

# STORAGE RADAR

R. M. Page

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## ABSTRACT

A new approach to echo ranging has been outlined, based on application of electronic storage or memory devices. Two uses are made of these devices, one for frequency conversion, the other for signal integration.

Some technical features of interest resulting from this approach are,

- (1) A high order of inherent frequency stability at intermediate frequency.
- (2) Pulse to pulse phase coherence at intermediate frequency,
- (3) Signal integration at intermediate frequency,
- (4) High duty factor operation of pulse systems, and
- (5) Exploitation of cross correlation techniques without compromise in function.

Among the functional properties added or improved over present practice may be included the following:

- (1) Sensitivity increased by orders of magnitude, with attendant increase in range and accuracy of tracking data in all coordinates,
- (2) Dynamic clutter rejection provided, independent of own ship's motion and antenna rotation,
- (3) Immunity to electronic jamming increased by orders of magnitude,
- (4) Range resolution for a given bandwidth increased by as much as a factor of four, and
- (5) Range rate introduced as a fourth and highly sensitive parameter of resolution, with range rate tracking of single targets and range rate display of multiple targets.
- (6) Variable resolution without loss in sensitivity introduced to permit freedom of exchange between resolution and rate of flow of information.

Examples are given in application to precision tracking and to Schnorkel detection by radar.

#### PROBLEM STATUS

This paper is the basis for a new program of research, and constitutes a first report on that program.

#### AUTHORIZATION

NRL Problem No. R14-01R

## STORAGE RADAR

### INTRODUCTION

The practical application of Radar to the detection and tracking of surface and airborne targets was demonstrated prior to the 1939 outbreak of World War II. Subsequently the military usefulness of radar was significantly enhanced by an unprecedented concentration of scientific talent on wartime electronic development. The high operational effectiveness of radar at war brought forth countermeasures which nullified that effectiveness with varying degrees of success. At war's end, although radar still had great value against existing targets, there remained several deficiencies which threatened to reduce to the vanishing point the military usefulness of radar against the anticipated targets of another war. The findings of the Radar Panel of the Research and Development Board indicated in 1947 that the anticipated requirements could not be met by the nominal improvements that result from refinement of existing techniques. Rather the indicated need was for a new approach which would yield improvement by orders of magnitude.

Significant improvement in radar angle error data has been accomplished through refinement in sequential lobing techniques, and application of monopulse techniques as proposed in an earlier report.<sup>1</sup> Present high-speed lobing and monopulse techniques, together with modern servo-mechanism developments, promise sufficiently accurate and smooth angle tracking to meet most anticipated tactical requirements.

The remaining improvements necessary if radar is to keep pace with its targets lie in several categories. Foremost among them is increase in range at which small high-speed targets of very low effective reflecting area may be detected and tracked. Almost equally important is increase in speed with which a large volume of space may be inspected and all targets in that volume resolved in at least three coordinates with relatively high coordinate accuracy. Here are four elements of search radar performance, 1. long range, 2. high scan rate, 3. three coordinate resolution, and 4. accuracy. These four elements are mutually in competition for signal energy, and all of them must be significantly improved.

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<sup>1</sup> "Accurate Angle Tracking by Radar" - R.M. Page NRL Report RA 2A 222A.  
28 December, 1944.



Another deficiency in present radar is its inability to single out a desired target that is surrounded by other reflecting objects such as terrain, sea, and atmospheric clutter. Much effort has been expended to achieve moving target indicating and anti-clutter circuits, with marked success under favorable circumstances, but with less success in four important applications, viz. anti-schnorkel radar, anti-tank radar, anti-personnel radar, and precision tracking fire-control radar.

The vulnerability of radar to jamming both by electronic means and by decoy or artificial clutter targets presents another problem whose solution is of vital importance. The success of the strategic bombing of Germany was made possible by the complete neutralization of German anti-aircraft fire-control radar through use of window. Electronic jamming has been used successfully against our own radars both in Europe and in the Pacific.

Still another factor of weakness which has been of concern to some is the large "circle of confusion" characteristic of radar angle tracking. Target resolution by radar falls far short of optical resolution in angle. It is partly compensated for by the high range resolution possibilities of radar, but further improvement is desirable.

Finally, the detectability of radar transmitter signals at targets which are beyond the echo detection range of the radar has been a severe limitation in some operations. This limitation has been particularly annoying in anti-submarine warfare.

It is the purpose of this report to outline a new approach to echo ranging which promises the desired orders of magnitude of improvement in at least some of the categories identified above. Not all of the limitations of this approach have been thoroughly explored. Some of the techniques basic to the approach are in a highly preliminary state of development. Yet sufficient study has been made to indicate that a new program of research based on this approach is justified.

The proposals here presented are based on the application of storage devices to echo ranging. Any storage or memory device having the requisite flexibility of recording and playing back, and adequate speeds of response for "writing" and "reading", and an acceptable dynamic range or signal to noise ratio, would suffice, but this presentation will be in terms of cathode ray memory tubes. In their present state of development these tubes are not adequate for all of the applications here envisioned, but they are adequate for some, and the present large development effort in the field of storage tubes may be expected to extend their applicability.

Let us first consider the problem of range capability. This is a matter of signal to noise ratio, and resolves into two factors: (1) sensitivity of the receiving system, and (2) average power radiated by the

transmitter, referred in both cases to the direction of the target. We shall now deal with receiving system sensitivity, which is here considered in the light of relative response of the system to desired signal available at the receiver input terminals, and noise either at the input terminals or generated within the receiving system. The maximum possible identification by integration techniques between signal and noise is limited by two factors which appear as functional requirements of the system. They are (1) the quantity of information required, and (2) the time available for obtaining that information. The first may be expressed in terms of information resolution, the second in terms of rate of flow of information.

Focussing attention now on the range parameter of a searchlighting type of pulse radar, resolution may be replaced by its reciprocal, range confusion, with the notation  $R_c$ , where

$$R_c = \frac{1}{\text{Range Resolution}} \quad (1)$$

In those cases where range confusion is determined by pulse length,  $\tau$ , it has been almost universally assumed that it is equal to pulse length, although in application it is actually considerably less than the pulse length. However, in conformity with accepted practice, it is here assumed that

$$R_c = \frac{c}{2} \tau = R_\tau \quad (2)$$

where  $c$  is the velocity of electromagnetic propagation.

It has been determined experimentally under a variety of conditions that the i.f. bandwidth,  $B_\tau$  giving the lowest value of minimum detectable input signal is equal to the reciprocal of the pulse length  $\tau$ .

$$B_\tau = \frac{1}{\tau} \quad (3)$$

from which may be derived

$$R_c = \frac{c}{2B_\tau} \quad (4)$$

Equation (4) expresses the relationship between an arbitrarily defined range confusion, and the corresponding I.F. bandwidth that gives the lowest signal threshold in the presence of noise ~~for the pulse length~~ involved. In the case under consideration,  $R_c$  is generally made as small as practicable as a functional requirement. The limit is imposed by engineering considerations involving both  $\tau$  and  $B_\tau$ , and results in a value of  $B_\tau$  on the order of  $5(10)^6$  cycles per second.

$$B_T = 5(10)^6 \quad (5)$$

The rate of flow of information will now be expressed in terms of its reciprocal, the time available for signal integration,  $T_i$ . It is desired to express  $T_i$  in terms of bandwidth, and for the purposes of this discussion it is sufficiently accurate to equate it to  $\frac{2}{B_i}$ , where  $B_i$  is the modulation bandwidth of a carrier frequency.

$$T_i = \frac{2}{B_i} \quad (6)$$

A representative rate of flow of information required is for automatic control and bandwidths corresponding to this functional requirement may be on the order of 5 cycles per second.

$$B_i = 5 \quad (7)$$

The ratio of bandwidth required for resolution to bandwidth required for rate of flow of information is derived from equations (5) and (7), and is found to be

$$\frac{B_T}{B_i} = 10^6 \quad (8)$$

Equation (8) expresses an order of magnitude which may be considered average for tracking pulse radar, and may vary from  $10^5$  to  $10^7$ .

One of the functions of the radar receiver is to so operate on the input signal that the bandwidth required for its transmission is reduced from  $B_T$  to  $B_i$  or by a factor on the order of  $10^6$ . If this could be accomplished without similarly compressing the noise spectrum, the theoretical improvement in signal to noise ratio by bandwidth narrowing would be 60 db. This limit may not be reached with any pulse system in which  $T$  is less than  $T_i$ , however, since in no such system can that degree of bandwidth narrowing be achieved without some corresponding operation on noise. This follows from the fact that  $B_T$  corresponds approximately to the spectrum of a pulse of length  $T$  while  $B_i$  corresponds to the spectrum of a pulse of length  $T_i$ . If there are many pulses in the interval  $T_i$ , as in radar, all pulses added sequentially into one long pulse would still result in a pulse length less than  $T_i$  by a factor equal to the duty factor of the transmitter. It follows therefore that with a duty factor of  $10^{-3}$ , which represents an average for tracking radar, the maximum theoretical improvement in signal to noise ratio due to signal integration (bandwidth narrowing) is on the order of 30 db.

The above figures are based on a linear system. If the system includes any non-linear element, the signal to noise ratio will be modi-

fied. For example, a square law detector will, to a first order approximation, square the signal to noise ratio. Detectors which convert from I.F. to video or audio approximate square law operation on weak signals. The minimum useful signal in pulse radar systems is usually a few decibels less than noise at the detector output at video frequency. Integration after detection is utilized to bring the signal to noise ratio up to the order of +10 db as required for automatic tracking. Suppose the full value of 30 db improvement by integration is achieved. This brings the minimum ratio at the detector output to -20 db, which corresponds to -10 db at the detector input, and -10 db at the pre-amplifier output. If the 30 db integration were performed before detection, then the detector output ratio would be +10 db, the input ratio on the order of +5db and the integrator input ratio, -25 db, which would be the pre-amplifier output ratio. In this case, transferring the integrator from the video channel to the I.F. channel reduces the signal threshold at the input by 15 db, or half the total integration value in db.

For other than electronic methods of video integration such as persistence of vision and human memory, the improvement in signal to noise ratio can never exceed the theoretically maximum value as derived above for integration after detection, and in general will be less. Thus in all cases, maximum sensitivity of a receiving system including a square law detector can be achieved only by performing the entire integration process before the detector operates on the signal. Since this is a well known principle no further proof is necessary here.

