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THESIS

STOCHASTIC AND SIMULATION MODELS OF MARITIME INTERCEPT OPERATIONS CAPABILITIES

by

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STOCHASTIC AND SIMULATION MODELS OF MARITIME INTERCEPT OPERATIONS CAPABILITIES

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ABSTRACT

The Japanese maritime forces (Blue) are required to detect, identify and intercept maritime terrorist threats (Reds) well before they reach Japanese shores. However, it is challenging for the limited number of Blue maritime assets to identify and intercept Reds out of large numbers of law-abiding neutral vessels (Whites) within the limited time available to intercept a Red; there is a need to estimate Blue Maritime Intercept Operation (MIO) capabilities (a series of detection, identification and interception capabilities), and to identify the significant factors influencing the MIO capabilities quantitatively in order to examine current programs and to study new, alternative programs.

This thesis formulates and exercises stochastic and simulation models to assess Blue MIO capabilities. The models focus on the surveillance operations of the Maritime Patrol Aircraft (MPA). The analysis using the models estimates the probability with which a Red is detected, correctly classified, and escorted for intensive investigation and neutralization before it leaves an area of interest (AOI). The difficulty of obtaining adequate interception of the Red depends upon the AOI size, the number of Whites in the AOI, detection and identification capabilities, information retention, and close coordination between the MPA and investigative maritime vessels in various situations. The analysis ultimately provides quantitative guidance on the relative importance of the MIO capabilities. Although the models focus on the MPA operations, the analysis additionally provides various insights and recommendations to other defense components.

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EXECUTIVE SUMMARY

The Japanese maritime forces (Blue) are required to detect, identify and intercept maritime terrorist threats (Reds) well before they reach Japanese shores. However, it is challenging for the limited number of Blue maritime assets to identify and intercept Reds out of large numbers of law-abiding neutral vessels (Whites) within the limited time available to intercept a Red; there is a need to estimate Blue Maritime Intercept Operation (MIO) capabilities (a series of detection, identification and interception capabilities), and to identify the significant factors influencing the MIO capabilities quantitatively in order to examine current programs and to study new alternative programs.

In this study, we formulate and exercise stochastic and simulation models in order to assess Blue MIO capabilities. Logistic regression models are used to summarize the simulation output. The models focus on the surveillance operations of the Maritime Patrol Aircraft (MPA). There is one MPA to patrol an area of interest (AOI), otherwise called the Domain. One Red enters the AOI at time 0. The analysis using the models estimates the probability with which the Red is detected, correctly classified, and escorted for intensive investigation and neutralization before it leaves the AOI. The difficulty of obtaining adequate interception of the Red depends upon the AOI size, the number of Whites in the AOI, detection and identification capabilities, information retention, and close coordination between the MPA and investigative maritime vessels in various situations.

The principal measures of effectiveness (MOEs) used to quantify the effectiveness of Blue MIO capabilities are

- P_C =Probability a typical Red is detected and correctly classified before leaving the AOI (Domain)
- P_E =Probability a typical Red is detected, correctly classified, and escorted before leaving the AOI (Domain)

Three types of models are used in this study: a simulation model, an analytical stochastic model, and logistic regression models. The analytical stochastic model is a special case of the simulation model. The MOEs for the stochastic model are expressed as closed-form formulas. The MOE formulas from the analytical stochastic model are used to check the output of the simulation in the special case. The formulas are also used to suggest useful independent variables for logistic regression models to summarize the simulation output in cases other than the special case. These independent variables include functions of the simulation input variables.

In order to analyze the factors influencing Blue MIO capabilities, we eventually focus on the seven factors: (1) Dimensions of the rectangular AOI, M_x and M_y (2) Constant number of Whites in the AOI, w (3) Speed of Whites and Reds, u (4) Processing time for each contacted vessel by the MPA, τ (5) Probability that a detected White is correctly classified as White, c_{ww} (Note that $c_{wr} = 1 - c_{ww}$ is the probability that a detected White is incorrectly classified as Red) (6) Probability that a detected Red is correctly classified as Red, c_{rr} (Note that $c_{rw} = 1 - c_{rr}$ is the probability that a detected Red is incorrectly classified as White), and (7) The MPA's information retention (length of time for which the MPA retains the classification information on a vessel; the mean time is $1/\psi$). For the MIO capabilities (P_C and P_E), the results of the thesis identify the following tendencies:

- When the AOI size (w, u, respectively) increases, the MIO capabilities decrease.
- When c_{ww} (c_{rr}, respectively) increases, the MIO capabilities increase.
- Changes to the value of τ or $1/\psi$ do not change the MIO capabilities as much as changing the values of the other factors: the AOI size, w, u, c_{ww}, and c_{rr}.
- There is an interaction between the AOI size, w, and c_{ww}.
- w is the most influential factor for the MIO capabilities among the considered factors.

- Relatively high MIO capabilities can be achieved when there are a few Whites in a relatively small AOI; however, the MIO capabilities quickly decrease by increasing w a small amount ($0 \rightarrow 10$).

For this analysis, we apply various assumptions (scenario, data, distributions, and modeling). ***If they are "reasonable," it would imply that a single MPA is operationally inadequate to intercept a Red before it leaves an AOI. If more MPAs are simultaneously available, they should be used in the surveillance operation (MIO). Otherwise, measures which reduce the number of unidentified Whites in an AOI should be applied as much as possible – such measures include intelligence operations, maritime traffic control, and additional surveillance operations by other defense assets (satellites, helicopters, and maritime vessels).***

The results of the analysis cannot be directly applied to plan real concepts of operations (CONOPS) and operational plans (OPLAN) because the research is based on various assumptions. But they provide very useful intuition, enhancement, and stimulation. As a result, available field data should be collected to assess the reasonableness of the model assumptions. Although we focus on the MPA operations, the effects of other components (intelligence, helicopters, unmanned surveillance systems, maritime vessels, and C4I systems) can be studied using this MPA-based analysis. For example, reliable information of threatening vessels enables assignment of a small size AOI to an MPA, and surveillance operations by helicopters and maritime vessels can be used to complement the MPA's surveillance operations.

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I. INTRODUCTION

Terrorism has become a serious threat to today's global security environment – as demonstrated in the United States, Indonesia, Spain, Russia, and England from 2001 to 2005. Japan has initiated several efforts to prevent terrorist attacks; however, its security is still vulnerable to threats (especially from the sea) resulting from its long coastlines, its large numbers of important facilities in coastal areas, and the large numbers of commercial ships and boats entering Japanese territorial waters.

To protect Japan from maritime terrorist threats, the Japanese maritime forces (Blue) are required to detect, identify, and intercept the threats well before they reach Japanese shores. However, Blue has a finite number of assets (aircraft and vessels) used to execute a series of the operations called Maritime Intercept Operations (MIO). On the other hand, there are large numbers of vessels coming into or navigating in the waters around Japan, and terrorist vessels attempt to sneak into Japanese shores by camouflaging and hiding among these vessels. Thus, it is challenging for Blue maritime assets to detect, correctly classify, and intercept the threats out of the large numbers of law-abiding neutral vessels within the limited time available to intercept a terrorist vessel.

The purpose of this thesis is to propose and analyze Blue surveillance, awareness, and neutralization capabilities to achieve Japanese maritime homeland security.

A. CURRENT SITUATION

For Japan's maritime security during peacetime situations, the Japan Coast Guard (JCG), a police agency, is primarily in charge of the maritime security operations, and the Japan Maritime Self-Defense Force (JMSDF) does not have this responsibility under the existing law. If it is beyond the JCG's power to perform maritime security operations, JMSDF shall take actions in a supporting position "in case it is particularly required."[1]

[1] Self-Defense Forces Law, Article 82

However, as a result of the revision of the Self-Defense Forces Law, (which followed the incident of suspicious boats off Noto Peninsula in 1999), a step is taken that JMSDF assets (aircrafts and maritime ships) can be ordered into operation from the beginning of an emergency situation against serious maritime threats, such as suspected spy vessels, armed boats, and terrorist vessels.[2] Moreover, since the possible intention of the maritime terrorist threats is to conduct military activities, JMSDF is being required to join maritime security operations against the threats more than ever.

Thus, the framework of Japan's maritime security is gradually changing to correspond with the current security environment. JMSDF and JCG created a manual for joint responses to suspicious ships in 1999, and have started several joint training exercises for security operations. However, the manual currently targets only relatively low-intensity situations (i.e. police operations) and restricts its focus to identifying respective responsibilities and establishing rules for information sharing. So, there are still no comprehensive concepts of operations (CONOPS) and operational plans (OPLAN) for the two agencies to jointly deter maritime terrorist threats in Japan.

To effectively implement maritime security operations or the challenging MIO, appropriate CONOPS and OPLAN – which include assets, systems, tactics, procedures, intelligence, and information sharing – are indispensable to the responsible maritime forces (JMSDF and JCG). Moreover, to enhance the MIO capabilities, the maritime forces are required to constantly examine current programs and to study new, alternative programs – such as system development, operational development, procurement, and training. Thus, it is essential for the forces to assess their own MIO capabilities in order to study and plan the proper CONOPS and OPLAN. It is also desirable for the forces to identify and assess the factors influencing the MIO capabilities to examine current programs and study new, alternative programs using the quantitative methods of operations research.

2 Furusawa, Tadahiko, "On Territorial Defense – Policing Sea Area under Japanese Jurisdiction," *DRC annual report,* (2002)

B. CURRENT STUDIES

So far, several studies of maritime security operations and the MIO have been performed, focusing on various aspects of the operations. [Salcedo Franco, 1997] studies the resource (patrolling vessels) allocation for the maritime traffic control in Venezuela in order to maximize the number of "bad" vessels encountered. [Komiya, 2000] analyzes the surveillance route which maximizes the number of vessels detected and processed based on the previous flight information. [Nagai, 2003] examines the MIO's decision-making process that was used to intercept the suspicious vessels that appeared off Amami Ohshima in 2001.

However, the previous studies only focus on specific aspects of the maritime security operations, such as how to increase the number of vessels the maritime assets can process effectively. To properly assess the challenging MIO capabilities and to properly estimate the significant factors influencing the success of the MIO as a whole, we should consider a series of surveillance and intercept operations – such as detection, identification, and interception.

C. OBJECTIVES

This study formulates and exercises stochastic and simulation models to assess the likely MIO capabilities (a series of detection, identification, and interception capabilities). Specifically, we focus on the surveillance operations of the Maritime Patrol Aircraft (MPA). Using the models, the analysis estimates the probability with which a terrorist vessel is detected, correctly classified, and escorted for intensive investigation and neutralization before the vessel is able to leave an area of interest (AOI) and achieve a lethal effect.

The difficulty of obtaining adequate interception of the hostile vessel depends upon the AOI size, the number of neutral vessels in the AOI, detection and identification capabilities, information retention, and close coordination between the MPA and investigating maritime vessels in various situations. The analysis ultimately provides quantitative guidance on the effectiveness of the MIO capabilities.

D. STRUCTURE OF THESIS

Chapter II describes the maritime intercept operations. Chapter III introduces the simulation model. An analytical stochastic model is formulated and studied. The analytical stochastic model is a special case of the simulation in that all random times are independent having exponential distributions. Formulas of the measures of effectiveness (MOEs) for the analytical stochastic model are presented. Results of a short study are displayed comparing the simulation output in this special case to that of the analytical stochastic model. Chapter IV presents the results from the simulation for cases in which the random times while independent are not necessarily exponentially distributed. Logistic regression is used to summarize the simulation output. The independent variables used in the logistic regression are suggested by the formulas from the analytical stochastic model. The independent variables include functions of the simulation input values. The operational implications of the models results are discussed. The thesis concludes with suggestions for further work. Appendices provide further details and model results.

II. SCENARIO DESCRIPTION

A. JAPANESE FEATURES

Japan is an island country surrounded by sea, having approximately 4,470,000 square kilometers of territorial waters (territorial sea, contiguous zones, and economic exclusive zones) and 33,889 kilometers of coastlines. The oceans and maritime entries to Japan are sea routes for both good and bad influences. On the waters, commercial ships carry over 90 percent of the natural resources that are indispensable to Japanese existence. The same waters give illegal or asymmetric threatening vessels access to Japan. In the coastal areas of the Japanese islands, there are large numbers of important facilities – economic and military ports, atomic power plants, and oil refinery complexes. More than 11,000 commercial vessels come into Japanese ports from various foreign countries each month to access these facilities.[3] There are also large numbers of domestic vessels and pleasure boats navigating in Japanese territorial waters.

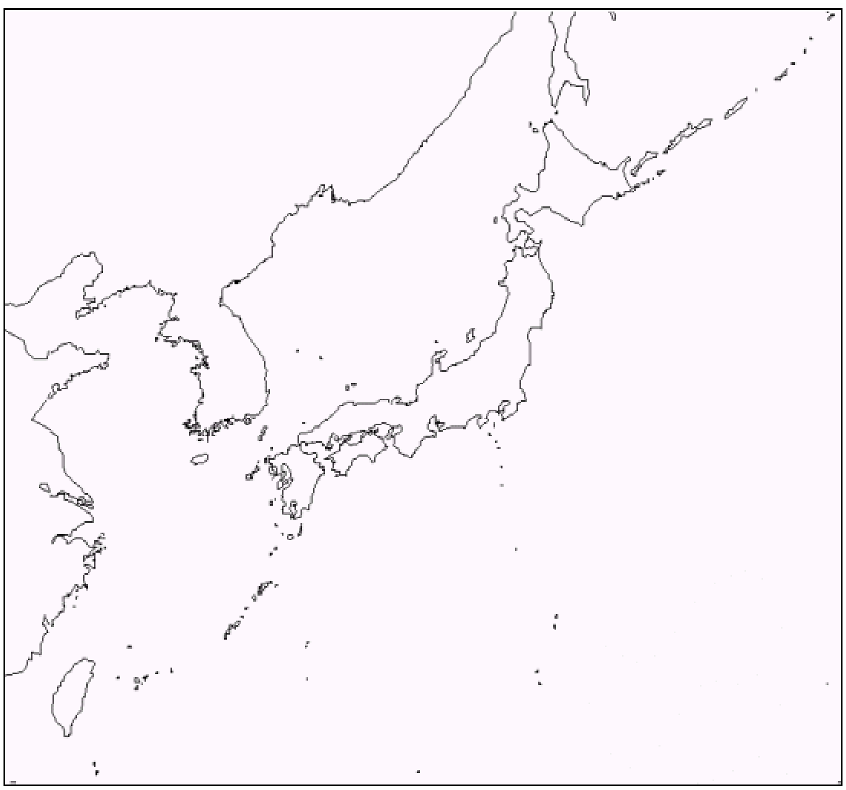

Figure 1. Area around Japan

[3] Ministry of Finance, "Entrance of Vessels by Nationality," *Trade Statistics of Japan,* (2005)

B. TERRORIST THREATS

Currently, there are several active terrorist groups: Al Qaeda, Jemaah Islamiyah, Basque Fatherland and Liberty (ETA), and Chechen separatists. Terrorists from these groups and others cause indiscriminate murders and subversive activities, both globally and regionally, in order to attain goals that are political, religious, or ideological in nature. In the vicinity of Japan, North Korean terrorist and guerrilla threats have become apparent through incidents such as the detection of suspicious boats appearing off Noto Peninsula in 1999 and off Amami Ohshima in 2001.

Japan is one of the leading economic powers in the world and also has close relationships with the United States politically and economically. As a result, Al Qaeda has listed Japan as a target country. Thus, it is possible that these terrorists will attack Japan in order to promote their influence or message to Japan and the rest of the world. There are roughly two ways for terrorist threats to intrude into Japan: airplanes and ships. To destroy important infrastructures and populations crowded in coastal areas, some terrorists may try to sneak into Japan by sea, by camouflaging and hiding among large numbers of law-abiding neutral vessels.

C. MARITIME INTERCEPT OPERATIONS

Japan's maritime security policy is to make its utmost efforts, by utilizing all available means, to prevent any threats from reaching Japan directly. To achieve this, several maritime assets (aircraft and maritime vessels) conduct constant surveillance, attempting to identify ships and submarines transporting guerillas and special operations units as early as possible in order to prevent them from advancing.[4] [5] For further support, Japan also consistently executes information sharing about possible threats with other nations and their intelligence agencies.

The MIO's surveillance operation by the Maritime Patrol aircraft (MPA) is the most vital in obtaining initial contact with threatening vessels, and is also the most vital in collecting information that can be used in the future by other defense assets. Thus, as a specific interest, we focus on the surveillance operations of the MPA of JMSDF.

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[4] Japan Defense Agency, "National Defense Program Guideline for FY 2005 and After," (2004)

[5] Japan Defense Agency, "Defense of Japan 2005 White Paper," (2005)

JMSDF has several squadrons using MPAs, which are designated to respective regions to execute maritime defense operations. In each region, the MPA constantly executes surveillance operations over the area of interest (AOI). All contacts within the AOI are tracked and identified. All available sensors (radar and visual) are used to detect, identify, and collect intelligence on contacts of interest. Every contact identified as a vessel is tracked, observed, and judged if it is a potential target. Maintaining accurate information on vessels previously processed is critical to prevent multiple interceptions of the same ships as they pass through specific areas.[6]

If the MPA classifies a vessel as a target, it requests inspection units (maritime ships) to come and interrogate the vessel, and tracks the suspicious vessel until the inspection units reach the vessel to take over from the MPA. To keep from losing sight of the vessel, the MPA may not process other vessels while tracking the suspicious vessel. Tracking a real target is essential, but tracking a misclassified non-threatening vessel can impede the tracked vessel's freedom of navigation, and also wastes time that is needed to detect and process real targets. Thus, it is important for the MPA coordinators to correctly identify each vessel contacted. After passing over the suspicious vessel to the inspection units, the MPA resumes the surveillance operations by returning to its flight route as soon as possible.

