

STRUCTURAL VIBRATION MODELING & VALIDATION

Modeling Uncertainty and Stochastic Control for Structural
Control

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ACRONYMS

FORM	First Order Reliability Method
FRF	Frequency Response Function
GIT	Generalized Information Theory
LQR	Linear-Quadratic Regulator
MIMO	Multi-Input, Multi-Output
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PDF	Probability-Density-Function
SORM	Second Order Reliability Method
SIM	Space Interferometer Mission
SISO	Single-Input, Single Output
SVMV	Structural Vibration Modeling and Verification
V&V	Verification and Validation

EXECUTIVE SUMMARY

A number of US Government agencies are sponsoring research and development efforts to improve the accuracy and frequency bandwidth of test verified structural dynamic modeling techniques for complex, precision space structure. The impetus behind these efforts is the projection that future precision spacecraft may be too large and flimsy to undergo system level testing in the deployed configuration prior to launch. Without a system level test, there will be increased reliance on performance prediction through modeling. A need to improve the capabilities for dynamic response predictions of precision structures, without large uncertainty factors, was identified as a critical technology to enable this paradigm, and research and development efforts were initiated. The objective of this paper was to assess control relevant uncertainty models, and make recommendations on their use in the context of the precision structures of interest in the Structural Vibration Modeling and Verification program.

1.0 INTRODUCTION

A number of US Government agencies are sponsoring research and development efforts to improve the accuracy and frequency bandwidth of test verified structural dynamic modeling techniques for complex, precision space structure. The impetus behind these efforts is the projection that future precision spacecraft may be too large and flimsy to undergo system level testing in the deployed configuration prior to launch. This may be due to a lack of inherent structural integrity of the deployed system in a 1-g gravity field or the cost and complexity of designing and building an adequate gravity offload system. It's envisioned that these structures will take on their true performance characteristics only in the weightlessness of space.

This represents a new paradigm for the spacecraft industry. Without a system level test, there will be increased reliance on performance prediction through modeling. A need to improve the capabilities for dynamic response predictions of precision structures, without large uncertainty factors, was identified as a critical technology to enable this paradigm, and research and development efforts were initiated. One such effort is the Structural Vibration Modeling and Verification (SVMV) program. The objective of this program was to develop and validate modeling capabilities and techniques using available complex testbed structures. Two contractors were competitively selected to carry out the work. A successful SVMV program will move us closer to the capability of designing, modeling, and building high performance structures in the frequency range of interest without large model uncertainty factors and enhance model based system performance predictions.

The AFRL/VSSV supported the sponsoring agency on the SVMV program by addressing four specific tasks:

A) Metric Evaluation and Plant Identification

The contractors proposed Frequency Response Function (FRF) metrics with which they will evaluate system identification model quality in terms of FRF data. An objective of this task was to gain insight into the contractor's efforts, and develop an understanding of threshold levels for the metrics. The contractors also used proprietary plant identification software during the program. A second objective of this task was to evaluate the performance of the plant identification tools with respect to commercially available software and published methods.

B) SVMV Technical Recommendations

The AFRL/VSSV worked with other members of the Sponsor's technical support team to capture information from the SVMV program and present it in suitable forms for use by

the Sponsor, future system program managers, and the technical community.

C) Model Uncertainty in Structural Control

An assumption of the new paradigm is that some form of structural control will be available, and necessary, to “tune” the structure on-orbit. This assumption has implications for structure design, modeling, fabrication as well as structural control systems. High-fidelity models may not be able to account for build-to-build variations, response differences between 0-g and 1-g environments, and structural changes due to the space environment. All of these factors must be captured in a control-relevant form in order to be used in the design and analysis of robust structural controllers. The objective of this task was to assess control relevant uncertainty models, and make recommendations on their use in the context of the precision structures of interest in the SVMV program.

D) Propagation of Errors

Since system level tests are omitted in the new paradigm, it is important to understand how subsystem model properties affect system level models and performance predictions. For example, the assumption of proportional damping carries implications about the physics of the structure captured in the model. However, proportional damping applied at the subsystem level may not lead to proportionally damped models at the system level using component mode synthesis techniques. Therefore the underlying assumptions of the physics have been affected. The objective of this task was to make recommendations on the propagation of substructure sensitivities to system level optical performance quantities.

This report contains results of task C. The results of Task B effort were reported to the SVMV Sponsor in a different form. Upon completion, results from Tasks A and D will be compiled in other reports. Chapter 2 lists some relevant articles in the area of model uncertainty representation and quantification for computational model performance prediction reliability enhancement. Chapter 3 lists some relevant articles in the stochastic robust control area that deal with the synthesis of performance reliability based controllers. Chapter 4 discusses activities for further investigation relative to control of uncertain structures. Budget and schedule limitations precluded the development of a more in-depth report.

