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ON-LINE ADAPTIVE ESTIMATION AND TRAJECTORY RESHAPING (PREPRINT)

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14. ABSTRACT

An Adaptive Trajectory Reshaping and Control (ATRC) system is envisioned for RLVs to avoid catastrophic failure when subjected to performance restricting damages and failures. The ATRC is a response system that continuously reshapes and optimizes the reference RLV trajectory, such that, if physically possible, the feasibility constraints are satisfied. The focus of this paper is on two important features of the ATRC system that allow (a) estimation of a parameter functional over the RLV flight envelope to determine feasibility constraints, and (b) real time reshaping of the RLV trajectory for feasibility and optimization of end goals. The knowledge of the effects of a failure at future flight condition is required to design and reshape feasible trajectories. Our approach uses regularization of the ill-posed learning problem by using fusion of existing knowledge and geometric structure in the functional to reduce the uncertainties of future flight conditions. The paper also addresses the difficult problem of real time on-line trajectory generation based on an inverse dynamics principle. An acceptable trajectory is a solution of a two-point boundary value problem for a non-flat (under-actuated) non-linear differential equation of motion. The inverse dynamics approach solves a set of algebraic equations, which strictly satisfies the non-linear differential equations of a non-flat system.

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On-Line Adaptive Estimation and Trajectory Reshaping

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An *Adaptive Trajectory Reshaping and Control* (ATRC) system is envisioned for RLVs to avoid catastrophic failure when subjected to performance restricting damages and failures. The ATRC is a response system that continuously reshapes and optimizes the reference RLV trajectory, such that, if physically possible, the feasibility constraints are satisfied. The focus of this paper is on two important features of the ATRC system that allow (a) estimation of a parameter functional over the RLV flight envelope to determine feasibility constraints, and (b) real time reshaping of the RLV trajectory for feasibility and optimization of end goals. The knowledge of the effects of a failure at future flight condition is required to design and reshape feasible trajectories. Our approach uses regularization of the ill-posed learning problem by using fusion of existing knowledge and geometric structure in the functional to reduce the uncertainties of future flight conditions. The paper also addresses the difficult problem of *real time* on-line trajectory generation based on an *inverse dynamics* principle. An acceptable trajectory is a solution of a two-point boundary value problem for a non-flat (under-actuated) non-linear differential equation of motion. The inverse dynamics approach solves a set of algebraic equations, which strictly satisfies the non-linear differential equations of a non-flat system.

I. Introduction

The large potential for space utilization is not being exploited as it is currently inhibited by the huge cost of launching operations. The benefit of advanced space utilization can be greatly increased by making space utilization more affordable The Reusable Launch Vehicle (RLV) programs are targeted towards affordable space utilization. However, to maintain the economical viability of RLVs, it is important to enhance operations safety and reliability by providing the RLV the capability to respond to various uncertainties and emerging emergency situations. Responding to an uncertain environment after a damage/failure presents many tough technical problems for this class of vehicle. These problems manifest in the following challenges that must be overcome: First, to adequately determine and model the dynamic characteristics of the vehicle in the altered state after a damage/failure; second, to estimate the new constraints and limitation(s) of the vehicle; third, to adopt and reconfigure the command, control and guidance of the vehicle to the modified system dynamics; and fourth, to design and plan a new feasible path with respect to the end goal maximization. A high percentage of such damage/failure cases leave the vehicle in an uncontrolled and uncertain environment with a highly likely probability of ultimately entering a state for catastrophic failure. The high cost of loss resulting from catastrophic failures has prompted researchers in the direction of developing technologies to assist in minimizing such failures.

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Damage to a vehicle or a sub-system failure affect the system characteristics. For example, a failure in control effectors may reduce the control power and hence, shrink the usable flight envelope where the vehicle can be trimmed. The AFRL study[1] examines the effect of control failure on trajectory retargeting. Although a control surface failure does affect the constraint boundaries, it does not bring the vehicle in the realm of uncertainty on the condition that the control surface failure is identified. However, the case is quite different when damage results in some change of the surface geometry and hence, unknown alteration of aerodynamic coefficients throughout the flight envelope. Observe that a RLV passes through a large range of flight conditions in Mach number and altitude. For trajectory retargeting and trajectory feasibility determination, it is necessary to re-estimate the aerodynamic coefficients for the entire flight envelope through which the remaining reference trajectory is expected to pass. This requirement is difficult because the available measurements correspond only to the current flight condition and it is not adequate for estimations in downstream flight conditions.

Figure 1 broadly summarizes the fundamental approach to address the problem of responding to undesirable events such as system/sub-system damage or failure resulting in uncertainty in the system model and environment. The approach consists of reducing the uncertainty through estimation of the system model and active constraints, defining and planning optimal and feasible operations based on the latest estimates, and ensuring the execution of the planned operations. The first step is to observe the system, and learn and estimate its characteristics. This step normally reduces the uncertainty, however its elimination altogether may not be guaranteed. Based on the best available current estimations, the second step consists of re-planning or re-targeting the reference flight path. The final step is to execute the new plan by tracking the reference flight path as closely as possible. The three steps must be repeated during the entirety of the mission to continuously reduce the uncertainty and adapt the flight plans for a realistic and feasible objective accordingly.