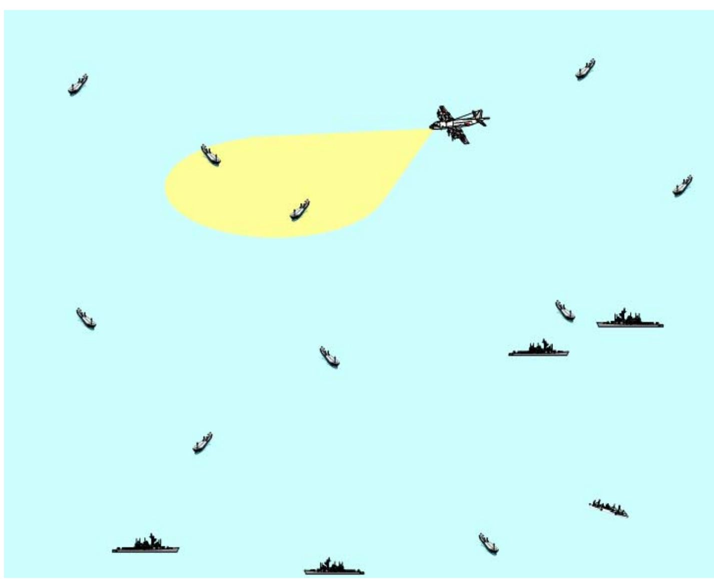

Figure 2. Maritime Intercept Operations

[6]Navy Tactical Support Activity, "Maritime Interdiction Force Procedures – Multi-National Maritime Manual," (1996)

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III. A MODEL OF MARITIME INTERCEPT OPERATIONS

A. OBJECTIVE

The objective of the model studied here is to estimate the Maritime Intercept Operations (MIO) capabilities – especially, the fraction of terrorist vessels that are detected, correctly classified, and escorted for intensive investigation and neutralization before the vessels leave an area of interest (AOI), otherwise called the Domain, and achieve a lethal effect. As a specific interest, we focus on the surveillance operations of the Maritime Patrol Aircraft (MPA). Obtaining adequate interception of hostiles depends upon the AOI size, the number of neutral vessels in the AOI, detection and identification capabilities, information retention, and close coordination between the MPA and investigative maritime vessels under various circumstances.

B. MEASURES OF EFFECTIVENESS

It is highly desirable for the Blue maritime assets to detect, identify, and intercept terrorist vessels before they leave an AOI. However, as a minimum, the MPA must detect and correctly classify hostile vessels before they pass through the AOI to Blue homeland (the coast of Japan). Thus, as measures of effectiveness (MOEs), we consider the following two measures:

- Long-run fraction of terrorist vessels being detected and correctly classified before leaving the AOI
- Long-run fraction of terrorist vessels being detected, correctly classified and escorted before leaving the AOI

C. ASSUMPTIONS

1. Intelligence

Accurate information, intelligence, and knowledge of maritime terrorist threats – signs of activities, ship personnel, maneuvers, destinations, and possible areas where threat vessels exist – are important for the success of the MIO. However, intelligence is not always available or incomplete, and terrorists may attempt to execute surprise attacks. In this study, precise and timely intelligence is assumed not to be available; this is a worst-case scenario.

2. Multi-Agency Operations

To implement the challenging MIO as successfully as possible, close coordination between the Japan Maritime Self-Defense Force (JMSDF) and the Japan Coast Guard (JCG) is required. The coordination results in layered defense and information sharing between the two agencies. In this study, we focus only on the JMSDF operations – especially, the surveillance operation of the Maritime Patrol Aircraft (MPA). The MPA performs the most vital role in the MIO to obtain an initial contact of threatening vessels and to collect information for future use by other defense assets.

3. JMSDF Operations

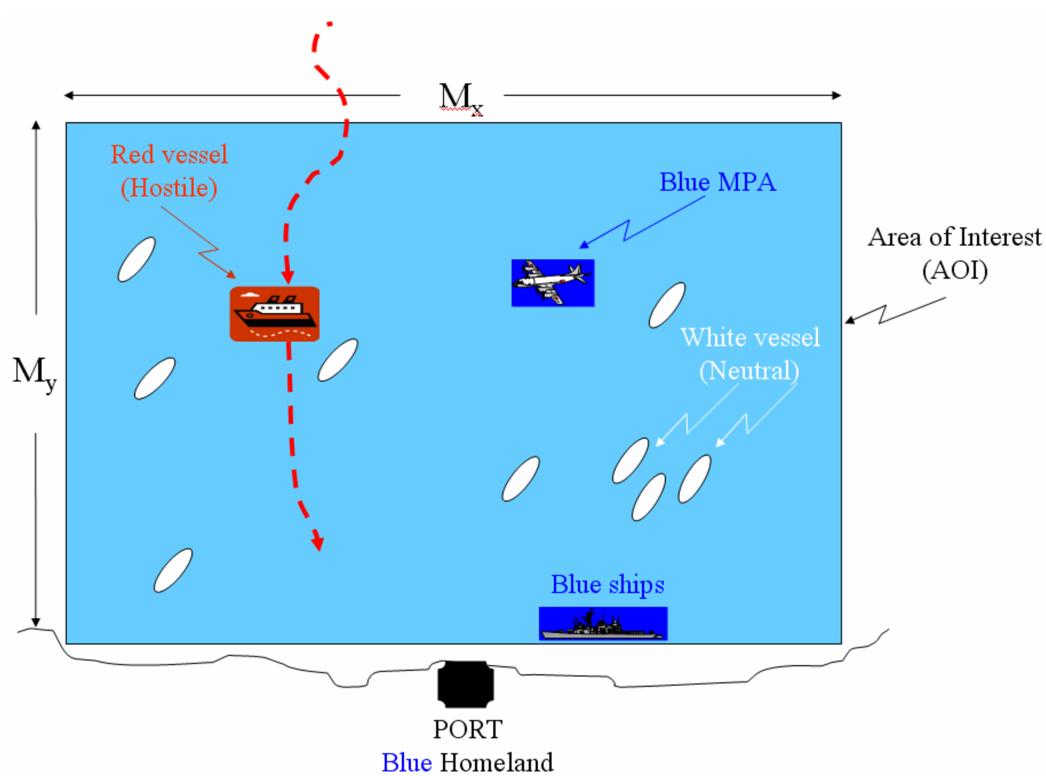

Figure 3. JMSDF operations

To implement the MIO, JMSDF organizes a Maritime Intercept Group (MIG) that is composed of the MPA and maritime vessels, and specifies an area of interest (AOI) for surveillance far from Japanese shores. In this study, the following assumptions are made:

- The AOI is a rectangle.
- There is one MPA patrolling the AOI at a time. And, as a result of successive rotation of MPAs, there is constantly one MPA on patrol in the AOI.
- Maritime vessels also execute (barrier) patrol at the lower boundary of the AOI (nearest the Japanese coasts). However, in this thesis, it is assumed that they are entirely engaged in the inspection operation of the vessels classified as "suspicious" by the MPA.
- The patrolling MPA executes surveillance (here, random search) over the AOI. Whenever it detects an unknown vessel (usually by its radar), it approaches the vessel and judges if it is suspicious. It takes several minutes for the MPA to process (approach and classify) each contacted/detected unknown vessel.
- Times between detections for unknown vessels are independent random variables having exponential distributions. The mean time between detections depends on the following factors:
  - Size of the AOI
  - Surveillance speed of the MPA
  - Radar coverage or radar sweep width of the MPA
  - Time to process (approach and classify) each detected vessel
  - Number of unknown vessels in the AOI

  Since the mean time between detections for unknown vessels depends on the number of unknown vessels in the AOI, the times between detections are not identically distributed: they tend to be longer if unknown vessels are few, and shorter if there are many unknown vessels.
- The number of unknown vessels changes as they are classified and the classification information is retained.

- The process (approach and classify) time for each detected vessel is assumed to be constant.
- If the MPA detects and classifies a vessel as "suspicious," it tracks the vessel until the inspection unit (maritime ships) arrives to investigate and possibly escort the suspicious vessel. While tracking a suspicious vessel, the MPA cannot process other vessels. However, the MPA resumes the surveillance and the processing of other vessels after turning over the suspicious vessel to the inspection unit. Here we do not consider the possibility that the inspection units may be limited: there is always one available unit on station in the AOI. This is an optimistic assumption to be relaxed in future work.
- The random time for an inspection unit to relieve the MPA tracking a suspicious vessel has an arbitrary distribution F_D. Successive MPA tracking times are independent and identically distributed. In this study the spatial dependence of relief times is not explicitly considered.
- If an MPA detects and classifies a vessel as "non-suspicious," it permits the vessel to pass through and resumes surveillance to detect and process other vessels. The MPA can retain the information (possible position and direction) of the vessels classified as non-suspicious for awhile. However, after some time passes, the MPA loses the classification information of the vessels. Thus, when the MPA detects the vessels somewhere again, the MPA may have to process them again.
- The random time for the MPA to lose the classification information of a classified vessel has an arbitrary distribution F_H. The times the MPA retains information for different vessels are assumed to be independent and identically distributed.
- The MPA has no learning capacity; that is, the MPA does not hold any information about a vessel after losing its classification information.
- The MPA tries to classify each vessel correctly. However, since its classification ability is not perfect, commercial vessels may be incorrectly classified as "suspicious." In this case, the MPA is occupied by tracking a misclassified commercial vessel for awhile and wastes time. Conversely, the

terrorist vessels may be erroneously processed as "non-suspicious." In this case, the MPA ignores the terrorist vessel crossing through the AOI until it next detects it as an unknown vessel.

4. Terrorist Vessels

In this thesis we call hostile terrorist vessels "Red vessels", "Reds" or "Rs". We assume that Rs operate independently; that is, Rs are assumed not to be executing joint operations. If an R is classified as "suspicious" and escorted by the maritime inspection units, it has been correctly identified as "threat." After identified as "threat," Rs may escape, destroy themselves, or attack inspection vessels. However, we do not consider such situations in this thesis.

Specifically, we focus on the interval of time that starts when one R enters the AOI and ends when it is either escorted while crossing the AOI or it successfully passes through the AOI. We assume there is one R entering the AOI at time 0. The random time for an R to pass through the AOI has an arbitrary distribution F_U. All of the random times whose distributions are referred to above are assumed to be independent in this study.

5. Commercial Vessels

We call (non-hostile) commercial vessels "White vessels", "Whites" or "Ws". There are large numbers of Ws coming into, going out of, navigating, or staying in the AOI. Each W operates independently. Some Ws may be classified as "suspicious" and escorted by the maritime inspection units erroneously. In this thesis, we make the simplifying assumption that the total number of Ws in the AOI is constant. For more generality, see the working paper [Gaver, Jacobs, and Sato, 2005][7].

At this point, we consider the legality of the MIO during peacetime situations. Subject to the United Nations Convention on the Law of the Sea, ships of all states, whether coastal or land-locked, enjoy the right of presumably innocent passage through the territorial sea.[8] Thus, the maritime forces are not permitted to subjectively and

[7] Gaver, Donald P., Jacobs, Patricia A, and Sato, Hiroyuki, "Assessing resource requirements for maritime domain awareness and protection," Working paper, Naval Postgraduate School (2005)

[8] United Nations Convention on the Law of the Sea, Article 17

haphazardly inspect a vessel with its domestic police law. However, to prevent the infringement of the customs, fiscal, immigration or sanitary laws and regulations of the coastal state, the coastal state can adopt laws and regulations that conform with the provisions of the convention and other rules of international law relating to innocent passage through the territorial sea.[9] Here we assume that the maritime assets are permitted to escort any suspicious vessels for intensive investigation by a specific law or regulation under an emergency situation declared by the authorities.

D. STRUCTURES

1. Notation

M_x : Length of x-direction side of AOI (See Figure3), which corresponds to the length of homeland coastal region defended.

M_y : Length of y-direction side of AOI (See Figure3)

v : Mean speed of the MPA

v_I : Mean speed of the inspection units (maritime vessels)

u : Mean speed of Ws and Rs

τ : Process (=approach and classify) time for each detected vessel

f : Radar coverage or radar sweep width of the MPA

c_{ww} : Probability that a W is correctly classified as W

c_{wr} : Probability that a W is incorrectly classified as R ($=1-c_{ww}$)

c_{rr} : Probability that an R is correctly classified as R

c_{rw} : Probability that an R is incorrectly classified as W ($=1-c_{rr}$)

t_j : Time at which the MPA finishes processing the j^{th} detected vessel

$W_u(t_j)$: The number of Ws unknown in the AOI at time t_j

[9] United Nations Convention on the Law of the Sea, Article 21

$W_i(t_j)$: The number of Ws identified as W in the AOI at time t_j

w: Total number of Ws in the AOI at time t_j $(w = W_u(t_j) + W_i(t_j))$

$R_u(t_j)$: The number of Rs unknown in the AOI at time t_j

$R_i(t_j)$: The number of Rs thought to be W in the AOI at time t_j

$R(t_j)$: Total number of Rs in the AOI at time t_j $(R(t_j) = R_u(t_j) + R_i(t_j))$

$N_C(t_j)$: Cumulative number of Rs correctly classified at time t_j

$N_E(t_j)$: Cumulative number of Rs being escorted at time t_j

$N_P(t_j)$: Cumulative number of Rs passing through the AOI at time t_j

T_C: Time at which an R is correctly classified before it leaves the AOI

T_E: Time at which an R is escorted before it leaves the AOI

$$I_{WC}(t_j, H) = \begin{cases} 1: \; \textit{If a W is correctly classified at time } t_j \\ \quad \textit{and returns to be unknown at time } t_j + H \\ 0: \; \textit{otherwise} \end{cases}$$

$$I_{RC}(t_j, H) = \begin{cases} 1: \; \textit{If an R is incorrectly classified as W at time } t_j \\ \quad \textit{and returns to be unknown at time } t_j + H \\ 0: \; \textit{otherwise} \end{cases}$$

$L(t_j)$: Time until the next detection after time t_j

U: Time an un-encountered R spends in the AOI

D: Time the MPA tracking a suspicious vessel is occupied

H: Time until the MPA loses classification information of a classified vessel

$\delta(t_j)$: Detection rate at time t_j

$1/\mu$: Mean time an un-encountered R spends in the AOI

$1/\phi$: Mean time the MPA tracking a suspicious vessel is occupied

$1/\psi$: Mean time until the MPA loses classification information of a classified vessel

P_C : Fraction of Rs being detected and correctly classified before leaving the AOI

P_E : Fraction of Rs being detected, correctly classified, and escorted before leaving the AOI

2. Simulation Formulations

a. Detection Rate

Most models for target detection are for situations in which there are one or few possible targets.[10] In the MIO situation, there are many possible targets, and each possible target takes time to process. For the Maritime Interdict/Intercept Operations or the Maritime Traffic Control situation, target detection (rate) is not principal, but is a valuable measure in representing the number of vessels processed in a period. There are few quantitative studies concerning the rate at which vessels are processed in an MIO setting. The model in Salcedo Franco [1997] treats the number of vessels processed by the maritime assets in a period (the processing rate) as constant.

In this section, we formulate a detection rate model to handle the number of unknown vessels encountered and processed. The detection rate model $\delta(t_j)$ depends on the five variables: size of the AOI, surveillance speed of the MPA, radar coverage or radar sweep width of the MPA, processing (classification) time for each detected vessel, and the number of unknown vessels in the AOI.

The MPA searches a rectangle region AOI with area $M_x \cdot M_y$. The MPA travels the AOI at a speed v. The radar coverage or radar footprint is assumed to be a square with sides of length f. The number of radar footprints necessary to cover the

10 Landa Borges, Jose Manuel, "Radar Search and Detection with the CASA 212 S43 Aircraft," *Naval Postgraduate School*, (2004)

whole AOI is $\left(M_x \cdot M_y\right)/ f^2$ (see Figure4); the time for the MPA to cross one footprint area is f / v. Hence, the time for the MPA to transit the entire AOI (with no processing time) is

$$\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v} \qquad (3.1)$$

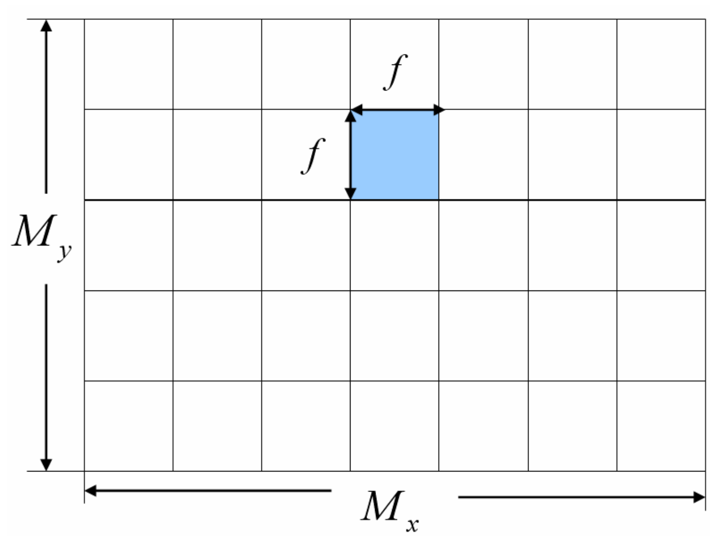

Figure 4. The number of footprints necessary to cover the whole AOI

We assume that the positions of Ws are uniformly distributed over the AOI. Our model considers the time interval starting with the entrance of one R into the AOI until the R leaves the AOI; no other Rs enter the AOI. During this time interval, the number of (unknown) vessels the MPA expects to process while transiting the entire AOI is (roughly) $w+1$. The processing time of each detected vessel is τ. Thus, the total time the MPA spends detecting (transiting) and processing vessels in the AOI during one pass through the AOI is

$$\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v}+\left(w+1\right)\tau \qquad (3.2)$$

The long run average rate of target detections/classifications is

$$\frac{w+1}{\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v}+(w+1)\tau} \qquad (3.3)$$

Hence, detection rate at time is t_j is taken to be

$$\boxed{\delta(t_j)=\frac{W_u(t_j)+R_u(t_j)}{\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v}+\left(W_u(t_j)+R_u(t_j)\right)\cdot\tau}} \qquad (3.4)$$

If τ =0 and there are always the same number of unknown vessels in the AOI, the model (3.4) is the common random search model[11]. However, in this study, the detection rate $\delta(t_j)$ depends on τ and the number of unknown vessels in the AOI at time t_j. Thus, it is not constant and will vary over time.

b. Mean Time an Un-encountered R Spends in the AOI

We assume that an R travels along the y-direction of the AOI. The mean time an un-encountered R spends in the AOI is

$$\boxed{1/\mu=\frac{M_y}{u}} \qquad (3.5)$$

c. Mean Time an MPA Tracking a Suspicious Vessel is Occupied

We approximate the y-position in the AOI where a vessel is detected and classified as suspicious by $M_y/2$. After being classified as R, the vessel keeps advancing toward the Japanese shores (the bottom of the AOI) until it meets the inspection maritime units. The x-position of the inspection units is assumed to be the same as that of the vessel. The relative speed the vessel and the inspection units approach is v_I+u. The mean time an MPA tracks a suspicious vessel is taken to be

11 Wagner, Daniel H., Mylander, Charles W., and Sanders, Thomas J., "Naval Operations Analysis, 3rd ed, " *Naval Institute Press*, (1999)

$$\boxed{1/\phi = \frac{M_y/2}{v_I + u}} \qquad (3.6)$$

d. Mean Time Until the MPA Loses Classification Information of a Classified Vessel

The mean time until the MPA loses classification information depends on the MPA's tactics, systems, human factors, C4ISR capabilities, and vessels behaviors. In this thesis the mean time until the MPA loses information concerning a previously classified vessel is treated as an external parameter.

e. Fraction of Rs Being Detected and Correctly Classified Before Leaving the AOI

The fraction of Rs being detected and correctly classified before leaving the AOI is taken to be

$$\boxed{P_C = \frac{N_C(T)}{N_C(T) + N_P(T)}, \quad T = min\left(T_C,\ U\right)} \qquad (3.7)$$

Below are further details of the simulation. We abbreviate the expression "with probability" as w.p.