2.0 UNCERTAINTY MODELS

Representing and quantifying uncertainties associated with computational models for performance prediction reliability enhancement is an area of active research. The application areas of this research vary from weather prediction to prediction of nuclear weaponry performance. This research has direct applicability to the SVMV paradigm in that SVMV decisions will be made to launch a space vehicle based solely on computational model predictions that the performance requirements are satisfied. This computational model will not have been validated, in the standard sense of validation, at the integrated vehicle level. Thus, quantitatively capturing in the computational model framework that which is known as well as that which is uncertain with regard to the space vehicle performance are equally important in establishing reliability in the computational model performance predictions.

Various forms of uncertainty representation are being considered to enhance computational model prediction reliability. The most widely known and developed methods are available within the mathematics of probability theory, such as frequentist and Bayesian estimation. Researchers are also exploring the utility of non-traditional methods of uncertainty representation and quantification for engineering modeling applications such as risk and reliability analysis [22]-[26]. Described broadly as Generalized Information Theory (GIT), these approaches are listed in Table 1:

Table 1 Generalized Information Theory Approaches

- | | |
|-----------------------------------|----------------------------|
| • Interval Analysis | • Imprecise Probabilities |
| • Dempster-Shafer Evidence Theory | • Nonadditive Measures |
| • Fuzzy Systems | • Random Sets |
| • Possibility Theory | • Probability Bounds |
| • Rough Sets | • Probabilistic Robustness |

This chapter lists some relevant articles, along with their abstracts and in some cases their introduction, from this area of research that are of interest to the SVMV paradigm.

[5] de Lima, B.S.L.P., and Ebecken, N. F.F., “A comparison of models for uncertainty analysis by the finite element method”, *Finite Elements in Analysis and Design* Vol 34 (2), 2000, pp. 211-232.

Abstract: Uncertainty in structural engineering analysis exists in the architecture of a structural system, its basic parameters, the information resulting from the abstracted aspects of the system, and the non-abstracted or unknown aspects of the system. Also, uncertainty is present as a result of prediction models, analysis and design of structures, and general lack of knowledge about the behavior of real structures. One of the important factors that lead to errors in numerical predictions

is the degree of precision in obtaining the relevant parameters. In this paper we discuss two different methodologies:

1. Classical probabilistic approach, in which the properties are treated as random variables. Stochastic Finite Element Methods are examined using both Monte Carlo Simulation and Perturbation Methods.
2. Possibilistic approach, by a model based on the theory of fuzzy sets.

Some results are presented to point out the main characteristics of the two methodologies.

[7] Helton, J.C., Johnson, J.D., Oberkampf, W.L., “An exploration of alternative approaches to the representation of uncertainty in model predictions,” *Reliability Engineering and System Safety*, Vol. 85, 2004, pp. 39-71.

Abstract: Several simple test problems are used to explore the following approaches to the representation of the uncertainty in model predictions that derives from uncertainty in model inputs: probability theory, evidence theory, possibility theory, and interval analysis. Each of the test problems has rather diffuse characterizations of the uncertainty in model inputs obtained from one or more equally credible sources. These given uncertainty characterizations are translated into the mathematical structure associated with each of the indicated approaches to the representation of uncertainty and then propagated through the model with Monte Carlo techniques to obtain the corresponding representation of the uncertainty in one or more model predictions. The different approaches to the representation of uncertainty can lead to very different appearing representations of the uncertainty in model predictions even though the starting information is exactly the same for each approach. To avoid misunderstandings and, potentially, bad decisions, these representations must be interpreted in the context of the theory/procedure from which they derive.

[8] Guest Editorial, “Alternative representations of epistemic uncertainty,” *Reliability Engineering and System Safety*, Vol. 85, 2004, pp. 1-10, A workshop on alternative representations of epistemic uncertainty was sponsored by Sandia National Laboratories in Albuquerque, NM, on August 6–7, 2002.

Abstract: This workshop was organized around the solution and discussion of a set of “Challenge Problems”. The goal of the workshop was to bring together a diverse group of individuals with an interest in the representation of epistemic uncertainty for interaction and discussion. The intent of the Challenge Problems was to provide a central focus around which techniques and structures for the representation of epistemic uncertainty could be presented, compared and

discussed. The following special issue of Reliability Engineering and System Safety contains papers prepared and, in most cases, presented in conjunction with this workshop.

- [9] Oberkampf, W.L., DeLand, S.M., Rutherford, B.M., Diegert, K.V., and Alvin, K.F., "Error and uncertainty in modeling and simulation," *Reliability Engineering and System Safety*, Vol. 75, 2002, pp. 333-357.

Abstract: This article develops a general framework for identifying error and uncertainty in computational simulations that deal with the numerical solution of a set of partial differential equations (PDEs). A comprehensive, new view of the general phases of modeling and simulation is proposed, consisting of the following phases: conceptual modeling of the physical system, mathematical modeling of the conceptual model, discretization and algorithm selection for the mathematical model, computer programming of the discrete model, numerical solution of the computer program model, and representation of the numerical solution. Our view incorporates the modeling and simulation phases that are recognized in the systems engineering and operations research communities, but it adds phases that are specific to the numerical solution of PDEs. In each of these phases, general sources of uncertainty, both aleatory and epistemic, and error are identified. Our general framework is applicable to any numerical discretization procedure for solving ODEs or PDEs. To demonstrate this framework, we describe a system-level example: the flight of an unguided, rocket-boosted, aircraft-launched missile. This example is discussed in detail at each of the six phases of modeling and simulation. Two alternative models of the flight dynamics are considered, along with aleatory uncertainty of the initial mass of the missile and epistemic uncertainty in the thrust of the rocket motor. We also investigate the interaction of modeling uncertainties and numerical integration error in the solution of the ordinary differential equations for the flight dynamics.

