Reducing Uncertainty in Effects-Based Operations

THESIS

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Abstract

Known as the *fog of war*, uncertainty has been prevalent in the conduct of military operations throughout human history. Intelligence collection efforts are tasked to reduce this uncertainty through the collection of information. Utilizing Shannon's entropy as a measure of the expected information gain due to an intelligence collection effort, a methodology is developed to prioritize and allocate intelligence assets in an efficient manner. Incorporated in this methodology are target priority and the requirement to reassess dynamic targets. The application area for the methodology is Effects-Based Operations. A generalized state model is developed to conduct adversary system-of-systems analysis. This model forms the basis for the entropy calculations and the resultant integer program to maximize the information gain.

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I. Introduction

1.1 Background

> [U]ncertainty seems to increase of its own accord unless something is done to reduce it. [14]

Known as the *fog of war*, uncertainty has been prevalent in the conduct of military operations throughout human history. As friendly forces act and react, an adversary also continues to act and react. Without continuous intelligence updates, the uncertainty associated with an assessment of the adversary's current state will continue to grow. Add to the continual need for intelligence a limited supply of intelligence resources, and the problem becomes one of asset allocation, the goal of which is to maximize information gain.

Increasingly, intelligence collection has become the limiting factor in performing military operations as there will always be less intelligence collection opportunities than intelligence targets. Vying for these limited intelligence collection opportunities are a number of agencies (e.g., Defense Intelligence Agency, Central Intelligence Agency, National Security Agency, etc.). The limited intelligence collection efforts allocated to military operations must be efficiently used in order to properly characterize the current state of an adversary.

The problem of allocating intelligence resources can be related to the field of computer adaptive tests (CAT) like the Graduate Record Examination (GRE) and Graduate Management Admission Test (GMAT). At the beginning of the test, the CAT has no information on the subject's mastery of the material. The CAT assumes a prior probability distribution on the subject's mastery level. Then, the test must select resources (e.g., questions) to reduce its uncertainty in assessing the subject's mastery level. To do so, the CAT selects the question that will reduce the expected uncertainty the most. Once the subject answers the question, the test updates the prior distribution and selects another question to maximize information gain (i.e., minimize uncertainty). The process is repeated until the subject's mastery level is known. [15]

The same kind of process can be applied to an intelligence collection effort. Intelligence analysts typically provide qualitative assessments of an adversary entity's current state using linguistic quantifiers, or "words of estimative probability" (e.g., *Almost certain*, *Probably*, etc.). For example, an intelligence analyst may give the precise location of an airfield, but may provide a judgement or estimate of the use of the airfield (e.g., "It is almost certainly a military airfield"). It is this linguistic estimate that is the source of the probabilistic uncertainty. [11] A prior probability distribution can be assigned to these linguistic quantifiers to describe an adversary's current state. Then based on the effectiveness of the available intelligence collection assets, the assets can be efficiently allocated to collect information on the adversary. After the intelligence collection efforts, the prior probability distributions are updated, and the process is repeated.

Military intelligence targets do not have equal priority. Certain entities will be high priority targets that require a reduction in the uncertainty of their current states (e.g., location of weapons of mass destruction, location of senior adversary leadership, etc.). Although not desired, higher uncertainty in the assessment of lesser priority intelligence targets' current states may be tolerated. Intelligence collection efforts must be directed to collect information on the most uncertain, highest priority adversary entities.

Additionally, an adversary is continually acting and reacting to friendly (blue) force actions; therefore, unlike CATs, an adversary's current state will never be known with total (or near) certainty. Management of the uncertainty is thus extremely important. Intelligence collection efforts must be repeatedly tasked to collect information on adversary systems (nodes) that have been assessed already.

Finally, in current operations, when an action is taken on an adversary entity, intelligence assets are tasked to collect information on that entity (e.g., bomb damage assessment (BDA)). This use of intelligence assets is warranted if the action taken on an entity has a low probability of success or has an unknown effect. For example, the probabilities of success associated with nonkinetic operations are often low or unknown. These nonkinetic operations thus require subsequent intelligence collection efforts to characterize the effectiveness of the operations. On the other hand, if an action is taken on an entity that has a high probability of success and has known outcomes, then using an intelligence asset to collect information on the entity may not be the best use of the intelligence asset (i.e., may not provide the greatest information gain). Therefore, prioritizing intelligence assets must account for actions that are planned against the entities.

1.2 Problem Statement

Perfect knowledge of an adversary is never truly obtained. Considerable uncertainty is always present in characterizing the current state of an adversary. Formally, uncertainty is defined below. [1]

uncertainty: estimated amount or percentage by which an observed or calculated value may differ from the true value

In particular, uncertainty is present in describing the current state of an adversary's entities (e.g., radars, tanks, leaders, economy, etc.). The goal is to reduce this uncertainty in an efficient manner. Thus, the problem is prioritizing intelligence assets to maximize information gain while bound to intelligence asset constraints. The prioritization of intelligence assets must take into consideration target priority and the need to reassess dynamic targets. To accomplish this, the following are required:

- Mathematical model of an adversary's entities
- Mathematical representation of linguistic quantifiers used by the intelligence community (e.g., *Probably*, *Almost certain*, etc.)

- Formal measure of the uncertainty associated with an adversary's entities
- Priority, planned action, and dynamic updating of the intelligence assessment associated with an adversary's entities
- Methodology to allocate intelligence collection assets to collect information on an adversary's entities.

1.3 Summary of Current Knowledge

1.3.1 Effects-Based Operations. The Effects-based Operations (EBO) process provides a framework from which adversary system uncertainty can be measured. According to the Joint Warfighting Center (JWFC) Pamphlet 7 on EBO, the current working definition of EBO is:

> Operations that are planned, executed, assessed, and adapted based on a holistic understanding of the operational environment in order to influence or change system behavior or capabilities using the integrated application of selected instruments of power to achieve directed policy aims. [10]

EBO aim to "promote synchronized, overlapping, near simultaneously executed actions" through a "commonly shared system understanding of the adversary and the operational environment by all members of the joint, interagency, multinational team." In order to gain further understanding and situational awareness of an adversary and the operational environment, a System-of-Systems Analysis (SoSA) is developed. Each node within the network represents a person, place, or physical thing that is a fundamental component of the adversary's system. The linkages connecting the nodes represent the behavioral, physical, or functional relationships between the nodes. By developing a interrelated network of a region or nation of interest, planners aim to take a holistic view of the operational environment. [10]

1.3.2 Uncertainty. Uncertainty may arise from making observations about ill-defined (or complex) concepts. In addition, uncertainty may arise from creating rules relating events, when the knowledge of the correlations between events is weak.

When a system (or concept) is too complex to accurately model, observing outputs based on inputs to the system can lead to drawing false conclusions about the system. Other sources of uncertainty include the aggregation of multiple sources. This often leads to inconsistencies or "over-estimat[ion] of the likelihood of events due to assumed, but untrue, dependencies." [14] Further uncertainty can come from trying to measure time-dependent variables. These variables change over time, and thus are subject to temporal uncertainties. Overall, the sources of uncertainty are application dependent. The two major sources of uncertainty for EBO are Haimes' top-level categorizations: natural variability and knowledge uncertainty. [8] Natural variability is due to population variances, and knowledge uncertainty rises from a lack of understanding or missing knowledge.

The major sources of EBO natural variability are temporal, spatial, and individual heterogeneous [8]. Temporal variability is value fluctuation due to changes over time. Spatial variability relates to fluctuations due to location or area. All other natural variability is captured by individual heterogeneous.

EBO knowledge uncertainty can be attributed to four main sources: decision uncertainty, incompleteness, inconsistency, and inaccuracy. Decision uncertainty is the human subjectivity associated with decision making. Missing data is captured by incompleteness. Inconsistency involves two pieces of contradicting information. The final source of knowledge uncertainty is inaccuracy, which relates to incorrect data on the current true state of a system.

1.3.3 Methods for Handling Uncertainty. The three most common models used by the uncertainty community to address uncertainty are probability, possibility and evidence theories. In addition, several other measures, like certainty factors, fuzzy sets, and rough sets, are used to a lesser degree. Probability, possibility and evidence theories are similar, but differ in "subtleties of meaning and application." [14] All three models are based on a distribution function that distributes some measure of uncertainty to the events of interest. This distribution can be based on statistical data, physical possibility, or subject matter expert (SME) assessment.

The techniques for handling uncertainty fall into two broad categories: quantitative and qualitative. The quantitative techniques require enumeration of all the required quantities; whereas, the qualitative techniques use either strictly qualitative data or a mixture of qualitative and quantitative data. Each type of technique has its benefits and drawbacks. The quantitative techniques maintain a high level of precision, but require vast amounts of data that is often times unattainable. On the other hand, the qualitative techniques require less specific data but are much less precise. In fact, the qualitative techniques can lead to no useful information being obtained due to lack of specificity.

1.3.4 Measuring Uncertainty. In order to maximize information gain through intelligence asset prioritization, it is necessary to measure the uncertainty present in a system. Once a distribution has been assigned to the current state of the system (or subsystem), a measure of uncertainty for the distribution is needed. The uncertainty community has used two measures to quantify the uncertainty associated with probability and possibility distributions. Shannon's entropy is the commonly used measure of uncertainty used for probabilities, and specificity is used to measure the uncertainty for possibility distributions. The entropy of a random variable measures its complexity, or degree of randomness. Given a higher entropy of a random variable, it is harder to predict the value of the random variable. [7] Specificity is an extension of Shannon's entropy to possibility distributions.

1.3.5 Intelligence Asset Assignment. The allocation of intelligence collection efforts from multiple intelligence sources against multiple adversary entities can be related to the integer program (IP) of assigning multiple resources to multiple containers, or knapsacks–the multiple knapsack problem. In allocating intelligence assets, however, the costs (i.e., used intelligence resources) and benefits (i.e., information gain) associated with each adversary entity is dependent upon the intelligence asset assigned to collect information on it. The generalized assignment problem (GAP), a more general IP, can be used to solve this problem. [12]

1.4 Assumptions

To prove out the methodology and maintain a practical level of analysis, the following assumptions are made in this thesis:

- The adversary or SoSA network developed during the EBO process is assumed to be a correct representation of an adversary. The existence of all nodes and linkages are assumed to be known with certainty.
- The nodes of the SoSA network are assumed to be independent.

Based on the first assumption, this thesis does not address the *you don't know what you don't know* problem. In military operations, this incompleteness source of uncertainty is quite prevalent. However, without known existence, allocating resources against these unknown nodes is not quantifiable. A simple solution is to allocate a portion of the available assets to investigate the unknown nodes of the system.

The second assumption limits the measure of information gain on the targeted node. The SoSA network includes linkages (i.e., influences) between the nodes. Incorporating these linkages would more accurately represent the information gain due to intelligence collection. For example, it may be easier to collect information on a neighboring node, which may reduce the uncertainty on the targeted node.

1.5 Thesis Organization

This thesis is composed of five chapters. The present chapter provides a brief background of the topic, a description of the problem to be addressed, and a summary of the current knowledge. Chapter 2 delves further into the field of data uncertainty as it relates to EBO. Methods for handling and measuring uncertainty, as well as linguistic quantifiers are explored. Finally, the generalized assignment problem is investigated.

Chapter 3 presents a mathematical model for SoSA networks, which lays the framework for a methodology to prioritize a single intelligence asset's collection efforts. This methodology is then applied to the problem of allocating multiple intelligence assets. Chapter 4 applies these methodologies to a notional 20-node, 4-asset example. Finally, Chapter 5 summarizes the results and provides recommendations for future research.

II. Summary of Current Knowledge

2.1 Overview

This chapter presents a review of the current literature on Effects-Based Operations (EBO), knowledge uncertainty, linguistic quantifiers, methods for handling uncertainty, uncertainty measures, and the generalized assignment problem (GAP). Each of these plays an integral part in designing a methodology for prioritizing intelligence assets during EBO, which is addressed in Chapter 3.

2.2 Effects-Based Operations

According to the Joint Warfighting Center (JWFC) Pamphlet 7 on EBO, the current working definition of EBO is:

> Operations that are planned, executed, assessed, and adapted based on a holistic understanding of the operational environment in order to influence or change system behavior or capabilities using the integrated application of selected instruments of power to achieve directed policy aims. [10]

EBO aim to "promote synchronized, overlapping, near simultaneously executed actions" through a "commonly shared system understanding of the adversary and the operational environment by all members of the joint, interagency, multinational team." This shared system understanding is enabled through a collaborative information environment (CIE) and a operational net assessment (ONA). A CIE is a virtual environment designed to improve collaboration and knowledge management across a combatant command. ONA is a process and product that develops a comprehensive system-of-systems understanding of the operational environment. [10]

In order to gain further understanding and situational awareness of an adversary and the operational environment, ONA begins with a System-of-Systems Analysis (SoSA). To develop a SoSA, the operational environment is assumed to be composed of political, military, economic, social, infrastructure, and information (PMESII) entities (or systems). These six interrelated PMESII systems are represented pictorially using a multi-dimensional network (see Figure 2.1 on the following page). Each node

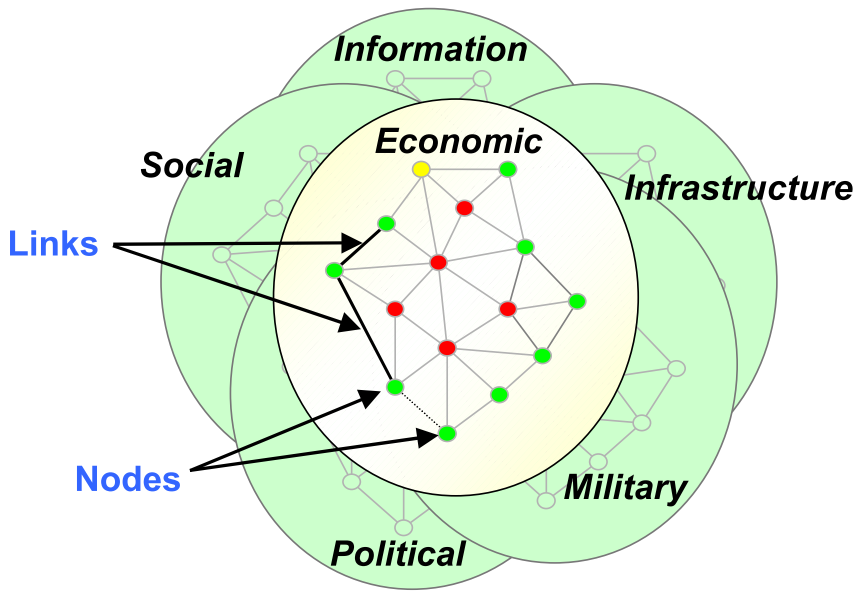

Figure 2.1: Systems-of-systems Analysis [10]

within the network represents a person, place, or physical thing that is a fundamental component of the system. The linkages connecting the nodes represent the behavioral, physical, or functional relationships between the nodes. By developing a interrelated network of a region or nation of interest, planners aim to take a holistic view of the operational environment. [10]

2.3 Uncertainty

According to the Merriam-Webster Concise Dictionary of English Language, uncertainty is defined in the following manner:

> uncertainty - 1: the quality or state of being uncertain; doubt, 2: something that is uncertain. [13]

and uncertain is defined as:

> uncertain - 1: indefinite, indeterminate, 2: not certain to occur; problematical, 3: not reliable; untrustworthy, 4 a: not known beyond doubt; dubious, b: not having certain knowledge; doubtful, c: not clearly identified or defined, 5: not constant; variable, fitful. [13]

In other words, knowledge can be uncertain due to inaccurate measurement, vagueness or ambiguity, variability, source reliability, etc. All of these uncertainties are easily found in EBO. Inaccurate measurement can be seen in estimations of enemy troop strengths. Vagueness is displayed in many intelligence assessments, where linguistic quantifiers such as *probable* or *possible* are used. Variability is evident in the time dependent nature of military operations, and source reliability is obviously evident in human intelligence (HUMINT) collection.

In order to address uncertainty, one must look at the sources of the uncertainty. To start, uncertainty may arise from making observations about ill-defined (or complex) concepts. In addition, uncertainty may arise from creating rules relating events, when the knowledge of the correlations between events is weak. When a system (or concept) is too complex to accurately model, observing outputs based on inputs to the system can lead to drawing false conclusions about the system. Often times a system is assumed to be a black box that responds with certain observable state changes. These observable state changes may be once, twice, or more times removed from the actual system state changes. The primary system state changes may be unobservable or indeterminate. Without further knowledge of the system, the conclusions drawn based on the observed changes can be uncertain. These sources of uncertainty are obviously present in EBO. The entire SoSA process is full of uncertainty due to unknown and inaccurate linkages. For example, when trying to plan information operations (IO), the forms of the adversary response (i.e., in what manner does an adversary respond to IO) may not be fully known, or even observable, making it difficult to capture the impact of friendly actions. In turn, assumptions are developed relating IO actions to planned effects. Obviously, there is a great deal of uncertainty associated with planning these actions.

Other sources of uncertainty include the aggregation of multiple sources. This often leads to inconsistencies or "over-estimat[ion] of the likelihood of events due to assumed, but untrue, dependencies." [14] Often in intelligence collection, this source of uncertainty is displayed. For example, given three very unreliable sources of the same or similar intelligence, the intelligence analyst is more likely to assign a higher probability to the event even though the independent sources only warrant a low probability. The fact that multiple sources point out the same intelligence is the only grounds the analyst has for increasing the assigned probability.

Further uncertainty can come from trying to measure time-dependent variables. These variables change over time, and thus are subject to temporal uncertainties. For example, intelligence data may take too long to analyze to remain valid or relevant. Or intelligence data may not be attainable because events happen too quickly to capture accurate and timely intelligence on them. Either way, temporal uncertainty is present in the system analysis.

Overall, the sources of uncertainty are application dependent. In turn, the techniques and methods for addressing data uncertainty are also application dependent. In *Qualitative Methods for Reasoning Under Uncertainty*, Parsons argues for an "eclectic school of thought", where different techniques may be needed to address different situations depending on the predominant source or sources of uncertainty. [14] To characterize the sources of uncertainty in EBO, an uncertainty taxonomy was developed.

2.3.1 EBO Taxonomy. The literature presents multiple, varying uncertainty taxonomies. Parsons, working in the field of artificial intelligence, analyzes several different taxonomies, and Haimes, working in the field of risk analysis, presents a single taxonomy. [14] [8] Parsons' analysis included Smithson's [18], Smet's [17], Bonissone and Tong's [2], and Bosc and Prade's [4] taxonomies. Each of these taxonomies presented a different construct for categorizing uncertainty. Parsons makes the argument that an overarching taxonomy for all uncertainty regardless of the application is unattainable. However, Parsons notes there are some commonalities among the uncertainty taxonomies. Each taxonomy contains some notion of subjective uncertainty, vagueness, imprecision, incompleteness, inconsistency, and ambiguity. [14] The taxonomies were reviewed, filtered and organized based on their application to EBO.

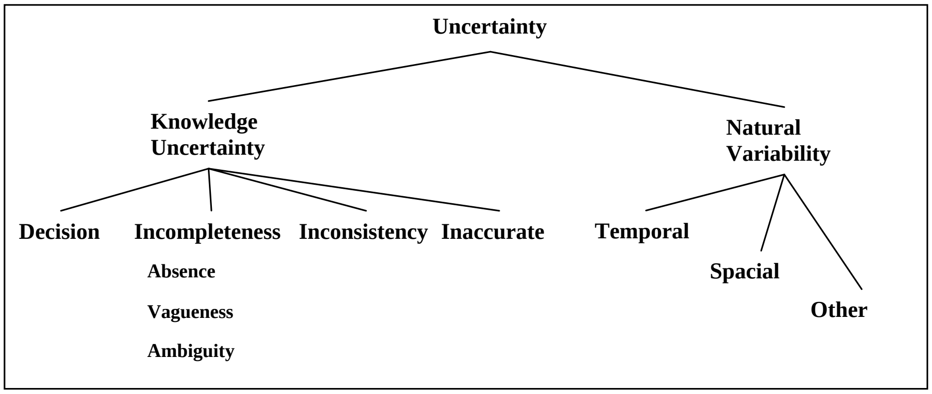

Figure 2.2: EBO Data Uncertainty Taxonomy

For application to EBO, a taxonomy was constructed taking portions of each of the reviewed taxonomies (see Figure 2.2). The two major sources of uncertainty for EBO are Haimes' top-level categorizations: natural variability and knowledge uncertainty. [8] Natural variability is due to population variances. A situation with a single entity would have no natural variability. For example, given a country of interest, not all locations within the country will react in the same manner to some action. Furthermore, the country may react differently given the timing of the action. Knowledge uncertainty rises from a lack of understanding or missing knowledge. In EBO, actions may result in unintended effects. This could be the result of planners not possessing complete knowledge of a situation.

The major sources of natural variability are temporal, spatial, and individual heterogeneous [8]. Temporal variability is value fluctuation due to changes over time. For example, the effectiveness of IO action can be drastically affected by timing. An operation that may be very successful on day 20 when civilian sentiment is fractionalized likely would not be as effective on day 1 when the population is unified. Spatial variability relates to fluctuations due to location or area. An example of spatial variability is the geographically separated religious sects in Iraq. All other natural variability is captured by individual heterogeneous. This thesis will not address natural variability. The SoSA network is assumed to be an accurate estimation of the real system.

EBO knowledge uncertainty can be attributed to four main sources: decision uncertainty, incompleteness, inconsistency, and inaccuracy. Decision uncertainty is the human subjectivity associated with decision making. If two people are given the same data, each might interpret the data differently resulting in different decisions. Furthermore, subjective judgment regarding the reliability of data is captured in decision uncertainty. Plus, different decision makers have different biases. This can be seen in EBO when planners project previous experiences to new situations. For example, a planner with extensive experience in operations in Southwest Asia may naively apply the same techniques to an operation in Southeast Asia. Again, we make the assumption that the SoSA network is constructed accurately and is not interpreted differently by decision makers. Note, this human subjectivity can be partially addressed through training and education.

Missing data is captured by incompleteness. As related to EBO, unknown nodes and/or unknown influences are categorized as incompleteness uncertainty. This thesis assumes that all nodes and influences are known. Data that is lacking required information or is at too broad of a resolution or fidelity is also classified as incomplete. This incompleteness is the target of this thesis effort. Intelligence data rarely provides a definitive probability for an event or state of a system. In contrast, intelligence usually gives linguistic quantifiers of the current state (i.e., confirmed, probable, unknown, etc.).

Inconsistency involves two pieces of contradicting information. For example, one intelligence source might indicate an enemy attack on position A whereas another intelligence source might point at position B. This type of uncertainty is inconsistency. The final source of knowledge uncertainty is inaccuracy, which relates to wrong data. This thesis assumes that neither of these are present in the SoSA network.

One important note is that all of these sources of data uncertainty are not mutually exclusive. In fact, the sources may be quite dependent on one another. For instance, take the intelligence rating of an enemy attack as *probable*. The uncertainty associated with the statement *probable* might be attributed to incompleteness or decision uncertainty. The rating is incomplete because it does not contain complete information regarding the event. This qualitative rating leads to further uncertainty, decision uncertainty, when multiple decision makers are presented the data. Various decision makers will most likely not interpret the rating the same.

2.4 Methods for Handling Uncertainty

2.4.1 Uncertainty Models. The three most commonly used models by the uncertainty community to address uncertainty are probability, possibility and evidence theories. In addition, several other measures, like certainty factors, fuzzy sets, and rough sets, are used to a lesser degree. Probability, possibility and evidence theories are similar, but differ in "subtleties of meaning and application." [14] All three models are based on a distribution function that distributes some measure of uncertainty to the events of interest. This distribution can be based on statistical data, physical possibility, or subject matter expert (SME) assessment.

2.4.1.1 Probability Theory. The basis for probability theory is the probability distribution, which allocates a probability measure to specific events. The probability distribution maps the events under consideration to the interval $[0, 1]$. An event that is known to occur, or sure event, has probability 1.

$$P(\text{sure event}) = 1$$

Conversely, an event that is known to not occur has probability 0.

$$P(\neg\text{sure event}) = 0$$

Total probability states that the probability of an event and its negation equals 1.

$$P(A) + P(\neg A) = 1$$

There are three axioms that define the behavior of the probability measure. First, the convexity law states that the probability of event A given information H is once again a probability measure.

$$0 \leq P(A|H) \leq 1$$

Second, the additive law relates the probability of two events to the probability of their union. Given two mutually exclusive events A and B,

$$P(A \cup B|H) = P(A|H) + P(B|H).$$

In turn, the sum of the probabilities of all mutually exclusive and collectively exhaustive events equals 1. Finally, the multiplication law relates the probability of two events happening together to their intersection.

$$P(A \cap B|H) = P(A|H) \cdot P(B|A \cap H)$$

Or more generally,

$$P(E_1, E_2, \ldots, E_n) = P(E_n|E_1, E_2, \ldots, E_{n-1}) \cdot \ldots \cdot P(E_2|E_1) \cdot P(E_1).$$

Some other useful properties of probability theory are the Law of Total Probability:

$$P(A) = \sum_{i=1}^{n} P(A|B_i) \cdot P(B_i)$$

and Bayes' Theorem:

$$P(a|c, x) = \frac{P(c|a, x) \cdot P(a, x)}{P(c, x)}.$$

2.4.1.2 Possibility Theory. According to Parsons, possibility theory is a "non-statistical means of quantifying uncertainty based on . . . fuzzy set theory." [14] The basis for possibility theory is the possibility distribution, which maps all the possible values of a variable to the interval $[0, 1]$. In possibility theory, an impossible event has possibility of 0.

$$\pi_x(u) = 0$$

Whereas, a possible event has possibility of 1.

$$\pi_x(u) = 1$$

In other words, setting π_x equal to 0 completely rules out x. Whereas, setting π_x equal to 1 just says that x is not ruled out. In the interval $(0, 1)$, π_x relates the possibility of events, where the possibility of some events is more possible than others. Given all the possible values, then according to the normalization condition, at least one is possible.

$$\exists u, \pi_x(u) = 1$$

Possibility theory captures complete knowledge by the following:

$$\exists u_0, \pi_x(u_0) = 1$$
$$\forall u \neq u_0, \pi_x(u) = 0.$$

And complete ignorance by:

$$\forall u, \pi_x(u) = 1.$$

Next, when given multiple sources of information, the principle of minimum specificity states the possibility distribution that accounts for these multiple sources is the least specific distribution. Thus, the joint possibility distribution $\pi_{x,y}$ is:

$$\pi_{x,y} = \min(\pi_x, \pi_y).$$

This is analogous to the principle of maximum entropy for probability theory. For example, given x can take on values $\{x_1, \ldots, x_n\}$, then without any knowledge on the probability or possibility of x, then

$$P(x_1) = \ldots = P(x_n) = \frac{1}{n}$$
$$\Pi(x_1) = \ldots = \Pi(x_n) = 1\,.$$

In contrast to probability measures, possibility measures are not additive. Possibility measures are subadditive:

$$\Pi(A \cup B) \leq \Pi(A) + \Pi(B).$$

Furthermore, a high possibility measure for an event does not imply a high probability measure, and a low probability measure does not imply a low possibility. A theoretical connection between probability and possibility exists [14]:

$$\forall x, P(x) \leq \Pi(x).$$

2.4.1.3 Evidence Theory. Evidence theory, which is often referred to as *Dempster-Shafer theory*, is based on a mass function or mass distribution function $m(\cdot)$. Given a frame of discernment, which is the set of hypotheses $\Theta = \{\theta_1, \ldots, \theta_n\}$, and its power set 2^Θ, which is the set of all possible subsets of Θ, a mass function assigns a measure of uncertainty to each member of the power set. Following are some properties of the mass function:

$$m : 2^\Theta \rightarrow [0, 1]$$
$$m(\emptyset) = 0$$
$$\sum_{A \subseteq \Theta} m(A) = 1.$$

From this mass function, a belief in a subset A of Θ can be defined as the sum of the basic belief masses that support A:

$$Bel(A) = \sum_{B \subseteq A} m(B).$$

Evidence theory can be thought of as a generalization of probability or possibility theory. Probability and possibility distributions have a corresponding mass assignment which gives a belief function. Furthermore, one can think of belief functions as a more tentative representation than probability distributions. In evidence theory, a mass can be assigned to a set of hypotheses and updated as further information is gathered.

