functions to lump the structural constraints now becomes clear. By employing KS functions, the number of structural constraints for the problem can be reduced from $\mathcal{O}(10^3)$ to just a few. Since the KS function is a lower bound envelope of all the constraint inequalities, this dramatic reduction in the number of structural constraints can in theory lead to conservative designs. In our experience and that of other researchers [2], the degree of conservativeness added by the KS functions is small.

# Results

In this section we present the results of the application of our sensitivity calculation method to the problem of aero-structural design of a supersonic, natural laminar flow, business jet. Before describing the results of our design experience, we present the aero-structural analysis framework and the results of a sensitivity validation study.
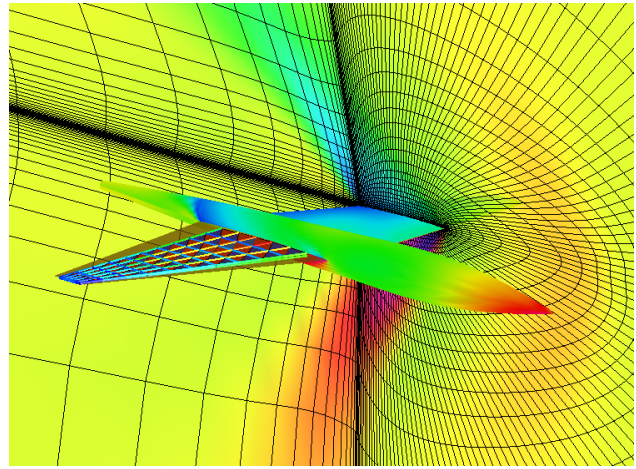


Fig. 5: Aero-structural model and solution of the supersonic business jet configuration, showing a slice of the grid and the internal structure of the wing.

**Aero-Structural Analysis**

The coupled-adjoint procedure was implemented as a module that was added to the aero-structural design framework previously developed by the authors [25, 18]. The framework consists of an aerodynamic analysis and design module (which includes a geometry engine and a mesh perturbation algorithm), a linear finite-element structural solver, an aero-structural coupling procedure, and various pre-processing tools that are used to setup aero-structural design problems. The multi-disciplinary nature of this solver is illustrated in Figure 5 where we can see the aircraft geometry, the flow mesh and solution, and the primary structure inside the wing.

The aerodynamic analysis and design module, SYN107-MB [23], is a multiblock parallel flow solver for both the Euler and the Reynolds Averaged Navier-Stokes equations that has been shown to be accurate and efficient for the computation of the flow around full aircraft configurations [26]. An aerodynamic adjoint solver is also included in this package in order to perform aerodynamic shape optimization in the absence of aero-structural interaction.

The structural analysis package is FESMEH, a finite element solver developed by Holden [12]. The package is a linear finite-element solver that incorporates two element types and computes the structural displacements and stresses of wing structures. Although this solver is not as general as some commercially-available packages, it is still representative of the challenges involved in using large models with tens of thousands of degrees of freedom. High-fidelity coupling between the aerodynamic and the structural analysis programs is achieved using a linearly consistent and conservative scheme [25, 7].

The structural model of the wing can be seen in Figure 5 and is constructed using a wing box with six spars evenly distributed from 15% to 80% of the chord.

Ribs are distributed along the span at every tenth of the semispan. A total of 640 finite elements were used in the construction of this model. Appropriate thicknesses of the spar caps, shear webs, and skins were chosen according to the expected loads for this design.
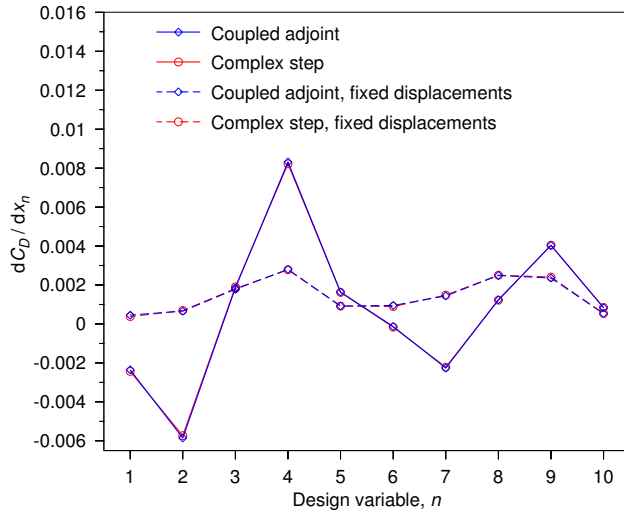
**Aero-Structural Sensitivity Validation**



Fig. 6: Sensitivities of the drag coefficient with respect to shape perturbations.