- [10] Oberkampf, W.L., Trucano, T.C., and Hirsch, C., "Verification, Validation, and Predictive Capability in Computational Engineering and Physics," *Proc. Foundations for Verification and Validation in the 21st Century Workshop*, Johns Hopkins University, Laurel, MD, 22-23 Oct 2002.

Summary: Computer simulations of physical processes are being relied on to an increasing degree for design, performance, reliability, and safety of engineered systems. Computational analyses have addressed the operation of systems at design conditions, off-design conditions, and accident scenarios. For example, the safety aspects of products or systems can represent an important, sometimes dominant, element of numerical simulations. The potential legal and liability costs of hardware failures can be staggering to a company, the environment, or the

public. This consideration is especially crucial, given that we may be interested in high-consequence systems that cannot ever be physically tested, including the catastrophic failure of a full-scale containment building for a nuclear power plant, explosive damage to a high-rise office building, ballistic missile defense systems, and a nuclear weapon involved in a transportation accident. Developers of computer codes, analysts who use the codes, and decision makers who rely on the results of the analyses face a critical question: How should confidence in modeling and simulation be critically assessed? Verification and validation (V&V) of computational simulations are the primary methods for building and quantifying this confidence. Briefly, verification is the assessment of the accuracy of the solution to a computational model. Validation is the assessment of the accuracy of a computational simulation by comparison with experimental data. In verification, the relationship of the simulation to the real world is not an issue. In validation, the relationship between computation and the real world, *i.e.*, experimental data, is the issue. This paper presents our viewpoint of the state of the art in V&V in computational physics. (In this paper we refer to all fields of computational engineering and physics, *e.g.*, computational fluid dynamics, computational solid mechanics, structural dynamics, shock wave physics, computational chemistry, etc., as computational physics.) We do not provide a comprehensive review of the multitudinous contributions to V&V, although we do reference a large number of previous works from many fields. We have attempted to bring together many different perspectives on V&V, highlight those perspectives that are effective from a practical engineering viewpoint, suggest future research topics, and discuss key implementation issues that are necessary to improve the effectiveness of V&V. We describe our view of the framework in which predictive capability relies on V&V, as well as other factors that affect predictive capability. Our opinions about the research needs and management issues in V&V are very practical: What methods and techniques need to be developed and what changes in the views of management need to occur to increase the usefulness, reliability, and impact of computational physics for decision making about engineering systems? We review the state of the art in V&V over a wide range of topics; for example, prioritization of V&V activities using the Phenomena Identification and Ranking Table (PIRT), code verification, software quality assurance (SQA), numerical error estimation, hierarchical experiments for validation, characteristics of validation experiments, the need to perform nondeterministic computational simulations in comparisons with experimental data, and validation metrics. We then provide an extensive discussion of V&V research and implementation issues that we believe must be addressed for V&V to be more effective in improving confidence in computational predictive capability. Some of the research topics addressed are

development of improved procedures for the use of the PIRT for prioritizing V&V activities, the method of manufactured solutions for code verification, development and use of hierarchical validation diagrams, and the construction and use of validation metrics incorporating statistical measures. Some of the implementation topics addressed are the needed management initiatives to better align and team computationalists and experimentalists in conducting validation activities, the perspective of commercial software companies, the key role of analysts and decision makers as code customers, obstacles to the improved effectiveness of V&V, effects of cost and schedule constraints on practical applications in industrial settings, and the role of engineering standards committees in documenting best practices for V&V.

[11] DeLaurentis, L.A., and Mavris, D.N., “Uncertainty modeling and management in multidisciplinary analysis and synthesis,” AIAA-2000-422, Aerospace Sciences Meeting and Exhibit, 38th, Reno, NV, Jan. 10-13, 2000.

Abstract: The complex, multidisciplinary nature of aerospace design problems, as well as the requirement to examine life-cycle characteristics, have exposed a need to model and manage uncertainty. In this paper, a formal approach for modeling uncertainty in such design problems is presented. The approach includes uncertainties associated with mathematical models, operation environment, response measurement, and input requirements. In addition, a new method for propagating this uncertainty (in an efficient manner) to find robust design solutions is developed and described. The uncertainty model combined with the probabilistic robust design technique is a critical advancement in multidisciplinary system design, in that it identifies solutions that have a maximum probability of success. Continued research in both uncertainty modeling and efficient robust design methods appears essential. Both the uncertainty model and robust design technique are demonstrated on an example problem involving the design of a supersonic transport aircraft using the relaxed static stability technology. At each step, validation studies are performed and initial results indicate that the robust design method represents an accurate depiction of the problem. This depiction provides critical insight into where and why uncertainty affects the family of design solutions.

