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Coordinating Networked Uninhabited Air Vehicles for Persistent Area Denial

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Abstract— **This paper explores the problem of cooperative control among multiple networked unmanned air vehicles (UAVs) for persistent area denial (PAD) mission. An adaptive Markov chain model is used to predict the locations of pop-up threats. The mixed information of predicted pop-up threats and actual pop-up targets is utilized to develop cooperative strategies for networked UAVs. The approach is illustrated by use of a simulation test bed for multiple networked UAVs and Monte Carlo simulation runs to evaluate our cooperative strategy. Both theoretical analysis and simulation results are presented to demonstrate the effectiveness of using predicted pop-up information in improving the overall PAD mission performance.**

I. INTRODUCTION

UNINHABITED air vehicles have been identified as potential valuable resources for future military, communications and earth-science efforts. Cooperation among multiple UAVs is a key capability for utilizing the full potential of UAV systems. With respect to military applications, one of the potential missions for UAVs is persistent area denial. The aim of multiple UAVs in PAD operations is to provide persistent surveillance, tracking, and rapid engagement with high-volume strike, against threats (e.g., enemy integrated air defense system) at various ranges in the adversarial terrains. Threats in the battle space, in general, are of two types: known and pop-up. The locations of known threats are identified by the UAVs at the beginning of the battle, while the locations of pop-up targets are not always observable during the entire PAD operation. In other words, the pop-up targets may appear and disappear at frequent and random intervals of time. Obviously, uncertainty introduced by pop-up targets presents the significant theoretical and technical challenges in designing and implementing cooperative control strategies for multiple UAVs.

The present paper is mainly concerned with the problem of coordinating multiple UAVs for pop-up threats in the PAD operation. In this work, the most closely related work is introduced in [1] and [2]. In [1], the author modeled the information flow and communication constraints between UAVs and developed cooperative strategy for path re-planning to deal with pop-up threats during the mission. The work in [1], however, does not consider that the pop-up targets appear and disappear in a stochastic manner. In [2], the locations of pop-up targets are modeled as a first-order Markov chain. Utilizing a mixed integer programming approach, a cooperative control strategy of multiple UAVs for PAD operation is developed in [2] as well. In this strategy, the trajectories of vehicles are optimized so that they can reach all targets in the shortest time. The predicted information, however, is not utilized in the computation of cooperative control strategies. In this paper, we modify this cooperative strategy in a manner that allows the UAVs to make use of both actual and predicted information about pop-up targets in coordinating their target assignments. Moreover, since recent advances in telecommunication have provided enabling technologies for achieving cooperative control of multiple UAVs via a communication network [3], we will consider networked UAVs in performing the PAD operations.

II. SYSTEM MODEL

A. Adaptive Markov-Chain Model of Pop-up Targets

Pop-up targets are ground assets that appear at unpredictable locations and at random instants of time. In addition, pop-up targets may stay for a regular interval time and then disappear. Suppose that the location of the next pop-up target only depends on the location of present targets in the area concerned. Clearly, such discrete-time random sequence of the appearance of pop-ups can be modeled as a first-order Markov chain [4]. To mathematically define this Markov-chain model, we need to specify its state space Q and one-step transition matrix P. First, we restrict the area of PAD operation to be a two-dimensional rectangular grid, shown in Fig. 1, with length L and width W. Divide the edge with length L into m segments and the edge with width W into n

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segments which will decompose the original rectangular region into mn smaller rectangles. Assume that there exists at most one target in each cell at one time and no target appears right between cells of partitioned area.

Let Z_k be the cell at which the pop-up target appears at discrete-event time index k. Note that the index k is the time when a pop-up target shows up. Let $Z = \{Z_k, k \geq 0\}$ represent a random sequence of locations of pop-up targets. In terms of our definition of the PAD operational area, the set of all the possible locations that the pop-up targets can be located at, i.e., the state space of Z, is composed of mn discrete cells, which is given by $Q = \{1,2,\cdots,N_Q\}$ and $N_Q = mn$. As described before, we model the predicted pop-up locations as a first-order Markov-chain such that, for any k, Z_{k+1} is conditionally independent of $Z_0, Z_1, \cdots, Z_{k-1}$ given Z_k. That is,

$$\begin{aligned}&\Pr\{Z_{k+1} = j \mid Z_k = i, Z_{k-1} = i_{k-1}, \cdots, Z_0 = i_0\} \\ &\qquad = \Pr\{Z_{k+1} = j \mid Z_k = i\}, \quad i, j \in Q\end{aligned}. \tag{1}$$