There are multiple schools of interpretation of evidence theory. One school thinks of evidence theory as a generalization of Bayesian subjective probability. This school of thought believes evidence theory can be related to probability theory by the following:

$$\forall A, Bel(A) \leq P(A).$$

In other words, the belief mass assigned to a hypothesis can never exceed its probability.

The second school of thought rejects the idea that evidence theory is related to probability theory. This school of thought interprets evidence theory with the transferable belief model. For this model, given any subset A of Θ, the belief mass $m(A)$ is the amount of belief assigned to A. This mass cannot be assigned to any subset of A due to lack of knowledge. As evidence is obtained, subsets of A can be excluded, and the original mass assigned to can be transferred to the remaining part of A.

Finally, evidence theory can be used to accomplish decision analysis. There are two possible means of using evidence theory for decision analysis. The first uses the interval interpretation of evidence theory. This interpretation is that $Bel(A)$ is a lower bound on the probability of A and $Pl(A)$ is an upper bound for the probability of A, where $Pl(A)$ is the total belief mass that may ever be assigned to A (i.e., the extent to which A might be true). $Pl(A)$ is defined as the probability mass not supporting $\neg A$.

$$Pl(A) = 1 - Bel(\neg A)$$

The second uses the transferable belief model to convert a mass distribution to a probability distribution. [14]

2.4.1.4 Other Uncertainty Models. Following are a few lesser used uncertainty models.

Certainty Factors. Certainty factors, which have been used to handle uncertainty, are based on rules of the form

IF evidence E
THEN hypothesis H.

Each hypothesis is characterized by two measures of uncertainty, MB and MD. MB is the degree which the evidence supports the belief of the hypothesis, and MD is the degree which the evidence supports the disbelief of the hypothesis. Then the certainty factor is computed

$$CF = \frac{MB - MD}{1 - \min(MB, MD)}.$$

Although widely used, certainty factors are not without flaws. People have challenged the independence assumptions and the evidence updating ability of the model.

Fuzzy Sets. Fuzzy sets are a generalization of classical set theory. Fuzzy sets aim to represent sets in which the boundaries of the set are not clear. Parsons, in [14], uses the set of all animals as an example for fuzzy sets. Obviously, dogs and cats are animals, but it is less clear whether bacteria and viruses are animals.

Some work was accomplished by Bonissone, et al. [3] in the late 1980s for combining fuzzy sets with probabilities into fuzzy probabilities for use in a military application. These fuzzy probabilities were shown to be useful in accomplishing naval and aerial situation assessment, where the goal is to detect, track and identify targets. Although useful for this military application, the applicability to EBO is not obvious. In fact, Parsons states that fuzzy sets are used to model vague information not uncertain information. [14]

Rough Sets. Rough sets are another generalization of classical set theory. Rough sets model objects that cannot be categorized into one set or another. These objects possess the qualities of being in the set, but also the qualities of being outside of the set. Thus, Parsons states that rough sets can be used to model ambiguity. [14]

2.4.2 Uncertainty Computational Techniques. Having discussed the potential models for dealing with uncertainty, it is important to take a look at the techniques used to manipulate these measures in order to arrive at useful information. These techniques fall into two broad categories: quantitative and qualitative. The quantitative techniques require enumeration of all the required quantities; whereas, the qualitative techniques use either strictly qualitative data or a mixture of qualitative and quantitative data. Each type of technique has its benefits and drawbacks. The quantitative techniques maintain a high level of precision, but require vast amounts of data that is often times unattainable. On the other hand, the qualitative techniques require less specific data but are much less precise. In fact, the qualitative techniques can lead to no useful information being obtained due to lack of specificity. Thus, it is imperative that a technique is chosen that will result in truly useful and attainable data.

2.4.2.1 Quantitative Techniques. Quantitative techniques were developed to address the large number of probabilities required of probabilistic systems. For example, given a system composed of n variables, to fully state the relationships of the system, 2^n probabilities are required. For many real world systems where n is quite large, this becomes computationally prohibitive. Thus, several quantitative techniques were suggested in order to overcome the computationally intensive nature of probability theory.

Causal Networks. Causal networks reduce the number of required probabilities/-possibilities by only representing the linkages between variables that have an actual causal relationship. These relationships are presented in a network model where the nodes are the variables and the relationships between the variables are depicted with arcs. Thus, a node is only connected with those nodes that directly affect it.

Although useful for reducing the number of computations, causal networks still require that probabilities/possibilities be obtained for each of the relevant relationships. Thus, for EBO, where linkages are sometimes vague or unknown, causal networks are limited in their ability to provide attainable answers (at least from a quantitative perspective).

Valuation Networks. Valuation networks are graphical representations of value-based systems. These systems represent the entities of the system as variables and the relationship between them as valuations. These valuations can be probability distributions, possibility distributions or belief mass assignments. Once again, the difficulty in using valuation networks for EBO is in the development of the respective distributions for each of the interactions between variables. This is almost always not possible due to the uncertainties of war.

Influence Diagrams. Influence diagrams, like causal networks, are a graphical representation of the interactions between a set of variables. Influence diagrams were developed as a decision analysis tool. Originally, in order to solve influence diagrams, they needed to be transformed into some other form and then solved. However, computational methods have been developed that evaluate influence diagrams. Probability distributions for the chance nodes and value structures for the value nodes are required; both of which can be difficult to obtain for EBO.

2.4.2.2 Qualitative Methods. The major drawback to all of the quantitative methods is: "where are all the numbers coming from?" [14] This is definitely the major concern when considering the use of the quantitative methods for EBO. The possible solutions are qualitative methods that use either partial quantitative data or strictly qualitative information. These qualitative methods do not provide as much detail as the quantitative models, but arrive at more robust results with much less effort. [6]

Qualitative Reasoning. Qualitative reasoning uses the abstraction of real numbers into positive, negative, and zero valued quantities. This method has been used widely to assess highly complex systems such as digital circuits and the human body. Instead of trying to obtain numerical data regarding a system, qualitative reasoning aims to identify the interesting features of system behavior. This method is useful in assessing systems with complexity or ambiguity problems. The results are not very detailed and for EBO would likely be ineffective.

An extension to qualitative reasoning aimed at reducing the level of abstraction is order of magnitude systems. For these systems, the qualitative values are further divided into orders of magnitude (i.e., A is negligible with respect to B, or A has the same order of magnitude as B). Once again, for EBO purposes the system is too coarse to be of much use.

Finally, qualitative algebras also seek to reduce the level of abstraction. Qualitative algebras use more quantitative information combined with the qualitative information. The quantitative information is used as extensive as possible, then translated into qualitative values. This makes it possible to gain more information than strictly converting the quantitative values to qualitative values prior to reasoning. Yet again, though, for EBO this method does not provide the fidelity of solution needed for planning or analysis.

Interval-Based Systems. Interval-based systems aim to overcome a lack of knowledge by relaxing the requirement for point values. These intervals, whether they are probability or another measure, make the imprecision of information clear and intuitive. This makes them attractive for application to EBO. For interval-based systems, the upper probability of a hypothesis A is defined among the set S of probabilities of A as:

$$P^*(A) = \sup_{P \in S} P(A)$$

and the lower bound is

$$P_*(A) = \inf_{P \in S} P(A).$$

Thus, if A and B are disjoint events

$$P^*(A \cup B) \leq P^*(A) + P^*(B)$$
$$P_*(A \cup B) \geq P_*(A) + P_*(B).$$

A specific approach of interval probabilities that may prove effective for EBO is the use of interval probabilities to depict the linguistic quantifiers provided by intelligence assessments. For example, the intelligence assessments of *probable* or *possible* could be mapped to the intervals $[a, b]$ and $[b, c]$, where $0 < a \leq b \leq c < 1$.

Abstraction of Quantitative Systems. Another technique that may be effective in EBO planning and assessment is Wellman's work using qualitative probabilistic networks (QPN) for decision analysis. QPNs are constructed based on influence diagrams and Bayesian belief networks. The difference is that the relationships among the variables are qualitative influences and synergies rather than precise probabilities, which are, as already stated, hard to obtain. [20] [6]

These qualitative influences and synergies make QPNs attractive for EBO. For example, given a desired effect, there may be several possible actions that can be taken to obtain that effect. The exact relation among the actions and effects will most likely not be attainable. However, a planner or intelligence analyst might be able to place an influence, either positive, negative, or zero, on the action-effect relationship. In addition, when multiple actions are considered, there might be synergies associated with the multiple actions. These also are addressed in QPNs.

A major drawback to QPNs is that in the basic model the variables are binary (i.e., two possible states). As variables with more than two values are considered, the concept becomes more complex. To account for this, Wellman suggests thinking of the multiple values of a variable as two sets of *higher* and *lower* values, where *higher* and *lower* are based on some defined order of the values. [14]

Defeasible Reasoning. Defeasible reasoning is reasoning where new information can invalidate old conclusions. Some times it is necessary to make hypotheses to account for incomplete information. As new information is obtained the hypotheses are retracted. In doing this, the problem of inconsistent information is added to the original problem of incomplete information. These methods are primarily used for systems which the incomplete information is known to exist. [14] In EBO, there is incomplete knowledge of how uncertain the information is. Thus, defeasible reasoning would most likely not be effective for EBO planning and assessment.

2.5 Measuring Uncertainty

In order to maximize information gain through intelligence asset prioritization, it is necessary to measure the uncertainty present in the system. Once a distribution has been assigned to the current state of the system (or subsystem), a measure of uncertainty for the distribution is needed. The uncertainty community has used two measurements of uncertainty to quantify the uncertainty associated with probability and possibility distributions. Shannon's entropy is the commonly used measure of uncertainty used for probabilities, and specificity is used to measure the uncertainty for possibility distributions. The entropy of a random variable measures its complexity, or degree of randomness. Given a higher entropy of a random variable, it is harder to predict the value of the random variable. [7] Specificity is an extension of Shannon's entropy to possibility distributions.

2.5.1 Shannon's Entropy. Shannon's entropy was developed in 1948 by Claude E. Shannon during his research on information theory. Shannon was concerned with the amount of information, or bandwidth, that could be passed by communication systems. Shannon's work was the basis for much of the work on data compression used in today's internet. Shannon developed a logarithmic measure to quantify the uncertainty associated with a discrete channel communication system represented by a Markov process. Following is Shannon's entropy for discrete probability distributions:

$$H = -K \sum_{i=1}^{n} p_i \log_2 p_i \tag{2.1}$$

where K is a positive constant and p_i are transition probabilities from the current state to the n possible states. Shannon also developed a continuous measure for use with signals or messages that are continuously variable. Following is Shannon's entropy for continuous probability distributions:

$$H = -\int \cdots \int p\left(x_1, \ldots, x_n\right) \log_2 p\left(x_1, \ldots, x_n\right) dx_1 \cdots dx_n. \tag{2.2}$$

2.5.1.1 Shannon's Entropy - Discrete Distributions. Suppose we are given n possible discrete events with known probabilities p_1, p_2, ... , p_n. Shannon states that if a measure of uncertainty, H, exists for the n events then it must have the following properties. [16]

1. H should be continuous in the p_i.
2. If all the p_i are equal, $p_i = \frac{1}{n}$, then H should be a monotonic increasing function of n. With equally likely events there is more choice, or uncertainty, when there are more possible events.
3. If a choice be broken down into two successive choices (see Figure 2.3 on the following page), the original H should be the weighted sum of the individual values of H...
$H\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right) = H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{1}{2} H\left(\frac{2}{3}, \frac{1}{3}\right)$.

Shannon presents the following result with proof. [16]

> Theorem: The only H satisfying the three above assumptions is of the form:
>
> $$H = -K \sum_{i=1}^{n} p_i \log_2 p_i$$

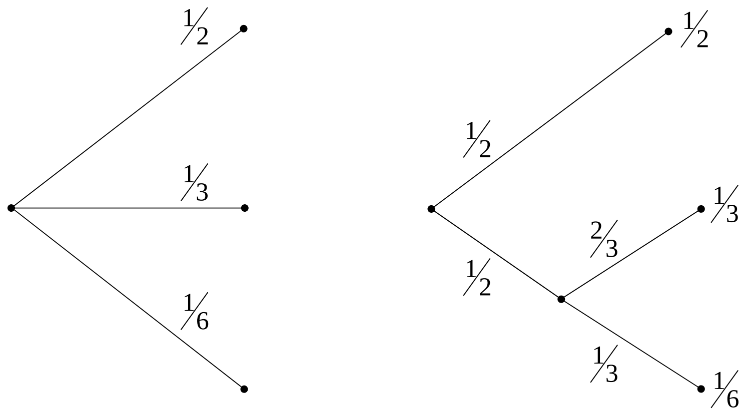

Figure 2.3: Entropy for Multiple Decision Points

where K is a positive constant.

The constant K is a matter of choice and is simply a unit of measure. K can be any positive value and is chosen based on application. For example, if one wanted to normalize the entropy measure, you could divide by the maximum possible entropy [19]

$$K = \frac{1}{\log_2 n}.$$

This results in a measure called *relative entropy.* [16] By normalizing the entropy, the effect of the number of possible states, n, is removed. For this thesis, K is assumed to equal $\frac{1}{\log_2 n}$ due to the need to compare the entropy of nodes with different values of n (i.e., with different number of states). For example, given a SoSA network where the most important node is represented by two states and the least important node is represented by three states, if no information is known regarding the current state of each node, then the Shannon entropy of each (using Equation (2.1) with $K = 1$) is

$$H_{\text{most important}} = -\left(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}\right) = 1$$

and

$$H_{\text{least important}} = -\left(\frac{1}{3}\log_2\frac{1}{3} + \frac{1}{3}\log_2\frac{1}{3} + \frac{1}{3}\log_2\frac{1}{3}\right) = 1.585.$$

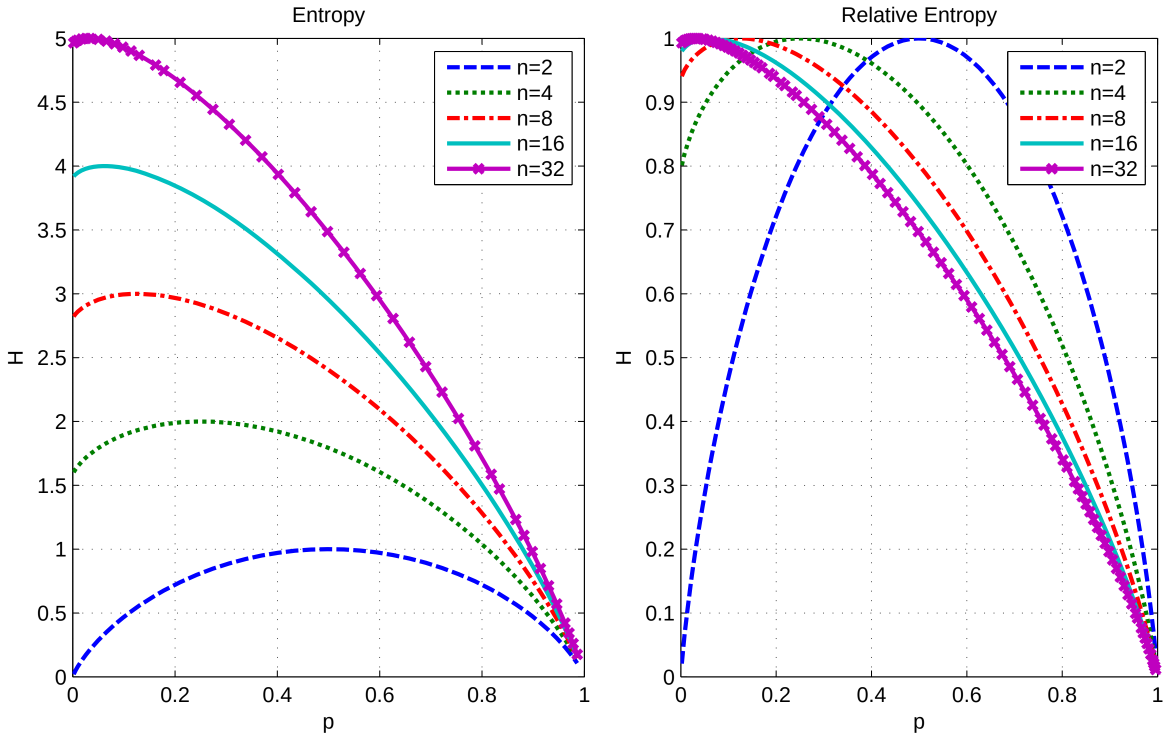

Figure 2.4: Entropy ($K = 1$) Versus Relative Entropy ($K = \frac{1}{\log_2 n}$)

If a node is then chosen for intelligence collection, one would choose the more uncertain node–the one with the highest entropy. In this case, it would be the least important node.

The effect of n is depicted in Figure 2.4. On the left side of the figure is a plot of (discrete) probability p versus entropy for various values of n. A system achieves its highest entropy value at $p = \frac{1}{n}$; however, this highest value does not remain constant, but grows logarithmically with n. On the right side of Figure 2.4, a plot of p versus relative (or normalized) entropy for various values of n is given. Again, the system achieves its highest entropy value at $p = \frac{1}{n}$, but this value is a constant of one regardless of the possible number of states, n.

The base of the logarithm corresponds to the choice of a unit for the measurement. Base 2 corresponds to binary digits, or bits. Base 2 relates the number of bits required to store or transmit the information. Using base 10 gives the entropy

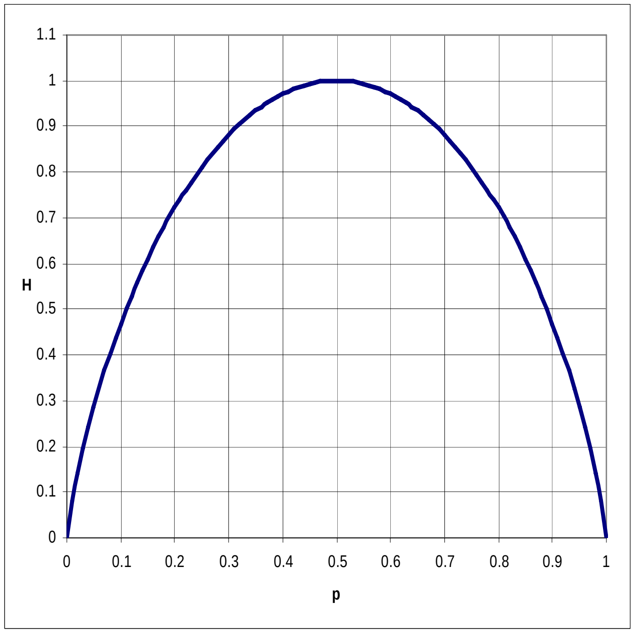

Figure 2.5: Entropy, H, as a Function of p For the Binary Case [16]

in decimal digits. In the uncertainty community, base 2 (bits) is predominantly used (presumably due to the community's close ties to information theory).

Consider the binary system where two mutually exclusive events are possible, with probabilities p and $q = 1 - p$. Thus, the entropy of the system is

$$H = -\left(p \log_2 p + q \log_2 q\right).$$

The entropy for this system is plotted as a function of p in Figure 2.5. Note, the maximum entropy is attained when $p = q = 0.5$. This corresponds to the values of p and q for which the least information (or maximum uncertainty) is known regarding the outcome of the system. Furthermore, minimum entropy is attained when either event is known to occur with probability 1, $p = 1$ or $q = 1$. This corresponds to perfect information (or no uncertainty) regarding the outcome of the system.

Next, consider the case where there are eight possible mutually exclusive events. Given a node, n_i, let N_i be the state of n_i. N_i is a discrete random variable defined

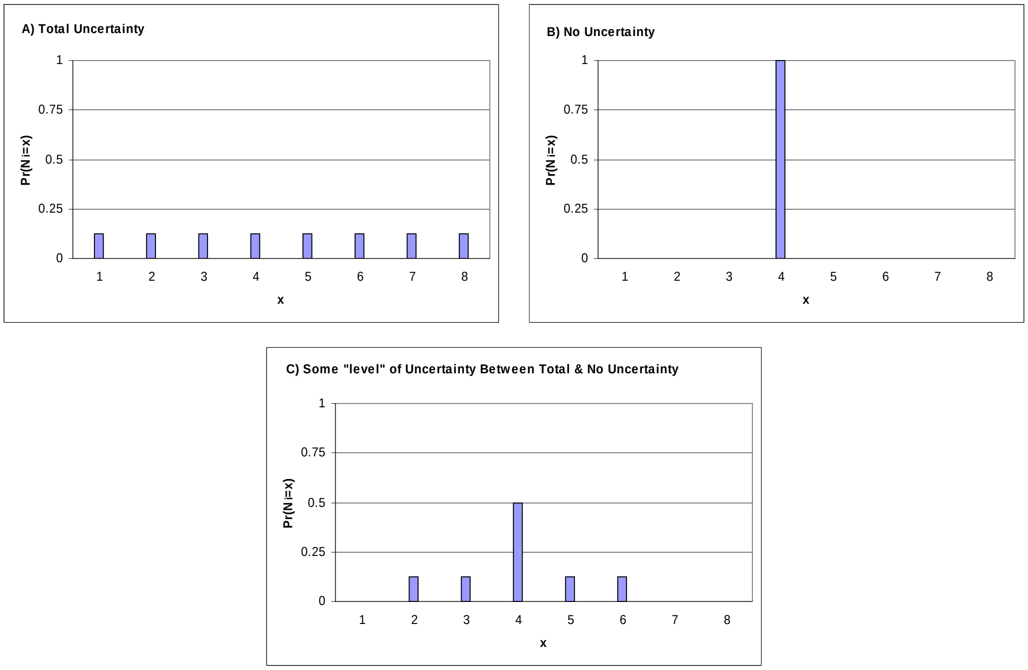

Figure 2.6: Discrete Distributions with Varying Uncertainties

on a finite set with an associated probability distribution, p_i, where $p_i(x) = P(N_i = x)$. Under total uncertainty, one cannot say that any one state is more probable than another state. Thus, based on the principle of maximum entropy, one has to assume a uniform distribution for the eight discrete states. [14] No uncertainty, or total certainty, implies that the random variable is known to equal a single state. Thus, all $p_i(x)$ equal zero except for one, which equals one. In between total uncertainty and total certainty, Shannon's relative entropy results in a measure of the varying *levels* of uncertainty.

To illustrate, consider the distributions presented in Figure 2.6. Now, using Shannon's relative entropy for discrete probability distributions (Equation (2.1)), we arrive at a measure for the uncertainty of each of these distributions. For A, the total uncertainty case, the relative entropy is:

$$\begin{aligned} H(x) &= -\frac{1}{\log_2 8} \sum_x p_i(x) \log_2 p_i(x) \\ &= -\frac{1}{3} [0.125 \times 8 (\log_2 0.125)] \\ &= 1. \end{aligned}$$

For B, the no certainty case, the relative entropy is:

$$\begin{aligned} H(x) &= -\frac{1}{\log_2 8} \sum_x p_i(x) \log_2 p_i(x) \\ &= -\frac{1}{3} [1 (\log_2 1)] \\ &= 0 \end{aligned}$$

and for C, the case in between total and no uncertainty, the relative entropy is:

$$\begin{aligned} H(x) &= -\frac{1}{\log_2 8} \sum_x p_i(x) \log_2 p_i(x) \\ &= -\frac{1}{3} [0.125 \times 4 (\log_2 0.125) + 0.5 (\log_2 0.5)] \\ &= 0.667. \end{aligned}$$

Entropy has some interesting properties that further substantiate it as a valid measure of uncertainty. [16]

1. H is not dependent upon the values of X. It is only dependent upon the probabilities, $P(X = x)$.

2. $H = 0$ if and only if all p_i equal 0 except one, which equals 1. Thus, only when we are certain of the outcome does $H = 0$. Otherwise, $H > 0$.

$$\begin{aligned} H &= -\sum_{i=1}^{n} p_i \log_2 p_i \\ &= -1 \log_2 (1) \\ &= 0 \end{aligned}$$

3. For a given n, H is maximized and equal to $\log_2 n$ when all p_i are equal (i.e., $p_i = \frac{1}{n}$).

$$\begin{aligned} H &= -\sum_{i=1}^{n} p_i \log_2 p_i \\ &= -n \left({}^{1}\!/\!_{n} \right) \log_2 \left({}^{1}\!/\!_{n} \right) \\ &= -\log_2 \left({}^{1}\!/\!_{n} \right) \\ &= \log_2 n \end{aligned}$$

Thus, the maximum H increases as the possible number of events increases. This is intuitive as the number of possible events increases so too does the uncertainty associated with the system.