$L\left(t_j\right) \sim$ An exponential distribution F_L with rate $\delta(t_j)$

$U \sim$ An arbitrary distribution F_U with mean $1/\mu$

$D \sim$ An arbitrary distribution F_D with mean $1/\phi$

$H \sim$ An arbitrary distribution F_H with mean $1/\psi$

All sample realizations of the above four random variables are mutually independent.

$$t_{j+1} = \begin{cases} t_j + L\left(t_j\right) & w.p.\ \dfrac{W_u(t_j)}{W_u(t_j) + R_u(t_j)} c_{ww} + \dfrac{R_u(t_j)}{W_u(t_j) + R_u(t_j)} c_{rw} \\ t_j + L\left(t_j\right) + D_j & w.p.\ \dfrac{W_u(t_j)}{W_u(t_j) + R_u(t_j)} c_{wr} + \dfrac{R_u(t_j)}{W_u(t_j) + R_u(t_j)} c_{rr} \end{cases}$$

D_j in the expression above is the MPA tracking time of the j^{th} classified vessel given the vessel is classified as a R; successive MPA tracking times are assume to be independent and identically distributed.

$$W_u(t_{j+1}) = \begin{cases} W_u(t_j) - 1 + \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{WC}(t_k, H_k) & w.p.\ \dfrac{W_u(t_j)}{W_u(t_j) + R_u(t_j)} \\ W_u(t_j) \quad + \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{WC}(t_k, H_k) & otherwise \end{cases}$$

$$W_i(t_{j+1}) = \begin{cases} W_i(t_j) \ + 1 - \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{WC}(t_k, H_k) & w.p.\ \dfrac{W_u(t_j)}{W_u(t_j) + R_u(t_j)} \\ W_i(t_j) \quad - \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{WC}(t_k, H_k) & otherwise \end{cases}$$

$$R_u(t_{j+1}) = \begin{cases} R_u(t_j) - 1 + \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{RC}(t_k, H_k) & w.p.\ \dfrac{R_u(t_j)}{W_u(t_j) + R_u(t_j)} \\ R_u(t_j) \quad + \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{RC}(t_k, H_k) & otherwise \end{cases}$$

$$R_i(t_{j+1}) = \begin{cases} R_i(t_j) \ + 1 - \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{RC}(t_k, H_k) & w.p.\ \dfrac{R_u(t_j)}{W_u(t_j) + R_u(t_j)} \\ R_i(t_j) \quad - \sum\limits_{\substack{k \le j \\ t_j < t_k + H_k \le t_{j+1}}} I_{RC}(t_k, H_k) & otherwise \end{cases}$$

$$I_{WC}(t_{k+1}, H_k) = \begin{cases} 1 & w.p.\ \dfrac{W_u(t_k)}{W_u(t_k) + R_u(t_k)} \\ 0 & otherwise \end{cases}$$

$$I_{RC}(t_{k+1}, H_k) = \begin{cases} 1 & w.p.\ \dfrac{R_u(t_k)}{W_u(t_k) + R_u(t_k)} \cdot c_{rw} \\ 0 & otherwise \end{cases}$$

H_k in the expressions above is the random time the MPA retains classification information concerning the k^{th} vessel classified. The information retention times are assumed to be independent and identically distributed.

$$
\begin{aligned}
&N_C(t_{j+1}) = \begin{cases} 1 & w.p.\ \dfrac{R_u(t_j)}{W_u(t_j)+R_u(t_j)} \cdot c_{rr} \ \ and\ \ if\ \ t_j + L\left(t_j\right) \le U \\ 0 & otherwise \end{cases} \\
&N_P(t_{j+1}) = \begin{cases} 1 & if\ \ t_{j+1} > U\ \ and\ \ N_C(t_{j+1}) = 0 \\ 0 & otherwise \end{cases} \\
&T_C = \begin{cases} t_{j+1} & if\ \ N_C(t_{j+1})\ becomes\ 1\ at\ time\ t_{j+1} \\ \infty & otherwise \end{cases}
\end{aligned} \tag{3.8}
$$

f. Fraction of Rs being Detected, Correctly Classified, and Escorted before Leaving the AOI

The fraction of Rs being detected, correctly classified, and escorted before leaving the AOI is taken to be

$$
P_E = \frac{N_E(T)}{N_E(T) + N_P(T)}, \quad T = min\left(T_E,\ U\right) \tag{3.9}
$$

This simulation is the same as the one above (Fraction of Rs being detected and correctly classified before leaving the AOI) except for using (3.10) instead of (3.8).

$$
\begin{aligned}
&N_E(t_{j+1}) = \begin{cases} 1 & w.p.\ \dfrac{R_u(t_j)}{W_u(t_j)+R_u(t_j)} \cdot c_{rr} \ \ and\ \ if\ \ t_{j+1} \le U \\ 0 & otherwise \end{cases} \\
&N_P(t_{j+1}) = \begin{cases} 1 & if\ \ t_{j+1} > U\ \ and\ \ N_E(t_{j+1}) = 0 \\ 0 & otherwise \end{cases} \\
&T_E = \begin{cases} t_{j+1} & if\ \ N_E(t_{j+1})\ becomes\ 1\ at\ time\ t_{j+1} \\ \infty & otherwise \end{cases}
\end{aligned} \tag{3.10}
$$

E. IMPLEMENTATION

To implement the simulation formulations, we develop a discrete event simulation model using Simkit (initially developed by K. Stork[12] and subsequently reviewed and

[12] Stork, Kirk A., "Sensors in Object Oriented Discrete Event Simulation," *Naval Postgraduate School*, (1996)

extended by Professor Arnold Buss[13], Naval Postgraduate School). Simkit has a number of pseudo-random generators. The default generator is a Mersenne Twister[14]. Additionally, a variant of the Mersenne Twister is also included, as well as some other generators – SIMSCRIPT's linear congruential generator (based on Peter Lewis' work[15]), a Tausworth generator, and several mixtures. The pooled random number generator in Simkit has a cycle length of approximately 2^{62}. The pooled generator guarantees that the cycle length of random numbers is the product of the two separate cycle lengths of the two underlying generators, as long as the two cycle lengths are relatively prime.[16] The following random variates are generated in Simkit according to the specified distributions. All random times are assumed to be independent.

- $L\left(t_j\right)$: Time until the next detection at time t_j
- U: Time an un-encountered R spends in the AOI
- D: Time the MPA tracking a suspicious vessel is occupied
- H: Time until the MPA loses classification information of a classified vessel

F. VALIDATIONS

We assume one R enters the AOI at time 0 and there is a constant number of Ws in the AOI, w. When the three distributions, $F_L, F_D, \text{ and } F_U$ are all exponential and the MPA has no memory of its classification information, the probability that a typical R is detected and correctly classified before the R leaves the AOI, P_C, and the probability that a typical R is detected, correctly classified, and escorted before the R leaves the AOI, P_E,

13 Buss, Arnold H., "Discrete Event Programming with Simkit," *Simulation News Europe*, (2001)

14 Matsumoto, Makoto and Nishimura, Takuji, "Mersenne Twister: A 623-Dimensionally Equidistributed Uniform Pseudo-Random Number Generator," Keio University, (1998)

15 Lewis, Peter A. W. and Learmonth, Gerard G., "Naval Postgraduate School random number generator package LLRANDOM," *Naval Postgraduate School*, (1973)

16 Hovda, Erik K., "A Simulation to determine the Effect that the Army Basic Officer Leadership Course will have on Accession Training," *Naval Postgraduate School*, (2002)

can be calculated analytically.[17] To verify the simulation model, we compare the results of the simulation model with those of the analytical model.

- F_L : Distribution of the time between detections
- F_U : Distribution of the time an un-encountered R spends in the AOI
- F_D : Distribution of the time the MPA tracking a suspicious vessel is occupied

1. Input Data

As an example, we consider the following case as input data:

M_x : Length of x-direction side of AOI (=400NM)

M_y : Length of y-direction side of AOI (=200NM)

v : Mean speed of the MPA (=200kt)

v_I : Mean speed of the inspection units (=30kt)

u : Mean speed of Ws and Rs (=15kt)

τ : Process (=approach and classify) time for each detected vessel (=4min)

f : Radar coverage or radar sweep width (=25NM)

c_{ww} : Probability that a W is correctly classified as W (=0.95, 0.99)

c_{wr} : Probability that a W is incorrectly classified as R (=$1-c_{ww}$)

c_{rr} : Probability that an R is correctly classified as R (=0.6, 0.8)

c_{rw} : Probability that an R is incorrectly classified as W (=$1-c_{rr}$)

w : Total number of Ws in the AOI (=100)

$L(t_j) \sim$ An exponential distribution F_L with rate δ (constant)

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[17] Donald P. Gaver and Patricia A. Jacobs, "A Stochastic Modeling of a Variety of Simple MDA Situations," (June), *Naval Postgraduate School (2005).*

$$\delta = \frac{w+1}{\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v} + (w+1)\cdot\tau}$$

$U \sim$ An exponential distribution F_U with mean $1/\mu = \frac{M_y}{u}$

$D \sim$ An exponential distribution F_D with mean $1/\phi = \frac{M_y/2}{v_I + u}$

2. Derivation of the Measures of Evaluation (MOEs) for the Analytical Model

a. Probability that one R is detected and correctly classified before leaving the AOI, P_C

$$P_C = \frac{\delta\frac{1}{w+1}c_{rr}}{\delta\frac{1}{w+1}c_{rr} + \delta\frac{w}{w+1}c_{wr} + \mu} + \frac{\delta\frac{w}{w+1}c_{wr}}{\delta\frac{1}{w+1}c_{rr} + \delta\frac{w}{w+1}c_{wr} + \mu}\cdot\frac{\phi}{\phi+\mu}P_C$$

Solving

$$P_C = \frac{\left(\frac{\delta\frac{1}{w+1}c_{rr}}{\delta\frac{1}{w+1}c_{rr} + \delta\frac{w}{w+1}c_{wr} + \mu}\right)}{1-\left(\frac{\delta\frac{w}{w+1}c_{wr}}{\delta\frac{1}{w+1}c_{rr} + \delta\frac{w}{w+1}c_{wr} + \mu}\right)\frac{\phi}{\phi+\mu}}$$

$$\boxed{Pc = \frac{\delta\frac{1}{w+1}c_{rr}}{\delta\frac{1}{w+1}c_{rr} + \delta\frac{w}{w+1}c_{wr}\frac{\mu}{\phi+\mu} + \mu}} \quad (3.11)$$

b. Probability that one R is detected, correctly classified, and escorted before leaving the AOI, P_E

$$P_E = \frac{\delta \frac{1}{w+1} c_{rr}}{\delta \frac{1}{w+1} c_{rr} + \delta \frac{w}{w+1} c_{wr} + \mu} \cdot \frac{\phi}{\phi+\mu} + \frac{\delta \frac{w}{w+1} c_{wr}}{\delta \frac{1}{w+1} c_{rr} + \delta \frac{w}{w+1} c_{wr} + \mu} \cdot \frac{\phi}{\phi+\mu} P_E$$

Solving

$$P_E = \frac{\left(\frac{\delta \frac{1}{w+1} c_{rr}}{\delta \frac{1}{w+1} c_{rr} + \delta \frac{w}{w+1} c_{wr} + \mu} \right) \frac{\phi}{\phi+\mu}}{1 - \left(\frac{\delta \frac{w}{w+1} c_{wr}}{\delta \frac{1}{w+1} c_{rr} + \delta \frac{w}{w+1} c_{wr} + \mu} \right) \frac{\phi}{\phi+\mu}}$$

$$\boxed{P_E = \frac{\delta \frac{1}{w+1} c_{rr} \frac{\phi}{\phi+\mu}}{\delta \frac{1}{w+1} c_{rr} + \delta \frac{w}{w+1} c_{wr} \frac{\mu}{\phi+\mu} + \mu}} \qquad (3.12)$$

3. Simulation Implementations

- For each case, 10,000 replications of the simulation are executed.
- Each replication begins with a warm-up period. Specifically, about one week of operations (surveillance) is simulated before one R enters the AOI at time 0. Thus, the R may enter the AOI when the MPA is tracking a misclassified W.

4. Comparison (Analytical Model vs. Simulation Model)

a. Probability that one R is detected and correctly classified before leaving the AOI, P_C

c_{ww}	c_{rr}	Analytical Model	Simulation model		
			Estimated Mean	95%LB	95%UB
0.95	0.6	0.1987	0.1976	0.1898	0.2054
	0.8	0.2485	0.2483	0.2398	0.2568
0.99	0.6	0.2451	0.2390	0.2306	0.2474
	0.8	0.3021	0.2951	0.2862	0.3040

Table 1. Analytical results vs. simulation results for P_C

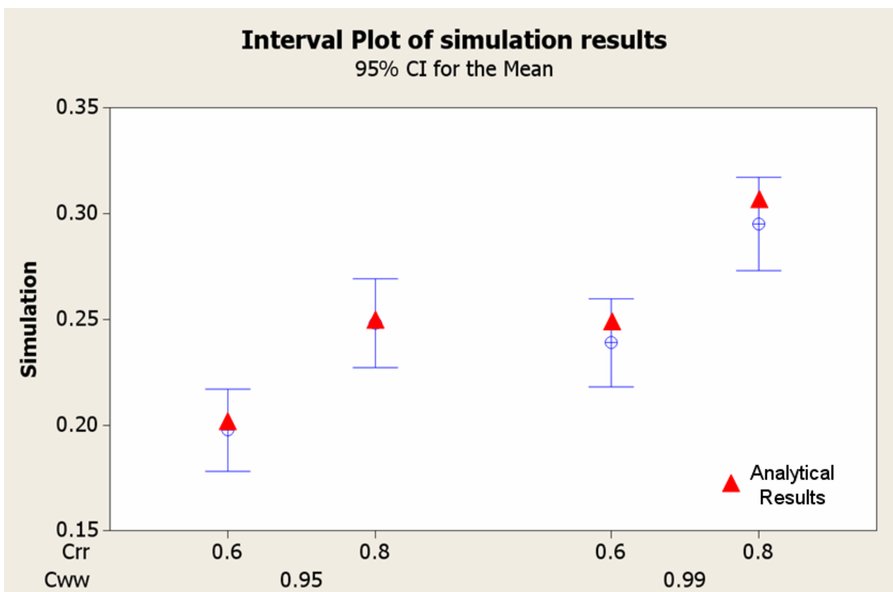

Figure 5. Analytical results vs. simulation results for P_C

Since each analytical result falls in the 95% confidence interval of the estimated probability obtained from the simulation model, the results of the simulation model and the analytical model are not statistically/significantly different. Thus, we validate the simulation model to study the MIO capabilities (P_C) in various situations.

b. Probability that one R is detected, correctly classified and escorted before leaving the AOI, P_E

c_{ww}	c_{rr}	Analytical model	Simulation model		
			Estimated Mean	95%LB	95%UB
0.95	0.6	0.1703	0.1687	0.1614	0.1760
	0.8	0.2130	0.2129	0.2049	0.2209
0.99	0.6	0.2101	0.2086	0.2006	0.2166
	0.8	0.2590	0.2524	0.2439	0.2609

Table 2. Analytical results vs. simulation results for P_E

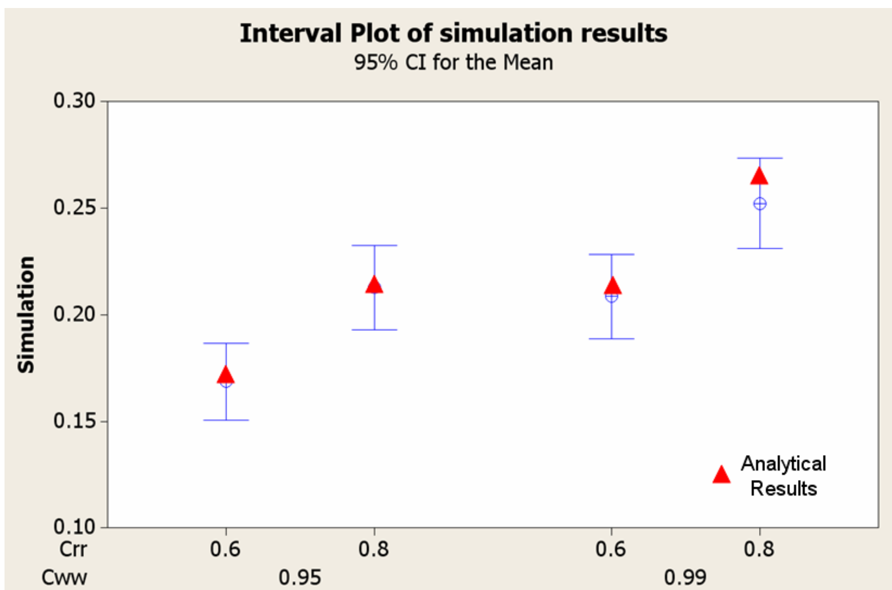

Figure 6. Analytical results vs. simulation results for P_E

Since each analytical result falls in the 95% confidence interval of the estimated probability obtained from the simulation model, the results of the simulation model and the analytical model are not statistically/significantly different. Thus, we validate the simulation model to study the MIO capabilities (P_E) for a variety of situations.

