Plume effect on Radar Cross Section of missiles at HF band

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Abstract – The Radar Cross Section of missiles at HF band is affected by the presence of the missile plume. In fact, although the plume is transparent at the microwave frequencies, it reflects almost the totality of the energy at the HF band.
In this paper we evaluate the effect of the missile plume on its Radar Cross Section (RCS) at HF band. In order to achieve the goal we: 1) verify that the missile plume can be physically represented by a plasma through a chemical analysis of the combustion; 2) define the time-varying non-linear differential equation that rules the electron density equilibrium in the plume; 3) solve the time-varying non-linear differential equation in order to evaluate the length of the plume that contributes to the RCS calculation; 4) evaluate the RCS of the missile+plume by means of the Method of Moments (MoM). As a numerical case, the application of both a surface and sky wave Over The Horizon (OTH) radar has been considered. Specifically, the RCS (as a function of the incidence angle) of the missile during the boost phase trajectory has been evaluated.

I. INTRODUCTION

Detection of ballistic missiles in the boost phase by Over the Horizon (OTH) HF radar is a very important task in a war scenarios to early activate the defence system. The visibility of a missile at long distance is mainly affected by its RCS, which is a parameter extremely important to correctly design the radar system. At HF band, the calculation of the RCS obtained by considering only the body of the missile would lead to an underestimated value. In fact, for frequencies ranging from 3 to 30 MHz and considering the boost phase, the plume behaves as plasma. To demonstrate that the plasma model is appropriate, the chemical reactions of the propulsion system combustion must be considered[1]. Specifically, the exothermic combustion process produces a rapid temperature augment that causes a split of the residual alkaline metals in positive and negative ions. Negative ions combine with the hydrogen and produce neutral molecules and electrons. At the end of the process, the exhaust gas is composed of neutral molecules and of a cloud of macroscopically neutral (“*Almost neutrality condition*”) positive and negative charges. This gas status can be defined as plasma, confirming the hypothesis.

As already known in the case of the ionosphere effect on the radio waves, a reflection of the incident e.m. field occurs at suitable frequencies and incidence angles. Therefore, the radar echo energy is in general affected by the presence of the missile plume.

The aim of the paper is to evaluate the contribution of the missile plume to its RCS. To obtain this result we 1) define the time-varying non-linear differential equation that rules the electron density equilibrium in the plume; 2) numerically solve the differential equation; 3) apply the Method of Moments (MoM) to the composition of missile body and plume, both modeled as cylinders. The numerical calculation of the RCS of a short range ballistic missile (length of 6 m) at 4 different frequencies and with incident angles (see θ_i in fig. 1) ranging from 90^0 to 0^o is presented and discussed in the last section.

II. PLUME ELECTROMAGNETIC MODEL

In first part of this section the chemical reactions due to the gas post-combustion are studied. By following chemical principles, the calculation of the electron concentration at the missile nozzle is then evaluated. In the second part of the section, the time-varying non-linear differential equation that rules the electron concentration in the missile plume is first defined and then solved.

II.1 Combustion chemical reactions

Various mechanisms occur in the post-combustion chemical reactions [1] (see Table I). The most important phenomena affecting the electron density are the charge production and charge recombination. In this section we focus on the chemical reactions of a solid fuel missile. For missile altitudes lower than 20 Km with respect to the sea level, the main electronic production mechanism is showed in eq. 1:

$$K + Cl \rightarrow K^+ + Cl^- \qquad (1)$$

followed by:

$$H + Cl^- \rightarrow HCl + e^- \qquad (2)$$

The dominant recombination reaction is showed in eq. 3:

$$K^+ + e^- + M \rightarrow K + M \qquad (3)$$

It is worth noting that, although the magnesium particles involved in the reaction of eq. 3 (originally present in the fuel) seem to not affect the production of electrons, they

[1] In this paper solid fuel rockets are considered. For liquid fuel rockets different reactions should be considered.

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14. ABSTRACT

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must be accounted for in order to satisfy impulse and energy conservation law.

The reactions showed in eq. 1-3 allow us to calculate the electron density at the missile nozzle, namely n_i. Specifically, we consider the molar fraction of chlorine in the combustion products, which is typically equal to N_{cl}=2.7*10^{-3}. Therefore, a mole of fuel contains $N_e = N_{cl} * N_{av}$=16.26 *10^{20} electrons/mole, where N_{av} is the Avogadro number.