4. Given two events, x and y. The entropy of the joint event is

$$H(x, y) = -\sum_{i,j} p(i, j) \log_2 p(i, j)$$

where

$$\begin{aligned} H(x) &= -\sum_{i,j} p(i, j) \log_2 \sum_{j} p(i, j) \\ H(y) &= -\sum_{i,j} p(i, j) \log_2 \sum_{i} p(i, j). \end{aligned}$$

Thus, the entropy of the joint event is less than or equal to the sum of the individual entropies.

$$H(x, y) \leq H(x) + H(y).$$

If the events x and y are independent, then

$$H(x, y) = H(x) + H(y).$$

5. Any change toward equalization of the probabilities $p_1, \ldots, p_n$ increases H. This property is demonstrated for the binary case in Figure 2.5 on page 29, and can be easily shown for other n.

2.6 Linguistic Quantifiers

In EBO, it is necessary to develop a methodology for describing the current states of the individual nodes and the overall network. The sources for characterizing the current states of the nodes and the network are intelligence data. This data is often times not exact, and as such contains much uncertainty. If an intelligence analyst provides a probability for the current state, then these probabilities can be plugged directly into the network. However, if an intelligence analyst provides linguistic quantifiers, then assigning a probability, possibility or belief to the current state of a node can be accomplished several different ways. However, for the goal of quantifying the uncertainty present in the assessment, some techniques are more acceptable than others.

Intelligence data usually makes use of linguistic quantifiers, or "words of estimative probability." For example, an intelligence analyst may give the precise location of an airfield, but may provide a judgement or estimate of the use of the airfield (i.e., "It is almost certainly a *military* airfield"). It is this linguistic estimate that is the source of the probabilistic uncertainty. [11]

Table 2.1: Sherman Kent's Words of Estimative Probability [11]

Possibility or Likelihood Terms	Synonyms
Possible	conceivable could may might perhaps
Almost Certain	virtually certain all but certain highly probable highly likely odds [or chances] overwhelming
Probable	likely we believe we estimate
50-50	chances about even chances a little better [or less] than even improbable unlikely
Probably Not	we believe that . . . not we estimate that . . . not we doubt, doubtful
Almost Certainly Not	virtually impossible almost impossible some slight chance highly doubtful

Kent [11] provides a list of linguistic quantifiers commonly used by intelligence analysts. This list is definitely not all inclusive but provides a good unclassified framework. Kent's list of synonyms is presented in Table 2.1. Kent also presents a suggestion for translating these linguistic quantifiers to probabilities (see Table 2.2 on the following page).

In dealing with Kent's interval probabilities, the idea of maximum entropy is quite useful. The principle of maximum entropy can be used to obtain a point estimate for a probability interval. If given a probability interval, $[a, b]$, for an event for which the distribution across the interval is unknown, the principle of maximum

Table 2.2: Kent's Probabilities Associated with Linguistic Quantifiers [11]

Probability	Linguistic Quantifier
100 %	Certainty
93 % ± 6 %	Almost certain
75 % ± 12 %	Probable
50 % ± 10 %	Chances about even
30 % ± 10 %	Probably not
7 % ± 5 %	Almost certainly not
0%	Impossibility

entropy distributes the probabilities equally across the interval (i.e., uniform distribution). According to Jaynes, this is the "least biased estimate possible based on the given information." [9] Thus, no single probability is more favored than any other. Therefore, the expected value for the interval, based on the principle of maximum entropy, is the midpoint of the interval $1/2 \cdot (a + b)$. [14]

This thesis uses the midpoint of Kent's interval probabilities as a point estimate for the discrete probabilities assigned to the linguistic quantifiers. As an example, given an eight-state node, the probability distribution for *Probably Not in state 5* is depicted in Figure 2.7 on the next page.

$$P(N_i = \neg 5) = 0.70$$

2.7 Generalized Assignment Problem

The allocation of intelligence collection efforts from multiple intelligence sources against multiple nodes can be related to the integer program (IP) of assigning multiple resources to multiple containers, or knapsacks–the multiple knapsack problem. The multiple knapsack problem takes the form of m containers with c_i capacities and n items with profits p_j. It makes use of a binary indicator variable x_{ij}, that takes value 1 when item j is placed in container i and 0 otherwise. The formulation of the problem

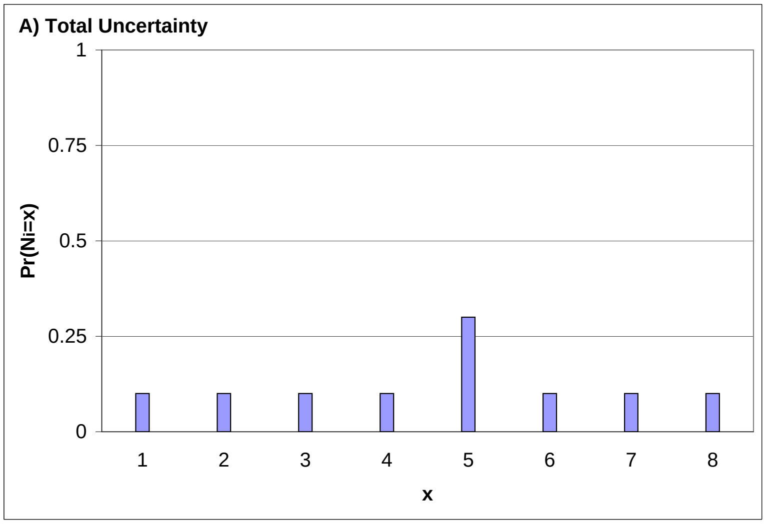

Figure 2.7: Probably Not in State 5

is

$$\text{maximize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} p_j x_{ij} \tag{2.3}$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_j x_{ij} \leq c_i, \qquad i = 1, \ldots, m, \tag{2.4}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \qquad j = 1, \ldots, n, \tag{2.5}$$

$$x_{ij} = 0 \text{ or } 1, \qquad i = 1, \ldots, m, j = 1, \ldots, n. \tag{2.6}$$

Equation (2.4) are the capacity constraints for the knapsacks, and Equation (2.5) ensures that item j is only added to a knapsack once.

In the objective function, Equation (2.3), the profit p_j is the profit (or benefit) obtained by adding item j to a knapsack. For the multiple knapsack problem, this value is a constant independent of the knapsack selected. For application to the intelligence asset allocation problem, the information gained (i.e., benefit) is dependent upon the intelligence asset selected to collect the information. Therefore, the GAP must be used to solve the allocation problem. Strictly speaking, the GAP is not a knapsack problem, but the algorithms used to solve it use the knapsack subproblems. [12]

The GAP can be formulated using the terminology of knapsack problems. Given n items and m knapsacks, with

$$\begin{aligned} p_{ij} &= \mathit{profit} \text{ of item } j \text{ if assigned to knapsack } i, \\ w_{ij} &= \mathit{weight} \text{ of item } j \text{ if assigned to knapsack } i, \\ c_i &= \mathit{capacity} \text{ of knapsack } i, \end{aligned}$$

maximize the total profit by assigning each item to at most one knapsack without assigning to any knapsack more than its total capacity. [12]

$$\text{maximize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} x_{ij} \tag{2.7}$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_{ij} x_{ij} \leq c_i, \qquad i = 1, \ldots, m, \tag{2.8}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \qquad j = 1, \ldots, n, \tag{2.9}$$

$$x_{ij} = 0 \text{ or } 1, \qquad i = 1, \ldots, m, j = 1, \ldots, n. \tag{2.10}$$

2.8 Chapter 2 Review

This chapter provided a review of the current literature on EBO, knowledge uncertainty, linguistic quantifiers, methods for handling uncertainty, uncertainty measures, and the GAP. In Chapter 3, probability theory is used to represent the uncertainty associated with the nodes of the SoSA network. This uncertainty is measured with Shannon's relative entropy, and the GAP is used to allocate intelligence resources to the nodes of the SoSA network with the goal of maximizing the information gain.

III. Methodology

3.1 Overview

This chapter presents a mathematical model for System-of-systems Analysis (SoSA) networks, a methodology for prioritizing a single intelligence asset's collection efforts, and a methodology for allocating multiple intelligence assets. The SoSA mathematical model is intended as a general state model which may be used for other applications dealing with SoSA. It defines notation and terminology and forms the foundation for the later methodologies for prioritizing and allocating intelligence collection efforts. The methodology for prioritizing a single asset's intelligence collection efforts uses Shannon's entropy [16] to measure the current uncertainty of the nodal states. This uncertainty is based on the latest intelligence assessments, which are typically provided using linguistic quantifiers, and the planned actions against the nodes. The expected reduction in entropy due to an intelligence collection effort is then calculated. Finally, intelligence collection efforts are selected based on the greatest possible reduction in uncertainty. This methodology is then extended to the allocation of multiple intelligence assets. A generalized assignment problem is used to maximize the amount of information gain from a limited number of intelligence collection opportunities.

3.2 SoSA Network Mathematical Model

According to the Joint Warfighting Center (JWFC) Pamphlet 7 on Effects-based Operations (EBO), a SoSA is accomplished in order to gain further understanding and situational awareness of an adversary and the operational environment. To accomplish a SoSA, the operational environment is assumed to be composed of political, military, economic, social, infrastructure, and information (PMESII) entities (or systems). These six interrelated PMESII systems are represented pictorially using a multi-dimensional network. Each node within the network represents a person, place, or physical thing that is a fundamental component of the system. The linkages connecting the nodes represent the behavioral, physical, or functional relationships

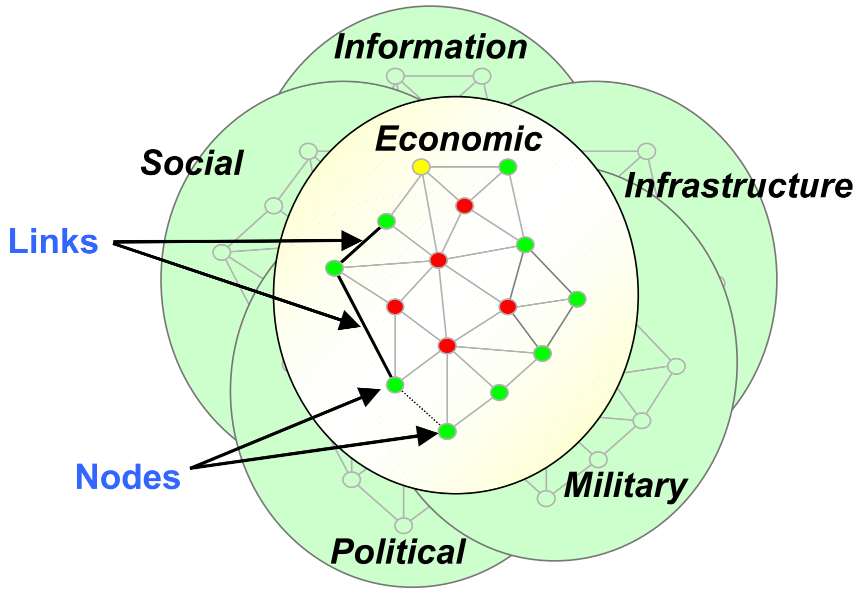

Figure 3.1: Systems-of-systems Analysis [10]

between the nodes. By developing an interrelated network of a region or nation of interest, planners aim to take a holistic view of the operational environment. See Figure 3.1 for a pictorial representation of a SoSA network. [10]

In order to mathematically represent the SoSA network, the following notation is introduced. Let *node* i represent the smallest, non-decomposable entity or node in the network (or at least at the highest resolution that one intends to represent the network). Let n_i denote the node, where i is a unique identifier for the node ($i \in \mathbb{N}^+$). Thus a SoSA network composed of k entities can be represented by the set of nodes, $\mathcal{N}$.

$$\mathcal{N} = \{n_1, n_2, \ldots, n_k\}$$

However, to fully characterize the network we must describe the current state of the k nodes, as well as the linkages and possible relationships among the states of the nodes. To describe the current state of a node, let N_i be defined as the current state of n_i. N_i may be composed of multiple characteristics or qualities associated with node n_i. Thus, for a node with m attributes, N_i is a vector of current attributes.

$$\mathbf{N_i} = \left[N_i^{(1)}, N_i^{(2)}, \ldots, N_i^{(m)}\right]$$

For EBO, these attributes will usually be assessed using linguistic quantifiers provided from intelligence (e.g., an enemy system could be defined as *operational*, *degraded*, *nonoperational*, etc.).

For example, a node, n_a, may possess physical, functional, and behavioral attributes. Then the current state is

$$N_a = \left[N_a^{(1)}, N_a^{(2)}, N_a^{(3)}\right]$$

where, $N_a^{(1)}$ is the physical state of the node, $N_a^{(2)}$ is the functional state of the node, and $N_a^{(3)}$ is the behavioral state of the node.

Due to possessing multiple attributes, each node can be in any one of u unique states, where

$$u = \prod_{z=1}^{m} \left[\# \text{ of possible linguistic quantifiers for } N_a^{(z)}\right].$$

Let $\mathcal{U}_i$ be the set of all discrete states for n_i.

Back to our example, assume each of the three attributes can be rated as follows

$$N_a^{(z)} = \begin{cases} 0 & \text{, if nonactive} \\ 1 & \text{, if active.} \end{cases}$$

Then n_a can be in any one of $2^3 = 8$ unique states.

For a SoSA network composed of k nodes, let $\mathcal{C}$ denote the set of current state vectors for the k nodes within the network ($\mathcal{N}$).

$$\mathcal{C} = \{N_1, N_2, \ldots, N_k\}$$

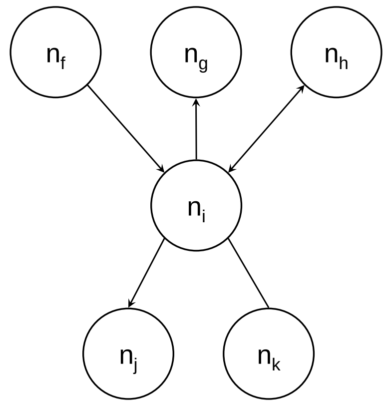

Figure 3.2: Multiple parent and child nodes

At this point, it must be noted that this notation describes a static network. If one is considering a dynamic network, the notation can easily be adapted to account for time. To accomplish this, let $N_i(t)$ be the state vector for node n_i at time t.

Next, notation is introduced for the linkages between nodes. Given two nodes, n_i and n_j, the linkage, l_{ij}, between states N_i and N_j denotes the existence of a relationship between the node states. This linkage, l_{ij}, can be unknown or directed. The unknown linkage arises when a relationship between the states N_i and N_j is known to exist, but quantifying, or even qualifying, the relation is not possible. The directed linkage exists when the state N_j is dependent upon the state N_i. For the directed linkage, the independent node is referred to as the parent node, and the dependent node is referred to as the child node. [5] [Note, the parent/child terminology only depicts a directed relation between two nodes. For EBO, no hierarchy or ownership is implied by the terminology. To say a node n_i is a parent to node n_j only indicates the state N_i has an influence on the state N_j.] A node may possess one or more parent or child nodes (see Figure 3.2). Thus, for any two nodes, n_i and n_j, $i \neq j$, there exists no relationship, an unknown relationship, or a directed relationship between their associated states, N_i and N_j. See Figure 3.3 on the following page.

To complete the directed graph terminology, the terms directed path, ancestor, and descendant are introduced. A directed path exists between two nodes, n_i and n_k,

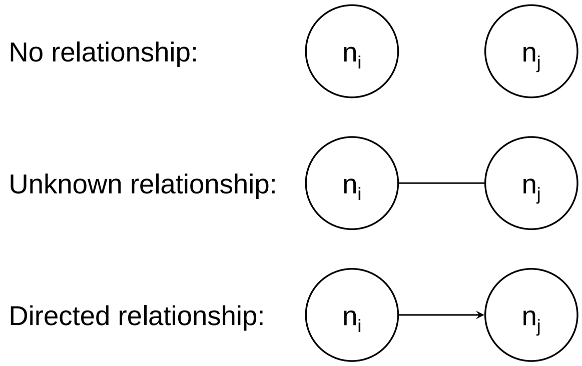

Figure 3.3: Possible linkages between n_i and n_j

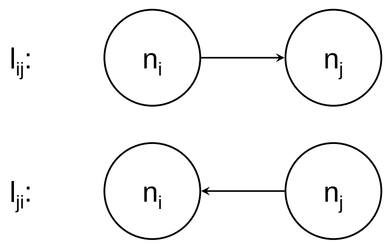

Figure 3.4: Linkage between n_i and n_j

if there exists directed linkages from node n_i to node n_k. For example, in Figure 3.2 on the page before, a directed path exists from node n_f to node n_j (i.e., l_{fi} from node n_f to node n_i and l_{ij} from node n_i to node n_j). If a directed path exists from node n_f to n_j, then node n_f is called an ancestor of node n_j, and n_j is referred to as a descendant of node n_f. [21]

It is important to note that linkages can occur in both directions between two nodes, n_i and n_j. In other words, there may exist an interdependence between the state N_i and the state N_j. To account for this, let l_{ij} denote the relationship from N_i to N_j, and l_{ji} denote the relationship from N_j to N_i (see Figure 3.4). If no relationship exists between the states of two nodes, then $l_{ij} = l_{ji} = \mathbf{0}$.

Thus,

$$l_{ij} = \begin{cases} 0 & \text{, if no relation exists between } n_i \text{ and } n_j \\ 1 & \text{, if a directed relation exists between } n_i \text{ and } n_j \\ [?] & \text{, if an unknown relation exists between } n_i \text{ and } n_j. \end{cases}$$

For a SoSA network composed of k nodes, let $\mathcal{L}$ denote the set of non-zero linkages between the k nodes. In other words, $\mathcal{L}$ denotes the set of all existing linkages between the k nodes.

$$\mathcal{L} = \{l_{ij} : l_{ij} \neq \mathbf{0}, i, j \in [1, k], i \neq j\}$$

$\mathcal{L}$ can also be represented using a $k \times k$ adjacency matrix, where the elements of the matrix, l_{ij}, represent linkages from node n_i to node n_j. In other words, the rows of $\mathcal{L}$ represent the parent nodes, and the columns represent the child nodes. For example, the adjacency matrix for Figure 3.2 on page 41 is

$$\mathcal{L} = \begin{bmatrix} 0 & l_{fg} & l_{fh} & l_{fi} & l_{fj} & l_{fk} \\ l_{gf} & 0 & l_{gh} & l_{gi} & l_{gj} & l_{gk} \\ l_{hf} & l_{hg} & 0 & l_{hi} & l_{hj} & l_{hk} \\ l_{if} & l_{ig} & l_{ih} & 0 & l_{ij} & l_{ik} \\ l_{jf} & l_{jg} & l_{jh} & l_{ji} & 0 & l_{jk} \\ l_{kf} & l_{kg} & l_{kh} & l_{ki} & l_{kj} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & [?] \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & [?] & 0 & 0 \end{bmatrix}.$$

Note, due to the unknown linkage between node n_i and node n_k, $l_{ik} = l_{ki} = [?]$. This holds for any unknown linkage.

Using the adjacency matrix, one can easily identify the linkages associated with a node. For example, *row* 1 indicates node n_f is a parent only to node n_i, and *column* 1 indicates n_f is not a child to any node.

Each existing linkage, $l_{ij} \neq \mathbf{0}$, may possess a function (or mapping) of the state of n_i, N_i, to the state of n_j, N_j. Let L_{ij} denote this function (or mapping).

$$L_{ij}(N_i) = N_j$$

L_{ij} may be quantitative or qualitative. A quantitative function (or mapping) exists if numerical data exist to relate the state N_i to the state N_j. In the absence of complete numerical data (i.e., partial numerical data or complete lack of numerical data), a qualitative mapping may exist. This mapping relates the state N_i to the state N_j by qualitatively describing the effect the state N_i has on state N_j. For example, given a positive (+) change in the state N_i, L_{ij} may qualitatively relate this change to a positive (+) or negative (−) change in the state N_j. Without more information, nothing more can be stated regarding the change in state N_j based on a change in state N_i. For the unknown linkage between the states N_i and N_j, L_{ij} returns an unknown relation. [14]

$$L_{ij}(N_i) = [?]$$

Taking a closer look at the linkages between the states of nodes, one can see that some attributes of a node may be more influential on child attributes than others. Whereas some attributes may have no influence on the state of the child node. To account for this, L_{ij} may be composed of individual attribute functions (or mappings) from parent attributes to child attributes.

$$L_{ij}\left(N_i\right) = \begin{bmatrix} L_{ij}^{(1)}\left(N_i^{(1)}\right) \\ L_{ij}^{(2)}\left(N_i^{(2)}\right) \\ \vdots \\ L_{ij}^{(m)}\left(N_i^{(m)}\right) \end{bmatrix}$$

Next, because each node, n_j, may possess more than one parent and thus more than one state mapped to its state, N_j, notation must be introduced to relate the p_j possible linkages, where p_j is defined as the number of parents to n_j. See Figure 3.5 on the following page. Also, it is possible there exist linkages between the p_j parent nodes. Thus, any mapping (or function) relating the p_j parent nodes to the state N_j must also take into account the linkages between the states of the parent nodes. Let f_j be the function that relates all the current states of the p_j parent nodes and the linkages among the parent nodes to the current state N_j.

$$N_j = f_j\left[N_d, N_e, \ldots, N_r, L_{de}, L_{df}, \ldots\right]$$

For a SoSA network composed of k nodes, let $\mathcal{F}$ denote the set of functions (or mappings) associated with the non-zero linkages between the k nodes.

$$\mathcal{F} = \left\{L_{ij}, f_j : l_{ij} \neq \mathbf{0}, i, j \in [1, k], i \neq j\right\}$$

Thus, an entire SoSA network, $\mathcal{A}$, is completely characterized by the nodes ($\mathcal{N}$), the current state of the nodes ($\mathcal{C}$), the linkages between the nodes ($\mathcal{L}$), and the

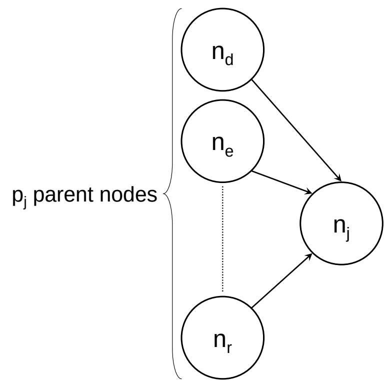

Figure 3.5: Node n_j with p_j parents

functions relating the states of the nodes ($\mathcal{F}$).

$$\mathcal{A} = \{\mathcal{N}, \mathcal{S}, \mathcal{L}, \mathcal{F}\}$$

In addition, the graphical model of the SoSA network, $\mathcal{G}$, can be entirely represented using $\mathcal{N}$ and $\mathcal{L}$

$$\mathcal{G} = (\mathcal{N}, \mathcal{L})$$

and the state model, $\mathcal{S}$, can be entirely represented using $\mathcal{C}$ and $\mathcal{F}$.

$$\mathcal{S} = (\mathcal{C}, \mathcal{F})$$

To allow planners to abstract or aggregate the network to the appropriately needed level, the concept of a system is introduced. A system, denoted $n_{[i]}$, is defined as a non-empty set of nodes and/or systems, current state vectors, linkages, and functions, which acts as a single entity.

$$n_{[i]} = \{\mathcal{N}', \mathcal{C}', \mathcal{L}', \mathcal{F}'\}$$

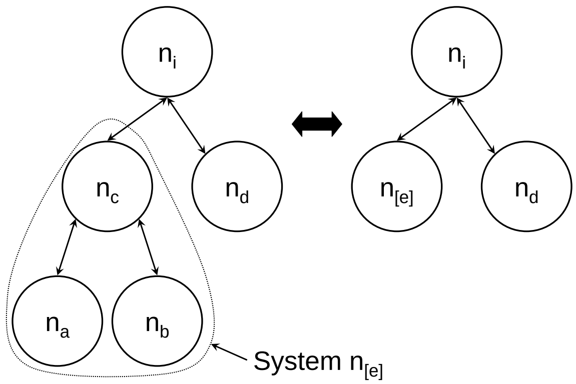

Figure 3.6: System notation

where, $\mathcal{N}' \subseteq \mathcal{N}$, $\mathcal{C}' \subseteq \mathcal{C}$, $\mathcal{L}' \subseteq \mathcal{L}$, $\mathcal{F}' \subseteq \mathcal{F}$. This notation allows one to represent complex systems as single nodes. In Figure 3.6, n_a, n_b, n_c, and their associated linkages are replaced with $n_{[e]}$.

As an example of a system, consider a surface-to-air missile (SAM) battalion within an enemy integrated air defense system (IADS). Planners may not desire to or may not be able to assess the current states of individual components of the SAM battalion (e.g., acquisition radar, tracking radar, command vehicle, transporter erector launcher (TEL). Thus, to represent the battalion within the network, the individual components, as well as their associated linkages, are encompassed in a system node, $n_{[i]}$. Planners can now define effects and actions in terms of the system, $n_{[i]}$, instead of each of the individual components.

3.3 Prioritizing A Single Intelligence Asset's Collection Efforts Based on Nodal Uncertainty

Given a mathematical model of the SoSA network, a methodology for prioritizing a single intelligence asset's collection efforts is developed. At first brush, one might decide to simply choose the most uncertain node (or nodes) (i.e., highest entropy) and task an asset to collect intelligence on it. However, the possible reduction in entropy due to an intelligence collection effort is dependent upon the intelligence asset available as well as the target node. For example, imagery intelligence (IMINT) may be very effective in assessing the current state of a tank or Army brigade but may provide little to no information on the current state of a political party or regime. Thus, any model developed must account for the dependence between an intelligence asset and target types. Presented below is a model that maximizes the information gain (i.e., the reduction in entropy) while accounting for the dependence between assets and targets.