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IV. ANALYSIS

Our interest is to estimate the results of likely Maritime Intercept Operations (MIO) capabilities (a series of detection, identification and interception capabilities) and to identify the significant factors influencing the results of MIO capabilities quantitatively. The MIO capabilities depend on several factors which are tactically controllable, systematically controllable, and uncontrollable. In this study, we consider the following factors: the size of an area of interest (AOI), the number of neutral vessels (Ws) in the AOI, detection and identification capabilities, information retention, and coordination between Maritime Patrol Aircraft (MPA) and maritime vessels.

First, we (hypothetically) specify the factor values for each factor considered in this thesis (called Phase1). Next, we design an input data acquisition set by the Nearly-Orthogonal Latin Hypercube (NOLH) design, and implement the simulations in order to establish appropriate regression models of the MIO capabilities (called Phase2). Last, based on the regression models, we examine the impact of each factor simultaneously by changing the specified factor values (called Phase3).

A. PHASE_1: FACTOR VALUES

1. Area Size

The size of the AOI is tactically controllable. It depends on the range of territorial waters, geographic features, the number of maritime assets (aircraft and maritime vessels) available, intelligence, and emergency levels. Here, we assume M_y is 200NM as constant. M_x is considered as being a value in the range 200NM– 400NM.

2. Mean Speed of the MPA

We define the mean speed of the MPA, v, as the MPA's transit speed between the location at which the MPA has processed a vessel and the next location the MPA contacts another unknown vessel. The maximum speed of the MPA is about 400kt. However, to process many vessels, the MPA is frequently forced to fly at low altitudes with a relatively low speed. Thus, we assume that v is 200kt and is constant.

3. Mean Speed of Inspection Units

When required, the inspection units (maritime ships) should advance toward the possible area the suspicious vessel is in as soon as possible. Since the maximum speed of a maritime vessel is about 30kt, we assume v_I is 30kt and is constant.

4. Rader Sweep Width

The radar sweep width depends on the radar performance, flight height, target strength, human factors, and environmental conditions.[18] We can estimate the radar sweep width in some specific conditions. In this section, we assume that f is estimated as 15NM and is constant during the MPA's surveillance operations. This value may realistically change with atmospherics and altitude.

5. Processing Time for Each Detected Vessel

We define this processing time τ as the time between when the MPA contacts an unknown vessel and when the MPA approaches and finishes classifying the vessel. Thus, τ depends on radar sweep width, density of vessels in the AOI, appearance of vessels, and operator's skill. Using the values of f and v considered above, τ can be considered as "$(f/2)/v + \textit{classification time}(=\alpha) \approx 3(\textit{min}) + \alpha$".Thus, we assume that τ is a constant value selected between 4min and 8min.

6. Probability That a W is Correctly Classified as W

The MPA operators have knowledge and experience about Ws navigating in Japanese territorial waters as a result of their constant surveillance operations. Thus, the probability of correctly classifying a White as W, c_{ww}, may be estimated to be fairly high. However, some Ws may not have been detected and classified before, or may behave strangely or may be required to be inspected. Thus, we assume c_{ww} is constant with a value between 0.8 and 1.0.

[18] Frederickson, P. A. and K. L. Davidson, "An operational bulk evaporation duct model," Working paper, Meteorology department, Naval Postgraduate School (2003)

7. Probability That an R is Correctly Classified as R

The MPA operators have knowledge and experience about Ws. Thus, if the MPA operators find an R behaving unusually among other Ws, the MPA operators may easily classify the R as suspicious. However, Red vessels (Rs) may camouflage themselves cleverly, or some Ws may be hijacked by terrorist groups. Thus, we assume c_{rr} is constant with a value between 0.6 and 1.0.

8. Mean Time Until the MPA Loses Classification Information of a Classified Vessel

The time until the MPA loses classification information depends on the MPA's tactics, systems, human factors, C4ISR capabilities, and target behaviors. In this section, we consider $1/\psi$ to be a constant with a value between 0 hrs (the MPA has no memory of its classification information) and 4 hrs (each classified vessel is tracked by satellite or is held in a database or C4ISR systems).

9. Mean Speed of Ws and Rs

Based on the common speed of commercial vessels, we consider u as a constant with a value between 15kt and 30kt.

10. Total Number of Ws in the AOI

In this study, we make the simplifying assumption that the total number of Ws in the AOI is constant. We can estimate the approximate number of vessels in the AOI by the previous flight information and several statistical data. Furthermore, under emergency situations, the maritime forces may control maritime traffic (the number of vessels) in the AOI. Thus, total number of Ws in the AOI can be known approximately. Here, we assume w is a constant with a value between 0 and 100.

11. The Distribution of Time Between Detections

The mean time between detections, which is the reciprocal of the detection rate $\delta(t_j)$, is

$$\frac{1}{\delta(t_j)} = \frac{\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v} + \left(W_u(t_j) + R_u(t_j)\right)\cdot\tau}{W_u(t_j) + R_u(t_j)} \quad (4.1)$$

In this study, we assume that the distribution of the times between detections (F_L) is exponential. The times between detections are independent. The following figure displays the cumulative distribution function of F_L when M_x=200NM M_y=200NM, f =15NM, v=200kt, $W_u(t_j)$=100, $R_u(t_j)$=1, and τ=4min. In this case, the mean time between detections is

$$\boxed{\frac{1}{\delta(t_j)} = \frac{\left(\frac{200\cdot 200}{15^2}\right)\frac{15}{200} + (100+1)\cdot\frac{4}{60}}{100+1} \approx 12min} \quad (4.2)$$

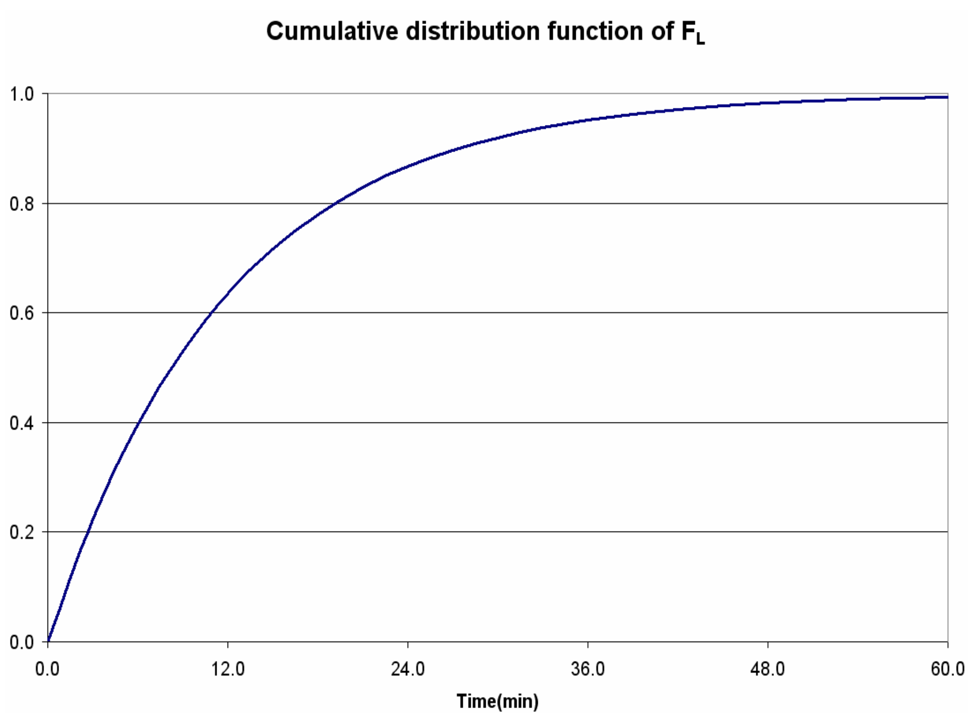

Figure 7. Cumulative distribution function of F_L

The assumption that the times between detections have an exponential distribution with mean 12 minutes implies that about 2/3 of the times between detections are less than 12 minutes.

12. The Distribution of Time an Un-encountered R Spends in the AOI

In this study, we assume that the random time an encountered R spends in the AOI has a Gamma distribution (F_U) with mean $1/\mu = \dfrac{M_y}{u}$ and shape parameter β_U =25. The following figure is the cumulative distribution function of F_U when M_y =200NM, u =20kt and β_U =25. In this case, the mean time an un-encountered R spends in the AOI is

$$\boxed{1/\mu = \frac{M_y}{u} = \frac{200NM}{20kt} = 10hrs} \quad (4.3)$$

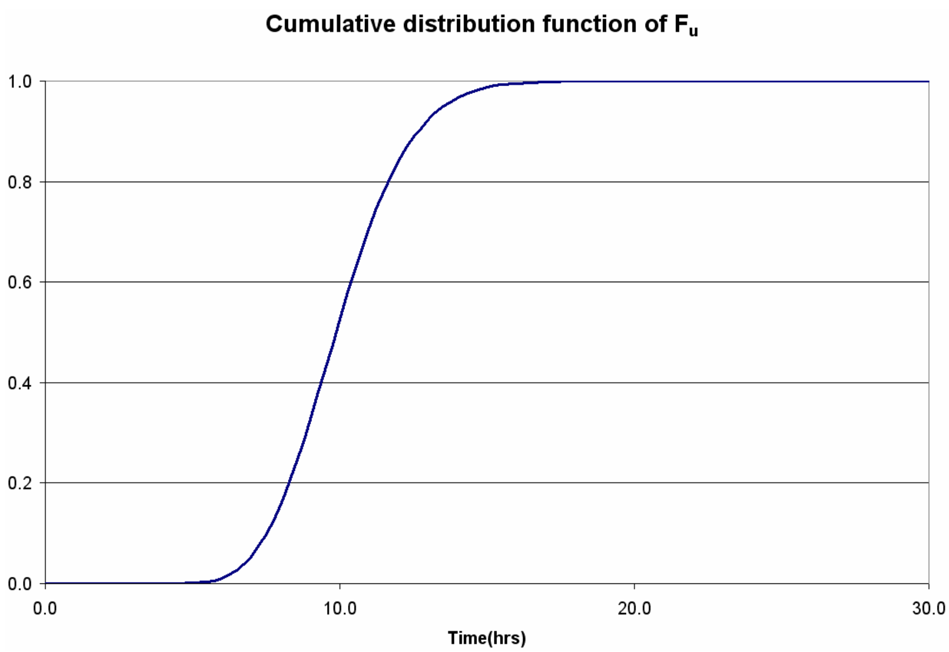

Figure 8. Cumulative distribution function of Fu

The figure implies that the time an encountered R spends in the AOI is a random value between 5hrs and 15 hrs. When the time is as small as 5hrs, we can consider the situation; the R grazes the AOI. On the other hand, when the time is as large as 15hrs, the R can be considered to have crossed the AOI diagonally.

13. The Distribution of Time the MPA Tracking a Suspicious Vessel is Occupied

We assume that the random time an MPA tracking a suspicious vessel is occupied has a Gamma distribution (F_D) with mean $1/\phi = \dfrac{M_y/2}{v_I + u}$ and shape parameter β_D=25. The successive tracking times are independent and identically distributed. The following figure displays the cumulative distribution function of F_D when M_y= 200NM, v_I=30kt, u=20kt, and β_D=25. In this case the mean time the MPA tracks a suspicious vessel is

$$\boxed{1/\phi = \frac{M_y/2}{v_I + u} = \frac{200NM/2}{(30kt + 20kt)} = 2hrs} \quad (4.4)$$

This is an approximate time for an inspection unit (maritime vessels) to meet the tracked suspicious vessel and release the MPA.

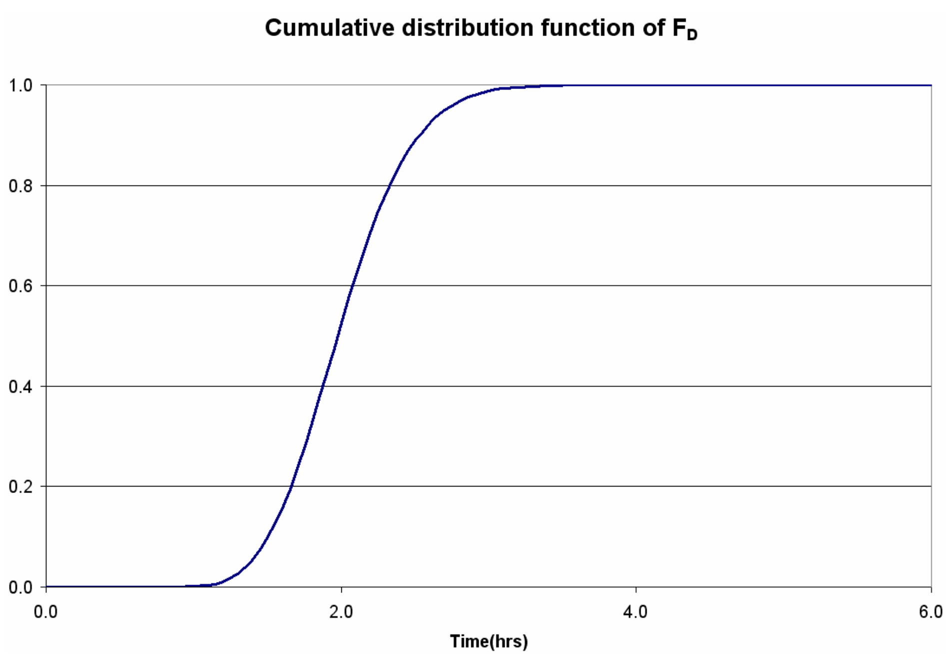

Figure 9. Cumulative distribution function of F_D

The figure implies that the time the MPA tracks a suspicious vessel is a random value between 1hr and 3hrs. When the time is as small as 1hr, we can consider the situation; a vessel is classified as R at a position close to the bottom of the AOI (near the Japanese shores). On the other hand, when the time is as large as 3hrs, a vessel is classified as R at a position far from the bottom of the AOI.

14. The Distribution of Time Until the MPA Loses Classification Information of a Classified Vessel

We assume that the random time until the MPA loses classification information of a classified vessel has a Gamma distribution (F_H) with mean $1/\psi$ and shape parameter β_H=50. The retention times for different classified vessels are independent and identically distributed random variables. The following figure displays the cumulative distribution function of F_H when $1/\psi = 4hrs$ and $\beta_H = 50$.

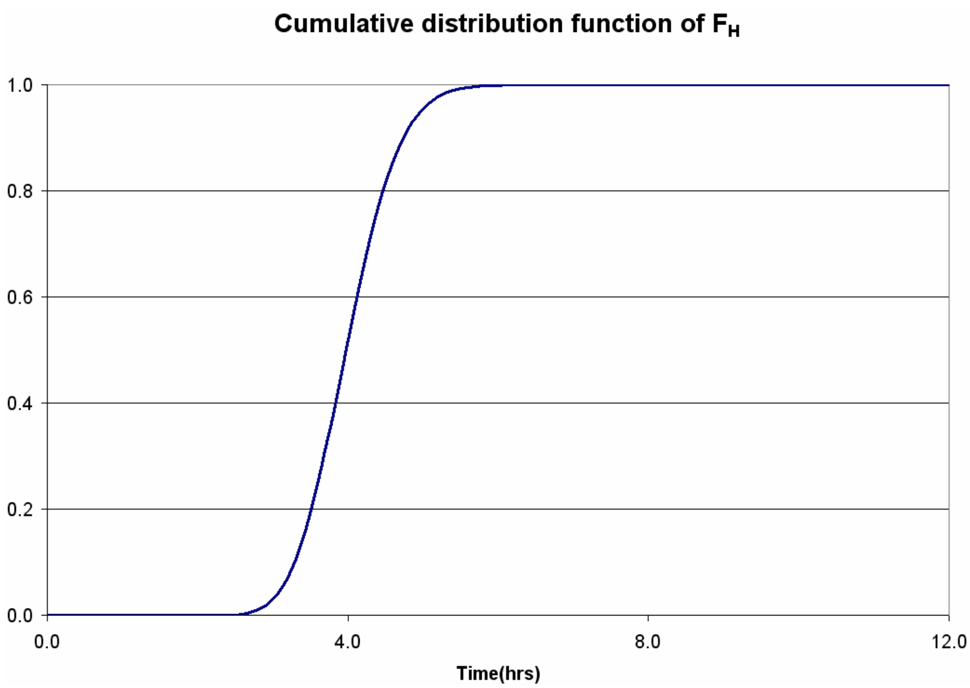

Figure 10. Cumulative distribution function of F_H

The time until the MPA loses classification information of a classified vessel is a random value between 2.5hrs and 5.5hrs. The MPA may lose its classification information before the mean time 4.0hrs resulting from the MPA's tactics and target behaviors. On the other hand, it is also possible that the information (possible position) of some vessels previously classified is available to the MPA for a long time by the renewal of the database (surface pictures) by other defense assets (Satellite, Automatic Identification System (AIS)[19]).