#### STORAGE FREQUENCY CONVERTER

Application of the principle to pulse echo ranging imposes severe requirements on frequency stability at intermediate frequency and pulse to pulse intermediate frequency phase coherence. Existing techniques are inadequate to meet these requirements. It is possible that the requirements in i.f. frequency stability and pulse to pulse phase coherence may be met by the frequency converter system illustrated in Figure 1. This diagram shows a pulse transmitter, a pulse receiver including input circuits, mixer, and intermediate frequency amplifier, and range unit, all conventional components of present tracking radar systems. Quite unconventionally, however, the local oscillator of the receiver is replaced by a storage tube with writing and reading sweeps controlled by the range unit. The storage system with its sweeps functions to record the transmitted signal during the sending of the pulse, and then at a later time determined by the range unit to "play back" the record for comparison with a received echo signal. The writing and reading rates are purposely made slightly different, in order to generate a reference signal differing slightly in frequency from the received signal, thus to produce by well known heterodyning principles the desired intermediate frequency. Should the average frequency of the generated I.F.

tend to differ from a desired value, the tendency may be neutralized by conventional A.F.C. operating on one of the storage tube sweeps, the control now being applied to sweep line speed rather than sweep recurrence rate or frequency. Each new pulse is preceded by an erasure of previous signals in the same storage area, to avoid confusing old records with new.

The storage frequency conversion system just described possesses several properties fundamentally different from those of a conventional local oscillator super-heterodyne system. These properties are discussed in the following paragraphs.

Variations in transmitter frequency or sweep speed, without A.F.C. are reflected into the I.F. on a percentage basis. This is true whether these variations be slow (pulse to pulse) or fast (modulation during the pulse). The effect is to render the I.F. relatively independent of transmitter frequency instability or frequency modulation, as compared to the case when a conventional local oscillator is used. Even the slight variation remaining may be removed, if desired, by use of identical writing and reading sweeps and use of an auxiliary local oscillator at I.F. to convert one component signal to the proper frequency for heterodyning. Thus the mechanism is at hand for an intermediate frequency of extraordinarily high stability.

Inasmuch as the stored record of each radiated pulse is in phase identity with the radiated signal, the relative phase between the received echo and the locally reproduced pulse is a function only of the delay time due to propagation of the radiated signal to the target and back, and the delay introduced by the range unit. As long as these two delays are matched in time, all echo pulses at intermediate frequency will have identical phase characteristics. Thus there exists a pulse to pulse phase coherence of a kind that is useful in the integration system that is contemplated. The range unit stability requirement imposed by this method of operation is not far removed from that already existing in modern precision tracking radar.

In addition to the two properties just described, the storage frequency converter possesses another property which is also useful. It was stated that frequency modulation of the transmitter during the pulse was essentially eliminated in the mixing process, since the frequency modulation characteristics of echo signal and heterodyne signal are essentially identical and therefore mutually cancel in the difference signal. If there is impressed on the transmitted signal a frequency modulation during the pulse of significant magnitude, the modulation will disappear in the mixing process only when the echo and the reproduced heterodyne signal are exactly in register. Furthermore, if the modulation is of appropriate character, such as a single cycle of any convenient waveform, preferably of complex shape, or, in the limit, a purely random function, there will be only one condition of complete register. If the two sig-

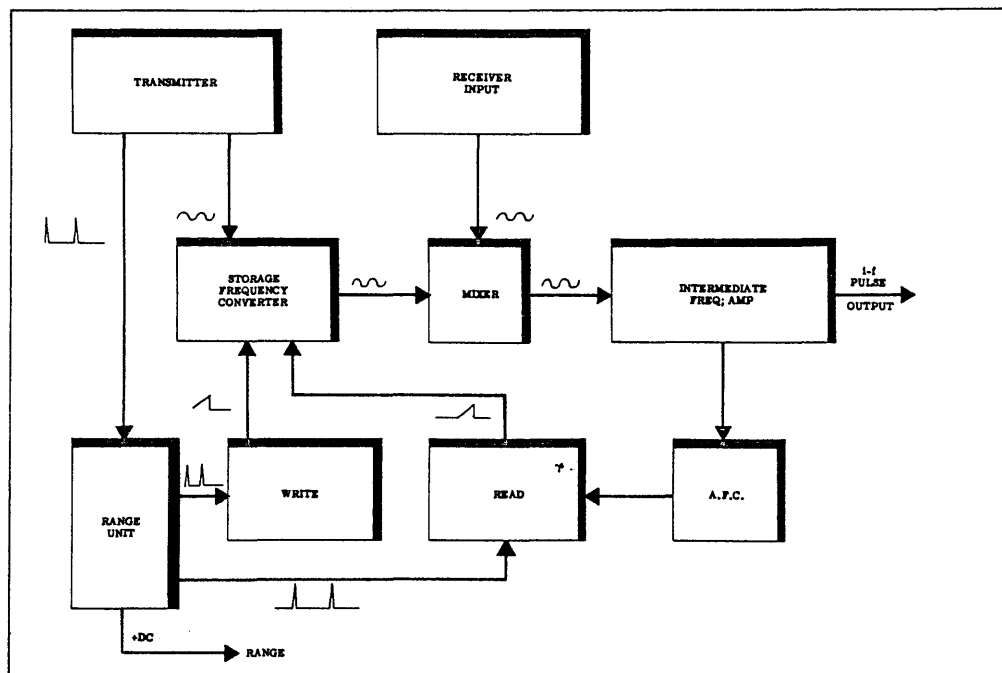


Fig. 1

nals depart from perfect register, the frequency modulation will not be completely neutralized. The intermediate frequency signal spectrum width due to frequency modulation will increase linearly with departure from register. As the spectrum increases beyond the i.f. bandwidth, the signal will deteriorate until ultimately it is lost. The amount, translated in terms of range, by which the two signals fed to the mixer may be out of register before intermediate frequency signal is lost is a measure of the range confusion of the system.

The rate of increase of intermediate frequency spectrum width due to frequency modulation as the mixer input signals depart from register will be proportional to the time rate of change of transmitted frequency  $\dot{F}_T$ . For any particular modulation wave form, including random noise,  $\dot{F}_T$  is proportional to the spectrum width of the transmitted signal, which may be called the transmitter bandwidth,  $B_T$ . Thus the greater  $B_T$ , the smaller the range confusion.

If the two signals are in perfect register, the intermediate frequency signal spectrum will be determined by the amplitude characteristics of the signal. Thus, if the signal is a pulse of length  $\tau$  eq. (3) expresses the minimum bandwidth required in the i.f. amplifier. The greater the value of  $\tau$ , the less the value of  $B_T$ . The less the value of  $B_T$ , the narrower are the tolerance limits for increase of signal spec-

trum due to frequency modulation. Thus the greater the value of  $\tau$ , the less the value of  $R_c$ . Equations (2) and (4) are now no longer applicable. At this point equation (4) may be replaced by equations (9) and (9a).

$$R_c = K/\tau \dot{F}_T \quad (9)$$

$$R_c = K' B_\tau / B_T \quad (9a)$$

Since resolution is now increased by increasing the pulse length, the value of  $\tau$  may be increased until some other limit is reached. If  $\tau$  is made ~~much longer~~ than present practice, as is quite feasible in the subject system, the i.f. bandwidth may be much less than  $B_T$ .

$$B_\tau \ll B_T \quad (10)$$

From this relationship emerges another property which may have considerable ultimate value. In order to obscure a desired signal by radiated interference; the interfering energy must lie within the pass band of the i.f. amplifier ( $B_\tau$ ). However, before it can reach the i.f. amplifier it must heterodyne with a locally generated signal which is frequency modulated with a spectrum bandwidth  $B_T$ . The jamming signal therefore, no matter how narrow its own spectrum, will be dispersed over a spectrum much greater than the transmission bandwidth of the receiving system. Further development of this proposal, which ultimately reduces the i.f. bandwidth approximately to  $B_i$ , enhances this anti-jam property. It must be recognized, of course, that any jamming signal that paralyzes any circuit preceding the mixer, including the mixer input, may still be effective, but it is no small gain over present systems to remove the entire i.f. amplifier out from under a similar restriction. The amount of amplification that precedes the mixer, even in relatively low frequency radar, is still sufficiently low that the increase in immunity to radiated noise jamming may be expressed in terms of bandwidth ratio

$$J = \frac{B_T}{B_\tau} \quad (11)$$

where  $J$  represents the factor by which jamming power must be multiplied to retain its effectiveness.

The storage frequency converter is shown in Fig. 1 as recording the radiated carrier frequency. The same functions may be accomplished at an intermediate frequency by operation on the storage input and output alike by a local oscillator in conventional fashion, for heterodyning the transmitter frequency down and back up again. Alternately the received signal may be heterodyned down by the same local oscillator to correspond with the stored signal. In neither case would the essential properties of the system be altered.

The storage frequency converter in the application just described

looks at one target at a time, and uses the time  $\tau$  to see the target. If  $\tau$  is very short relative to the time corresponding to maximum range, the stored pulse may be used repeatedly during the interval between transmitted pulses,  $T_p$ , each use corresponding to a "look" at a particular range. The maximum number of looks possible, when each look uses the entire time  $\tau$ , is equal to the ratio  $T_p/\tau$

$$n_R = T_p/\tau \quad (12)$$

where  $T_p$  is the pulse repetition period. The range interval that is seen at each look is equal to the range confusion,  $R_c$ . The total value of range rendered visible by  $n_R$  looks is the product of  $R_c$  and  $n_R$ , or

$$R_v = n_R R_c \quad (13)$$

Following equation (2), the range  $R_p$  corresponding to  $T_p$  is

$$R_p = \frac{c}{2} T_p \quad (14)$$

From equations (12), (13) and (14) it follows that

$$\frac{R_v}{R_p} = \frac{R_c}{R_p \tau} = \frac{2 R_c}{c \tau} \quad (15)$$

This states that the ratio of the range made visible by continuously repeated reading of the stored transmitted pulse to the total range corresponding to the time between transmitted pulses is proportional to the range confusion divided by the pulse length. The ratio still holds if  $R_p$  is replaced by any range interval of interest, as long as it is greater than  $R_\tau$ .

Repeated use of the stored pulse in the fashion just described may be useful for certain applications. Equation (15) gives an important criterion which may be used in determination of the extent of this usefulness for a specific application. The method is not given any further consideration in this report.