3.3.1 Overall Model. The overall approach to prioritizing the intelligence (information) collection effort is:

1. Based upon current intelligence assessments, calculate the uncertainty associated with each node within the network, $n_i \in \mathcal{N}$, using Shannon's entropy measure, h_i.

$$h_i = -K_i \sum_{l=1}^{u_i} p_{i,l}'' \log_2(p_{i,l}'') \tag{3.1}$$

   where,

$$K_i = \frac{1}{\log_2(u_i)}$$

   $p_{i,l}''$ = the probability that node i is in state l, $N_i = l$, adjusted for the *freshness* of the intelligence update and planned actions on n_i.

2. Next, based on the available intelligence asset, the expected entropy given an intelligence information collection effort on each node, $E[h_i']$, is calculated. For these calculations, the states of the nodes are assumed to be independent. The expected change in entropy for node n_i, δ_i, is

$$\delta_i = \rho_i (h_i - E[h_i']) \tag{3.2}$$

Table 3.1: Kent's Probabilities Associated with Linguistic Quantifiers [11]

Probability	Linguistic Quantifier
100 %	Certainty
93 % ± 6 %	Almost certain
75 % ± 12 %	Probable
50 % ± 10 %	Chances about even
30 % ± 10 %	Probably not
7 % ± 5 %	Almost certainly not
0%	Impossibility

where ρ_i is a weighting factor for n_i based on the previously determined nodal priority (i.e., *Priority I*, *Priority II*, etc.). Refer to Section 3.3.2 for a discussion on nodal priorities.

3. Finally, intelligence collection efforts are selected based on the maximum δ_i.

Given a m-node SoSA network, let p_i be defined as the prior probability distribution of the state of node i, N_i. If n_i can be in any one of u discrete states, then

$$p_i = \begin{bmatrix} p_{i,1} \\ p_{i,2} \\ \vdots \\ p_{i,u} \end{bmatrix}.$$

This prior probability distribution is based upon the latest intelligence assessment on n_i. Using Kent's [11] mapping of linguistic quantifiers to probabilities (see Table 3.1), discrete probability distributions are assigned to the states of the nodes. This thesis uses the midpoint of Kent's intervals as point estimates for the linguistic quantifiers. If an intelligence assessment only provides information on a single state of the node (i.e., N_i is *almost certainly* in state 0), Kent's mapping is used to assign a probability mass to that state. Because no further information is given regarding the remaining states, the unassigned probability mass is evenly distributed to the remaining states based on the principle of maximum entropy.

This prior probability distribution is then adjusted based on the *freshness* of the intelligence assessment. The need to adjust the prior probability distribution is due to the dynamic nature of the nodal states. As time progresses since the last intelligence update, the uncertainty in the prior probability distribution may grow. The prior probability distribution can *erode* toward total uncertainty dependent upon the amount of time since the last intelligence assessment. Intelligence collection efforts will need to be directed more often at the nodes that change state more frequently than those that remain static for longer periods of time. The *eroded* prior probability distribution, $p_i{}'$, is

$$p_i{}' = (\omega_i) \begin{bmatrix} p_{i,1} \\ p_{i,2} \\ \vdots \\ p_{i,u} \end{bmatrix} + (1 - \omega_i) \begin{bmatrix} 1/u \\ 1/u \\ \vdots \\ 1/u \end{bmatrix} \tag{3.3}$$

where ω_i is the *erosion* factor of the prior probability distribution for n_i. ω_i is chosen based on an assessment of how frequently N_i will possibly change states. Although other weighting schemes exist for ω_i, a straight-forward method is to make ω_i inversely proportional to the time since the last intelligence assessment on n_i. Thus,

$$\omega_i = \frac{1}{t_i}$$

where t_i is the amount of time since the last intelligence update on n_i. The units of t_i may be defined differently for different nodes. Some nodes will possibly change state more frequently than others requiring the units of t_i to be hours or days, whereas others that remain static (assuming no action is taken against the nodes) for longer periods of time may require the units of t_i to be weeks or even months. For example, a electrical power plant will likely remain in a constant state unless acted upon. Thus, required intelligence collection efforts will be needed less frequently. On the other extreme, a political regime may change states often resulting in frequently needed intelligence updates.

To illustrate, consider a *Priority 1* node, n_1, where

$$N_1 = \begin{cases} 0 & \text{if } n_1 \text{ is } \textit{nonoperational} \\ 1 & \text{if } n_1 \text{ is } \textit{degraded} \\ 2 & \text{if } n_1 \text{ is } \textit{operational} \end{cases} .$$

Based on the latest intelligence assessment given one day prior, n_1 was assessed to be *almost certainly operational.* Thus, based on Kent's mapping [11] and the principle of maximum entropy,

$$p_1 = \begin{bmatrix} 0.035 \\ 0.035 \\ 0.93 \end{bmatrix}.$$

Using Equation (3.1), the current relative entropy given p_1 is

$$\begin{aligned} h_1 &= -\frac{1}{\log_2(3)}\left(0.035\log_2(0.035) + 0.035\log_2(0.035) + 0.93\log_2(0.93)\right) \\ &= 0.275. \end{aligned}$$

If N_1 is assessed to possibly change state daily (i.e., the units of t_1 are days) and the prior probability distribution was assessed one day prior ($\omega_1 = 1$), then by Equation (3.3)

$$p_1{}' = (1)\begin{bmatrix} 0.035 \\ 0.035 \\ 0.93 \end{bmatrix} + (0)\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0.035 \\ 0.035 \\ 0.93 \end{bmatrix}.$$

The relative entropy of $p_1{}'$ (assuming an intelligence update one day prior) is again

$$\begin{aligned} h_1 &= -\frac{1}{\log_2(3)}\left(0.035\log_2(0.035) + 0.035\log_2(0.035) + 0.93\log_2(0.93)\right) \\ &= 0.275. \end{aligned}$$

If the intelligence assessment was provided three days prior ($\omega_1 = \frac{1}{3}$), then based on Equation (3.3)

$$p_1{}' = \left({}^1\!/_3\right) \begin{bmatrix} 0.035 \\ 0.035 \\ 0.93 \end{bmatrix} + \left({}^2\!/_3\right) \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0.2339 \\ 0.2339 \\ 0.5322 \end{bmatrix}.$$

The relative entropy of $p_1{}'$ (assuming an intelligence update three days prior) is

$$\begin{aligned} h_1 &= -\frac{1}{\log_2(3)} \left(0.2339 \log_2(0.2339) + 0.2339 \log_2(0.2339) + 0.5322 \log_2(0.5322)\right) \\ &= 0.9242. \end{aligned}$$

Therefore, given the assessment was given three days later, the prior probability distribution was updated resulting in more uncertainty.

After the prior probability distribution has been updated based on the time since the last intelligence update, it is necessary to update this distribution based on actions taken against the node since the last intelligence update, or planned prior to the next intelligence collection opportunity. Currently in operations, intelligence assets are tasked to collect information on lots of targets for which actions have been taken. This is a plausible allocation of intelligence assets if the action against a node has a low or unknown probability of success. There is a valid need to verify the effect of the action. However, given a high probability action against a node, it may be less vital to verify the effect of the action of the node. The following updating scheme updates the prior probability distribution based on an action's probability of success and estimated posterior probability distribution.

Given $p_i{}'$ and a set of actions against a subset of $\mathcal{N}$, the $p_i{}'$'s need to be updated based on the probability of success of the actions, α_i, and the estimated posterior probability distributions based on these actions, $p_i{}^\alpha$. α_i and $p_i{}^\alpha$ may be obtained from previous real world data or by subject matter expert (SME) assessments. The prior probability distribution updated for actions is

$$p_i'' = (1 - \alpha_i)\, p_i' + (\alpha_i)\, p_i^{\alpha}. \tag{3.4}$$

If no action is executed or planned against n_i, then $\alpha_i = 0$ and $p_i' = p_i''$.

Continuing with the example from above, prior to the next intelligence collection opportunity an action is planned against n_1. It is estimated the action will change N_1 to *nonoperational* with probability 0.75 or to *degraded* with probability 0.25.

$$p_1^{\alpha} = \begin{bmatrix} 0.75 \\ 0.25 \\ 0 \end{bmatrix}$$

The probability of success of the action is 0.75. Thus, by Equation (3.4)

$$p_1'' = (0.25) \begin{bmatrix} 0.2339 \\ 0.2339 \\ 0.5322 \end{bmatrix} + (0.75) \begin{bmatrix} 0.75 \\ 0.25 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.6210 \\ 0.2460 \\ 0.1330 \end{bmatrix}.$$

The new relative entropy given p_1'' is

$$h_1 = 0.8276.$$

Once the prior probability distributions have been updated based on the timing of the latest intelligence assessments and the planned actions, $E\left[h_i'\right]$ is calculated next. In order to calculate $E\left[h_i'\right]$, estimates for the effectiveness of the intelligence asset against n_i must be obtained. Specifically, estimates for intelligence asset effectiveness against each state of each node must be obtained. For example, an asset may be very effective at assessing *operational* if the node is in an operational state but only marginally capable of assessing *nonoperational* if the same node is in a nonoperational state. These estimates may be obtained from previous real world data or by SME assessments. The needed estimates are

$$P\left\{\text{asset indicates } N_i = a | N_i = a\right\}, \; \forall n_i \in \mathcal{N}, \; a \in \mathcal{U}_i$$

where $\mathcal{U}_i$ is the set of discrete states for n_i.

For the example from above, the available intelligence asset is assessed to possess the following effectiveness against n_1.

$$P\left\{\text{asset indicates } N_1 = 0 | N_1 = 0\right\} = 0.95$$
$$P\left\{\text{asset indicates } N_1 = 1 | N_1 = 1\right\} = 0.75$$
$$P\left\{\text{asset indicates } N_1 = 2 | N_1 = 2\right\} = 0.75$$

and, thus

$$P\left\{\text{asset indicates } N_1 \neq 0 | N_1 = 0\right\} = 0.05$$
$$P\left\{\text{asset indicates } N_1 \neq 1 | N_1 = 1\right\} = 0.25$$
$$P\left\{\text{asset indicates } N_1 \neq 2 | N_i = 2\right\} = 0.25.$$

Note, not all the required probabilities are given above. Using the principle of maximum entropy, the probability mass associated with $P\left\{\text{asset indicates } N_i \neq a | N_i = a\right\}$ is uniformly distributed among the remaining states.

$$P\left\{\text{asset indicates } N_1 = 1 | N_1 = 0\right\} = \frac{0.05}{2} = 0.025$$

$$P\left\{\text{asset indicates } N_1 = 2 | N_1 = 0\right\} = \frac{0.05}{2} = 0.025$$

$$P\left\{\text{asset indicates } N_1 = 0 | N_1 = 1\right\} = \frac{0.25}{2} = 0.125$$

$$P\left\{\text{asset indicates } N_1 = 2 | N_1 = 1\right\} = \frac{0.25}{2} = 0.125$$

$$P\left\{\text{asset indicates } N_1 = 0 | N_1 = 2\right\} = \frac{0.25}{2} = 0.125$$

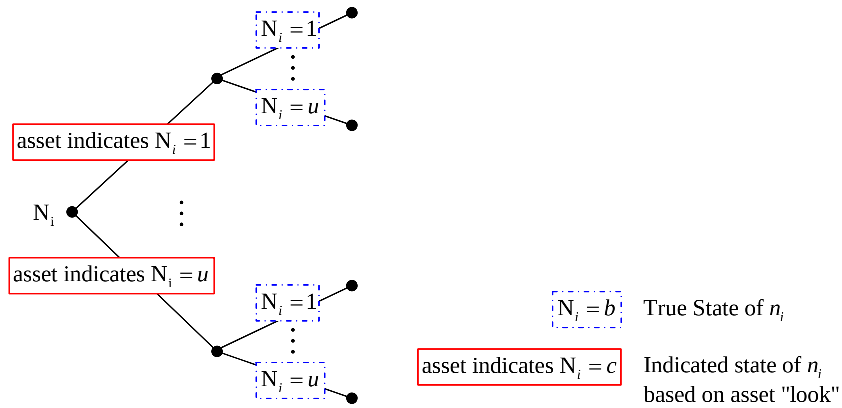

Figure 3.7: Decomposition of an Intelligence Collection Effort on n_1

$$P\left\{\text{asset indicates } N_1 = 1 | N_1 = 2\right\} = \frac{0.25}{2} = 0.125$$

Figure 3.7 depicts the decomposition of an intelligence collection effort on n_i. Using the estimates for the asset effectiveness, $P\left\{\text{asset indicates } N_i = a\right\}$ is calculated using the Law of Total Probability.

$$P\left\{\text{asset indicates } N_i = a\right\} = \sum_{l=1}^{u} P\left\{\text{asset indicates } N_i = a | N_i = l\right\} P\left\{N_i = l\right\} \tag{3.5}$$

where $P\left\{N_i = l\right\}$ is based on the updated prior probability distribution, p_i''. Thus, for the example from above

$$\begin{aligned}
P\left\{\text{asset indicates } N_1 = 0\right\} &= P\left\{\text{asset indicates } N_1 = 0 | N_1 = 0\right\} P\left\{N_1 = 0\right\} + \\
&\quad P\left\{\text{asset indicates } N_1 = 0 | N_1 = 1\right\} P\left\{N_1 = 1\right\} + \\
&\quad P\left\{\text{asset indicates } N_1 = 0 | N_1 = 2\right\} P\left\{N_1 = 2\right\} \\
&= (0.95)(0.6210) + (0.125)(0.2460) + (0.125)(0.1330) \\
&= 0.6373.
\end{aligned}$$

Similarly,

$$P\{\text{asset indicates } N_1 = 1\} = 0.21665$$
$$P\{\text{asset indicates } N_1 = 2\} = 0.1460.$$

From this, for all $b, c \in \mathcal{U}_i$, $P\{N_i = b|\text{asset indicates } N_i = c\}$ follows using Bayes' Theorem.

$$\begin{aligned} P\{N_i = b|\text{asset indicates } N_i = c\} &= \frac{P\{N_i = b \cap \text{asset indicates } N_i = c\}}{P\{\text{asset indicates } N_i = c\}} \\ &= \frac{P\{\text{asset indicates } N_i = c|N_i = b\}\, P\{N_i = b\}}{P\{\text{asset indicates } N_i = c\}} \end{aligned} \tag{3.6}$$

Thus, for the example

$$\begin{aligned} P\{N_1 = 0|\text{asset indicates } N_1 = 0\} &= \frac{P\{\text{asset indicates } N_1 = 0|N_1 = 0\}\, P\{N_1 = 0\}}{P\{\text{asset indicates } N_1 = 0\}} \\ &= \frac{(0.95)\,(0.6210)}{0.6373} \\ &= 0.9257. \end{aligned}$$

Similarly,

$$P\{N_1 = 1|\text{asset indicates } N_1 = 0\} = 0.0482$$
$$P\{N_1 = 2|\text{asset indicates } N_1 = 0\} = 0.0261$$
$$P\{N_1 = 0|\text{asset indicates } N_1 = 1\} = 0.0717$$
$$P\{N_1 = 1|\text{asset indicates } N_1 = 1\} = 0.8516$$

$$P\{N_1 = 2|\text{asset indicates } N_1 = 1\} = 0.0767$$
$$P\{N_1 = 0|\text{asset indicates } N_1 = 2\} = 0.1063$$
$$P\{N_1 = 1|\text{asset indicates } N_1 = 2\} = 0.2106$$
$$P\{N_1 = 2|\text{asset indicates } N_1 = 2\} = 0.6831.$$

Now, $E[h_i{}']$ is calculated.

$$E[h_i{}'] = \sum_{j=1}^{u_i} P\{\text{asset indicates } N_i = j\}\, h_i^{\text{asset indicates } N_i = j} \tag{3.7}$$

where $h_i^{\text{asset indicates } N_i=0}$ is the entropy of n_i given the asset indicates $N_i = a$ (i.e., the entropy of each branch of Figure 3.7. Thus,

$$\begin{aligned} h_1^{\text{asset indicates } N_1=0} &= -\frac{1}{\log(3)}\left(0.9257\log(0.9257) + 0.0482\log(0.0482) + 0.0261\log(0.0261)\right) \\ &= 0.2847 \end{aligned}$$

$$\begin{aligned} h_1^{\text{asset indicates } N_1=1} &= -\frac{1}{\log(3)}\left(0.0717\log(0.0717) + 0.8516\log(0.8516) + 0.0767\log(0.0767)\right) \\ &= 0.4758 \end{aligned}$$

$$\begin{aligned} h_1^{\text{asset indicates } N_1=2} &= -\frac{1}{\log(3)}\left(0.1063\log(0.1063) + 0.2106\log(0.2106) + 0.6831\log(0.6831)\right) \\ &= 0.7525 \end{aligned}$$

$$
\begin{aligned}
E\left[h_1{}'\right] = &\left(P\left\{\text{asset indicates } N_1 = 0\right\}\right)\left(h_1^{\text{asset indicates } N_1=0}\right) + \\
&\left(P\left\{\text{asset indicates } N_1 = 1\right\}\right)\left(h_1^{\text{asset indicates } N_1=1}\right) + \\
&\left(P\left\{\text{asset indicates } N_1 = 2\right\}\right)\left(h_1^{\text{asset indicates } N_1=2}\right) \\
= &\,(0.6373)\,(0.2847) + (0.21665)\,(0.4758) + (0.1460)\,(0.7525) \\
= &\,0.3944.
\end{aligned}
$$

Next, the weighted change in entropy is calculated.

$$\delta_i = \rho_i\left(h_i - E\left[h_i{}'\right]\right) \tag{3.8}$$

Thus,

$$
\begin{aligned}
\delta_1 &= \rho_1\left(h_1 - E\left[h_1{}'\right]\right) \\
&= (8)\,(0.8276 - 0.3944) \\
&= 3.4656.
\end{aligned}
$$

Finally, the maximum δ_i is selected. For the example above, only one node was examined. If multiple nodes were analyzed, the node corresponding to the maximum δ_i would be selected for the intelligence asset effort. Section 3.3.3 demonstrates a three-node example for a single intelligence asset.

In summary, based on a single intelligence asset, the methodology for selecting intelligence collection efforts is composed of the following steps. For all nodes n_i in the SoSA network ($i = 1, \ldots, u_i$)

1. Establish the prior probability distributions

   (a) Based on latest intelligence assessments (p_i)

   (b) Adjust based on the *freshness* of the intelligence assessments ($p_i{}'$)

(c) Update based on planned action's estimated probability of success and posterior probability distributions (p_i'')

2. Calculate the current entropy, h_i
3. Calculate the expected entropy based on the estimated intelligence asset effectiveness ($E[h_i']$)

    (a) Calculate $P\{\text{asset indicates } N_i = a\}$

    (b) Calculate $P\{N_i = b|\text{asset indicates } N_i = a\}$

    (c) Calculate $h_i^{\text{asset indicates } N_i=a}$

    (d) Calculate $E[h_i']$

4. Calculate δ_i
5. Select node corresponding to maximum δ_i

3.3.2 Nodal Priority. Based on strategic and/or operational objectives, the nodes of the SoSA network possess different priorities. To incorporate nodal priorities into the model presented in Section 3.3.1, the different priorities are simply assigned weights, ρ_i. Figure 3.8 on page 61 depicts the expected change in entropy given an intelligence collection effort, δ_i, for three weighting schemes for different priority nodes (e.g., *Priority 1*, *Priority 2*, *Priority 3*). The graphs were constructed using `Matlab`® -generated random numbers from $[0, 1]$ to simulate possible expected changes in relative entropy. These random numbers were assigned to sixty nodes–20 *Priority 1*, 20 *Priority 2*, and 20 *Priority 3*. The nodes with higher δ_i's would be selected for intelligence collection efforts.

The top graph shows equal priority weighting for each node type (i.e., $\rho_i = 1$). If a limited number of intelligence collection efforts from a single asset was available, then a quick look at the higher δ_i's indicates the nodes that would be selected for intelligence collection. The middle and bottom graphs show unequal weights for the priority types with the higher priority nodes weighted more heavily. These schemes successfully differentiate the δ_i's based on nodal priority. The higher δ_i's are composed predominantly of *Priority 1* nodes with only the highest *Priority 2* nodes near the top.

3.3.3 Three-Node Example. To illustrate the methodology for prioritizing the collection efforts of a single intelligence asset, consider a three-node network composed of two binary-state nodes and one three-state node,

$$N_1 = \begin{cases} 0 \\ 1 \end{cases} \qquad N_2 = \begin{cases} 0 \\ 1 \end{cases} \qquad N_3 = \begin{cases} 0 \\ 1 \\ 2 \end{cases}.$$

Nodes n_1 and n_3 are *Priority 1*, and n_2 is a *Priority 2* node. According to planner assessments, *Priority 1* nodes should be weighted twice as much as *Priority 2* nodes. Thus, $\rho_1 = \rho_3 = 2$ and $\rho_2 = 1$.

Available is one intelligence collection effort from an intelligence asset possessing the estimated effectiveness against the nodes listed in Table 3.2 on page 62. Note, not all required probabilities are given in Table 3.2. The principle of maximum entropy is used to obtain the remaining effectiveness estimates. For example,

$$P\{\text{asset indicates } N_3 = 0 | N_3 = 0\} = 0.95$$
$$P\{\text{asset indicates } N_3 = 0 | N_3 \neq 0\} = 0.05.$$

Thus, using the principle of maximum entropy,

$$P\{\text{asset indicates } N_3 = 0 | N_3 = 1\} = \frac{0.05}{2} = 0.025$$

$$P\{\text{asset indicates } N_3 = 0 | N_3 = 2\} = \frac{0.05}{2} = 0.025.$$

Intelligence assessments for the current nodal states, as well as the *freshness* of the assessments, are given in Table 3.3 on page 62.

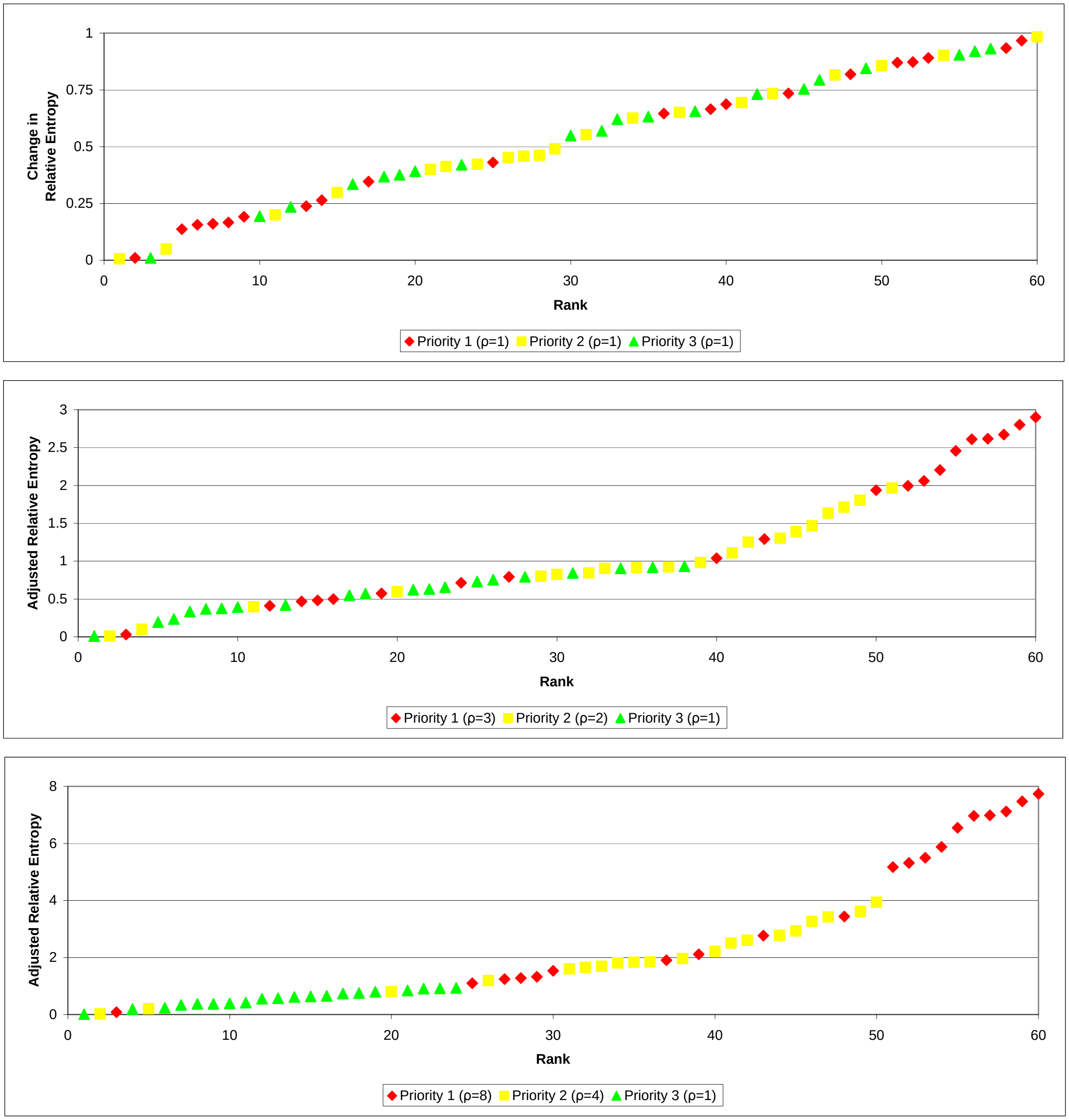

Figure 3.8: Various Priority Weighting Schemes

Table 3.2: Three-node Example: Estimates for Intelligence Asset Effectiveness

n_i	Priority	$N_i = a$	Asset Effectiveness	$P\{\text{asset indicates } N_i = a \mid N_i = a\}$
1	1	0	Not Effective	0.50
		1	Marginally Effective	0.75
2	2	0	Effective	0.95
		1	Marginally Effective	0.75
3	1	0	Effective	0.95
		1	Marginally Effective	0.75
		2	Not Effective	0.33

Table 3.3: Three-node Example: Prior Intelligence Assessment

Node (n_i)	Intelligence Assessment	Days Prior (t_i)	p_i
1	*Probably* in State 1	2	$\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$
2	*Almost certainly* in State 0	2	$\begin{bmatrix} 0.93 \\ 0.07 \end{bmatrix}$
3	*Probably Not* in State 2	1	$\begin{bmatrix} 0.35 \\ 0.35 \\ 0.30 \end{bmatrix}$

Table 3.4: Three-node Example: Actions Against n_i

Node (n_i)	Estimated Probability of Success (α_i)	Estimated Posterior Probability Distribution (${p_i}^{\alpha}$)
1	0	n/a
2	0.75	$\begin{bmatrix} 0.07 \\ 0.93 \end{bmatrix}$
3	0.90	$\begin{bmatrix} 0 \\ 0.20 \\ 0.80 \end{bmatrix}$

Using Equation (3.3), the prior probability distributions are updated based on the time since the last intelligence update. For this example, t is in days for all nodes.

$$p_1{}' = \left(1/2\right) \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} + \left(1/2\right) \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

$$p_2{}' = \left(1/2\right) \begin{bmatrix} 0.93 \\ 0.07 \end{bmatrix} + \left(1/2\right) \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 0.715 \\ 0.285 \end{bmatrix}$$

$$p_3{}' = (1) \begin{bmatrix} 0.35 \\ 0.35 \\ 0.30 \end{bmatrix} + (0) \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0.35 \\ 0.35 \\ 0.30 \end{bmatrix}$$

Actions are planned against two of the nodes, n_2 and n_3. The estimated probabilities of success and estimated posterior probability distributions are presented in Table 3.4.

