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THESIS

THE FAST THEATER MODEL (FATHM): OPTIMIZATION OF AIR-TO-GROUND ENGAGEMENTS AS A DEFENDER-ATTACKER MODEL

by

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December 2005

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OPTIMIZATION OF AIR-TO-GROUND ENGAGEMENTS AS A DEFENDER-
ATTACKER MODEL**

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ABSTRACT

The FAst THeater Model (FATHM) is a joint theater-level attrition model that combines a Lanchester ground combat model with a linear program, hereafter the Air model, that determines the optimal allocation of air strikes against ground forces. FATHM models time phased ground battles between two forces BLUE and RED, and calls the Air model based on the outcomes of the most recent ground battle, assuming BLUE air supremacy. This thesis develops an enhanced Air model that endows RED with the ability to actively prepare for BLUE air attacks by deploying dummy targets and anti-aircraft artillery as two augmenting defense plans with the goal to more realistically reduce BLUE effectiveness in killing RED targets and simultaneously increase attrition to attacking BLUE aircraft. This Air model is a mixed integer program (MIP), a defender-attacker model, with RED as the defender and BLUE as the attacker. The MIP is a cost- and resource-interdicted model, combining interdiction-induced costs with restrictions on resources for some constraints. This new defender-attacker model provides an optimal defense plan by RED in anticipation of optimized BLUE air attacks without changing FATHM's basic concept or structure. We demonstrate defensive actions by RED that can significantly reduce the BLUE attacker's effectiveness.

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EXECUTIVE SUMMARY

The FAst THeater Model (FATHM) is a joint theater level attrition model that combines ground combat between two forces, BLUE and RED, with air strikes by BLUE aircraft on RED ground targets. FATHM conducts a war in phases. Each phase divides into three-day time periods. For each time period, FATHM fights many battles on the ground (typically ten per day, or thirty in three days) and calls the air-to-ground model, hereafter the Air model, a large linear program that determines the optimal allocation of BLUE air strikes against RED ground forces based on the outcome of the most recent ground battles. The linear program prioritizes sorties against RED ground platforms with high value that are most successful in killing BLUE ground forces. FATHM assumes BLUE air supremacy and does not model RED air force, air combat and air defense. Prior to this thesis, defensive preparations by RED in anticipation of BLUE air strikes have not been represented.

We develop a defender-attacker model as an enhanced Air model, with RED as the defender and BLUE as the attacker. We endow RED with the ability to actively prepare for BLUE air attacks. We formulate a mixed integer program that expresses the situation where RED must lead with its preparations, and then BLUE attacks having seen these preparations. RED goals are to reduce BLUE effectiveness in killing RED target value and simultaneously increase attrition to attacking BLUE aircraft. BLUE as the attacker has to pay penalties, increased cost and effort, for attacking RED's defense. We call our enhanced optimization model a cost- and resource-interdicted model.

BLUE's main objective is to maximize target value destroyed by killing as many targets with high target value as possible. Simultaneously, BLUE wants to minimize its own casualties, minimize deficiencies in achieving kill goals for target classes, and minimize deviations from goals to equitably use Air Force, Marine, Navy, and allied air force components. RED's objectives are the reverse of BLUE's.

The main RED defense plan we model is the use of dummy targets. Given these dummy operations, the attacker has to increase the number of sorties to achieve the same effectiveness. If BLUE still chooses to attack the target, a penalty in the form of a

reduced effectiveness is incurred. The number of available dummy targets for specific target types is limited, as is the number of dummy operations (setting up or building dummy targets, limited by factors like material, personnel and time).

Our second plan represents enhanced air defense for specific targets by augmenting anti-aircraft artillery with the purpose of enhancing tactical air defense, and countering BLUE's second objective to minimize its own losses. Attrition increases for every platform that attacks a protected and actively defended target. If BLUE still attacks that target, a penalty in the form of higher losses of aircraft is incurred.

We implement our model using GAMS (General Algebraic Modeling System) and solve for both RED's optimal defense and BLUE's optimal attack plan using the input data for the first time period of a FATHM war. We also solve the legacy (one-sided) Air model and compare results. We analyze the effects of both defense plans on the objective: the attacker's effectiveness in killing RED target value.

Our results show that RED dummy operations have the most significant impact on BLUE effectiveness in killing RED target value. RED concentrates its efforts on prioritized target classes. Even a few dummy targets result in a large increase in our objective function incurred by interdiction-induced cost for BLUE attacks. Our model achieves significant reductions of the number of killed targets by deploying dummy operations and increases BLUE's underachievement of preset kill goals. We also see that the presence of dummies increases the number of BLUE attacks. This increase in the number of sorties affects BLUE capacity goals, stressing some BLUE components. BLUE also suffers higher casualties because more aircraft are shot down by RED. The number of weapons used and hours flown by BLUE aircraft increase proportionately with the number of BLUE attacks. BLUE effectiveness in killing RED target value is reduced by more than 30% when RED deploys a sufficient number of dummies (about one dummy target for every two real targets) for the prioritized target class.

Our defender-attacker model achieves the intended objective of reducing BLUE effectiveness by increasing his efforts in launching air strikes against RED's defense. Dummy operations turn out to have the most significant effect when comparing the two augmenting plans.

Based on our results, we propose the defender-attacker model as an enhanced Air model within FATHM that more realistically represents what an intelligent RED adversary would do in combat.

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I.   THE FAST THEATER MODEL FATHM

The FAst THeater Model (FATHM) [Brown and Washburn 2005], a large-scale, aggregated joint theater level attrition model, combines a two-sided Lanchester ground combat between two forces, BLUE and RED, with air strikes by BLUE aircraft on RED ground targets. FATHM conducts a war in phases. Each phase divides into three-day time periods. A war phase completes either by achieving specific levels of targets killed in target classes or by reaching a limiting phase duration. Each time period, FATHM fights many battles on the ground and calls the air-to-ground model, a linear program that determines the optimal allocation of BLUE air strikes against RED ground forces based on the outcome of the most recent ground battles. The linear program prioritizes air attacks against ground platforms that are most successful in killing BLUE ground forces. FATHM does not model RED air force and air combat, air defense or other counter measures.

This thesis proposes a defender-attacker model [e.g., Brown, Carlyle, Salmeron and Wood 2005] as an enhanced air-to-ground model, with RED as the defender and BLUE as the attacker. We endow RED with the ability to actively anticipate BLUE air attacks, and to employ dummy targets as well as anti-aircraft artillery with the goals of reducing BLUE's effectiveness in killing selected targets and simultaneously increasing attrition to attacking aircraft. Understanding RED's defense preparations as a frustration of BLUE's attacks, we describe our model as a cost- and resource-interdicted model.

A.   BACKGROUND – THE FAST THEATER MODEL

FATHM models the ground-to-ground battle as a Lanchester fire exchange model that uses attrition rates computed by the Army's COmbat SAmple GEnerator (COSAGE) model [Jones, 1995]. COSAGE's *killer-victim scoreboards* provide the input to FATHM for each phase of the modeled war to obtain appropriate Lanchester coefficients for the modeled aimed (direct) and un-aimed (indirect) fire.

FATHM's air-to-ground model, hereafter the Air model, is a large linear program that is called each time period after the ground battles have been fought. It determines an

optimized set of BLUE air strikes, similar to the Air Force's Conventional Forces Assessment Model (CFAM) [e.g., Brown and Washburn 2005]. The Air model uses the same kind of data CFAM uses. Dynamic target values, derived from the status of the preceding ground battles, drive the optimization to maximize the target value destroyed. The target value reflects the performance of RED platforms, the targets in the Air model, in killing BLUE platforms on the ground.

B.   KEY FEATURES AS A JOINT COMBAT MODEL

FATHM fuses two separate types of combat models, a ground combat model and an Air model. FATHM calls the Air model after the ground battles of each time period to perform optimal air strikes based on the progress of the battle on the ground. Management of data describing the state of ground and air action of the two opposing forces becomes vital in this joint combat model. The outputs of the two models have to provide the needed information for appropriate time period updates.

These updates include crucial logistical data and performance parameters for both sides. Logistic computations combine the results of both, ground battle and air assets, and consider pre-scheduled reinforcements for both sides, regeneration of destroyed or damaged targets. Performance parameters that are subject to learning with ongoing battle duration, attrition to BLUE aircraft and kill expectations by those, are recalculated.

The following diagram shows FATHM's basic structure. The only possibility for human interaction with the model is in the initialization phase.
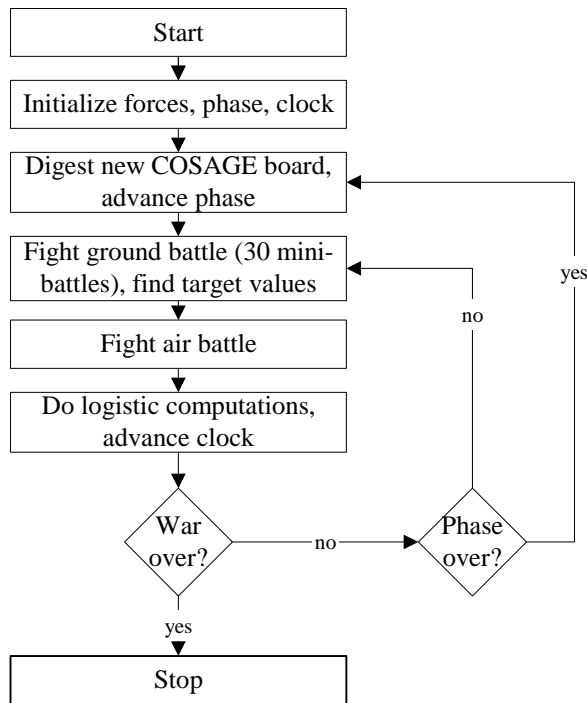
Figure 1.   Basic Flow Diagram for FATHM [from Brown and Washburn 2005]

Based on the logistic calculations, FATHM computes a dynamic value after each ground battle for each RED ground platform that reflects its performance in killing BLUE ground platforms. This dynamic value represents the target value (see Figure 1) from BLUE's perspective and is the most significant parameter provided to the Air model. It allows prioritizing the RED ground targets for the next air-to-ground engagement. The dynamic target value is the linking parameter between ground battles and air strikes.

C.   THE IDEA OF AN ENHANCED AIR MODEL

FATHM assumes BLUE air supremacy. Therefore, the Air model only considers platforms that are directly involved in an air attack: the attacked RED ground platforms and the attacking BLUE aircraft. RED's air defenses and their suppression, air combat and electronic counter measures are not modeled, but rather are assumed to be static exogenous conditions that may lead to attrition of BLUE aircraft. Possible anticipation

by RED of BLUE's air-to-ground attacks and use of this to program defensive actions are not represented.

FATHM's Air model determines the optimal allocation of air strikes by BLUE aircraft on RED ground systems with respect to

- assigned RED target values
- attrition of BLUE platforms
- BLUE kill goals and
- efforts of the services

in a variety of conditions and subject to limiting resources. The objective function evaluates aircraft sorties in an attempt to simultaneously kill targets, avoid attrition and equalize the efforts among the involved services, and to end a war phase as soon as possible by quickly achieving preset phase goals.

This thesis presents a defender-attacker model that solves at once for an optimal defense plan by RED against anticipated optimal BLUE air attacks without changing FATHM's basic concept and structure, and by maintaining the basic objectives of the existing Air model. We call the enhanced optimization model a cost- and resource-interdicted model, combining a cost interdiction model [e.g., Brown, Carlyle, Salmeron and Wood 2005] with changes on the resources governing constraints. We give RED the ability to prepare to frustrate BLUE air attacks with tactically reasonable counter measures. In FATHM, RED wants to reduce the number of targets killed and to increase the number of BLUE platforms lost. In this way, BLUE has to pay a penalty, increased cost and effort, if attacking any given target hardened by RED defensive measures.

II. FATHM'S UNIQUENESS: JOINING GROUND COMBAT WITH OPTIMIZED AIR STRIKES

A. OPTIMIZATION WITHIN A DYNAMIC COMBAT MODEL

The Air model, the optimization model within FATHM, allocates air strikes against RED ground forces involved in the preceding ground battle. The data are either static, direct input by the planner, or dynamic, computed by FATHM at the end of ground battles in each time period. Dynamic data, in particular the target values of RED ground platforms, is most significant. Because FATHM computes these dynamic target values based on the result of the most-recent ground battles, the Air model directly supports the ground model. FATHM can provide optimal planning guidance for decision makers concerned with theater level combat such as joint staffs and commanders.

B. THE AIR MODEL

The Air model is a large linear program that combines and balances objectives of air models commonly used by the U.S. Air Force for decades to maximize target value destroyed and minimize losses of BLUE aircraft [e.g., Yost 1996].