Figure 1. Response in Uncertainty

In this paper, we first introduce the architecture of an Adaptive Trajectory Reshaping and Control (ATRC) system for a general class of RLV systems, which is based on the principles as described in Figure 1. Next we focus on two specific features of ATRC related to adaptive functional learning and feasible reference trajectory reshaping.

II. Adaptive Trajectory Reshaping and Control (ATRC) System

The ATRC system enhances RLV capability to avoid catastrophic failure when subjected to performance restricting damages and failures. The overall goal of ATRC translates into specific requirements for design and development of functionalities related to adaptable and reconfigurable command, control, and guidance systems for the RLVs.

Figure 2 shows the general architecture of the envisioned ATRC system for RLVs. Note that the above structure is specific to longitudinal motion of the vehicle; however, it can be easily extended to include lateral motion as well. The main components of the envisioned ATRC system requires:

1. On-line system identification that includes parameter estimation and parameter projection for constraint boundary determination. The constraint boundaries influence the trajectory reshaping of the vehicle.
2. Real time trajectory determination for reshaping the reference trajectory under feasibility constraints.
3. Adaptive, closed loop control and guidance system for reference trajectory tracking[2,3,4,5].

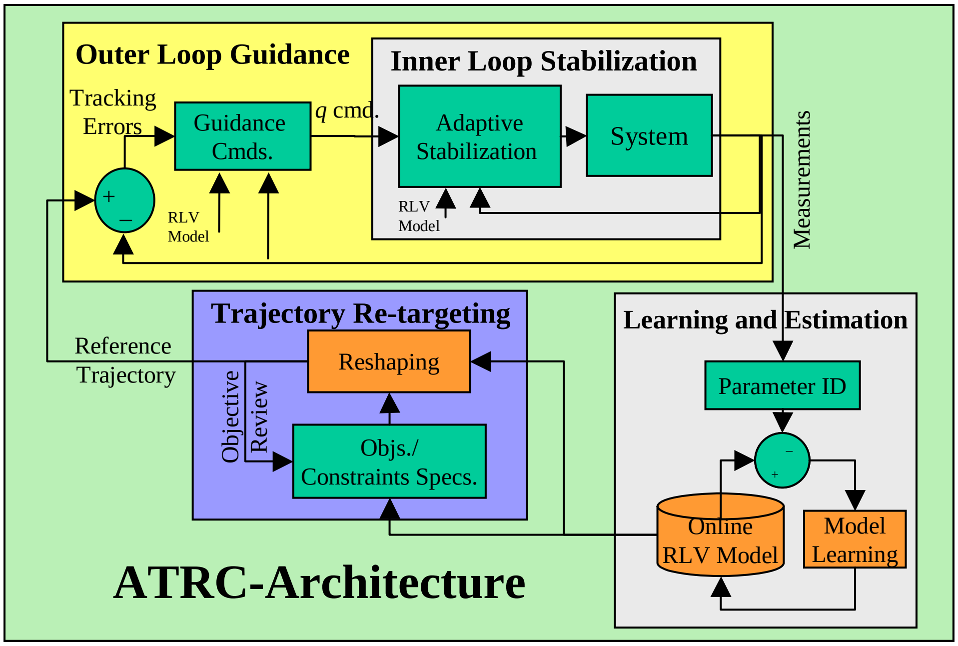

Figure 2. Architecture of ATRC

III. Aerodynamic Coefficient Function Estimation

An important goal of the ATRC system is the adaptive reshaping of the RLV trajectory in the presence of altered dynamic characteristics of the vehicle when unexpected damage occurs in the various operating scenarios. Any damage to a vehicle that has an impact on the external shape of the vehicle, or that creates an impediment in normal functioning of the control surfaces, results in alteration of the vehicle's aerodynamic characteristics. Figure 3 and Figure 4 show a few examples of the pitching moment coefficient variation in the presence of various damage scenarios. Since the aerodynamic behavior of the vehicle is captured in aerodynamic coefficients that are used for the design of vehicle control and trajectory planning, it becomes mission critical to adapt the reference trajectory for the altered vehicle dynamics. In this section, we first present the functional representation scheme and then concentrate on the estimation of the altered aerodynamic coefficients using Tikhonov regularization.