Reactions	Chemical components	Products
Hydrogen Carbon Oxide combustion	O+O+M	O_2+M
	O+H+M	OH+M
	H+H+M	H_2+M
	H+OH+M	H_20+M
	CO+O+M	CO_2+M
	OH+OH	H_2O+O
	OH+ H_2	H_2O+H
	O+ H_2	OH+H
	H+ O_2	OH+H
	CO+OH	C O_2+H
Reactions containing chlorine	H+Cl+M	HCl+M
	Cl+ H_2	HCl+H
	H_2O+Cl	HCl+OH
	OH+Cl	HCl+O
Neutral Potassium	K+HCl	KCl+H
Electrons production and recombination	K^++ e^- +M	K+M
	K^++Cl^-	K+Cl
Charge exchange	Cl+M+ e^-	M+Cl^-
	HCl+ e^-	H+ Cl^-

Table I - Main post-combustion chemical reactions

To evaluate n_i, the volume of a mole of exhaust gas at the pressure and temperature present at the burner exit must be evaluated. Hence, in order to calculate the pressure at the burner exit, we should evaluate the pressure distribution along the plume by assuming an adiabatic expansion (this hypothesis is realistic because the pressure gradient should be intuitively higher than the temperature gradient). Although this approach is physically consistent, we prefer to drastically simplify the problem by assuming that the electron density is not affected by variations in fluid pressure. Therefore, we make the assumption that it only depends on electron-ion recombination. In this condition, the electron density at the burner exit is assumed to be equal to the one at great distance, where the recombination effect is negligible. In other words, we evaluate n_i assuming the pressure at the burner exit equal to the atmospheric pressure. Although this assumption produces and underestimation of n_i, it does not significantly affect the evaluation of the missile RCS. This statement will be demonstrated in section III.

Assuming that the burner exit temperature is equal to 2000 K [1] and applying the perfect gas equation, we obtain a value of the volume $V = nRT/P$ = 16.629*10^3 m^3, where R is the Boltzmann constant and n is the number of moles (set to 1). In formula, the electron density n_i is showed in eq. 4:

$$n_i = \frac{N_e}{V} \cong 10^{11}\ cm^{-3} \qquad (4)$$

It is worth noting that when a solid fuel is used the chemical reactions change. Nevertheless, the electron density at the burner exit can be evaluated by means of an equivalent chemical analysis.

II.2 Plume electron density evaluation

Referring to Fig.1, let the origin of a reference system be located at the center of the missile burner nozzle, and the x coordinate be directed along the missile axis.

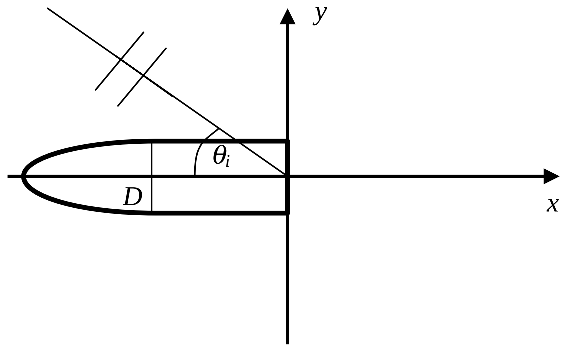

Fig.1 – System geometry

If we assume that for each value of x, the electron density is uniformly distributed on a circular plane with diameter D equal to the one of the missile[2], the differential equation that rules the spatial and time evolution of the electron density $n_e(t,x)$ is given by eq. 5:

$$\frac{d\, n_e(t,x)}{dt} + \alpha\, n_e^2(t,x) = q(t,x) \qquad (5)$$

where $q(t,x)$ is the forcing term for the generic section x, v is the electron conveying speed and α is the recombination factor. Note that the eq. 5, when the forcing term is equal to zero ($q(t,x)=0$), represents the electron recombination law for a generic plasma [2]. The definition of the forcing term will be further given. It is worth noting that the electron density $n_e(t,x)$ is non-zero after $\tau = x/v$ seconds. In fact, τ represents the time interval necessary for the electrons to reach the coordinate x from the origin. For the sake of simplicity, let the coordinate x be divided in M sub-cells: let be $x_m = m\,\Delta x$ where m=0,1,2,…, M-1. The quantity $L=M\Delta x$ represents the plume length to be analyzed. By denoting with $n_e(t,x_m) = n_{e,m}(t)$ the electron density of the m-th sub-cell,