To gain confidence in the effectiveness of the aero-structural coupled-adjoint sensitivities for use in design optimization, we must ensure that the values of the gradients are accurate. For validation purposes, we use four sets of sensitivities. Results from the adjoint method are compared to the exact discrete value of these sensitivities using the complex-step derivative approximation [20, 21].

In this sensitivity study two different functions are considered: the aircraft drag coefficient, $C_D$, and the KS function (6). The sensitivities of these two quantities with respect to both OML shape design variables and structural design variables are computed and discussed.

*$C_D$ with Respect to OML Variables*

The values of the aero-structural sensitivities of the drag coefficient with respect to shape perturbations are shown in Figure 6. The ten shape perturbations were chosen to be Hicks–Henne bumps distributed chordwise on the upper surface of two adjacent airfoils around the quarter span. The plot shows very good agreement between the coupled-adjoint and the complex-step results, with an average relative error between the two of only 3.5%. Note that all these sensitivities are *total* sensitivities in the sense that they account for the coupling between aerodynamics and structures.

To verify the need for taking the coupling into account, the same set of sensitivities was calculated for fixed structural displacements, where the displacement

field is frozen after the aero-structural solution. This is similar to assuming that the wing, after the initial aeroelastic deformation, is held rigid as far as the computation of sensitivities is concerned. The calculation of the sensitivities only takes into account variations related to the aerodynamics. Figure 6 shows that the single-system sensitivities exhibit significantly lower magnitudes and even opposite signs for many of the design variables, when compared with the coupled sensitivities. The use of single-discipline sensitivities would clearly lead to erroneous design decisions.
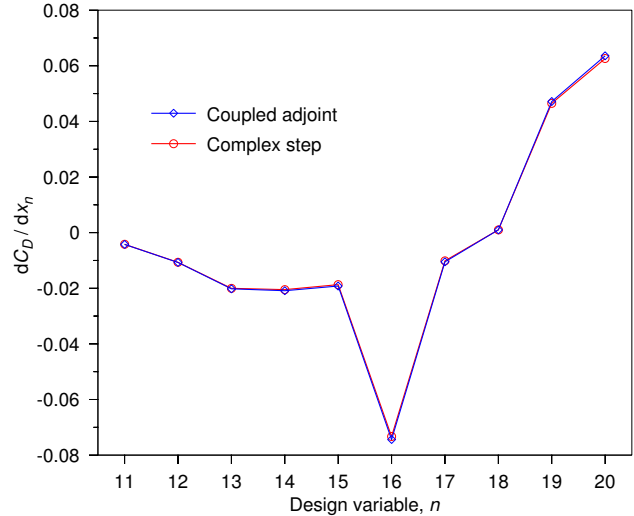


Fig. 7: Sensitivities of the drag coefficient with respect to structural thicknesses.

*$C_D$ with Respect to Thickness Variables*

Figure 7 also shows the sensitivity of the drag coefficient, this time with respect to the thicknesses of five skin groups and five spar groups distributed along the span. The agreement in this case is even better; the average relative error is only 1.6%. Even though these are sensitivities with respect to internal structural variables that do not modify the jig OML, the non-zero values in Figure 7 demonstrate that coupled sensitivity analysis is needed.

*KS with Respect to OML and Thickness Variables*

The sensitivities of the KS function with respect to the two sets of design variables described above are shown in Figures 8 and 9. The results show that the coupled-adjoint sensitivities are extremely accurate, with average relative errors of 2.9% and 1.6%. In Figure 9 we observe that the sensitivity of the KS function with respect to the first structural thickness is much higher than the remaining sensitivities. This markedly different magnitude is due to the fact that this particular structural design variable corresponds to the thickness of the top and bottom skins of the wing bay closest to the root, where the stress is the highest.