Using Equation (3.4), $p_i{}'$ is updated with the expected probability distributions given action is taken against the nodes.

$$p_1{}'' = p_1{}' = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

$$\begin{aligned} p_2{}'' &= (1 - \alpha_2)\, p_2{}' + (\alpha_2)\, p_2{}^{\alpha} \\ &= (0.25) \begin{bmatrix} 0.715 \\ 0.285 \end{bmatrix} + (0.75) \begin{bmatrix} 0.07 \\ 0.93 \end{bmatrix} \\ &= \begin{bmatrix} 0.231 \\ 0.769 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} p_3{}'' &= (1 - \alpha_3)\, p_3{}' + (\alpha_3)\, p_3{}^{\alpha} \\ &= (0.10) \begin{bmatrix} 0.35 \\ 0.35 \\ 0.30 \end{bmatrix} + (0.90) \begin{bmatrix} 0 \\ 0.20 \\ 0.80 \end{bmatrix} \\ &= \begin{bmatrix} 0.035 \\ 0.215 \\ 0.750 \end{bmatrix} \end{aligned}$$

Nodes n_1 and n_2 can be in two discrete states, and n_3 has three discrete states. Thus, for Shannon's relative entropy,

$$K_1 = K_2 = \frac{1}{\log_2 2} = 1$$

$$K_3 = \frac{1}{\log_2 3}.$$

Using Equation (3.1), the current entropies of the probability distributions are

$$\begin{aligned} h_1 &= -\left(0.375 \log_2 0.375 + 0.625 \log_2 0.625\right) \\ &= 0.9544 \end{aligned}$$

$$\begin{aligned} h_2 &= -\left(0.231 \log_2 0.231 + 0.769 \log_2 0.769\right) \\ &= 0.7798 \end{aligned}$$

$$\begin{aligned} h_3 &= -\frac{1}{\log_2 3}\left(0.035 \log_2 0.035 + 0.215 \log_2 0.215 + 0.750 \log_2 0.750\right) \\ &= 0.6040. \end{aligned}$$

From the estimates for asset effectiveness and the prior probability distributions, the $P\left\{\text{asset indicates } N_i = a\right\}$ is calculated using Equation (3.5). The results are presented in Table 3.5 on the following page. Now, using Equation (3.6) the $P\left\{N_i = b | \text{asset indicates } N_i = c\right\}$ are calculated for each of the branches of Figure 3.7 on page 55. The results are presented in Table 3.6.

For each branch (i.e., $P\left\{\text{asset indicates } N_i = 0\right\}$, $P\left\{\text{asset indicates } N_i = 1\right\}$, and $P\left\{\text{asset indicates } N_i = 2\right\}$), the relative entropies are calculated. Then, the expected relative entropy, $E\left[h_i{}'\right]$, is computed (see Table 3.7). Figure 3.9 depicts the $E\left[h_1{}'\right]$ calculation for n_1. $E\left[h_2{}'\right]$ and $E\left[h_3{}'\right]$ are similarly calculated for n_2 and n_3.

Table 3.5: Three-node Example: $P\{\text{Asset indicates } N_i = a\}$

Node (n_i)	Discrete States ($N_i = a$)	$P\{\text{Asset indicates } N_i = a\}$
1	0	0.3438
	1	0.6563
2	0	0.4117
	1	0.5883
3	0	0.3101
	1	0.4121
	2	0.2778

Table 3.6: Three-node Example: $P\{N_i = b|\text{asset indicates } N_i = c\}$

Node ($n_i = i$)	Asset Indicates $N_i = c$	$N_i = b$	$P\{N_i = b|\text{asset indicates } N_i = c\}$
1	0	0	0.5455
		1	0.4545
	1	0	0.2857
		1	0.7143
2	0	0	0.5330
		1	0.4670
	1	0	0.0196
		1	0.9804
3	0	0	0.1072
		1	0.0867
		2	0.8061
	1	0	0.0021
		1	0.3913
		2	0.6066
	2	0	0.0032
		1	0.0968
		2	0.9001

Table 3.7: Three-node Example: Expected Entropies

Node (n_i)	Asset Indicates $N_i = c$	$h_i^{\text{asset indicates } N_i=c}$	$E[h_i']$
1	0	0.9940	0.9081
	1	0.8631	
2	0	0.9969	0.4923
	1	0.1392	
3	0	0.5690	0.5185
	1	0.6220	
	2	0.3087	

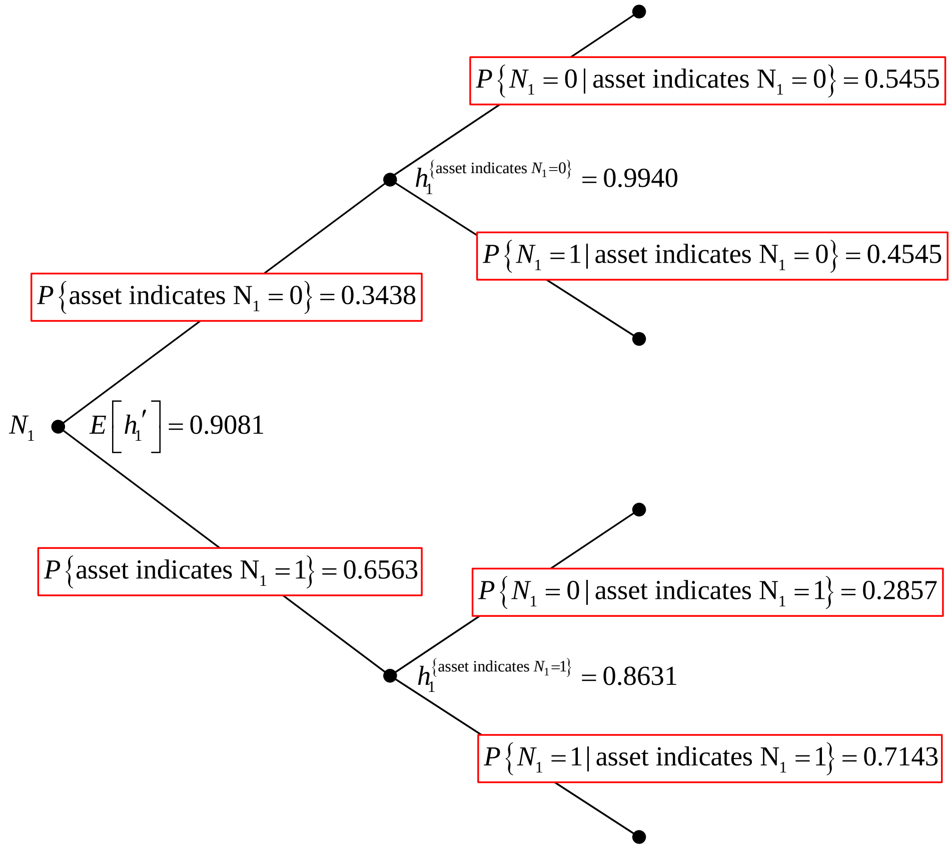

Figure 3.9: Three-node Example: Expected Entropy, $E\left[h_i{}'\right]$, Calculation for n_1

Finally, using Equation (3.8), the weighted expected changes in entropy, based on the asset effectiveness estimates, the prior probability distributions, and nodal priorities, are

$$
\begin{aligned}
\delta_1 &= \rho_1 \left(h_1 - E\left[h_1{}'\right]\right) \\
&= 2\left(0.9544 - 0.9081\right) \\
&= 0.0926
\end{aligned}
$$

$$\begin{aligned}
\delta_2 &= \rho_2 \left(h_2 - E\left[h_2{}'\right]\right) \\
&= 0.7798 - 0.4923 \\
&= 0.2875
\end{aligned}$$

$$\begin{aligned}
\delta_3 &= \rho_3 \left(h_3 - E\left[h_3{}'\right]\right) \\
&= 2\left(0.6040 - 0.5185\right) \\
&= 0.1710.
\end{aligned}$$

δ_2 is the maximum weighted change in expected entropy based on an intelligence collection effort. Thus, the intelligence asset should be directed to collect on n_2. This is an interesting result because n_2 was the only *Priority 2* node. In addition, h_1 is the highest entropy value, which implies there is the greatest opportunity to reduce the entropy. However, looking at Table 3.2 on page 62, the result checks out as the intelligence asset was most effective versus n_2 and least effective versus n_1. Therefore, even though n_2 is a *Priority 2* node, the intelligence asset would be better used being directed to collect intelligence on n_2 as opposed to n_1 or n_3. If a second intelligence effort became available, then intelligence should be collected on n_3, as it has the next highest δ.

3.3.4 Sensitivity to Prior Probabilities and Asset Effectiveness Estimates. The change in expected entropy, δ_i, is clearly dependent upon the prior probability distributions, asset effectiveness estimates, and nodal priorities. Figures 3.10 and 3.11 depict the dependence of δ_i on the prior probability distributions and asset effectiveness estimates, respectively. Both figures were constructed using a binary node like n_1 and n_2 from the previous example.

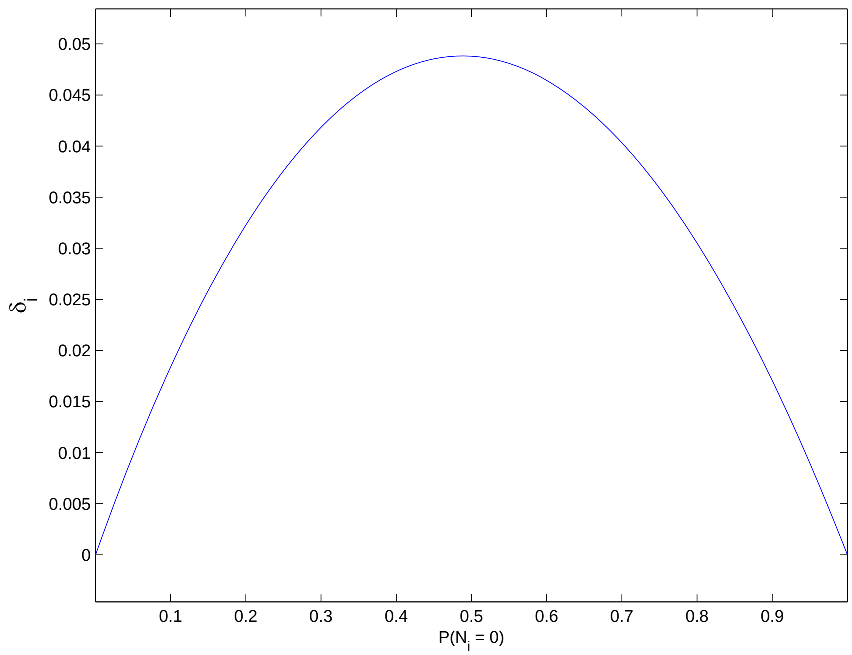

Figure 3.10: Dependence of δ_i on Prior Probabilities

Figure 3.10 plots δ_i versus $P\{N_i = 0\}$ using

$$p_i = \begin{bmatrix} P\{N_i = 0\} \\ 1 - P\{N_i = 0\} \end{bmatrix}.$$

For Figure 3.10, the intelligence asset was estimated to be not effective at assessing if $N_i = 0$ (i.e., $P\{\text{asset indicates } N_i = 0 | N_i = 0\} = 0.50$) and marginally effective at assessing if $N_i = 1$ (i.e., $P\{\text{asset indicates } N_i = 0 | N_i = 0\} = 0.75$). Notice that the maximum δ_i is obtained when

$$\begin{aligned} P\{N_i = 0\} &= 1 - P\{N_i = 0\} \\ &= 0.5 \end{aligned}$$

and the minimum when

$$P\{N_i = 0\} = 0$$

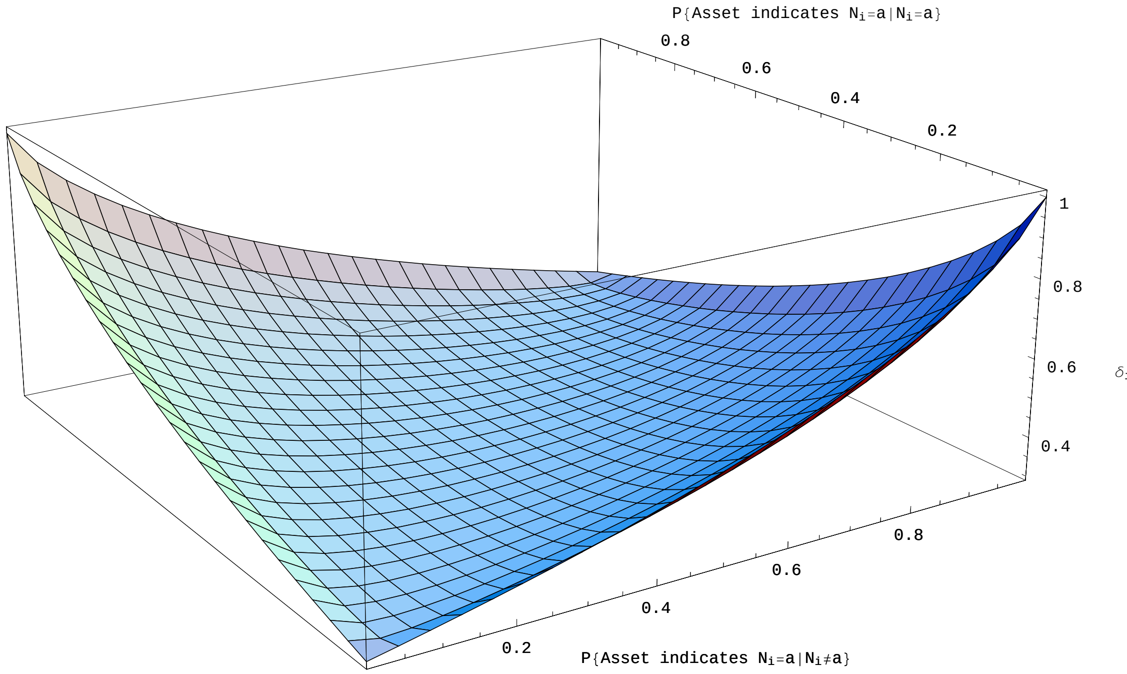

Figure 3.11: Dependence of δ_i on Asset Effectiveness Estimates

or

$$P\{N_i = 0\} = 1.$$

The maximum change in expected entropy corresponds to the case where no prior information (i.e., perfect uncertainty) on the current nodal state is possessed. Any new information provided by an intelligence collection effort is expected to reduce the entropy, h_i. Likewise, if the node is known to exist in either state 0 or 1 with probability 1, then assuming the state of the node did not change, any new intelligence collection efforts are expected to have no impact on the entropy. The implication for this sensitivity is that given a node that has little uncertainty associated with its nodal state (i.e., $h_i \approx 0$), then performing additional intelligence collection efforts on the node will have minimal impact on the level of uncertainty associated with its nodal state.

Figure 3.11 on the page before plots δ_i versus the intelligence asset effectiveness estimates. For Figure 3.11,

$$p_i = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

The maximum δ_i is obtained when

$$P\{\text{asset indicates } N_i = 0 | N_i = 0\} = 1$$
$$P\{\text{asset indicates } N_i = 0 | N_i \neq 0\} = 0$$

or

$$P\{\text{asset indicates } N_i = 0 | N_i = 0\} = 0$$
$$P\{\text{asset indicates } N_i = 0 | N_i \neq 0\} = 1.$$

This implies the intelligence asset is able to perfectly discern between $N_i = 0$ and $N_i \neq 0$. Note, the second set of probabilities seem to indicate the intelligence asset is providing misinformation (i.e., indicating $N_i = 0$ when $N_i \neq 0$); however, the intelligence asset estimates are a priori information. Thus, the intelligence analyst knows the intelligence asset indication and the true state of the node are perfectly negatively correlated. Therefore, when the asset indicates $N_i = 0$, the intelligence analyst knows that the true state is $N_i \neq 0$.

The minimum δ_i's are obtained when

$$P\{\text{asset indicates } N_i = 0 | N_i = 0\} = P\{\text{asset indicates } N_i = 0 | N_i \neq 0\}.$$

In other words, the $P\{\text{asset indicates } N_i = 0\}$ is independent of the true state of n_i. When the intelligence effectiveness estimates are equal (or nearly equal) then no (or little) information is provided on N_i. For example, consider a binary node like the ones used in Section 3.3.3, where

$$P\{\text{asset indicates } N_i = 0 | N_i = 0\} = 0.75$$
$$P\{\text{asset indicates } N_i = 1 | N_i = 1\} = 0.25.$$

Thus,

$$P\{\text{asset indicates } N_i = 0 | N_i \neq 0\} = 0.75$$

which results in $\delta_i = 0$.

3.4 Allocating Multiple Intelligence Assets Based on Nodal Uncertainty

In military operations, there will generally always be less intelligence collection opportunities than the number of nodes within the SoSA network. Thus, the problem becomes allocating assets to collect intelligence on the *highest payoff* nodes. The goal then is to maximize the amount of information gain from a limited number of intelligence collection opportunities. Now that a measure of the expected change in entropy given an intelligence collection effort on a node is developed, a methodology for allocating multiple intelligence assets can be developed.

3.4.1 General Methodology for Allocating Multiple Intelligence Assets .

First, the notation from the methodology presented in Section 3.3.1 must be adjusted to account for the multiple available intelligence assets. To accomplish this, a second subscript, j, is added to $E[h_i']$ and δ_i. Now,

$E[h_{ij}']$ = the expected entropy of n_i given an intelligence collection effort by asset j

δ_{ij} = the weighted expected change in entropy of n_i given an intelligence collection effort by asset j

In addition, estimates for the effectiveness of asset j against N_i need to be obtained for all the available intelligence assets and nodes in the SoSA network.

$$P\{\text{asset } j \text{ indicates } N_i = a | N_i = a\}$$

Once, these estimates are established, the methodology from Section 3.3.1 can be applied to obtain the δ_{ij}'s for each intelligence asset and node combination.

From this, an integer program (IP), the generalized assignment problem (GAP), can be accomplished [12]. Given m nodes and n intelligence assets, with

$$\begin{aligned} \delta_{ij} &= \text{weighted expected change in entropy of } n_i \text{ given an intelligence} \\ &\quad\ \text{collection effort by asset } j \\ w_{ij} &= \text{amount of asset } j \text{ capacity required to accomplish intelligence} \\ &\quad\ \text{collection on } n_i \\ c_j &= \text{capacity of asset } j. \end{aligned}$$

Then,

$$\text{maximize} \qquad z = \sum_{j=1}^{n} \sum_{i=1}^{m} \delta_{ij} x_{ij} \tag{3.9}$$

$$\text{subject to} \qquad \sum_{i=1}^{m} w_{ij} x_{ij} \leq c_j, \qquad j \in \{1, \ldots, n\} \tag{3.10}$$

$$\sum_{j=1}^{n} x_{ij} \leq 1, \qquad i \in \{1, \ldots, m\} \tag{3.11}$$

$$x_{ij} \in \{0, 1\}, \qquad i \in \{1, \ldots, m\},\ j \in \{1, \ldots, n\} \tag{3.12}$$

where

$$x_{ij} = \begin{cases} 1 & \text{if asset } j \text{ collects intelligence on node } i \\ 0 & \text{otherwise} \end{cases}.$$

Equation (3.10) is the capacity constraint for the intelligence asset. Equation (3.11) ensures that at most one asset looks at a node, and Equation (3.12) makes the x_{ij}'s indicator variables for whether or not asset j is collecting information on node i.

3.5 Chapter 3 Summary

Chapter 3 presented a mathematical model for SoSA networks, a methodology for prioritizing a single intelligence asset's intelligence collection efforts, and a methodology for allocating multiple intelligence assets. The SoSA mathematical model defined notation and terminology forming the foundation for the later methodologies for prioritizing and allocating intelligence collection efforts. The methodology for prioritizing a single intelligence asset's collection efforts was demonstrated for a simple three-node example. The methodology for allocating multiple intelligence asset's collection efforts makes use of a generalized assignment problem. This methodology is demonstrated in Chapter 4 on a 20-node, 4-asset example.

IV. 20-Node, 4-Asset Example

4.1 Overview

In Chapter 3, a methodology for allocating multiple intelligence resources to multiple nodes based on nodal uncertainty was developed. Calculation of the current nodal uncertainty, h_i, was performed using Shannon's relative entropy.

$$h_i = -K_i \sum_{l=1}^{u_i} p_{i,l}'' \log_2(p_{i,l}'')$$

where,

$$K_i = \frac{1}{\log_2(u_i)}$$

$p_{i,l}''$ = the probability $N_i = l$ adjusted for the *freshness* of the intelligence update and planned actions on n_i.

A priority-weighted expected reduction in uncertainty was obtained by estimating the intelligence asset's effectiveness versus the specific nodal states.

$$\delta_{ij} = \rho_i \left(h_i - E\left[h'_{ij}\right]\right)$$

where ρ_i is node i's priority weighting and $E\left[h'_{ij}\right]$ is the expected entropy of node i given an intelligence update by asset j. A generalized assignment problem (GAP) was then formulated to allocate multiple intelligence assets to multiple nodes resulting in the maximum expected reduction in uncertainty (i.e., the maximum expected information gain).

This chapter presents an application of the multiple intelligence asset methodology to a notional 20-node, 4-asset example. This example demonstrates the maximization of information gain through proper allocation of four intelligence assets.

4.2 20-Node, 4-Asset Scenario

20 nodes are present within the scenario. Each node is one of five generic nodal category types: person, radar, communications, economic, or vehicle. Using `Matlab`® , the 20 nodes were assigned:

- a random number of discrete states (2, 4, or 6),
- nodal priority (1, 2, 3, or 4),
- *freshness* of the intelligence data (1-6) (i.e., days since the last intelligence update),
- intelligence assessment (*Almost Certainly Not in 1*, etc.), and
- whether or not an action is planned against the node (0 or 1).

For the nodes for which actions are planned, notional posterior probabilities given an action were provided. See Figure 4.1 on the following page for the given nodal data. It was assumed that *Priority 1* nodes are twice as important as *Priority 2* nodes, which are twice as important as *Priority 3* nodes and four times as important as *Priority 4* nodes (i.e., $\rho_{Priority\ 4} = 1$, $\rho_{Priority\ 3} = 2$, $\rho_{Priority\ 2} = 4$, and $\rho_{Priority\ 1} = 8$).

Available are four intelligence assets: an imagery intelligence (IMINT) source, a electronic intelligence (ELINT) source, a human intelligence (HUMINT) source, and a communications intelligence (COMINT) source. Within a 24-hour period, a limited number of collection opportunities is available from each of these assets. Table 4.1 on page 78 presents the collection capacities of each asset. In addition, only one asset can be allocated per node. Note, this is a simplifying assumption to maintain independence between the intelligence assets (and their effectiveness estimates). If multiple assets were allowed to collect information on a single node, the resulting information gain would be dependent upon the interaction between the assets.