Accepting for the time being the limitation of inspecting a single target, the properties of the storage frequency converter of greatest immediate interest may be summarized as follows:

1. I.F. frequency stability of high order is inherent in the system.
2. Modulation characteristics of the transmitted signal are reduced or removed.
3. I.F. pulse to pulse phase coherence is available.



4. Immunity to radiated noise jamming may be made very great.
5. Range resolution may be increased rather than reduced by increasing pulse length.

#### STORAGE INTEGRATOR

Having provided for the necessary I.F. frequency stability and pulse to pulse phase coherence, it is now appropriate to consider pulse to pulse integration at I.F. The signal to be integrated consists of pulses of length  $\tau$  and repetition period  $T_p$ , with essentially no modulation apart from the normal amplitude modulation of pulse echoes. The bandwidth necessary for their transmission is  $B\tau$ . Arbitrary reduction of bandwidth from this value would operate similarly on signal and noise, and no gain in ratio could result, since equation (3) still holds.

Let the output of the i.f. amplifier of Figure 1 be recorded on another storage tube, and let each pulse of duration  $\tau$  be recorded on an area of the screen adjacent to the last pulse recorded. Let this process continue until all the pulses that occur in the integration time interval  $T_i$  are recorded. Then as each additional pulse is received, the oldest pulse in the record is erased and replaced by the new pulse. Thus there is produced a stored record which continuously contains all the most recently received pulses extending back for the time  $T_i$ . There are two fundamental methods of combining all these recorded signals into one new signal having a signal to noise ratio corresponding to the integrated record. One is to combine all pulses synchronously into one pulse of the same duration. In this process, noise energy would add while signal amplitude would add. The resulting signal to noise ratio would be greater than the input signal to noise ratio by the energy summation of the original signal. Thus if the integration includes  $10^3$  pulses, the improvement in signal to noise ratio would be 30 db.

The other method is to combine all pulses sequentially in phase coherence, so as to produce one pulse whose length is the summation of the lengths of all the pulses integrated. The result would be no change in signal to noise ratio, but a reduction in the frequency spectrum width of the signal with no change in noise spectrum. The signal width, being inversely proportional to the pulse length, would now require a transmission bandwidth inversely proportional to the number of pulses integrated. Again, if  $10^3$  pulses are added, the bandwidth may be reduced by a factor of  $10^3$  without loss of signal energy, but with a 30 db reduction in noise energy. Thus the signal to noise ratio would be improved by 30 db in this case also. In either case the improvement in signal to noise ratio S/N due to integration depends only on the number of pulses integrated. Its value in decibels is

$$\Delta S/N = 10 \log \frac{T_i}{T_p} \text{ db} \quad (16)$$

In choosing between these two methods of integration consideration must first be given to the information contained in the original signal and the affects of the integration process on this information. In conventional pulse ranging where range resolution depends on  $\tau$  (eq. 2) all range information is present in each stored pulse, and remains present on the synchronously integrated pulse. This is essential if all the range information is not extracted prior to the integration process. If a storage frequency converter is used in conjunction with a long, modulated pulse, the range information is deteriorated in the mixing process, and there is no possibility of restoring it in the integration process. It will be shown later that all range information can be extracted at the mixer input, which represents the widest bandwidth point in the system. There is therefore no need to preserve range information as such in the integrated signal, and sequential integration is therefore permissible.

The input signal also contains range rate information. In radar this information is most apparent in the form of pulse to pulse variation, since a single cycle of rate information may span many pulses. Synchronous pulse integration obviously would destroy this information. There appears to be no simple way of extracting this information before integration without loss of sensitivity. It will be shown later that this information is readily available after integration when sequential integration is used. Further development here is therefore based on sequential integration.

A method of performing the sequential integration is illustrated in Figures 2 and 3, in which are shown respectively a system block diagram and a signal storage pattern. The intermediate frequency amplifier of Figure 2 is the same one as is shown in Figure 1. Each pulse of length  $\tau$  is recorded as a horizontal line across the storage tube under influence of the horizontal writing sweep. Each recorded pulse is displaced vertically from the preceding recorded pulse by the vertical writing sweep. These sweeps operate under control of the normal radar range unit, which also is shown common to Figures 1 and 2, and from which indication of range setting is obtained. The components thus far identified in Figure 2, together with all of Figure 1, constitute one complete unit operating on a common time scale, which is the normal echo ranging time scale and may be identified as the input time scale.

The rest of Figure 2, consisting of C.W. amplifier, A.F.C. and horizontal and vertical reading sweeps, are independent of the input time scale, and may operate on a scale of their own, which may be identified as the output time scale. The reading sweeps are so designed that the stored record is played back continuously, with breaks between lines only on the order of a fraction of a cycle of recorded carrier frequency. This fraction of a cycle break is necessary to maintain phase coherence from line to line on the output time scale, and must be adjusted to the particular pulse length  $\tau$  so as to make  $\tau$  plus the gap equal an integral

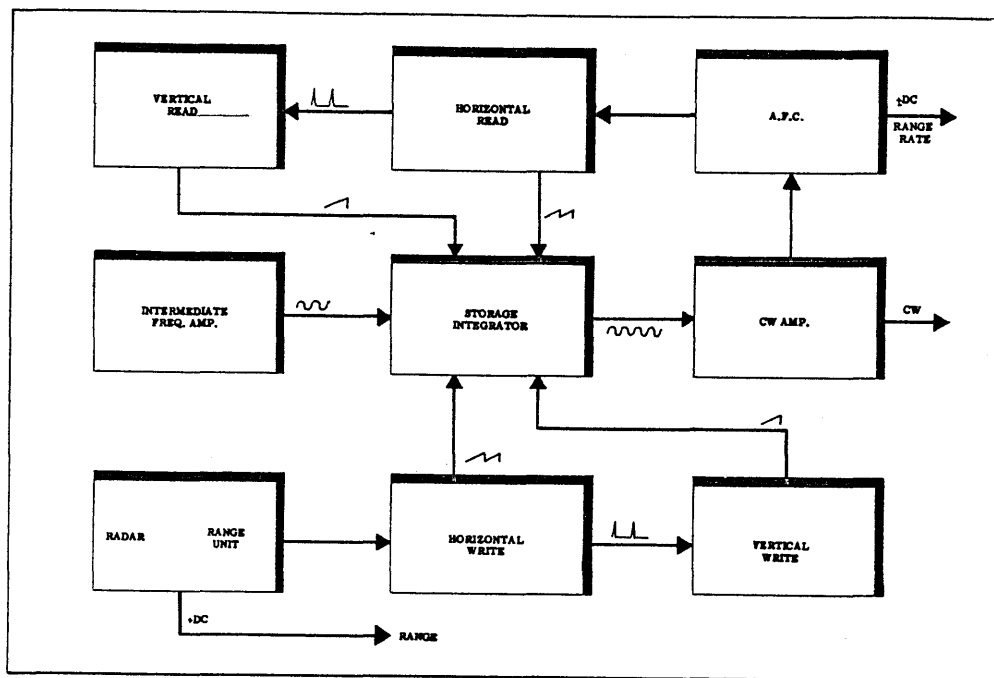


Fig. 2

number of cycles of recorded carrier frequency. If the gap is incorrect, a discontinuity in phase will occur between lines. Since all lines are similar, the phase discontinuities will also be similar and will integrate into a shift in frequency. The A.F.C. may therefore operate on the length of the horizontal reading sweep, thus maintaining constant frequency by maintaining line to line phase coherence in the reading process. The resulting output from the storage tube, will be a continuous wave whose amplitude is a function of the received echo signal strength, whose frequency is constant, and whose spectrum width is a function of signal fading and tracking perturbations.

Time and frequency relationships between the input and output time scales are not always in one to one correspondence. Let frame frequency be defined as the number of times per second the entire stored record is scanned. The input frame frequency  $F_i$  is then equal to the reciprocal of the integration time

$$F_i = \frac{1}{T_i} \quad (17)$$

The output frame frequency  $F_o$  would ordinarily never be lower than  $F_i$ ,

since if it were some of the stored signal would be erased before it was used, so it may be written that

$$F_o \geq F_i \quad (18)$$

If the limiting case is considered, where  $F_o = F_i$ , then the corresponding line frequencies would also be equal, or the line recurrence periods would be equal and of value  $T_p$ . However, each line is written in the time  $\tau$ , with an inactive interval of  $T_p - \tau$  between line writings. Since each line is read in time  $T_p$ , it follows that the output frequency will be lower than the input frequency in the ratio of  $\tau$  to  $T_p$ , or

$$\frac{f_o}{f_i} = \frac{\tau}{T_p} \quad (19)$$

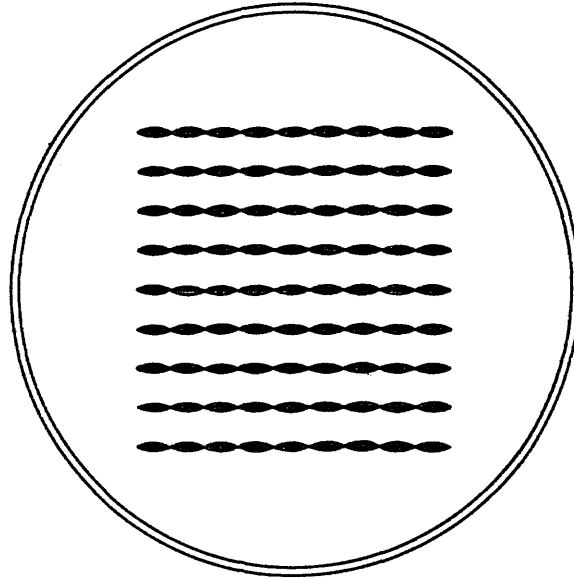


Fig. 3.

If this ratio is  $10^{-3}$ , as has been assumed for illustrative purposes, then the output frequency, all modulations which depend solely on this frequency as a carrier, and therefore the bandwidth required for their transmission, are reduced by a factor of  $10^3$  from corresponding input frequencies. It must be observed, however, that this reduction in bandwidth is not accompanied by a corresponding increase in signal to noise ratio, since signal and noise are operated on similarly. For the general case, where  $F_o > F_i$ , the I.F. frequency ratio is obviously the ratio of writing and reading line speeds.

$$\frac{f_o}{f_i} = \frac{L_{so}}{L_{si}} \quad (20)$$

Frequencies which depend on the pulse repetition rate as a carrier transfer from the input to the output time scales on the basis of line frequency, rather than line speed,

$$M_{po}/M_{pi} = L_o/L_i \quad (21)$$

In the case of radar, doppler frequencies fall in this category.