[12] Oberkampf, W.L., “Methodology for the Estimation of Uncertainty and Error in Computational Simulation,” *Proc. Nondeterministic Approaches and Their Potential for Future Aerospace Systems*, NASA Langley Research Center, Hampton, VA, Sep 2001.

Summary: Our focus is on developing a framework for identifying and estimating error and uncertainty in nondeterministic computational simulation. This framework is composed of six phases, which represent a synthesis of the activities

recognized in the systems engineering (operations research) community, the probabilistic risk assessment community, and the numerical methods community. Our framework emphasizes models that are given by a set of partial differential equations (PDEs) that must be solved numerically, although the framework is also applicable to modeling in general. We stress a clear distinction between the specification of the system, which is modeled by a set of PDEs, and the environment, which should be representative of the boundary conditions and excitation for the PDEs. We make a distinction between error and uncertainty so that the issues of representation and propagation of each is aided. The issue of numerical solution error is generally ignored in risk assessment analyses and nondeterministic simulations. Neglecting numerical solution error can be particularly detrimental to uncertainty estimation when the mathematical models of interest are cast in terms of nonlinear PDEs. Types of numerical error that are of concern in the numerical solution of PDEs are spatial discretization error in finite element and finite difference methods, temporal discretization error in time-dependent simulations, and error due to discrete representation of strongly nonlinear interactions.

[13] Trucano, T.G., “Prediction and Uncertainty in Computational Modeling of Complex Phenomena: A Whitepaper”, SAND98-2776, Computational Physics Research and Development, Albuquerque, NM, December 1998.

Abstract: This report summarizes some challenges associated with the use of computational science to predict the behavior of complex phenomena. As such, the document is a compendium of ideas that have been generated by various staff at Sandia. The report emphasizes key components of the use of computational to predict complex phenomena, including computational complexity and correctness of implementations, the nature of the comparison with data, the importance of uncertainty quantification in comprehending what the prediction is telling us, and the role of risk in making and using computational predictions. Both broad and more narrowly focused technical recommendations for research are given. Several computational problems are summarized that help to illustrate the issues we have emphasized. The tone of the report is informal, with virtually no mathematics. However, we have attempted to provide a useful bibliography that would assist the interested reader in pursuing the content of this report in greater depth.

3.0 STOCHASTIC ROBUST CONTROL

The work in this area consisted of reviewing the state-of-the art in stochastic robust control and determining the applicability of the technique within the SVMV paradigm. An assumption of the new paradigm is that some form of structural control will be available, and necessary, to “tune” the structure on-orbit. This assumption has implications for structure design, modeling, fabrication as well as structural control systems. High-fidelity models may not be able to account for build-to-build variations, response differences between 0-g and 1-g environments, and structural changes due to the space environment. All of these factors must be captured in a control-relevant form in order to be used in the design and analysis of robust structural controllers.

Of the many uncertainty representations and models discussed in the papers of the previous chapter, the most widely known and developed methods are available within the mathematics of probability theory. There uncertainty is described in terms of probability density functions and propagated through the computational model resulting in probability distributions for the performance metrics of interest. This approach is currently being applied in industry and will probably be the preferred method of dealing with uncertainties in the near future. Probability theory gives a way of representing uncertainties but not a method of quantifying them. Quantifying uncertainties is still a tricky area and even more so when very few samples are available to derive statistics. Engineering judgment may be required to fill this gap. The following references show that probability theory tends to be relied upon when performing practical design and analyses:

[1] Hasselman, T., “Quantification of Uncertainty in Structural Dynamic Models,” *Journal of Aerospace Engineering*, Vol. 14, No. 4, 2001, pp. 158-165.

Summary: This paper looks at “quantification of modeling uncertainty” due to experimental uncertainty, parametric uncertainty, and model form uncertainty. Uncertainty derived by comparing analysis and test modes. It derives linear perturbation result equations and discusses damping uncertainty. The authors apply uncertainty propagation (linear covariance, interval propagation using the vertex method, numerical simulation using Monte Carlo method). Predictive accuracy is discussed. Model uncertainty quantification general methodology is based on the correlation of modal analysis and test data.

[2] Bourgault, F., “Model Uncertainty and Performance Analysis for Precision Controlled Space Structures,” MSc. Dissertation, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Dec. 2000.