Original FATHM (see the Appendix) features passenger variables $TGTKILLS_k$, $PLTSLOST_p$, $HRSUSED_{pw}$, $WEPUSED_m$ and $SVCCAP_s$ that are each functions of X_{apmklw}. We replace these variables by their defining functions and reformulate the model so that our intended RED frustrations can be recognized more easily. For consistency, we maintain as much of the original notation as possible.

1. Subscripts and Sets

$s \in S$ set of services

$p \in P$ set of aircraft platforms

P_s partition of P, $s \in S$

$m \in M$ set of weapon types

$k \in K$ set of target types

$a \in A$ set of attack profiles

$l \in L$ set of loadouts

$w \in W$ set of weather states

$j \in J$ set of target classes

$k \in K_j$ subset of target types k referenced in target class j

2.   Data [units]

$mxtargk_k$ upper bound on target type k kills (the number available) [targets]

$kvalue_k$ target value for each target of type k [value/target]

$mxplats_p$ upper bound on aircraft platforms type of p

$mxhours_{pw}$ upper bound on aircraft type p hours used in weather state w [hours]

$used_{kpw}$ hours required for an attack on target k by aircraft p in weather w [hours]

$mxwepns_m$ upper bound on weapon type m use [weapons]

cap_p capacity of aircraft p (used only in service equity computations) [value/platform]

λ multiplier for attrition in objective function

e_{apmklw} expected kills per sortie of attack profile a, aircraft p, weapon m on target k with loadout l in weather w [targets/attack]

att_{apmklw} expected attrition per sortie as above [platforms/attack]

c_{apmklw} weapons used per sortie as above [weapons/attack]

$\underline{jgoal}_j$, $\overline{jgoal}_j$ lower and upper goals for kills of target class j [targets]

$\underline{jpen}_j, \overline{jpen}_j$ lower and upper penalties for violating kill goals for target class j [value/target]

$\underline{sgoal}_s, \overline{sgoal}_s$ lower and upper goals for capacity used by service s [capacity]

$\underline{spen}_s, \overline{spen}_s$ lower and upper penalties for violating capacity goals for service s [value/capacity]

3.   Variables (all nonnegative)

Variable

X_{apmklw} attacks in attack profile a with platform p, weapon m and loadout l on target k in weather w [attacks]

Deficiency Variables

$UNDERKILLS_j, MIDKILLS_j, OVERKILLS_j$

under, slack, and over-kills with respect to kill goals of target class j [targets]

$UNDERCAP_s, MIDCAP_s, OVERCAP_s$

under, slack, and over-achievement with respect to service s goals [capacities]

4.   Formulation

The objective balances the efforts between killing target value, avoiding attrition to BLUE aircraft, achieving kill goals of target classes critical to the current war phase and equitably stressing the BLUE air services.

Minimize $-\sum_{apmklw} kvalue_k \, e_{apmklw} \, X_{apmklw}$

$+\lambda \sum_{apmklw} cap_p \, att_{apmklw} \, X_{apmklw}$

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$$+\sum_{j}\left(\underline{jpen_j}\,UNDERKILLS_j + \overline{jpen_j}\,OVERKILLS_j\right)$$

$$+\sum_{s}\left(\underline{spen_s}\,UNDERCAP_s + \overline{spen_s}\,OVERCAP_s\right)$$

The numbers of targets killed, linear functions of the numbers of attacks X_{apmklw} assigned, are subject to not killing more targets than exist. The constraints on the number of platforms lost, hours and weapons used are similar.

$TGTKILLS_k$

$$\sum_{apmlw} e_{apmklw}\,X_{apmklw} \leq mxtargk_k \qquad \forall\, k$$

$PLTSLOST_p$

$$\sum_{amklw} att_{apmklw}\,X_{apmklw} \leq mxplats_p \qquad \forall\, p$$

$HRSUSED_{pw}$

$$\sum_{amkl} used_{kpw}\,X_{apmklw} \leq mxhours_{pw} \qquad \forall\, pw$$

$WEPUSED_m$

$$\sum_{apklw} c_{apmklw}\,X_{apmklw} \leq mxwepns_m \qquad \forall\, m$$

For each target class j, upper and lower soft goals mark the desired range of the number of targets to kill in that class.

Kill goals $JGOAL_j$

$$\sum_{apmk\in K_j lw} e_{apmklw}\,X_{apmklw} + UNDERKILLS_j + MIDKILLS_j - OVERKILLS_j = \overline{jgoal_j} \qquad \forall\, j$$

$$MIDKILLS_j \leq \overline{jgoal_j} - \underline{jgoal_j} \qquad\qquad \forall\, j$$

Similarly, for each service capacity, there are lower and upper soft goals in order to avoid overstressing the respective BLUE air service components. The service capacity $SVCCAP_s$ is a linear function of $HRSUSED_{pw}$, which itself is a linear function of the number of sorties X_{apmklw}.

Service goals $SGOAL_s$

$$\sum_{ap \in P_s mklw} cap_p \; used_{kpw} \; X_{apmklw} + UNDERCAP_s + MIDCAP_s - OVERCAP_s = \overline{sgoal_s} \qquad \forall \, s$$

$$MIDCAP_s \qquad \leq \overline{sgoal_s} - \underline{sgoal_s} \qquad\qquad\qquad \forall \, s$$

All variables are nonnegative.

$$X_{apmklw} \geq 0 \qquad\qquad\qquad\qquad\qquad \forall apmklw$$
$$UNDERKILLS_j, \; MIDKILLS_j, \; OVERKILLS_j \geq 0 \qquad \forall j$$
$$UNDERCAP_s, \; MIDCAP_s, \; OVERCAP_s \geq 0 \qquad \forall s$$

The possible combinations of aircraft, weapons and loadout, attack profiles on specific targets in different weather states are summarized in a catalog of system specific missions. These missions are pre-determined static input data $mission_{napmklw}$ provided by the planner. The index n herein represents mission number, an identifier. $mission_{napmklw}$ not only ensures realistic attack configurations, it also restricts the number of variables in the linear program for mission n.

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III.   THE DEFENDER – ATTACKER FATHM

A.   ENHANCEMENT TO A TWO-SIDED DEFENDER-ATTACKER MODEL

Building upon the existing Air model, we have identified two possible RED preparations in anticipation of BLUE's attack intentions. BLUE's main objective is to maximize target value destroyed by killing as many targets with high target values as possible. Simultaneously, BLUE wants to minimize its own losses.

We employ the basic concept of an intelligent attacker and defender, and sequential actions by each side [e.g., Brown, Carlyle, Salmeron and Wood 2005]. We classify our model as a defender-attacker model with target value destroyed as the more critical component for both opponents. We seek an optimal defense plan with limited resources. Two clearly distinguishable and separate preparations may augment RED's defense plan.

The main plan, hereafter plan Y_k, represents dummy operations taken by RED in order to reduce the effectiveness of the launched attacks against target k by setting up, if available, or building dummy targets (Figure 2). Given that these dummy targets are indistinguishable from the real target, the BLUE attacker has to increase sorties to achieve the same effectiveness on real targets.
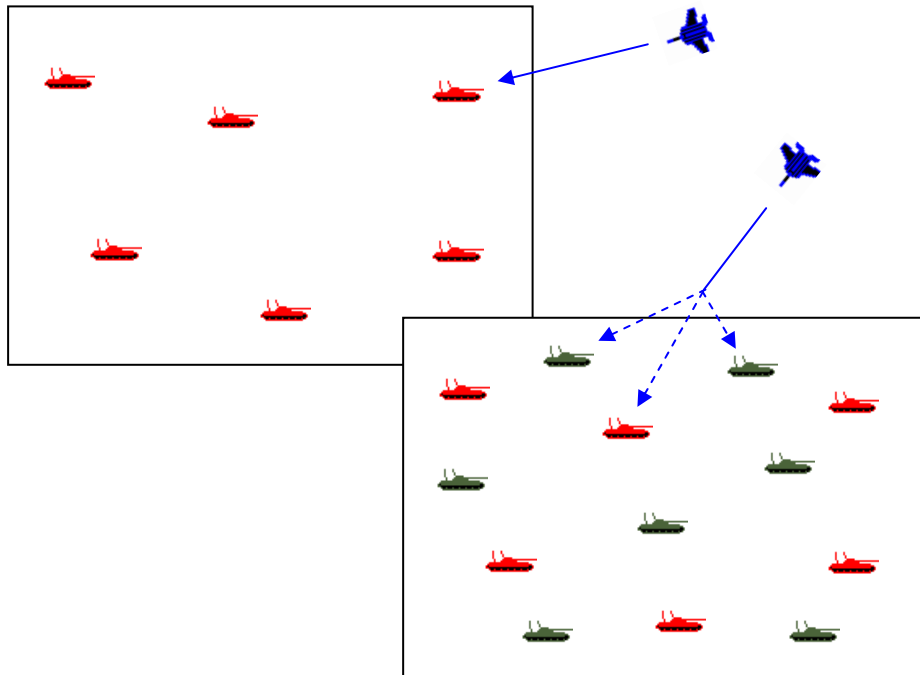
Figure 2.   Illustration of an attack without and with defense plan Y_k (dummy targets)

Dummy targets, indistinguishable from real ones, complicate BLUE's attacks by requiring more of them.

Y_k is subject to limited resources: the number of available dummy targets for specific target types k and the number of dummy operations (setting up or building dummy targets) that that are limited by factors like material, personnel, units and time.

Our second plan, hereafter plan Z_k, represents an enhanced air defense plan for specific targets by augmenting anti-aircraft artillery (hereafter AAA) in order to enhance tactical air defense in anticipation of the attacker's plan (Figure 3). The plan Z_k counters BLUE's second objective to minimize own losses. Z_k is intended to increase attrition to every BLUE platform that attacks the protected or actively defended RED target k. If BLUE still attacks a protected target, a penalty in the form of higher attrition and consequently higher losses of aircraft is incurred.
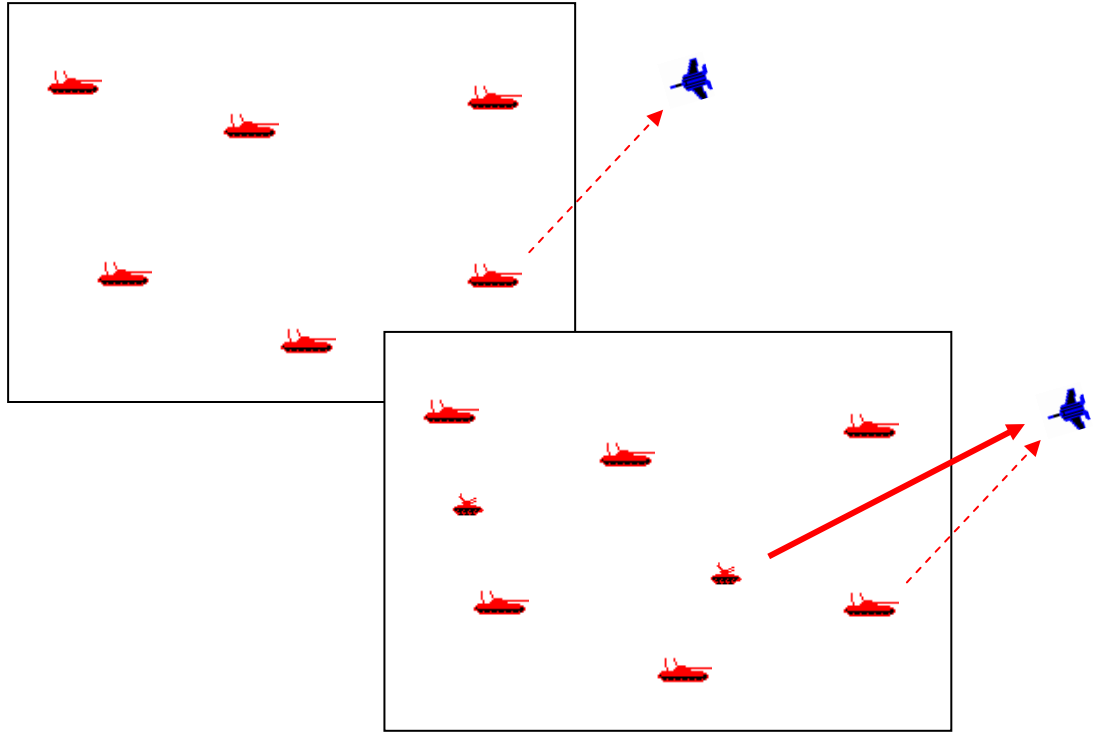
Figure 3. Illustration of attrition without and with defense plan Z_k

An aircraft attacking a target with enhanced defenses suffers increased attrition.

Besides AAA, FATHM also models surface-to-air missile systems (SAM). The missile air defense is in control and command at a division or higher echelon. They build the air defense umbrella for the entire area of operation. AAA units are transferred to the fighting mechanized regiments to provide close air defense support to the frontline armored and infantry battalions [e.g., Russia/Soviet Special Weapons Agencies 2005 and Russia Military Guide 2005].