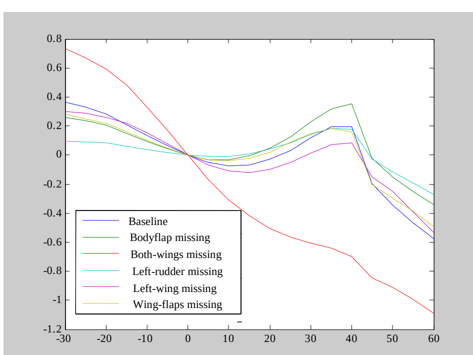

Figure 3. Moment Coefficient with AOA for Nominal and Various Failure Cases

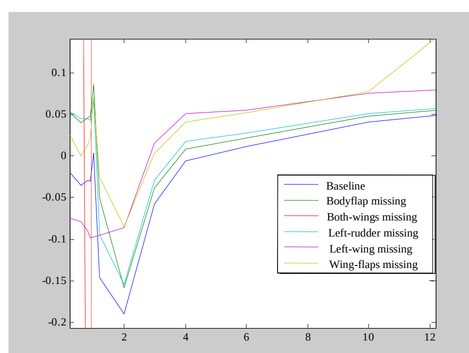

Figure 4. Moment Coefficient with Mach for Nominal and Various Failure Cases

A. Finite Element Function Approximation

We first formulate the aerodynamic coefficient function with a set of parameters that will be estimated on-line. We used a finite element modeling approach so that the approximation function would capture local variations in an efficient manner. Jancaitis, et. al.[6] and Junkins[7] demonstrate the use of a finite element piecewise approximation for mapping geopotential. Verma[8] applied the technique to aerodynamic coefficients representation. First, the argument space of the function is divided to form a grid with one control point at every grid point. At each control point we use a local polynomial function that is determined using a weighted, least square method from a given set of nominal data. The scope of each local polynomial centered at the control point lies in between the two adjacent grid points (see Figure 5). Notice the overlapping of local polynomials, which helps in obtaining a smooth function over the entire range. Once local approximations are determined, a smooth global approximation is obtained as a weighted combination of the local approximations. The smoothness of the approximation function implies that the function, as well as its first derivative, is continuous. Notice that grids are not restricted to be equidistant. To capture nonlinearities effectively, more control points should be placed near highly nonlinear regions. Figure 5 uses a second order polynomial function for local approximations.

Figure 6 shows the final approximated function that is composed of the weighted combinations of the local function approximations. A highlight of this finite element approximation is that it preserves the local function value and its first derivative at its control point. This is achieved by a smooth weighting function that smoothly goes from unity to zero, from one control point to another, without contributing to a first derivative at both control points.

B. Learning Process

A function is a mapping from the independent domain corresponding to an input vector, to a dependent range of the observable outputs. For example the mapping $C_m = C_m(M, \alpha)$ is a map $\Re^2 \rightarrow \Re$, where Mach M and angle of attack α form the input vector and C_m is the observable output. The goal of learning is to discover the mapping between the input domain and its range from a given set of pairs of inputs and output measurements. Hence, the learning of the relationship from the given example relations is an inverse problem that must be solved. For example, consider the least square method for learning a functional relationship *f* expanded on a finite set of basis functions ϕ as

$$f(x) = \sum_{i=1}^{p} c_i \phi_i(x) \quad (1)$$

Let the data set consisting of pairs be given as

$$(x_i, \bar{y}_i), \ \ i = 1, N$$

where x_i is the input vector and $\bar{y}_i$ is the measurement of the observable output. If **e** is the error vector between measurement and estimation, then we define the cost function to be minimized as

$$\mathcal{E}(f) = \tfrac{1}{2} \sum_{i=1,N} \|e_i\|^2 , \quad (2)$$

$$e_i = \left(\bar{y}_i - \sum_{j=1}^{p} c_j \phi_j(x_i) \right) . \quad (3)$$

The coefficients, **c**, of the functions using the least squares method are found as

$$\mathbf{c} = \left[\boldsymbol{\varphi}^T(x_i)\right]^{+}_{p\times N} \mathbf{e}_{N\times 1} , \quad (4)$$

where $\left[\boldsymbol{\varphi}^T(x_i)\right]^{+}$ is the pseudo inverse.

For any process that requires inversion, we must ensure that the problem is *well-posed*. A *well-posed* problem satisfies the conditions of existence, uniqueness and continuity. For an *ill-posed* problem, there is always a difficulty of singularity during inversion. Basically, an *ill-posed* problem is created when the given data sets may contain a small or insufficient amount of information about the desired solution. As stated by Lanczos[9] that "lack of information cannot be remedied by any mathematical trickery."

C. Tikhonov Regularization

For a RLV, the coefficient function learning is a difficult and ill-posed problem because of insufficient observable data for the domain of interest. Figure 7 illustrates an example showing the difficulty of an ill-posed problem. When there are insufficient data points that cover the problem domain for which functional learning is required, it is very difficult to find a stable solution in an iterative scheme. The instability is the result of inherent extrapolation in the method. The learning takes place by minimizing the error between the function and the given data points. However, as there are no data points in a large range of the domain, there is great uncertainty in the functional for that range. The large uncertainty results in higher sensitivity and the instability in the iterative learning process. To illustrate the sensitivity and the instability of the process typical to an RLV problem, consider two sets of data at two time steps, where the second set includes the first set and additional data. Figure 7 shows the two cases; first, an approximate solution was found for a small set of data points using the least squares method. Next, the data set was augmented by additional data, and the solution was updated. Notice the variation between two approximations. For a RLV, this type of variation is totally unacceptable. When more data was added to span the whole range of interest, we achieved a more accurate solution as shown in Figure 8. Notice the difference between the true solution and the approximate solution for the ill-posed condition. This highlights the severity or the difficulty of the problem being attempted. For an acceptable solution there is a need to introduce "regularization" in the learning process.