Frequencies which depend on frame frequency as a carrier are unmodified. These are the frequencies containing target presence, signal amplitude, and target tracking information.

The net result of this integration method, in addition to achieving the desired improvement in signal to noise ratio, is to reduce the i.f. carrier frequency and spectrum width by approximately the duty factor, increase the absolute frequency shift due to the doppler effects of moving targets, and transmit unchanged the frequencies corresponding to desired target information. The total bandwidth narrowing is from  $B_T$  to  $B_i$ , whose ratio in present tracking radar may be on the order of  $10^6$  as an average

$$B_T/B_i \approx 10^6 \quad (22)$$

That part of the bandwidth reduction which is effective in improving signal to noise ratio is represented by the ratio  $T_i/T_p$ , whose present radar value is on the order of  $10^3$ , representing 30 db improvement relative to the information available from a single pulse. (See eq. 16). This part of the process also results in reduction of  $R_c$  as given in eq. (9) and (9a) by the factor  $T_i/T_p$ , as well as increase of  $J$  as given in eq. (11) by the same factor. The remainder of the bandwidth narrowing is a consequence of translation from the input to the output time scales of the storage integrator of Fig. 2, and is expressed in eq. (20).

Attention is now directed to doppler frequency shift due to range variation. Signal spectrum compression and doppler shift expansion have been carried to the point where the transmission bandwidth necessary for the desired target information is much less than the doppler frequency spectrum corresponding to the velocity spread of targets. For example, the CW amplifier of Fig. 2 may logically have a bandwidth of five cycles per second or less, while the doppler frequency shift in carrier frequency at this amplifier may be ten to a hundred cycles per second per knot relative range rate. Since doppler frequency shift appears as a modulation which depends on pulse repetition frequency as a carrier, it appears in the stored record as an intermediate frequency phase discontinuity between lines. The A.F.C. loop of Figure 2 operates on exactly this type of phase discontinuity, and will therefore keep the signal from the target being tracked in the center of the pass band of this amplifier. Other targets in the antenna beam at the same range having range rates differing from that of the target being tracked will give rise to frequencies which will not be passed by C.W. amplifier. Thus range rate appears as a new resolution parameter, and, as seen from the illustrative figures just given, range rate resolution may be on the order of small fractions of a knot. The value of the range rate of the target being tracked may be observed in terms of the control voltage necessary to track the signal frequency, as shown in Figure 2, when the range gate is not tracking. When the range gate is tracking in range, the range rate control voltage of Figure 2 will indicate rate error in

range tracking, since the range rate of the gate would compensate for the range rate of the target.

The range rate resolution just described forms an ideal moving target indicator, since any target that is moving relative to clutter, even by so much as a few inches per second, will not be confused with that clutter. If the clutter itself is moving with an appreciable velocity spread, and the target velocity lies within that spread, the target will then compete only with that portion of the clutter which produces doppler frequencies within the pass band of the C.W. amplifier. If high resolution under these conditions were of paramount importance, C.W. amplifier bandwidth might be made still narrower, thus sacrificing rate of flow of information for increased discrimination against clutter.

It is of interest to note that the storage integrator system of Figure 2 places no special requirements on the input signal except that it possess pulse to pulse phase identity. It therefore does not depend on the storage frequency converter of Figure 1, providing the phase identity may be accomplished in some other manner. The storage frequency converter, however, is ideally suited to work into the storage integrator, the unique properties of which may now be summarized.

- (1) Signal integration to the limit of available integration time is accomplished at intermediate frequency.
- (2) Moving targets are separated from clutter with great effectiveness.
- (3) Range rate is introduced as a new parameter of target resolution.
- (4) Automatic tracking in range rate is provided.
- (5) Apart from range information, none of the previously listed properties of the storage frequency converter are compromised.

## RANGE TRACKING

The range information that may be extracted from the input signals is range to any target, direction and amount of deviation of that range from a range comparison or reference signal, and the range distribution of multiple targets. All this information is lost in the storage integrator, due to the sequential integration process. As stated earlier, however, the range information may be extracted before integration. The range unit indicates range to any gated target. The storage frequency converter, by virtue of transmitter modulation bandwidth, excludes all other targets from the system within the limits of range confusion. Range distribution of multiple targets will be dealt with later under

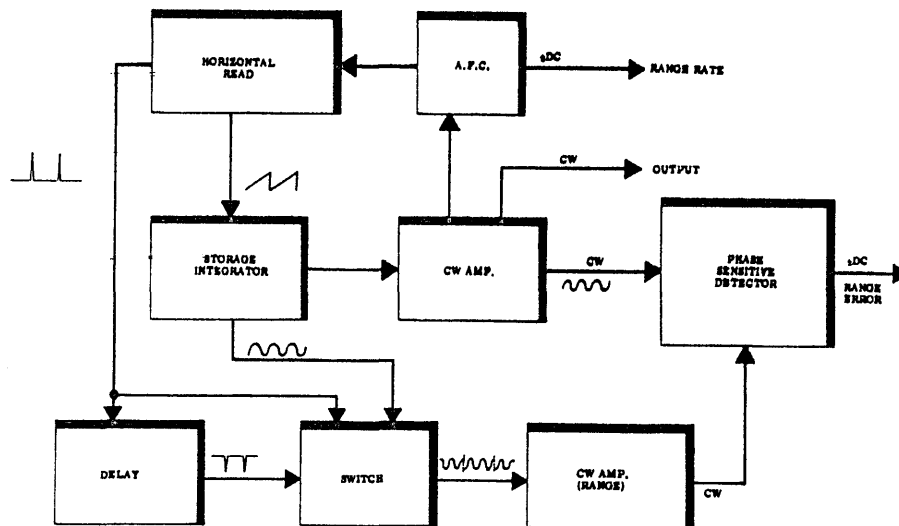


Fig. 4

target acquisition and search.

Attention is now directed to indication of amount and direction of departure of target range from a range reference signal, such as that which controls gate position or storage frequency converter reading time. As just shown in the treatment of range rate tracking, a voltage is available that is highly sensitive to difference in range rate between target and radar range unit. Once the range unit is set on proper range, the range rate error voltage may be used to prevent the range unit from deviating from the target range. This can be made to produce very stable tracking as long as no acceleration occurs during signal fades. However, since there is no sensitivity to position error, there is no guarantee that tracking is free from bias error. If sensitivity to position error were added, the high quality of the rate error signal available makes it ideally suited for tracking stabilization, and makes the requirements on position error sensitivity much less severe.

There are several methods of providing sensitivity to position error in range. Reference is made to Figure 4, in which the storage unit, C.W. amplifier, A.F.C. and horizontal reading sweep are the same units as shown similarly identified in Figure 2. The new elements in Figure 4 are a delay, a switch, a range C.W. amplifier, and a phase sensitive

detector. The switch is a simple phase reversing switch, operating to control the phase of the carrier frequency output of the storage unit as it is fed to the range C.W. amplifier. The switch is actuated by the horizontal reading sweep so as to start reading all lines in the same phase, and to reverse phase at the center of each line. The object is to read the first half of all lines in one phase, and the last half of all lines in the opposite phase. The delay unit assists in the switching process as one way to obtain the reversal at the center of the line.

Since the bandwidth of the range C.W. amplifier may be very much less than the spectrum represented by the switching rate, the amplifier will not follow the phase reversals, but will merely respond to the integrated resultant of the switch output. If the same energy is present on each phase, the net output will be zero. This corresponds to the condition when the echo pulse is exactly centered on the storage line, which occurs when the "gate" is properly delayed, or when the stored transmitted pulse (Fig. 1) is played back at exactly the right time to be in perfect register with the received echo. Any departure from this condition will produce a signal in the range amplifier. The amplitude of this signal will be proportional to the amount of departure, and the phase relative to the signal output of the storage unit will indicate the direction of error. The phase sensitive detector combines the outputs of the two C.W. amplifiers in sum and difference fashion to extract this information and yield a D.C. range error signal. This technique is the same as that used in the angle tracking circuits of the TAB radar.

It will be noted that range error signal derived as just outlined is based on pulse length  $\tau$  instead of the ultimate resolution based on transmitter modulation bandwidth  $B_T$ . This will suffice for many cases, even when  $B\tau$  is considerably less than  $B_T$ , so long as the pulse is so recorded that first and last halves are easily separated at switching rates that are easily integrated. However, if  $\tau$  were to be increased to approach  $T_i$ , the necessary switching rate does not permit easy integration and if  $\tau$  were increased indefinitely, corresponding to the theoretically possible transmission of modulated CW, there remains no basis for the method. Another approach to the range tracking problem may therefore be desirable in some applications.

Let range tracking be based on transmitter modulation. The error signal may then be originated at the point of maximum range resolution, which is the point of maximum sensitivity to changes in range. Figure 5 represents a single recorded modulation cycle envelope as applied to the transmitter frequency, shown in solid line as a triangular waveform. Under the stored envelope is shown in dashed line a portion of received echo in perfect register. Also shown in dotted line is a similar portion of received echo slightly out of register. Vertical lines by their length represent the frequency difference between echo and locally pro-



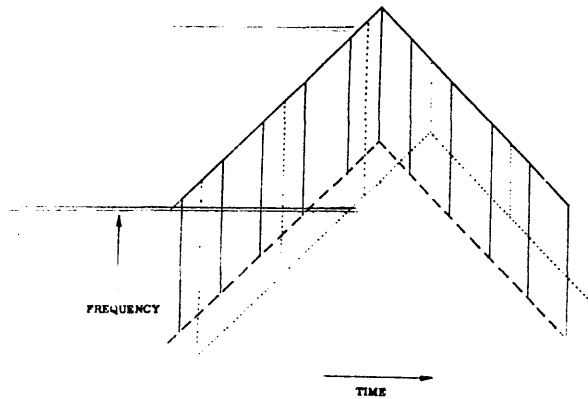


Fig. 5

duced signal, which is the intermediate frequency. The echo in register is seen to produce one constant frequency I.F. which is the correct frequency,  $f_1$ . The echo out of register is seen to produce two frequencies, equally spaced above and below the correct frequency. The two frequencies would normally have equal energy content. The frequency separation,  $\Delta f_1$ , of each of these frequencies from the correct frequency is proportional to the product of the amount of departure of the echo from perfect register,  $\Delta R$ , and the rate of change of transmitter frequency  $\dot{F}_T$ .