Each nodal type requires a specific number of intelligence collection *looks* (w_{ij}) to gather the required information for an intelligence assessment of the current nodal state. The required number of intelligence *looks* is dependent upon the type of asset

Node	Node Type	Number of Discrete States	Priority	Priority Weight	"Freshness" of Intel (days)	Intel Assessment	State	Prior Probability	Action	Probability of Success of Action	Posterior Probability Given Action
1	Vehicle	4	1	8	3	Almost Certainly Not in 1	0	0.31	1	0.75	0.6
							1	0.07			0.3
							2	0.31			0.1
							3	0.31			0
2	Radar	4	2	4	2	Almost Certainly in 3	0	0.02333	1	0.75	0.4
							1	0.02333			0.4
							2	0.02333			0.2
							3	0.93			0
3	Economic	2	4	1	4	Probably Not in 1	0	0.7	1	0.75	0.9
							1	0.3			0.1
4	Comm	4	1	8	2	Probably in 2	0	0.23333	1	0.75	0.45
							1	0.23333			0.45
							2	0.3			0.1
							3	0.23333			0
5	Vehicle	4	1	8	6	Almost Certainly Not in 3	0	0.31	0	0	0
							1	0.31			0
							2	0.31			0
							3	0.07			0
6	Economic	4	2	4	2	Chances Even in 2	0	0.16667	1	0.5	0.8
							1	0.16667			0.1
							2	0.5			0.1
							3	0.16667			0
7	Comm	2	2	4	2	Probably Not in 0	0	0.3	0	0	0
							1	0.7			0
8	Person	2	3	2	2	Chances Even in 0	0	0.5	0	0	0
							1	0.5			0
9	Vehicle	4	2	4	2	Probably in 1	0	0.08333	1	0.75	0.75
							1	0.75			0.2
							2	0.08333			0.05
							3	0.08333			0
10	Comm	6	2	4	2	Almost Certainly in 3	0	0.014	1	0.9	0.3
							1	0.014			0.3
							2	0.014			0.3
							3	0.93			0.1
							4	0.014			0
							5	0.014			0
11	Economic	6	1	8	3	Almost Certainly in 4	0	0.014	0		0
							1	0.014			0
							2	0.014			0
							3	0.014			0
							4	0.93			0
							5	0.014			0
12	Economic	2	4	1	2	Probably Not in 0	0	0.3	0		0
							1	0.7			0
13	Vehicle	2	3	2	2	Probably in 1	0	0.25	1	0.9	0.9
							1	0.75			0.1
14	Economic	6	4	1	5	Chances Even in 3	0	0.1	0		0
							1	0.1			0
							2	0.1			0
							3	0.5			0
							4	0.1			0
							5	0.1			0
15	Person	2	3	2	2	Almost Certainly in 0	0	0.93	0		0
							1	0.07			0
16	Comm	4	3	2	2	Probably Not in 2	0	0.23333	1	0.75	0.8
							1	0.23333			0.1
							2	0.3			0.1
							3	0.23333			0
17	Vehicle	6	4	1	2	Probably in 3	0	0.05	0		0
							1	0.05			0
							2	0.05			0
							3	0.75			0
							4	0.05			0
							5	0.05			0
18	Vehicle	4	3	2	4	Almost Certainly Not 1	0	0.31	0		0
							1	0.07			0
							2	0.31			0
							3	0.31			0
19	Comm	6	2	4	2	Almost Certainly Not in 4	0	0.186	0		0
							1	0.186			0
							2	0.186			0
							3	0.186			0
							4	0.07			0
							5	0.186			0
20	Vehicle	2	3	2	2	Chances Even in 1	0	0.5	1	0.25	0.95
							1	0.5			0.05

Figure 4.1: 20-Node, 4-Asset Example: Given Data

Table 4.1: 20-Node, 4-Asset Example: Intelligence Source Capacities

Source (j)	Maximum Number of Collection Opportunites Per 24-hour Period (c_j)
IMINT	4
ELINT	6
HUMINT	1
COMINT	6

Table 4.2: 20-Node, 4-Asset Example: Nodal Intelligence Requirements (w_{ij})

	Intelligence Requirements (w_{ij})			
Nodal Type	IMINT	ELINT	HUMINT	COMINT
Person	4	5	1	2
Radar	2	1	1	2
Communications	2	3	1	1
Economic	4	5	1	2
Vehicle	1	2	1	4

allocated to collect the information on the node. For example, a radar node may require only one *look* from an ELINT source but may require multiple *looks* from an IMINT source to capture the required information to perform an intelligence assessment. Table 4.2 presents the intelligence requirements for each nodal type. Note, the units of these intelligence *looks* are number of *looks* in a 24-hour period.

In order to calculate the expected change in entropy, δ_{ij}, for each node/asset combination, intelligence asset effectiveness estimates were also provided (see Figure 4.2 on the next page). These estimates were mapped to probabilities using Table 4.3.

Table 4.3: Intelligence Asset Effectiveness Estimates

Asset Effectiveness	$P\{\text{asset indicates } N_i = a \mid N_i = a\}$
Not Effective	1/u
Marginally Effective	0.75
Effective	0.90
Very Effective	0.98

			Asset Effectiveness			
Nodal Type	Number of States	States	IMINT	ELINT	HUMINT	COMINT
Person	2	0	Not Effective	Not Effective	Very Effective	Effective
		1	Marginally Effective	Not Effective	Very Effective	Very Effective
	4	0	Not Effective	Not Effective	Very Effective	Effective
		1	Not Effective	Not Effective	Effective	Effective
		2	Not Effective	Not Effective	Effective	Effective
		3	Marginally Effective	Not Effective	Very Effective	Very Effective
	6	0	Not Effective	Not Effective	Very Effective	Effective
		1	Not Effective	Not Effective	Effective	Effective
		2	Not Effective	Not Effective	Effective	Effective
		3	Not Effective	Not Effective	Effective	Effective
		4	Not Effective	Not Effective	Effective	Effective
		5	Marginally Effective	Not Effective	Very Effective	Very Effective
Radar	2	0	Effective	Effective	Effective	Marginally Effective
		1	Very Effective	Very Effective	Effective	Effective
	4	0	Effective	Effective	Effective	Marginally Effective
		1	Effective	Effective	Marginally Effective	Marginally Effective
		2	Effective	Very Effective	Marginally Effective	Effective
		3	Very Effective	Very Effective	Effective	Effective
	6	0	Effective	Effective	Effective	Marginally Effective
		1	Effective	Effective	Effective	Marginally Effective
		2	Effective	Effective	Marginally Effective	Marginally Effective
		3	Effective	Very Effective	Marginally Effective	Effective
		4	Effective	Very Effective	Marginally Effective	Effective
		5	Very Effective	Very Effective	Effective	Effective
Communications	2	0	Effective	Marginally Effective	Effective	Very Effective
		1	Marginally Effective	Effective	Effective	Very Effective
	4	0	Effective	Marginally Effective	Effective	Very Effective
		1	Marginally Effective	Marginally Effective	Marginally Effective	Very Effective
		2	Marginally Effective	Marginally Effective	Marginally Effective	Very Effective
		3	Marginally Effective	Effective	Effective	Very Effective
	6	0	Effective	Marginally Effective	Effective	Very Effective
		1	Effective	Marginally Effective	Marginally Effective	Very Effective
		2	Marginally Effective	Marginally Effective	Marginally Effective	Very Effective
		3	Marginally Effective	Marginally Effective	Marginally Effective	Very Effective
		4	Marginally Effective	Effective	Marginally Effective	Very Effective
		5	Marginally Effective	Effective	Effective	Very Effective
Economic	2	0	Effective	Not Effective	Very Effective	Effective
		1	Marginally Effective	Not Effective	Effective	Effective
	4	0	Effective	Not Effective	Very Effective	Effective
		1	Marginally Effective	Not Effective	Effective	Effective
		2	Marginally Effective	Not Effective	Effective	Effective
		3	Marginally Effective	Not Effective	Effective	Effective
	6	0	Effective	Not Effective	Very Effective	Effective
		1	Marginally Effective	Not Effective	Effective	Effective
		2	Marginally Effective	Not Effective	Effective	Effective
		3	Marginally Effective	Not Effective	Effective	Effective
		4	Marginally Effective	Not Effective	Effective	Effective
		5	Marginally Effective	Not Effective	Effective	Effective
Vehicle	2	0	Very Effective	Not Effective	Effective	Marginally Effective
		1	Very Effective	Marginally Effective	Effective	Marginally Effective
	4	0	Very Effective	Not Effective	Effective	Marginally Effective
		1	Effective	Not Effective	Marginally Effective	Not Effective
		2	Effective	Not Effective	Marginally Effective	Not Effective
		3	Very Effective	Marginally Effective	Effective	Marginally Effective
	6	0	Very Effective	Not Effective	Effective	Marginally Effective
		1	Very Effective	Not Effective	Marginally Effective	Not Effective
		2	Effective	Not Effective	Marginally Effective	Not Effective
		3	Effective	Not Effective	Marginally Effective	Not Effective
		4	Very Effective	Marginally Effective	Marginally Effective	Not Effective
		5	Very Effective	Marginally Effective	Effective	Marginally Effective

Figure 4.2: 20-Node, 4-Asset Example: Intelligence Asset Effectiveness Estimates

Table 4.4: 20-Node, 4-Asset Example: Expected Changes in Entropy (δ_{ij})

Expected Change in Entropy (δ_{ij})				
	Intelligence Asset			
Node	IMINT	ELINT	HUMINT	COMINT
1	5.312	0.197	3.545	1.017
2	2.767	2.946	2.033	1.930
3	0.230	0.000	0.488	0.342
4	3.359	2.860	3.548	6.197
5	6.353	0.526	4.263	1.218
6	1.736	0.000	2.694	2.364
7	1.301	1.347	2.050	3.327
8	0.098	0.000	1.717	1.380
9	2.288	0.063	1.526	0.449
10	1.959	1.542	1.752	2.965
11	3.534	0.000	5.388	5.196
12	0.325	0.000	0.655	0.512
13	1.016	0.049	0.590	0.200
14	0.498	0.000	0.753	0.722
15	0.078	0.000	1.464	1.118
16	0.718	0.581	0.765	1.290
17	0.718	0.088	0.456	0.088
18	1.600	0.149	1.082	0.328
19	2.198	2.152	2.198	3.690
20	1.649	0.092	1.015	0.359

4.3 20-Node, 4-Asset Model

The methodology for calculating the δ_{ij}'s presented in Chapter 3 was applied to each node/asset combination. See Appendix A for the calculations. The calculated δ_{ij}'s are summarized in Table 4.4. Next, using the provided w_{ij}'s and calculated δ_{ij}'s, a GAP was formulated to allocate the intelligence assets.

$$
\begin{aligned}
\text{maximize} \quad & z = \sum_{j=1}^{4} \sum_{i=1}^{20} \delta_{ij} x_{ij} \\
\text{subject to} \quad & \sum_{i=1}^{20} w_{i\,\mathrm{IM}} x_{i\,\mathrm{IM}} \leq 4 \\
& \sum_{i=1}^{20} w_{i\,\mathrm{EL}} x_{i\,\mathrm{EL}} \leq 6 \\
& \sum_{i=1}^{20} w_{i\,\mathrm{HUM}} x_{i\,\mathrm{HUM}} \leq 1 \\
& \sum_{i=1}^{20} w_{i\,\mathrm{COM}} x_{i\,\mathrm{COM}} \leq 6 \\
& \sum_{j=1}^{4} x_{ij} \leq 1, \qquad i \in \{1, \ldots, 20\} \\
& x_{ij} \in \{0, 1\}, \qquad i \in \{1, \ldots, 20\},\ j \in \{1, \ldots, 4\}
\end{aligned}
$$

where

$$
x_{ij} = \begin{cases} 1 & \text{if asset } j \text{ collects intelligence on node } i \\ 0 & \text{otherwise} \end{cases} .
$$

4.4 20-Node, 4-Asset Example: Results and Analysis

The results to the GAP are summarized in Table 4.5 on the following page, Table 4.6 on page 83, and Table 4.7 on page 83. Microsoft Excel's Solver was used to solve the GAP. The maximum expected reduction in entropy obtained was 43.05976. All *Priority 1* and *Priority 2* nodes were assigned an intelligence asset. Three of the four intelligence assets were fully tasked, while the ELINT source was under tasked (i.e., the ELINT capacity constraint was not binding) (refer to Table 4.6 on page 83). A quick look at the scenario lends insight into why the ELINT source was undertasked. First, the intelligence requirements, w_{ij}, for the ELINT source were higher for all node types except *radar*. There is only one *radar* in the scenario. In addition, the ELINT source was estimated to be *Not Effective* or *Marginally Effective*

Table 4.5: 20-Node, 4-Asset Example: GAP Results

Indicator Variable (x_{ij})					
	Intelligence Asset (j)				
Node (i)	IMINT	ELINT	HUMINT	COMINT	$\sum_j \delta_{ij} x_{ij}$
1	1	0	0	0	5.312294
2	0	1	0	0	2.945542
3	0	0	0	0	0
4	0	0	0	1	6.197174
5	1	0	0	0	6.352986
6	0	0	0	1	2.363909
7	0	0	0	1	3.327207
8	0	0	0	0	0
9	1	0	0	0	2.287929
10	0	0	0	1	2.964695
11	0	0	1	0	5.388013
12	0	0	0	0	0
13	0	0	0	0	0
14	0	0	0	0	0
15	0	0	0	0	0
16	0	1	0	0	0.580939
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	1	3.689797
20	1	0	0	0	1.649275
				$z =$	43.05976

against all node types except *radar*. Thus, collecting intelligence on these node types is more effectively performed by the other intelligence assets.

4.5 Extensions to the General Methodology

The GAP presented in Section 3.4.1 and illustrated in the 20-node, 4-asset example is intended to be a general methodology for allocating intelligence assets. A couple extensions to the methodology could greatly increase the utility of the methodology. First, Constraint (3.11) restricts the number of intelligence assets allocated to a single node to one. This restriction does not allow for the increased effectiveness of tasking multiple assets to collect information on a node. For example, consider the

Table 4.6: 20-Node, 4-Asset Example: Resource Usage

Resource Usage ($w_{ij}x_{ij}$)				
	Intelligence Asset(j)			
Node(i)	IMINT	ELINT	HUMINT	COMINT
1	1	0	0	0
2	0	1	0	0
3	0	0	0	0
4	0	0	0	1
5	1	0	0	0
6	0	0	0	2
7	0	0	0	1
8	0	0	0	0
9	1	0	0	0
10	0	0	0	1
11	0	0	1	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0
16	0	3	0	0
17	0	0	0	0
18	0	0	0	0
19	0	0	0	1
20	1	0	0	0
Total	4	4	1	6
Capacity (c_{ij})	4	6	1	6

Table 4.7: 20-Node, 4-Asset Example: Intelligence Asset Allocations

Intelligence Source	Node	Type	Priority
IMINT	1	Vehicle	1
	5	Vehicle	1
	9	Vehicle	2
	20	Vehicle	3
ELINT	2	Radar	2
	16	Communications	3
HUMINT	11	Economic	1
COMINT	4	Communications	1
	6	Economic	2
	7	Communications	2
	10	Communications	2
	19	Communications	2

event where information on a surface-to-air missile (SAM) system is to be collected. Using an IMINT source or an ELINT source by itself would not be as effective as combining the information provided by both intelligence assets to assess the current state of the SAM. Next, many intelligence assets are capable of collecting information on multiple nodes during a single intelligence collection effort. For example, a HUMINT source may be able to simultaneously collect intelligence on more than one node.

To address the need for packaging intelligence assets together to collect information on a node, the GAP may be reformulated using the variables x_{ig}, δ_{ig}, and ${w_{ig}}^j$, where

$$
\begin{aligned}
g &\in \mathcal{J} \\
\mathcal{J} &= \text{the set of all possible collections of intelligence assets} \\
x_{ig} &= \text{a binary indicator variable for whether } g \text{ is tasked to collect} \\
&\quad\ \text{intelligence on } N_i \\
\delta_{ig} &= \text{weighted expected change in entropy of } n_i \text{ given an intelligence} \\
&\quad\ \text{collection effort by the collection of intelligence assets } g \\
{w_{ig}}^j &= \text{amount of asset } j \text{ capacity required to accomplish intelligence} \\
&\quad\ \text{collection on } n_i \text{ given an intelligence collection effort by the} \\
&\quad\ \text{collection of intelligence assets } g \\
c_j &= \text{capacity of asset } j.
\end{aligned}
$$

The GAP is now

$$\text{maximize} \qquad z = \sum_{g \in \mathcal{J}} \sum_{i=1}^{m} \delta_{ig} x_{ig}$$

$$\begin{aligned} \text{subject to} \qquad \sum_{g \in \mathcal{J}} \sum_{i=1}^{m} w_{ig}{}^{j} x_{ig} &\leq c_j, & j &\in \{1, \ldots, n\} \\ \sum_{g \in \mathcal{J}} x_{ig} &\leq 1, & i &\in \{1, \ldots, m\} \\ x_{ig} &\in \{0, 1\}, & i &\in \{1, \ldots, m\}, \; g \in \mathcal{J} \end{aligned}$$

where

$$x_{ig} = \begin{cases} 1 & \text{if } g \text{ collects intelligence on node } i \\ 0 & \text{otherwise} \end{cases} .$$

Note, if $\mathcal{J}$ is confined to single-asset collections, then the problem is the same as the one presented in Section 3.4.1. However, if an operational restriction exists that no more than k assets can (or will) be tasked against a single asset, then the GAP consists of v indicator variables, where

$$v = \binom{n}{k} + \binom{n}{k-1} + \cdots + \binom{n}{1}.$$

This accounts for all possible ways of packaging k, $k-1$, $\ldots$, and 1 intelligence assets. Obviously, the number of variables could be greatly trimmed down by various elimination schemes. One example is to eliminate any collection of assets g that contains more than one of any single type of intelligence asset (e.g., g can not contain more than one IMINT source).

Similarly, to address the capability of a single asset to collect on multiple nodes, the indicator variables could be redefined to include sets of nodes. Then,

$$
\begin{aligned}
x_{hj} = {} & \text{a binary indicator variable for whether asset } j \text{ collects} \\
& \text{intelligence on the set of nodes } h
\end{aligned}
$$

where,

$$
\begin{aligned}
& h \in \mathcal{I} \\
& \mathcal{I} = \text{the set of possible collections of node(s).}
\end{aligned}
$$

To illustrate, given the 20-node, 4-asset example from Section 4.2, add the requirement that no more than two intelligence assets can be tasked against a single node. Furthermore, through geographical analysis, it is given that the ELINT source can collect information on n_4, n_7, and n_8 simultaneously. Let

$$
\begin{aligned}
\mathcal{I} &= \{1, \ldots, 20, (4, 7, 8)\} \\
\mathcal{J} &= \{(IM), (EL), (HUM), (COM), (IM, EL), (IM, HUM), \\
& \quad (IM, COM), (EL, HUM), (EL, COM), (HUM, COM)\} \\
x_{hg} &= \begin{cases} 1 & \text{if the collection of assets } g \text{ collects intelligence on node(s) } h \\ 0 & \text{otherwise} \end{cases}.
\end{aligned}
$$

Then,

$$\text{maximize} \qquad z = \sum_{h\in\mathcal{I}} \sum_{g\in\mathcal{J}} \delta_{hg} x_{hg} \tag{4.1}$$

$$\text{subject to} \qquad \sum_{h\in\mathcal{I}} \sum_{g\in\mathcal{J}} {w_{hg}}^{\text{IM}} x_{hg} \leq 4 \tag{4.2}$$

$$\sum_{h\in\mathcal{I}} \sum_{g\in\mathcal{J}} {w_{hg}}^{\text{EL}} x_{hg} \leq 6 \tag{4.3}$$

$$\sum_{h\in\mathcal{I}} \sum_{g\in\mathcal{J}} {w_{hg}}^{\text{HUM}} x_{hg} \leq 1 \tag{4.4}$$

$$\sum_{h\in\mathcal{I}} \sum_{g\in\mathcal{J}} {w_{hg}}^{\text{COM}} x_{hg} \leq 6 \tag{4.5}$$

$$\sum_{g\in\mathcal{J}} x_{hg} \leq 1, \qquad h \in \mathcal{I} \tag{4.6}$$

$$x_{hg} \in \{0,1\}, \qquad h \in \mathcal{I},\ g \in \mathcal{J}. \tag{4.7}$$

Equations (4.2) - (4.5) are the intelligence assets' capacity constraints, which are summed across all possible collections of nodes and all possible collections of assets. Equation (4.6) limits the number of collections of assets that may be allocated to a single node (or collection of nodes) to one. Finally, Equation (4.1) is the objective function for all possible collections of nodes and assets.

4.6 20-Node, 4-Asset Example: Summary

This example successfully demonstrated the application of the multiple intelligence asset allocation methodology. Although proven on a rather small example, the methodology may easily be applied to much larger scenarios.

V. Conclusions and Recommendations

5.1 Overview

This chapter presents the conclusions from this research effort and the recommendations for future research.

5.2 Conclusions

The primary goal of this research was to develop a methodology to prioritize the limited supply of intelligence collection efforts. Intelligence collection has become the limiting factor in performing military operations as there will always be less intelligence collection opportunities than intelligence targets. These intelligence collection efforts must be efficiently used in order to properly characterize an adversary's current state. This research's methodologies proved to be effective in prioritizing intelligence assets to maximize information gain while bound to intelligence asset constraints.

Due to the estimations for mapping of linguistic quantifiers to probabilities and asset effectiveness versus nodes, the methodology is not intended to be the final solution to prioritizing intelligence collection efforts. However, it will provide a good first-cut at prioritizing the assets, which can then be adjusted by planners. Much like the SoSA network within the EBO process, the major benefit of the method is to provide insight to planners. SoSA networks are constructed of numerous nodes and linkages; thus, when planning operations, it is difficult for an individual planner (or even a team of planners) to keep track of all the information contained in the network as well as available intelligence assets. Matching the two up is a daunting task without the assistance of a methodology like the one presented in this research.

In the development of this research's methodology, a mathematical model of the SoSA network and a method for translating intelligence linguistic quantifiers to probability distributions were developed. The SoSA mathematical model laid the framework for the subsequent methodology and may be used for other EBO applications. Using Kent's mapping of linguistic quantifiers to interval probabilities and the principle of maximum entropy, probability distributions were obtained for the linguistic quantifiers.

These probability distributions were then updated using a methodology to incorporate the priorities of the nodes, the dynamic nature of the nodes, and any actions planned against the nodes. A simple weighting scheme was used to account for nodal priorities. Although simple, the weighting scheme was effective in stratifying the different priority nodes and ensuring that intelligence assets were allocated to the most uncertain, highest priority nodes. The dynamic nature of the nodes was incorporated into the model using a time-dependent *erosion* of the probability distribution towards total uncertainty. Thus, as time passed since the last intelligence update, additional uncertainty was introduced into the probability distributions. Finally, estimates for an action's probability of success and posterior probability distributions given an action were used to update the probability distribution.

Once the prior probability distributions were updated, Shannon's relative entropy was used to quantify the uncertainty associated with the probability distributions. Then, using estimates for the intelligence asset effectiveness versus the nodes, the expected entropy of the posterior probability distributions given an intelligence collection effort was calculated. Next, the weighted expected reductions in entropy given a intelligence collection effort were calculated.

These expected reductions in entropy were then used in an integer program (IP) (specifically, the generalized assignment problem (GAP)) to allocate the intelligence assets in an efficient manner. To formulate the GAP, the *profits* per asset/node combination were the expected reductions in entropy calculated above. The *costs* of the asset/node combinations were defined to be the number of intelligence collection efforts required to perform an assessment of the node. The total *costs* for an asset type were constrained to the maximum capacity of the asset (i.e., maximum number of collection opportunities in a 24-hour period). The final constraint for the GAP was that at most one asset could collect information on a node. For a notional 20-

node, 4-asset example, the GAP was formulated and successfully solved providing an allocation of the intelligence assets.

Extensions to the GAP were presented in Chapter 4. The GAP presented in Chapter 3 is limited in that it does not permit multiple assets (i.e., collections of assets) to be tasked against a single asset. The synergistic effects of tasking multiple assets to collect information are not accounted for. In addition, the methodology does not allow for an asset to collect information on more than one node simultaneously, which may be accomplished operationally (i.e., based on geography or proximity). To address these limitations, the GAP was reformulated to include the capability to allocate sets of assets against sets of nodes.

5.3 Recommendations for Future Research

For this research, the nodes were assumed to independent. If an intelligence collection effort were directed at a node, the resulting reduction in uncertainty had no impact on neighboring nodes. Therefore, additional research should investigate the dependence between nodes. One of the SoSA goals is to "determine direct and indirect relationships between nodes within and across systems." [10] An investigation into the propagation of uncertainty across nodes may prove very useful and insightful in the prioritization methodology. For example, gathering information on a neighboring node may provide information on a target node at a lower cost than looking directly at the node.

Next, the midpoints of Kent's interval probabilities were used as point estimates to quantify the linguistic quantifiers. Although a good estimate of the linguistic quantifiers, using point estimates does not capture all the uncertainty associated with these linguistic quantifiers. Further research should investigate using Kent's interval probabilities to quantify the linguistic quantifiers. In turn, the continuous case of Shannon's entropy would need to be added to the methodology. Another technique that may be useful is using the upper and lower probabilities of Kent's intervals as bounds for the uncertainty present in the intelligence assessment.

Chapter 4 demonstrated a 20-node, 4-asset example which used notional asset effectiveness estimates. These estimates were then mapped to probabilities using a notional mapping scheme (i.e., *Effective* = 0.90). The sensitivity of this mapping needs further investigation. In addition, obtaining these estimates in a real-world application may prove to be difficult. Methods for obtaining these estimates, which are critical to the presented methodology, should be developed.

Also in Chapter 4, extensions to the GAP were discussed. These extensions included reformulating the problem to account for the packaging of multiple intelligence assets to collect information on a single node and/or the ability of a single intelligence asset to collect information on more than one node simultaneously. Research is needed to prove out the presented extensions to the methodologies. Additionally, the intelligence asset effectiveness estimates would need further exploration as packaging the assets would require additional estimates.

The nodal priority weighting scheme presented in this research used a simple mapping to assign weights to the various priority types. Further research should investigate other weighting schemes. The priority weights are highly influential in deciding the final intelligence asset allocation. Weighting schemes could be developed that are dependent upon the type of conflict or operation.

Finally, a quantitative comparison between the presented methodology and the current methodologies used in operations for prioritizing intelligence assets should be accomplished. Based on the allocation of intelligence assets from each methodology, the total reduction in entropy (i.e., information gain) across the entire network could be used as a measure of effectiveness to compare the two.