In the expression above, $Z_k = i$ means that the pop-up target appears in cell i at discrete event index k. We assume that one-step transition probabilities $\Pr\{Z_{k+1} = j \mid Z_k = i\}, i, j \in Q$ depend only on the states i and j and are independent of the index k, i.e.,

$$\Pr\{Z_{k+1} = j \mid Z_k = i\} = p_{ij} \qquad \text{for all } i, j \in Q. \tag{2}$$

In other words, we are concerned with homogeneous Markov chains only in this paper. Next, we define two maps to simplify some notations for our later discussion. First, we define a map to transform a two-dimensional coordinate into the corresponding cell, i.e.,

$$ce : [0, L] \times [0, W] \to Q \tag{3}$$

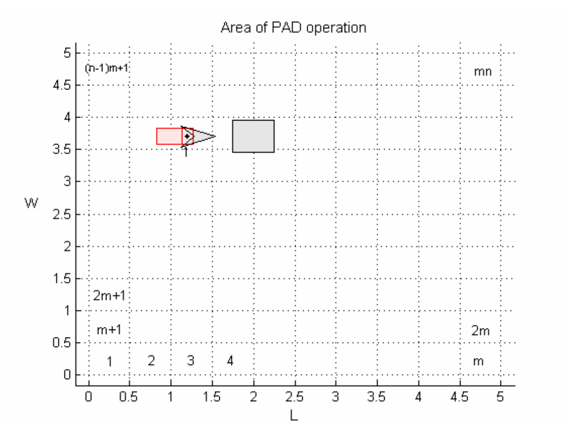

Fig. 1 Theater of PAD operations

Let $x = [x_1, x_2]^T$ be the coordinate of the pop-up target with x_1 as its horizontal coordinate and x_2 as its vertical coordinate. This transformation can be expressed as

$$ce(x) := \text{int}\left(\frac{x_2}{W/n}\right) \times m + \text{int}\left(\frac{x_1}{L/m}\right) + 1, \tag{4}$$

where $\text{int}(\cdot)$ is the operator which takes the integer part of its operand. It is easy to verify that $ce(x) \in Q$. On the other hand, we can define a map to convert a given cell where a target is located to a two-dimensional coordinate, which is given by

$$cr : Q \to [0, L] \times [0, W]. \tag{5}$$

To complete this definition, we assume that the pop-up target is located at the center of the cell. Thus, if the target is at cell $j \in Q$, its corresponding coordinate $x = [x_1, x_2]^T$, or $x = cr(j)$, can be determined as

$$\begin{cases} x_1 = (\text{mod}(j, m) - 1) \times \dfrac{L}{m} + \dfrac{L}{2m} \\ x_2 = \text{int}(\dfrac{j}{n}) \times \dfrac{W}{n} + \dfrac{W}{2n} \end{cases} \quad \text{if } \text{mod}(j, m) \neq 0\,; \tag{6}$$

where mod(a,b) is the operator which takes the value of the modulus after dividing a by b.

In what follows, we will briefly review the adaptive algorithm to estimate transition probability $\hat{p}_{ij}(k+1)$, which is determined as

$$\hat{p}_{ij}(k+1) = \frac{N(i, j, k+1)}{N(i, k+1)}. \tag{7}$$

where $N(i,k)$ represent the number of pop-up targets having appeared in the cell $i \in Q$ in the random sequence Z by the discrete event time k, and $N(i, j, k)$ the number of transitions from the current pop-up target being at cell $i \in Q$ to the next pop-up target at cell $j \in Q$ in the random sequence Z until k, which are calculated as

$$N(i, j, k+1) = \begin{cases} N(i, j, k) + 1 & \text{if } j = j' \\ N(i, j, k) & \text{otherwise} \end{cases}, \tag{8}$$

$$N(i, k+1) = N(i, k) + 1, \tag{9}$$

Essentially, this update procedure increases the transition probability to a location where a pop up has appeared and reduces it for the other locations.

