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Non-Linear Control Allocation Using Piecewise Linear Functions

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Abstract

A novel method is presented for the solution of the non-linear control allocation problem. Historically, control allocation has been performed by assuming that a linear relationship exists between the control induced moments and the control effector displacements. However, aerodynamic databases are discrete-valued and almost always stored in multi-dimensional look-up tables where it is assumed that the data is connected by piecewise linear functions. The approach that is presented utilizes the piecewise linear assumption for the control effector moment data. This assumption allows the non-linear control allocation problem to be cast as a piecewise linear program. The piecewise linear program is ultimately cast as a mixed-integer linear program, and it is shown that this formulation solves the control allocation problem exactly. The performance of a re-usable launch vehicle using the piecewise linear control allocation method is shown to be markedly improved when compared to the performance of a more traditional control allocation approach that assumes linearity.

Introduction

The utilization of dynamic inversion control laws for flight control, coupled with new aircraft configurations with redundant control effectors, has resulted in an increased interest in the subject of control allocation. Dynamic inversion control laws require a control allocation algorithm when the number of control effectors exceeds the number of control variables. This is because there are typically only a small number of desired moments or control variables and a large number of control effectors may be used to generate the desired commands. It is quite common that the desired commands can be achieved in many different ways, and a control allocation algorithm is necessary in order to find solutions that meet some desired objective. Satisfactory control allocation requires an accurate estimation of the control effector's moment producing capabilities. This is especially important if the vehicle has experienced a control effector failure. Control allocation has historically been performed by assuming that a perfectly linear relationship exists between the control moments and the control effector displacements, despite the fact that the forces and moments produced by aircraft control surfaces are almost always non-linear functions of control surface displacement.

Control allocation is vital to the adaptive/reconfigurable flight control systems that are now being developed. These control systems are gaining favor due to their robustness properties, especially when an aircraft experiences control effector failures. Several examples of dynamic inversion based adaptive/re-configurable flight control systems can be found in the literature.[1–3] Control allocation algorithms also play an important part in the online determination of an accurate footprint for a reusable launch vehicle that has experienced a control effector failure.[4] Buffington[5] has demonstrated the application of a dynamic inversion control law along with a control allocation algorithm to a tailless fighter application. Tailless aircraft have reduced directional stability due to the lack of a vertical tail, and hence they lack a rudder for directional control. Ailerons or spoilers are examples of conventional control surfaces that can be used to provide directional control; however, these control effectors lack the control authority that a rudder would have, requiring that a mix of control effectors be used to generate the appropriate moments. Furthermore, left-right pairs of effectors such as ailerons and elevators typically have highly non-linear contributions to the yawing moment. This is especially true at low angles-of-attack where the effects of parasitic drag dominate induced drag. Non-linear control alloca-

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tion is required to make beneficial use of the non-linear effects that are rejected as disturbances by the control system.

Comprehensive surveys of existing control allocation techniques have been presented by Bodson[6] as well as Page and Steinberg .[7,8] Page and Steinberg[7,8] have performed extensive simulation studies and documented the open and closed-loop performance of the most common linear control allocation algorithms. On the other hand, Bodson[6] compared constrained, numerical-based optimization methods for control allocation to determine their feasibility for use in a real-time flight control system.

There are numerous linear control allocation algorithms[5,9–11] currently available and well documented in the literature. Buffington's[5] approach was to solve to the control allocation problem using a multi-branch approach. The first step of this multi-branch control allocation is a simultaneous check of control feasibility and control deficiency. The control feasibility and control deficiency determine whether the control effectors are capable of producing a given moment command without violating control effector limits. Control feasibility and deficiency are checked by solving an optimization problem that minimizes the ℓ_1 norm of the error between the desired moment and the moments produced by control effectors. The value of the performance index indicates the feasibility of the commanded moment. If the commanded moment is determined to be feasible, there exists at least one and possibly more feasible solutions to the problem; therefore, a second branch is considered. This control sufficiency branch provides solution uniqueness by minimizing the ℓ_1 norm of the control effector positions with respect to some "preferred" effector positions. Durham[9,10] developed a linear control allocation method called direct allocation that makes use of geometric concepts that relate to an attainable moment set (AMS). The AMS is a volume in moment space that represents all physically realizable moments. Direct allocation attempts to preserve the direction of the commanded moment by finding the maximum moment on the boundary in the same direction as the commanded moment. For commanded moments that lie outside of the boundary, the commanded moments are clipped to the boundary of the AMS volume. If the commanded moment is interior to the AMS, the control surface deflections are simply scaled back to produce the commanded moment. Other efforts include those of Ikeda , et.al.,[11] who used ℓ_1 optimization to determine the maximum attainable moment set. Hodel and Shtessel[12] have used linear programming to calculate a "local estimate" of the attainable moment set with respect to the current control surface positions.

Recently, there have been several efforts put forth towards improving upon the linear approximation. To begin, Doman and Oppenheimer[13] have implemented a linear control allocator that uses the local slope of the control moment curve with an added intercept term to more accurately account for the non-linear behavior of aerodynamic control effectors. They have shown an improvement in tracking performance without significantly increasing computation time. The shortcoming of the intercept correction method is that the control moments must be monotonic functions of effector position. If there are slope reversals present, it is possible for the control allocator to want to drive the effectors in a direction opposite to that where the actual solution lies. Doman and Sparks[14] have provided a method for the determination of the non-linear attainable moment set (AMS) for the two-dimensional case. Their effort considered control effectors that produced a control moment that was a quadratic function of effector position about one axis and a linear function about a second axis. More recently, Bolender and Doman[15] extended the work of Doman and Sparks to three dimensions for the computation of the non-linear AMS volume when the control moments about the third control axis were linear functions of effector position. At the present time, non-linear control allocation methods are computationally intensive and do not lend themselves to be applied in a real-time control system.

The objective of this paper is to demonstrate that the control allocation problem can be posed in a manner such that the moments that result from the corresponding control surface deflections are exactly the moments that are returned from the aerodynamic database. Aerodynamic data is discrete-valued and typically stored in large, multi-dimensional arrays that are functions of flight condition- typically angle-of-attack, sideslip angle, and Mach number. For flight control system design and handling qualities analysis, it is commonplace to assume that the data is connected by piecewise linear functions. The method that we are proposing assumes that the control moments are piecewise linear functions of control surface deflection and flight condition. We will then pose the control allocation problem as a piecewise linear program. The piecewise linear program will account for the non-linearities of the aerodynamic data, and the solution to this problem will produce moments that are exact with respect to the aerodynamic data. This approach results in improved command tracking performance when compared to the linear programming approaches discussed above. Results will be presented for a lifting-body type aircraft with redundant control effectors.

Dynamic Inversion Flight Control

Dynamic inversion controllers attempt to cancel and replace the dynamics of the plant being controlled with those of some pre-selected reference model. If the fidelity of the reference model is high enough, then the dynamic inversion control law results in a closed-loop system that behaves like a decoupled bank of integrators. In the context of flight control, a common objective of a dynamic inversion control law is to provide good body-axis angular rate tracking.