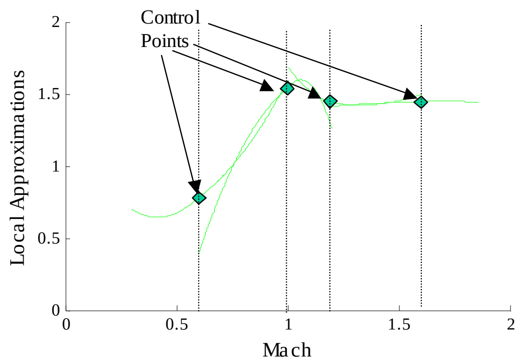

Figure 5: Local Approximations Centered at Control Points

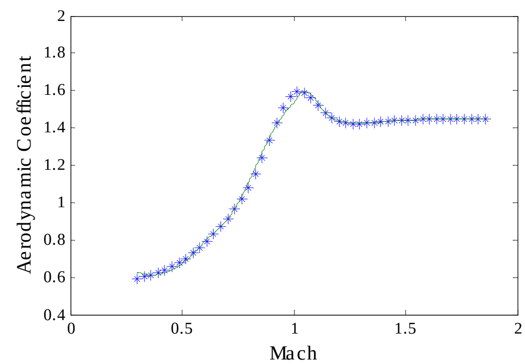

Figure 6: Smooth Functional Approximation using Weighted Local Approximations

Regularization is a method of imposing additional conditions for solving inverse problems with optimization methods. When model parameters are not fully constrained by the problem (the inverse problem is mathematically ill-posed), regularization limits the variability of the model and guides the iterative optimization to the desired solution by adding assumptions about the model power, smoothness, predictability, etc. In other words, it constrains the model null space to an *a priori* chosen pattern. Tikhonov[10] introduced his regularization technique in 1963 for the stability of the solution of ill-conditioned differential equations. Similar problems are encountered in the functional learning and hence Tikhonov regularization can easily be adopted in this case. We refer to a thorough mathematical theory of regularization in works of Tikhonov's school[10, 11].

Introducing regularization in inverse problems allows us to fuse *a priori* knowledge about the coefficient and the current information being received from the sensors and parameter ID for a given flight condition. In the standard learning process, Tikhonov regularization is introduced by adding another term in the cost function as

$$\mathcal{E}(f) = \mathcal{E}_s(f) + \lambda\, \mathcal{E}_c(f) \tag{5}$$

where λ is a regularization parameter, $\mathcal{E}_s(f)$ is a standard error term and $\mathcal{E}_c(f)$ is a Tikhonov regularization term given as

$$\mathcal{E}_c(f) = \tfrac{1}{2}\|Df\|^2 . \tag{6}$$

Here D is a differential operator acting on the function *f*. The optimal solution for this problem satisfies the optimality condition given as

$$d\,\mathcal{E}(f,h) = d\,\mathcal{E}_s(f,h) + \lambda d\,\mathcal{E}_c(f,h) = 0 , \tag{7}$$

where

$$d\,\mathcal{E}(f,h) = \left[\frac{d}{d\beta}\mathcal{E}(f+\beta h)\right]_{\beta=0} \tag{8}$$

is defined as a Frechet differential of a function. In essence, the Tikhonov term introduces geometric properties in the learning process. In other words, as mentioned earlier, we introduce or *fuse* prior knowledge of the solution in the learning process. This is very reasonable for the problem that we address in this paper, as the nature of aerodynamic coefficient functions is generally known. For example, see the pitching moment coefficient functional in Figure 3 and Figure 4 that show a similar trend except for one severe damage condition. This geometric *a priori* information about the pitching moment functional can be introduced through a Tikhonov term in the learning process facilitating regularization of the problem. Note that without regularization, the in-flight coefficient learning for a RLV is an ill-posed problem that suffers from near singularity due to insufficiency of data, as there is no way to generate on-line knowledge from measurements for the far away flight conditions.