Abstract: The purpose of this thesis is to provide confidence for the designer that

a concept of a future space-based telescope will meet its very stringent requirements. More specifically, our goal is to predict the amount of uncertainty in the performance prediction made through out the design process. Also, given a statistical database for structural uncertainty, the methodology presented will establish the probability of success of a particular architecture. The traditional design process starts by evaluating and comparing the performance of different concepts by using simplified structural and disturbance models. As the process progresses the different solutions are evaluated and the most promising concept is retained and refined. Later on, some preliminary structural testing is performed, and the finite element model is updated to reflect the reality more accurately. Eventually, when the design process approach completion and is moving toward production, most of the structural elements have been tested, and the performance predictions of the model should converge to the actual system performance. Large flexible space structures present a problem in using this approach because they are often too flexible to support their own weight and/or too large to fit inside any laboratory facilities to be tested fully assembled. For example, it would be impractical to test the whole assembly of the International Space Station or SIM on the ground. Also, during the preliminary design phase, no test data are available to update the models. Nevertheless, even when the model is very mature and has been updated after experimental testing, a discrepancy remains between the predicted and actual performance of the system. These uncertainties are due to various sources of variability in the system: variable noises (sources and levels), testing conditions and environmental factors, assembly/reassembly, shipset, disturbance levels, and others. How then, can we have confidence that a particular concept will meet the requirements if the only tool we have are finite element models that may not be accurate? The solution is to try to estimate the range of uncertainty around our nominal model performances. Since in the early design phase no test data are available, our best bet will be to use past experience to predict the expected uncertainty range on the performances of a new design. Using sensitivity information and statistical uncertainties from the literature (i.e.: modal mass and stiffness parameters uncertainties, as well as modal damping ratios uncertainties), we demonstrate with different techniques how to obtain estimates of the performance predictions uncertainty ranges. We also obtain the probability distribution function of the performance of the system and use it to deduce its “probability of success” (i.e. the probability that once built the actual structure will satisfy the performance requirements). This last result, which can be obtained without too much computation, has a great useful potential and might become an integral design step for high performance controlled structures as it promises to help build confidence in the model predictions and could be used in the so called error budgeting phase. The techniques are demonstrated on a 2

degree-of-freedom sample case and on a more realistic system: the Space Interferometer Mission (SIM) Classic model.

- [3] Campbell, M. E., and Grocott, C. O., "Parametric Uncertainty Model for Control Design and Analysis," *IEEE Transactions on Control System Technology*, Vol. 7, No. 1, 1999, pp. 85-96.

Summary: Research focused on characterization of model uncertainties for control design and analysis. Objective: accurate model development of parametric uncertainties for control design and analysis of a structural system. Parametric uncertainties include mismodeling of material, and geometric properties, mismodeled damping, discretization effects, and nonlinearities. Paper assumes a linear stochastic perturbation with first-and second-order statistics of frequencies damping ratio and mode shapes. Run repeated test and use system identification to get parameters, uses modal coordinates, used collocated actuator/sensors to scale model shapes correctly for uncertainty model development. Uncertain mode shapes model not utilized in 0-g due to sensitivity of system to this and lack of confidence in uncertainty model.

- [4] Crawley, E.F., Barlow, M.S., van Schoor, M.C., Masters, B., and Bicos, A.S., "Measurement of the Modal Parameters of a Space Structure in Zero Gravity," *Journal Of Guidance, Control, and Dynamics*, Vol. 18, No. 3, May-June 1995, pp. 385-394.

Abstract: An analytic and experimental study of the changes in the modal parameters of space structural test articles from 1 to 0 g is presented. Deployable, erectable, and rotary modules were assembled to form three one- and two-dimensional structures in which variations in bracing wire and rotary joint preload could be introduced. The structures were modeled as if hanging from a suspension system in 1 g, and unconstrained, as if free floating in 0 g. The analysis is compared with ground experimental measurements made on a springwire suspension system with a nominal plunge frequency of 1 Hz and with measurements made on the Shuttle middeck. The degree of change in linear modal parameters, as well as the change in nonlinear nature of the response, is examined. Trends in modal parameters are presented as a function of force amplitude, joint preload, and ambient gravity level.

For the normal engineering approach, the drawings and handbook properties leads to hardware development and build. After the hardware is built, a reconciliation process is begun. This process compares results from the hardware and the computational model. Detected discrepancies are corrected by either tuning up the computational model or the hardware, whichever is deemed to be erroneous after analysis of the results from each. The validation process includes all of the anomalies listed in the flow path. However in that this setting computational

numerical inaccuracies and other inaccuracies are coupled. Successful validation requires a good knowledge of material properties, geometry, boundary conditions, and small computational inaccuracies.

Since probability theory likely will be used for representing and quantifying uncertainty in computational model prediction problems, including structural finite-element models, having a control system synthesis methodology for such plant models would be beneficial. Given that the computational model is probabilistic in its parameters, a consistent control system synthesis methodology would be probabilistic yielding probability of success on the performance metrics of interest. In the literature, such a control system synthesis methodology is loosely referred to as stochastic robust control. Typical stochastic control is a framework for synthesizing controllers for systems with stochastic inputs. Stochastic robust control is a framework for synthesizing controllers for systems that have stochastic parameters in addition to stochastic inputs. There are not many published results in this area. A few relevant publications, along with their abstracts and in some cases their introductions are listed.