It is assumed that a pilot or an outer-loop guidance system generates body-axis angular velocity commands P_c, Q_c, R_c. The inner-loop dynamic inversion control law is designed so that the aircraft tracks these body-rate commands (see Figure 1). The rotational dynamics for an aircraft can be written as:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{P}) + \boldsymbol{g}(\boldsymbol{P}, \boldsymbol{\delta}) \tag{1}$$

where $\boldsymbol{\omega} = \begin{bmatrix} P & Q & R \end{bmatrix}^T$ and $\boldsymbol{P}$ denotes measurable or estimable quantities that influence the body-rate states. The parameter vector $\boldsymbol{P}$ includes variables such as Mach number, angle of attack, sideslip angle and vehicle mass properties such as moments of inertia. Equation 1 expresses the body-axis rotational accelerations as a sum that includes control dependent accelerations, $\boldsymbol{g}(\boldsymbol{P}, \boldsymbol{\delta})$, and accelerations that are due to the wing-body aerodynamics and propulsion system. We will collectively refer to the latter as the base moments. It is assumed that the mass properties of the aircraft change slowly when compared to the body-axis rates so that $\dot{\boldsymbol{I}} \approx 0$ and

$$\dot{\boldsymbol{\omega}} = \boldsymbol{I}^{-1}(\boldsymbol{G}_B - \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega}) \tag{2}$$

where

$$\boldsymbol{G}_B = \boldsymbol{G}_{BAE}(\boldsymbol{\omega}, \boldsymbol{P}) + \boldsymbol{G}_{\boldsymbol{\delta}}(\boldsymbol{P}, \boldsymbol{\delta}) = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{BAE} + \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{\boldsymbol{\delta}} \tag{3}$$

where $\boldsymbol{G}_{BAE}(\boldsymbol{\omega}, \boldsymbol{P})$ is the moment generated by the base engine-aerodynamic system and $\boldsymbol{G}_{\boldsymbol{\delta}}$ is the sum of the moments produced by the control effectors. Thus

$$\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{P}) = \boldsymbol{I}^{-1}(\boldsymbol{G}_{BAE}(\boldsymbol{\omega}, \boldsymbol{P}) - \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega}) \tag{4}$$

and

$$\boldsymbol{g}(\boldsymbol{P}, \boldsymbol{\delta}) = \boldsymbol{I}^{-1}\boldsymbol{G}_{\boldsymbol{\delta}}(\boldsymbol{P}, \boldsymbol{\delta}) \tag{5}$$

The model used for the design of the dynamic inversion control law becomes:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{P}) + \boldsymbol{G}_{\boldsymbol{\delta}}(P, \boldsymbol{\delta}) \tag{6}$$

and our objective is to find a control law that provides direct control over $\dot{\boldsymbol{\omega}}$ so that $\dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_{des}$, i.e.

$$\dot{\boldsymbol{\omega}}_{des} = \boldsymbol{f}(\boldsymbol{\omega}, P) + \boldsymbol{G}_{\boldsymbol{\delta}}(\boldsymbol{P}, \boldsymbol{\delta}) \tag{7}$$

therefore, the inverse control must satisfy:

$$\dot{\boldsymbol{\omega}}_{des} - \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{P}) = \boldsymbol{G}_{\boldsymbol{\delta}}(\boldsymbol{P}, \boldsymbol{\delta}) \tag{8}$$

Since there are more control effectors than controlled variables, a control allocation algorithm must be used to obtain a unique solution. Solution of this control allocation problem will be discussed in detail in a later section. Equation 8 states that the control effectors are to be used to correct for the difference between the desired accelerations and the accelerations due only to the base moments.

Piecewise Linear Programming

Piecewise linear programming is an optimization method that allows non-linear programming problems that are comprised of separable functions to be approximated by a linear program. The resulting linear program can subsequently be solved using a modified simplex method.[16] The piecewise linear program may also be reformulated and solved as a mixed-integer linear program.[17] The utility of piecewise linear programming allows one to cast an ℓ_1 optimization problem as a piecewise linear program.[17] In fact, Buffington[5] poses linear control allocation problems as ℓ_1 optimization problems and transforms these into linear programs. The resulting multi-branch control allocation problem minimizes the ℓ_1 norm of the control moment error (control feasibility and control deficiency branch) and subsequently minimizes the ℓ_1 norm of the control error with respect to some preferred effector positions (the control sufficiency branch). The restriction of approximating separable functions by piecewise linear functions may appear to be overly restrictive. For most aircraft, the control induced moments can be considered as separable since, in many cases, there are no significant aerodynamic interactions among the control effectors. There are instances of physical systems, (e.g., automobiles) where there exists a cross-coupling of steering and braking effectors. This coupling may take the form $\delta_1\delta_2$, and at first glance, may not appear to be separable. However, the product $\delta_1\delta_2$ can be written in separable form as

$$\delta_1\delta_2 = \delta_3^2 - \delta_4^2 \tag{9}$$

where

$$\delta_3 = \frac{1}{2}(\delta_1 + \delta_2) \tag{10}$$

$$\delta_4 = \frac{1}{2}(\delta_1 - \delta_2) \tag{11}$$

and then approximated using a piecewise linear function for δ_3^2 and δ_4^2. Our interest in piecewise linear programming is motivated by a desire to improve upon current linear control allocation techniques and to enable the solution of non-linear control allocation problems.

For purposes of illustration, we will approximate a single-valued function, $f(x)$, by its piecewise linear approximation and show how to formulate the minimization of $f(x),\ x \in [a,b]$ as a piecewise linear program. The approach given below for a single variable function can be generalized for multi-variable, separable functions rather easily. Furthermore, we are not restricted to only approximating the objective function by a piecewise linear approximation since it is also possible to consider piecewise linear approximations of the constraints, if they are separable, within the same framework. A detailed discussion can be found in Reklaitis, et.al.[16]

Without a loss of generality, we begin by considering a function, $f(x)$, of a single variable, defined on an interval, $[a,b]$. Begin by defining a grid of K points spaced in the interval $[a,b]$ and denote these points as $x^{(k)},\ k = 1,\dots,K$ where $a = x^{(1)} < x^{(2)} < \cdots < x^{(k)} < \cdots < x^{(K)} = b$. Note that we are not restricted to a uniform spacing of the $x^{(k)}$. Furthermore, let $f^{(k)}$ denote the value of $f(x^{(k)})$. A piecewise linear approximation of $f(x)$ can then be constructed by connecting $(x^{(k)}, f^{(k)})$ and $(x^{(k+1)}, f^{(k+1)})$ with a straight line as shown in Figure 2. The equation connecting the points $(x^{(k)}, f^{(k)})$ and $(x^{(k+1)}, f^{(k+1)})$ is given by

$$\tilde{f}(x) = f^{(k)} + \frac{f^{(k+1)} - f^{(k)}}{x^{(k+1)} - x^{(k)}}(x - x^{(k)}) \tag{12}$$

where $x \in [x^{(k)}, x^{(k+1)}]$. There will be $K-1$ such equations, one for each subinterval. Observe that on a given subinterval, x can be written as

$$x = \lambda^{(k)}x^{(k)} + \lambda^{(k+1)}x^{(k+1)} \tag{13}$$

where $\lambda^{(k)} \geq 0$ and $\lambda^{(k+1)} \geq 0$. The $\lambda^{(k)}$ are normalized such that