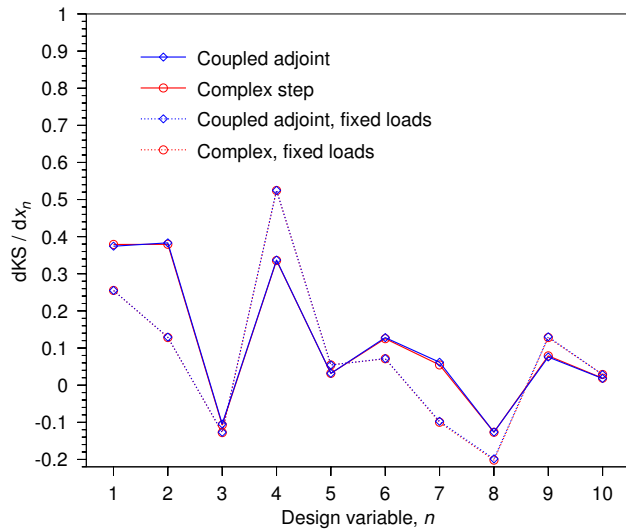The sensitivities of the KS function for fixed loads

Fig. 8: Sensitivities of the KS function with respect to shape perturbations.
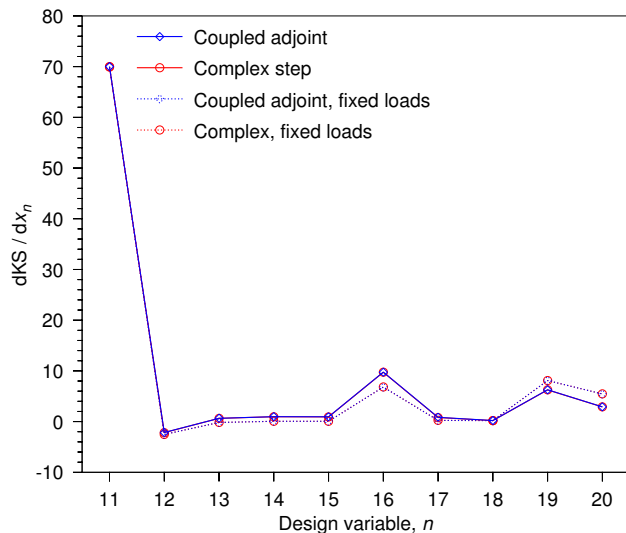


Fig. 9: Sensitivities of the KS function with respect to structural thicknesses.

are also shown in Figures 8 and 9. Using the complex-step method, these sensitivities were calculated by calling only the structural solver after the initial aero-structural solution. The approach is equivalent to using just equations (17, 18) without the partial derivatives of $\mathcal{A}_k$. The difference in these sensitivities when compared to the coupled ones is not as dramatic as in the fixed displacements case shown in Figure 6, but it is still significant.

### Computational Efficiency

The cost of calculating a gradient vector using either the finite-difference or the complex-step methods is expected to be linearly dependent on the number of design variables. This expectation is confirmed in Figure 10 where the gradient calculation times are shown for an increasing number of design variables. The time

axis is normalized with respect to the time required for a single aero-structural solution (98 seconds on 9 processors of an SGI Origin 2000). The symbols indicate timings measured from actual calculations: 2,000 design variables were tested for the coupled-adjoint method.
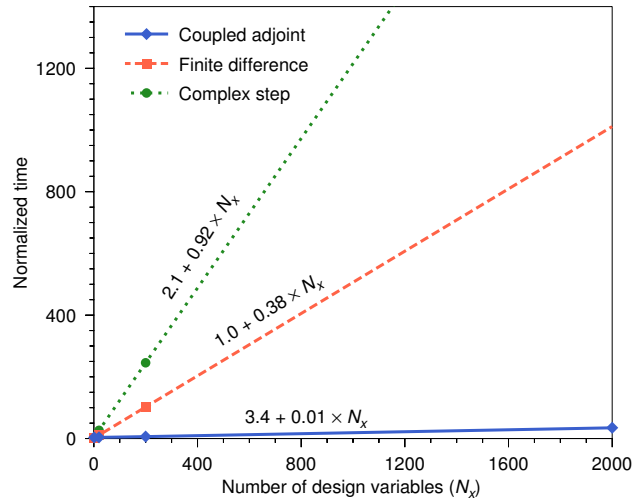


Fig. 10: Computational time vs. number of design variables for finite differencing, complex step and coupled adjoint. The time is normalized with respect to the time required for one aero-structural solution.