[6] Zang, T.A., Hensch, M.J., Hilburget, M.W., Kenny, S.P., Luckring, J.M., Maghami, P, Padula, S.L., and Stroud, W.J., “Needs and Opportunities for Uncertainty-Based Multidisciplinary Design Methods for Aerospace Vehicles”, NASA/TM-2002-211462, July 2002, Langley Research Center, Hampton, VA.

Summary: This report consists of a survey of the state of the art in uncertainty-based design together with recommendations for a Base research activity in this area for the NASA Langley Research Center. In particular, it focuses on the needs and opportunities for computational and experimental methods that provide accurate, efficient solutions to nondeterministic multidisciplinary aerospace vehicle design problems. We use the term *uncertainty-based design* to describe this type of design method. The two major classes of uncertainty-based design problems are robust design problems and reliability-based design problems. A *robust design* problem seeks a design that is relatively insensitive to small changes in the uncertain quantities. A *reliability-based design* seeks a design that has a probability of failure that is less than some acceptable (invariably small) value.

Traditional design procedures for aerospace vehicle structures are based on combinations of factors of safety and knockdown factors. The aerodynamic design procedures used by the industry are exclusively deterministic. There has been considerable work on “robust controls,” but this work has been limited to using norm bounds on the uncertain variables. Reliability-based design methods have been used within civil engineering for several decades and in aircraft engine design for about a decade. Applications to the structural design of airframes are

only now starting to emerge. Only academic studies of reliability-based design methods within the aerodynamics and controls disciplines are known to the authors.

To use uncertainty-based design methods, the various uncertainties associated with the design problem must be characterized and managed, and these characterizations must be exploited. In the context of computational modeling and simulation, two complementary categorizations of uncertainties are useful. One categorization distinguishes between parameter uncertainties and model form uncertainties. *Parameter uncertainties* are those uncertainties associated either with the input data (boundary conditions or initial conditions) to a computational process or with basic parameters that define a given computational process, such as the coefficients of phenomenological models. *Model form uncertainties* are uncertainties associated with model validity, i.e., whether the nominal mathematical model adequately captures the physics of the problem. Systematic procedures for characterizing and managing uncertainties in experimental activities include design of experiment methods and statistical process control techniques. The former focuses more on characterizing the uncertainties and the latter more on managing them.

Parameter uncertainties are typically specified in terms of probability density functions, membership functions, or interval bounds. Model form uncertainties are very difficult to characterize. Generic techniques are available for assessing the effects of uncertainties on discipline and system performance predictions, and some optimization methods can account for uncertainties. However, better and less resource-intensive methods are needed for both uncertainty propagation and optimization under uncertainty. Certainly, the deployment of existing and new techniques within the aerodynamic, controls, structures, and systems analysis disciplines for applications to aerospace vehicles is critically needed.

- [14] Crespo, L. G., “Probabilistic Formulations to Robust Optimal Control”, AIAA-2004-1667-CP, *Proceedings, AIAA Structures, Structural Dynamics, and Materials Conference*, Palm Springs, CA, April 2004, pp. 1-21.

Abstract: This paper presents a study on the design of robust compensators by using random variables to model parametric uncertainty. In this framework, all plants in the uncertain set are weighted according to their chance of occurrence. This allows us to assess and reduce the conservatism in which conventional robust control techniques unnecessarily incur. The propagation of the uncertain plant through conventional control analysis tools leads to probabilistic metrics of stability and performance. Several control formulations, in which some type of robust optimality in the probabilistic sense is aimed, are presented herein. Examples that admit closed form expressions for the random variables and

processes that determine the closed-loop stability and performance are used to elucidate the nature of the problem at hand.

- [15] May, B. S., and Beck, J. L., “Probabilistic Control For The Active Mass Driver Benchmark Structural Model,” *Earthquake Engineering And Structural Dynamics*, Vol 27, 1998, pp. 1331–1346.

Summary: A probability-based robust control design methodology is presented that is applied to the ‘benchmark system’, which is a high-fidelity model of an active-mass-driver laboratory structure. For the controller design, the objective is to maximize the probability that the uncertain structure/controller system achieves satisfactory performance when subject to uncertain excitation. The controller’s robust performance is computed for a set of possible models by weighting the conditional performance probability for a particular model with the probability of that model, then integrating over the set of possible models. This is accomplished in an efficient manner using an asymptotic approximation. The probable performance is then maximized over the class of constant-gain acceleration-feedback controllers to find the optimal controller. This control design method is applied to a reduced-order model of the benchmark system to obtain four controllers, two that are designed on the basis of a ‘nominal’ system model and two ‘robust’ ones that consider model uncertainty. The performance is evaluated for the closed-loop systems that are subject to various excitations.

The probabilistic robust control approach creates controllers that incorporate probabilistic descriptions of the model uncertainties into the design of the optimal controller. The controllers are designed to satisfy probable performance over the class of uncertain models, and may be less conservative than those designed using methods based on the worst-case performance (e.g. H_∞ -control and its derivatives), where the ‘worst’ model may be quite improbable. Probabilistic uncertainty descriptions can arise when models of the system are identified using response data, or when the modeling uncertainties in describing the system are quantified based on engineering experience.