Node	Number of Discrete States	Priority	Priority Weight	"Freshness" of Intel (days)	Intel Assessment	State	Prior Probability	Action	Probability of Success of Action	Posterior Probability Given Action	Asset Effectiveness Estimate	P{Asset indicates N_i=x | N_i=x}	K	p'	p''	h	P{asset indicates N_i=x}	P{N_i=y | Asset Indicates N_i =x}						Branch Entropy	Expected Entropy	delta_i
																		0	1	2	3	4	5			
1	4	1	8	3	Almost Certainly Not in 1	0	0.310	1	0.75	0.600	Very Effective	0.980	0.50	0.270	0.518	0.833	0.521	0.973	0.017	0.009	0.001	-	-	0.106	0.169	5.312
						1	0.070			0.300	Effective	0.900		0.190	0.273		0.254	0.014	0.966	0.019	0.002	-	-	0.128		
						2	0.310			0.100	Effective	0.900		0.270	0.143		0.141	0.024	0.064	0.908	0.003	-	-	0.269		
						3	0.310			0.000	Very Effective	0.980		0.270	0.068		0.083	0.041	0.109	0.057	0.793	-	-	0.520		
2	4	2	4	2	Almost Certainly in 3	0	0.023	1	0.75	0.400	Effective	0.900	0.50	0.137	0.334	0.957	0.319	0.943	0.035	0.019	0.003	-	-	0.192	0.265	2.767
						1	0.023			0.400	Effective	0.900		0.137	0.334		0.319	0.035	0.943	0.019	0.003	-	-	0.192		
						2	0.023			0.200	Effective	0.900		0.137	0.184		0.189	0.059	0.059	0.877	0.005	-	-	0.344		
						3	0.930			0.000	Very Effective	0.980		0.590	0.148		0.173	0.064	0.064	0.035	0.836	-	-	0.448		
3	2	4	1	4	Probably Not in 1	0	0.700	1	0.75	0.900	Effective	0.900	1.00	0.550	0.813	0.696	0.778	0.940	0.060	-	-	-	-	0.328	0.466	0.230
						1	0.300			0.100	Marginally Effective	0.750		0.450	0.188		0.222	0.366	0.634	-	-	-	-	0.948		
4	4	1	8	2	Probably in 2	0	0.233	1	0.75	0.450	Effective	0.900	0.50	0.242	0.398	0.852	0.408	0.877	0.081	0.029	0.012	-	-	0.344	0.433	3.359
						1	0.233			0.450	Marginally Effective	0.750		0.242	0.398		0.329	0.040	0.908	0.036	0.015	-	-	0.290		
						2	0.300			0.100	Marginally Effective	0.750		0.275	0.144		0.159	0.083	0.208	0.677	0.032	-	-	0.654		
						3	0.233			0.000	Marginally Effective	0.750		0.242	0.060		0.104	0.128	0.320	0.116	0.437	-	-	0.894		
5	4	1	8	6	Almost Certainly Not in 3	0	0.310	0	0.00	0.000	Very Effective	0.980	0.50	0.260	0.260	0.998	0.274	0.931	0.032	0.032	0.005	-	-	0.226	0.204	6.353
						1	0.310			0.000	Effective	0.900		0.260	0.260		0.246	0.007	0.952	0.035	0.006	-	-	0.166		
						2	0.310			0.000	Effective	0.900		0.260	0.260		0.246	0.007	0.035	0.952	0.006	-	-	0.166		
						3	0.070			0.000	Very Effective	0.980		0.220	0.220		0.235	0.007	0.037	0.037	0.919	-	-	0.258		
6	4	2	4	2	Chances Even in 2	0	0.167	1	0.50	0.800	Effective	0.900	0.50	0.208	0.504	0.873	0.495	0.917	0.026	0.040	0.018	-	-	0.270	0.439	1.736
						1	0.167			0.100	Marginally Effective	0.750		0.208	0.154		0.161	0.104	0.719	0.123	0.054	-	-	0.641		
						2	0.500			0.100	Marginally Effective	0.750		0.375	0.238		0.216	0.078	0.059	0.823	0.040	-	-	0.473		
						3	0.167			0.000	Marginally Effective	0.750		0.208	0.104		0.128	0.132	0.101	0.155	0.612	-	-	0.785		
7	2	2	4	2	Probably Not in 0	0	0.300	0	0.00	0.000	Effective	0.900	1.00	0.400	0.400	0.971	0.510	0.706	0.294	-	-	-	-	0.874	0.646	1.301
						1	0.700			0.000	Marginally Effective	0.750		0.600	0.600		0.490	0.082	0.918	-	-	-	-	0.408		
8	2	3	2	2	Chances Even in 0	2	0.500	0	0.00	0.000	Not Effective	0.500	1.00	0.500	0.500	1.000	0.375	0.667	0.333	-	-	-	-	0.918	0.951	0.098
						3	0.500			0.000	Marginally Effective	0.750		0.500	0.500		0.625	0.400	0.600	-	-	-	-	0.971		
9	4	2	4	2	Probably in 1	0	0.083	1	0.75	0.750	Very Effective	0.980	0.50	0.167	0.604	0.716	0.604	0.980	0.015	0.004	0.000	-	-	0.080	0.144	2.288
						1	0.750			0.200	Effective	0.900		0.500	0.275		0.254	0.016	0.973	0.010	0.001	-	-	0.106		
						2	0.083			0.050	Effective	0.900		0.167	0.079		0.085	0.048	0.108	0.841	0.003	-	-	0.397		
						3	0.083			0.000	Very Effective	0.980		0.167	0.042		0.057	0.071	0.162	0.047	0.721	-	-	0.621		
10	6	2	4	2	Almost Certainly in 3	0	0.014	1	0.90	0.300	Effective	0.900	0.39	0.090	0.279	0.800	0.279	0.901	0.020	0.050	0.026	0.002	0.002	0.244	0.310	1.959
						1	0.014			0.300	Effective	0.900		0.090	0.279		0.279	0.020	0.901	0.050	0.026	0.002	0.002	0.244		
						2	0.014			0.300	Marginally Effective	0.750		0.090	0.279		0.229	0.024	0.024	0.916	0.032	0.002	0.002	0.221		
						3	0.930			0.100	Marginally Effective	0.750		0.548	0.145		0.135	0.041	0.041	0.104	0.807	0.003	0.003	0.396		
						4	0.014			0.000	Marginally Effective	0.750		0.090	0.009		0.040	0.141	0.141	0.352	0.183	0.171	0.011	0.884		
						5	0.014			0.000	Marginally Effective	0.750		0.090	0.009		0.040	0.141	0.141	0.352	0.183	0.011	0.171	0.884		
11	6	1	8	3	Almost Certainly in 4	0	0.014	0	0.00	0.000	Effective	0.900	0.39	0.116	0.116	0.900	0.148	0.702	0.039	0.039	0.039	0.142	0.039	0.576	0.458	3.534
						1	0.014			0.000	Marginally Effective	0.750		0.116	0.116		0.128	0.018	0.681	0.045	0.045	0.165	0.045	0.588		
						2	0.014			0.000	Marginally Effective	0.750		0.116	0.116		0.128	0.018	0.045	0.681	0.045	0.165	0.045	0.588		
						3	0.014			0.000	Marginally Effective	0.750		0.116	0.116		0.128	0.018	0.045	0.045	0.681	0.165	0.045	0.588		
						4	0.930			0.000	Marginally Effective	0.750		0.421	0.421		0.341	0.007	0.017	0.017	0.017	0.925	0.017	0.213		
						5	0.014			0.000	Marginally Effective	0.750		0.116	0.116		0.128	0.018	0.045	0.045	0.045	0.165	0.681	0.588		
12	2	4	1	2	Probably Not in 0	0	0.300	0	0.00	0.000	Effective	0.900	1.00	0.400	0.400	0.971	0.510	0.706	0.294	-	-	-	-	0.874	0.646	0.325
						1	0.700			0.000	Marginally Effective	0.750		0.600	0.600		0.490	0.082	0.918	-	-	-	-	0.408		
13	2	3	2	2	Probably in 1	0	0.250	1	0.90	0.900	Very Effective	0.980	1.00	0.375	0.848	0.616	0.834	0.996	0.004	-	-	-	-	0.035	0.108	1.016
						1	0.750			0.100	Very Effective	0.980		0.625	0.153		0.166	0.102	0.898	-	-	-	-	0.475		
14	6	4	1	5	Chances Even in 3	0	0.100	0	0.00	0.000	Effective	0.900	0.39	0.153	0.153	0.992	0.180	0.765	0.043	0.043	0.065	0.043	0.043	0.513	0.494	0.498
						1	0.100			0.000	Marginally Effective	0.750		0.153	0.153		0.153	0.020	0.753	0.050	0.076	0.050	0.050	0.524		
						2	0.100			0.000	Marginally Effective	0.750		0.153	0.153		0.153	0.020	0.050	0.753	0.076	0.050	0.050	0.524		
						3	0.500			0.000	Marginally Effective	0.750		0.233	0.233		0.209	0.015	0.037	0.037	0.838	0.037	0.037	0.388		
						4	0.100			0.000	Marginally Effective	0.750		0.153	0.153		0.153	0.020	0.050	0.050	0.076	0.753	0.050	0.524		
						5	0.100			0.000	Marginally Effective	0.750		0.153	0.153		0.153	0.020	0.050	0.050	0.076	0.050	0.753	0.524		
15	2	3	2	2	Almost Certainly in 0	0	0.930	0	0.00	0.000	Not Effective	0.500	1.00	0.715	0.715	0.862	0.429	0.834	0.166	-	-	-	-	0.649	0.823	0.078
						1	0.070			0.000	Marginally Effective	0.750		0.285	0.285		0.571	0.626	0.374	-	-	-	-	0.954		
16	4	3	2	2	Probably Not in 2	0	0.233	1	0.75	0.800	Effective	0.900	0.50	0.242	0.660	0.716	0.623	0.955	0.018	0.019	0.008	-	-	0.167	0.357	0.718
						1	0.233			0.100	Marginally Effective	0.750		0.242	0.135		0.141	0.157	0.722	0.085	0.036	-	-	0.616		
						2	0.300			0.100	Marginally Effective	0.750		0.275	0.144		0.146	0.151	0.077	0.738	0.034	-	-	0.594		
						3	0.233			0.000	Marginally Effective	0.750		0.242	0.060		0.091	0.243	0.125	0.132	0.500	-	-	0.878		
17	6	4	1	2	Probably in 3	0	0.050	0	0.00	0.000	Very Effective	0.980	0.39	0.108	0.108	0.871	0.119	0.894	0.004	0.018	0.077	0.004	0.004	0.241	0.154	0.718
						1	0.050			0.000	Very Effective	0.980		0.108	0.108		0.119	0.004	0.894	0.018	0.077	0.004	0.004	0.241		
						2	0.050			0.000	Effective	0.900		0.108	0.108		0.108	0.004	0.004	0.899	0.085	0.004	0.004	0.219		
						3	0.750			0.000	Effective	0.900		0.458	0.458		0.416	0.001	0.001	0.005	0.991	0.001	0.001	0.036		
						4	0.050			0.000	Very Effective	0.980		0.108	0.108		0.119	0.004	0.004	0.018	0.077	0.894	0.004	0.241		
						5	0.050			0.000	Very Effective	0.980		0.108	0.108		0.119	0.004	0.004	0.018	0.077	0.004	0.894	0.241		
18	4	3	2	4	Almost Certainly Not 1	0	0.310	0	0.00	0.000	Very Effective	0.980	0.50	0.265	0.265	0.996	0.277	0.937	0.025	0.032	0.006	-	-	0.212	0.196	1.600
						1	0.070			0.000	Effective	0.900		0.205	0.205		0.197	0.009	0.937	0.045	0.009	-	-	0.205		
						2	0.310			0.000	Effective	0.900		0.265	0.265		0.249	0.007	0.027	0.958	0.007	-	-	0.151		
						3	0.310			0.000	Very Effective	0.980		0.265	0.265		0.277	0.006	0.025	0.032	0.937	-	-	0.212		
19	6	2	4	2	Almost Certainly Not in 4	0	0.186	0	0.00	0.000	Effective	0.900	0.39	0.176	0.176	0.995	0.195	0.816	0.018	0.045	0.045	0.030	0.045	0.427	0.445	2.198
						1	0.186			0.000	Effective	0.900		0.176	0.176		0.195	0.018	0.816	0.045	0.045	0.030	0.045	0.427		
						2	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.163	0.022	0.022	0.812	0.054	0.036	0.054	0.430		
						3	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.163	0.022	0.022	0.054	0.812	0.036	0.054	0.430		
						4	0.070			0.000	Marginally Effective	0.750		0.118	0.118		0.122	0.029	0.029	0.072	0.072	0.726	0.072	0.561		
						5	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.163	0.022	0.022	0.054	0.054	0.036	0.812	0.430		
20	2	3	2	2	Chances Even in 1	0	0.500	1	0.25	0.950	Very Effective	0.980	1.00	0.500	0.613	0.963	0.608	0.987	0.013	-	-	-	-	0.098	0.139	1.649
						1	0.500			0.050	Very Effective	0.980		0.500	0.388		0.392	0.031	0.969	-	-	-	-	0.201		

Figure A.1: 20-Node, 4-Asset Example: δ_{ij} Calculations–IMINT

Node	Number of Discrete States	Priority	Priority Weight	"Freshness" of Intel (days)	Intel Assessment	State	Prior Probability	Action	Probability of Success of Action	Posterior Probability Given Action	Asset Effectiveness Estimate	P{Asset indicates N_i=x | N_i=x}	K	p'	p''	h	P{asset indicates N_i=x}	P{N_i=y | Asset Indicates N_i =x}						Branch Entropy	Expected Entropy	delta_i
																		0	1	2	3	4	5			
1	4	1	8	3	Almost Certainly Not in 1	0	0.310	1	0.75	0.600	Not Effective	0.250	0.50	0.270	0.518	0.833	0.239	0.542	0.285	0.149	0.024	-	-	0.766	0.808	0.197
						1	0.070			0.300	Not Effective	0.250		0.190	0.273		0.239	0.542	0.285	0.149	0.024	-	-	0.766		
						2	0.310			0.100	Not Effective	0.250		0.270	0.143		0.239	0.542	0.285	0.149	0.024	-	-	0.766		
						3	0.310			0.000	Marginally Effective	0.750		0.270	0.068		0.284	0.456	0.240	0.126	0.178	-	-	0.915		
2	4	2	4	2	Almost Certainly in 3	0	0.023	1	0.75	0.400	Effective	0.900	0.50	0.137	0.334	0.957	0.314	0.957	0.035	0.004	0.003	-	-	0.144	0.220	2.946
						1	0.023			0.400	Effective	0.900		0.137	0.334		0.314	0.035	0.957	0.004	0.003	-	-	0.144		
						2	0.023			0.200	Very Effective	0.980		0.137	0.184		0.204	0.055	0.055	0.886	0.005	-	-	0.325		
						3	0.930			0.000	Very Effective	0.980		0.590	0.148		0.168	0.066	0.066	0.007	0.860	-	-	0.379		
3	2	4	1	4	Probably Not in 1	0	0.700	1	0.75	0.900	Not Effective	0.500	1.00	0.550	0.813	0.696	0.500	0.813	0.188	-	-	-	-	0.696	0.696	0.000
						1	0.300			0.100	Not Effective	0.500		0.450	0.188		0.500	0.813	0.188	-	-	-	-	0.696		
4	4	1	8	2	Probably in 2	0	0.233	1	0.75	0.450	Marginally Effective	0.750	0.50	0.242	0.398	0.852	0.346	0.864	0.096	0.035	0.006	-	-	0.359	0.495	2.860
						1	0.233			0.450	Marginally Effective	0.750		0.242	0.398		0.346	0.096	0.864	0.035	0.006	-	-	0.359		
						2	0.300			0.100	Marginally Effective	0.750		0.275	0.144		0.176	0.188	0.188	0.612	0.011	-	-	0.707		
						3	0.233			0.000	Effective	0.900		0.242	0.060		0.133	0.250	0.250	0.090	0.410	-	-	0.920		
5	4	1	8	6	Almost Certainly Not in 3	0	0.310	0	0.00	0.000	Not Effective	0.250	0.50	0.260	0.260	0.998	0.213	0.305	0.305	0.305	0.086	-	-	0.936	0.933	0.526
						1	0.310			0.000	Not Effective	0.250		0.260	0.260		0.213	0.305	0.305	0.305	0.086	-	-	0.936		
						2	0.310			0.000	Not Effective	0.250		0.260	0.260		0.213	0.305	0.305	0.305	0.086	-	-	0.936		
						3	0.070			0.000	Marginally Effective	0.750		0.220	0.220		0.360	0.181	0.181	0.181	0.458	-	-	0.927		
6	4	2	4	2	Chances Even in 2	0	0.167	1	0.50	0.800	Not Effective	0.250	0.50	0.208	0.504	0.873	0.250	0.504	0.154	0.238	0.104	-	-	0.873	0.873	0.000
						1	0.167			0.100	Not Effective	0.250		0.208	0.154		0.250	0.504	0.154	0.238	0.104	-	-	0.873		
						2	0.500			0.100	Not Effective	0.250		0.375	0.238		0.250	0.504	0.154	0.238	0.104	-	-	0.873		
						3	0.167			0.000	Not Effective	0.250		0.208	0.104		0.250	0.504	0.154	0.238	0.104	-	-	0.873		
7	2	2	4	2	Probably Not in 0	0	0.300	0	0.00	0.000	Marginally Effective	0.750	1.00	0.400	0.400	0.971	0.360	0.833	0.167	-	-	-	-	0.650	0.634	1.347
						1	0.700			0.000	Effective	0.900		0.600	0.600		0.640	0.156	0.844	-	-	-	-	0.625		
8	2	3	2	2	Chances Even in 0	2	0.500	0	0.00	0.000	Not Effective	0.500	1.00	0.500	0.500	1.000	0.500	0.500	0.500	-	-	-	-	1.000	1.000	0.000
						3	0.500			0.000	Not Effective	0.500		0.500	0.500		0.500	0.500	0.500	-	-	-	-	1.000		
9	4	2	4	2	Probably in 1	0	0.083	1	0.75	0.750	Not Effective	0.250	0.50	0.167	0.604	0.716	0.243	0.621	0.283	0.081	0.014	-	-	0.662	0.700	0.063
						1	0.750			0.200	Not Effective	0.250		0.500	0.275		0.243	0.621	0.283	0.081	0.014	-	-	0.662		
						2	0.083			0.050	Not Effective	0.250		0.167	0.079		0.243	0.621	0.283	0.081	0.014	-	-	0.662		
						3	0.083			0.000	Marginally Effective	0.750		0.167	0.042		0.271	0.558	0.254	0.073	0.115	-	-	0.804		
10	6	2	4	2	Almost Certainly in 3	0	0.014	1	0.90	0.300	Marginally Effective	0.750	0.39	0.090	0.279	0.800	0.245	0.855	0.057	0.057	0.030	0.001	0.001	0.321	0.414	1.542
						1	0.014			0.300	Marginally Effective	0.750		0.090	0.279		0.245	0.057	0.855	0.057	0.030	0.001	0.001	0.321		
						2	0.014			0.300	Marginally Effective	0.750		0.090	0.279		0.245	0.057	0.057	0.855	0.030	0.001	0.001	0.321		
						3	0.930			0.100	Marginally Effective	0.750		0.548	0.145		0.151	0.092	0.092	0.092	0.720	0.001	0.001	0.510		
						4	0.014			0.000	Effective	0.900		0.090	0.009		0.057	0.243	0.243	0.243	0.126	0.142	0.003	0.886		
						5	0.014			0.000	Effective	0.900		0.090	0.009		0.057	0.243	0.243	0.243	0.126	0.003	0.142	0.886		
11	6	1	8	3	Almost Certainly in 4	0	0.014	0	0.00	0.000	Not Effective	0.167	0.39	0.116	0.116	0.900	0.167	0.116	0.116	0.116	0.116	0.421	0.116	0.900	0.900	0.000
						1	0.014			0.000	Not Effective	0.167		0.116	0.116		0.167	0.116	0.116	0.116	0.116	0.421	0.116	0.900		
						2	0.014			0.000	Not Effective	0.167		0.116	0.116		0.167	0.116	0.116	0.116	0.116	0.421	0.116	0.900		
						3	0.014			0.000	Not Effective	0.167		0.116	0.116		0.167	0.116	0.116	0.116	0.116	0.421	0.116	0.900		
						4	0.930			0.000	Not Effective	0.167		0.421	0.421		0.167	0.116	0.116	0.116	0.116	0.421	0.116	0.900		
						5	0.014			0.000	Not Effective	0.167		0.116	0.116		0.167	0.116	0.116	0.116	0.116	0.421	0.116	0.900		
12	2	4	1	2	Probably Not in 0	0	0.300	0	0.00	0.000	Not Effective	0.500	1.00	0.400	0.400	0.971	0.500	0.400	0.600	-	-	-	-	0.971	0.971	0.000
						1	0.700			0.000	Not Effective	0.500		0.600	0.600		0.500	0.400	0.600	-	-	-	-	0.971		
13	2	3	2	2	Probably in 1	0	0.250	1	0.90	0.900	Not Effective	0.500	1.00	0.375	0.848	0.616	0.462	0.917	0.083	-	-	-	-	0.411	0.591	0.049
						1	0.750			0.100	Marginally Effective	0.750		0.625	0.153		0.538	0.787	0.213	-	-	-	-	0.746		
14	6	4	1	5	Chances Even in 3	0	0.100	0	0.00	0.000	Not Effective	0.167	0.39	0.153	0.153	0.992	0.167	0.153	0.153	0.153	0.233	0.153	0.153	0.992	0.992	0.000
						1	0.100			0.000	Not Effective	0.167		0.153	0.153		0.167	0.153	0.153	0.153	0.233	0.153	0.153	0.992		
						2	0.100			0.000	Not Effective	0.167		0.153	0.153		0.167	0.153	0.153	0.153	0.233	0.153	0.153	0.992		
						3	0.500			0.000	Not Effective	0.167		0.233	0.233		0.167	0.153	0.153	0.153	0.233	0.153	0.153	0.992		
						4	0.100			0.000	Not Effective	0.167		0.153	0.153		0.167	0.153	0.153	0.153	0.233	0.153	0.153	0.992		
						5	0.100			0.000	Not Effective	0.167		0.153	0.153		0.167	0.153	0.153	0.153	0.233	0.153	0.153	0.992		
15	2	3	2	2	Almost Certainly in 0	0	0.930	0	0.00	0.000	Not Effective	0.500	1.00	0.715	0.715	0.862	0.500	0.715	0.285	-	-	-	-	0.862	0.862	0.000
						1	0.070			0.000	Not Effective	0.500		0.285	0.285		0.500	0.715	0.285	-	-	-	-	0.862		
16	4	3	2	2	Probably Not in 2	0	0.233	1	0.75	0.800	Marginally Effective	0.750	0.50	0.242	0.660	0.716	0.521	0.951	0.022	0.023	0.004	-	-	0.172	0.426	0.581
						1	0.233			0.100	Marginally Effective	0.750		0.242	0.135		0.171	0.323	0.595	0.070	0.012	-	-	0.658		
						2	0.300			0.100	Marginally Effective	0.750		0.275	0.144		0.176	0.312	0.064	0.612	0.011	-	-	0.643		
						3	0.233			0.000	Effective	0.900		0.242	0.060		0.133	0.415	0.085	0.090	0.410	-	-	0.835		
17	6	4	1	2	Probably in 3	0	0.050	0	0.00	0.000	Not Effective	0.167	0.39	0.108	0.108	0.871	0.141	0.128	0.128	0.128	0.540	0.038	0.038	0.765	0.784	0.088
						1	0.050			0.000	Not Effective	0.167		0.108	0.108		0.141	0.128	0.128	0.128	0.540	0.038	0.038	0.765		
						2	0.050			0.000	Not Effective	0.167		0.108	0.108		0.141	0.128	0.128	0.128	0.540	0.038	0.038	0.765		
						3	0.750			0.000	Not Effective	0.167		0.458	0.458		0.141	0.128	0.128	0.128	0.540	0.038	0.038	0.765		
						4	0.050			0.000	Marginally Effective	0.750		0.108	0.108		0.217	0.083	0.083	0.083	0.352	0.374	0.025	0.808		
						5	0.050			0.000	Marginally Effective	0.750		0.108	0.108		0.217	0.083	0.083	0.083	0.352	0.025	0.374	0.808		
18	4	3	2	4	Almost Certainly Not 1	0	0.310	0	0.00	0.000	Not Effective	0.250	0.50	0.265	0.265	0.996	0.206	0.322	0.249	0.322	0.107	-	-	0.949	0.922	0.149
						1	0.070			0.000	Not Effective	0.250		0.205	0.205		0.206	0.322	0.249	0.322	0.107	-	-	0.949		
						2	0.310			0.000	Not Effective	0.250		0.265	0.265		0.206	0.322	0.249	0.322	0.107	-	-	0.949		
						3	0.310			0.000	Marginally Effective	0.750		0.265	0.265		0.383	0.173	0.134	0.173	0.520	-	-	0.878		
19	6	2	4	2	Almost Certainly Not in 4	0	0.186	0	0.00	0.000	Marginally Effective	0.750	0.39	0.176	0.176	0.995	0.165	0.803	0.054	0.054	0.054	0.014	0.021	0.441	0.457	2.152
						1	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.165	0.054	0.803	0.054	0.054	0.014	0.021	0.441		
						2	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.165	0.054	0.054	0.803	0.054	0.014	0.021	0.441		
						3	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.165	0.054	0.054	0.054	0.803	0.014	0.021	0.441		
						4	0.070			0.000	Effective	0.900		0.118	0.118		0.145	0.061	0.061	0.061	0.061	0.733	0.024	0.557		
						5	0.186			0.000	Effective	0.900		0.176	0.176		0.196	0.045	0.045	0.045	0.045	0.012	0.808	0.437		
20	2	3	2	2	Chances Even in 1	0	0.500	1	0.25	0.950	Not Effective	0.500	1.00	0.500	0.613	0.963	0.403	0.760	0.240	-	-	-	-	0.796	0.917	0.092
						1	0.500			0.050	Marginally Effective	0.750		0.500	0.388		0.597	0.513	0.487	-	-	-	-	1.000		