We need another procedure to estimate the uptime at a particular location (or cell). Let $t_{e\sup}(i,k)$ represent the estimated uptime of a pop-up target stored at previously update index k and $t_{up}(i,k)$ the real uptime of the target staying at cell i at previously update time k. As the targets start popping up, the uptime for a particular pop-up target is re-calculated when the target either is destroyed or disappears at next update time $k+1$, which is given by:

$$t_{e\sup}(i,k+1)=\frac{t_{e\sup}(i,k)N(i,k)+t_{up}(i,k)}{N(i,k)+1} \quad (10)$$

B. Networked Vehicles Model

Suppose that there are N_v UAVs and that the i^{th} UAV is modeled as a Dubins' car model [5] and flies at a constant altitude and at a constant velocity, which has a continuous time kinematics given by

$$\begin{cases} \dot{x}_{v1}^i = v\cos(\theta_v^i) \\ \dot{x}_{v2}^i = v\sin(\theta_v^i) \\ \dot{\theta}_v^i = \omega_{\max} u_v^i \end{cases} \quad (11)$$

where x_{v1}^i is its horizontal position, x_{v2}^i is its vertical position, v is its transitional (constant) velocity, θ_v^i is its heading direction, $\omega_{\max}$ is its maximum angular velocity, and $-1 \le u_v^i \le 1$ is the steering input. Rather than use this continuous time representation we assume that vehicles will either travel on the minimum turn radius or on straight lines. It is then possible to analytically write down the formulas for the vehicle trajectories (e.g. in terms of arc segments on circles and line segments [5]). All UAVs are connected via communication network [6] and assumed to have perfect knowledge of threats that have been popped up.

C. Multi-UAV Simulation Test bed

In order to implement and evaluate cooperative control strategy, we use a Matlab simulation first developed at AFRL [7] and then extended by the authors as a test bed. In this paper, we consider the area of PAD operation in Fig.1 with length of $L = 5000$ meters and width of $W = 5000$ meters, and let $m = n = 10$ so that the entire area is divided into 100 cells. Here we assume that, for each current pop-up location, the maximum number of next possible pop-up locations is five and each of the next pop-up locations is not identical to the current pop-up location. We also assume that the first-order Markov chain model and the estimated uptime of pop-up targets at each cell have been obtained using the adaptive approach before UAVs start their PAD mission.

III. Maneuvering UAVs To Pop-up Threats

To aid in the development of cooperative control strategies among UAVs, first we need to address the issue of how to maneuver UAV to pop-up targets. In the absence of knowing the information about predicted pop-up threats, a UAV often waits for the next actual pop-up target to appear and then maneuvers to that target. Such strategy is often referred to as a simplistic and inactive strategy. We call this type of strategy "WA" strategy. On the other hand, a UAV may decide to move to an intermediate position even before the real one shows up. This could be done if the predicted information about pop-up targets is available.

A. Strategies for locating UAVs intermediate destination

Consider a present pop-up target at cell i, and the corresponding next pop-up targets predicted from the Markov-chain model as illustrated in Fig. 2. Let S denote the set of cells where those predicted pop-up targets are located, which is $S = \left\{ j_i^1, j_i^2, \cdots, j_i^s \right\}$, where $p_{ij_i^l} > 0$ for all $l = 1, \cdots, s$. Moreover, let $x^l = [x_1^l, x_2^l]^T$ be the coordinate of the l^{th} predicted pop-up targets and $x^l = cr(j_i^l)$, for $l = 1, 2, \cdots, s$.

Definition 1. A predicted location $x^{ML} = [x_1^{ML}, x_2^{ML}]^T$ selected as

$$x^{ML} = x^{l^*} \text{ with } l^* = \arg\max_l \{p_{ij_i^l}\}$$

is called maximum likelihood (ML) locating strategy.

Definition 2. A predicted location $x^{BW} = [x_1^{BW}, x_2^{BW}]^T$ given by

$$x^{BW} = \sum_{l=1}^{s} p_{ij_i^l} \times x^l$$

is called a Bayesian weighted (BW) locating strategy.