- [16] Yuen, K. V., and Beck, J. L., “Reliability-based robust control for uncertain dynamical systems using feedback of incomplete noisy response measurements,” *Earthquake Engineering And Structural Dynamics*, Vol 32, 2003, pp. 751-770.

Summary: A reliability-based output feedback control methodology is presented for controlling the dynamic response of systems that are represented by linear state-space models. The design criterion is based on a robust failure probability for the system. This criterion provides robustness for the controlled system by considering a probability distribution over a set of possible system models with a stochastic model of the excitation so that robust performance is expected. The

control command signal can be calculated using incomplete response measurements at previous time steps without requiring state estimation. Examples of robust structural control using an active mass driver on a shear building model and on a benchmark structure are presented to illustrate the proposed method.

Because complete information about a dynamical system and its environment are never available, the system and excitation cannot be modeled exactly. Classical control design methods based on a single nominal model of the system may fail to create a control system that provides satisfactory performance. Robust control methods (e.g. H_2 , H_∞ , and μ -synthesis, etc.) were therefore proposed so that the optimal controller can provide robust performance and stability for a set of ‘possible’ models of the system. In a probabilistic robust control approach, an additional ‘dimension’ is introduced by using probabilistic descriptions of all the possible models when selecting the controller to achieve optimal performance. These probability distributions give a measure of how plausible the possible parameter values are, and they may be obtained from engineering judgment or Bayesian system identification methods.

Over the last decade or so, there has been increasing interest in probabilistic, or stochastic, robust control theory. Monte Carlo simulations methods have been used to synthesize and analyze controllers for uncertain systems. First and second-order reliability methods have been incorporated to compute the probable performance of linear-quadratic regulator controllers (LQR). On the other hand, an efficient asymptotic expansion has been used to approximate the probability integrals that are needed to determine the optimal parameters for a passive tuned mass damper and the optimal gains for an active mass driver for robust structural control. The proposed controller feeds back output measurements at the current time only, where the output corresponds to certain response quantities that need not be the full state vector of the system. However, there is additional information from past output measurements which may improve the performance of the control system.

- [17] Spencer, B. F., Sain, M.K., Won, C.H., Kaspari, D.C., and Sain, P. M., “Reliability-Based Measures Of Structural Control Robustness,” *Structural Safety*, Vol. 15, 1994, pp. 111-129.

Abstract: Because of the uncertainty inherent in engineering structures, consistent probabilistic stability/performance measures are essential to accurately assessing and comparing the robustness of structural control systems. An approach is presented herein for calculating such probabilistic measures for a controlled structure. First and second order reliability methods (FORM/SORM) are shown to be appropriate for the required calculations. The concepts are illustrated through several examples of seismically excited structures with active protective systems.

[18] Yuen, K-V., “Model Selection Identification and Robust Control for Dynamical Systems,” Ph.D. Dissertation, Department of Civil Engineering, California Institute of Technology, Apr. 2002.

Abstract: To fully exploit new technologies for response mitigation and structural health monitoring, improved system identification and controller design methodologies are desirable that explicitly treat all the inherent uncertainties. In this thesis, a probabilistic framework is presented for model selection identification and robust control of smart structural systems under dynamical loads, such as those induced by wind or earthquakes. First, a probabilistic based approach is introduced for selecting the most plausible class of models for a dynamical system using its response measurements. The proposed approach allows for quantitatively comparing the plausibility of different classes of models among a specified set of classes. Then, two probabilistic identification techniques are presented. The first one is for modal identification using nonstationary response measurements and the second one is for updating nonlinear models using incomplete noisy measurements only. These methods allow for updating of the uncertainties associated with the values of the parameters controlling the dynamic behavior of the structure by using noisy response measurements only. The probabilistic framework is very well suited for solving this nonunique problem and the updated probabilistic description of the system can be used to design a robust controller of the system. It can also be used for structural health monitoring. Finally, a reliability based stochastic robust control approach is used to design the controller for an active control system. Feedback of the incomplete response at earlier time steps is used without any state estimation. The optimal controller is chosen by minimizing the robust failure probability over a set of possible models for the system. Here, failure means excessive levels of one or more response quantities representative of the performance of the structure and the control devices. When calculating the robust failure probability, the plausibility of each model as a representation of the system’s dynamic behavior is quantified by a probability distribution over the set of possible models; this distribution is initially based on engineering judgment but it can be updated using the aforementioned system identification approaches if dynamic data become available from the structure. Examples are presented to illustrate the proposed controller design procedure, which includes the procedure of model selection, identification and robust control for smart structures.

[19] Cresp, L.G., “Stochastic Control Synthesis of Systems with Structured Uncertainty”, NASA/CR-2003-212167 (NIA Report No. 2003-01), Dec 2003, National Institute of Aerospace, Hampton, VA.