[2] As a first approximation it is possible to neglect the quantity of electrons that exit the cylinder.

the differential equation that rules the equilibrium in the m-th sub-cell can be written as follows:

$$\frac{d\, n_{e,m}(t)}{dt} + \alpha\, n_{e,m}^2(t) = q(t, x_m) \qquad (6)$$

with the condition: $n_{e,m}(t) = 0$ when $t \le \tau_m = x_m / v$. To completely describe eq. 6 we have to define the forcing term $q(t,x_m)$ and the recombination coefficient α.

The forcing term $q(t,x_m)$ accounts for the variation of the electron concentration due to the electrons propagation along the x axis. In fact, if we apply the mass conservation principle to the generic m–th sub-cell, we have to consider the nunmber of electrons that enter and exit the sub-cell. In order to calculate the variation of the electron concentration per time unit we refer to fig. 2.

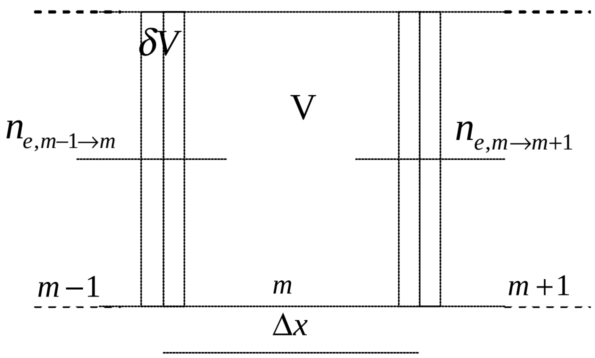

Fig.2 – Sub-cell electron density equilibrium

We first consider the interface between the m-th and $(m+1)$-th sub-cells. Let δV be the infinitive volume of a cylinder of diameter D and height δx that includes the interface between the two sub-cells. The infinitive height δx can be written as $\delta x = v \cdot \delta t$, where δt is an infinitive time interval that allows us to consider the electron concentration time-unvarying. The volume δV can be calculated through eq. 7:

$$\delta V = \frac{\pi D^2}{4} v \delta t \qquad (7)$$

The number of electrons that exit the m–th sub-cell (enter the $(m+1)$-th sub-cell) in the infinitive time interval δt can be written as follows:

$$N_{e,m\to m+1}(t) = n_{e,m}(t) \frac{\pi D^2}{4} v \delta t \qquad (8)$$

In order to consider the variation of the electron concentration in the sub-cell, the number of electrons that exit the m–th sub-cell must be:

1) divided by the volume of the sub-cell, which is equal to $V = \frac{\pi D^2}{4} \Delta x$;
2) divided by the infinitive time interval δt and take the limit for $\delta t \to 0$, as follows:

$$\frac{d}{dt}\left[n_{e,m\to m+1}(t)\right] = \lim_{\delta t \to 0} \frac{1}{\delta t\, V} N_{e,m}(t) = \frac{v}{\Delta x} n_{e,m}(t) \qquad (9)$$

The calculation of the number of electrons that enter the m–th sub-cell (exit the $(m$-1)-th sub-cell) is equivalent. The only exception is represented by the concentration of the electrons, which is evaluated for the $(m$-1)-th sub-cell at the time t.

By avoiding mathematical passages, we can write:

$$\frac{d}{dt}\left[n_{e,m-1\to m}(t)\right] = \frac{v}{\Delta x} n_{e,m-1}(t) \qquad (10)$$

Hence, the forcing term $q(t,x_m)$ can be written as follows:

$$q(t, x_m) = \frac{v}{\Delta x}\left[n_{e,m-1}(t) - n_{e,m}(t)\right] \qquad (11)$$

By combining eq. 6 and eq. 11 we obtain the differential equation that rules the equilibrium of the electron concentration for the generic m–th sub-cell, as follows in eq. 12:

$$\frac{d\, n_{e,m}(t)}{dt} + \alpha\, n_{e,m}^2(t) + \frac{v}{\Delta x} n_{e,m}(t) = \frac{v}{\Delta x} n_{e,m-1}(t) \qquad (12)$$

with the condition: $n_{e,m}(t) = 0$ when $t \le \tau_m = x_m / v$.