$$\lambda^{(k)} + \lambda^{(k+1)} = 1 \tag{14}$$

It can then be shown that Equation 12 can be written as

$$\tilde{f}(x) = \lambda^{(k)}f^{(k)} + \lambda^{(k+1)}f^{(k+1)} \tag{15}$$

Therefore, in the interval $[x^{(1)}, x^{(K)}]$, each x and the approximate value $\tilde{f}(x)$ can be determined by assigning appropriate values to $\lambda^{(k)}$ and $\lambda^{(k+1)}$ that correspond to the subinterval in which x lies. Since x can only be defined on a single subinterval, all the $\lambda^{(k)}$ which are not associated with that particular interval all must be equal to zero. As a result, we can express Equations 13 and 15 as

$$x = \sum_{k=1}^{K}\lambda^{(k)}x^{(k)} \tag{16}$$

$$\tilde{f}(x) = \sum_{k=1}^{K}\lambda^{(k)}f^{(k)} \tag{17}$$

subject to the following conditions:

$$\sum_{k=1}^{K}\lambda^{(k)} = 1 \tag{18}$$

$$\lambda^{(k)} \geq 0,\ k = 1,\dots,K \tag{19}$$

and

$$\lambda^{(i)}\lambda^{(j)} = 0 \quad \text{if } j > i+1;\ i = 1,\dots,K-1 \tag{20}$$

Equation 20 in necessary to insure that only points lying on the piecewise linear segments are considered as part of the approximating function. For example, given a value of x, no more than two of the

$\lambda^{(k)}$'s are allowed to be positive, and the two $\lambda^{(k)}$'s must be adjacent. If we consider a value of x where $\lambda^{(3)}$ and $\lambda^{(4)}$ are positive, with $\lambda^{(1)} = \lambda^{(2)} = 0$ and $\lambda^{(k)} = 0, k = 5, \ldots, K$, then the value of $\tilde{f}(x)$ lies on the approximating function between $x^{(3)}$ and $x^{(4)}$. On the other hand, if $\lambda^{(4)} > 0$ was to be replaced by $\lambda^{(6)} > 0$, and all other $\lambda^{(k)} = 0$, then a line connecting $x^{(3)}$ and $x^{(6)}$ would be generated that is not part of the approximating function. Furthermore, if we chose a value of x such that $x = x^{(k)}$ and $\tilde{f}(x) = f(x)$, then from Equation 18 $\lambda^{(k)} = 1$ and all other values of $\lambda = 0$. Lastly, it is important to note that one can always obtain a more accurate approximation of $f(x)$ by increasing the number of gridpoints; however, there is a resulting increase in problem size.

Given that we now have a piecewise linear approximation to $f(x)$ and the additional constraints that result from the transformation, we are able to state the Piecewise Linear Program that corresponds to the minimization of $f(x)$ on the interval $a \leq x \leq b$.

$$\min \tilde{f}(x) = \sum_{k=1}^{K} \lambda^{(k)} f^{(k)} \tag{21}$$

subject to

$$\sum_{k=1}^{K} \lambda^{(k)} = 1 \tag{22}$$

$$\lambda^{(k)} \geq 0 \tag{23}$$

Once the solution to the piecewise linear program is obtained, one uses Equation 16 to find the corresponding value of x that gives an approximate minimum to $f(x)$. Finding a solution to a piecewise linear program requires an approach that ensures that Equation 20 is satisfied. Recall that Equation 20 requires that no more than two adjacent $\lambda^{(k)}$'s are allowed to be non-zero. Therefore, to find an optimal feasible solution to the piecewise linear program, one of two approaches must be taken. One approach is to solve the problem using the simplex method with a restricted basis entry rule.[16] A second approach is to formulate Equation 20 using binary decision variables[17] to constrain x to be on only one subinterval, resulting in another increase in the size of the problem beyond what was necessary for the piecewise linear approximation. The addition of the binary variables transforms the piecewise linear programming problem into a mixed-integer linear program (MILP). We take the latter approach as it is sufficient for demonstrating the validity of our approach, and also because of the in-house availability of a commercial branch-and-bound code (TOMLAB ®)[18] for solving mixed-integer linear programs.

Transformation of the Piecewise Linear Program to a Mixed-Integer Linear Program

Begin by considering the piecewise linear approximation shown in Figure 2. Note that if there are K breakpoints, then there are $K-1$ linear segments. We assign a variable $y^{(k)}$ that corresponds to the k^{th} linear segment of the piecewise linear approximation such that

$$y^{(k)} = \begin{cases} 1 & \text{if } \lambda^{(k)} \neq 0 \text{ and } \lambda^{(k+1)} \neq 0, \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

for $k = 1, \ldots, K-1$. Next, we make the observation that if $\lambda^{(1)} \neq 0$ and $\lambda^{(2)} \neq 0$, then

$$\lambda^{(1)} \leq y^{(1)} \tag{25}$$

$$\lambda^{(2)} \leq y^{(1)} \tag{26}$$

where $y^{(1)} = 1$. However, if we are on the segment where $\lambda^{(2)} \neq 0$ and $\lambda^{(3)} \neq 0$, such that $y^{(2)} = 1$, then

$$\lambda^{(2)} \leq y^{(2)} \tag{27}$$

$$\lambda^{(3)} \leq y^{(2)}. \tag{28}$$

If we proceed in this manner, we observe that the following restrictions can be imposed upon the $\lambda^{(k)}$

$$\lambda^{(1)} \leq y^{(1)}, \tag{29}$$

$$\lambda^{(k)} \leq y^{(k-1)} + y^{(k)}, \; k = 2, \ldots, K-1 \tag{30}$$

$$\lambda^{(K)} \leq y^{(K-1)}. \tag{31}$$

The rationale behind Equation 30 is as follows: the $\lambda^{(k)}$ that correspond to points that are interior to the interval (i.e., they are not the endpoints of the interval on which x is defined) can be associated with one of two line segments. A particular $\lambda^{(k)}$ is the endpoint for the line segment immediately preceding it in addition to the line segment that comes immediately after it. Only one of these two line segments may be "active" at any time; therefore, the right-hand side of Equation 30 is never greater than one. In addition to Equations 29-31, we have an additional constraint to ensure that only one of the

$K-1$ line segments is active, hence only one of the $y^{(k)}$ can be equal to one:

$$\sum_{k=1}^{K-1} y^{(k)} = 1 \tag{32}$$

By including Equations 29-32 into the piecewise linear program, we transform it into a mixed integer linear program. The transformed optimization problem is stated as follows:

$$\min \tilde{f}(x) = \sum_{k=1}^{K} \lambda^{(k)} f^{(k)} \tag{33}$$

subject to

$$\sum_{k=1}^{K} \lambda^{(k)} = 1 \tag{34}$$

$$\lambda^{(k)} \geq 0 \tag{35}$$

$$\lambda^{(1)} \leq y^{(1)} \tag{36}$$

$$\lambda^{(k)} \leq y^{(k-1)} + y^{(k)},\ k = 2, \ldots, K-1 \tag{37}$$

$$\lambda^{(K)} \leq y^{(K-1)} \tag{38}$$

$$\sum_{k=1}^{K-1} y^{(k)} = 1 \tag{39}$$

$$y^{(k)} \in \{0, 1\} \tag{40}$$

By including the additional constraints which are necessary to complete the transformation of the piecewise linear program, we have added an additional $K-1$ decision variables to the problem. This does not include any slack or surplus variables that may be required by the solver. The slack and surplus variables will further increase the number of decision variables. The solution to the mixed integer linear program is obtained by using a branch-and-bound algorithm. Technical details on the branch-and-bound algorithm can be found in Bertsimas.[17]