Definition 3. A predicted location $x^{UW} = [x_1^{UW}, x_2^{UW}]^T$ given by

$$x^{UW} = \frac{1}{s}\sum_{l=1}^{s} x^l$$

is called a uniformly weighted (UW) locating strategy.

We define completion time as the time that it takes a UAV to reach the next pop-up target after it becomes visible. Clearly, the completion time depends on when this pop-up target shows up and which predicted target in the set S will be appeared as a real target. Clearly, the completion time is a random variable. Hence, we will use the expected value of completion time as a performance index to evaluate various locating strategies developed so far.

Let t_{WA} be the completion time when using WA strategy, and let $E(t_{WA})$ denote the expected value of t_{WA}, which can be calculated as

$$E(t_{WA}) = \sum_{l=1}^{s} p_{ij_i^l} d(i, j_i^l)/v \quad (12)$$

where $d(i, j)$ is the minimum flying distance for UAV from cell i to cell j. Similarly, we define t_{ML}, t_{BW} and t_{UW} as the completion times using ML locating strategy, BW locating strategy and UW locating strategy, respectively.

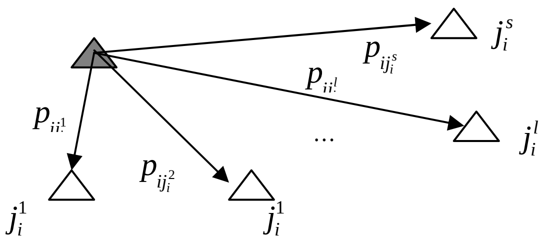

Fig. 2. Present target at cell i and the corresponding predicted pop-up targets in the set *S*.

The expected values of these completion times can be computed as

$$E(t_{ML}) = \sum_{\substack{l=1,\\ l\neq l^*}}^{s} p_{ij_i^l}\, d(j_i^{l*}, j_i^l)/v \text{ where } l^* = \arg\max_l \{p_{ij_i^l}\} \quad (13)$$

$$E(t_{BW}) = \sum_{l=1}^{s} p_{ij_i^l}\, d(j^{BW}, j_i^l)/v \text{ where } j^{BW} = ce(x^{BW}) \quad (14)$$

$$E(t_{UW}) = \sum_{l=1}^{s} p_{ij_i^l}\, d(j^{UW}, j_i^l)/v \text{ where } j^{UW} = ce(x^{UW}) \quad (15)$$

With the expressions (12)-(15), we can easily figure out under what conditions the performances of three locating strategies are better than the WA strategy. Obviously, if

$$p_{ij_i^{l*}}\, d(i, j_i^{l*}) + \sum_{\substack{l=1,\\ l\neq l^*}}^{s} p_{ij_i^l}\left(d(i, j_i^l) - d(j_i^{l*}, j_i^l)\right) > 0\,. \quad (16)$$

Similarly, the condition

$$\sum_{l=1}^{s}\left(d(i, j_i^l) - d(j^*, j_i^l)\right) > 0 \quad \text{where } x_{j*} = x^{BW} \text{ or } x^{UW} \quad (17)$$

Ensures that the performance of the weighted locating strategies is better than WA strategy.

We can also obtain other conditions for evaluating the performance of the ML locating strategy and the weighted locating strategies in a similar way. In general, the hold of those conditions will depend upon either the transition probabilities or spatial distribution of the predicted targets, or both of them.

B. *Monte Carlo Evaluations*

In this section, we would like to perform the Monte Carlo simulations to evaluate various locating strategies. It is clear that transition probabilities and spatial distribution of the predicted targets are two extremely important factors in determining which strategy works best.

Experiment 3.1 In this experiment, we consider the effects of transition probabilities on the performance of various strategies. First, we define

$$\delta_{pr}^{i} = \max_l \{p_{ij_i^l}\} - \min_l \{p_{ij_i^l}\} \text{ where } p_{ij_i^l} > 0$$

as the variation of the transition probabilities with respect to the present target at cell i. For example, if $\delta_{pr}^i = 0$, this means $p_{ij_i^l} = p_{ij_i^{l'}} > 0$ for $l \neq l'$. We choose a scenario with 100 sets of pop-up locations and perform the 10,000 Monte Carlo runs for each set letting the variation of the transition probabilities vary from 1% to 35%. The results are shown in Fig. 3.