We characterize our defender-attacker model as a cost- and resource interdicted model with BLUE as the attacker and RED as the defender. With the target value destroyed, based on the dynamic target values, as the most critical component in the attack plan, we solve for an optimal defense plan with the components Y_k and Z_k as the augmenting plans. We introduce binary variables Y_k and Z_k, whose values represent the optimal preparation for the attack subject to limited resources or decisions to interdict attacks against specific targets k. Fixing the binary variables to their optimal values, we solve for an optimal attack plan with a linear program in order to determine an optimal

strategy for BLUE given RED optimal defense. We expect the strategy to change compared to the non-interdicted battle.

The objective of our defender-attacker model remains the same for BLUE as the attacker, to balance efforts in killing target value destroyed, avoiding casualties, and achieving kill goals and capacity goals for the services, but now includes the defender's optimal preparations. In terms of objective function values, BLUE seeks the minimum, whereas RED seeks to maximize BLUE's optimal solution.

Our defender-attacker model objective is:

$$
\underset{Y,Z}{Maximize}\ \underset{X}{Minimize}\ \ -\sum_{apmklw} kvalue_k \left(e_{apmklw} - penaltyY\ Y_k \right) X_{apmklw}
$$

$$
+\lambda \sum_{apmklw} cap_p \left(att_{apmklw} + penaltyZ\ Z_k \right) X_{apmklw}
$$

$$
+\sum_{j} \left(\underline{jpen}_j UNDERKILLS_j + \overline{jpen}_j OVERKILLS_j \right)
$$

$$
+\sum_{s} \left(\underline{spen}_s UNDERCAP_s + +\overline{spen}_s OVERCAP_s \right)
$$

B. ASSUMPTIONS

The basic assumption is transparency. In order to solve for an optimal defense plan against anticipated BLUE attacks, RED requires knowledge of the underlying data, including the most critical factor, the target value for each RED ground platform. Once the optimal defense plan is obtained, it is the crucial input for the model to solve again for an optimal, but changed attack plan, given RED defense.

We introduce additional data and parameters. Our intention for this thesis is either to treat these data as static, not subject to changes, or dynamic, derived as functions of FATHM's original dynamic input data. This enables our model to perform the same updates with our new data that FATHM performs with the original data at the end of the last ground battle in each time period.

We maintain the basic concept of FATHM's original Air model for the choice of RED targets by BLUE. The dynamic target values in combination with preset kill goals determine BLUE's target selection.

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We assume RED dummy targets to be indistinguishable from real RED targets. In other words, they have the same kill probability and thus expected number of kills per BLUE sortie as real targets, e.g., e_{apmklw}.

FATHM has no direct concept of geographic position of targets. So we are not able to position AAA to protect specific geographic positions. We assume the average attrition of all available AAA against each platform p (see aa_p defined below), as the measure of the augmented air defense capability when Z_k is deployed for air defense of specific targets k.

C.   ADDITIONAL SETS, DATA AND DECISION VARIABLES

The formulation of our defender-attacker model requires the introduction of additional sets, data and decision variables.

1.   Subscripts and Sets

$k \in K_{AAA}$ subset of target types k that are AAA

2.   Data

We first introduce the additional data needed to initialize the state for every new time period.

$mxdumk_k$	upper bound on dummy target type k kills (the number available)
$dumops$	upper bound on dummy operations Y_k
$aaops$	upper bound on AAA enhancements Z_k

All the following are dynamic data and computed by our model as functions of FATHM's original input data.

$edec_{apmklw}$	expected reduction of e_{apmklw}, if defense plan Y_k is deployed; penalty in the objective function

aa_p

average attrition by AAA $k \in K_{AAA}$ to platform p, for all attack configurations a,m,l,w;

$$aa_p = \frac{\sum\limits_{amk \in K_{AAA} \, lw} att_{apmklw}}{\sum\limits_{amk \in K_{AAA} \, lw} 1} \qquad \forall p$$

aaa_{apmklw}

additional attrition by augmented AAA to platform p with loadout l and weapon m in attack profile a and weather state w attacking actively defended target type k, if defense plan Z_k is active; penalty in the objective function

3. Variables (all binary)

To represent RED defense plans as discussed in Chapter II, we introduce the binary variables

Y_k

1, if RED sets up dummy targets for target type k, 0 otherwise,

Z_k

1, if RED augments AAA to enhance air defense capabilities for target k, 0 otherwise.

4. Data Pre-processing

In a first step, we analyze simple scenarios in order to derive appropriate expressions for our additional data. In a second step, we transfer the results to parameters with the full set of indices in order to be able to pre-compute the dynamic data our defender-attacker model requires as additional input.

a.   Defense Plan Y_k – Dummy Targets

Defense plan Y_k, setting up dummy targets for specific target types k, has the purpose to reduce BLUE's effectiveness against these targets.   Without RED defensive actions Y_k, all sorties launched by BLUE are directed against real targets, which results in killing real targets only.   Given that dummy targets are deployed, the total number of BLUE attacks against RED targets divides into a proportion against real targets and a proportion against dummy targets.

We consider the case that the attacker is able to kill all targets.   We assume (without indices)

$\quad$ e = 0.5 $\qquad$ (interpretation: need two attacks to kill a target)

$\quad$ mxtargk = 10

$\quad$ mxdumk = 10

$\quad$ kvaluek = 1.

For a non-interdicted attack, $Y = 0$, the number of attacks needed to kill all targets is $X = \dfrac{mxtargk}{e} = \dfrac{10}{0.5} = 20$. This solution has an objective function value of $value = kvaluek \cdot e \cdot X = 1 \ \cdot \ 0.5 \ \cdot \ 20 = 10$.

Given a defense plan $Y = 1$, the number of attacks needed to kill all (real) targets becomes $X = \dfrac{mxtargk}{e} + \dfrac{mxdumk}{e} = \dfrac{10}{0.5} + \dfrac{10}{0.5} = 20 + 20 = 40.$ $\qquad$ With $mxtargk_k = mxdumk_k$, BLUE has to launch twice as many sorties in order to kill all real targets. Half of the attacks are directed against real targets, the other half against indistinguishable dummies.   Because the objective function measures target value destroyed, its value has to remain the same and calculates as $value = kvaluek \ \cdot \ e' \ \cdot \ X = 1 \cdot e' \cdot 30 = 10$, where e' is the reduced effectiveness caused by defense plan Y. We derive an expression for e' as

$$e' = e\frac{X \,|\, Y = 0}{X \,|\, Y = 1} = 0.5\frac{20}{40} = \frac{1}{4}$$

or, using the declared parameters

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$$e' = e\ \cfrac{\cfrac{mxtargk}{e}}{\cfrac{mxtargk}{e} + \cfrac{mxdumk}{e}}\quad \text{which is equivalent to}$$

$$e' = e\ \frac{mxtargk}{mxtargk + mxdumk}.$$

For the objective function in our defender-attacker model, we want this reduction to be the penalty that is incurred, if BLUE attacks the selected target. We set $e' = e - edec$ and obtain the reduction in effectiveness, the desired penalty, as

$$edec = e - e' = e\left(1 - \frac{mxtargk_k}{mxtargk_k + mxdumk_k}\right)\quad \text{or equivalently}$$

$$edec = e\ \frac{mxdumk}{mxtargk + mxdumk}.$$

We can now formulate the objective function with the full set of indices as

$$\textit{Minimize}\qquad -\sum_{apmklw} kvalue_k \left(e_{apmklw} - edec_{apmklw}Y_k\right) X_{apmklw}$$

$$\ldots$$

$$\text{where}\qquad edec_{apmklw} = e_{apmklw}\ \frac{mxdumk_k}{mxtargk_k + mxdumk_k}.$$

When dummies are deployed by Y_k, $\dfrac{mxdumk_k}{mxtargk_k + mxdumk_k}$ computes the proportion of attacks that are directed against dummies.

Modeling defensive actions Y_k, the number of real targets killed is

$$\sum_{apmlw} \left(e_{apmklw} - edec_{apmklw}\ Y_k\right) X_{apmklw}\qquad \forall k.$$

The number of dummy kills is

$$\sum_{apmlw} edec_{apmklw}\ Y_k\ X_{apmklw}\qquad \forall k.$$

To prevent killing more targets than exist on the battlefield, we include the constraint

$$\sum_{apmlw} e_{apmklw}\, X_{apmklw} \leq mxtargk_k + mxdumk_k\, Y_k \qquad\qquad \forall\, k.$$

The physical increase of the total number of targets available by dummy operations Y_k also affects BLUE's desired soft goals for the number of targets killed for each target class j. For each of these target classes there are lower and upper goals, $\overline{jgoal_j}$ and $\underline{jgoal_j}$ respectively. These goals now have to include the number of dummy targets that have to be killed in order to achieve the *jgoal*, if $Y_k = 1$.

In this case, the goals become

$$\underline{jgoal_j} \Leftarrow \underline{jgoal_j} + \frac{\underline{jgoal_j}}{\sum_{k \in K_j} mxtargk_k} \sum_{k \in K_j} mxdumk_k Y_k \qquad\qquad \forall j$$

and

$$\overline{jgoal_j} \Leftarrow \overline{jgoal_j} + \frac{\overline{jgoal_j}}{\sum_{k \in K_j} mxtargk_k} \sum_{k \in K_j} mxdumk_k Y_k \qquad\qquad \forall j$$

where $\dfrac{\underline{jgoal_j}}{\sum_{k \in K_j} mxtargk_k}$ and $\dfrac{\overline{jgoal_j}}{\sum_{k \in K_j} mxtargk_k}$ equal the relative soft goal h_j for the number of targets killed for each target class j in each war phase h. ($h_j * 100$) is the kill goal in percent for each target class j relative to the maximum number of targets available in that class, which equals the sum of all targets $k \in K_j$, $\sum_{k \in K_j} mxtargk_k$.

The expressions for the soft goals can be simplified to

$$\underline{jgoal}_j \Leftarrow \underline{jgoal}_j + h_j \sum_{k \in K_j} mxdumk_k Y_k \qquad \forall j$$

and

$$\overline{jgoal}_j \Leftarrow \overline{jgoal}_j + h_j \sum_{k \in K_j} mxdumk_k Y_k \qquad \forall j$$

The $JGOAL_j$ constraint set in our defender-attacker model is:

$$\sum_{apmk \in K_{jlw}} e_{apmklw} X_{apmklw} + UNDERKILLS_j + MIDKILLS_j - OVERKILLS_j$$

$$= \overline{jgoal}_j + h_j \sum_{k \in K_j} mxdumk_k Y_k \qquad \forall j$$

$$MIDKILLS_j \leq \overline{jgoal}_j - \underline{jgoal}_j \qquad \forall j$$

The number of dummy operations deployed by Y_k is limited by RED's respective resources and represented in our model by *dumops*:

$$\sum_k Y_k \leq dumops.$$

b.   *Defense Plan Z_k – Dynamic Air Defense*

Defense plan Z_k augments AAA to better protect high value targets from BLUE air strikes and has the purpose to increase BLUE losses when attacking these targets. We define this increase of attrition as aaa_{apmklw}, the penalty that is incurred if BLUE aircraft p attack targets k with weapons m and loadout l in attack profile a and weather w. We require $Z_k = 0$ for $k \in K_{AAA}$, because our goal is to enhance the air defense capability of RED platforms without organic air defense weapons that are more vulnerable against BLUE aircraft, i.e., artillery, tanks, armored personnel carriers and infantry fighting vehiles. Following our main assumption regarding FATHM's spatial management of platforms, we set $aaa_{apmklw} = aa_p$. Every BLUE attack will suffer the additional average attrition by RED AAA.

Consider the following example:

$att = 0.1$ (interpretation: loss of 1 platform when BLUE launches 10 attacks);

$aaa = 0.5$ (interpretation: loss of 1 platform when BLUE launches 2 attacks);

$mxplats = 10$;

Without defense plan Z, $Z = 0$, BLUE loses all platforms, if launching $\frac{mxplats}{att} = \frac{10}{0.1} = 100$ sorties. The augmentation of AAA, $Z = 1$ and $(att + aaa) X = (0.1 + 0.5) X = 10$ yields $X = \frac{10}{0.6} = 16.667$. RED performs six times better with active defense of a selected target. BLUE's objective to minimize own losses discourages sorties against AAA protected targets.

We formulate the respective part of the objective function

Minimize ...

$$\lambda \sum_{apmklw} cap_p \left(att_{apmklw} + aaa_{apmklw} Z_k \right) X_{apmklw} .$$

...

Because we do not physically change resources, i.e., aircraft available $mxplats_p$, and overall losses are small compared to these limits, the penalized objective function fully captures the effect of the defense Z_k and no modification of constraints is required.