The solution of the non-linear differential equation with time-varying coefficients of eq. 12 can be obtained through an iterative numerical algorithm. Specifically, considering the first sub-cell (m=0), we have to solve the following differential equation:

$$\frac{d\, n_{e,0}(t)}{dt} + \alpha\, n_{e,0}^2(t) + \frac{v}{\Delta x} n_{e,0}(t) = \frac{v}{\Delta x} n_i \qquad (13)$$

with the condition: $n_{e,0}(t) = 0$ when $t \le 0$. The initial concentration n_i is given in eq. 4. In the second sub-cell (m=1), the differential equation to be solved is the following:

$$\frac{d\, n_{e,1}(t)}{dt} + \alpha\, n_{e,1}^2(t) + \frac{v}{\Delta x} n_{e1}(t) = \frac{v}{\Delta x} n_{e,0}(t) \qquad (14)$$

with the condition: $n_{e,1}(t) = 0$ when $t \le \tau_1 = x_1 / v$. It is worth noting that the term at the second member has been calculated at the previous step. This procedure is then iterated for the next sub-cells (m=2,3,…,M-1).

To solve eq. 12, three parameters must be defined:

1) The electron conveying speed v;
2) The cell length Δx;
3) The recombination coefficient α.

The electron conveying speed v is a parameter that depends on the missile propulsion system and it must be evaluated case by case. Concerning the cell length problem, to reduce the errors introduced by the x coordinate sampling, we would be tempted to take a value of Δx approaching to zero. Nevertheless, the validity of the differential equations of eq. 12 is constrained by the almost neutrality condition. Specifically, the minimum size of the cell must be larger than the Debye radius, in formula:

$$\Delta x > r_D \cong 6.8678 \sqrt{\frac{T_e}{N}} \qquad (15)$$

where T_e is the temperature and N is electron density in the specific cell.

The recombination coefficient α depends on the electron temperature and concentration according to the following relationship [2]:

$$\alpha = \frac{10^{-25}}{T_e^{4.5}} n_e(t) \qquad (16)$$

Eq. 16 states that the recombination effect:

1) increases when the electronic density increases (the larger the electron density the higher the recombination)

2) decreases when the temperature increases.

In fact, the higher temperature increases the kinetic energy of particles, which overcome the ions attraction. Therefore, it produces a reduction of the ion-electron recombination.

By substituting eq. 15 in eq. 12 we obtain:

$$\frac{d\, n_{e,m}(t)}{dt} + \frac{10^{-25}}{T_e(m)} n_{e,m}^3(t) + \frac{v}{\Delta x} n_{e,m}(t) = \frac{v}{\Delta x} n_{e,m-1}(t) \quad (17)$$

where the function $T_e(m)$ represents the temperature profile along the discrete x coordinate

III. MISSILE RCS CALCULATION

In this section we propose a numerical example relative to the calculation of the RCS of a missile in the following scenario:

1) Use of an OTH radar (at HF band);

2) Missile in the boost phase.

In order to calculate the RCS we first calculate the plume length that affect the RCS evaluation and than we apply the MoM to evaluate the missile+plume RCS.

III.1 Plume length evaluation

Before starting with the plume length evaluation some parameters must be defined. Specifically, the electron conveying speed within the plume cylinder v and the temperature profile $T_e(m)$. The value of the electron conveying speed v is set to be equal to v=100 m/s and the temperature profile $T_e(m)$ is plotted in fig. 3. It is worth noting that the specific values of the two parameters have been chosen on a heuristic basis. The electron density distribution can be obtained y solving eq. 17. In fig. 4 we show the electron density distribution over the x axis for 6 different time instants. It is worth noting that the regime is reached after roughly 0.30 s. Therefore, it is correct to consider the electron density profile at t=1s to evaluate the plume length. To this purpose we introduce the parameter n_c, which represents the critical electron density. To simplify the calculus of the plume length we assume that: the plume reflects all the incident e.m. energy when the electron density is higher than n_c and is transparent when the electron density is lower than n_c.[3]