[19] United States, Government Accountability Office, "Maritime Security: Partnering could reduce federal costs and facilitate implementation of automatic vessel identification system " (2004)

B. PHASE_2: LOGISTIC REGRESSION MODELS

In this phase, we describe and explore appropriate regression models of the MIO capabilities by designing the input data by the Nearly-Orthogonal Latin Hypercube (NOLH) designs and implementing the simulations.

1. SIMULATION IMPLEMENTATION

a. Multi-level Factor Values

Table 3 shows the specified factor values of the multi-level factors.

	Low	High
M_x	200NM	400NM
w	0	100
u	15kt	30kt
τ	4min	8min
c_{ww}	0.8	1.0
c_{rr}	0.6	1.0
$1/\psi$	0hrs	4hrs

Table 3. Multi-level factor values

b. Design of Experiment

For the experiment involving less than 23 factors, NOLH designs are available for examining the impact on the simulated MOEs of simultaneously changing the specified factor values.[20] Since the number of factors of this case is only 7 as shown in Table 3, we apply the NOLH to produce our basic design of experiments. However, the independent variables in the logistic regression models for P_C and P_E, are obtained from the analytical Markov model.[21] Details are in Appendix_1.

[20] Kleijnen, Jack P.C., Sanchez, Susan M., Lucas, Thomas W., and Cioppa, Thomas M., "A User's Guide to the Brave New World of Designing Simulation Experiments," *INFORMS Journal on Computing,* (2005)

[21] Gaver, Donald P. and Jacobs, Patricia A., "A Stochastic Modeling of a Variety of Simple MDA Situations," *Naval Postgraduate School*, (2005)

Table 4 displays the 34 design points. These design points are used as input to the simulations. Design points (#1 – 17) are obtained from the NOLH based on Table 3 using the NOLH design spreadsheet[22]. The spreadsheet uses the designs based on the Cioppa's basic NOLH designs[23]. The Design points (#18 – 34) are intentionally added to obtain relatively higher probabilities for P_C and P_E by using the other NOLH based on Table 5.

Design Point ID	Mx (NM)	W	u (kt)	tau (min)	Cww	Crr	1/psi (hrs)
1	263	100	27.2	5.5	0.85	0.98	2.3
2	213	25	28.1	6.3	0.80	0.73	2.5
3	225	44	15.9	5.0	0.93	0.93	4.0
4	238	63	19.7	8.0	0.91	0.65	3.0
5	350	94	21.6	4.5	0.86	0.60	3.3
6	400	31	20.6	7.3	0.81	0.90	3.5
7	325	19	30.0	5.3	0.98	0.78	3.8
8	313	88	26.3	7.8	0.96	0.85	2.8
9	300	50	22.5	6.0	0.90	0.80	2.0
10	338	0	17.8	6.5	0.95	0.63	1.8
11	388	75	16.9	5.8	1.00	0.88	1.5
12	375	56	29.1	7.0	0.88	0.68	0.0
13	363	38	25.3	4.0	0.89	0.95	1.0
14	250	6	23.4	7.5	0.94	1.00	0.8
15	200	69	24.4	4.8	0.99	0.70	0.5
16	275	81	15.0	6.8	0.83	0.83	0.3
17	288	13	18.8	4.3	0.84	0.75	1.3
18	216	20	19.1	4.8	0.96	0.99	2.3
19	203	5	19.4	5.1	0.95	0.93	2.5
20	206	9	15.3	4.5	0.98	0.98	4.0
21	209	13	16.6	6.0	0.98	0.91	3.0
22	238	19	17.2	4.3	0.97	0.90	3.3
23	250	6	16.9	5.6	0.95	0.98	3.5
24	231	4	20.0	4.6	0.99	0.94	3.8
25	228	18	18.8	5.9	0.99	0.96	2.8
26	225	10	17.5	5.0	0.98	0.95	2.0
27	234	0	15.9	5.3	0.99	0.91	1.8
28	247	15	15.6	4.9	1.00	0.97	1.5
29	244	11	19.7	5.5	0.97	0.92	0.0
30	241	8	18.4	4.0	0.97	0.99	1.0
31	213	1	17.8	5.8	0.98	1.00	0.8
32	200	14	18.1	4.4	1.00	0.93	0.5
33	219	16	15.0	5.4	0.96	0.96	0.3
34	222	3	16.3	4.1	0.96	0.94	1.3

Table 4. Design of experiment

[22] Sanchez, Susan M., Sanchez, Paul J, Lucas, Thomas W., "NOLH designs spreadsheet," *Naval Postgraduate School*,(2005). Available online via http://diana.cs.nps.navy.mil/SeedLab/

[23] Cioppa, Thomas M. "Efficient Nearly Orthogonal and Space-filling Experimental Designs for High-dimensional Complex Models," *Naval Postgraduate School*, (2002)

	Low	High
M_x	200NM	250NM
w	0	20
u	15kt	20kt
τ	4min	6min
c_{ww}	0.95	1.0
c_{rr}	0.90	1.0
$1/\psi$	0hrs	4hrs

Table 5. Multi-level factor values additionally considered

2. OUTPUT DATA

a. Probability that the one R is detected and correctly classified before leaving the AOI, P_C

The probability P_C is estimated as the fraction of replications for which the one R is detected and correctly classified before leaving the AOI. Table 6, which is sorted in descending order based on P_C, displays the outputs of the simulation model. Under the favorable situations (Design points #18 – 34), P_C is estimated as in the range 0.40 to 0.58 (Note: P_C can not be more than about 0.6 because the design point #20 is nearly the most favorable situation for the Blue force). On the other hand, under the moderate situations (Design points #1 – 17), P_C is estimated as in the range 0.10 to 0.37.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	1/psi (hrs)	Simulation model			
												Pc			Logit
												Mean	95%LB	95%UB	
20	206	200	9	200	30	15.3	15	4.5	0.98	0.98	4.0	0.5809	0.5712	0.5906	0.3265
31	213	200	1	200	30	17.8	15	5.8	0.98	1.00	0.8	0.5389	0.5291	0.5487	0.1559
34	222	200	3	200	30	16.3	15	4.1	0.96	0.94	1.3	0.5211	0.5113	0.5309	0.0845
27	234	200	0	200	30	15.9	15	5.3	0.99	0.91	1.8	0.5187	0.5089	0.5285	0.0748
33	219	200	16	200	30	15.0	15	5.4	0.96	0.96	0.3	0.5086	0.4988	0.5184	0.0344
32	200	200	14	200	30	18.1	15	4.4	1.00	0.93	0.5	0.5076	0.4978	0.5174	0.0304
21	209	200	13	200	30	16.6	15	6.0	0.98	0.91	3.0	0.4983	0.4885	0.5081	-0.0068
28	247	200	15	200	30	15.6	15	4.9	1.00	0.97	1.5	0.4931	0.4833	0.5029	-0.0276
26	225	200	10	200	30	17.5	15	5.0	0.98	0.95	2.0	0.4821	0.4723	0.4919	-0.0716
19	203	200	5	200	30	19.4	15	5.1	0.95	0.93	2.5	0.4805	0.4707	0.4903	-0.0780
23	250	200	6	200	30	16.9	15	5.6	0.95	0.98	3.5	0.4698	0.4600	0.4796	-0.1209
30	241	200	8	200	30	18.4	15	4.0	0.97	0.99	1.0	0.4577	0.4479	0.4675	-0.1696
24	231	200	4	200	30	20.0	15	4.6	0.99	0.94	3.8	0.4533	0.4435	0.4631	-0.1873
18	216	200	20	200	30	19.1	15	4.8	0.96	0.99	2.3	0.4445	0.4348	0.4542	-0.2229
25	228	200	18	200	30	18.8	15	5.9	0.99	0.96	2.8	0.4393	0.4296	0.4490	-0.2440
22	238	200	19	200	30	17.2	15	4.3	0.97	0.90	3.3	0.4318	0.4221	0.4415	-0.2745
29	244	200	11	200	30	19.7	15	5.5	0.97	0.92	0.0	0.4006	0.3910	0.4102	-0.4030
14	250	200	6	200	30	23.4	15	7.5	0.94	1.00	0.8	0.3722	0.3627	0.3817	-0.5228
3	225	200	44	200	30	15.9	15	5.0	0.93	0.93	4.0	0.3701	0.3606	0.3796	-0.5318
17	288	200	13	200	30	18.8	15	4.3	0.84	0.75	1.3	0.2794	0.2706	0.2882	-0.9474
11	388	200	75	200	30	16.9	15	5.8	1.00	0.88	1.5	0.2684	0.2597	0.2771	-1.0028
10	338	200	0	200	30	17.8	15	6.5	0.95	0.63	1.8	0.2682	0.2595	0.2769	-1.0038
15	200	200	69	200	30	24.4	15	4.8	0.99	0.70	0.5	0.2467	0.2383	0.2551	-1.1163
13	363	200	38	200	30	25.3	15	4.0	0.89	0.95	1.0	0.2001	0.1923	0.2079	-1.3857
7	325	200	19	200	30	30.0	15	5.3	0.98	0.78	3.8	0.1996	0.1918	0.2074	-1.3888
2	213	200	25	200	30	28.1	15	6.3	0.80	0.73	2.5	0.1879	0.1802	0.1956	-1.4637
6	400	200	31	200	30	20.6	15	7.3	0.81	0.90	3.5	0.1858	0.1782	0.1934	-1.4775
9	300	200	50	200	30	22.5	15	6.0	0.90	0.80	2.0	0.1852	0.1776	0.1928	-1.4815
16	275	200	81	200	30	15.0	15	6.8	0.83	0.83	0.3	0.1773	0.1698	0.1848	-1.5347
4	238	200	63	200	30	19.7	15	8.0	0.91	0.65	3.0	0.1700	0.1626	0.1774	-1.5856
8	313	200	88	200	30	26.3	15	7.8	0.96	0.85	2.8	0.1516	0.1446	0.1586	-1.7221
1	263	200	100	200	30	27.2	15	5.5	0.85	0.98	2.3	0.1352	0.1285	0.1419	-1.8557
12	375	200	56	200	30	29.1	15	7.0	0.88	0.68	0.0	0.1106	0.1045	0.1167	-2.0846
5	350	200	94	200	30	21.6	15	4.5	0.86	0.60	3.3	0.0998	0.0939	0.1057	-2.1994

Table 6. Output of P_C

b. Probability that the one R is detected, correctly classified, and escorted before leaving the AOI, P_E

The probability P_E is estimated as the fraction of replications for which the one R is detected, correctly classified, and escorted before leaving the AOI. Table 7, which is sorted in descending order based on P_E, displays the outputs of the simulation model. Under the favorable situations (Design points #18 – 34), P_E is estimated as in the range 0.34 to 0.51 (Note: P_E can not be more than about 0.53 because the design point #20 is nearly the most favorable situation for the Blue force). On the other hand, under the moderate situations (Design points #1 – 17), P_E is estimated as in the range 0.08 to 0.32.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	1/psi (hrs)	Simulation model			
												P_E			Logit
												Mean	95%LB	95%UB	
20	206	200	9	200	30	15.3	15	4.5	0.98	0.98	4.0	0.5094	0.4996	0.5192	0.0376
31	213	200	1	200	30	17.8	15	5.8	0.98	1.00	0.8	0.4688	0.4590	0.4786	-0.1250
34	222	200	3	200	30	16.3	15	4.1	0.96	0.94	1.3	0.4540	0.4442	0.4638	-0.1845
27	234	200	0	200	30	15.9	15	5.3	0.99	0.91	1.8	0.4525	0.4427	0.4623	-0.1906
33	219	200	16	200	30	15.0	15	5.4	0.96	0.96	0.3	0.4496	0.4398	0.4594	-0.2023
32	200	200	14	200	30	18.1	15	4.4	1.00	0.93	0.5	0.4364	0.4267	0.4461	-0.2558
21	209	200	13	200	30	16.6	15	6.0	0.98	0.91	3.0	0.4340	0.4243	0.4437	-0.2655
28	247	200	15	200	30	15.6	15	4.9	1.00	0.97	1.5	0.4328	0.4231	0.4425	-0.2704
26	225	200	10	200	30	17.5	15	5.0	0.98	0.95	2.0	0.4136	0.4039	0.4233	-0.3491
19	203	200	5	200	30	19.4	15	5.1	0.95	0.93	2.5	0.4115	0.4019	0.4211	-0.3578
23	250	200	6	200	30	16.9	15	5.6	0.95	0.98	3.5	0.4061	0.3965	0.4157	-0.3801
30	241	200	8	200	30	18.4	15	4.0	0.97	0.99	1.0	0.3901	0.3805	0.3997	-0.4469
24	231	200	4	200	30	20.0	15	4.6	0.99	0.94	3.8	0.3814	0.3719	0.3909	-0.4836
18	216	200	20	200	30	19.1	15	4.8	0.96	0.99	2.3	0.3741	0.3646	0.3836	-0.5147
25	228	200	18	200	30	18.8	15	5.9	0.99	0.96	2.8	0.3739	0.3644	0.3834	-0.5155
22	238	200	19	200	30	17.2	15	4.3	0.97	0.90	3.3	0.3672	0.3578	0.3766	-0.5442
29	244	200	11	200	30	19.7	15	5.5	0.97	0.92	0.0	0.3389	0.3296	0.3482	-0.6682
3	225	200	44	200	30	15.9	15	5.0	0.93	0.93	4.0	0.3171	0.3080	0.3262	-0.7671
14	250	200	6	200	30	23.4	15	7.5	0.94	1.00	0.8	0.3027	0.2937	0.3117	-0.8345
17	288	200	13	200	30	18.8	15	4.3	0.84	0.75	1.3	0.2331	0.2248	0.2414	-1.1909
11	388	200	75	200	30	16.9	15	5.8	1.00	0.88	1.5	0.2234	0.2152	0.2316	-1.2460
10	338	200	0	200	30	17.8	15	6.5	0.95	0.63	1.8	0.2228	0.2146	0.2310	-1.2494
15	200	200	69	200	30	24.4	15	4.8	0.99	0.70	0.5	0.1972	0.1894	0.2050	-1.4039
13	363	200	38	200	30	25.3	15	4.0	0.89	0.95	1.0	0.1585	0.1513	0.1657	-1.6694
6	400	200	31	200	30	20.6	15	7.3	0.81	0.90	3.5	0.1531	0.1460	0.1602	-1.7105
7	325	200	19	200	30	30.0	15	5.3	0.98	0.78	3.8	0.1531	0.1460	0.1602	-1.7105
16	275	200	81	200	30	15.0	15	6.8	0.83	0.83	0.3	0.1513	0.1443	0.1583	-1.7244
9	300	200	50	200	30	22.5	15	6.0	0.90	0.80	2.0	0.1493	0.1423	0.1563	-1.7401
2	213	200	25	200	30	28.1	15	6.3	0.80	0.73	2.5	0.1488	0.1418	0.1558	-1.7440
4	238	200	63	200	30	19.7	15	8.0	0.91	0.65	3.0	0.1357	0.1290	0.1424	-1.8515
8	313	200	88	200	30	26.3	15	7.8	0.96	0.85	2.8	0.1153	0.1090	0.1216	-2.0377
1	263	200	100	200	30	27.2	15	5.5	0.85	0.98	2.3	0.1077	0.1016	0.1138	-2.1145
12	375	200	56	200	30	29.1	15	7.0	0.88	0.68	0.0	0.0844	0.0790	0.0898	-2.3840
5	350	200	94	200	30	21.6	15	4.5	0.86	0.60	3.3	0.0818	0.0764	0.0872	-2.4181

Table 7. Output of P_E

3. LOGISTIC REGRESSION MODELS

a. Preliminary Study

As a preliminary study, we explore appropriate logistic regression models for the following two cases. Details are in Appendix1.

<u>Notation</u>:

F_L : Distribution of the time between detections

F_U : Distribution of the time an un-encountered R spends in the AOI

F_D : Distribution of the time for the MPA to track a suspicious vessel

CASE-1:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape 1 (i.e. Exponential distribution)
- F_D is a Gamma distribution with mean $1/\phi$ and shape 1 (i.e. Exponential distribution)
- The MPA has no memory of its previous classification information (pessimistic assumption)

CASE-2:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape β (=5 to 50)
- F_D is a Gamma distribution with mean $1/\phi$ and shape β which is the same as that of F_U. (U and D are statistically independent.)
- The MPA has no memory of its previous classification information (pessimistic assumption)

Our situation assumed in the Phase_1 is a specific case of CASE-2 except that the MPA has memory of its previous classification information. Thus, first, we explore appropriate logistic regression models which do not consider the MPA's information retention (do not include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$), based on the following statistically fitted logistic regression models (4.5) and (4.6) introduced in CASE2 of Appendix1; (this attempt is called "Not considering the MPA's information retention"). After that, we explore appropriate logistic regression models which consider the MPA's information retention; (this attempt is called "Considering the MPA's information retention").

- Probability that the one R is detected and correctly classified before leaving the AOI, P_C

$$\ln\left(\frac{P_C}{1-P_C}\right)=13.737+2.119\ln c_{rr}-1.773\ln u+0.287\ln\beta$$
$$+7.530\ln\left(\frac{w(1-c_{ww})}{2v_I+3u}+\frac{M_x}{f\cdot v}+\frac{(w+1)\tau}{M_y}\right)$$
$$+1.510\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u}+\frac{M_x}{f\cdot v}+\frac{(w+1)\tau}{M_y}\right)\right)^2 \quad (4.5)$$

This model is the same as (A.13) in Appendix1.