The number of AAA augmentations by Z_k, represented by aaops, is limited by the availability of RED AAA:

$$\sum_k Z_k \leq aaops .$$

We assume that RED wants to actively defend as many targets k as possible with its given resources. So we only allow one defensive operation at a time for one target k:

$$Z_k + Y_k \leq 1 \qquad \forall k .$$

5.   Formulation as a Nonlinear-Integer Program

Our aim is still to optimize the attacker's (BLUE) air strikes, but now against a defender (RED) with an optimal defense plan. BLUE seeks to minimize the objective function by the choice of attacks X, RED wants to maximize BLUE's optimal solution by the choice of the defense plans Y and Z. This leads to the following maximin formulation, with dual variables shown in brackets. Thus:

$$\underset{Y,Z}{Maximize}\begin{bmatrix} \underset{X}{Minimize} & -\sum_{apkmlw} kvalue_k \left(e_{apmklw} - edec_{apmklw} Y_k \right) X_{apmklw} \\[2mm] & + \lambda \sum_{apmklw} cap_p \left(att_{apmklw} + aaa_{apmklw} Z_k \right) X_{apmklw} \\[2mm] & + \sum_j \left(\underline{jpen}_j\, UNDERKILLS_j + \overline{jpen}_j\, OVERKILLS_j \right) \\[2mm] & + \sum_s \left(\underline{spen}_s\, UNDERCAP_s + \overline{spen}_s\, OVERCAP_s \right) \end{bmatrix}$$

$s.t.$ $KILLS_k$

$$\sum_{apmlw} e_{apmklw}\, X_{apmklw} \le mxtargk_k + mxdumk_k\, Y_k \qquad\qquad \forall k \quad [\alpha_k]$$

$PLTSLOST_p$

$$\sum_{amklw} att_{apmklw}\, X_{apmklw} \le mxplats_p \qquad\qquad \forall p \quad [\beta_p]$$

$HRSUSED_{pw}$

$$\sum_{amkl} used_{kpw}\, X_{apmklw} \le mxhours_{pw} \qquad\qquad \forall pw \quad \left[\gamma_{pw}\right]$$

$WEPUSED_m$

$$\sum_{apklw} c_{apmklw}\, X_{apmklw} \le mxwepns_m \qquad\qquad \forall m \quad [\delta_m]$$

$JGOAL_j$

$$\sum_{apmk\in K_j lw} e_{apmklw}\, X_{apmklw} + UNDERKILLS_j + MIDKILLS_j - OVERKILLS_j$$

$$= \overline{jgoal}_j + h_j \sum_{k\in K_j} mxdumk_k Y_k \qquad\qquad \forall j \quad \left[\varepsilon_j\right]$$

$$MIDKILLS_j \quad \le \overline{jgoal}_j - \underline{jgoal}_j \qquad\qquad \forall j \quad \left[\varphi_j\right]$$

$SGOAL_s$

$$\sum_{ap\in P_s mklw} cap_p\, used_{kpw}\, X_{apmklw} + UNDERCAP_s + MIDCAP_s - OVERCAP_s$$

$$= \overline{sgoal}_s \qquad\qquad \forall s \quad \left[\eta_s\right]$$

$$MIDCAP_s \quad \le \overline{sgoal}_s - \underline{sgoal}_s \qquad\qquad \forall s \quad \left[\kappa_s\right]$$

$$X_{apmklw} \ge 0 \qquad\qquad \forall apmklw$$

$$UNDERKILLS_j,\ MIDKILLS_j,\ OVERKILLS_j \ge 0 \qquad \forall j$$

$$UNDERCAP_s,\ MIDCAP_s,\ OVERCAP_s \quad \ge 0 \qquad \forall s$$

$$\sum_k Y_k \quad \le dumops$$

$$Z_k + Y_k \le 1 \qquad\qquad \forall k$$

$$\sum_k Z_k \quad \le aaops$$

$$Y_k \in \{0,1\} \qquad\qquad \forall k$$

$$Z_k \in \{0,1\} \qquad\qquad \forall k$$

Fixing the binary variables and taking the dual of the interior linear program yields a linear program with the fixed binary variables appearing in the objective function and the right hand side. Releasing the binary variables, we obtain a non-linear integer program.

Maximize

$$\sum_k mxtargk_k\, \alpha_k + \sum_k mxdumk_k\, Y_k\, \alpha_k + \sum_p mxplats_p\, \beta_p + \sum_{pw} mxhours_{pw}\, \gamma_{pw} + \sum_m mxwepns_m\, \delta_m +$$

$$\sum_j \overline{jgoal}_j\, \varepsilon_j + \sum_{j,k\in K_j} h_j\, mxdumk_k Y_k\, \varepsilon_j + \sum_j \left(\overline{jgoal}_j - \underline{jgoal}_j \right) \varphi_j + \sum_s \overline{sgoal}_s\, \eta_s + \sum_s \left(\overline{sgoal}_s - \underline{sgoal}_s \right) \kappa_s$$

subject to :

$$e_{apmklw}\, \alpha_k + att_{apmklw}\, \beta_p + used_{kpw}\, \gamma_{pw} + c_{apmklw}\, \delta_m + e_{apmklw}\, \varepsilon_j + cap_p\, used_{kpw}\, \eta_s \le$$
$$-kvalue_k \left(e_{apmklw} - edec_{apmklw} Y_k \right) + \lambda\, cap_p \left(att_{apmklw} + aaa_{apmklw} Z_k \right) \qquad \forall amlw, j, s, p \in P_s, k \in K_j \quad \left[X_{apmklw} \right]$$

$\varepsilon_j \le \underline{jpen}_j$	$\forall j$	$\left[UNDERKILL_j \right]$
$\varepsilon_j + \varphi_j \le 0$	$\forall j$	$\left[MIDKILL_j \right]$
$-\varepsilon_j \le \overline{jpen}_j$	$\forall j$	$\left[OVERKILL_j \right]$
$\eta_s \le \underline{spen}_s$	$\forall s$	$\left[UNDERCAP_s \right]$
$\eta_s + \kappa_s \le 0$	$\forall s$	$\left[MIDCAP_s \right]$
$-\eta_s \le \overline{spen}_s$	$\forall s$	$\left[OVERCAP_s \right]$

$$
\begin{aligned}
\alpha_k &\le 0 & \forall k \\
\beta_p &\le 0 & \forall p \\
\gamma_{pw} &\le 0 & \forall pw \\
\delta_m &\le 0 & \forall m \\
\varphi_j &\le 0 & \forall j \\
\kappa_s &\le 0 & \forall s \\
\varepsilon_j &\text{ unrestricted} & \forall j \\
\eta_s &\text{ unrestricted} & \forall s
\end{aligned}
$$

$$\sum_k Y_k \le dumops$$
$$Z_k + Y_k \le 1 \qquad\qquad\qquad\qquad\qquad \forall k$$
$$\sum_k Z_k \le aaops$$

$$Y_k \in \{0,1\} \qquad \forall k$$
$$Z_k \in \{0,1\} \qquad \forall k$$

6.   Formulation as an Mixed Integer Program (dual MIP)

We reformulate the nonlinear-integer program as a mixed integer program (dual MIP). The only features violating an integer linear program form are the nonlinear terms in the objective function $Y_k \alpha_k$ and $Y_k \varepsilon_j$. These are products of binary Y_k with dual variables. We reformulate these terms.

We introduce two continuous non-positive variables A_k, $B_k \leq 0$, and substitute the first product $Y_k \alpha_k = A_k$. The resulting expression in the objective function becomes $\sum_k mxdumk_k \ A_k$, which is linear in A_k.

If $Y_k = 0$, we want $A_k = 0$, and if $Y_k = 1$, we want $A_k = \alpha_k$. We are able to achieve the appropriate setting of A_k's values by adding the constraints

$$A_k + B_k = \alpha_k \qquad\qquad \forall\, k$$

$$(1 - Y_k)(negative\,\#) \leq B_k \qquad\qquad \forall\, k.$$

B_k may attain a nonzero value if $Y_k = 0$. Because B_k is not an element of our substitution, it is not part of the objective function.

We choose the lower bound of the dual variable α_k as the $(negative\,\#)$. α_k represents the change in the objective function if the right hand side of the constraint $KILLS_k$ is increased by one unit, i.e., the value of one additional target. We estimate this conservatively by $\alpha_k = \min\limits_{apmlw} \left\{ -\dfrac{kvalue_k}{e_{apmklw}} \right\}$.

To substitute the second product, $Y_k \varepsilon_j$, where this time ε_j is unrestricted in sign, we follow a similar approach. We introduce four non-negative continuous variables $EY_{jk}^+,\ EY_{jk}^-,\ E_{jk}^+, E_{jk}^- \geq 0$ and substitute $Y_k \varepsilon_j = EY_{jk}^+ - EY_{jk}^-$.

Depending on the value of Y_k and the sign of ε_j, we want the new variables to have the values shown in Table 1.

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$Y_k = 1$		$Y_k = 0$	
$\varepsilon_j \geq 0$	$\varepsilon_j < 0$	$\varepsilon_j \geq 0$	$\varepsilon_j < 0$
$EY_{jk}^+ = \varepsilon_j$	$-EY_{jk}^- = \varepsilon_j$	$E_{jk}^+ = \varepsilon_j$	$-E_{jk}^- = \varepsilon_j$
$EY_{jk}^- = E_{jk}^+ = E_{jk}^- = 0$	$EY_{jk}^+ = E_{jk}^+ = E_{jk}^- = 0$	$EY_{jk}^+ = EY_{jk}^- = E_{jk}^- = 0$	$EY_{jk}^+ = EY_{jk}^- = E_{jk}^+ =$

Table 1.   Variable values for Y_k and ε_j combinations

We can get the appropriate values for the four new variables by adding the constraints

$$E_{jk}^+ + EY_{jk}^+ - E_{jk}^- - EY_{jk}^- = \varepsilon_j \qquad \forall\ jk$$
$$EY_{jk}^+ \leq (big\ \#)\,Y_k \qquad \forall\ jk$$
$$E_{jk}^+ \leq (big\ \#)\big(1 - Y_k\big) \qquad \forall\ jk$$
$$EY_{jk}^- \leq (big\ \#)\,Y_k \qquad \forall\ jk$$
$$E_{jk}^- \leq (big\ \#)\big(1 - Y_k\big) \qquad \forall\ jk$$

$$EY_{jk}^+,\ EY_{jk}^-,\ E_{jk}^+, E_{jk}^- \geq 0 \qquad \forall\ jk.$$

Analyzing the objective function, we identify the penalties $\underline{jpen}_j$ and $\overline{jpen}_j$ for not achieving the corresponding kill goals $\underline{jgoal}_j$ and $\overline{jgoal}_j$ as the parameters that cause the change in the objective function, if the right-hand side of the constraint $JGOAL_j$ is increased by one unit, i.e., target. This increase implicitly incurs a respective increase of $UNDERKILLS_j$ or $OVERKILLS_j$. Keeping the optimal attack plan X_{apmklw}^* constant, e.g., keeping the number of targets killed constant, an increase of the right-hand side results in an increase of $UNDERKILLS_j$, which causes the objective function to change by the respective penalty $\underline{jpen}_j$.

Because the penalties for under achieving the soft goals are no less than the penalties for over achievement, we assign the maximum value of $\underline{jpen}_j$ to the big

positive number (*big #*) needed in our additional constraints. We now perform the substitution and reformulate the expression as

$$\sum_{j,k \in K_j} h_j \ mxdumk_k \left(EY_{jk}^+ - EY_{jk}^- \right),$$

which now is linear in the difference $\left(EY_{jk}^+ - EY_{jk}^- \right)$.

In substituting the non-linear expressions in the objective function, we can reformulate our previous model and obtain our defender-attacker model in form of a mixed integer program.