$$\Delta f_1 \propto \dot{F}_T \Delta R \quad (23)$$

Since the proportionality factor is that factor which relates range to time (see eq. 14), it follows that

$$\Delta f_1 = \frac{2 \dot{F}_T}{c} \Delta R \quad (24)$$

Let it be assumed that the out-of-register echo is close enough to register so that the two resultant frequencies lie within the pass band of the i.f. amplifiers and discriminators.

$$2\Delta f_1 < B_T \quad (25)$$

Their combined effect in the discriminators (A.F.C. circuits of Figures 1 and 2) is then the same as if the echo were in perfect register. The automatic alignment functions of the A.F.C. circuits would therefore remain undisturbed. It is now desired to obtain an element operative at the output of the CW amplifier of Figure 2 which is sensitive to  $\Delta f_1$ , and also sensitive to the order in which the two out-of-register frequencies are generated. Sensitivity to  $\Delta f_1$  may be had with two frequency-sensitive detectors having complementary frequency-amplitude characteristics AA and BB as shown in Figure 6. The crossover point is at the frequency  $f_1$ . The two outputs will now be equal and of op-

posite polarity, arising from input signals proportional to  $\Delta f_i$  plus a reference amplitude  $A_0$ .

$$A = -B = K_1 \Delta f_i + A_0 \quad (26)$$

Sensitivity to the order in which the two out-of-register frequencies are generated is necessary to obtain the direction of departure of the echo from register. Eq. (24) shows that  $\Delta f_i$  is proportional to  $\dot{F}_T$ . If the transmitter frequency  $F_T$  is differentiated to obtain a voltage of waveform  $\dot{F}_T$ , this may be used to amplitude modulate  $f_i \pm \Delta f_i$ . The frequency value of  $\Delta f_i$  would be unchanged, but  $f_i + \Delta f_i$  and  $f_i - \Delta f_i$  would differ in amplitude. The amplitude,  $a$ , of  $f_i + \Delta f_i$  would be

$$a = K_2 S (1 + m \dot{F}_T) \quad (27)$$

and the amplitude,  $b$ , of  $f_i - \Delta f_i$  would be

$$b = K_2 S (1 - m \dot{F}_T) \quad (28)$$

where  $M$  is the amplitude modulation factor. If the amplification preceding the range tracking detectors is automatically controlled to maintain the average signal constant, then  $S$  becomes invariant, and may be replaced by a constant, or

$$K_2 S = K_3 \quad (29)$$

It is now only necessary that the two frequency sensitive detectors be also amplitude sensitive. The amplitude variation at detector A input due to frequency sensitivity of the input circuits will be, from eq's. (24) and (26)

$$A_f = \frac{2 K_1}{c} \dot{F}_T \Delta R + A_0 \quad (30)$$

The amplitude variation of the signal at the input to the frequency sensitive circuits, from eq's. (27) and (29) will be

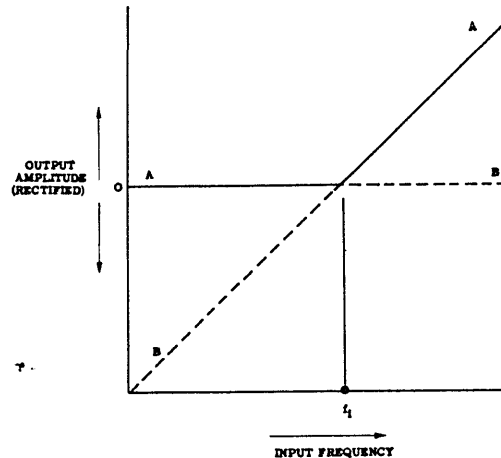


Fig. 6

$$A_a = K_3(1 + m\dot{F}_T) \quad (31)$$

The total output amplitude of detector A will then be

$$A = A_d K_3 \left[ \frac{2 K_1}{c} \dot{F}_T (1 + m\dot{F}_T) \Delta R + A_o (1 + m\dot{F}_T) \right] \quad (32)$$

where  $A_d$  is the detector characteristic. Likewise the total output from detector B is

$$B = -A_d K_3 \left[ \frac{2 K_1}{c} \dot{F}_T (1 - m\dot{F}_T) \Delta R + A_o (1 - m\dot{F}_T) \right] \quad (33)$$

These outputs may be combined in addition to give

$$A + B = 2A_d K_3 m\dot{F}_T \left[ \frac{2 K_1}{c} \dot{F}_T \Delta R + A_o \right] \quad (34)$$

It is possible to assign a constant magnitude to  $\dot{F}_T$ . When this is done, eq. (34) may be re-written in the form

$$A + B = K_4 \Delta R + K_5 A_o \quad (35)$$

This equation shows that the combined outputs of the two range tracking detectors give a signal which is linearly related to range tracking error in both magnitude and direction. In order to maintain pulse to pulse phase coherence for the frequencies  $f_i \pm \Delta f_i$ , as is necessary in order to retain full integration advantage, not only must  $\dot{F}_T$  be constant in magnitude for each pulse, but the modulation wave form of all pulses must be identical. The symmetrical triangular form shown in Figure 5 is therefore specified. The register reference point is the apex of the triangle.

A block diagram illustrating application of the method is shown in Figure 7. This figure combines parts of Figures 1 and 2, and adds the three range tracking elements differentiator, high pass detector and low pass detector, together with A.G.C.

From equation (34) it may be seen that range error sensitivity is equal to

$$S_{\Delta R} = \frac{4 A_d K_1 K_3}{c} m\dot{F}_T^2 \quad (36)$$

The factors K involve only signal amplification, which is limited ultimately by noise. The full integration advantage is effective, as is evident from Figure 7, in making K as large as possible.  $\dot{F}_T$  for the waveform of Figure 5, depends on pulse length and transmitter bandwidth  $B_T$ , and is equal to twice their ratio when  $\tau$  is very long compared to



$\frac{1}{B_T}$  (frequency modulation index very low)

$$\dot{F}_T = 2 B_T / \tau \quad (37)$$

Combining (36 and (37) yields the interesting expression for range error signal sensitivity

$$S_{\Delta R} = \frac{16 A_d K_1 K_3 m}{c} (B_T / \tau)^2 \quad (38)$$

It is now possible to give further consideration to range confusion. Targets appearing out of register generate frequencies  $f_i \pm \Delta f_i$ . When  $2\Delta f$  exceeds the i.f. bandwidth  $B_T$ , the target signal is rejected and no confusion exists with another target which is in register. The range confusion limit is therefore reached when

$$\Delta f_i = \frac{B_T}{2} \quad (39)$$

at which point  $\Delta R = R_c$ . By successive substitution of equations (24), (37) and (3) and rearrangement of terms, it is found that

$$R_c = \frac{c}{8 B_T} \quad (40)$$

Equation (40) is subject to the limitation of equation (37), which is based on the wave form of Figure 5. It indicates a range confusion of 40 yards for a bandwidth of  $10^6$ . In present systems, where equation (4) is valid, a bandwidth of  $10^6$  corresponds to a range confusion of 160 yards. Range confusion for a given bandwidth is therefore reduced by a factor of four.

#### ANGLE TRACKING

Consideration may now be given to angle tracking with the storage integrator. The first approach to consider, of course, is monopulse. It is at once obvious that normal application of RF amplitude comparison techniques would involve an extensive duplication of apparatus if the full integration capabilities are to be retained. Various devices for separating the signals of two or more channels in such a way that they can all be operated on by the same apparatus is possible, but not without compromise of one sort or another.

The compromise of high speed lobing relative to monopulse is not a serious one as long as a certain randomness may be introduced into lobing sequence in order to defeat angle jamming. High speed lobing may be applied very simply to the storage integration system. One device for

accomplishing this purpose is illustrated in Figure 8. The receiving antenna configuration must provide the necessary directional lobes with electronically controlled switches, as in the present high speed lobing laboratory radar system. These are illustrated in the upper left hand corner of Figure 8, with antenna bearing and elevation coordinates indicated. The receiving antenna would normally feed a receiver such as shown in Figure 1. The storage integrator, - CW amp., -A.F.C.- horizontal reading sweep loop is the same one shown in Figures 2, 4 and 7, since Figure 7 implies the storage sweeps to be included in the component marked "storage integrator." The lobe switch normally operates to sample a different lobe on each consecutive pulse, pairing the two lobes of each coordinate. Thus the switching order might be Right-Left-Up-Down, each lobe being used for the duration of one pulse interval,  $T_p$ . Each four consecutive lines recorded on the storage integrator will thus be a record of one each of four consecutive lobes as selected by the lobe switch. The amplitude distribution among these four lines will correspond to the signal distribution among the four receiving antenna lobes. Directional information may therefore be derived from the stored record.

Switch 1 of Figure 8 is a phase reversing switch similar to the switch of Figure 4. It is operated in synchronism with the horizontal reading sweep so as to read out adjacent lines mutually in opposite phase. Switch 2 serves as a double pole circuit switch to transfer the output of switch 1 to one or the other of the bearing and elevation CW amplifiers. Switch 2 is synchronized on one half the frequency of switch 1, so as to feed two adjacent lines to the bearing CW amplifier, and the next two adjacent lines to the elevation CW amplifier. The outputs of the bearing and elevation amplifiers will be of the same character as the output of the range CW amplifier of Figure 4, and may be similarly utilized to derive bearing and elevation error information, as shown in Figure 8.

It remains necessary to so synchronize the switches with the stored record that only bearing lines are switched into the bearing CW amplifier and only elevation lines are switched into the elevation CW amplifier. This may be accomplished by recording and integrating a number of pulses equal to an integral multiple of four, and using the vertical sweep fly-backs to over-ride lobing and switching controls so as to start all sequences with the same lobe on the first line of each frame. It is then apparent that as long as the same sequence is used for antenna lobe switching and playback output switching for each recorded frame, any desired sequence may be used, and the sequence may even be changed from frame to frame if the reading and writing frame frequencies are equal.