- Probability that the one R is detected, correctly classified, and escorted before leaving the AOI, P_E

$$\ln\left(\frac{P_E}{1-P_E}\right)=1.871\ln c_{rr}-2.689\ln u+3.911\ln\left(2v_I+2u\right)+0.302\ln\beta$$
$$-9.810\ln\left(c_{rr}+w(1-c_{ww})+\left(\frac{M_x}{f\cdot v}+\frac{w+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right)$$
$$+2.128\left(\ln\left(c_{rr}+w(1-c_{ww})+\left(\frac{M_x}{f\cdot v}+\frac{w+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right)\right)^2 \quad (4.6)$$

This model is the same as (A.17) in Appendix1.

Note: The form of the independent/explanatory variables are derived from an analytical model. Simple use of the raw parameter values results in uselessly inferior predictions. See Appendix1.

b. *Not Considering the MPA's Information Retention*

To find appropriate logistic regression models which do not include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$, we consider all independent variables of (4.5) (respectively (4.6)) except for $\ln\beta$ (because we assume β=25 as constant in the Phase-1). The statistically fitted logistic regression models are follows

- Probability that the one R is detected and correctly classified before leaving the AOI, P_C

$$\ln\left(\frac{P_C}{1-P_C}\right) = 0.881 + 1.117\ln c_{rr} - 1.134\ln u - 0.440\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right) + 0.199\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\right)^2 \quad (4.7)$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999
R Square	0.998
Adjusted R Square	0.998
Standard Error	0.032
Observations	34

ANOVA

	df	SS	MS	F	Significance F
Regression	4	18.581	4.645	4534	0.000
Residual	29	0.030	0.001		
Total	33	18.611			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.881	0.177	4.984	0.000	0.519	1.242
b1	1.117	0.046	24.089	0.000	1.022	1.212
b2	-1.134	0.033	-34.722	0.000	-1.201	-1.068
b3	-0.440	0.179	-2.460	0.020	-0.806	-0.074
b4	0.199	0.045	4.429	0.000	0.107	0.290

Since the R-square value of the regression model (4.7) is 0.998, and the p-value of each independent variable is less than 0.05, the four independent variables of the model (4.7) are statistically significant in the logistic regression. Figure 11 and Figure 12 display the outputs of the estimated regression model (4.7) and those of the simulation model.

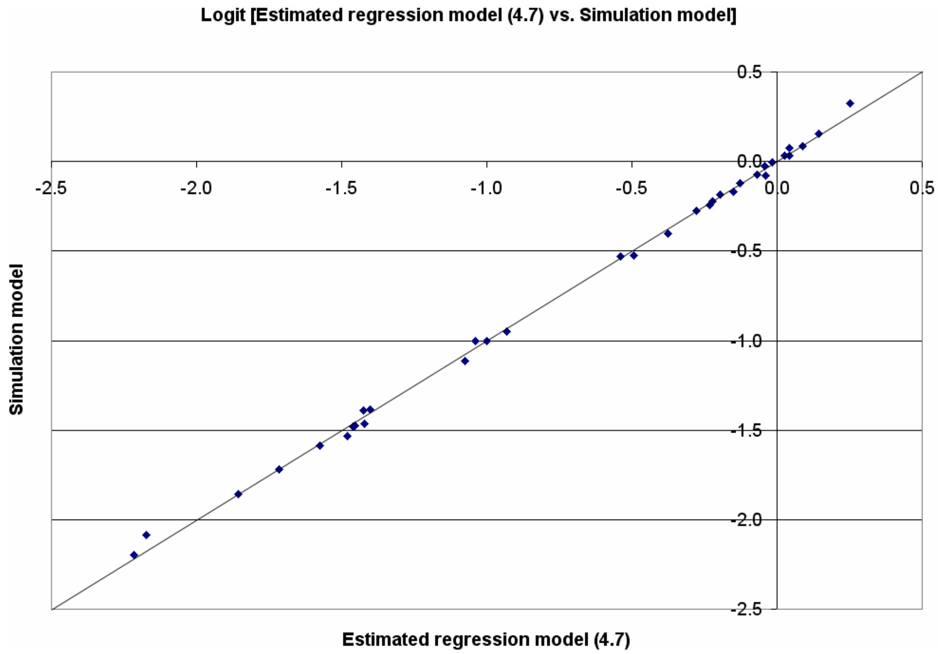

Figure 11. Logit [Estimated regression model (4.7) vs. Simulation model]

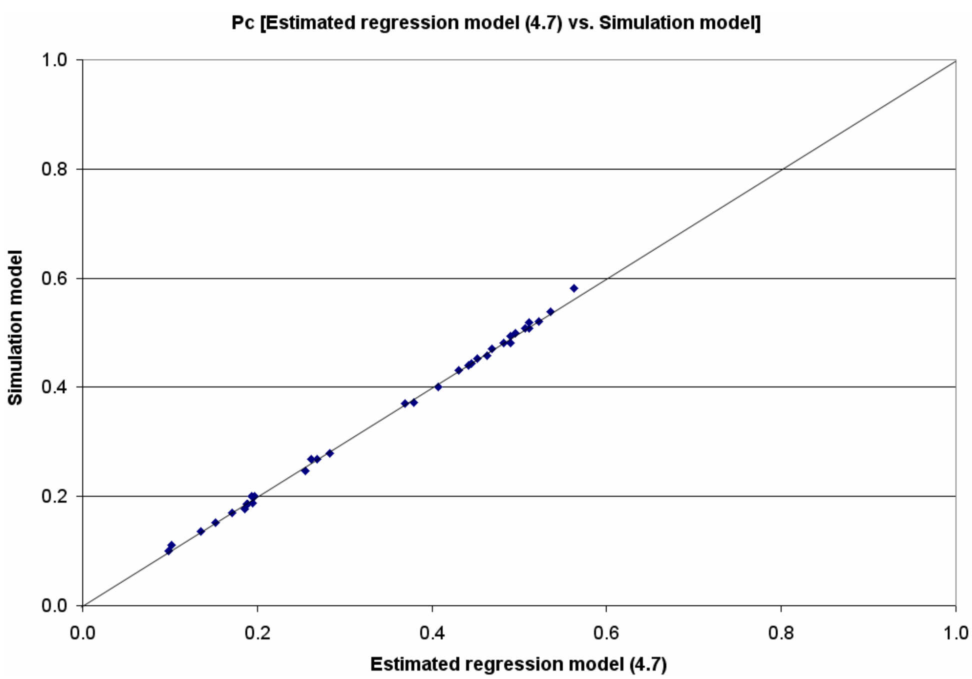

Figure 12. P_C [Estimated regression model (4.7) vs. Simulation model]

Even though the estimated logistic regression model (4.7) does not include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$, it summarizes the simulation output well for the input values considered.

- Probability that the one R is detected, correctly classified, and escorted before leaving the AOI, P_E

$$\ln\left(\frac{P_E}{1-P_E}\right) = 1.122\ln c_{rr} - 1.474\ln u + 1.965\ln\left(2v_I + 2u\right) - 2.790\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right) + 0.269\left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)\right)^2 \quad (4.8)$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	1.000
Adjusted R Square	0.965
Standard Error	0.022
Observations	34

ANOVA

	df	SS	MS	F	Significance F
Regression	5	51.477	10.295	22242	0.000
Residual	29	0.013	0.000		
Total	34	51.490			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.000	-	-	-	-	-
b1	1.122	0.031	35.804	0.000	1.058	1.186
b2	-1.474	0.036	-40.780	0.000	-1.548	-1.400
b3	1.965	0.063	31.403	0.000	1.837	2.093
b4	-2.790	0.156	-17.828	0.000	-3.110	-2.470
b5	0.269	0.027	9.896	0.000	0.214	0.325

Since the R-square value of this regression model (4.8) is 1.000, and the p-value of each independent variable is 0, the five independent variables of the model (4.8) are statistically significant in the logistic regression model. Figure 13 and Figure 14 display the outputs of the estimated regression model (4.8) and those of the simulation model.

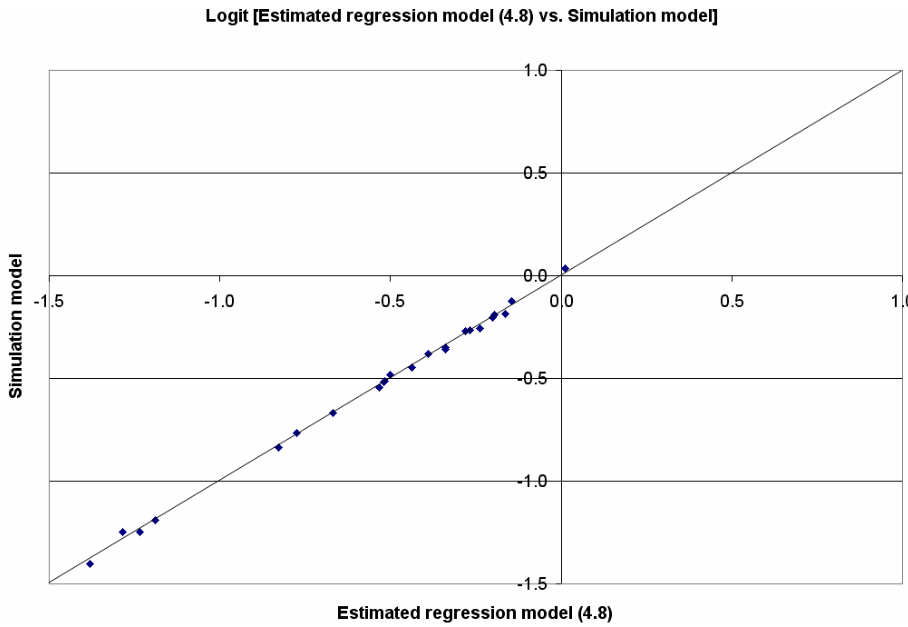

Figure 13. Logit [Estimated regression model (4.8) vs. Simulation model]

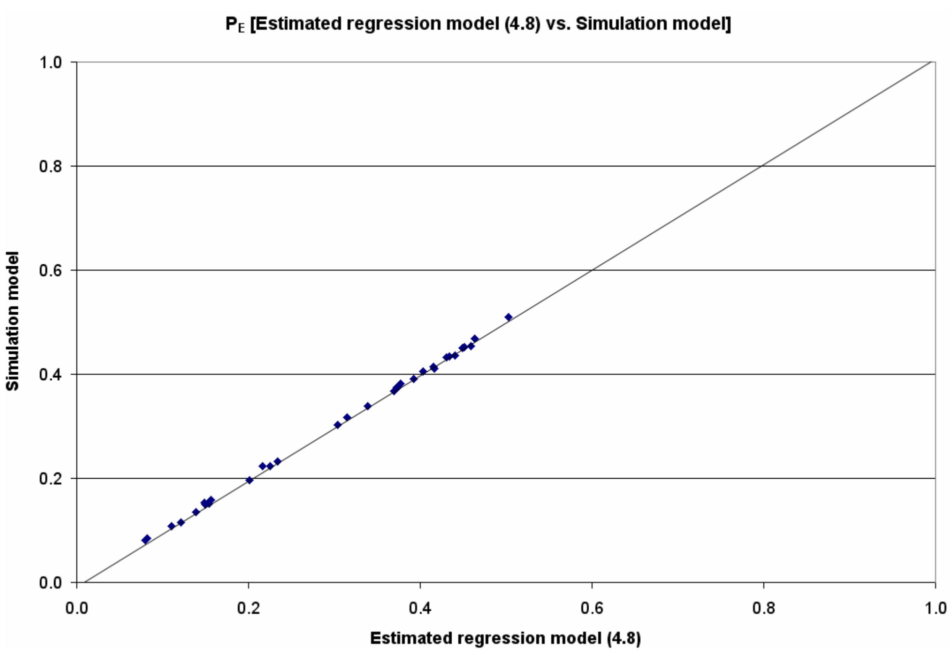

Figure 14. P_E [Estimated regression model (4.8) vs. Simulation model]

Even though the estimated logistic regression model (4.8) does not include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$, it summarizes the simulation output well for the input values considered.

c. *Considering the MPA's Information Retention*

The logistic regression equations (4.7) and (4.8) do not have the mean time of information retention, $1/\psi$, as an independent variable. In Appendix 3, we consider logistic regression models which include it as an independent variable. For the logistic regression models and input values considered, the estimated coefficient of the mean time of information retention, $1/\psi$, is not statistically different from 0. Thus, the regression models (4.7) and (4.8) are used to summarize the simulation output.

C. PHASE_3: ANALYSIS OF THE SIGNIFICANT FACTORS

1. Significant Factors

We initially consider seven multi-level factors (M_x, w, u, τ, c_{ww}, c_{rr}, and $1/\psi$), however, $1/\psi$ is omitted because its estimated coefficients in the logistic regression models are not statistically different from 0 in Phase_2 (Details are in Appendix3). Since M_y=200NM, M_x represents the size of the AOI. Table 8 displays the six significant factors that are analyzed in this phase and their values specified in Phase_1.

	Low	High
M_x	200NM	400NM
w	0	100
u	15kt	30kt
τ	4min	8min
c_{ww}	0.8	1.0
c_{rr}	0.6	1.0

Table 8. Significant factors and specified values

2. Logistic Regression Models of the MIO Capabilities

We use the logistic regression models (4.7) and (4.8) to analyze the significant factors influencing the Maritime Intercept Operations (MIO) capabilities. In Appendix4, these models are cross-validated by other input values.

3. Data Analysis

CASE-0:

Initially, we consider a specific situation (M_x=300NM, w=50, u=22.5kt, τ= 6min, c_{ww}=0.9, and c_{rr}=0.8). Each factor value is the middle value of the specified factor values in Table 8. This situation is called CASE-0.

Probability that the one R is detected and correctly classified before leaving the AOI, P_C

The probability P_C is estimated as "**0.189**" using the model (4.7). Fugure15 displays the sensitivity of the probability P_C by changing each factor value when other factors values are fixed. Figure16 also displays the interaction of any two factors when other factor values are fixed. ***The value P_C =0.189 is impractically low, and would be unacceptable operationally. The AOI size and the number of distracting Ws are simply too large for an unaided MPA to cover adequately. The Blue search capabilities must be enhanced in various ways.***

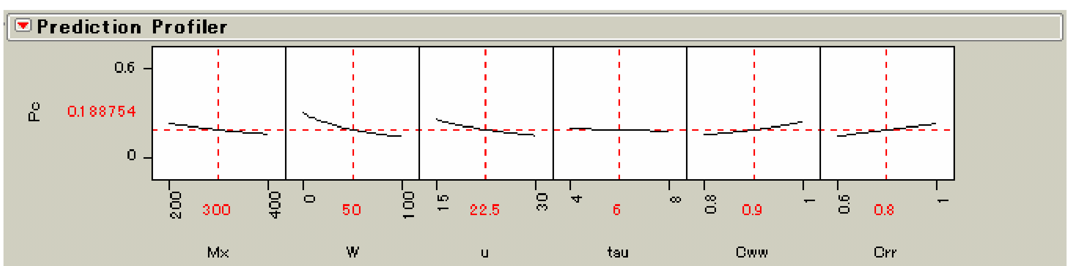

Figure 15. Prediction Profiler (CASE-0: P_C)

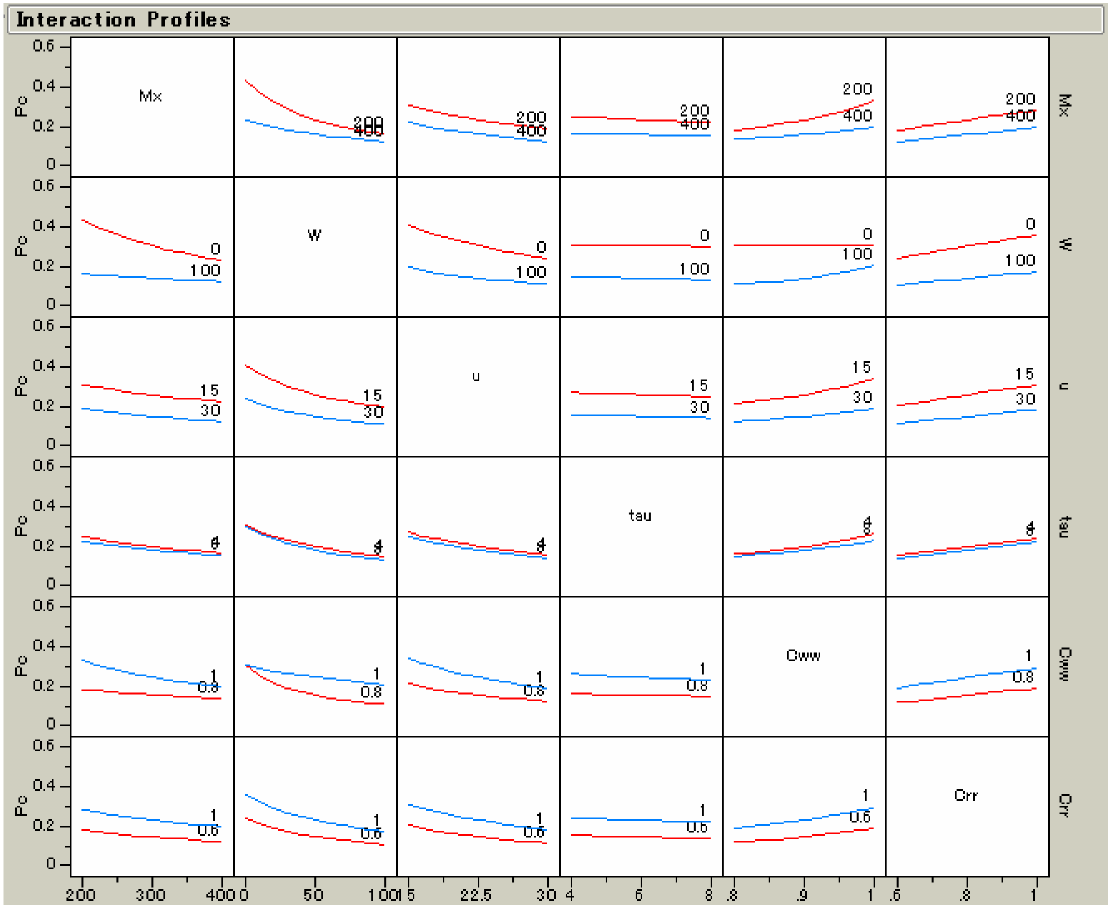

Figure 16. Interaction Profiler (CASE-0: P_C)

The following tendencies are suggested by Figure 15 and 16:

- When M_x (w, u, respectively) increases, the probability P_C decreases.
- When c_{ww} (c_{rr}, respectively) increases, the probability P_C increases.
- Changes to the value of the mean time to classify a detected vessel, τ, do not change P_C as much as changing the values of the other factors.
- There is an interaction between M_x, w, and c_{ww}.
- w is the most influential factor for the probability P_C among the considered factors.
- A relatively high P_C value can be achieved when there are a few Ws and a relatively small AOI; however, the P_C value quickly decreases when w increases a small amount (0→10).

a. Probability that the one R is detected, correctly classified, and escorted before leaving the AOI, P_E

The probability P_E is estimated as "**0.150**" using the model (4.8). Fugure17 displays the sensitivity of the probability P_E by changing each factor value when other factors values are fixed. Figure18 also displays the interaction of any two factors when other factor values are fixed.