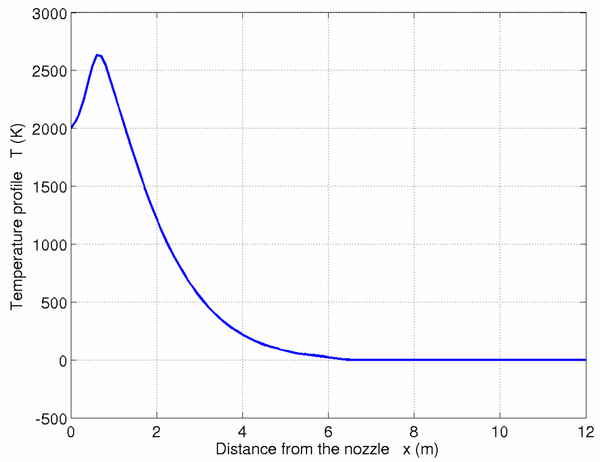

Fig. 3 – Gas Temperature profile

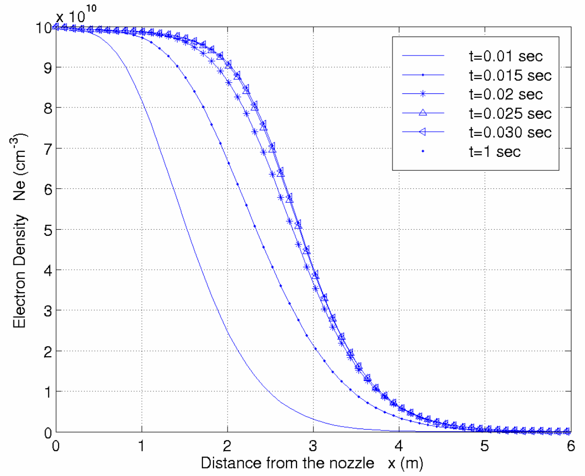

Fig. 4 – Electron density $n_e(t, x)$ at various time instants

The expression of n_c is showed in eq. 18 [2]:

$$n_c = m * \frac{4\pi^2 f^2}{q^2} * \varepsilon_0 \qquad (18)$$

where m and q are the mass and charge of the electron and ε_0 is the free space permittivity. Values of n_c for frequencies in the HF Band are reported in table II. The equivalent electric length l of the plume is calculated by using the electron density profile in a regime condition, the analytical expression is given by eq. 19:

$$l = \max\left\{x : n_e\left(t_{regime}, x\right) \geq n_c\right\} \qquad (19)$$

[3] It is worth noting that the assumption made is quite strong. It is the authors' intention to give an approximated result, which can be refined by taking into account the e.m. returns due to contribution of the plume tail, where the electron density is below the threshold.

Frequency	Critical electron density n_c	Plume length l
10 MHz	$1.2439*10^5$ cm^{-3}	6.377 m
15 MHz	$1.8658*10^5$ cm^{-3}	6.342 m
20 MHz	$2.4878*10^5$ cm^{-3}	6.307 m
25 MHz	$3.1097*10^5$ cm^{-3}	6.291 m

Table II - Plume length at various frequencies

By zooming in Fig.4 around the intersections between the electron density curve relative to the time t=1s and the horizontal lines which represent the values of n_c relative to the frequencies of 10MHz and 25MHz, we note that (see fig.5):

1) the plume length is roughly the same for both the frequencies (hence for all the intermediate frequencies);
2) the plume length is roughly equal to l = 6.5 m.