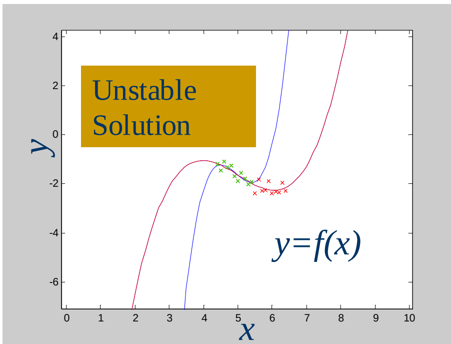

Figure 7: Ill-Posed Problem. Instability in Function Approximation

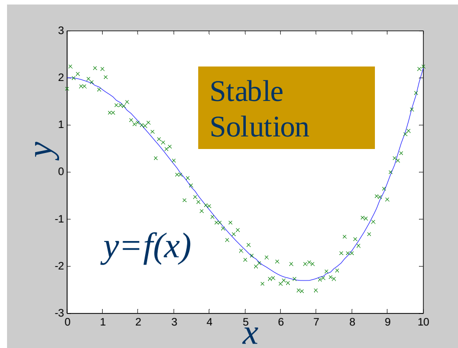

Figure 8: Well-Posed Problem. Stable Function Approximation

Observe that the regularization term is added just like constraint terms are added in a general optimization problem. When constraint terms are added, λ is a Lagrange multiplier, which is not known *a priori*, and it is determined as a part of the solution. But in this case, λ is a regularization parameter, whose value is *a priori* assigned. If λ is large, then more importance is given to the prior knowledge about the functional and less importance is given to the information obtained from the data set. Similarly, if λ is small, then the prior knowledge is weighted less in relation to the data set. Hence, a regularization parameter is a balancing term between the information data and *a priori* knowledge about the functional.

To demonstrate the concept of functional learning, we simulated the adaptive learning process. In the adaptive learning process, the existing estimate of the functional is continuously refined. The approximation functional is updated as new information or data points are made available. In the simulation, we used a representative RLV model. We considered the case of the bodyflap missing but did not assume any prior information about the damage. We only assumed the knowledge of the nominal pitching moment coefficient. Here it must be stated that any advance knowledge of the failure will further help to improve the accuracy of the learning of approximation solution.

Figure 9 shows the various stages of learning for the pitch moment coefficient for the RLV. In the figure we have a base line curve, which is the nominal pitching moment coefficient for the RLV. We also plotted the actual curve after the failure corresponding to the missing bodyflap. The in-flight data that becomes available for the coefficient learning is plotted with a '+' sign. Finally, we have the curve for the current approximation. Figure 9 shows the progression of the available approximation of the pitch coefficient. In the beginning, the approximation is close to the nominal curve. As more data points become available, the approximation curve closely matches the failure case near the flight conditions. Additionally, we also observed the effect of Tikhonov regularization at far away flight conditions. And, as more data points become available, the coefficient curve matches the actual case corresponding to the missing bodyflap.