We observe that the performances of three locating strategies (ML, BW and UW) are significantly better than the WA strategy for each probability distribution. Since the predicted information is not utilized in the WA strategy, the performance of this strategy does not vary much as we vary the transition probability distribution. When the variation of the transition probabilities is small, both UW and BW locating strategies work better than ML locating strategy. As the variation δ_{pr}^i increases, the averaged completion time using the ML locating strategy falls off constantly and results in much better performance than weighted locating strategies. The error bars show the standard deviation of every data point which corresponds to the average completion time of 100 independent runs.

Experiment 3.2 In this experiment, we examine the effects of the other factor, i.e., the spatial distribution of predicted targets, on the performance of various strategies. We fix the location of the present pop-up target at cell 1 and take five sets of spatial distributions of predicted targets as shown in Fig. 4.

For each configuration of distribution, we assume that the predicted pop-up targets are uniformly distributed in the area composed of the corresponding set of cells. Hence, the predicted pop-up targets are spread out as the set number increases. Fig. 5 compares the outcome of simulations with 5000 sets of pop-up targets for each distribution and 1000 Monte Carlo runs for each set.

We observe that three locating strategies present better performance than that using WA strategy. As the targets are more spread out, we observe that the difference between two weighted locating strategies becomes larger. In particular, the ML locating strategy works best in this simulation. However, the standard deviation of the performance of the ML locating strategy is worse than those two weighted locating strategies. This can be seen in Fig 6.

In most cases, the ML locating strategy works best in terms of minimizing the expected time to reach the next pop-up target after it appears.

IV. Cooperative Control Strategies of Networked UAVs for PAD Mission

Basically, it is desirable to have more than one UAV performing persistent area denial since multiple UAVs can cooperate by sharing relevant information and try to accomplish the task more efficiently. In this section, we will define specific version of our cooperative control strategy by involving the information of predicted pop-up targets in our development.

A. *Cooperative Strategy Using Mixed Information*

1) Initial Cooperative Target Assignment: First, let us consider the allocation of multiple UAVs to the targets that are currently observed. This can be done by using integer programming (IP) approach [8]. The cost function and constraints are formulate

$$\min_{\psi(i,j)} \left\{ \sum_{j=1}^{n_1(k)} \sum_{i=1}^{n_1(k)} d(x_v^i, x_r^j)\Omega(i,j) \right\} \quad (18)$$

subject to the constraints

$$\sum_{j=1}^{n_1(k)} \Omega(i,j) = 1 \quad (19)$$

$$\sum_{i=1}^{n_1(k)} \Omega(i,j) = 1 \quad (20)$$

where $\Omega(i,j)$ is the assignment function which is given by

$$\Omega(i,j) = \begin{cases} 1 & \text{if UAV } i \text{ is assigned to target } j \\ 0 & \text{otherwise} \end{cases}. \quad (21)$$

The constraints (19) and (20) means that each UAV can be assigned to a single target only and vise versa. For the purpose of illustration, we currently consider two UAVs, i.e., $N_v = 2$. This algorithm actually assigns each UAV to the target closest to its current location in the area. Let *AV* (*UV*) and *AT* (*UT*) denote the set of assigned (unassigned) UAVs and assigned (unassigned) pop-up targets, respectively. In this algorithm, a UAV (target) assigned to a dummy target (UAV) is called an unassigned UAV (target). *2) Cooperative Targeting Adjustment:* It is possible that in certain situation, a single UAV is sufficient to destroy two or more targets close to each other. In addition, the allocation of UAVs to targets depends on the remaining time of the appearance of targets also, i.e. $\{t_{esup}(i), i \in Q\}$. Consider the situation where $Dim(AT) = 2$ and $AT = \{j_1, j_2\}$. This means all the UAVs are assigned to attack the two targets because $N_v = 2$. We claim that two targets are close to each other if the distance between them is less than a threshold d_{th}. In this case, one UAV, say UAV i^*, can be assigned to attack both of them at the same time. If the set *UT* is not empty, then the other UAV $i' \neq i^*$ will be assigned to a target in *UT* closest to it. If $Dim(UT) \geq 2$, UAV i' may determine how many targets it can deal with using the adjustment algorithm. If $UT = \varnothing$, the sets *AV* and *UV* are updated as : $AV = AV / \{UAV_{i'}\}$ and $UV = UV \bigcup \{UAV_{i'}\}$.