Maximize

$$\sum_k mxtargk_k\, \alpha_k + \sum_k mxdumk_k\, A_k + \sum_p mxplats_p\, \beta_p + \sum_{pw} mxhours_{pw}\, \gamma_{pw} + \sum_m mxwepns_m\, \delta_m +$$

$$\sum_j \overline{jgoal}_j\, \varepsilon_j + \sum_{j,k\in K_j} h_j\, mxdumk_k\left(EY_{jk}^+ - EY_{jk}^-\right) + \sum_j\left(\overline{jgoal}_j - \underline{jgoal}_j\right)\varphi_j + \sum_s \overline{sgoal}_s\, \eta_s + \sum_s\left(\overline{sgoal}_s - \underline{sgoal}_s\right)\kappa_s$$

subject to :

$$e_{apmklw}\, \alpha_k + att_{apmklw}\, \beta_p + used_{kpw}\, \gamma_{pw} + c_{apmklw}\, \delta_m + e_{apmklw}\, \varepsilon_j + cap_p\, used_{kpw}\, \eta_s \le$$

$$-kvalue_k\left(e_{apmklw} - edec_{apmklw}Y_k\right) + \lambda\, cap_p\left(att_{apmklw} + aaa_{apmklw}Z_k\right) \qquad \forall amlw, j, s, p\in P_s, k\in K_j \quad \left[X_{apmklw}\right]$$

$\varepsilon_j \le \underline{jpen}_j$	$\forall j$	$\left[UNDERKILL_j\right]$
$\varepsilon_j + \varphi_j \le 0$	$\forall j$	$\left[MIDKILL_j\right]$
$-\varepsilon_j \le \overline{jpen}_j$	$\forall j$	$\left[OVERKILL_j\right]$
$\eta_s \le \underline{spen}_s$	$\forall s$	$\left[UNDERCAP_s\right]$
$\eta_s + \kappa_s \le 0$	$\forall s$	$\left[MIDCAP_s\right]$
$-\eta_s \le \overline{spen}_s$	$\forall s$	$\left[OVERCAP_s\right]$

$$A_k + B_k = \alpha_k \qquad \forall k$$

$$(1-Y_k)\left(\min_{apmlw}\left\{-\frac{kvalue_k}{e_{apmklw}}\right\}\right) \le B_k \qquad \forall k$$

$$E_{jk}^+ + EY_{jk}^+ - E_{jk}^- - EY_{jk}^- = \varepsilon_j \qquad \forall j, k\in K_j$$

$$EY_{jk}^+ \le \max\left\{\underline{jpen}_j\right\}Y_k \qquad \forall j, k\in K_j$$

$$E_{jk}^+ \le \max\left\{\underline{jpen}_j\right\}(1-Y_k) \qquad \forall j, k\in K_j$$

$$EY_{jk}^- \le \max\left\{\underline{jpen}_j\right\}Y_k \qquad \forall j, k\in K_j$$

$$E_{jk}^- \le \max\left\{\underline{jpen}_j\right\}(1-Y_k) \qquad \forall j, k\in K_j$$

α_k	$\le$	0	$\forall k$
β_p	$\le$	0	$\forall p$
γ_{pw}	$\le$	0	$\forall pw$
δ_m	$\le$	0	$\forall m$
φ_j	$\le$	0	$\forall j$
κ_s	$\le$	0	$\forall s$
$A_k, B_k \le$		0	$\forall k$

ε_j	*unrestricted*	$\forall j$
η_s	*unrestricted*	$\forall s$

$$EY_{jk}^+, EY_{jk}^-, E_{jk}^+, E_{jk}^- \ge 0 \qquad \forall jk$$

$$\sum_k Y_k \le dumops$$

$$Z_k + Y_k \le 1 \qquad \forall k$$

$$\sum_k Z_k \le aaops$$

$$Y_k \in \{0,1\} \qquad \forall k$$

$$Z_k \in \{0,1\} \qquad \forall k$$

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IV.   DEFNDER-ATTACKER RESULTS

A.   MODEL IMPLEMENTATION IN GAMS

We generate the defender-attacker model using the General Algebraic Modeling System (GAMS) [Brooke, Kendrick, Meeraus and Raman 1998] and solve it using CPLEX version 9.0 [GAMS Development Corporation, CPLEX 9 2005].

Figure 4 shows the basic flow of the defender-attacker model. We first solve the dual MIP. By fixing the values of Y_k and Z_k from the dual MIP, the optimal defense plan, we solve the maximin program, a nonlinear integer program, as a linear program (LP) for the attack plan X_{apmklw}.
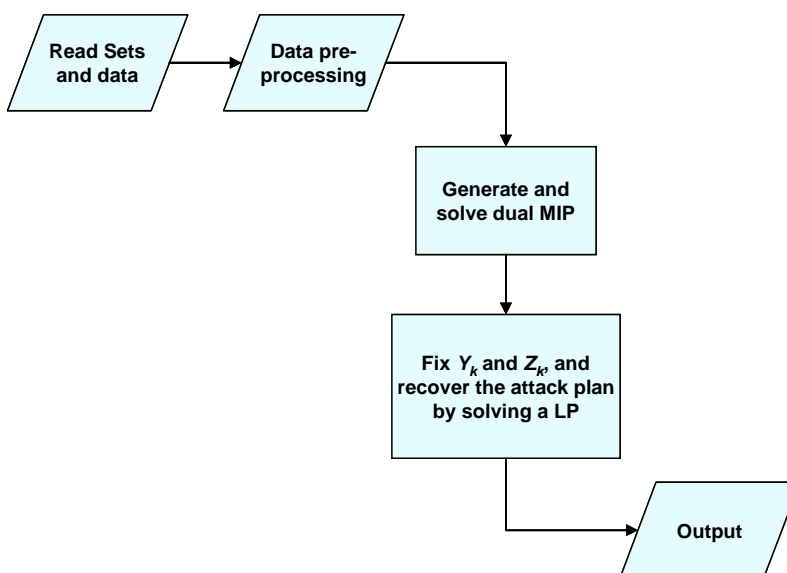
Figure 4.   Basic program structure and flow for the defender-attacker model

Our mixed integer program (MIP) has about 1,000 continuous variables, 200 binary variables, and 44,000 constraints. We solve for optimality within 1% (GAMS option optimality criterion optcr = 0.01) in about a minute on a 2.0 GHz personal computer.

The linear program has about 44,000 continuous variables and 200 constraints. It solves in a couple of seconds.

B. CRITICAL PARAMETERS AND ASSUMPTIONS FOR ACTIVE RED DEFENSE OPERATIONS

We compare our defender-attacker version results with the legacy Air model for the first time period of a FATHM war.

We have FATHM's input data for a selected unclassified war scenario. We see the target values, generated by FATHM, and the soft goals for killing targets in each target class j as the critical input data. For the first time period, the target values for all targets k are set to one, implying no specific target is prioritized. The kill proportion for target class $j1$ equals 1, whereas all other classes are 0.01. We expect the legacy Air model to concentrate attacks on targets in target class $j1$. We expect our defender-attacker model to actively defend exactly these targets.

We assume RED capable of conducting 10 dummy operations and three air defense augmentations. With each dummy operation, RED is able to set up or build slightly more than half the number of real targets as dummies.

Defender-Attacker Model specific input data:

$$mxdumk_k = \left\lfloor \frac{5}{8} mxtargk_k \right\rfloor \qquad \forall k$$
$$dumops = 10$$
$$aaops = 3.$$

C. COMPARISON WITH THE AIR MODEL AND ANALYSIS OF RESULTS

We expect that deployment of defense plans Y_k and Z_k will reduce the number of targets killed, the deviation from the kill goals to increase and the number of platforms shot down by RED to increase. We concentrate on the attack plan X_{apmklw}, the number of RED targets killed $KILLS_k$, the number of BLUE aircraft lost $PLTSLOST_p$, and the deficiency variables $TKILLS_j$, with $UNDERKILLS_j$, $MIDKILLS_j$ and $OVERKILLS_j$ as the kill deficiencies regarding the soft kill goals, and $SHOURS_s$ with $UNDERCAP_s$, $MIDCAP_s$ and $OVERCAP_s$ for the capacity soft goals.

1.   Attack Summaries

We summarize results in Table 2. The defender-attacker model reduces the number of targets killed by 20% (from 449.53 to 361.75), and increases the number of attacks by 6% (from 302.41 to 319.62). The Air model kills 1.5 targets per sortie compared to 1.1 targets with the defender-attacker model. BLUE's casualties are 33% higher (from 4.93 to 6.55 platforms). Furthermore, BLUE needs 13% more weapons to kill 20% fewer targets. The objective function conveys these results. The effectiveness of BLUE air strikes in killing targets against a RED defense decreases by 33% or, in other words, BLUE's efforts in killing even fewer targets increases by one third. The value of the objective function increases from 10,666.5 to 14,154.3.

	Air model	defender-attacker model
OBJECTIVE	10666.5	14154.3
TOTALKILLS	449.5	361.8
TOTALATTACKS	302.4	319.6
TOTALPLATSLOST	4.9	6.6
TOTALWEPUSED	2771.2	3140.3
TOTALHRSUSED	302.4	319.6

Table 2.   Air strike comparison

The defender-attacker model causes a significant decrease of BLUE's effectiveness when attacking RED's active defenses.

2.   Effects of Active Defenses

Facing active RED defenses Y_k and Z_k, BLUE achieves a smaller portion of the preset kill goals to a higher degree than without any defensive actions (Table 3). There is a concentration of RED defense efforts on target class *j1* resulting in an increase to *UNDERKILLS_j*. This increase of *UNDERKILLS_j* reveals BLUE's diminished success in killing targets.

Target class	Air model			defender-attacker model		
	under	mid	over	under	mid	over
j1	330.2			662.2		
j4			10.0			
j13	0.7			0.7		
j14	613.0			613.0		

Table 3.   Comparison of *JGOAL$_j$* deficiencies *TKILLS$_j$*

The defender-attacker model defends targets in high-priority target class j1, and this leads to increased under kills (from 330.2 to 662.2) for BLUE.

Services	Air model			defender-attacker model		
	under	mid	over	under	mid	over
Air Force			56.7			73.9
Navy			243.9			243.9
Army	0.1	0.8		0.1	0.8	

Table 4.   Comparison of *SGOAL$_s$* deficiencies *SHOURS$_s$*

The defender-attacker model increases the over capacity (from 56.7 to 73.9) for the Air Force service component.

Our defender-attacker model generally forces an increase in the number of BLUE attacks and thus increases weapons needed and hours flown. This causes more deviation from equitable BLUE service use. As Table 4 shows, the Air Force's contribution required to perform the optimal attacks increases from 56.71 by 30% to 73.92 and incurs a higher penalty in the objective function.

3.   Target Kill Accounting

The most influential factor attracting attacks should be the attacker's preset phase goals for the current phase, h_j. As shown in Table 5, RED concentrates defense plans, Y_k and Z_k, on all targets in target class *j1*, for which the phase goal equals one. Dummies are set up for all but one target (RED resources are limited by *dumops*). The remaining

target, *k69*, is assigned AAA by plan Z_k as protection against attacking aircraft. Thus, all targets in target class *j1* are considered in RED's defense plan.

We note significant reductions in the number of targets killed of 70% (from 80.8 to 23.7) and 30% (from 69 to 48.4) for *k1* and *k17* respectively. The defender-attacker model is only able to kill a total of 361.8 targets in that class, 20% less than the Air model's 449.5 target kills (Table 2). In addition to these 361.8 (real) targets, a total of 191.6 dummy targets are attacked and "killed," which adds up to a total of 553.4 attacked and destroyed targets. We observe that the main plan Y_k succeeds not only in reducing the number of targets destroyed, but also in increasing BLUE's efforts to achieve the documented kills.

Targets					Dummy targets		Variable	Variable
			killed		killed		Y_k	Z_k
class j	type k	available	Air model	defender-attacker model	defender-attacker model	available		
j1	k1	288	80.8	23.7	14.8	180	**1**	
	k2	32	32	32	20	20	**1**	
	k17	69	69	48.4	29.8	43	**1**	
	k18	69	69	69	43.0	43	**1**	
	k19	64	64	64	40	40	**1**	
	k38	36				22	**1**	
	k69	20	20	20		12		**1**
	k74	61				38	**1**	
	k75	35	35	35	21	21	**1**	
	k79	26				16	**1**	
	k80	37	37	37	23	23	**1**	
j2	k3	54	9.1	9.1		33		**1**
	k16	43				26		
	k39	125				78		
	k68	21				13		
	k71	1				0		
	k72	12				7		
	k73	8				5		
	k76	3				1		
	k77	15				9		
	k78	7				4		
	k84	264				165		
	k85	88				55		
	k86	9				5		
	k93	264				165		
j3	k64	4				2		
	k65	10	0.2	0.2		6		
	k66	3				1		
j4	k4	44				27		
	k5	44				27		
	k6	2				1		
	k7	10				6		
	k8	9				5		
	k9	18				11		
	k10	17				10		
	k11	26				16		
	k13	44				27		
	k14	40	14.2	4.2		25		**1**
	k32	20				12		
	k44	6				3		
	k45	39				24		
	k50	20				12		
	k51	20				12		
	k81	19				11		
	k82	1				0		
	k94	40				25		

Table 5.   Target and dummy kill accounting and comparison for selected target classes j

The defender-attacker model concentrates active defenses by Y_k for target class j1 and kills fewer RED targets in that prioritized target class. A one for Y_k indicates that dummy targets are deployed for that target. A one for Z_k indicates that AAA is deployed for enhanced air defense. For instance, there are 69 targets type k17. The Air model kills 69, all of them. The defender-attacker model is only able to kill 48.4 targets, but additionally kills 29.8 of a total available 43 dummy targets.