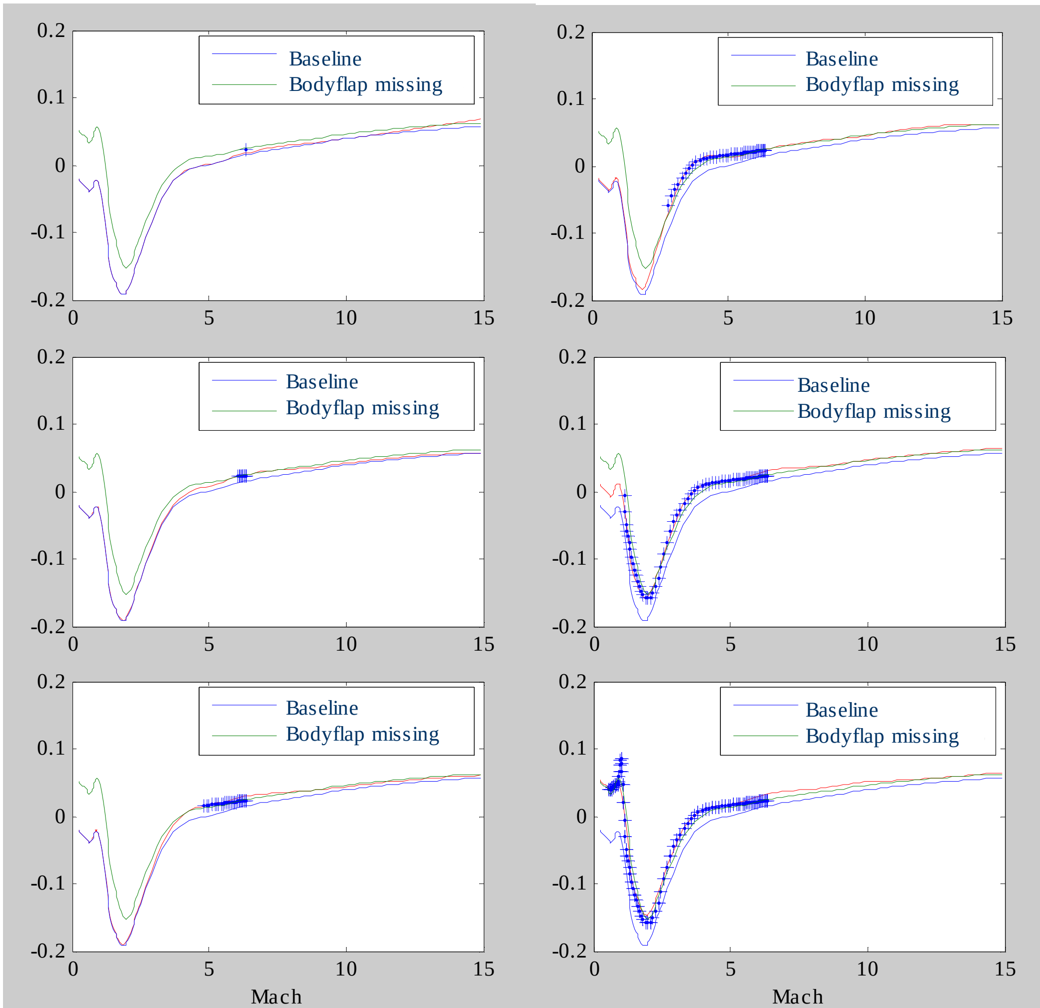

Figure 9. Functional Learning at Various Stages

IV. Reference Trajectory Design Using Inverse Dynamics

To determine a trajectory for a non-linear dynamic system, a solution must be found that satisfies the set of differential equations governing the dynamics of the system. Further, the trajectory solution should not violate some non-linear constraints, which limits the operational capability of the system. For an aircraft, the constraints arise due to the "angle of attack," "load factor," maximum "sideslip angle," and actuator saturation limitations.

There are two primary approaches for trajectory generation and these have been classified in the literature as the "integral approach" and the "differential approach[12]." In any approach, where generation of a trajectory involves the integration of the equations of motion[13], this approach is classified as the "integral approach." In a differential approach, an assumed functional form for the trajectory is differentiated to obtain algebraic functions for the higher derivatives, which are required to impose constraints on the control inputs for the "inverse dynamics" solution. There are various applications where inverse dynamics have been used, such as spacecraft trajectories and path planning in robotics[14] and overhead cranes[15]. In the inverse dynamics approach, the algebraic equations are solved instead of integrating ODEs. Historically, the inverse dynamics approach has been used for "differentially flat" systems. A system is "differentially flat"[16,17] if there exists a set of outputs, known as "flat outputs," such that there is a one-to-one correspondence between the trajectories of flat outputs and the full state and control inputs of the system. Verma, et. al.[18,19] and Verma[8] first presented an approach that uses inverse dynamics for a non-flat system with the help of pseudo forces. With the inverse dynamics approach for aircraft trajectories, a problem arises due to inherent under-actuation in most of the aircrafts. For a six-degree of freedom aircraft, there are normally four controls: thrust, elevator, aileron, and rudder. We implemented a novel trajectory generation scheme[8], which uses pseudo forces for inverse dynamic computation. In this work we use another innovative approach where trajectory computation consists of a two-step process that helps in a faster convergence of the solution. The first step is a fast process that solves a simpler, point mass trajectory problem for path planning, ensuring that most of the acceleration constraints are satisfied. In the second step, a rotational degree of freedom is also included to compute the full trajectory and corresponding reference controls.

A. RLV Trajectory Reshaping

Observe that the dynamic inversion requires as many independent control parameters as the degrees of freedom. For longitudinal motion, there are three degrees of freedom that require three independent trajectory profiles to be prescribed. Here we present the ways to divide the trajectory-reshaping problem into two sub problems so as to simplify the complexity and obtain a faster solution.

For longitudinal motion of a RLV, our goal is to determine a gliding trajectory that does not use any thrust. In the new approach we solve this problem in two steps. In the first step, we only solve the 2-DOF trajectory, leaving out the pitch rotation motion. The momentum level governing equations of motion for 2-DOF are given as

$$\begin{aligned} m\dot{V} &= -D(T,\alpha,\delta_e) - mg\,sin(\gamma) \\ mV\dot{\gamma} &= L(T,\alpha,\delta_e) - mg\,cos(\gamma), \end{aligned} \tag{9}$$

and the kinematics are defined as

$$\begin{aligned} \dot{X} &= V\,cos(\gamma) \\ \dot{H} &= V\,sin(\gamma). \end{aligned} \tag{10}$$

Here *m* is the mass, g is the acceleration due to gravity, δ_e is the control surface deflection, *V* is the vehicle velocity, γ is the flight path angle, *H* is altitude and *X* is the forward axis. The force terms like drag *D* and lift *L* are functions of angle of attack α. Note that kinematic relations are always exact while the momentum level equations have force terms, *D* and *L* that are normally approximated and have some uncertainties. For the simplified 2-DOF problem we ignore elevon contribution and use angle of attack as a real control variable and positive thrust as the pseudo control force. Note that for an RLV, negative thrust is available through speed brakes. For feasibility, the trajectory is perturbed until the thrust required is negligible. In the second step, the solution for angle of attack is assumed to be an angle of attack trajectory. The pitch trajectory is obtained from angle of attack and flight path angle using the relation