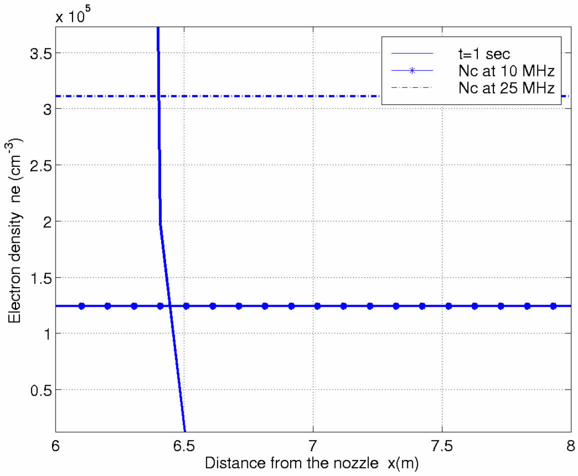

Fig.5. - Zoom in of the electron density curve of Fig.4

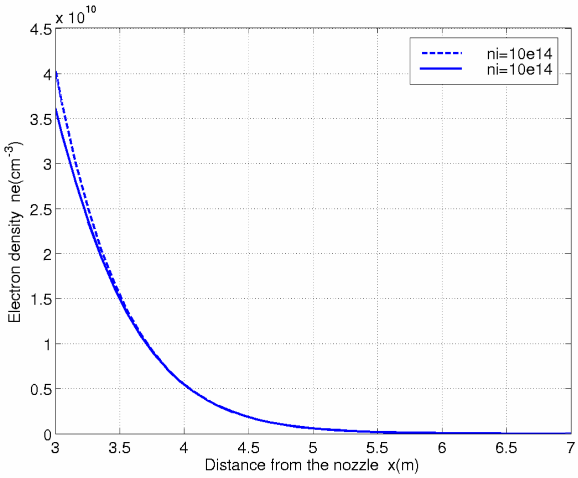

Fig.6 - Electron density profiles at t=1 s

Before evaluating the RCS of the missile, it is worth discussing the assumption made in section 2. Specifically, the initial electronic density n_i of eq. 4 is calculated by ignoring the real pressure profile outside the missile nozzle. This approximation led to a value of $n_i=10^{11}$. In order to justify such an assumption let the initial electron density be $n_i=10^{14}$[4]. In other words, we assume to have an error of three magnitude order larger than the proposed value. In Fig. 6 we report the graph of the electron density profiles for both the values of n_i in regime condition (t=1s). It is clearly visible that when the plume reaches the length of roughly 3.6 m the two curves are undistinguishable. In particular, when the length is equal to l=6.5 m, we can consider the plume lengths identical.

III.2 RCS evaluation

To evaluate the RCS of the missile, the contribution of the body and plum must be accounted for. Assuming that 1) the reflectivity of the body and plume are independent of each other and 2) the plume is as a perfect electromagnetic conductor (PEC), the RCS of the missile is given by eq. 20[3]:

$$\sigma_M = \sigma_{Body} + \sigma_{Plum} \quad (20)$$

To numerically evaluate the RCS of the missile we have used a commercial electromagnetic code [4],[5], namely *Numerical Electromagnetic Code* (NEC). The NEC estimates the RCS of objects by using the Method of Moments. The results relative to the frequencies 10MHz, 15MHz, 20MHz and 25MHz are reported in Fig.8-11, respectively. The incident angle is defined in fig. 1.

We can note:

1) Within a wide range of incident angles the RCS of the missile with plume is larger than the RCS of the only missile body. In particular, when the frequency is equal to 10 MHz, the presence of the plume provokes a RCS gain of 24 dB. Nevertheless, in the other cases the RCS is at least 5 dB larger.

2) At the frequencies of 20,25 MHz, at particular values of the incidence angles, the plume produces a loss in the RCS because of resonance phenomena. In fact, if we consider the curve of the RCS relative to 25MHz (in fig. 11), we note a dumping around a value of the incidence angle equal to θ_d=70°. Considering that 1) the projection of the length of the missile body + plume to the axis orthogonal to the incident e.m. wave is roughly equal to l_{tot}=11.74 m and 2) the wavelength correspondent to the frequency f=25MHz is equal to 12, we can conclude that the dumping is due to the resonance effect.

IV. Conclusions

The main contribution of this paper is due to the development of a method for estimating the RCS of missiles, taking into account also the plume effect. In order to achieve the goal, we have first verified that the missile plume can be physically represented by a plasma model. In order to verify

4 $n_i=10^{14}$ is the concentration of the electrons in a thermonuclear reactor, hence, it represents a conservative value

the correctness of the assumption the chemical reactions of the post-combustion process have been considered. In order to calculate the effect of the plume on the missiles RCS we have first studied the equilibrium of the electron density in the plasma and then defined the non-linear time-varying differential equation of the equilibrium. Consequently, a numerical iterative solution of the differential equation has been carried out. The use of the MoM for the RCS calculation of the missile+plume system has finally led to the following conclusions:

1) the effect of the plume generally provokes an increase in the value of the RCS;

2) the resonance effect can be studied accordingly with the particular system specification.