The cost of a finite-difference gradient evaluation can be linearly approximated by the equation $1.0 + 0.38 \times N_x$, where $N_x$ is the number of design variables. Notice that one might expect this method to incur a computational cost equivalent to one aero-structural solution per additional design variable. The cost per additional design variable is lower than unity because each additional aero-structural calculation does not start from a uniform flow-field initial condition, but from the previously converged solution.

The same applies to the cost of the complex-step method. Because the function evaluations require complex arithmetic, the cost of the complex step method is, on average, 2.4 times higher than that of finite differencing. However, this cost penalty is worthwhile since there is no need to find an acceptable step size *a priori*, as is the case for finite-difference approximations [20, 21].

The cost of computing sensitivities using the coupled-adjoint procedure is in theory independent of the number of variables. Using our implementation, however, some of the partial derivatives in the total sensitivity equation (18) are calculated using finite differences involving mesh perturbations and therefore, there is a small dependence on the number of variables. The line representing the cost of the coupled adjoint in Figure 10 has a slope of 0.01 which is between one and two orders of magnitude less than the slope for the other two lines.

The constant terms for the straight lines in Figure 10

represent the upfront cost of each procedure when no sensitivities are required. For the finite-difference case, this is equivalent to one aero-structural solution, and hence the constant is 1.0. When performing the aero-structural solution using complex arithmetic, the cost rises to 2.1 times the real arithmetic solution. The cost of computing the coupled-adjoint vectors (without computing the gradients) is 3.4. This cost includes the aero-structural solution, which is necessary before solving the adjoint equations, and hence the aero-structural adjoint computation alone incurs a cost of 2.4. Since the solution of the aero-structural adjoint equation incurs a computational cost similar to that of the aero-structural analysis, the equivalent of 1.4 aero-structural analyses is spent in the computation of the forcing terms introduced in equations (16) and (17). In particular, the last term in equation (17) requires that the full CFD mesh be perturbed for each of the degrees of freedom on the surface of the CSM mesh. The calculation of this term can increase the computational cost for very large structural models.

The main conclusion still remains that the cost of computing sensitivities with respect to hundreds or even thousands of variables is acceptable when using the coupled-adjoint approach, while it is impractical to use finite-differences, the complex-step method, or the direct method for such a large number of design variables.

**Aero-Structural Design**

The objective in this optimization is to solve the optimization problem that we previously described, i.e.,

$$\begin{aligned} \text{minimize} \quad & I = \alpha C_D + \beta W \\ x_n \in \mathbb{R}^n \\ \text{subject to} \quad & C_L = C_{L_T} \\ & \text{KS} \geq 0 \\ & x_n \geq x_{n_{\min}}. \end{aligned}$$

In our example the value of $C_D$ corresponds to that of the cruise condition, which has a target lift coefficient of 0.1. The structural stresses, in the form of the KS function, correspond to a single maneuver condition, for which $C_{L_T} = 0.2$.

All optimization work is carried out using the nonlinear constrained optimizer NPSOL [10]. Euler calculations are performed on a wing-body 36-block mesh that is constructed from the decomposition of a $193 \times 33 \times 49$ C-H mesh. During the process of the optimization, all flow evaluations are converged to 5.3 orders of magnitude of the average density residual and the $C_L$ constraint is achieved to within $10^{-6}$.

In order to parameterize the shape of the aircraft, we have chosen sets of design variables that apply to both the wing and the fuselage. The wing shape is modified by the design optimization procedure at six defining stations uniformly distributed from the side-of-body

to the tip of the wing. The shape modifications of these defining stations are linearly lofted to a zero value at the previous and next defining stations. On each defining station, the twist, the leading and trailing edge camber distributions, and five Hicks–Henne bump functions on both the upper and lower surfaces are allowed to vary. The leading and trailing edge camber modifications are not applied at the first defining station. This yields a total of 76 OML design variables on the wing. Planform modifications, which are permitted by our software, were not used in the present calculations. Planform optimization is only meaningful if additional disciplines and constraints are taken into account.