$$\theta = \alpha + \gamma\ . \tag{11}$$

The rotational pitch dynamics and the kinematics are governed by the equations

$$I\dot{q} = M(\alpha,\delta_e), \quad q = \dot{\theta}\ , \tag{12}$$

where *I* is the moment of inertia and q is the pitch rate. At this stage we have two alternatives: i) For a rigorous solution, the 3-DOF trajectory should be solved simultaneously with two pseudo forces and one real control given by elevons; ii) An approximate solution can be solved by only solving the rotational degree of freedom for the given 1-D angle of attack trajectory. The second alternative is very fast due to reduced complexity. However, note that the elevon control parameter solution obtained by considering only rotational dynamics also has an effect on the translational motion dynamics, more prominently about the heave in 'Lift' axis. Since the contribution of elevons, δ_e, in the 2-DOF translational motion was not considered, the original 2-DOF trajectory solution is no longer rigorous. In the first approach, as we consider all 3-DOF dynamics together, the solution is rigorous, however, for all practical purposes the second approach provides a feasible solution.

B. Trajectory Reshaping Example

Figure 10 presents the two-degree of freedom trajectory solution for two nautical miles of forward motion of a RLV. The starting point for all these trajectories is 15,000 ft. The goal was to determine a feasible solution. In an iterative approach, various end flight conditions ranging from final altitude of 7,000ft to 12,000ft were considered to determine the feasible path. The feasibility is determined by observing the requirement of *Pseudo Thrust*. From the figure, we observe that for final altitudes greater than 10,000 ft. at two nautical miles, the pseudo control is not negligible and hence the trajectory is not feasible. To achieve a feasible solution, the altitude must be lower than 10,000 ft.

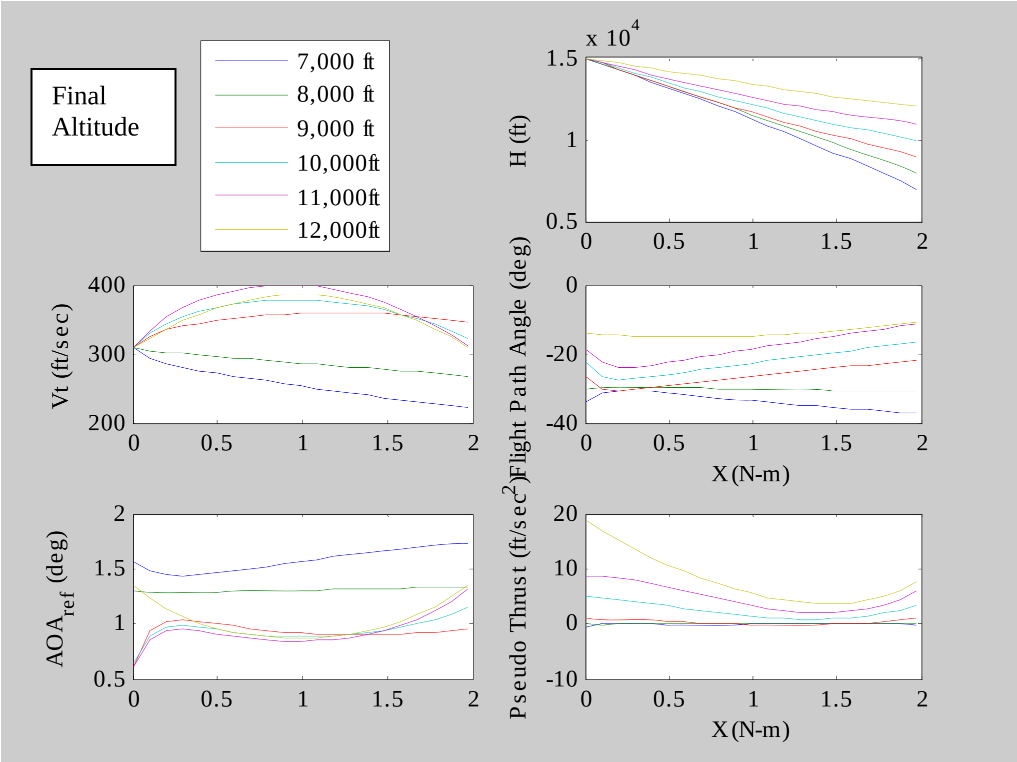

Figure 10. Inverse Solution for 2-DOF Trajectory

Figure 11 demonstrates an example for a 3-DOF trajectory determination using an inverse dynamic computation. In this example, a thousand feet of forward motion was considered. The starting point for the vehicle is 15,000ft. Three cases of altitude drop of 300ft, 400ft and 600ft were considered. The goal was to determine a feasible inverse dynamic solution for a gliding trajectory that uses only one real control given by elevons. To solve the 3-DOF inverse problem, two pseudo forces consisting of Pseudo thrust and Δ AOA were considered. It can be observed from Figure 11, that pseudo Δ AOA is always negligible. This is because Δ AOA is absorbed in the actual AOA trajectory variable during successive iterations. From Figure 11, we also observe that for the case where the altitude drop is 400ft, pseudo thrust is negligible and hence the gliding trajectory is feasible. For a 600ft drop in altitude, pseudo thrust is negative. If we assume the usual case of the availability of air brakes in the RLV, a negative thrust is achievable. In that case all trajectories requiring negative thrust are considered feasible. However, the figure shows that a 300ft drop in the given X-range is not a feasible solution.