Formulation of Control Allocation Problems as Mixed Integer Linear Programs

For typical aircraft, there are three controlled variables (moments) and three control surfaces, resulting in a square system of equations that form a unique mapping of the control moments to the control surfaces while obeying position and rate limits on the actuators. On the other hand, aircraft such as the X-33, X-37, F/A-18 HARV, F-15 ACTIVE, and AFTI/F-16 have more control surfaces than controlled variables. The resulting underdetermined system requires that a control allocation algorithm be used to ensure that Equation 8 be satisfied. There are often an infinite number of solutions for given values of the controlled variables; therefore, control allocation is often cast as an optimization problem in order to obtain a solution with some desired properties. Such objectives may include the minimization of control effector displacement or the minimization of the control moment error. The control allocation formulation that is used in this paper follows the multi-branch approach, similar to that posed by Buffington;[5] however, the assumption that the control induced moments are linear functions of the control moments has been removed.

In this section, the control allocation problems are formulated as piecewise linear programs. The PLP approach minimizes a linear performance index subject to piecewise linear constraints. Linear inequality constraints are used to ensure that effector position limits are not violated. The control allocation problem is broken-down into a control deficiency branch and a control sufficiency branch. We will begin with a discussion of the linear control allocation problem. We will derive the piecewise linear programs that achieve the same objectives as the linear programming formulations of the multi-branch control allocation approach. The piecewise linear programs are then subsequently transformed and solved as mixed-integer linear programs using a branch-and-bound algorithm. The resulting mixed-integer program is much more complex and difficult to solve when compared to the traditional linear programming algorithms; however, we are now able to achieve exactly the moments that we desire, something that the linear programming algorithms cannot achieve.

Control Deficiency Branch

The control deficiency branch is used to test the feasibility of satisfying Equation 8. For convenience we will refer to the left-hand side of (8) as $\boldsymbol{d}_{des}$:

$$\boldsymbol{d}_{des} \triangleq \dot{\boldsymbol{\omega}}_{des} - \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{P}) = \boldsymbol{G}_{\boldsymbol{\delta}}(\boldsymbol{P})\boldsymbol{\delta} \triangleq \boldsymbol{B}\boldsymbol{\delta} \tag{41}$$

If it is not feasible to obtain $\boldsymbol{d}_{des} = \boldsymbol{B}\boldsymbol{\delta}$ due to control effector constraints, then the difference between the desired and actual effector-induced body-axis accelerations is minimized. Thus the objective can be summarized in terms of minimizing a 1-norm performance index subject to constraints:

$$\min_{\boldsymbol{\delta}} J_D = \|\boldsymbol{B}\boldsymbol{\delta} - \boldsymbol{d}_{des}\|_1 \tag{42}$$

subject to:

$$\underline{\boldsymbol{\delta}} \leq \boldsymbol{\delta} \leq \overline{\boldsymbol{\delta}} \tag{43}$$

where $\underline{\boldsymbol{\delta}}$ and $\overline{\boldsymbol{\delta}}$ are the most restrictive lower bounds and upper bounds on the control effector deflection.

$$\begin{aligned} \overline{\boldsymbol{\delta}} &= \min(\boldsymbol{\delta}_u, \Delta T \dot{\boldsymbol{\delta}}_r + \boldsymbol{\delta}) \\ \underline{\boldsymbol{\delta}} &= \max(\boldsymbol{\delta}_l, -\Delta T \dot{\boldsymbol{\delta}}_r + \boldsymbol{\delta}) \end{aligned} \tag{44}$$

where $\boldsymbol{\delta}_u$ is the upper position limit vector, $\boldsymbol{\delta}_l$ is the lower position limit vector, $\dot{\boldsymbol{\delta}}_r$ is a vector of effector rate limits and ΔT is the inner-loop flight control system update rate. The optimization problem posed in Equation 43 may be transformed into the following linear programming problem:[5]

$$\min_{\boldsymbol{\delta}} J_D = \begin{bmatrix} 0 & \cdots & 0 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}_s \end{bmatrix} \tag{45}$$

subject to:

$$\begin{bmatrix} \boldsymbol{\delta}_s \\ -\boldsymbol{\delta} \\ \boldsymbol{\delta} \\ -\boldsymbol{B}\boldsymbol{\delta} + \boldsymbol{\delta}_s \\ \boldsymbol{B}\boldsymbol{\delta} + \boldsymbol{\delta}_s \end{bmatrix} \geq \begin{bmatrix} \mathbf{0} \\ -\overline{\boldsymbol{\delta}} \\ \underline{\boldsymbol{\delta}} \\ -\boldsymbol{d}_{des} \\ \boldsymbol{d}_{des} \end{bmatrix} \tag{46}$$

where $\boldsymbol{\delta}_s$ which is the same dimension as the set of controlled variables. If $J_D = 0$ then the commanded controlled variable rates are achievable and there may be excess control power available that can be used to optimize sub-objectives. If $J_D \neq 0$, the commanded controlled variable rates are not achievable and the control allocation algorithm provides a vector of effector commands that minimize the deficiency.

Control Deficiency Branch as a MILP

To transform the ℓ_1 optimization of the control error to a piecewise linear program, we will focus on the transformed linear program as defined in Equations 45 and 46. The transformation of the control allocation problem to the piecewise linear program will involve the control effectors, $\delta_i,\ i = 1, \ldots, m$ and the terms containing $\boldsymbol{B}\boldsymbol{\delta}$. We want to replace $\boldsymbol{B}\boldsymbol{\delta}$, where an element in the i^{th} row of $\boldsymbol{B}$ is a linear approximation of the control moment produced by δ_i, by a piecewise linear approximation of the control moments as a function of control effector position.