4.   Optimal Sortie Allocation and Effectiveness

The defender-attacker model allocates sorties differently.   We compute the aircraft-target assignments based on optimal solutions for each attack plan X_{apmklw} (Table 6).   We see a change in assignments of aircraft on targets in all cases except for the platforms $p3$, $p4$ and $p7$. Defense plan Z_k assigns AAA to targets $k3$, $k14$ and $k69$. The number of sorties by $p2$ against $k14$ is reduced by 70% (from 6.7 to 2.0). The Air model attacks $k69$ with 7.2 sorties by $p11$ compared with 3.8 sorties by $p2$. The number of sorties decreases by 47%.   The Air model attacks $k3$ with aircraft $p8$ whereas the defender-attacker model assigns platform $p9$ to target $k3$.

We also see increases in sorties against specific targets. For example, aircraft $p2$ and $p11$ are assigned to $k19$ in the Air model and fly 12.5 sorties in total. The defender-attacker model changes the assignment and assigns 30.4 attacks by $p2$ and now $p9$ against $k19$, which is an increase of 143% in the number of sorties.

aircraft p	Air model		defender-attacker model	
	target	sorties	target	sorties
p2	k14	6.7	k14	2.0
			k17	16.4
	k19	11.9	k19	8.1
	k21	0.03	k21	0.03
	k22	0.1	k22	0.1
	k28	0.1	k28	0.1
	k37	0.1	k37	0.1
	k52	0.03	k52	0.03
			k69	3.8
	k83	0.8	k83	0.8
	k88	0.6	k88	0.6
p3	k17	9.1	k17	9.1
p4	k17	28.2	k17	33.8
p7	k1	1.4	k1	1.4
	k17	36.5	k17	36.5
	k61	0.5	k61	0.5
p8	k1	45.8	k1	48.0
	k3	2.2		
p9	k1	69.1	k1	4.2
			k3	2.2
			k18	22.0
			k19	22.3
			k75	0.6
	k80	2.9	k80	20.8
p11	k1	3.5	k1	3.5
	k2	39.3	k2	63.8
	k18	13.6		
	k19	0.6		
	k65	0.1	k65	0.1
	k69	7.2		
	k75	12.2	k75	19.1
	k80	10.1		

Table 6.   Comparison of the optimal sortie allocation of aircraft p on targets k

The defender-attacker model reduces the number of sorties against actively defended targets with defense plan Z_k. This table shows how the defender-attacker model changes assignments of sorties by some aircraft on specific targets. For instance, the Air model attacks with aircraft p2 target k14 and assigns 6.7 sorties, the defender attacker model still assigns p2 on k14, but reduces the number of attacks to 2.0.

Table 6 shows that RED defenses change the strategy for BLUE attacks not only in the number of assigned sorties, but also in the aircraft-target combinations. We compute the total number of sorties against specific targets in order to summarize this effect and identify Y_k's effects (Table 7). We see an increase of the number of sorties against targets *k2*, *k17*, *k18*, *k19*, *k75* and *k80* with a minimum of 30% to 95.7 for *k17*, a

maximum of 144% to 30.4 for *k19* and roughly 60% increases in between. Referring to the results in Table 5, this increase of sorties can be attributed to the reduced effectiveness of BLUE's attacks. BLUE is still able to kill all targets *k18*, *k19*, *k75* and *k80*, but also has to kill all dummies set up by defense plan Y_k. While BLUE kills all targets *k17* in the Air model, active defenses reduce the number of kills by 30% even though 30% more sorties are flown.

We see a reduction of sorties against target *k1* by 52% to 57.1 attacks that are only able to kill 23.7 targets according to Table 5. The effectiveness of these attacks decreases from 80.8/119.9 = 0.67 kills per sortie to 23.7/57.1 = 0.41 kills per sortie, or by 40%.

class j	target k	attacks		Variable Y_k	Variable Z_k
		Air model	defender-attacker model		
j1	k1	119.9	57.1	1	
	k2	39.3	63.8	1	
	k17	73.8	95.7	1	
	k18	13.6	22.0	1	
	k19	12.5	30.4	1	
	k38			1	
	k69	7.2	3.8		1
	k74			1	
	k75	12.2	19.6	1	
	k79			1	
	k80	12.9	20.8	1	
j2	k3	2.2	2.2		1
j3	k65	0.1	0.1		
j4	k14	6.7	2.0		1
j5	k61	0.5	0.5		
j6	k28	0.1	0.1		
j7	k37	0.1	0.1		
j8	k83	0.8	0.8		
	k88	0.5	0.6		
j9	k22	0.1	0.1		
j10	k21	0.03	0.03		
j12	k52	0.03	0.03		

Table 7.   Comparison of the number of sorties against targets k

The defender-attacker model increases the total number of sorties against prioritized target class j1 when dummy targets are installed by defense plan Y_k. Table 7 shows the differences in the number of sorties against attacked targets between the two models. For instance, belonging to target class j1, target k1 is attacked by 119.9 sorties in the Air model compared to 57.1 attacks of the defender-attacker model. The value one for Y_k indicates that RED deploys dummy targets. With no entry for Z_k, target k1 is not defended by augmented AAA.

5.   BLUE Aircraft Attrition

The augmentation of AAA by defense plan Z_k actively defends targets $k3$, $k14$ and $k69$ (Table 7).   BLUE aircraft that attack RED target $k3$ suffer increased attrition aaa_{apmklw} between 0.014 and 0.034 platforms per sortie. This results in increases of attrition between 11% and 736% compared to the attrition when AAA is not augmented. For BLUE platforms attacking k14 and k69, the incurred penalty attritions are between 0.0002 and 0.066 aircraft per sortie, and between 0.008 and 0.066 respectively. The resulting increases for k14 are in a range from 32% to 2,303% (up to 23 times higher), for k69 from 32% to 1,112% (up to 11 times higher).

Aircraft $p2$, assigned to attack AAA-defended targets $k14$ (TANK) and $k69$ (IFV) according to Table 6, suffers an attrition almost eight times as high as in the Air model. Because our model still attacks AAA $k19$, $k21$ and $k22$ but reduces the number of sorties against these targets, we see deployed air defense Z_k as the main cause for this high attrition.

Aircraft $p9$, attacking $k3$ in our model, also suffers an increase of losses from 0.9 to 2.9 (220%) compared to the Air model. We see two reasons for this high attrition to aircraft $p9$. Our defender-attacker model assigns BLUE platform $p9$ to primarily attack AAA $k1$, $k18$ and $k19$ (Table 6). In addition, $p9$ now attacks target $k3$, which is assigned AAA by the dynamic air defense Z_k

aircraft p	aircraft lost	
	Air model	defender-attacker model
p2	0.05	0.4
p3	0.03	0.03
p4	0.9	1.0
p7	0.3	0.3
p8	0.7	0.6
p9	0.9	2.9
p11	2.2	1.3

Table 8.   Comparison of attrition per BLUE aircraft

The defender-attacker model increases BLUE casualties by deploying optimal defense operations. Table 8 shows where attrition per BLUE aircraft changes with the defender-attacker model. BLUE platform p2, for instance, suffers 0.05 expected losses in the Air model compared to 0.4 lost aircraft with the defender-attacker model.

6.   Conclusion

Comparison of our defender-attacker model with the legacy Air model shows that an optimal, active defense plan by RED reduces BLUE effectiveness in killing target value by reducing the number of targets killed, increasing casualties and thus influencing the degree (and rate) of goal achievements. In terms of the objective function, BLUE's progress is frustrated and its efforts are significantly increased.

D.   FURTHER COMPUTATIONS

We further explore the effects of RED's defense plans to gain insights about our defender-attacker model. We want to learn how changing the number and characteristics of the defense plans Z_k and Y_k affect the attacker's performance. We vary the number of air defense operations *aaops*, the number of dummy operations *dumops* and $mxdumk_k$, the number of dummy targets available for targets k as our factors in this study. We analyze the effect on the number of aircraft lost $PLTSLOST_p$, targets killed $TGTKILLS_k$, the deficiencies $TKILLS_j$ and $SHOURS_s$, and the value of the objective function as our measurements for RED's performance.

1.   Dynamic Air Defense Z_k

We analyze the effect of the plan Z_k separately (without Y_k) for this part of our study. Using the data for the first time period as in Section IV.C., we solve the defender-attacker model for different numbers of air defense operations and compare BLUE casualties (Table 9). We observe a large increase of attrition to attacking BLUE aircraft when RED deploys only one AAA action. RED is able to increase BLUE's casualties from 4.9 to 7.3 platforms. RED can increase BLUE losses to about 7.7 with a second AAA augmentation, and to 8.1 with a third one. This yields a total relative increase of 56% or 64% respectively.   We notice that the relative marginal increase by the deployment of two and three air defense operations declines to only 5.5% and 5.2%.

aircraft p	RED air defense operations *aaops*										
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
p2 (lost)	0.05	0.05	0.05	0.05	0.08	0.08	0.13	0.13	0.13	0.14	0.14
p3 (lost)	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
p4 (lost)	0.86	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71
p7 (lost)	0.25	0.75	0.75	0.75	0.75	0.75	0.75	0.76	0.76	0.76	0.76
p8 (lost)	0.65	0.65	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55
p9 (lost)	0.92	0.92	1.14	1.14	1.15	1.18	1.18	1.18	1.18	1.18	1.18
p11 (lost)	2.17	2.17	2.45	2.85	2.85	2.89	2.89	2.89	2.89	2.89	2.89
SUM	4.93	7.29	7.69	8.09	8.13	8.20	8.25	8.26	8.26	8.27	8.27
Total increase		47.9%	56.0%	64.1%	64.9%	66.3%	67.3%	67.5%	67.5%	67.7%	67.7%
Marginal increase		47.9%	5.5%	5.2%	0.5%	0.9%	0.6%	0.1%	0.0%	0.1%	0.0%

Table 9.   Effect of RED air defense operations on BLUE casualties

Z_k is most effective when deploying up to three air defense operations. The marginal increases for more than three air defense operations are less than 1%.

Figure 5 shows the number of aircraft shot down by RED. We see that the deployment of more than three reinforcements by AAA, i.e. *aaops > 3*, only yields a marginal increase of less than 1%. We interpret this result as a rapid decrease in the efficiency of defense plan Z_k with an increasing number of air defense operations *aaops*.
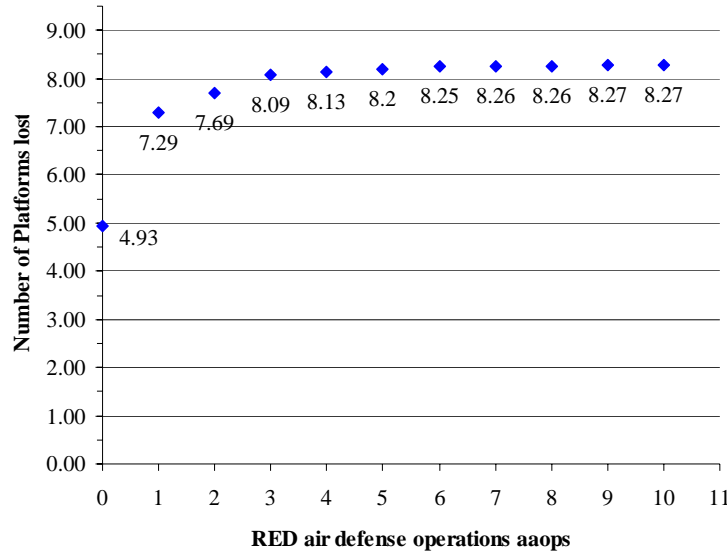

Figure 5.   Number of BLUE casualties for different numbers of target reinforcements by Z_k

Z_k is most incrementally effective when deploying AAA reinforcements for no more than three targets.

40

Our model primarily assigns AAA to targets in target class $j1$, the class that is prioritized for attacks by the phase goals h_j in the current time period. The targets $k17$ (TANK), $k80$ (RAD), $k75$ (ARTY) and $k69$ (IFV) are assigned AAA with the first four *aaops* as active defense reinforcement against the attacking BLUE aircraft. These primary targets are subject to 106 attacks (73.8 attacks on $k17$, 12.9 on $k80$, 12.2 on $k75$ and 7.2 attacks on $k69$) compared to the total number of 11 attacks on targets not belonging to the target class $j1$. Because the number of aircraft lost is a function of the attacks X_{apmklw}, these numbers imply the highest losses when AAA is assigned to targets under massive attack by the three or four *aaops* as shown above.

Z_k does increases the objective function modestly from 10,666.5 to 10,670. We conclude that Z_k achieves its intended purpose, even though the effect on the objective function is not of large magnitude: Z_k causes more BLUE casualties.