The shape of the fuselage is parameterized in such a way that its camber is allowed to vary while the total volume remains constant. This is accomplished with 9 bump functions evenly distributed in the streamwise direction starting at the 10% fuselage station. Fuselage nose and trailing edge camber functions are added to the fuselage camber distribution in a similar way to what was done with the wing sections.

The structural sizing is accomplished with 10 design variables, which correspond to the skin thicknesses of the top and bottom surfaces of the wing. Each group is formed by the plate elements located between two adjacent ribs. All structural design variables are constrained to exceed a pre-specified minimum gage value.

The complete configuration is therefore parameterized with a total of 97 design variables. As mentioned in an earlier section, the cost of aero-structural gradient information using our coupled-adjoint method is effectively independent of the number of design variables: in more realistic full configuration test cases that we are about to tackle, 500 or more design variables will be necessary to describe the shape variations of the configuration (including nacelles, diverters, and tail surfaces) and the sizing of the structure.

The initial application of our design methodology to the aero-structural design of a supersonic business jet is simply a proof-of-concept problem meant to validate the sensitivities obtained with our method. Current work is addressing the use of multiple realistic load conditions, dynamic loads, aeroelastic constraints, and the addition of diverters, nacelles, and empennage.

In the present design case, we use $\alpha = 10^4$, $\beta = 3.226 \times 10^{-3}$. Note that the scalars that multiply the structural weight, $W$, and the coefficient of drag, $C_D$, reflect the correct trade-off between drag and weight that was previously mentioned, i.e. that one count of drag is worth 310 pounds of weight.

Figure 11 shows the evolution of this aero-structural design case for successive major design iterations. The figure shows the values of the coefficient of drag (in counts), the wing structural weight (in lbs), and the value of the KS function. Note that the structural constraints are satisfied when the KS function is positive.
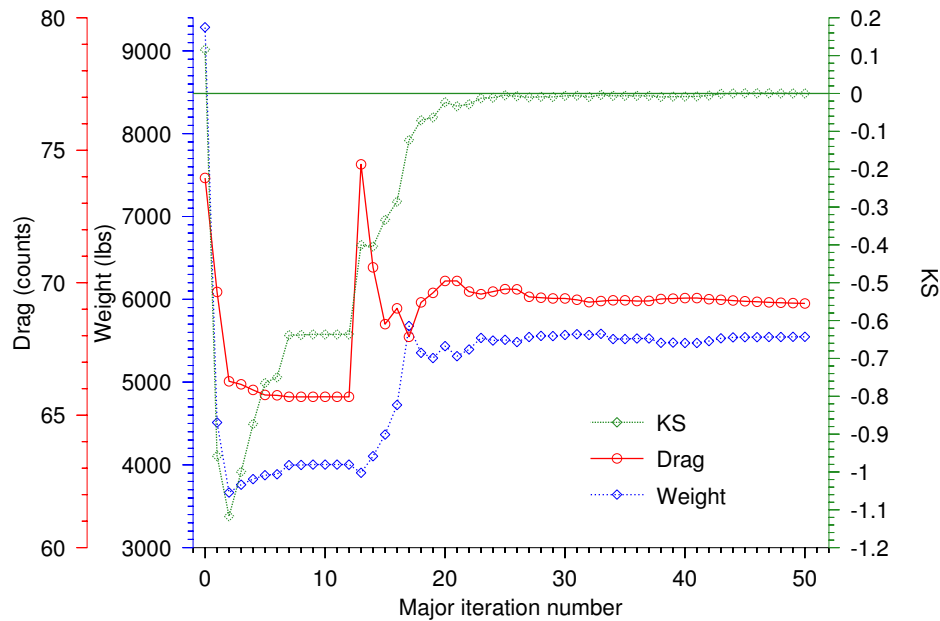
Fig. 11: Convergence history of the aero-structural optimization.

Because of the approximate nature of the KS function, all structural constraints may actually be satisfied for small but negative values of the KS function.