Let $L_i(\delta_i)$, $M_i(\delta_i)$, $N_i(\delta_i)$ denote the rolling, pitching, and yawing moments produced by deflecting the i^{th} control surface, δ_i. The piecewise linear approximation of $L_i(\delta_i)$, can be written as

$$L_i(\delta_i) = \sum_{k=1}^{K_i} L_i^{(k)} \lambda_i^{(k)} \tag{47}$$

$$= \begin{bmatrix} L_i^{(1)} L_i^{(2)} \ldots L_i^{(K_i)} \end{bmatrix} \begin{bmatrix} \lambda_i^{(1)} \\ \lambda_i^{(2)} \\ \vdots \\ \lambda_i^{(K_i)} \end{bmatrix} \tag{48}$$

where K_i is the number of breakpoints chosen to approximate the rolling moment due to δ_i, and the $\lambda_i^{(k)}$ are the normalized coefficients introduced previously. The piecewise linear approximations for $M_i(\delta_i)$ and $N_i(\delta_i)$ follow accordingly. Furthermore, we have the following expression for δ_i given $\lambda_i^{(k)}$:

$$\delta_i = \sum_{k=1}^{K_i} \lambda_i^{(k)} \delta_i^{(k)} \tag{49}$$

We are now able to re-write the $\boldsymbol{B}$ matrix as

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} L_1^{(1)} & L_1^{(2)} & \ldots & L_i^{(k)} & \ldots & L_m^{(K_m)} \\ M_1^{(1)} & M_1^{(2)} & \ldots & M_i^{(k)} & \ldots & M_m^{(K_m)} \\ N_1^{(1)} & N_1^{(2)} & \ldots & N_i^{(k)} & \ldots & N_m^{(K_m)} \end{bmatrix} \tag{50}$$

We also define a vector $\boldsymbol{\Lambda}$ as

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1^{(1)} \\ \lambda_1^{(2)} \\ \vdots \\ \lambda_i^{(k)} \\ \vdots \\ \lambda_m^{(K_m)} \end{bmatrix} \tag{51}$$

such that $\boldsymbol{B}\boldsymbol{\delta}$ is replaced by $\tilde{\boldsymbol{B}}\boldsymbol{\Lambda}$. The vector $\boldsymbol{\Lambda}$ is of length $\sum_{i=1}^{m} K_i$ and $\tilde{\boldsymbol{B}}$ is a matrix of size $n_{c_r} \times \sum_{i=1}^{m} K_i$ where n_{c_r} is the number of controlled variables. In formulating the piecewise linear control allocation problem, we no longer consider the case where the actuator rate limits and the sample rate of the flight control system set the upper and lower position limits on the control effectors. This is due to the fact that we have made the assumption that the control moments are functions of control deflection only; therefore, there is no provision within the piecewise linear formulation of the problem for including actuator rates. In addition, the upper and lower position limits for each effector are now accounted for in the *a priori* selection of each control effector's breakpoints and are therefore automatically included in the problem formulation. Since we have replaced an explicit dependence on δ_i with an implicit one, λ_i^k, we only need to impose the following bounds: $\lambda_i^{(k)} \geq 0$ $k = 1, \ldots, K_i$. Recall that we do not need to explicitly define the upper bounds on the $\lambda_i^{(k)}$ since we have the constraint that $\sum_{k=1}^{K_i} \lambda_i^{(k)} = 1$, $i = 1, \ldots, m$ and the constraints associated with the binary decision variables, $y_i^{(k)}$ that will restrict $\lambda_i^{(k)} \leq 1$. Once we obtain an optimal solution to the problem, we compute each δ_i using Equation 49. There are also an additional m constraints of the form

$$\sum_{k=1}^{K_i} \lambda_i^{(k)} = 1, \quad i = 1, \ldots, m \tag{52}$$

Transformation to the MILP form requires the additional constraints involving the binary variables, $y_i^{(k)}$.

$$\lambda_i^{(1)} \leq y_i^{(1)}, \quad i = 1, \ldots, m \tag{53}$$

$$\lambda_i^{(k)} \leq y_i^{(k-1)} + y_i^{(k)}, \quad i = 1, \ldots, m, \quad k = 2, \ldots, K_i - 1 \tag{54}$$

$$\lambda_i^{K_i} \leq y_i^{(K_i-1)}, \quad i = 1, \ldots, m \tag{55}$$

$$\sum_{k=1}^{K_i-1} y_i^{(K_i-1)} = 1, \quad i = 1, \ldots, m \tag{56}$$

$$y_i^{(k)} \in \{0, 1\} \tag{57}$$

The control feasibility and control deficiency branch in the form of a mixed-integer linear program is

$$\min_{\boldsymbol{\delta}_s} J_D = \begin{bmatrix} 0 & \cdots & 0 & \vdots & 0 & \cdots & 0 & \vdots & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda} \\ \hdashline \boldsymbol{y} \\ \hdashline \boldsymbol{\delta}_s \end{bmatrix} \tag{58}$$

subject to:

$$\boldsymbol{\delta}_s \geq \mathbf{0} \tag{59}$$

$$\tilde{\boldsymbol{B}}\boldsymbol{\Lambda} + \boldsymbol{\delta}_s \geq \boldsymbol{d}_{des} \tag{60}$$

$$-\tilde{\boldsymbol{B}}\boldsymbol{\Lambda} + \boldsymbol{\delta}_s \geq -\boldsymbol{d}_{des} \tag{61}$$

$$\lambda_i^{(k)} \geq 0, \quad i = 1, \ldots, m, \quad k = 2, \ldots, K_i \tag{62}$$

$$\sum_{k=1}^{K_i} \lambda_i^{(k)} = 1, \quad i = 1, \ldots, m \tag{63}$$

$$\lambda_i^{(1)} \leq y_i^{(1)}, \quad i = 1, \ldots, m \tag{64}$$

$$\lambda_i^{(k)} \leq y_i^{(k-1)} + y_i^{(k)} \quad i = 1, \ldots, m, \quad k = 2, \ldots, K_i - 1 \tag{65}$$

$$\lambda_i^{K_i} \leq y_i^{(K_i-1)}, \quad i = 1, \ldots, m \tag{66}$$

$$\sum_{k=1}^{K_i-1} y_i^{(K_i-1)} = 1, \quad i = 1, \ldots, m \tag{67}$$

$$y_i^{(k)} \in \{0, 1\} \tag{68}$$

Note the vector $\boldsymbol{y}$ is of length $-m + \sum_{i=1}^{m} K_i$ and is defined in the same manner as $\boldsymbol{\Lambda}$:

$$\boldsymbol{y} = \begin{bmatrix} y_1^{(1)} & \ldots & y_1^{(K_1)} & \ldots & y_i^{(k)} & \ldots & y_m^{(K_m)} \end{bmatrix}^T \tag{69}$$

Control Sufficiency Branch

If there is sufficient control power available such that $J_D = 0$, then there may be excess control power available to optimize a sub-objective. One sub-objective could involve driving the control effectors to a preferred position $\boldsymbol{\delta}_p$. A linear optimization problem reflecting this objective is given by:

$$\min_{\boldsymbol{\delta}_s} J_S = \| \boldsymbol{W}_{\boldsymbol{\delta}}(\boldsymbol{\delta} - \boldsymbol{\delta}_p) ||_1 \tag{70}$$

subject to:

$$\boldsymbol{B}\boldsymbol{\delta} = \boldsymbol{d}_{des} \tag{71}$$

$$\underline{\boldsymbol{\delta}} \leq \boldsymbol{\delta} \leq \overline{\boldsymbol{\delta}} \tag{72}$$

where $\boldsymbol{W}_{\boldsymbol{\delta}}$ is a vector that allows one to weight one preference over another. This optimization problem can be cast into the LP framework as follows:

$$\min_{\boldsymbol{\delta}} J_S = \boldsymbol{W}_{\boldsymbol{\delta}}^T \boldsymbol{\delta}_s \tag{73}$$

subject to:

$$\begin{bmatrix} \boldsymbol{\delta}_s \\ -\boldsymbol{\delta} \\ \boldsymbol{\delta} \\ -\boldsymbol{\delta}+\boldsymbol{\delta}_s \\ \boldsymbol{\delta}+\boldsymbol{\delta}_s \end{bmatrix} \geq \begin{bmatrix} \mathbf{0} \\ -\overline{\boldsymbol{\delta}} \\ \underline{\boldsymbol{\delta}} \\ -\boldsymbol{\delta}_p \\ \boldsymbol{\delta}_p \end{bmatrix} \tag{74}$$

$$\boldsymbol{B}\boldsymbol{\delta} = \boldsymbol{d}_{des} \tag{75}$$

where $\boldsymbol{\delta}$, $\boldsymbol{\delta}_s$,$\boldsymbol{\delta}_p$ and $\boldsymbol{W}_{\boldsymbol{\delta}}$ are of the same dimension as the number of control effectors. The preference vector $\boldsymbol{\delta}_p$ is used in this case to de-correlate the control effectors to enable on-line system identification of the control effectiveness matrix $\boldsymbol{B}$. More detail can be found in Chandler, et.al.[19]

Control Sufficiency Branch as a MILP

The differences between the MILP formulation of the control deficiency branch and MILP formulation of the control sufficiency branch are relatively minor. The primary difference is that the control sufficiency branch is a slightly larger problem since that the objective function is being optimized with respect to the control effectors rather than the moments. This results in additional constraints due to the presence of two inequality constraints of the form $\boldsymbol{\delta}+\boldsymbol{\delta}_s \geq \boldsymbol{\delta}_p$ in the linear program. The number of binary variables, $y_i^{(k)}$ and the parameters $\lambda_i^{(k)}$ are the same as for the control deficiency branch.

The control sufficiency branch can be stated as the following MILP:

$$\min_{\boldsymbol{\delta}_s} J_S = \left[\begin{array}{ccc:ccc:ccc} 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 1 \end{array}\right] \begin{bmatrix} \boldsymbol{\Lambda} \\ \hdashline \boldsymbol{y} \\ \hdashline \boldsymbol{\delta}_s \end{bmatrix} \tag{76}$$

subject to:

$$\boldsymbol{\delta}_s \geq \mathbf{0} \tag{77}$$

$$\sum_{k=1}^{K_i} \lambda_i^{(k)} \delta_i^{(k)} + \delta_{s,i} \geq \delta_{p,i}, \quad i = 1,\ldots,m \tag{78}$$

$$-\sum_{k=1}^{K_i} \lambda_i^{(k)} \delta_i^{(k)} + \delta_{s,i} \geq -\delta_{p,i}, \quad i = 1,\ldots,m \tag{79}$$

$$\tilde{\boldsymbol{B}}\boldsymbol{\Lambda} = \boldsymbol{d}_{des} \tag{80}$$

$$\lambda_i^{(k)} \geq 0, \quad i = 1,\ldots,m, \quad k = 2,\ldots,K_i \tag{81}$$

$$\sum_{k=1}^{K_i} \lambda_i^{(k)} = 1, \quad i = 1,\ldots,m \tag{82}$$

$$\lambda_i^{(1)} \leq y_i^{(1)}, \quad i = 1,\ldots,m \tag{83}$$

$$\lambda_i^{(k)} \leq y_i^{(k-1)} + y_i^{(k)}, \quad i = 1,\ldots,m, \quad k = 2,\ldots,K_i - 1 \tag{84}$$

$$\lambda_i^{K_i} \leq y_i^{(K_i-1)}, \quad i = 1,\ldots,m \tag{85}$$

$$\sum_{k=1}^{K_i-1} y_i^{(K_i-1)} = 1, \quad i = 1,\ldots,m \tag{86}$$

$$y_i^{(k)} \in \{0,1\} \tag{87}$$

Results

The mixed-integer linear programs for the multi-branch control allocation discussed above were implemented in a Simulink simulation of a lifting body type vehicle in the approach and landing phases. There are six control surfaces on the vehicle: left and right rudders, left and right flaperons, a body flap, and a speed brake. The simulation models the descent, final approach, and touchdown of the vehicle.

The performance of the piecewise linear approach is compared to a "worst-case" linear control allocation method. The linear control allocation method utilizes a global slope approximation of the control moments as a function of deflection. At each update of the flight control system, a linear least-squares fit to each control moment curve is computed. The corresponding slopes make up the elements of the $\boldsymbol{B}$ matrix.

For the control sufficiency branch of the control allocator, a "preferred" control position, $\boldsymbol{\delta}_p$ is required.

There are several different $\boldsymbol{\delta}_p$ which may be used including minimum control deflection ($\boldsymbol{\delta}_p = \mathbf{0}$), minimum ℓ_2 norm of deflection, and null-space injection. The preference vector that results from using the null-space injection approach is the solution to a weighted constrained least-squares problem where the weights are scaled by sampling from a uniform random distribution on the interval $[-1, 1]$. A random weighting of the control preference vector can aid with on-line identification of the control effectiveness parameters by de-correlating the control effectors. The approach that is followed here is to assume that the weighting matrix is constant and equal to the identity matrix. The preference vector in this case is $\boldsymbol{\delta}_p = \boldsymbol{B}^T(\boldsymbol{B}\boldsymbol{B}^T)^{-1}\boldsymbol{d}_{des}$. This $\boldsymbol{\delta}_p$ minimizes $\boldsymbol{\delta}^T\boldsymbol{\delta}$ subject to $\boldsymbol{B}\boldsymbol{\delta} = \boldsymbol{d}_{des}$ in a least-squares sense. Such a preference vector facilitates robustness analysis with the control allocator in the loop. For the piecewise linear control allocation, we found that it is sufficient to compute the right pseudo-inverse solution, $\boldsymbol{\delta}_p$, with a $\boldsymbol{B}$ matrix that uses local slopes of the control moments at the last control surface position.

The results that follow show the closed-loop vehicle performance when there are two failures injected into the flight control system. The first failure occurs 30s into the simulation, and involves fixing the body flap at a displacement of -5 deg. This failure contributes a constant pitching moment to the aircraft. A second failure, where the right rudder is fixed at 1 deg, occurs at 40s. This adds an additional pitching moment to the aircraft in addition to constant rolling and yawing moments. Also, since the dynamic-inversion control law is trying to track normal load factor, N_z, the gain on N_z is adjusted to ensure that the aircraft can track the commanded load factor.

We will measure the closed-loop performance of the four control allocators by the ability of the dynamic inversion control law to track the commanded normal load factor. A second performance measure is a "truth model" comparison of the control moments estimated by the control allocator effectiveness model to the moments that actually result from the non-linear aerodynamic database given the control effector positions returned by the control allocation algorithm. Together these two performance measures will provide an indication of the ability of a particular control allocation algorithm to produce moments that have no unanticipated effects on the aircraft.