V. Conclusion

We presented a framework for an Adaptive Trajectory Reshaping and Control (ATRC) System for RLVs that respond to performance restricting damages and failures to the vehicle by reshaping the vehicle's trajectory to accommodate new constraint boundaries. The challenge arises in learning constraints when available data is limited. We addressed the problem using Tikhonov regularization that allows fusion of *a priori* knowledge in the learning process. Next, we presented an innovative approach for online trajectory reshaping using an inverse dynamics method that uses pseudo forces to facilitate dynamic inversion of an under-actuated system. The method was demonstrated with a longitudinal trajectory reshaping example.

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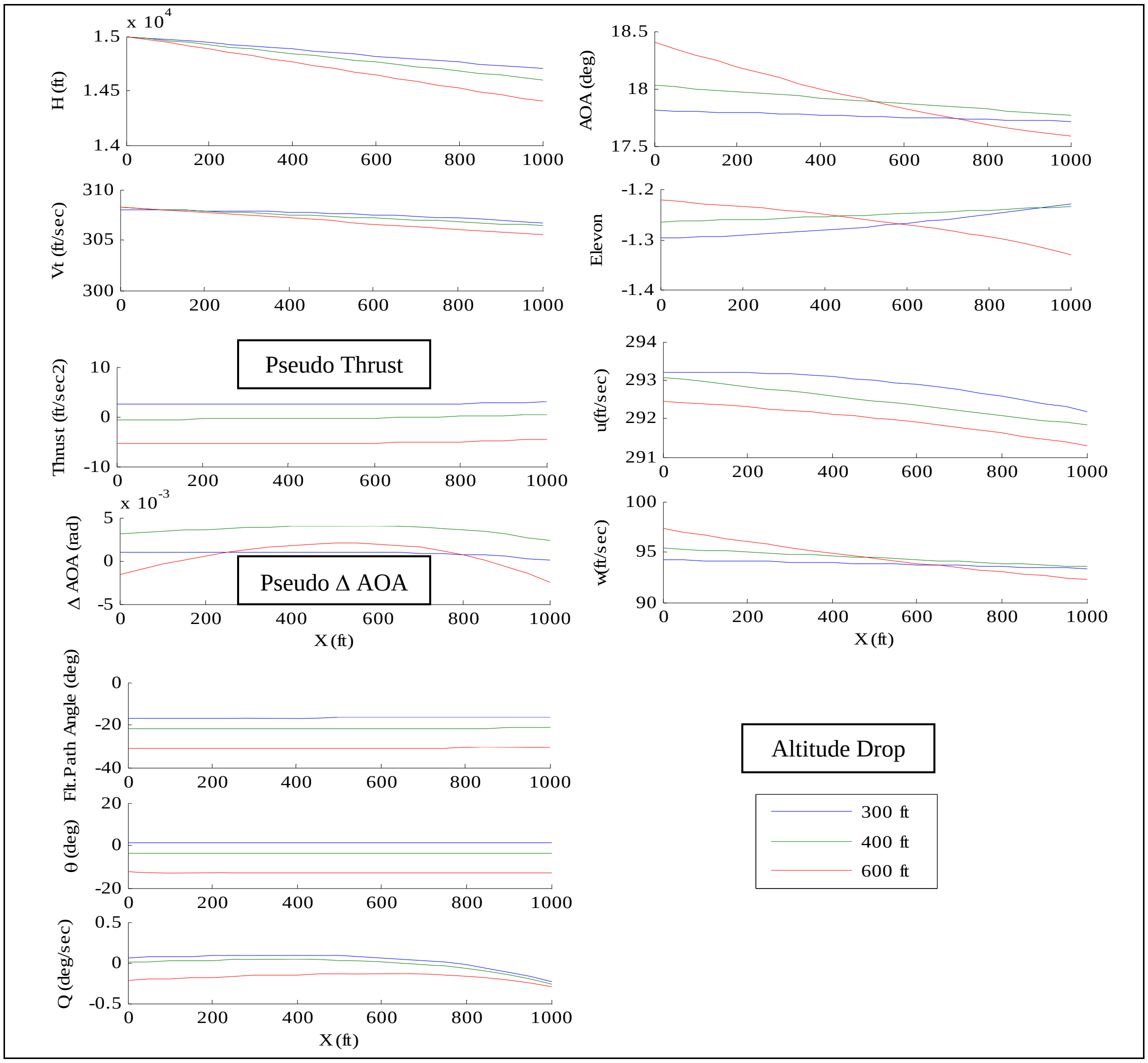

Figure 11. Inverse Solution for 3-DOF Trajectory

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