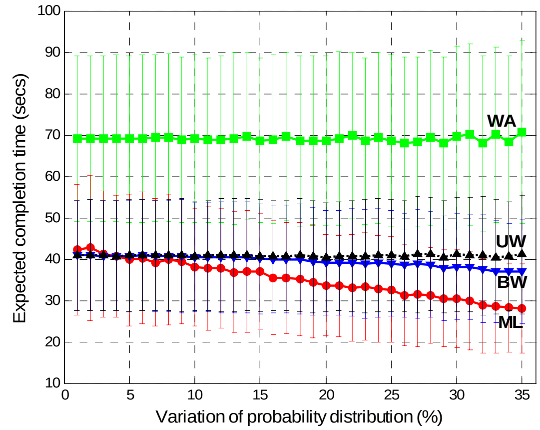

Fig. 3. Completion time for various variations of transition probabilities

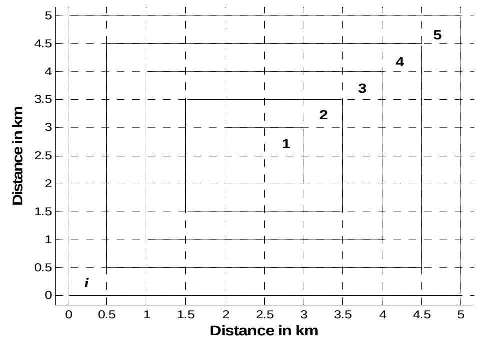

Fig. 4. Five sets of spatial distributions

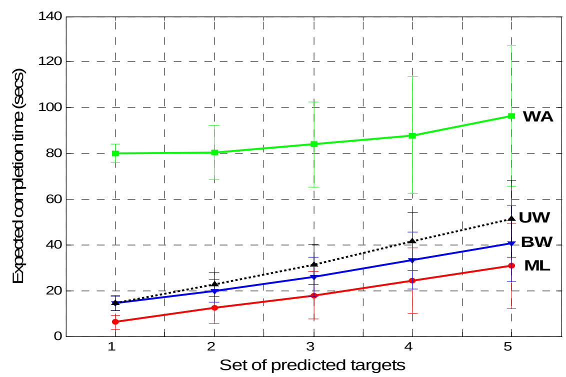

Fig. 5. Completion time for various spatial distribution of the predicted pop-up targets

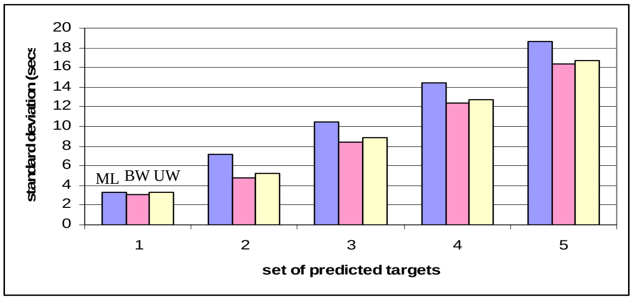

Fig. 6. Standard deviations for three locating strategies

3) Predicted Target Assignment: In the case that $UT = \varnothing$ but $UV \neq \varnothing$, the UAVs in set UV will determine an intermediate location to move by using the locating strategies developed in step 2) based on the predicted information obtained from Markov model, instead of waiting for the next pop-up target to appear.

B. *Illustrative Examples*

The cooperative control strategies developed in the previous section are tested using the simulation performed from the AFRL software. The simulations here will have a group of two UAVs assigned for four pop-up targets in total in the given area and the initial situation is shown in Fig. 7. Note that the present targets 1 and 2 are close to each other. As a result, UAV 1 will be assigned to reach both of them. Hence, *UV*={UAV 2}.