Simulation Results

The results for the piecewise linear control allocator are shown in Figures 3-5. We see in Figure 3 that we can track the commanded load factor extremely well up to the time that the second failure is introduced at the 40s mark. After the second failure is introduced, we see that about 10s passes before the measured load factor is reasonably close to that which is commanded. It was observed that during the 10s window spanning 40s to 50s that the $\boldsymbol{d}_{des}$ resulted in a control deficiency in the yawing moment coefficient. Above 60s the tracking performance is not as well behaved due to the flare maneuver used to set-up for landing. Around 65s the landing gear is extended and the control system is not able to track the N_z command very accurately after that time; however, the tracking performance exceeds that of the linear control allocation algorithms (see Figures 7- 9). Control surface commands, as output from the control allocator, are given in Figure 4. We see that after the second failure is injected into the simulation that the flaperon deflections approach their upper limit. Above 60s the speed brake becomes active and the command oscillates between 30 and 70 deg. Finally, the modelling error that results from approximating the aerodynamic data by piecewise linear functions is shown in Figure 5. As is expected, the piecewise linear approximation more accurately describes the nonlinear aerodynamic data as compared to a purely linear fit of the moment coefficients. The aerodynamic data that were used in this simulation were neural network fits of an aerodynamic data base. The neural networks allow for the data to be stored quite compactly. Note that there are a few minor discrepancies between the piecewise linear approximation and the neural net fit since the latter was smoothed in order to provide a continuous approximation to the data.

It should be noted that the simulation with the branch-and-bound solver was extremely slow. It was observed that the solver returned a solution in 3-5s, which is unacceptable for it to be considered for use in a real-time, digital flight control system. Research is currently underway to determine if performance gains can be achieved using the simplex method with the restricted basis entry rule.

The results shown in Figures 7-9 are for a linear control allocator that uses a least-squares linear fit model of the control moment data. The slopes of each least-squares estimate comprise the elements of the $\boldsymbol{B}$ matrix. The performance is initially adequate with the presence of some lag because of a 50% reduction on the load factor error prefilter gain (Figure 6) as compared to that used for the piecewise linear control allocation. Here we see that the load factor is not tracked (Figure 7) after the right rudder failure is introduced into the simulation at 40s. This can be accounted for by examining the modelling error time histories as shown in Figure 8. The modelling errors for this particular control allocation approach is two orders of magnitude worse on average than for the piecewise linear control allocator. The effect of the modelling errors are evident in Figure 9. Specifically, the ruddervators are deflected in the opposite direction of those generated by the piecewise linear control allocator. We also see that there are very large errors in the control pitching and yawing moment coefficients after the second failure. This is a direct result of the modelling errors that result from attempting to describe the aircraft's true moment producing capability using a linear least-squares fit.

Conclusions

A novel method was presented for the solution of the control allocation problem. Control allocation has historically been performed by assuming that a linear relationship exists between the control induced moments and the control effector displacements. Since aerodynamic data almost always exhibits some non-linear behavior, such assumptions can lead to degraded performance or vehicle loss when secondary non-linear effects must be used to control a vehicle, particularly after control effector failures have occurred. Aerodynamic databases are usually discrete-valued and almost always stored in multi-dimensional look-up tables where it is assumed that the data is connected by piecewise linear functions. The approach that was presented assumes that the control effector moment data is piecewise linear. This assumption allows us to cast the control allocation problem as a piecewise linear program. In order to solve the piecewise linear program, it was re-formulated as a mixed-integer linear program and solved using a commercial, off-the-shelf branch-and-bound algorithm. Analysis showed that the piecewise linear program formulation results in improved tracking performance when compared to a more traditional control allocation approach that uses a linear approximation of the control moments as a function of control surface deflection, especially when the aircraft is subjected to control effector failures. The piecewise linear control allocator was able to maintain control of the aircraft and safely land after the failures were introduced while the control allocator that assumed a simple linear relationship did not.

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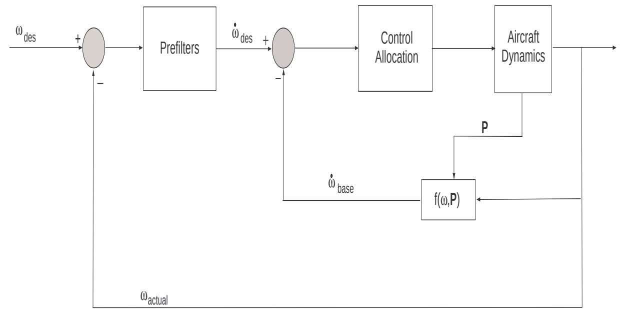

Figure 1: Block Diagram of Inner-loop Dynamic Inversion Control Law

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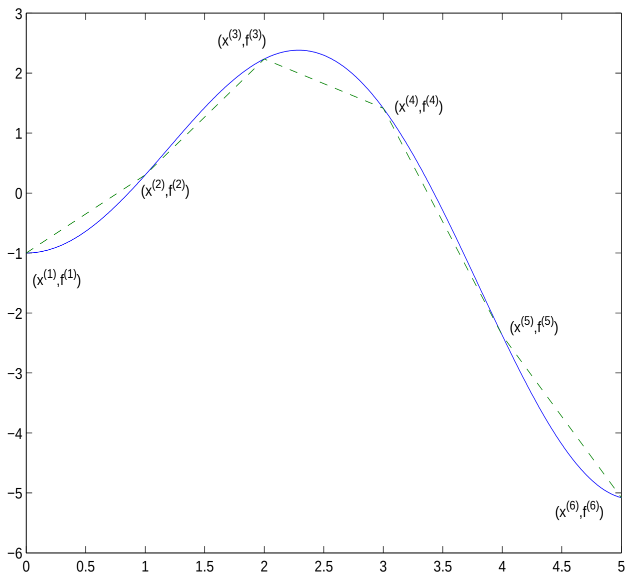

Figure 2: Piecewise Linear Function Approximation

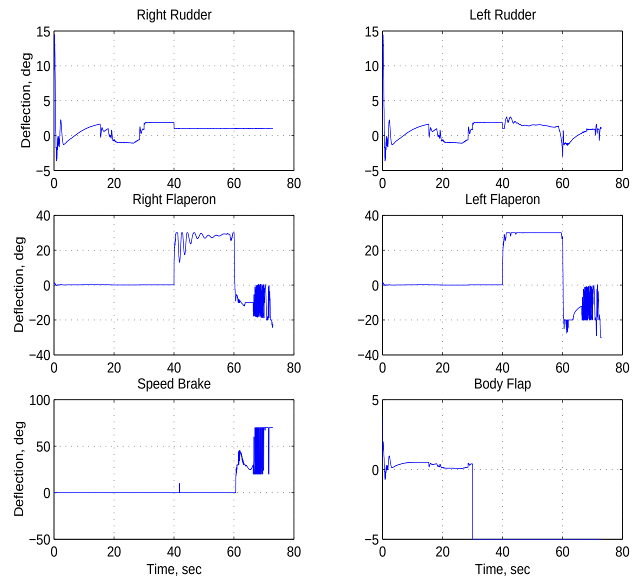

Figure 4: Control Effector Commands from Piecewise Linear Control Allocation

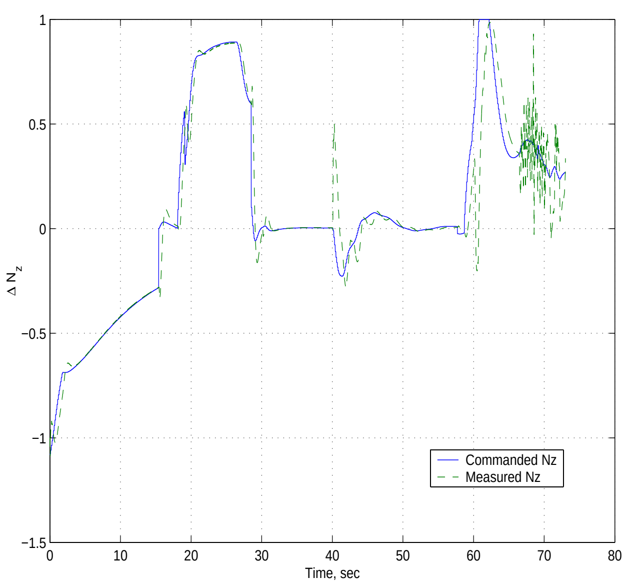

Figure 3: Commanded and Measured Normal Load Factor for Piecewise Linear Control Allocation