2.   Number of Dummy Targets

Dummy targets are the main element in our defense plan Y_k. We start by conducting only one dummy operation, *dumops = 1*, without any defense plans Z_k. We calculate the number of dummy targets as stepwise increasing percentages of the respective maximal available numbers of real targets k. We choose a step size of ten percent between 0 and 100%. Beyond 100%, we only consider 120% and 150%. Solving our defender-attacker model, we observe that dummy operations are conducted for target $k1$, an AAA, which belongs to targets class $j1$. The reason for the selection of $k1$ is seen in the number of available targets, $mxtargk_{k1} = 288$, the maximum among all targets in the prioritized class. Table 10 shows a steady reduction of the number of killed targets as the result of deployed dummies. The third row shows the reduction of BLUE's effectiveness in killing RED targets with increasing numbers of dummy targets. This reduction also represents the relative reduction of sorties against real targets that is caused by the respective number of dummies. For instance, deploying half of the number of real targets as dummies reduces the effectiveness by exactly one third, equivalently to the reduction of the attack ratio against real targets $\dfrac{mxdumk_k}{mxtargk_k + mxdumk_k}$. Setting up the

same number of dummies as there are real targets yields a 50% reduction in BLUE effectiveness.

target *k1*	number of dummy targets in percent of mxtargk$_k$										
	0	10	20	30	40	50	60	70	80	90	100
target kills	80.8	73.6	67.4	62.2	57.7	53.9	50.6	47.6	44.9	42.5	40.4
dummy kills	0.0	7.2	13.4	18.6	23.1	26.9	30.2	33.2	35.9	38.3	40.4
Total reduction		-8.9%	-16.6%	-23.0%	-28.6%	-33.3%	-37.4%	-41.1%	-44.4%	-47.4%	-50.0%
Marginal reduction		-8.9%	-8.4%	-7.7%	-7.2%	-6.6%	-6.1%	-5.9%	-5.7%	-5.3%	-4.9%

target *k1*	number of dummy targets in percent of mxtargk$_k$	
	120	150
target kills	36.8	32.3
dummy kills	44.0	48.5
Total reduction	-54.4%	-60.0%
Marginal reduction	-8.9%	-12.2%

Table 10.   Number of targets killed for increasing number of dummies deployed for target k1

The Defender-Attacker Model steadily reduces the number of targets killed with increasing number of dummies deployed for target k1.

Figure 6 shows the relation between real target kills and dummy kills. We recognize a steeper descent for the number of targets $k1$ killed in the graph for the range 0% to about 50%. We see the same behavior for the number of killed dummy targets in the opposite, increasing direction.
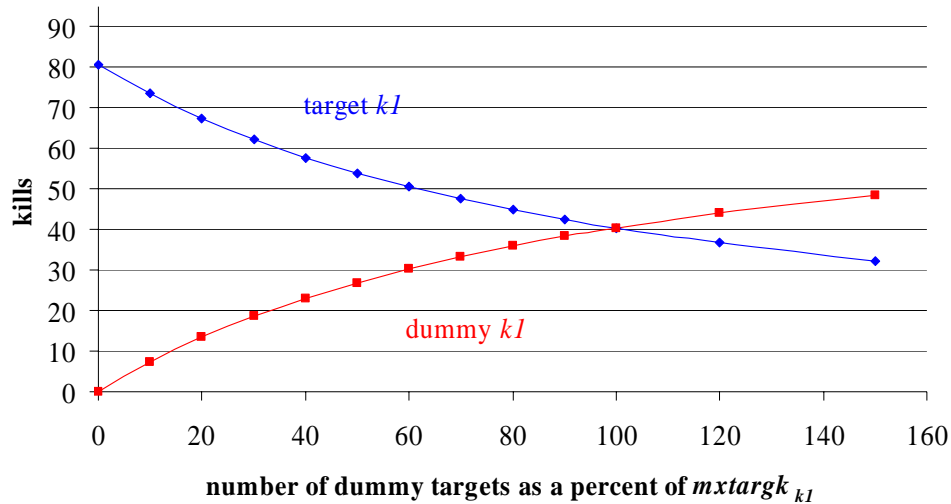

Figure 6.   Target and dummy kills for increasing number of available dummy targets in percent of $mxtargk_{k1}$

The number of dummy targets killed increases as the number of killed targets decreases. When as many dummies are set up as there are targets, BLUE kills approximately as many dummies as targets.

Dummy targets increase the total number of potential targets. This increase also affects soft goals for killing targets of specific target classes, *JGOALS$_j$*. Consequently, dummy operations increase *UNDERKILLS$_j$* at the same time as fewer targets are killed (Table 11). The effect of the number of dummy targets deployed is even bigger for the deficiency *UNDERKILLS$_{j1}$*. An addition of 40% as dummies increases *UNDERKILLS$_{j1}$* by about one third; an addition of 60% increases it by about one half compared to the undefended case. *UNDERKILLS$_{j1}$* doubles with 120% of *mxtargk$_{k1}$* built up as dummy targets.

target class $j1$	number of dummy targets in percent of mxtargk$_{k1}$										
	0	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
UNDERKILLS $_{j1}$	330.2	358.2	387.2	416.2	445.2	474.2	502.2	531.2	560.2	589.2	618.2
Total increase		8.5%	17.3%	26.0%	34.8%	43.6%	52.1%	60.9%	69.6%	78.4%	87.2%
Marginal increase		8.5%	8.1%	7.5%	7.0%	6.5%	5.9%	5.8%	5.5%	5.2%	4.9%

target class $j1$	number of dummy targets in percent of mxtargk$_{k1}$	
	120	**150**
UNDERKILLS $_{j1}$	675.2	762.2
Total increase	104.5%	130.8%
Marginal increase	9.2%	12.9%

Table 11.　Deficiency *UNDERKILLS$_{j1}$* for target class *j1* with increasing number of dummies

UNDERKILLS for the prioritized target class j1 increases rapidly. If 60% of the available number of (real) targets are set up as dummies, UNDERKILLS$_{j1}$ increases by approximately 50%.

Plotting the data of Table 11, we see in that *UNDERKILLS$_{j1}$* continuously increases and seems to be nearly linear in the number of deployed dummy targets for *k1* (Figure 7).
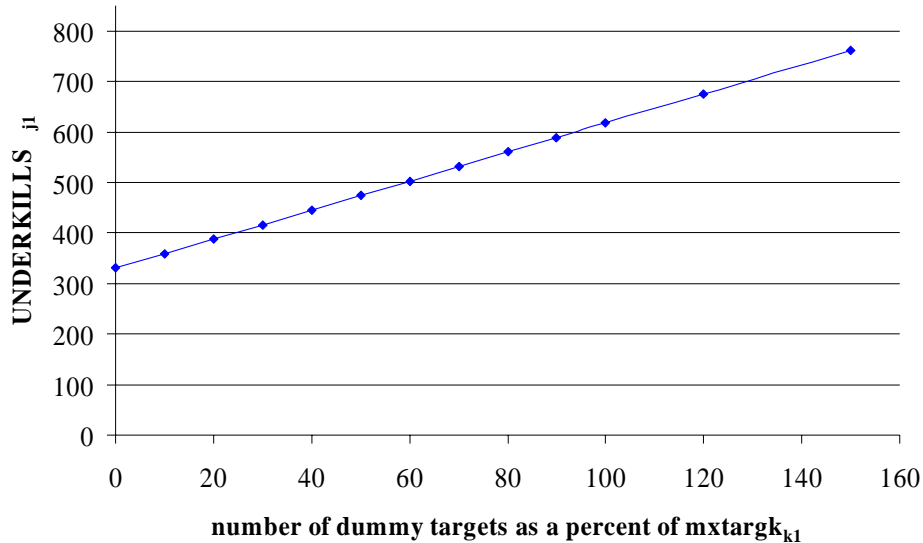

Figure 7.　*UNDERKILLS$_{j1}$* versus available dummy targets in percent of *mxtargk$_{k1}$*

We recognize a continuous, nearly linear increase with growing number of dummy targets deployed.

The attacker's effectiveness in killing targets or target value decreases with an increasing number of dummy targets. This has a significant effect on the objective function (Table 12). The objective function values steadily increase with the increasing number of dummy targets. We see an approximate increase of 25% if 90% of the number of available targets are set up as dummies and almost a 50% increase when 1.50 times the number of real targets are dummies.

objective Function	number of dummy targets in percent of mxtargk$_{k1}$										
	0	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
values	10666.5	10953.6	11249.8	11545.1	11839.5	12133.4	12416.7	12709.7	13002.4	13294.7	13586.9
Total increase		2.7%	5.5%	8.2%	11.0%	13.8%	16.4%	19.2%	21.9%	24.6%	27.4%
Marginal increase		2.7%	2.7%	2.6%	2.5%	2.5%	2.3%	2.4%	2.3%	2.2%	2.2%

Objective Function	Number of dummy targets in percent of mxtargk$_{k1}$	
	120	**150**
Values	14160.5	15350.0
Total increase	32.8%	43.9%
Marginal increase	4.2%	8.4%

Table 12.   Objective function value for increasing numbers of dummies

The objective function increases continuously approximately with an average rate of 2.5% for every 10% increase in the number of dummy targets.

3.   Dummy Operations Only

We now solve our model for different numbers of dummy operations. We do not allow air defense operations ($Z_k = 0 \ \forall k$). We assume the number of dummy targets as about 60% (5/8 *mxtargk$_k$*) of the available real targets *mxtargk$_k$* for the respective targets *k*. We solve our model for up to 15 dummy operations, *dumops = 15*.

All performance measures are affected by dummy operations. The value of the objective function increases due to a reduction of targets killed by a simultaneously increasing number of attacks and an increase in the number of platforms shot down, causing increased penalties with respect to the soft goals that can no longer be achieved on a par with the success of the Air model.

The number of killed (real) targets significantly decreases with the number of dummy operations. We see three effects for the observed BLUE actions. Either BLUE does not attack a target type at all, consequently no dummies are killed; or BLUE is driven by the kill goals and kills all targets, in which case all dummy targets are killed, too; or BLUE attacks targets k and the respective dummies, and reduces the number of sorties, i.e., changes attack strategy. Figure 8 shows the observed evolution of RED's defensive operations Y_k with increasing number of dummy operations.

target class j	target type k	number of dummy operations *dumops*															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
j1	k1	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d
	k2	t	t	t	t	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d
	k17	t	t	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d
	k18	t	t	t	t	t	t	t	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d
	k19	t	t	t	t	t+d	t	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d	t+d
	k38						d	d	d	d	d	d	d	d	d	d	d
	k69	t	t	t	t	t	t	t	t	t	t	t	t+d	t+d	t+d	t+d	t+d
	k74			d	d	d	d	d	d	d	d	d	d	d	d	d	d
	k75	t	t	t	t	t	t	t	t	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d
	k79						d	d	d	d	d	d	d	d	d	d	d
	k80	t	t	t	t	t	t	t	t	t	t+d	t+d	t+d	t+d	t+d	t+d	t+d
j2	k3	t	t	t	t	t	t	t	t	t	t	t	t	t	t+d	t+d	t+d
	k93																d
j3	k65	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j4	k14	t	t	t	t	t	t	t	t	t	t	t	t	t+d	t+d	t+d	t+d
j5	k61	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j6	k28	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j7	k37	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j8	k83	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
	k88	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j9	k22	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j10	k21	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j12	k52	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t	t
j13	k25															d	d

Legend:
t	target attacked, no dummies deployed, some killed	t+d target and dummies attacked, some killed
t	target attacked, no dummies deployed, all killed	t+d target and dummies attacked, all killed
	target not attacked	d dummies deployed, not attacked

Figure 8.   Evolution of RED's optimal defense strategy for dummy operations

The Defender-Attacker Model prioritizes dummy operations in accordance with the phase and kill goals and concentrates efforts on target class j1.

The chart clearly shows the prioritization of target class *j1*, where only few attacks are launched against targets of other classes due to the respective soft goals. Our defender-attacker model concentrates BLUE sorties on all RED targets in this class and

kills all targets *k2*, *k17*, *k18*, *k19*, *k69*, *k75* and *k80* when $Y_k = 0 \quad \forall k$. We notice that this success can be maintained when dummies are deployed for the respective target for all but one target type. By setting up dummies for *k17*, BLUE is no longer able to kill all of that target. The attacks show partial success against *k1*, and *k38*, *k74* and *k79* are not attacked at all. We observe in our outputs, that the more *dumops* are deployed, the fewer targets type *k1* are killed. Dummies do not affect the BLUE strategy when they are set up for targets that originally are not attacked.