The diagram of Figure 8 may be used in combination with those of Figures 1 and 2, simultaneous with either Figure 4 or 7. The performance of each of these component systems is in no way compromised by putting them all together. The combined system therefore retains all the desirable properties of each component system, and now possesses the following additional properties not previously identified in summary form:

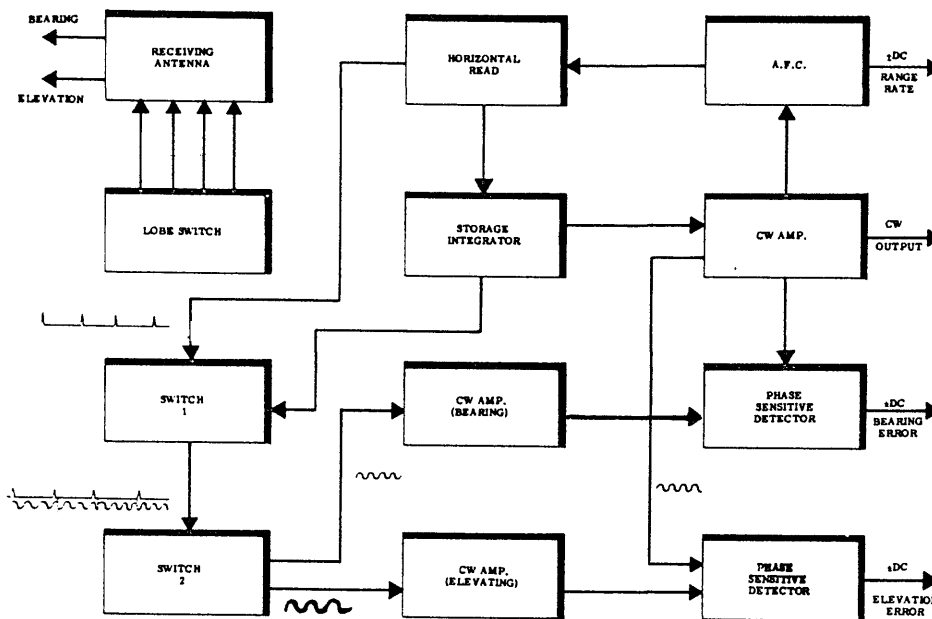


Fig. 8

1. Range resolution for a given bandwidth increased over conventional pulse radar by a factor of four.
2. Range error data for range tracking of high precision.
3. Angle error data for angle tracking on a high speed lobing basis, with all the flexibility of the present high speed lobing radar system.

#### TARGET ACQUISITION

The discussion this far has been limited to a pulse echo tracking system that will track automatically in four independent coordinates any single target to which the system has been adjusted. Consideration will now be given to information from other targets than the one being tracked. Such information is helpful in acquiring targets, and is necessary in search functions.

Figure 9 illustrates a basic diagram for performing search functions. It represents a re-arrangement of the techniques described for tracking, and replaces Figures 1 and 2. Two storage tubes are used.

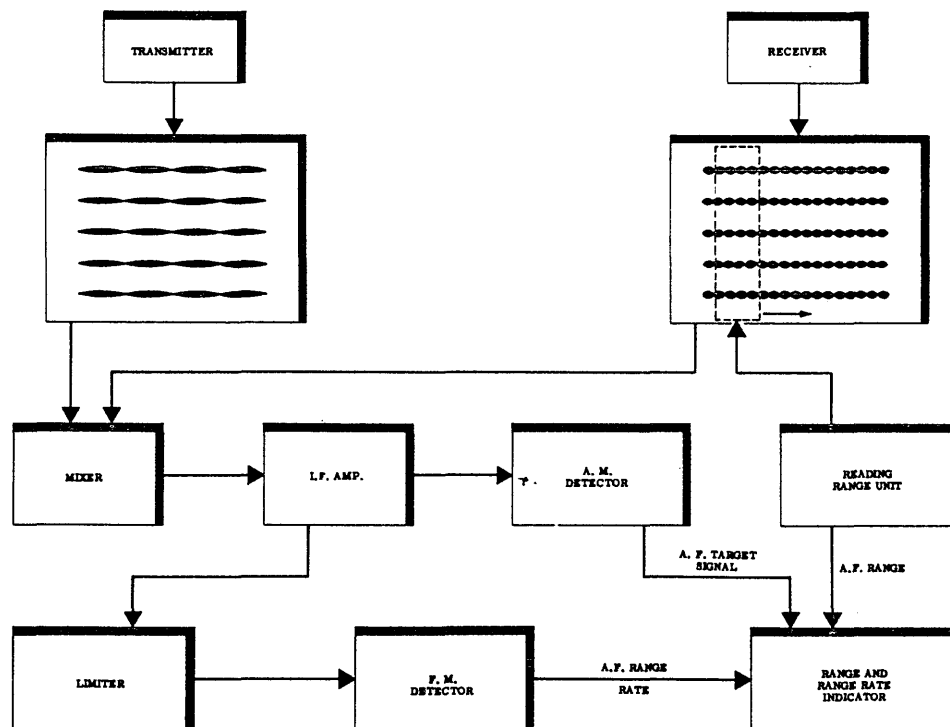


Fig. 9

One records only transmitted signals. The other records only uncorrelated received signals. Both record multiple lines, each line corresponding to one pulse interval. Each line of the transmitter record consists of the transmitted pulse, of length  $\tau$ . Each line of the receiver record consists of an extended range interval many times  $\tau$ , and possibly even the entire range interval represented by  $T_p$ . The number of lines recorded is  $T_i/T_p$ . There now exists a complete record on one storage tube of all signals received, for the entire integration time interval  $T_i$ , and on another storage tube a complete record of all transmitted signals during that same time interval. One is at liberty to inspect these records in any fashion that may prove fruitful.

#### RANGE SEARCH

Let the reading sweeps of the two storage tubes be synchronized so that an interval equal to  $\tau$  on one line of receiver record will be scanned synchronously with the corresponding pulse on the transmitter record. Let this partial inspection of one receiver line be followed by a similar inspection of the same range interval on the next adjacent line, and let this process continue until all lines have been inspected



at one range interval of length  $\tau$ . Let the outputs of the two storage tubes, after amplification if desired, be combined in a mixer to produce a difference frequency resonant to the i.f. amplifier. The difference frequency may be produced as in Figure 1. If the reading process just described were repeated continuously at the same range, or with range changed only to track a target, the system would correspond exactly in function to a combination of Figures 1 and 2, the i.f. amplifier of Figure 9 corresponding to the CW amplifier of Figure 2. Now, however, the range interval of length  $\tau$  that is subjected to inspection may be scanned by a reading range unit from one end to the other of the entire receiver record, the total scanning time, or reading frame period, being much longer than the vertical scan time required to inspect one range interval  $\tau$ . Signal from a target will generate a pulse as the reading range unit scans the reading sweeps across the record of this signal. The bandwidth of the i.f. amplifier must be sufficient to pass the spectrum of the generated pulse, plus the frequency spread due to all anticipated doppler shifts. The pulse will be longer than the input radar pulse by a factor on the order of the number of lines integrated. The frequency spectrum of this pulse will therefore be narrower than that of the input radar pulse by that same factor. The doppler frequencies, on the other hand, will be increased in the same manner as for Figure 2. The magnitude of the increase, however, will be much greater, since the frame frequency of Figure 2 corresponds to the vertical scan frequency of Figure 9. The vertical scan frequency must be greater than the frame frequency of Figure 9 by a factor on the order of the ratio of range interval recorded to range resolution. If all stored information is to be used, the reading frame frequency of Figure 9, as of Figure 2, must be not less than the writing frame frequency. Therefore, the reading line frequency during each vertical scan of Figure 9 will be greater than the reading line frequency of Figure 2 by a factor on the order of the ratio of range interval recorded to range resolution. Doppler frequency shifts will correspondingly be increased by the same ratio, and will therefore normally occupy a spectrum width much greater than the output pulse spectrum width. The bandwidth of the i.f. amplifier of Figure 9, therefore, is determined by the doppler frequency spectrum it is to pass. The output carrier frequencies of the storage tubes of Figure 9 will normally be somewhat greater than their input carrier frequencies, and unless the recorded range interval is much less than the total range corresponding to  $T_p$ , the output frequencies may be much greater. It is obvious that while both carrier frequency and doppler shifts are much higher in the i.f. amplifier of Figure 9 than in the CW amplifier of Figure 2, their ratios are the same, since they are fixed in the stored record, and reproduced in the same reading sequence.

The A.M. detector, reading range unit and range indicator of Figure 9 perform the functions respectively of detector, range sweep, and range indicator of normal radar, but on a much reduced time scale. The range indicator will display target pulses properly placed in the range interval corresponding to the stored record. If the record spans the entire

range corresponding to  $T_p$ , the indicator will display all targets in the beam within that range, regardless of range rate. If the transmitted signal is unmodulated, the range confusion is given by equation 2. If the signal is modulated, equation 9 applies, and if the modulation is of the form of Figure 5, equation 40 applies.

#### RANGE RATE SEARCH

A loss in sensitivity is now suffered due to the increase of bandwidth necessary to include the doppler spectrum. This loss may be neutralized by the addition of range rate resolution. The ultimate is reached when the doppler spectrum corresponding to range rate confusion equals the pulse spectrum. In Figure 9, the pulse spectrum is determined by  $T_i$ . No new fundamental limit, therefore, is placed on sensitivity by the inclusion of all target rates.

One method of indicating target range rate is shown in Figure 9. The output of the i.f. amplifier may be put through a voltage limiter and then to a frequency detector. The frequency detector may be so balanced that frequencies corresponding to one particular range rate will give no output. This range rate will be identified hereafter as the reference rate. Targets closing relative to the reference rate will then give rectified pulses of one polarity, and targets opening relative to the reference rate will give rectified pulses of the opposite polarity. The amplitude of the rectified pulses will be proportional to the range rate relative to reference rate. These signals may then be displayed on a suitable indicator. A "Pisa" indicator, where normal "A scope" echoes lean in proportion to range rate, is one example of how range and rate may be simultaneously displayed.

The range rate sensing device just described will not resolve the separate rates of three or more targets appearing at the same range, but rather will indicate one range rate which is an average of the group. Two rates will be resolved, however, if one is the reference rate. The average rate of clutter could therefore be made the reference rate, and then individual targets moving relative to the clutter would be displayed and their correct rates indicated. Two or more targets moving relative to clutter but at the same range would give rise to a rate indication representing an average of the group. If the rates differed widely, the targets would not remain long at the same range, and would then be resolved in the range coordinate. If the rate differences were small, the average would be not far from the individually correct values. The range rate resolution of multiple targets having very nearly the same range rate ultimately depends fundamentally on frequency isolation by frequency selective circuits, not shown in Figure 9.