Summary: This paper presents a study on the design of robust controllers by using random variables to model structured uncertainty for both SISO and MIMO feedback systems. Once the parameter uncertainty is prescribed with probability density functions, its effects are propagated through the analysis leading to stochastic metrics for the system's output. Control designs that aim for satisfactory performances while guaranteeing robust closed loop stability are attained by solving constrained non-linear optimization problems in the frequency domain. This approach permits not only to quantify the probability of having unstable and unfavorable responses for a particular control design but also to search for controls while favoring the values of the parameters with higher chance of occurrence. In this manner, robust optimality is achieved while the characteristic conservatism of conventional robust control methods is eliminated. Examples that admit closed form expressions for the probabilistic metrics of the output are used to elucidate the nature of the problem at hand and validate the proposed formulations.

The main requirement of feedback control is to achieve acceptable levels of performance in the presence of uncertainty. Fundamental trade offs and compromises between these two aspects motivate the entire body of feedback theory. While performance concerns aspects such as reference tracking, disturbance rejection, bounded control effort, etc., uncertainty appears as a result of the inevitable discrepancies between the physical problem and its deterministic mathematical model. Ignorance on the system's exact dynamics, on the actual operating conditions and the purposeful choice of a simplified representation of the physical problem exemplify this aspect. In this context, uncertainty can be classified as *structured* (or *parametric*) and *unstructured*. The first kind corresponds to inaccuracies on the parameters of the model while the second one corresponds to unmodeled dynamics. Uncertainty can be modeled in many ways depending upon the desired quality of its mathematical description. Differential sensitivity, multi-models, interval analysis, perturbations, fuzzy sets and probabilistic methods have been used. The effects of uncertainty on the stability associated with the prescribed control solutions have been studied by both deterministic and stochastic means. These analysis tools however, have not been integrated to the control design process. The methods most commonly used for robust control design are μ -synthesis and H_∞ optimization. In these, uncertainty is modeled with norm-bounded complex perturbations of fixed but arbitrary structure about its nominal form. This treatment is extensively used primarily because it leads to a tractable set of sufficient conditions for robust stability. Such approaches however, have the following drawbacks: (i) the crudeness of the uncertainty description usually leads to redundant and physically impossible plants, then to highly conservative designs, (ii) it is not feasible to favor scenarios

with higher chance of occurrence among all the possible ones, and as a result, robust optimality is precluded, (iii) a quantitative description of the robustness of the solution is unattainable and (iv) the resulting controllers are so complex that model reduction techniques are usually required. While such perturbations account for unstructured uncertainty coarsely, an augmented plant model with structured uncertainty can be used to conciliate the uncertainty representation with the physics of the problem. While robust optimization has been studied in various disciplines using different uncertainty models, stochastic control synthesis remains, to a large extent, unexplored. This paper studies the control design of plants with structured uncertainty for both single-input-single-output (SISO) and multiple-input-multiple-output (MIMO) systems using a probabilistic approach. The joint probability-density-function (PDF) of the parameters is prescribed *a priori*, and then propagated, leading to a probabilistic description of the metrics of the controlled response. Control design, involving decoupling, performance and stability aspects, is carried out by solving constrained non-linear optimization problems in the frequency domain.

4.0 CONCLUSIONS AND RECOMMENDED FUTURE TASKS

Robust control methods provide a methodology for developing structural control systems that attain robust performance in the presence of norm-bounded model uncertainty. Because of the limitations incurred in the design of system controllers for norm-bounded uncertainties mentioned in the papers of the previous chapter, this form of uncertainty was not considered in this study. Their primary shortcoming is their overly conservative nature often resulting in synthesized controllers that are unable to achieve stringent performance requirements. There are some nonconservative norm-bounded uncertainty forms but their applicability to computational model performance prediction enhancement has not been demonstrated.

One form of *Stochastic Robust Control* aims at developing control systems that attain some “probability of success level in robust performance” for plants subject to stochastic model parameter uncertainty, stochastic input disturbances, and representation uncertainty. Examples of representation uncertainty include approximating an infinite dimensional system by a finite dimensional one, or modeling a non-linear system as a linear one. Some representation uncertainties may be handled as stochastic disturbances acting directly on the performance measure. Stochastic Robust Control may apply Bayesian Model Averaging over a set of possible plant models, with the plant models being generated by sampling the parameter space. Bayesian Model Averaging is used to identify a likely subspace of plant models based on the available measured data. The stochastic inputs then drive these models and probability is implanted to synthesize an optimal controller that maximizes the probability of success, minimizes the probability of failure, based on a performance measure. Some areas of application for this methodology are economics and structural control. This approach appears suitable for the structural control problems of interest and is useful for DOD certification by analysis.

Further research into the practical applications and developments of stochastic robust control for enhancing the reliability of performance requirements validation based on computational models is warranted. An extended literature survey should be performed with an eye toward applications of stochastic robust control methodologies to high order systems such as uncertain structural models, applicability to decentralized control and substructure synthesis, and rapid uncertainty propagation routines or methodologies. In the absence of published results in these and other areas of interest, publishable results should be generated including pros or cons with respect to this control synthesis methodology.

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