In the first simulation, we test the performance of the cooperative strategy without using the predicted information (i.e., WA strategy). By viewing a movie of Fig. 7, we see that the unassigned UAV 2 is just hovering around its initial location to wait for the appearance of the next pop-up target, i.e., target 3. As a result, when pop-up target 3 appears, UAV 2 starts moving to it and finally, the UAV 2 misses its target. Similarly, UAV 1 fails to reach the pop-up target 4 in time. In the end, we notice that the team of UAVs missed two pop-up targets in total and it took them 1501 seconds for the overall simulation.

We then performed the same simulation except that the unassigned UAV now decides to move to an intermediate location using the ML locating strategy. In this simulation, we see that the UAV 2 flies to the destination where the predicted pop-up target with the highest transition probability is located. When the pop-up target 3 shows up, UAV 2 is much closer to it and thus continues to move to it. As a result, UAV 2 catches the target 3 successfully. We also observe that UAV 1, after accomplishing the pre-assignment, then heads towards the predicted location using the ML locating strategy, and reach the target 4 in a timely manner. In the end, all the four pop-up targets are reached successfully by taking advantage of the predicted information and the overall mission takes the group of UAVs 1401 seconds, a shorter simulation time, to complete the PAD task.

V. Concluding Remarks

This research addresses the coordination of the multiple UAVs in PAD operations for pop-up threats. We present various locating strategies to move a UAV to an intermediate position using predicted information. Monte Carlo runs show that the ML locating strategy has an average performance superior to other locating strategies in many cases. Cooperative strategies considered in this paper are designed to achieve the objectives of UAVs based on

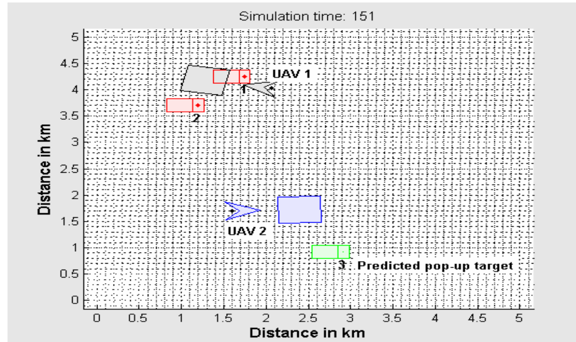

Fig. 7 Initial Situation

available information about predicted pop-up targets, thus enabling better results for their PAD operations. Our simulation results have demonstrated that the developed cooperative strategies using mixed information about pop-up targets in an appropriate way can reduce mission time and increase the number of the targets reached, and thus improve the overall performance of PAD operation.

References

[1] Jiecai Luo, "Some New Optimal Control Problems in UAV Cooperative Control with Information Flow Constraints," *Proceedings of the American Control Conference*, Denver, Colorado, June 4-6, 2003, pp. 2181-2186.

[2] Shankar K. Subramanian and Jose B. Cruz, "Adaptive Models of Pop-Up Threats for Multi-Agent Persistent Area Denial", *Proceedings of the 42nd IEEE Conference Decision and Control*, Hawaii, December, 2003, pp. 510-515.

[3] A. Gil, S. Ganapathy, K. Passino, and A. Sparks, "Cooperative scheduling of tasks for networked autonomous vehicles," *Proceedings of the IEEE Conference on Decision and Control*, Hawaii, 2003, pp.522-527.

[4] U. Narayan Bhat, *Elements of applied stochastic processes*, second edition, John Wiley& Sons, 1984.

[5] L. E. Dubins, "On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and targets," *American Journal of Mathematics*, Vol. 79, No. 3, July, 1957, pp. 497-516.

[6] J. Finke, K. M. Passino, S. Ganapathy, and A. Sparks, "Modeling and analysis of cooperative control systems for uninhabited autonomous vehicles," in *Cooperative Control* (S.Morse, N. Leonard, and V. Kumar, eds.), Springer-Verlag, 2004.

[7] S. J. Rasmussen and P. R. Chandler, "MultiUAV: A multiple UAV simulation for investigation of cooperative control," *Proceedings of the 2002 Winter Simulation Conference*, San Diego, CA, Vol.1, pp. 869-877.

[8] H. Fourer, D. M. Gay, and B.W. Kernighan, *AMPL, A modeling language for mathematical programming*, The Scientific Press, 1993.