An exception exists in the case of two targets where the rate of one is the reference rate. Here the limiting factor is noise, and the

rate indicator of Figure 9, as well as the range rate follower of Figure 2, is particularly advantageous in this regard. Noise voltages in the frequency spectrum corresponding to closing target rates are rectified and balanced against similarly rectified noise voltages in the frequency spectrum corresponding to opening target rates, to give a resultant output. Fluctuations in this output appear as noise on the reference rate. To be resolved in rate from the reference rate, the rate signal from a second target must be discernable in this noise. As shown above, the amplitude of the rate signal is proportional to the rate difference from reference rate, therefore rate resolution is limited basically by rate noise on the reference rate.

When clutter is used for determination of reference rate, a further limitation arises from the doppler spectrum of the clutter. Reference rate will be an average of clutter rates, assuming a finite spread of clutter doppler frequencies, and that average may itself have a fluctuating value. A target whose rate is within the spread of clutter rates but different from the average clutter rate will be displayed when that difference is discernable in the variations of average clutter rate.

The search system of Figure 9 possesses many of the properties of the tracking systems of Figures 1 to 8. There are also some basic differences. The range scanning operation of the reading range unit will introduce a frequency shift corresponding to an extremely high range rate. This may be compensated for within the system by any of several fairly simple and more or less obvious devices. Corresponding modification of the writing or reading sweep speed for the transmitter signal provides a fairly direct compensation.

Inasmuch as targets are to be displayed whose rates are not tracked, the matter of handling doppler frequencies equal to multiples of half the pulse recurrence frequency takes on new significance. Doppler shifts equal to a multiple, including one, of the pulse recurrence frequency will be confused with reference rate, while those equal to an odd multiple of one half the recurrence frequency will have zero amplitude, because of the line to line phase reversal in the stored record. By using a line to line phase reversing switch, as is used for angle measurement in Figure 8, and operating a second range rate channel on the output of this switch, zero amplitude will occur on doppler shifts equal to multiples of the recurrence frequency, and maximum amplitude on shifts equal to odd multiples of one half the recurrence frequency. This corresponds exactly to an identical limitation of present MTI systems. The two channels together would leave no gaps in coverage, but all range rates would appear as less than that corresponding to a doppler shift of  $1/2 T_p$ , and range rates corresponding to a doppler shift of  $N/T_p$  would appear as zero reference rate, and therefore be confused with clutter. If necessary, points of confusion may be resolved by noting effects when pulse recurrence frequency is changed.

These limitations may be overcome by recording the radiated carrier frequency on the storage devices, and repeatedly playing back the record, once for each group of range rates corresponding to the bandwidths  $1/2 T_p$ . The vertical reading scan would then be tilted at an angle corresponding to range rate. Each time the record is scanned, the angle of tilt would be changed, the change corresponding to one half cycle per line of the record. If the total doppler spectrum to be inspected is  $n/2T_p$ , then the complete inspection would require  $n$  playings of the record, and the output frame frequency would be at least  $n$  times the input frame frequency. The output carrier frequency would also be at least  $n$  times the input carrier frequency, and the method of operation is applicable only when radiated carrier frequency is recorded. Obviously, radar application of such a system with present storage devices has its limitations, but applications involving much lower carrier frequencies than common in radar may be quite appropriate.

The presence in the stored record of frequencies shifted by doppler effects as required in Figure 9, rules out the method of coordinate separation for angle error measurement illustrated in Figure 8. While it is true that lobe switching for angle measurement has had little use in search radar, it may still be desirable in some applications. The cross-talk that would occur between angle error and range rate if Figures 8 and 9 were to be combined can be prevented by amplifying and rectifying separately the storage tube outputs corresponding to each lobe, and then comparing the rectified outputs.

#### AZIMUTH SEARCH

Search radar generally includes azimuth coverage greater than antenna beam width, which is accomplished by azimuth scan. Generally this takes the form of 360 degree plan position indicating systems. When the system of Figure 9 is used for PPI, the integrating time  $T_i$  is the time required for the antenna to rotate one beamwidth. High sensitivity, as well as sensitivity to small range rates or high clutter rejection, dictates wide beams and slow rotation speeds in order to make target illumination time as long as possible. The high integration efficiency of storage techniques, together with the high quality of rate information and clutter rejection, may justify re-examination of the problem of surveillance. This report does not include exploitation of that lead. In the case of airborne radar for schnorkel detection, target velocities are low and speed of azimuth scan is limited at its low end only by range of detection and speed of flight of searching aircraft. Application of the system of Figure 9 in this field should therefore prove particularly fruitful, especially when it is remembered that automatic signal tracking on clutter to be removed also automatically compensates for own ships motion and antenna rotation.

The basic search system of Figure 9, even when the lowest possible

intermediate frequency is used for recording, imposes much more severe requirements on storage tubes than does the tracking systems of Figures 1 to 8. First, the recording frequency may be higher, since the recording bandwidth is determined by  $B_T$  rather than  $B_f$ , and the recording carrier must be greater than the modulation bandwidth. Second, a much greater range interval is recorded. Both factors combine to increase the resolution requirements, since both add to the number of cycles that must be recorded. Third, the frequencies generated on reading are much greater for Figure 9, and in general would be greater than the writing frequencies. The development of storage tubes is in a very early stage, however, and the definition of requirements set forth by these systems will provide guides for a direction of effort that storage tube development engineers are seeking. It is not unreasonable to expect that as the need grows for particular qualities in storage devices, progress will continue toward meeting the need.

#### HIGH DUTY FACTOR OPERATION

Up to this point only maximum utilization of echo energy available at the receiver input has been considered. The only theoretical limitations imposed on minimum detectable signal at the output are the fundamental relationships of the Hartley law relating quantity of information per unit time to transmission bandwidth. The bandwidth reduction from that required for information resolution to that sufficient for rate of flow of information has been accomplished in the theoretically most advantageous manner, within the limits imposed by an arbitrary form of radiated signal consisting of very short pulses with very low duty factor.

The integrating method used has opened up the possibility of modifying the form of the radiated signal. Since range measurement and range resolution have been made independent of pulse length, and transmitter frequency modulation during the pulse is no deterrent to the integrating process, the way is opened for increase of transmitter pulse length. Since over-all system sensitivity is proportional to the average power radiated, two directions of improvement in transmitter design are made available. (1) Peak power may be reduced, keeping average power constant, without loss in system sensitivity, or (2) Average power may be increased, keeping peak power constant, with corresponding increase in system sensitivity. As a practical matter, both may be exploited at once, giving some reduction in peak power, some increase in average power, and some increase in sensitivity. Investigation will now be made into the degree to which this approach may be carried.

If sending and receiving could take place simultaneously, obviously there would be no limit, and the sender could radiate continuously. This mode of operation may not be permitted in some applications, particularly when the same antenna is used for sending and receiving. If the

restriction is imposed that sender and receiver cannot operate simultaneously, even for close in echoes, then the sender must be operated so as not to block out any echoes in the design range of the system. In present pulse radar systems this would limit the pulse length to that corresponding to the minimum range at which targets must be detectable. In present practice, that minimum range is not very much greater than the range resolution, so it at first appears that there is little to be gained in this direction. There appears to be no good reason, however, why range measurement cannot be made from the trailing edge of the radiated pulse as a reference point. When this is done, the receiver may start properly indicating echoes as soon as it recovers from the sender signal. The length of time the sender has been on the air before it is cut off is of no consequence to the receiver in indicating the time of cut-off of echo signals from nearby targets. The cut-off of echo signals from more distant targets will occur at a later time, and the time from sender cut-off to echo cut-off is available for reception of energy from that particular echo. Thus, the greater the range to the target, the greater the sensitivity of the system, up to the limit determined by sender pulse length.

The pulse length may now be made to correspond to the maximum design range of the system, and the duty factor becomes 0.5 or 50%. The method of range tracking illustrated in Figures 2 and 4 may be used, providing compensation is made for the varying effective length of received pulses, and providing further that  $T_p$  is sufficiently less than  $T_i$ . In fact, the method may be only slightly modified to operate satisfactorily when  $T_p = T_i$ . Modulation waveform therefore need not be restricted by range tracking requirements, and random modulation may be utilized. Thus no confusion will exist for multiple range targets.

This mode of operation introduces a new set of limitations on search functions. It will be noted that inspection of the stored record now involves use of the same stored information many times over. This arises from the fact that signal detection involves a process of cross correlation between two functions of time which differ principally only by a linearly added time delay, and a separate inspection of all or an appreciable portion of the record is necessary for each value of time delay for which inspection is made. Each time delay corresponds to a unique value of range, so that each inspection corresponds to one look at a range interval equal to  $R_c$ , the range confusion. The total number of inspections required to cover the total range of the system,  $R_p$  is therefore  $R_p/R_c$ .

One obvious consequence is that the output carrier frequency will be much higher than the input carrier frequency. It will not be greater by the factor  $R_p/R_c$  since the entire record is inspected only for the longest ranges, and for shorter ranges, shorter portions of the record are utilized. However, ratios of  $R_p/R_c$  considered acceptable for pre-

sent radar are on the order of  $10^3$ , so that output frequency may still be several hundred times the input frequency. This may be an annoying inconvenience, if not even a limiting factor, in application of the method.

Another consequence arises from doppler phenomena. Doppler shifts occur not only in carrier frequency, but also in modulation frequencies, and they are cumulative. The doppler shift therefore introduces time as a multiplying factor in one of the functions in the cross correlation process. The effect is to restrict the time interval during which the two functions can remain in correlation within the limits defined by  $R_c$ . A simple way to visualize the physical significance is to observe that when a doppler-shifted signal is properly matched to the delayed reference signal at the beginning of a long pulse, the match will become less perfect as one moves along the pulse until ultimately the high frequency components of modulation will become out of phase. Long before this point is reached, however, the difference frequency from the combination of the two storage tube outputs will become multiple valued and its spectrum will increase until it exceeds the bandwidth of the following amplifiers.

#### VARIABLE RESOLUTION

It will be noted that both output frequency and doppler shift range are related to  $R_c$ . It so happens that increase in  $R_c$  will reduce the high frequency requirements and increase the doppler shift range. It therefore becomes appropriate to re-examine the range resolution requirements of echo ranging apparatus. In the case of single target precision tracking the limitations do not apply, since concentration of interest in range and range rate to a single target permits omission of range scanning and compensation for range rate. In the case of search functions, resolution requirements are not the same for all ranges. It is very desirable to retain high resolution for targets at close range, but early warning detection of targets at long range does not of itself require high resolution at long range. In those search functions which require high resolution at long range, there will be applications where it is permissible to obtain it momentarily at the expense of some other parameter, such as sensitivity to targets at other ranges in the same direction. It is therefore operationally admissible to propose that resolution for search purposes be modified as a function of range, so that high resolution is obtained at short range, and relatively lower resolution is obtained at long range. Such operation is technically feasible in the method under consideration, and permits at least some relief from the high frequency requirements, while at the same time providing the desired range of doppler shifts for all applications at present envisioned.