The baseline design is feasible, with a cruise drag coefficient of 74.04 counts and a structural weight of 9,285 lbs. The KS function is slightly positive indicating that all stress constraints are satisfied at the maneuver condition. In the first two design iterations, the optimizer takes large steps in the design space, resulting in a drastic reduction in both $C_D$ and $W$. However, this also results in a highly infeasible design that exhibits maximum stresses 2.1 times the yield stress of the material. After these initial large steps, the optimizer manages to decrease the norm of the constraint violation. This seems to have been accomplished by increasing the structural skin thicknesses, since the weight increases while the drag is further reduced. Towards major iteration 10, there is no visible progress for several iterations while the design remains infeasible. A large step is taken in iteration 13 that results in a sudden increase in feasibility accompanied by an equally sudden increase in $C_D$. The optimizer has established that the only way to obtain a feasible design is by increasing the wing thickness (with the consequent increases in $C_D$ and weight) and the structural thicknesses. From that point on, the optimizer rapidly converges to the optimum. After 43 major iterations, the KS constraint is reduced to $\mathcal{O}(10^{-4})$ and all stress constraints are satisfied. The aero-structurally optimized result has $C_D = 0.006922$ and a total wing structure weight of 5,546 lbs.

Visualizations of the baseline and optimized configurations are shown in Figures 12 and 13. Measures of performance and feasibility are written in the first section of Table 1. The left halves of Figures 12 and 13

show the surface flow density distribution with the corresponding structural deflections at the cruise condition for both the initial and optimized designs. The right halves show an exploded view of the stress distribution on the structure (spar caps, spar shear webs, and skins, from top to bottom) at the $C_L = 0.2$ maneuver condition. From these Figures one can appreciate that not only have the surface density distributions changed substantially at the cruise point, but so have the element stresses at the maneuver condition. In fact, as expected from a design case with a single load condition, the optimized structure is nearly fully-stressed, except in the outboard sections of the wing, where the minimum gage constraints are active. It is also worth noting that about half of the improvement in the $C_D$ of the optimized configuration results from drastic changes in the fuselage shape: both front and aft camber have been added to distribute the lift more evenly in the streamwise direction in order to reduce the total lift-dependent wave drag.

A total of 50 major design iterations including aero-structural analyses, coupled adjoint solutions, gradient computations, and line searches were performed in approximately 20 hours of wall clock time using 18 processors of an SGI Origin 3000 system (R12000, 400 Mhz processors). Since these are not the fastest processors currently available we feel confident that much larger models can be optimized with overnight turnaround in the near future.

**Comparison With Sequential Optimization**

The usefulness of a coupled aero-structural optimization method can only be measured in comparison with the results that can be obtained using current state-of-the-art practices. In the case of aero-structural design, the typical approach is to carry out

aerodynamic shape optimization with *artificial* thickness constraints meant to represent the effect of the structure, followed by structural optimization with a fixed OML. It is well known that sequential optimization cannot be guaranteed to convergence to the true optimum of a coupled system. In order to determine the difference between the optima achieved by fully-coupled and sequential optimizations, we have also carried out one cycle of sequential optimization within our analysis and design framework.

To prevent the optimizer from thinning the wing to an unreasonable degree during the aerodynamic shape optimization, 5 thickness constraints are added to each of the 6 defining stations for a total of 30 linear constraints. These constraints are such that, at the points where they are applied, the wing box is not allowed to get any thinner than the original design.

After the process of aerodynamic shape optimization is completed, the initial $C_D$ has decreased to $C_D = 0.006992$ as can be seen in the second half of Table 1. After fixing the OML, structural optimization is performed using the maneuver loads for the baseline configuration at $C_L = 0.2$. The optimized structural design reduces the empty weight of the structure to $6,567$ lbs.

We can now compare the results of the fully coupled optimization in the previous section and the outcome of the process of sequential optimization. The differences are clear: the coupled aero-structural optimization was able to achieve an optimized design with a weight of only $5,546$ lbs when compared with the $6,567$ lbs of the sequential optimization. This represents a relative difference of nearly 16%. Although it is barely distinguishable, the coefficient of drag for the coupled optimization approach is slightly lower than for sequential optimization.

Finally, notice that since sequential optimization neglects the aero-structural coupling in the computation of maneuver loads, there is no guarantee that the resulting design will be feasible. In fact, the aero-structural analysis shows that the value of the KS function is slightly negative.