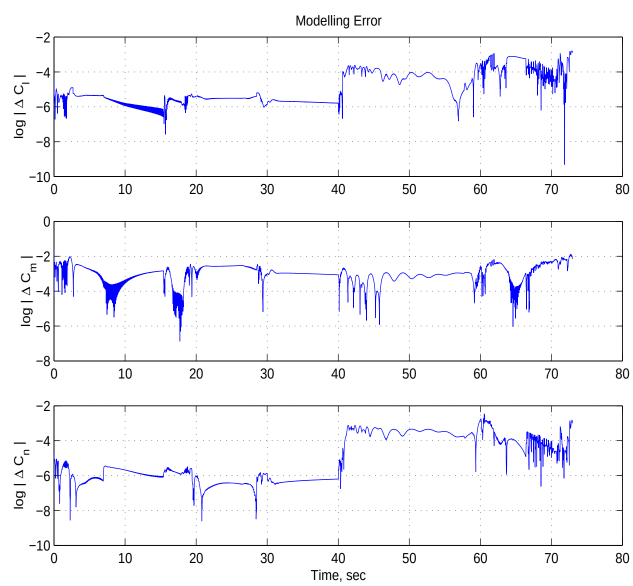

Figure 5: Moment Coefficient Modelling Error for the Piecewise Linear Control Allocator

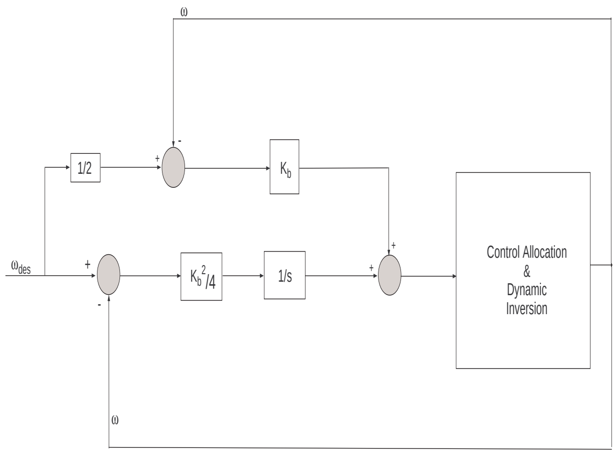

Figure 6: Block Diagram of Prefilter Structure for Implicit Model Following Control Law

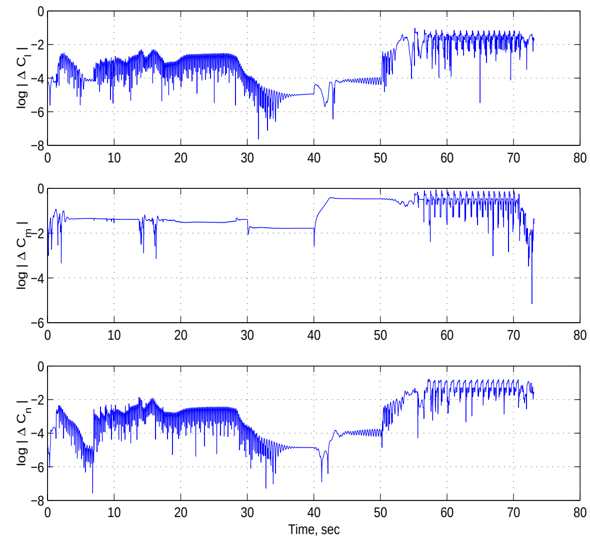

Figure 8: Control Moment Modelling Errorfor the Linear Control Allocator with Least-Squares Fit

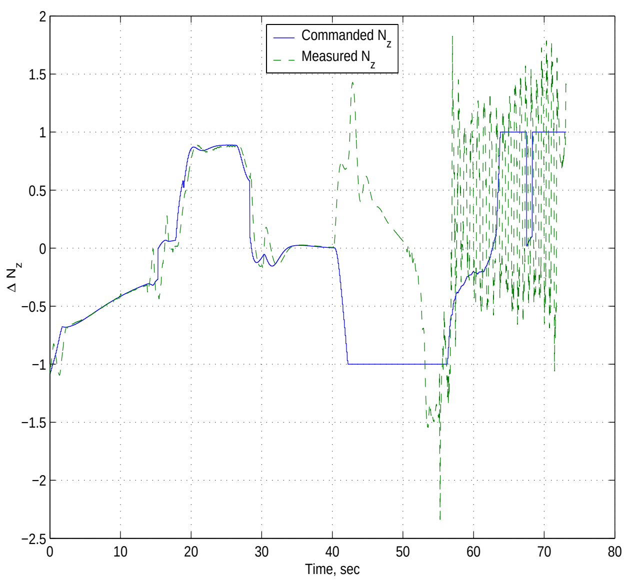

Figure 7: Commanded and Normal Load Factor for Linear Control Allocator Using a Linear Least-Squares Fit of Control Moment Data

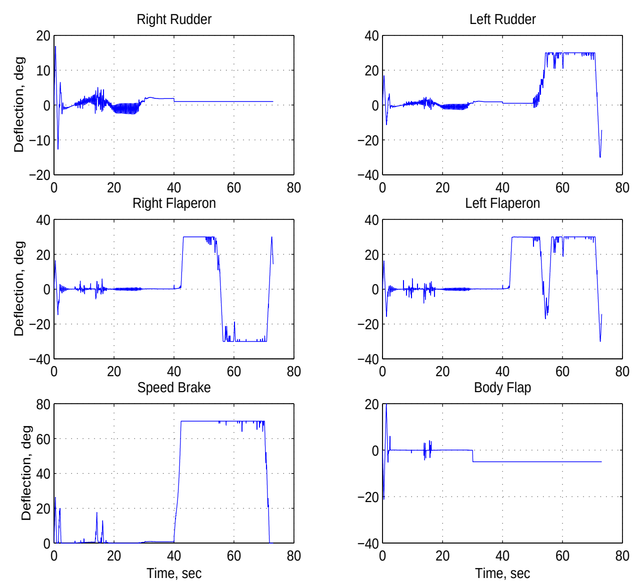

Figure 9: Control Effector Commands from Linear Control Allocator Using a Linear Least-Squares Fit of Control Moment Data