The parameter that is to be varied with range is the modulation bandwidth of the transmitter. Let the radiated pulse start with a narrow

modulation bandwidth corresponding to the maximum permissible value of  $R_c$  at maximum range. Then let the bandwidth be increased with time until, at the end of the pulse, the bandwidth is attained which corresponds to the value of  $R_c$  required at short range. The law of variation of bandwidth with time may be set up arbitrarily to fit any desired type of performance, even to the inclusion of bandwidth discontinuities. Since the trailing end of the pulse only is utilized at short ranges,  $B_T$  is great and  $R_c$  is small for targets at short ranges. At greater ranges, the effective value of  $B_T$  for the whole range to a particular target will be less than at short range, so  $R_c$  will be greater. In the range scanning process, therefore, successive scans will be very close together in range at short range where the length of record inspected is short, and may be spaced much farther apart at longer ranges where the length of record inspected on each scan is long. As the successive scans become more widely separated, the correlation at the wide band end of the pulse will be lost, but since in practical application the length of signal requiring wide bandwidth modulation is a very small fraction of the total signal, the loss in sensitivity from this cause may be kept well below one decibel, which is insignificant in terms of range difference. The net result of this mode of operation relative to constant bandwidth modulation is three fold. (1) Output frequencies are lowered, (2) doppler shift range is increased, and (3) range resolution is varied with range, the higher resolution occurring at the shorter ranges. The functional significance is that range resolution at long range has been traded for increased sensitivity and ability to register targets having greater range rates.

It is of considerable interest that change in transmitter modulation bandwidth does not require corresponding change in receiver bandwidth to retain maximum sensitivity so long as the receiver input bandwidth is sufficient for the greatest transmitter bandwidth encountered. This is because the receiver bandwidth before integration is not a factor in receiver output signal to noise ratio. Thus there is presented a flexibility in manipulation of transmitter bandwidth that may be turned to a number of uses. For example, range resolution may be traded for speed of search in a tracking equipment for target acquisition, or for field of view in a search system for close inspection of a particular target area, all with no change in sensitivity, and at the choice of the equipment operator.

#### EXAMPLES

Typical examples indicating theoretical limits of performance with ideal circuits may be instructive in clarifying some of the concepts presented in this paper. Consider the radar Mark 25, a medium power precision tracking X-band system now in service use. The parameters of this set are nominally



$T_i = 2 \text{ Sec.}$                       range resolution, 40 yards  
 $T_p = 5 (10)^{-4} \text{ sec}$   
 duty factor =  $5 (10)^{-4}$   
 Peak power = 50 kw  
 $\tau = 2.5 (10)^{-7}$

From equation 16, the signal to noise improvement over a single pulse is 36 db. Assuming weak signal operation of a square law detector, wherein ~~signal to noise~~ ratio is squared in the detection process, present improvement over a single pulse by integration after detection can be no greater than 18 db. The net increase in sensitivity is therefore 36-18, = 18 db. This corresponds to increasing range capabilities by a factor of 2.5. It arises entirely from integration before detection instead of after detection, and is obtained without modification of the radiated signal.

The duty factor may now be increased to 0.5. If this is accompanied by decrease in peak power to maintain constant average power, the peak power is brought down to 50 watts and the factor of 2.5 increase in range is undisturbed. If it is desired to reduce the average power and present sensitivity is adequate, the peak power falls to less than one watt, average less than 1/2 watt. Thus, by employment of integration before detection plus 50% duty factor operation, the transmitting oscillator of the radar Mark 25 could be replaced by a receiver local oscillator without loss in range. In addition the set would be anti-clutter, have a range resolution on the order of 10 yards instead of the present 40 yards, and a range rate resolution on the order of .03 kt.

A practical compromise to favor sensitivity would be to increase average power to several hundred watts, reduce peak power to twice average power, retaining all other factors. Net increase in range over the present set would then approach the factor of 5.

Another example of interest is the AN/APS-20-A as used for schnorkel detection. The significant parameters are

Pulse frequency	= 300/sec
antenna beam width	= $3.5^\circ$ in azimuth
antenna rotation speed	= 6 rpm
average power	= 600 watts
frequency	$\approx 3000 \text{ mc}$
$\tau$	= $2 (10)^{-6}$

From the parameters it is seen that target illumination time is 0.1 sec. so  $T_i = 0.1$ .  $T_p = 1/300$ , so  $T_i/T_p = 30$ . Gain over single

pulse operation is therefore approximately 15 db. Present gain with integration after detection is on the order of 7.5 db. Net gain in sensitivity over the present set due to integration before detection would be about 7.5 db, corresponding to a 25% increase in range. This appears at first to be of little value, but with integration before detection it becomes possible to juggle other parameters for further net gain. For example, antenna beamwidth may be increased without loss in gain, since the resulting added time of target illumination is utilized for effective signal integration. Increased antenna beamwidth means a smaller antenna, therefore less load on the aircraft. Also conversion to 50% duty factor removes the high voltage, high peak power, and high filament power requirements for the transmitter. Conceivably the average power of the transmitter might therefore be doubled without increasing over-all aircraft loading. This gains 3 db. If the range of detection were doubled, the antenna scan rate may be halved. Thus  $T_1$  is doubled, giving another 3 db. The total improvement of  $7.5+3+3$  db is 13.5 db, or more than enough to double the range.

Retaining the present transmitter bandwidth, range resolution would be 80 yards, rather than the present 320 yards, and range rate resolution on the order of .05 kt., or one inch per second. In this application, the dynamic clutter rejection would be a particularly significant factor.

#### SUMMARY

A new approach to echo ranging has been outlined, based on application of electronic storage or memory devices. Two uses are made of these devices, one for frequency conversion, the other for signal integration.

Some technical features of interest resulting from this approach are:

- (1) A high order of inherent frequency stability at intermediate frequency.
- (2) Pulse to pulse phase coherence at intermediate frequency.
- (3) Signal integration at intermediate frequency.
- (4) High duty factor operation of pulse systems, and
- (5) Exploitation of cross correlation techniques without compromise in function.

Among the functional properties added or improved over present practice may be included the following:

- (1) Sensitivity increased by orders of magnitude, with attendant

increase in range and accuracy of tracking data in all coordinates.

- (2) Dynamic clutter rejection provided, independent of own ship's motion and antenna rotation,
- (3) Immunity to electronic jamming increased by orders of magnitude,
- (4) Range resolution for a given bandwidth increased by as much as a factor of four, and
- (5) Range rate introduced as a fourth and highly sensitive ~~parameter~~ of resolution, with range rate tracking of single targets and range rate display of multiple targets.
- (6) Variable resolution without loss in sensitivity introduced to permit freedom of exchange between resolution and rate of flow of information.

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## APPENDIX

### NOTATION

<u>Symbol</u>	<u>Units</u>	<u>Definition</u>
$[A_a]$	= V	Amplitude of input to frequency sensitive circuits of one side of range tracking detector.
$[a,b]$	= V	Amplitudes of i.f. signals arising from range tracking position error.
$[A,B]$	= V	Amplitudes of the inputs to the two sides of the range tracking detector.
$[A_d]$	= $V^{-1}$	Detector characteristic.
$[A_f]$	= V	Amplitude of input to one side of range tracking detector due to frequency sensitivity of its input circuits.
$[A_o]$	= V	Amplitude of the input to each side of the range tracking detector when there is no range error.
$[B_i]$	= $T^{-1}$	I.F. bandwidth corresponding to integration time.
$[B_\tau]$	= $T^{-1}$	I.F. bandwidth corresponding to minimum detectable signal for pulse length $\tau$ .
$[B_T]$	= $T^{-1}$	Modulation bandwidth of transmitted signal.
$[c]$	= $LT^{-1}$	Velocity of propagation of radiated signal.
$[F_i]$	= $T^{-1}$	Input frame frequency
$[f_i]$	= $T^{-1}$	Input I.F.
$[\Delta f_i]$	= $T^{-1}$	Change in intermediate frequency at integrator input due to position error of <del>radar range track-</del> ing unit.
$[F_o]$	= $T^{-1}$	Output frame frequency.
$[f_o]$	= $T^{-1}$	Output I.F.
$[F_T]$	= $T^{-2}$	Time rate of change of transmitter frequency as a result of frequency modulation.

<u>Symbol</u>	<u>Units</u>	<u>Definition</u>
[J]	= 1	Jamming power multiplying factor.
[K]	= $LT^{-1}$	
[K']	= L	
[K <sub>1</sub> ]	= VT	
[K <sub>2</sub> ]	= 1	
[K <sub>3</sub> ]	= V	
[K <sub>4</sub> ]	= $VL^{-1}$	
[K <sub>5</sub> ]	= 1	
[L <sub>i</sub> ]	= $T^{-1}$	Input line frequency.
[L <sub>o</sub> ]	= $T^{-1}$	Output line frequency.
[L <sub>si</sub> ]	= $LT^{-1}$	Input line speed.
[L <sub>so</sub> ]	= $LT^{-1}$	Output line speed.
[m]	= $T^2$	Modulation factor.
[M <sub>pi</sub> ]	= $T^{-1}$	Frequency of modulation which depends on input line frequency as a carrier.
[M <sub>po</sub> ]	= $T^{-1}$	Frequency of modulation which depends on output line frequency as a carrier.
[N]	= V	Input noise.
[n]	= 1	Any integer.
[n <sub>R</sub> ]	= 1	Number of range intervals inspected per transmitted pulse.
[R <sub>c</sub> ]	= L	Range Confusion.
[R <sub>p</sub> ]	= L	Range corresponding to the time interval $T_p$ .
[R <sub>T</sub> ]	= L	Range corresponding to the time $\tau$ .
[R <sub>v</sub> ]	= L	Total range inspected per transmitted pulse.

<u>Symbol</u>	<u>Units</u>	<u>Definition</u>
$[\Delta R]$	= L	Position error of radar range tracking unit in terms of range.
$[S]$	= V	Input signal.
$[S_{\Delta R}]$	= $VL^{-1}$	Sensitivity of range tracking unit to position error in range tracking.
$[T_1]$	= T	Integration time.
$[\tau]$	= T	Transmitted pulse length.