Figure A.2: 20-Node, 4-Asset Example: δ_{ij} Calculations–ELINT

Node	Number of Discrete States	Priority	Priority Weight	"Freshness" of Intel (days)	Intel Assessment	State	Prior Probability	Action	Probability of Success of Action	Posterior Probability Given Action	Asset Effectiveness Estimate	$P\{\text{Asset indicates } N_i=x \mid N_i=x\}$	K	p'	p''	h	$P\{\text{Asset indicates } N_i=x\}$	$P\{N_i=y \mid \text{Asset Indicates } N_i=x\}$						Branch Entropy	Expected Entropy	delta_i
																		0	1	2	3	4	5			
1	4	1	8	3	Almost Certainly Not in 1	0	0.310	1	0.75	0.600	Effective	0.900	0.50	0.270	0.518	0.833	0.503	0.927	0.045	0.024	0.004	-	-	0.233	0.390	3.545
						1	0.070			0.300	Marginally Effective	0.750		0.190	0.273		0.236	0.073	0.867	0.050	0.010	-	-	0.368		
						2	0.310			0.100	Marginally Effective	0.750		0.270	0.143		0.149	0.116	0.152	0.717	0.015	-	-	0.605		
						3	0.310			0.000	Effective	0.900		0.270	0.068		0.113	0.153	0.202	0.105	0.540	-	-	0.852		
2	4	2	4	2	Almost Certainly in 3	0	0.023	1	0.75	0.400	Effective	0.900	0.50	0.137	0.334	0.957	0.349	0.862	0.080	0.044	0.014	-	-	0.380	0.449	2.033
						1	0.023			0.400	Marginally Effective	0.750		0.137	0.334		0.282	0.039	0.889	0.054	0.017	-	-	0.333		
						2	0.023			0.200	Marginally Effective	0.750		0.137	0.184		0.182	0.061	0.153	0.759	0.027	-	-	0.552		
						3	0.930			0.000	Effective	0.900		0.590	0.148		0.187	0.060	0.149	0.082	0.710	-	-	0.649		
3	2	4	1	4	Probably Not in 1	0	0.700	1	0.75	0.900	Very Effective	0.980	1.00	0.550	0.813	0.696	0.815	0.977	0.023	-	-	-	-	0.158	0.208	0.488
						1	0.300			0.100	Effective	0.900		0.450	0.188		0.185	0.088	0.912	-	-	-	-	0.429		
4	4	1	8	2	Probably in 2	0	0.233	1	0.75	0.450	Effective	0.900	0.50	0.242	0.398	0.852	0.405	0.884	0.082	0.030	0.005	-	-	0.321	0.409	3.548
						1	0.233			0.450	Marginally Effective	0.750		0.242	0.398		0.326	0.041	0.916	0.037	0.006	-	-	0.262		
						2	0.300			0.100	Marginally Effective	0.750		0.275	0.144		0.156	0.085	0.212	0.690	0.013	-	-	0.613		
						3	0.233			0.000	Effective	0.900		0.242	0.060		0.113	0.118	0.294	0.106	0.482	-	-	0.867		
5	4	1	8	6	Almost Certainly Not in 3	0	0.310	0	0.00	0.000	Effective	0.900	0.50	0.260	0.260	0.998	0.285	0.822	0.076	0.076	0.026	-	-	0.467	0.465	4.263
						1	0.310			0.000	Marginally Effective	0.750		0.260	0.260		0.233	0.037	0.838	0.093	0.032	-	-	0.433		
						2	0.310			0.000	Marginally Effective	0.750		0.260	0.260		0.233	0.037	0.093	0.838	0.032	-	-	0.433		
						3	0.070			0.000	Effective	0.900		0.220	0.220		0.250	0.035	0.087	0.087	0.792	-	-	0.523		
6	4	2	4	2	Chances Even in 2	0	0.167	1	0.50	0.800	Very Effective	0.980	0.50	0.208	0.504	0.873	0.511	0.968	0.010	0.016	0.007	-	-	0.127	0.200	2.694
						1	0.167			0.100	Effective	0.900		0.208	0.154		0.154	0.022	0.904	0.052	0.023	-	-	0.298		
						2	0.500			0.100	Effective	0.900		0.375	0.238		0.226	0.015	0.023	0.947	0.015	-	-	0.191		
						3	0.167			0.000	Effective	0.900		0.208	0.104		0.110	0.031	0.047	0.072	0.851	-	-	0.415		
7	2	2	4	2	Probably Not in 0	0	0.300	0	0.00	0.000	Effective	0.900	1.00	0.400	0.400	0.971	0.420	0.857	0.143	-	-	-	-	0.592	0.458	2.050
						1	0.700			0.000	Effective	0.900		0.600	0.600		0.580	0.069	0.931	-	-	-	-	0.362		
8	2	3	2	2	Chances Even in 0	2	0.500	0	0.00	0.000	Very Effective	0.980	1.00	0.500	0.500	1.000	0.500	0.980	0.020	-	-	-	-	0.141	0.141	1.717
						3	0.500			0.000	Very Effective	0.980		0.500	0.500		0.500	0.020	0.980	-	-	-	-	0.141		
9	4	2	4	2	Probably in 1	0	0.083	1	0.75	0.750	Effective	0.900	0.50	0.167	0.604	0.716	0.575	0.946	0.040	0.011	0.002	-	-	0.178	0.335	1.526
						1	0.750			0.200	Marginally Effective	0.750		0.500	0.275		0.234	0.086	0.880	0.028	0.006	-	-	0.328		
						2	0.083			0.050	Marginally Effective	0.750		0.167	0.079		0.104	0.194	0.221	0.572	0.013	-	-	0.742		
						3	0.083			0.000	Effective	0.900		0.167	0.042		0.087	0.231	0.263	0.076	0.430	-	-	0.900		
10	6	2	4	2	Almost Certainly in 3	0	0.014	1	0.90	0.300	Effective	0.900	0.39	0.090	0.279	0.800	0.287	0.875	0.049	0.049	0.025	0.002	0.001	0.289	0.362	1.752
						1	0.014			0.300	Marginally Effective	0.750		0.090	0.279		0.237	0.024	0.884	0.059	0.031	0.002	0.001	0.272		
						2	0.014			0.300	Marginally Effective	0.750		0.090	0.279		0.237	0.024	0.059	0.884	0.031	0.002	0.001	0.272		
						3	0.930			0.100	Marginally Effective	0.750		0.548	0.145		0.143	0.039	0.098	0.098	0.761	0.003	0.001	0.455		
						4	0.014			0.000	Marginally Effective	0.750		0.090	0.009		0.048	0.117	0.293	0.293	0.152	0.142	0.004	0.868		
						5	0.014			0.000	Effective	0.900		0.090	0.009		0.049	0.113	0.283	0.283	0.147	0.009	0.165	0.883		
11	6	1	8	3	Almost Certainly in 4	0	0.014	0	0.00	0.000	Very Effective	0.980	0.39	0.116	0.116	0.900	0.131	0.865	0.018	0.018	0.018	0.064	0.018	0.327	0.226	5.388
						1	0.014			0.000	Effective	0.900		0.116	0.116		0.120	0.004	0.868	0.019	0.019	0.070	0.019	0.312		
						2	0.014			0.000	Effective	0.900		0.116	0.116		0.120	0.004	0.019	0.868	0.019	0.070	0.019	0.312		
						3	0.014			0.000	Effective	0.900		0.116	0.116		0.120	0.004	0.019	0.019	0.868	0.070	0.019	0.312		
						4	0.930			0.000	Effective	0.900		0.421	0.421		0.389	0.001	0.006	0.006	0.006	0.975	0.006	0.086		
						5	0.014			0.000	Effective	0.900		0.116	0.116		0.120	0.004	0.019	0.019	0.019	0.070	0.868	0.312		
12	2	4	1	2	Probably Not in 0	0	0.300	0	0.00	0.000	Very Effective	0.980	1.00	0.400	0.400	0.971	0.452	0.867	0.133	-	-	-	-	0.565	0.316	0.655
						1	0.700			0.000	Effective	0.900		0.600	0.600		0.548	0.015	0.985	-	-	-	-	0.110		
13	2	3	2	2	Probably in 1	0	0.250	1	0.90	0.900	Effective	0.900	1.00	0.375	0.848	0.616	0.778	0.980	0.020	-	-	-	-	0.139	0.321	0.590
						1	0.750			0.100	Effective	0.900		0.625	0.153		0.222	0.382	0.618	-	-	-	-	0.959		
14	6	4	1	5	Chances Even in 3	0	0.100	0	0.00	0.000	Very Effective	0.980	0.39	0.153	0.153	0.992	0.167	0.899	0.018	0.018	0.028	0.018	0.018	0.273	0.239	0.753
						1	0.100			0.000	Effective	0.900		0.153	0.153		0.152	0.004	0.905	0.020	0.031	0.020	0.020	0.254		
						2	0.100			0.000	Effective	0.900		0.153	0.153		0.152	0.004	0.020	0.905	0.031	0.020	0.020	0.254		
						3	0.500			0.000	Effective	0.900		0.233	0.233		0.223	0.003	0.014	0.014	0.942	0.014	0.014	0.172		
						4	0.100			0.000	Effective	0.900		0.153	0.153		0.152	0.004	0.020	0.020	0.031	0.905	0.020	0.254		
						5	0.100			0.000	Effective	0.900		0.153	0.153		0.152	0.004	0.020	0.020	0.031	0.020	0.905	0.254		
15	2	3	2	2	Almost Certainly in 0	0	0.930	0	0.00	0.000	Very Effective	0.980	1.00	0.715	0.715	0.862	0.706	0.992	0.008	-	-	-	-	0.068	0.130	1.464
						1	0.070			0.000	Very Effective	0.980		0.285	0.285		0.294	0.049	0.951	-	-	-	-	0.281		
16	4	3	2	2	Probably Not in 2	0	0.233	1	0.75	0.800	Effective	0.900	0.50	0.242	0.660	0.716	0.620	0.959	0.018	0.019	0.003	-	-	0.150	0.334	0.765
						1	0.233			0.100	Marginally Effective	0.750		0.242	0.135		0.138	0.160	0.738	0.087	0.015	-	-	0.571		
						2	0.300			0.100	Marginally Effective	0.750		0.275	0.144		0.143	0.154	0.079	0.753	0.014	-	-	0.549		
						3	0.233			0.000	Effective	0.900		0.242	0.060		0.100	0.221	0.113	0.120	0.546	-	-	0.841		
17	6	4	1	2	Probably in 3	0	0.050	0	0.00	0.000	Effective	0.900	0.39	0.108	0.108	0.871	0.139	0.702	0.039	0.039	0.165	0.039	0.016	0.553	0.416	0.456
						1	0.050			0.000	Marginally Effective	0.750		0.108	0.108		0.119	0.018	0.681	0.045	0.192	0.045	0.018	0.561		
						2	0.050			0.000	Marginally Effective	0.750		0.108	0.108		0.119	0.018	0.045	0.681	0.192	0.045	0.018	0.561		
						3	0.750			0.000	Marginally Effective	0.750		0.458	0.458		0.364	0.006	0.015	0.015	0.944	0.015	0.006	0.169		
						4	0.050			0.000	Marginally Effective	0.750		0.108	0.108		0.119	0.018	0.045	0.045	0.192	0.681	0.018	0.561		
						5	0.050			0.000	Effective	0.900		0.108	0.108		0.139	0.016	0.039	0.039	0.165	0.039	0.702	0.553		
18	4	3	2	4	Almost Certainly Not 1	0	0.310	0	0.00	0.000	Effective	0.900	0.50	0.265	0.265	0.996	0.287	0.832	0.060	0.077	0.031	-	-	0.451	0.455	1.082
						1	0.070			0.000	Marginally Effective	0.750		0.205	0.205		0.194	0.046	0.795	0.114	0.046	-	-	0.514		
						2	0.310			0.000	Marginally Effective	0.750		0.265	0.265		0.234	0.038	0.073	0.851	0.038	-	-	0.416		
						3	0.310			0.000	Effective	0.900		0.265	0.265		0.287	0.031	0.060	0.077	0.832	-	-	0.451		
19	6	2	4	2	Almost Certainly Not in 4	0	0.186	0	0.00	0.000	Effective	0.900	0.39	0.176	0.176	0.995	0.195	0.816	0.045	0.045	0.045	0.030	0.018	0.427	0.445	2.198
						1	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.163	0.022	0.812	0.054	0.054	0.036	0.022	0.430		
						2	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.163	0.022	0.054	0.812	0.054	0.036	0.022	0.430		
						3	0.186			0.000	Marginally Effective	0.750		0.176	0.176		0.163	0.022	0.054	0.054	0.812	0.036	0.022	0.430		
						4	0.070			0.000	Marginally Effective	0.750		0.118	0.118		0.122	0.029	0.072	0.072	0.072	0.726	0.029	0.561		
						5	0.186			0.000	Effective	0.900		0.176	0.176		0.195	0.018	0.045	0.045	0.045	0.030	0.816	0.427		
20	2	3	2	2	Chances Even in 1	0	0.500	1	0.25	0.950	Effective	0.900	1.00	0.500	0.613	0.963	0.590	0.934	0.066	-	-	-	-	0.350	0.456	1.015
						1	0.500			0.050	Effective	0.900		0.500	0.388		0.410	0.149	0.851	-	-	-	-	0.608		

Figure A.3: 20-Node, 4-Asset Example: δ_{ij} Calculations–HUMINT

Node	Number of Discrete States	Priority	Priority Weight	"Freshness" of Intel (days)	Intel Assessment	State	Prior Probability	Action	Probability of Success of Action	Posterior Probability Given Action	Asset Effectiveness Estimate	P{Asset indicates N_i=x | N_i=x}	K	p'	p''	h	P{asset indicates N_i=x}	P{N_i=y | Asset Indicates N_i =x}						Branch Entropy	Expected Entropy	delta_i
																		0	1	2	3	4	5			
1	4	1	8	3	Almost Certainly Not in 1	0	0.310	1	0.75	0.600	Marginally Effective	0.750	0.50	0.270	0.518	0.833	0.498	0.780	0.137	0.072	0.011	-	-	0.509	0.706	1.017
						1	0.070			0.300	Not Effective	0.250		0.190	0.273		0.153	0.283	0.447	0.234	0.037	-	-	0.850		
						2	0.310			0.100	Not Effective	0.250		0.270	0.143		0.153	0.283	0.447	0.234	0.037	-	-	0.850		
						3	0.310			0.000	Marginally Effective	0.750		0.270	0.068		0.198	0.218	0.345	0.180	0.256	-	-	0.979		
2	4	2	4	2	Almost Certainly in 3	0	0.023	1	0.75	0.400	Marginally Effective	0.750	0.50	0.137	0.334	0.957	0.290	0.866	0.096	0.021	0.017	-	-	0.361	0.474	1.930
						1	0.023			0.400	Marginally Effective	0.750		0.137	0.334		0.290	0.096	0.866	0.021	0.017	-	-	0.361		
						2	0.023			0.200	Effective	0.900		0.137	0.184		0.226	0.123	0.123	0.732	0.022	-	-	0.597		
						3	0.930			0.000	Effective	0.900		0.590	0.148		0.195	0.143	0.143	0.032	0.682	-	-	0.668		
3	2	4	1	4	Probably Not in 1	0	0.700	1	0.75	0.900	Effective	0.900	1.00	0.550	0.813	0.696	0.750	0.975	0.025	-	-	-	-	0.169	0.354	0.342
						1	0.300			0.100	Effective	0.900		0.450	0.188		0.250	0.325	0.675	-	-	-	-	0.910		
4	4	1	8	2	Probably in 2	0	0.233	1	0.75	0.450	Very Effective	0.980	0.50	0.242	0.398	0.852	0.394	0.990	0.007	0.002	0.001	-	-	0.047	0.078	6.197
						1	0.233			0.450	Very Effective	0.980		0.242	0.398		0.394	0.007	0.990	0.002	0.001	-	-	0.047		
						2	0.300			0.100	Very Effective	0.980		0.275	0.144		0.147	0.018	0.018	0.961	0.003	-	-	0.144		
						3	0.233			0.000	Very Effective	0.980		0.242	0.060		0.065	0.041	0.041	0.015	0.904	-	-	0.298		
5	4	1	8	6	Almost Certainly Not in 3	0	0.310	0	0.00	0.000	Marginally Effective	0.750	0.50	0.260	0.260	0.998	0.343	0.568	0.189	0.189	0.053	-	-	0.799	0.846	1.218
						1	0.310			0.000	Not Effective	0.250		0.260	0.260		0.170	0.127	0.382	0.382	0.108	-	-	0.893		
						2	0.310			0.000	Not Effective	0.250		0.260	0.260		0.170	0.127	0.382	0.382	0.108	-	-	0.893		
						3	0.070			0.000	Marginally Effective	0.750		0.220	0.220		0.317	0.068	0.205	0.205	0.521	-	-	0.846		
6	4	2	4	2	Chances Even in 2	0	0.167	1	0.50	0.800	Effective	0.900	0.50	0.208	0.504	0.873	0.470	0.965	0.011	0.017	0.007	-	-	0.136	0.282	2.364
						1	0.167			0.100	Effective	0.900		0.208	0.154		0.167	0.101	0.831	0.047	0.021	-	-	0.440		
						2	0.500			0.100	Effective	0.900		0.375	0.238		0.239	0.070	0.021	0.894	0.015	-	-	0.311		
						3	0.167			0.000	Effective	0.900		0.208	0.104		0.124	0.136	0.042	0.064	0.758	-	-	0.569		
7	2	2	4	2	Probably Not in 0	0	0.300	0	0.00	0.000	Very Effective	0.980	1.00	0.400	0.400	0.971	0.404	0.970	0.030	-	-	-	-	0.193	0.139	3.327
						1	0.700			0.000	Very Effective	0.980		0.600	0.600		0.596	0.013	0.987	-	-	-	-	0.103		
8	2	3	2	2	Chances Even in 0	2	0.500	0	0.00	0.000	Effective	0.900	1.00	0.500	0.500	1.000	0.460	0.978	0.022	-	-	-	-	0.151	0.310	1.380
						3	0.500			0.000	Very Effective	0.980		0.500	0.500		0.540	0.093	0.907	-	-	-	-	0.445		
9	4	2	4	2	Probably in 1	0	0.083	1	0.75	0.750	Marginally Effective	0.750	0.50	0.167	0.604	0.716	0.545	0.831	0.126	0.036	0.006	-	-	0.409	0.604	0.449
						1	0.750			0.200	Not Effective	0.250		0.500	0.275		0.142	0.354	0.483	0.139	0.024	-	-	0.782		
						2	0.083			0.050	Not Effective	0.250		0.167	0.079		0.142	0.354	0.483	0.139	0.024	-	-	0.782		
						3	0.083			0.000	Marginally Effective	0.750		0.167	0.042		0.170	0.296	0.404	0.116	0.184	-	-	0.929		
10	6	2	4	2	Almost Certainly in 3	0	0.014	1	0.90	0.300	Very Effective	0.980	0.39	0.090	0.279	0.800	0.276	0.990	0.004	0.004	0.002	0.000	0.000	0.039	0.059	2.965
						1	0.014			0.300	Very Effective	0.980		0.090	0.279		0.276	0.004	0.990	0.004	0.002	0.000	0.000	0.039		
						2	0.014			0.300	Very Effective	0.980		0.090	0.279		0.276	0.004	0.004	0.990	0.002	0.000	0.000	0.039		
						3	0.930			0.100	Very Effective	0.980		0.548	0.145		0.145	0.008	0.008	0.008	0.976	0.000	0.000	0.078		
						4	0.014			0.000	Very Effective	0.980		0.090	0.009		0.013	0.087	0.087	0.087	0.045	0.691	0.003	0.586		
						5	0.014			0.000	Very Effective	0.980		0.090	0.009		0.013	0.087	0.087	0.087	0.045	0.003	0.691	0.586		
11	6	1	8	3	Almost Certainly in 4	0	0.014	0	0.00	0.000	Effective	0.900	0.39	0.116	0.116	0.900	0.122	0.855	0.019	0.019	0.019	0.069	0.019	0.346	0.250	5.196
						1	0.014			0.000	Effective	0.900		0.116	0.116		0.122	0.019	0.855	0.019	0.019	0.069	0.019	0.346		
						2	0.014			0.000	Effective	0.900		0.116	0.116		0.122	0.019	0.019	0.855	0.019	0.069	0.019	0.346		
						3	0.014			0.000	Effective	0.900		0.116	0.116		0.122	0.019	0.019	0.019	0.855	0.069	0.019	0.346		
						4	0.930			0.000	Effective	0.900		0.421	0.421		0.391	0.006	0.006	0.006	0.006	0.970	0.006	0.101		
						5	0.014			0.000	Effective	0.900		0.116	0.116		0.122	0.019	0.019	0.019	0.019	0.069	0.855	0.346		
12	2	4	1	2	Probably Not in 0	0	0.300	0	0.00	0.000	Effective	0.900	1.00	0.400	0.400	0.971	0.420	0.857	0.143	-	-	-	-	0.592	0.458	0.512
						1	0.700			0.000	Effective	0.900		0.600	0.600		0.580	0.069	0.931	-	-	-	-	0.362		
13	2	3	2	2	Probably in 1	0	0.250	1	0.90	0.900	Marginally Effective	0.750	1.00	0.375	0.848	0.616	0.674	0.943	0.057	-	-	-	-	0.314	0.516	0.200
						1	0.750			0.100	Marginally Effective	0.750		0.625	0.153		0.326	0.649	0.351	-	-	-	-	0.935		
14	6	4	1	5	Chances Even in 3	0	0.100	0	0.00	0.000	Effective	0.900	0.39	0.153	0.153	0.992	0.155	0.891	0.020	0.020	0.030	0.020	0.020	0.290	0.269	0.722
						1	0.100			0.000	Effective	0.900		0.153	0.153		0.155	0.020	0.891	0.020	0.030	0.020	0.020	0.290		
						2	0.100			0.000	Effective	0.900		0.153	0.153		0.155	0.020	0.020	0.891	0.030	0.020	0.020	0.290		
						3	0.500			0.000	Effective	0.900		0.233	0.233		0.225	0.014	0.014	0.014	0.932	0.014	0.014	0.200		
						4	0.100			0.000	Effective	0.900		0.153	0.153		0.155	0.020	0.020	0.020	0.030	0.891	0.020	0.290		
						5	0.100			0.000	Effective	0.900		0.153	0.153		0.155	0.020	0.020	0.020	0.030	0.020	0.891	0.290		
15	2	3	2	2	Almost Certainly in 0	0	0.930	0	0.00	0.000	Effective	0.900	1.00	0.715	0.715	0.862	0.649	0.991	0.009	-	-	-	-	0.073	0.303	1.118
						1	0.070			0.000	Very Effective	0.980		0.285	0.285		0.351	0.204	0.796	-	-	-	-	0.730		
16	4	3	2	2	Probably Not in 2	0	0.233	1	0.75	0.800	Very Effective	0.980	0.50	0.242	0.660	0.716	0.649	0.997	0.001	0.001	0.001	-	-	0.019	0.071	1.290
						1	0.233			0.100	Very Effective	0.980		0.242	0.135		0.138	0.032	0.958	0.007	0.003	-	-	0.146		
						2	0.300			0.100	Very Effective	0.980		0.275	0.144		0.147	0.030	0.006	0.961	0.003	-	-	0.138		
						3	0.233			0.000	Very Effective	0.980		0.242	0.060		0.065	0.067	0.014	0.015	0.904	-	-	0.284		
17	6	4	1	2	Probably in 3	0	0.050	0	0.00	0.000	Marginally Effective	0.750	0.39	0.108	0.108	0.871	0.217	0.374	0.083	0.083	0.352	0.083	0.025	0.808	0.784	0.088
						1	0.050			0.000	Not Effective	0.167		0.108	0.108		0.141	0.038	0.128	0.128	0.540	0.128	0.038	0.765		
						2	0.050			0.000	Not Effective	0.167		0.108	0.108		0.141	0.038	0.128	0.128	0.540	0.128	0.038	0.765		
						3	0.750			0.000	Not Effective	0.167		0.458	0.458		0.141	0.038	0.128	0.128	0.540	0.128	0.038	0.765		
						4	0.050			0.000	Not Effective	0.167		0.108	0.108		0.141	0.038	0.128	0.128	0.540	0.128	0.038	0.765		
						5	0.050			0.000	Marginally Effective	0.750		0.108	0.108		0.217	0.025	0.083	0.083	0.352	0.083	0.374	0.808		
18	4	3	2	4	Almost Certainly Not 1	0	0.310	0	0.00	0.000	Marginally Effective	0.750	0.50	0.265	0.265	0.996	0.338	0.587	0.151	0.196	0.065	-	-	0.790	0.832	0.328
						1	0.070			0.000	Not Effective	0.250		0.205	0.205		0.162	0.137	0.317	0.410	0.137	-	-	0.919		
						2	0.310			0.000	Not Effective	0.250		0.265	0.265		0.162	0.137	0.317	0.410	0.137	-	-	0.919		
						3	0.310			0.000	Marginally Effective	0.750		0.265	0.265		0.338	0.065	0.151	0.196	0.587	-	-	0.790		
19	6	2	4	2	Almost Certainly Not in 4	0	0.186	0	0.00	0.000	Very Effective	0.980	0.39	0.176	0.176	0.995	0.176	0.981	0.004	0.004	0.004	0.003	0.004	0.069	0.072	3.690
						1	0.186			0.000	Very Effective	0.980		0.176	0.176		0.176	0.004	0.981	0.004	0.004	0.003	0.004	0.069		
						2	0.186			0.000	Very Effective	0.980		0.176	0.176		0.176	0.004	0.004	0.981	0.004	0.003	0.004	0.069		
						3	0.186			0.000	Very Effective	0.980		0.176	0.176		0.176	0.004	0.004	0.004	0.981	0.003	0.004	0.069		
						4	0.070			0.000	Very Effective	0.980		0.118	0.118		0.119	0.006	0.006	0.006	0.006	0.970	0.006	0.101		
						5	0.186			0.000	Very Effective	0.980		0.176	0.176		0.176	0.004	0.004	0.004	0.004	0.003	0.981	0.069		
20	2	3	2	2	Chances Even in 1	0	0.500	1	0.25	0.950	Marginally Effective	0.750	1.00	0.500	0.613	0.963	0.556	0.826	0.174	-	-	-	-	0.667	0.784	0.359
						1	0.500			0.050	Marginally Effective	0.750		0.500	0.388		0.444	0.345	0.655	-	-	-	-	0.930		

Figure A.4: 20-Node, 4-Asset Example: δ_{ij} Calculations–COMINT

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14. ABSTRACT

Known as the *fog of war*, uncertainty has been prevalent in the conduct of military operations throughout human history. Intelligence collection efforts are tasked to reduce this uncertainty through the collection of information. Utilizing Shannon's entropy as a measure of the expected information gain due to an intelligence collection effort, a methodology is developed to prioritize and allocate intelligence assets in an efficient manner. Incorporated in this methodology are target priority and the requirement to reassess dynamic targets. The application area for the methodology is Effects-Based Operations. A generalized state model is developed to conduct adversary system-of-systems analysis. This model forms the basis for the entropy calculations and the resultant integer program to maximize the information gain.

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