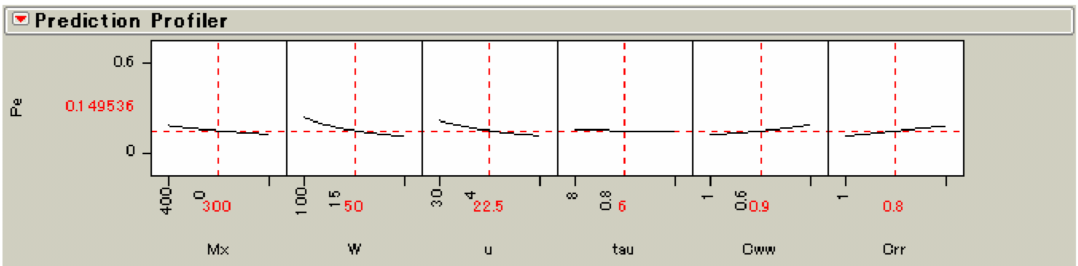

Figure 17. Prediction Profiler (CASE-0: P_E)

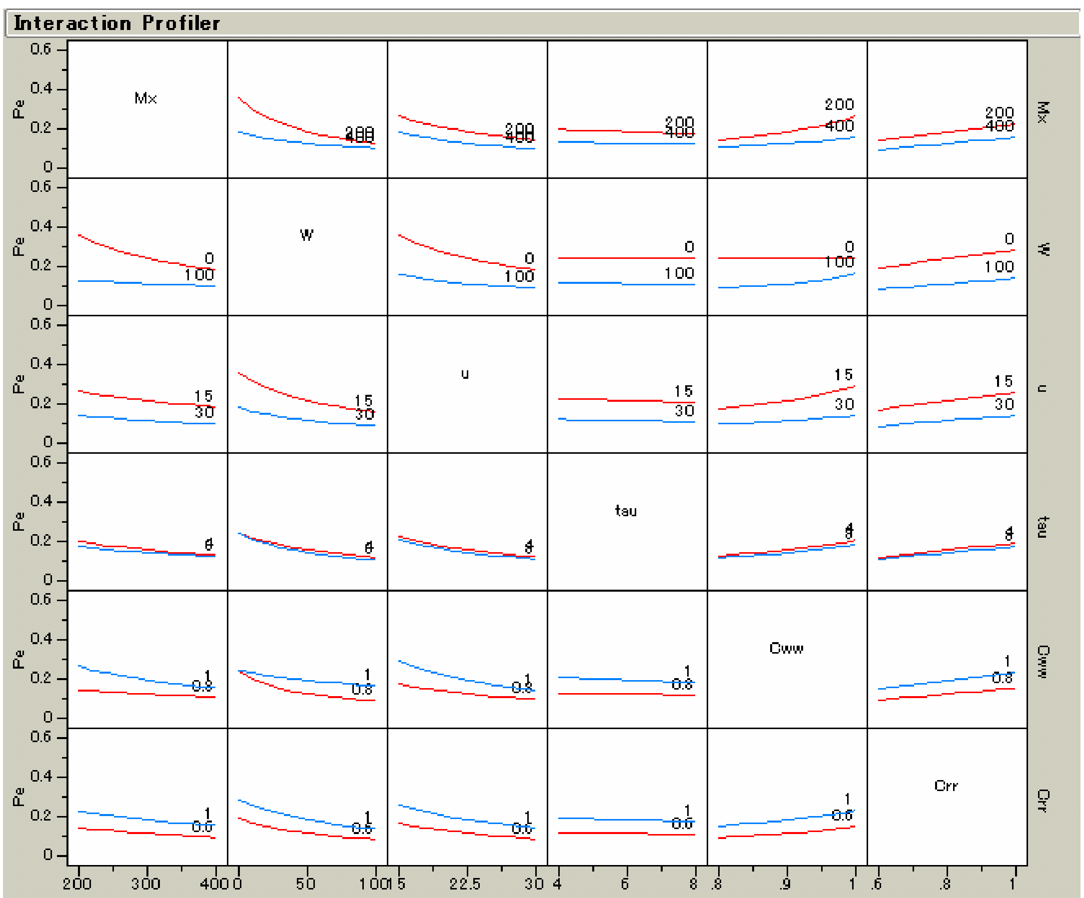

Figure 18. Interaction Profiler (CASE-0: P_E)

The tendencies of the probability P_E are quite similar to those of the probability P_C. The second most influential factor for the probability P_E can be identified as the velocity of the R, u.

<u>CASE-1</u>:

In CASE-1, we consider a specific situation [M_x=**200NM**, w=**33**, u=22.5kt, τ= 6min, c_{ww}=0.9, and c_{rr}=0.8]. w is the most influential factor in the regression models. To decrease w, we reduce the AOI size for an MPA. Here M_x and w are assumed to be reduced to 2/3 of the CASE-0 values (M_x=300NM→200NM, w=50→33). To implement this change, more MPAs or other surveillance assets are required in operation to cover the original size of the area ($M_x \times M_y$=300NM×200NM); otherwise more reliable intelligence about the path of Rs is required to designate a specific AOI whose size is relatively small. For these parameter values, the probabilities P_C and P_E are estimated as **0.278** and **0.222** respectively. By reducing the AOI size by 2/3, P_C (respectively P_E) becomes 1.47 times [**0.189→0.278**] (respectively 1.48 times [**0.150→0.222**]) of that of CASE-0.

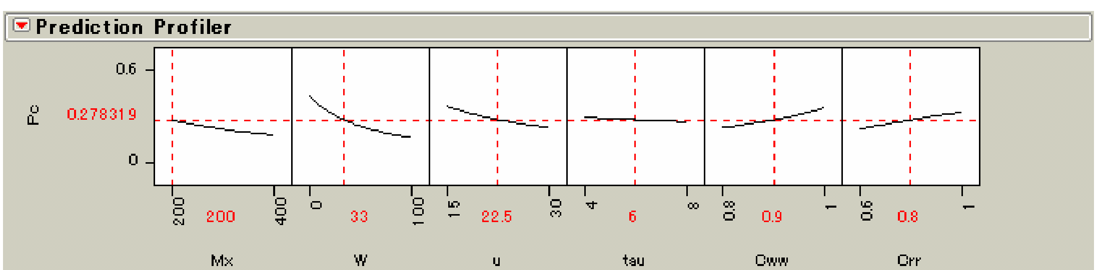

Figure 19. Prediction Profiler (CASE-1: P_C)

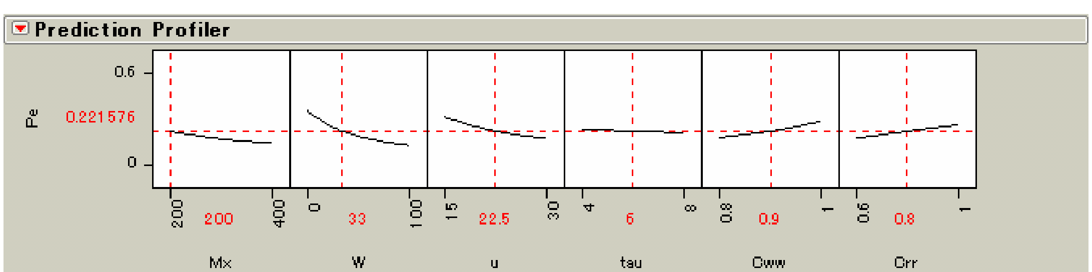

Figure 20. Prediction Profiler (CASE-1: P_E)

CASE-2:

In CASE-2, we consider a specific situation [M_x =300NM, w =50, u =22.5kt, τ = 6min, c_{ww} =**0.95**, and c_{rr} =**0.90**] in which the AOI size per an MPA can not be reduced (because additional MPAs or other surveillance assets are not available or because more specific intelligence about the path of Rs is not available): however, high identification capabilities c_{ww} and c_{rr} are attainable using identification systems (database, experiences, and training.) Here we assume that c_{ww} =0.90→0.95, c_{rr} =0.80→0.90. In this case, the probabilities P_C and P_E are estimated as **0.236** and **0.187** respectively. By enhancing the identification capabilities as mentioned above, P_C (respectively P_E) becomes 1.25 times [**0.189→0.236**] (respectively 1.25 times [**0.150→0.187**]) of that of CASE-0. However, the effect obtained by enhancing identification capabilities is smaller than that obtained by reducing the AOI size. ***Such low values are operationally unsatisfactory***.

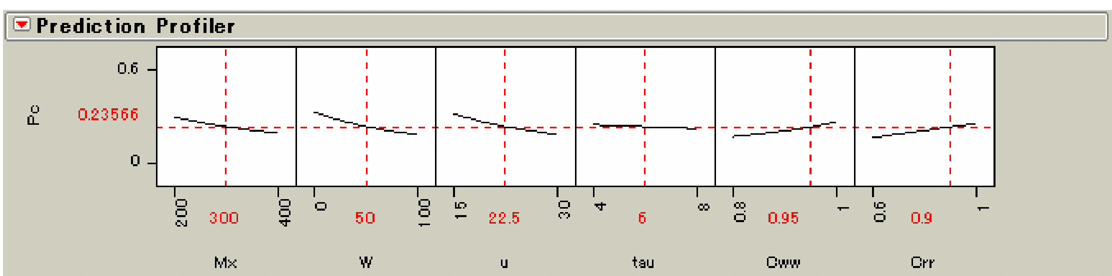

Figure 21. Prediction Profiler (CASE-2: P_C)

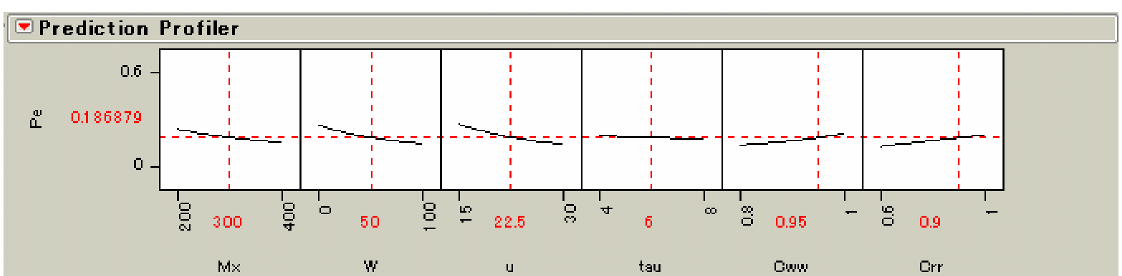

Figure 22. Prediction Profiler (CASE-2: P_E)

CASE-3:

In Case-3, we consider a specific situation [M_x=**200NM**, w=**33**, u=22.5kt, τ= 6min, c_{ww}=**0.95**, and c_{rr}=**0.90**] in which the changes in parameter values considered in CASE-1 and CASE-2 are applied simultaneously. In this case, the probabilities P_C and P_E are estimated as **0.343** and **0.274** respectively. By considering both measures (tactically reducing the AOI size per an MPA, and enhancing the identification capabilities), P_C (respectively P_E) becomes 1.81 times [**0.189→0.343**] (respectively 1.83 times [**0.150→0.274**]) of that of CASE-0. However, P_C=**0.343** and P_E=**0.274** are still not operationally acceptable.

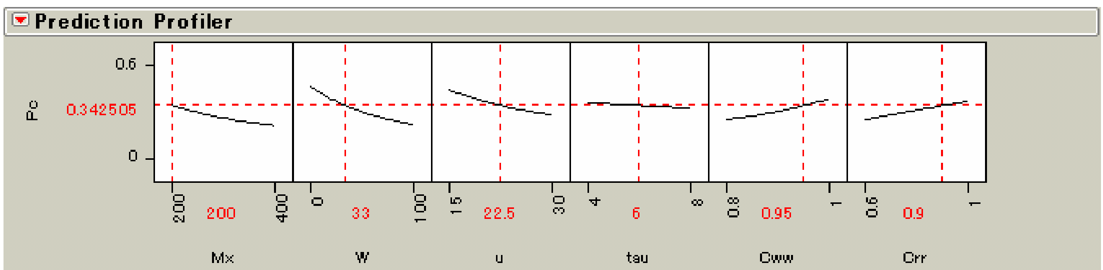

Figure 23. Prediction Profiler (CASE-3: P_C)

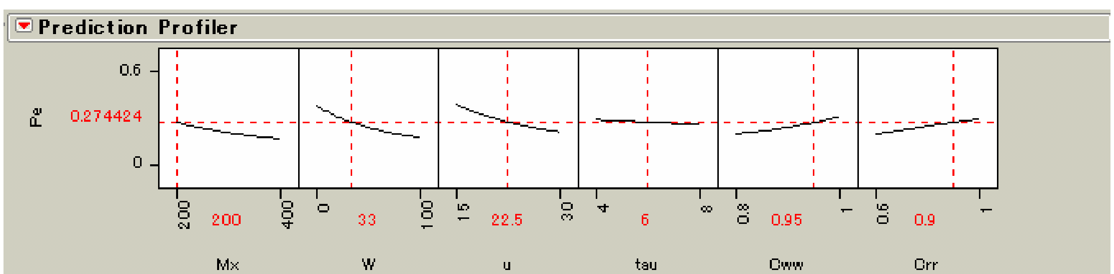

Figure 24. Prediction Profiler (CASE-3: P_E)

For this analysis, we apply various assumptions (scenario, data, distributions, and modeling). If they are "reasonable," it would imply that a single Maritime Patrol Aircraft (MPA) is operationally inadequate to intercept an R before it leaves an AOI. If more MPAs are simultaneously available, they should be used in the surveillance operation. Otherwise, measures which reduce the number of unidentified Ws in an AOI should be applied as much as possible. These measures include intelligence operations, maritime traffic control, and additional surveillance operations by other defense assets (satellites, helicopters, and maritime vessels).

V. CONCLUSION AND FUTURE IMPROVEMENTS

A. GENERAL

This research formulates and exercises stochastic and simulation models to assess the Maritime Intercept Operations (MIO) capabilities and to quantitatively identify the significant factors influencing the MIO capabilities to detect and interdict hostile vessels. The research is based on various assumptions: scenario, data, distributions of the events, and models. It also focuses explicitly on the surveillance operations of the Maritime Patrol Aircraft (MPA) in the MIO. The model is also not spatial. Thus, the results of the analysis can not be directly applied to plan real concepts of operations (CONOPS) and operational plans (OPLAN). But they provide very useful intuition, enhancement, and stimulation. As a result, available field data should be collected to assess the reasonableness of the model assumptions. Although we focus on the MPA operations, the effects of other components (intelligence, helicopters, unmanned surveillance systems, maritime vessels, and C4I systems) can be studied using this MPA-based analysis. For example, reliable information concerning presence of threatening vessels enables assignment of smaller size of Area of Interest (AOI) to an MPA, and surveillance operations by helicopters and maritime vessels can be used to complement the MPA's surveillance operations.

B. CONCLUSIONS

1. MIO Capabilities

If our assumptions (scenario, data, distributions of the events, parameter values, and models) are "reasonable," the probability that one R is detected and correctly classified before leaving the AOI, P_C, is estimated as 0.189 (CASE-0) to 0.343 (CASE-3); the probability that one R is detected, correctly classified, and escorted before leaving the AOI, P_E, is estimated as 0.150 (CASE-0) to 0.274 (CASE-3). ***This would be operationally unacceptable and would imply that a single unassisted MPA is operationally inadequate. If more MPAs are simultaneously available, they should be used in the surveillance operation. Otherwise, the complementary surveillance***

operations by other defense components (intelligence, helicopters, maritime vessels, C4ISR systems) are surely indispensable for the success of the MIO capabilities.

- CASE-0 (M_x =300NM, w =50, u =22.5kt, τ = 6min, c_{ww} =0.90, and c_{rr} =0.80)
- CASE-3 (M_x =200NM, w =33, u =22.5kt, τ = 6min, c_{ww} =0.95, and c_{rr} =0.90)

2. Factors Influencing the MIO Capabilities

We initially consider more than ten factors that possibly influence the MIO capabilities. Since the length of y -direction side of the AOI, mean speed of the MPA, mean speed of Inspection Units, radar sweep width, and the distributions on the events (which are surely influential to the MIO capabilities) are assumed to be constant, we eventually consider the following seven factors which are important to evaluate current programs and study new, alternative programs.