Next research steps are:

1) calculate the effects of exhausts stream, taking into account the spatial distributions of conductivity, permittivity and magnetic permeability;

2) Improve the geometrical model of the plume, by taking into account for the non-cylindrical shape.

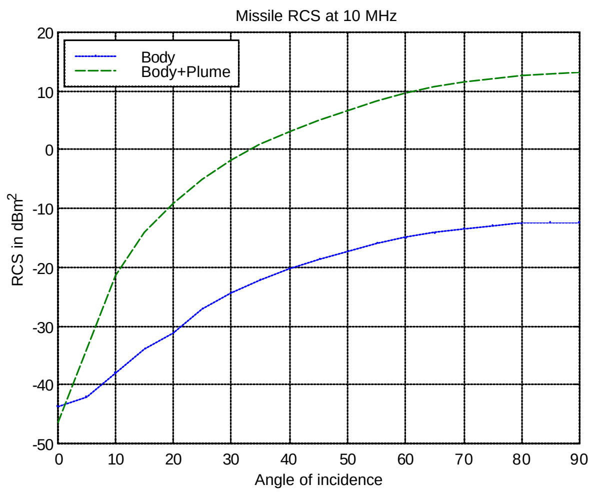

Fig.8 - Missile RCS at 10 MHz

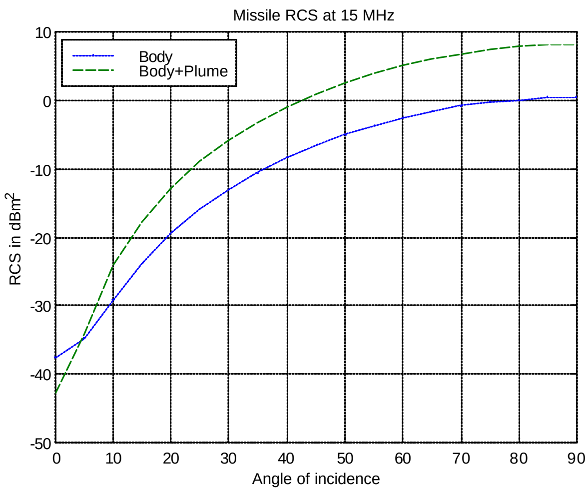

Fig.9 - Missile RCS at 15 MHz

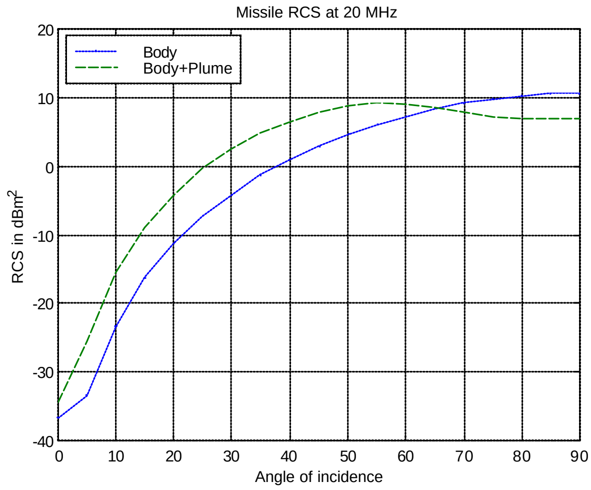

Fig.10 - Missile RCS at 20 MHz

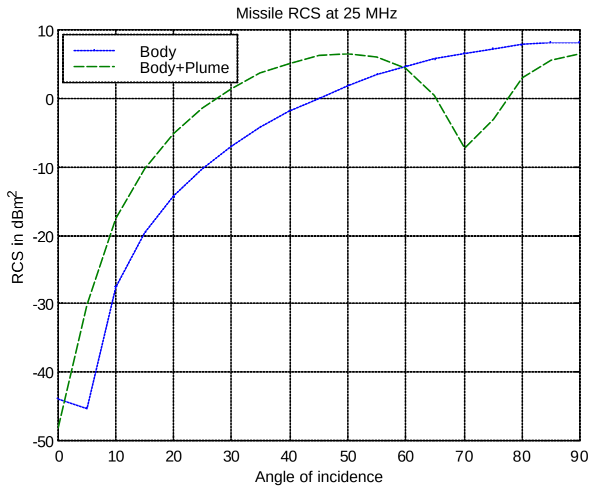

Fig.11 - Missile RCS at 25 MHz

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