## Conclusions

A methodology for coupled sensitivity analysis of high-fidelity aero-structural systems was presented. The sensitivities computed by the lagged-coupled-adjoint method were compared to sensitivities given by the complex-step derivative approximation and shown to be extremely accurate, having an average relative error of 2%. Moreover, significant differences in the values and signs of the sensitivities were found when aero-structural values were compared to rigid ones.

In realistic aero-structural design problems with hundreds of design variables, there is a considerable reduction in computational cost when using the coupled-adjoint method as opposed to either finite-differences or the complex-step approaches. This improvement is due to the fact that the cost associated with the adjoint method is practically independent of the number of design variables.

Sensitivities computed using the presented methodology were successfully used to optimize the design of a supersonic business jet that was parameterized with a large number of aerodynamic and structural variables. The outcome of this optimization was compared with the traditional method of sequential optimization and it was found to improve the structural weight by an additional 16%.

## Acknowledgments

| | $C_D$ (counts) | KS | $\sigma_{\max}/\sigma_{\mathrm{yield}}$ | Weight (lbs) | Objective |
|---|---|---|---|---|---|
| **Integrated approach** | | | | | |
| Baseline | 73.95 | $1.15 \times 10^{-1}$ | 0.87 | $9,285$ | |
| Optimized | 69.22 | $-2.68 \times 10^{-4}$ | 0.98 | $5,546$ | 87.12 |
| **Sequential approach** | | | | | |
| Aerodynamic optimization | | | | | |
| Baseline | 74.04 | | | | |
| Optimized | 69.92 | | | | |
| Structural optimization | | | | | |
| Baseline | | $1.02 \times 10^{-1}$ | 0.89 | $9,285$ | |
| Optimized | | $1.45 \times 10^{-8}$ | 0.98 | $6,567$ | |
| Aero-structural analysis | 69.95 | $-9.01 \times 10^{-3}$ | 0.99 | $6,567$ | 91.14 |

Table 1: Comparison between the integrated and sequential approaches to aero-structural optimization.