Dummy operations significantly affect BLUE's effectiveness. The presence of dummies reduces the number of killed targets and increases in particular the number of *UNDERKILLS* because the air strikes kill dummy as well as real targets (Figure 9). We see that the number of UNDERKILLS doubles with ten and reaches its maximum when RED conducts eleven dummy operations. RED reaches this point when all targets of *j1* are defended by Y_k. We recognize the greatest increase already with one dummy operation that sets up dummies for target *k1*. Given equal target values, we see the reason in the largest number of available targets *mxtargk$_{k1}$* = 288 and dummies, *mxdumk$_k$* = 180 in that class. A significant decrease in the number of killed targets is caused by this operation.
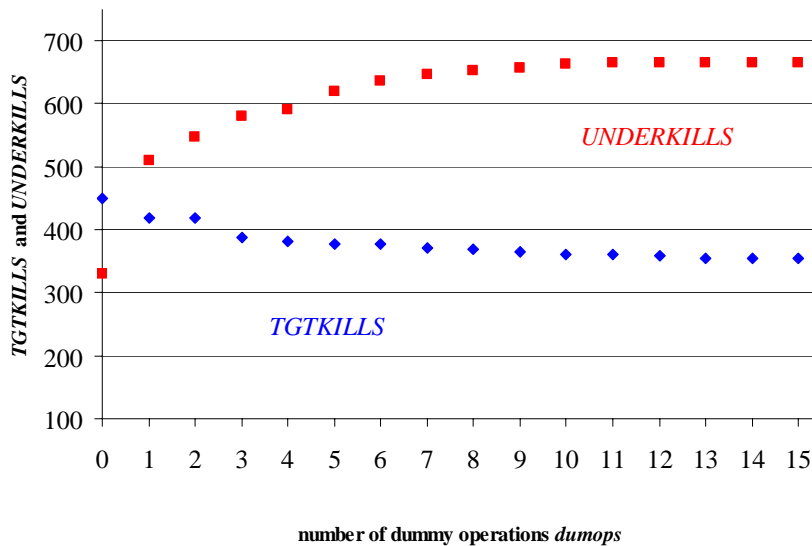

Figure 9.   Total *TGTKILLS* and *UNDERKILLS* versus number of dummy operations
The deficiency UNDERKILLS grows significantly faster than TGTKILLS decreases.

We also see how Y_k changes the number of sorties (Figure 10). BLUE launches 17.2 attacks more than without defense plan Y_k. We realize that this increase of the number of sorties causes an increase of over capacities $OVERCAP_s$ of one of the services by exactly the same number: The Air Force has to carry all additional effort. The increase of the number of the attacks is caused by the third dummy operation. With this operation RED deploys dummies for two attacked targets. The second dummy action does not have any influence, because it covers $k74$, which is not attacked (Figure 8). This absolute increase of both, attacks and $OVERCAP$ of 17.2 attacks represents only 6% with respect to the number of attacks without defense, but it increases the deficiency variable OVERCAP by 30 %.
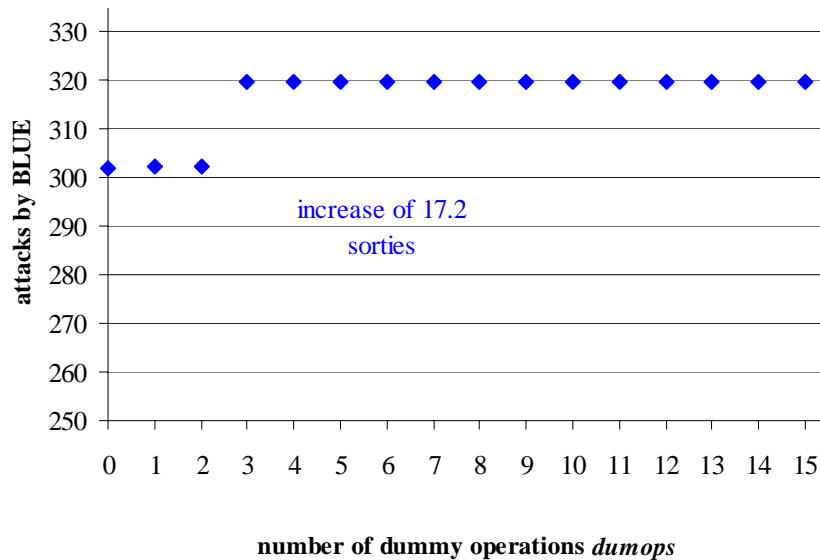

Figure 10.   Number of attacks with increasing number of RED dummy operations

With three dummy operations, the Defender-Attacker Model increases its sorties by 17.21 attacks. More dummy operations have no further effect.

The reduced number of targets killed and the increase in the number of sorties have the greatest effect on our objective (Figure 11). We see the steepest ascent with only one dummy operation.
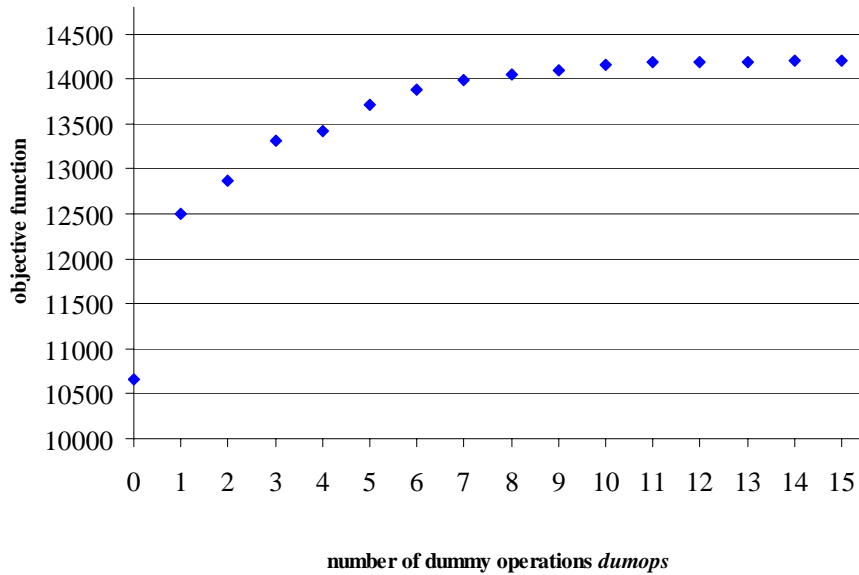

Figure 11.   Defender-Attacker Model objective with increasing number of RED dummy operations

One dummy operation causes an increase of the objective function value by 17%. Deploying further dummy operations, the Defender-Attacker Model is able to double that first increase to 33% with dumops = 10.

Comparing the results for deployed decoy operations with those of the enhanced air operations by Z_k, we conclude that deploying dummy targets with the defense plan Y_k has a larger impact on the attacker's effectiveness and performance.

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V. CONCLUSIONS

We enhance the FATHM Air model and formulate a defender-attacker model to give RED the capability to take defensive actions against anticipated BLUE air strikes. Our model solves an optimal defense plan for RED, augmented by dummy operations Y_k and enhanced air defense operations Z_k, that change BLUE's attack plan.

Our model reduces the attacker's effectiveness: we increase BLUE efforts in launching air strikes against RED defense. Given objective function weights we use, RED dummy operations in the defense plan Y_k, turn out to be more significant than active air defense plans.

Dummy operations reduce the number of targets killed by setting up dummy targets for specific targets k. In reducing the number of kills, BLUE is no longer able to achieve preset kill goals for specific target classes j. BLUE fails to a higher degree and must launch more sorties. This increase in the number of sorties affects BLUE capacity goals, stressing some BLUE service components. Finally, the increased number of sorties results in higher casualties for the attacker. The number of weapons used and hours flown by the aircraft are proportional to the sorties assigned and increase. Summarizing these effects, dummy operations result in a very significant reduction of BLUE effectiveness in killing RED targets by addressing all critical elements of performance and resources, represented in the objective function. The value of the objective function, which BLUE seeks to minimize, is successfully increased by RED's main plan Y_k.

The second defense plan Z_k, enhances air defense to actively defend prioritized targets against attacking BLUE aircraft by augmenting AAA. This does not show nearly the results that Y_k does on BLUE's effectiveness. The objective function increases by about 1% compared to the Air model. On the other hand, Z_k significantly affects the attacker's most critical resource, the number of available aircraft. We conclude that Z_k has a far greater impact on BLUE attack plans with ongoing battle duration over several phases and time periods.

To represent an intelligent defender, we think our defender-attacker model has more to recommend it than traditional, one-sided models such as the FATHM Air model.

APPENDIX

THE AIR MODEL: ORIGINAL FORMULATION (EXTRACT)

[from: Brown and Washburn 2005]

Variables (all nonnegative)

X_{apmklw} attacks

$TGTKILLS_k$ targets k killed

$HRSUSED_{pw}$ aircraft platform p hours used in weather state w

$PLTSLOST_p$ aircraft p lost

$WEPUSED_m$ air_weapons m used

$SVCCAP_s$ capacity used by service s

$UNDERKILLS_j, MIDKILLS_j, OVERKILLS_j$

       under, slack, and over-kills of target class j

$UNDERCAP_s, MIDCAP_s, OVERCAP_s$

       under, slack, and over-achievement of service s goals

Formulation

The number of targets killed is a linear function of the number of sorties assigned, subject to not killing more targets than exist, and similarly for the number of platforms lost, hours used, and weapons used.

$$KILLS_k: \qquad TGTKILLS_k = \sum_{apmlw} e_{apmklw} X_{apmklw} \qquad \forall k$$

$$TGTKILLS_k \leq mxtargk_k \qquad \forall k$$

$$PLATS_p: \qquad PLTSLOST_p = \sum_{amklw} att_{apmklw} X_{apmklw} \qquad \forall p$$

$$PLTSLOST_p \leq mxplats_p \qquad \forall p$$

$$WXHOURS_{pw}: \qquad HRSUSED_{pw} = \sum_{amkl} used_{kpw} X_{apmklw} \qquad \forall pw$$

$$HRSUSED_{pw} \leq mxhours_{pw} \qquad \forall pw$$

$$WEPNS_m: \qquad WEPUSED_m = \sum_{apklw} c_{apmklw} X_{apmklw} \qquad \forall m$$

$$WEPUSED_m \leq mxwepns_m \qquad \forall m$$

The amount of a service's capacity used is a linear function of the number of hours used by the service's platforms.

$$SERVICE_s: \qquad SVCCAP_s = \sum_{p \in P_s, w} cap_p HRSUSED_{pw} \qquad \forall s$$

For each class of target j, there are soft goals for the number of targets killed and for each service's capacity.

$$JGOAL_j:$$

$$\sum_{k \in K_j} TGTKILLS_k + UNDERKILLS_j + MIDKILLS_j - OVERKILLS_j =$$

$$\overline{jgoal}_j \qquad \forall j$$

$$MIDKILLS_j \leq \overline{jgoal}_j - \underline{jgoal}_j \qquad \forall j$$

$$SGOAL_s:$$

$$SVCCAP_s + UNDERCAP_s + MIDCAP_s - OVERCAP_s =$$

$$\overline{sgoal}_s \qquad \forall s$$

$$MIDCAP_s \leq \overline{sgoal}_s - \underline{sgoal}_s \qquad \forall s$$

The objective offers terms to balance effort between killing target value, avoiding Blue platform attrition, achieving kills of target classes critical to the war phase, and equitably stressing the services. Attrition is emphasized when λ is large.

Minimize
$$-\sum_k kvalue_k TGTKILLS_k$$

$$+\lambda \sum_p cap_p PLTSLOST_p$$

$$+\sum_j (\underline{jpen}_j UNDERKILLS_j + \overline{jpen}_j OVERKILLS_j)$$

$$+\sum_s (\underline{spen}_s UNDERCAP_s + \sum_s \overline{spen}_s OVERCAP_s)$$

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LIST OF REFERENCES

Brooke A., Kendrick D., Meeraus A. and Raman R. 1998, "GAMS: User's Guide," GAMS Development Corporation, Washington, D.C., September.

Brown, G., Carlyle, M., Salmeron, J., and Wood, K. 2005, "Analyzing the Vulnerability of Critical Infrastructure to Attack and Planning Defenses," **Tutorials in Operations Research**, Institute for Operations Research and Management Science, Hanover, Maryland, pp.102-123..

Brown, G. and Washburn, A. 2005, "The Fast Theater Model (FATHM)," 2005 revised version of the 2001 Naval Postgraduate School Operations Research Technical Report NPS-OR-01-002-PR, May.

GAMS Development Corporation 2005, "GAMS: The Solver Manuals," CPLEX 9, Washington, D.C., last modified December 2003.

Jones, H. 1995, "COSAGE User's Manual, Volume 1 – Main Report. Revision 4."

Russia/Soviet Special Weapons Agencies, http://www.fas.org/nuke/guide/russia/agency/, accessed November 2005.

Russia Military Guide, www.globalsecurity.org/military/world/russia/index.html, accessed November 2005.

Yost, K. 1996, "The Time Strike Munitions Optimizations Model," Naval Postgraduate School Operations Research Technical Report NPS-OR-96-001, January.

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