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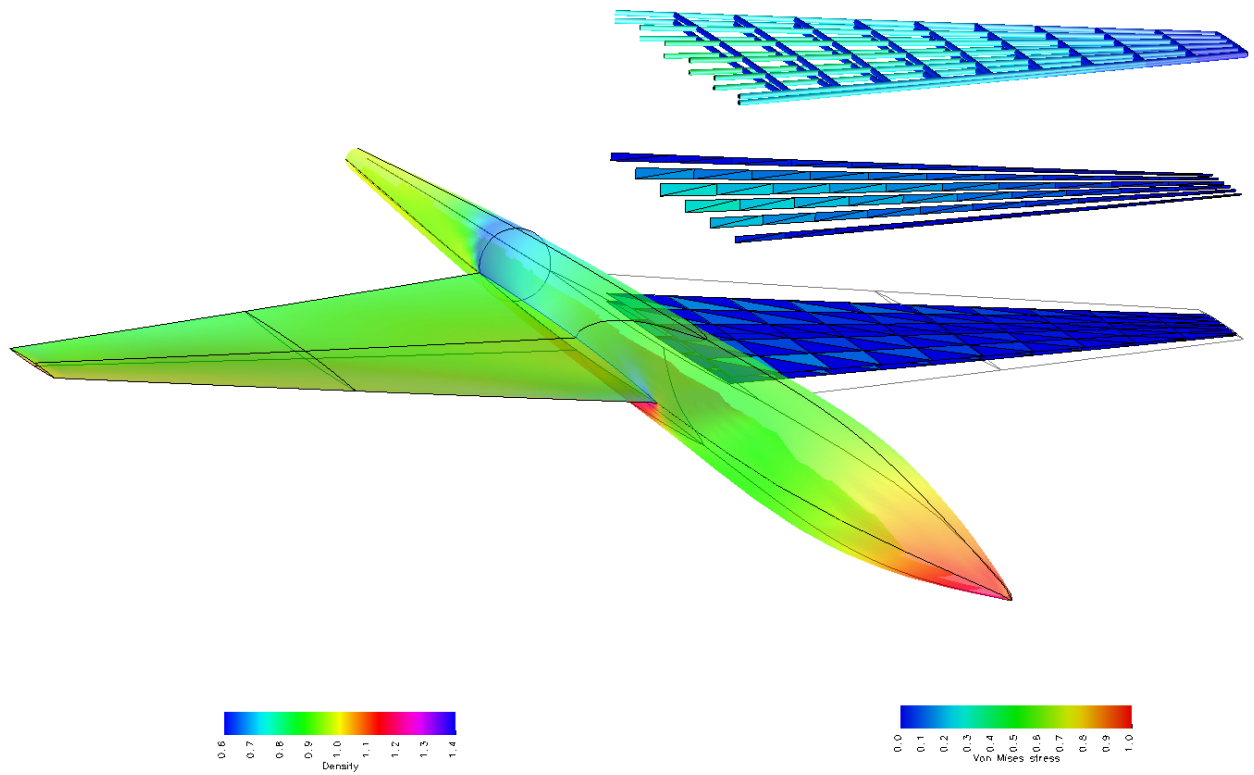
Fig. 12: Baseline configuration for the supersonic business jet showing surface densities at the cruise condition and structural stresses at the maneuver condition. The density is normalized by the freestream value and the von Mises stresses are normalized by the material yield stress.
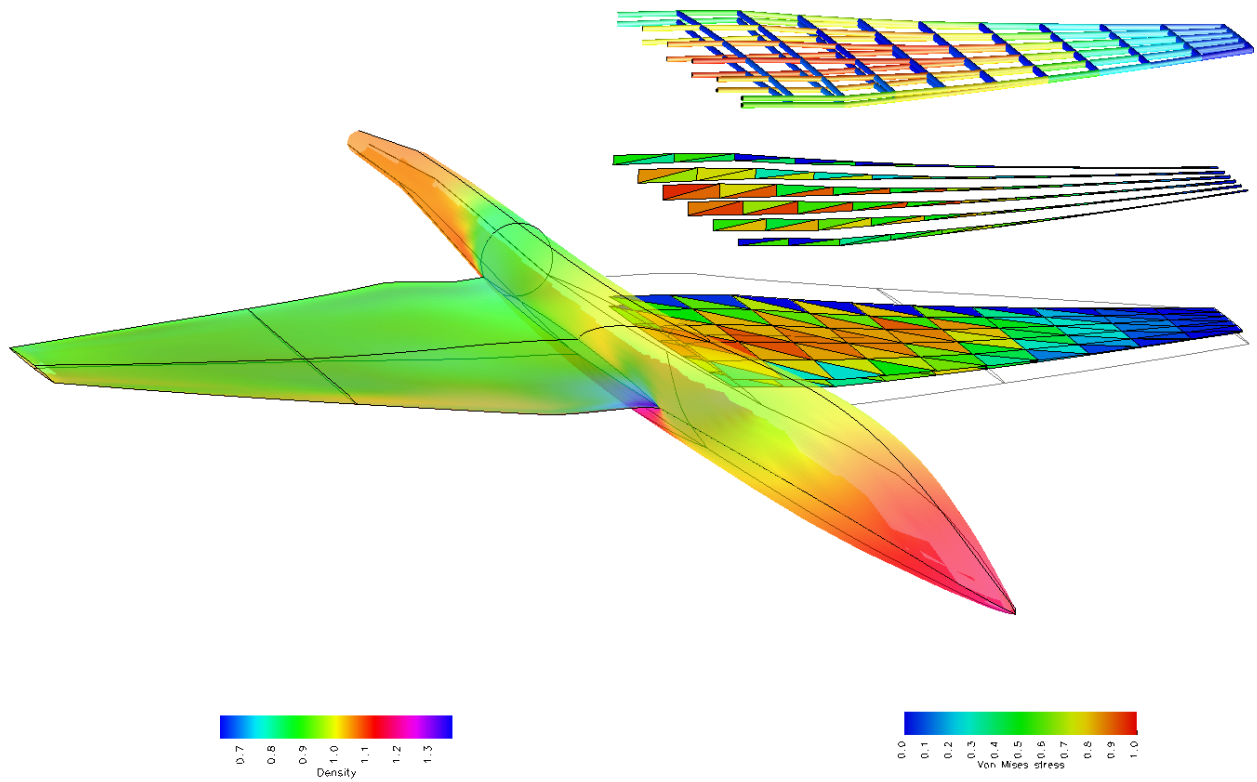


Fig. 13: Optimized configuration for the supersonic business jet.