- w : The number of Ws in an AOI
- M_x : Length of x -direction side of the AOI
- u : Mean speed of Ws and Rs
- τ : Process time for each detected vessel
- c_{ww} : Probability that a W is correctly classified as W
- c_{rr} : Probability that a R is correctly classified as R
- $1/\psi$: Mean time until the MPA loses classification information of a classified vessel.

The most significant factor for the MIO capabilities is " w " that must be classified by one MPA. Relative high probabilities P_C and P_E can be achieved when there are few Ws (0 to 10) and a relative small AOI; however, the capabilities quickly decrease by increasing w to more than 10. Thus, for the success of the MIO, we should plan possible programs that enable restriction of w per one MPA.

- Increasing the number of the MPAs and reducing the number of unidentified Ws which should be processed by an MPA (i.e. reducing M_x of the AOI per one MPA)
- Enhancing intelligence operations and specifying a small AOI (or choke point area) where Rs probably pass through.
- If traffic control is available for vessels entering Japanese territorial areas, it should be applied as often as possible.

The mean speed of the Ws and Rs is probably the second most significant factor for the MIO capabilities. Thus, for Rs, transiting with a high speed may be a good way to intrude Japanese shores successfully. However, their unusual behavior among other Ws may make them easily identifiable as R using overhead sensors (satellites, the MPAs, and helicopters).

The AOI size (here, represented by M_x with a constant M_y=200NM) influences w which should be processed per one MPA. The large AOI forces an MPA not only to process more vessels and but also to take more time to cover the whole AOI. Thus, the AOI size can be considered as significant as w.

The identification capabilities c_{ww} and c_{rr} are also important factors to enhance the MIO capabilities: however, their effects are smaller than the programs to restrict w which should be processed by one MPA – such as increasing the number of the MPAs in operation and enhancing intelligence operations. Since it is usually difficult to procure more MPAs or to assign many MPAs simultaneously to surveillance operations, our interests may focus on enhancing the identification capabilities to deter Rs. However, since their effects are moderate, we should consider the cost effectiveness of procuring new identification systems or to applying new programs.

In this study, the process time for each detected vessel τ and the mean time until the MPA loses classification information of a classified vessel $1/\psi$ are identified as operationally insignificant factors. However, more time spent identifying the vessel may increase the MPA's identification capability for an unknown vessel; this can enhance the

MIO capabilities; see the working paper [Gaver, Jacobs, and Sato, 2005][24]. Moreover, although information retention may not be significant for one MPA itself, it is important when we consider joint operations with other defense components for which information sharing is required.

C. FUTURE IMPROVEMENTS

The analysis provides several insights on the MIO capabilities even though we focus only on MPA surveillance operations. However, to support the planning of appropriate CONOPS and OPLAN practically, we must not only review our arbitrary assumptions (scenario, data, distributions and models) by analyzing available field data but we must also study the MIO capabilities more intensively by considering/ coordinating other defense components (intelligence, helicopters, maritime vessels and C4ISR systems) and developing more comprehensive large-scale models to study the missions/functions of the defense components, the coordination of the components, the information sharing among the components, and the significance of each component and its sub systems.

[24] Gaver, Donald P., Jacobs, Patricia A, and Sato, Hiroyuki, "Assessing resource requirements for maritime domain awareness and protection," Working paper, Naval Postgraduate School (2005)

APPENDIX 1 LOGISTIC REGRESSION MODELS

A. OBJECTIVE

The objective of this appendix is to describe and explore appropriate logistic regression models for summarizing simulation output of the Maritime Intercept Operations (MIO) capabilities.

B. MEASURES OF EFFECTIVENESS

1. Probability that a typical R is detected and correctly classified before leaving the Area of Interest (AOI), P_C

2. Probability that a typical R is detected, correctly classified and escorted before leaving the AOI, P_E

C. NOTATION

M_x : Length of x-direction side of the AOI

M_y : Length of y-direction side of the AOI

v : Mean speed of the Maritime Patrol Aircraft (MPA)

v_I : Mean speed of the inspective maritime vessels

u : Mean speed of Ws and Rs

τ : Process (=approach and identify) time for each detected vessel

f : Radar coverage or radar sweep width

c_{ww} : Probability that a W is correctly classified as W

c_{wr} : Probability that a W is incorrectly classified as R (=$1-c_{ww}$)

c_{rr} : Probability that an R is correctly classified as R

c_{rw} : Probability that an R is incorrectly classified as W (=$1-c_{rr}$)

t_j: Time at which the MPA finishes processing the j^{th} detected vessel

$W_u(t_j)$: The number of Ws unknown in the AOI at time t_j

$W_i(t_j)$: The number of Ws identified as W in the AOI at time t_j

w: Total number of Ws in the AOI at time t_j ($w = W_u(t_j) + W_i(t_j)$)

$R_u(t_j)$: The number of Rs unknown in the AOI at time t_j

$R_i(t_j)$: The number of Rs thought to be W in the AOI at time t_j

$R(t_j)$: Total number of Rs in the AOI at time t_j ($R(t_j) = R_u(t_j) + R_i(t_j)$)

$L(t_j)$: Time until the next detection at time t_j

U: Time an un-encountered R spends in the AOI

D: Time the MPA tracking a suspicious vessel is occupied

F_L: Distribution of the time between detections

F_U: Distribution of the time an un-encountered R spends in the AOI

F_D: Distribution of the time for the MPA to track a suspicious vessel

$\delta(t_j)$: Detections rate at time t_j

$$\boxed{\delta(t_j) = \frac{W_u(t_j) + R_u(t_j)}{\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v} + \left(W_u(t_j) + R_u(t_j)\right)\cdot \tau}} \qquad \text{(A.1)}$$

$1/\mu$: Mean time an un-encountered R spends in the AOI

$$\boxed{1/\mu = \frac{M_y}{u}} \qquad \text{(A.2)}$$

$1/\phi$: Mean time for the MPA to track a suspicious vessel

$$\boxed{1/\phi = \frac{M_y/2}{v_I + u}} \quad \text{(A.3)}$$

D. CASE STUDIES

In this appendix, we consider the following two cases:

CASE-1:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape 1 (i.e. Exponential distribution)
- F_D is a Gamma distribution with mean $1/\phi$ and shape 1 (i.e. Exponential distribution)
- The MPA has no memory of its previous classification information (pessimistic assumption)

CASE-2:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape β (=5 to 50)
- F_D is a Gamma distribution with mean $1/\phi$ and shape β which is the same as that of F_U. (U and D are statistically independent.)
- The MPA has no memory of its previous classification information (pessimistic assumption)

E. MODEL IMPLEMENTATION

Input parameters:

1. Constant parameters

$M_y = 300NM$, $v = 300kt$, $v_I = 30kt$, $f = 15NM$

2. Multi-level parameters

For the following six parameters/factors, we consider the range between Low and High respectively.

	Low	High
M_x	50NM	300NM
w	0	50
u	10kt	30kt
τ	4min	8min
c_{ww}	0.9	1.0
c_{rr}	0.8	1.0

Table 9. Multi-level parameters

Design of experiments:

For the experiment involving less than 23 factors, Nearly-Orthogonal Latin Hypercube (NOLH) designs are available for examining the impact on the simulated MOE when simultaneously changing the specified factor values.[25]

Since the number of factors in CASE-1 is 6 (M_x, w, u, τ, C_{ww}, C_{rr}) and the number of factors in CASE-2 is 7 (M_x, w, u, τ, C_{ww}, C_{rr}, β), we apply the NOLH to produce our basic designs of experiment. However, the independent variables in the logistic regression models of P_C and P_E are those obtained for the logit of the probabilities obtained from the analytical Markov model.[26] Details are in Appendix_2. The logit of the analytical Markovian model for P_C (respectively P_E) appears in (A.4) (respectively (A.5)).

$$\boxed{\ln\left(\frac{P_C}{1-P_C}\right) = \ln c_{rr} - \ln u - \ln\left(\frac{w(1-c_{ww})}{2v_I + 3u} + \frac{M_x}{f \cdot v} + \frac{w+1}{M_y} \cdot \tau\right)} \quad \text{(A.4)}$$

[25] Kleijnen, Jack P.C., Sanchez, Susan M., Lucas, Thomas W., and Cioppa, Thomas M, "A User's Guide to the Brave New World of Designing Simulation Experiment," *INFORMS Journal on Computing* (2005)

[26] Gaver, Donald P. and Jacobs, Patricia A., "A Stochastic Modeling of a Variety of Simple MDA Situations," (June), *Naval Postgraduate School* (2005)

$$\ln\left(\frac{P_E}{1-P_E}\right) = \ln c_{rr} - \ln u + \ln\left(2v_I + 2u\right) - \ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right) \quad (A.5)$$

CASE-1:

Table 10 displays the 42 design points. These design points are used as input to the simulations. Design points (#1 – 34) are obtained from NOLH and design points (#35 – 42) are intentionally added to obtain higher probabilities for P_C and P_E. For each design point, 10^4=10,000 replications of the simulation are executed.

Design Point ID	Mx (NM)	W	u (kt)	tau (min)	Cww	Crr
1	128	50	26.3	5.5	0.93	0.99
2	66	13	27.5	6.3	0.90	0.86
3	81	22	11.3	5.0	0.96	0.96
4	97	31	16.3	8.0	0.96	0.83
5	238	47	18.8	4.5	0.93	0.80
6	300	16	17.5	7.3	0.91	0.95
7	206	9	30.0	5.3	0.99	0.89
8	191	44	25.0	7.8	0.98	0.93
9	175	25	20.0	6.0	0.95	0.90
10	222	0	13.8	6.5	0.98	0.81
11	284	38	12.5	5.8	1.00	0.94
12	269	28	28.8	7.0	0.94	0.84
13	253	19	23.8	4.0	0.94	0.98
14	113	3	21.3	7.5	0.97	1.00
15	50	34	22.5	4.8	0.99	0.85
16	144	41	10.0	6.8	0.91	0.91
17	159	6	15.0	4.3	0.92	0.88
18	300	41	17.5	5.0	0.99	0.91
19	113	44	21.3	4.0	0.93	0.93
20	159	3	15.0	6.5	0.98	1.00
21	206	16	30.0	6.3	0.91	0.95
22	284	22	12.5	5.3	0.90	0.96
23	128	19	26.3	4.3	0.98	0.98
24	97	50	16.3	7.5	0.94	0.99
25	269	38	28.8	7.3	0.96	0.94
26	175	25	20.0	6.0	0.95	0.90
27	50	9	22.5	7.0	0.91	0.89
28	238	6	18.8	8.0	0.97	0.88
29	191	47	25.0	5.5	0.92	0.80
30	144	34	10.0	5.8	0.99	0.85
31	66	28	27.5	6.8	1.00	0.84
32	222	31	13.8	7.8	0.93	0.83
33	253	0	23.8	4.5	0.96	0.81
34	81	13	11.3	4.8	0.94	0.86
35	50	0	10.0	4.0	1.00	1.00
36	50	1	10.0	4.0	1.00	1.00
37	50	0	12.5	4.0	1.00	1.00
38	50	1	12.5	4.0	1.00	1.00
39	75	0	10.0	4.0	1.00	1.00
40	75	1	10.0	4.0	1.00	1.00
41	75	0	12.5	4.0	1.00	1.00
42	75	1	12.5	4.0	1.00	1.00

Table 10. Design of experiment (CASE-1)

CASE-2:

Table 11 displays the 42 design points. These design points are used as input data sets for the simulation. Design points (#1 – 34) are obtained by NOLH and design points (#35 – 42) are intentionally added to obtain higher probabilities for P_C and P_E. For each design point, 10^4=10,000 replications of the simulation are executed.

Design Point ID	Mx (NM)	W	u (kt)	tau (min)	Cww	Crr	Beta
1	128	50	26.3	5.5	0.93	0.99	30
2	66	13	27.5	6.3	0.90	0.86	33
3	81	22	11.3	5.0	0.96	0.96	50
4	97	31	16.3	8.0	0.96	0.83	39
5	238	47	18.8	4.5	0.93	0.80	42
6	300	16	17.5	7.3	0.91	0.95	44
7	206	9	30.0	5.3	0.99	0.89	47
8	191	44	25.0	7.8	0.98	0.93	36
9	175	25	20.0	6.0	0.95	0.90	28
10	222	0	13.8	6.5	0.98	0.81	25
11	284	38	12.5	5.8	1.00	0.94	22
12	269	28	28.8	7.0	0.94	0.84	5
13	253	19	23.8	4.0	0.94	0.98	16
14	113	3	21.3	7.5	0.97	1.00	13
15	50	34	22.5	4.8	0.99	0.85	11
16	144	41	10.0	6.8	0.91	0.91	8
17	159	6	15.0	4.3	0.92	0.88	19
18	300	41	17.5	5.0	0.99	0.91	19
19	113	44	21.3	4.0	0.93	0.93	8
20	159	3	15.0	6.5	0.98	1.00	11
21	206	16	30.0	6.3	0.91	0.95	13
22	284	22	12.5	5.3	0.90	0.96	39
23	128	19	26.3	4.3	0.98	0.98	50
24	97	50	16.3	7.5	0.94	0.99	33
25	269	38	28.8	7.3	0.96	0.94	30
26	175	25	20.0	6.0	0.95	0.90	28
27	50	9	22.5	7.0	0.91	0.89	36
28	238	6	18.8	8.0	0.97	0.88	47
29	191	47	25.0	5.5	0.92	0.80	44
30	144	34	10.0	5.8	0.99	0.85	42
31	66	28	27.5	6.8	1.00	0.84	16
32	222	31	13.8	7.8	0.93	0.83	5
33	253	0	23.8	4.5	0.96	0.81	22
34	81	13	11.3	4.8	0.94	0.86	25
35	50	0	10.0	4.0	1.00	1.00	47
36	50	1	10.0	4.0	1.00	1.00	19
37	50	0	12.5	4.0	1.00	1.00	42
38	50	1	12.5	4.0	1.00	1.00	11
39	75	0	10.0	4.0	1.00	1.00	5
40	75	1	10.0	4.0	1.00	1.00	39
41	75	0	12.5	4.0	1.00	1.00	25
42	75	1	12.5	4.0	1.00	1.00	33

Table 11. Design of experiment (CASE-2)

F. DATA ANALYSIS

CASE-1:

1. Probability that the one R is detected and correctly classified before leaving the AOI, P_C

Output data:

The probability, P_C, is estimated as the fraction of replications for which an R is detected and correctly classified before leaving the AOI. Table 12 displays the outputs of the analytical Markov model (A.4) and those of the Exponential simulation model (all random times in the simulation are independent and have exponential distributions). Table 12 is sorted in descending order based on the outputs of the Exponential simulation model. The maximum value of P_C is estimated as about 0.9.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Analytical Markov model		Exponential simulation model			
													Pc			Logit
											Logit	Pc	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	2.1774	0.8982	0.9038	0.8980	0.9096	2.2402
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	2.1580	0.8964	0.8995	0.8936	0.9054	2.1917
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	1.9543	0.8759	0.8799	0.8735	0.8863	1.9915
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	1.9349	0.8738	0.8763	0.8698	0.8828	1.9578
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	1.7785	0.8555	0.8598	0.8530	0.8666	1.8136
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	1.7654	0.8539	0.8591	0.8523	0.8659	1.8078
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	1.5554	0.8257	0.8259	0.8185	0.8333	1.5568
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	1.5423	0.8238	0.8242	0.8167	0.8317	1.5451
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	0.9228	0.7156	0.7226	0.7138	0.7314	0.9574
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	0.9296	0.7170	0.7136	0.7047	0.7225	0.9129
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	0.6165	0.6494	0.6584	0.6491	0.6677	0.6562
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	0.5913	0.6437	0.6477	0.6383	0.6571	0.6089
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	0.5794	0.6409	0.6449	0.6355	0.6543	0.5967
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	0.5347	0.6306	0.6286	0.6191	0.6381	0.5262
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	0.4914	0.6204	0.6270	0.6175	0.6365	0.5194
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	0.3443	0.5852	0.5888	0.5792	0.5984	0.3590
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	0.1665	0.5415	0.5464	0.5366	0.5562	0.1861
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	0.1758	0.5438	0.5438	0.5340	0.5536	0.1757
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	0.0862	0.5215	0.5209	0.5111	0.5307	0.0836
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	0.0356	0.5089	0.5187	0.5089	0.5285	0.0748
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	0.0766	0.5191	0.5176	0.5078	0.5274	0.0704
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	0.0237	0.5059	0.5117	0.5019	0.5215	0.0468
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	-0.0063	0.4984	0.5084	0.4986	0.5182	0.0336
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	-0.1470	0.4633	0.4711	0.4613	0.4809	-0.1157
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	-0.1854	0.4538	0.4527	0.4429	0.4625	-0.1898
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	-0.2065	0.4486	0.4509	0.4411	0.4607	-0.1970
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.2533	0.4370	0.4425	0.4328	0.4522	-0.2310
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.2533	0.4370	0.4314	0.4217	0.4411	-0.2761
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	-0.3174	0.4213	0.4301	0.4204	0.4398	-0.2814
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	-0.3412	0.4155	0.4218	0.4121	0.4315	-0.3154
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	-0.4643	0.3860	0.3939	0.3843	0.4035	-0.4309
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	-0.4552	0.3881	0.3904	0.3808	0.4000	-0.4456
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	-0.5064	0.3760	0.3819	0.3724	0.3914	-0.4815
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	-0.5084	0.3756	0.3818	0.3723	0.3913	-0.4819
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	-0.5212	0.3726	0.3763	0.3668	0.3858	-0.5053
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	-0.6092	0.3522	0.3552	0.3458	0.3646	-0.5963
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	-0.6100	0.3521	0.3541	0.3447	0.3635	-0.6011
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	-0.6610	0.3405	0.3413	0.3320	0.3506	-0.6575
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	-0.7835	0.3136	0.3210	0.3118	0.3302	-0.7492
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	-0.9686	0.2752	0.2776	0.2688	0.2864	-0.9564
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	-0.9765	0.2736	0.2753	0.2665	0.2841	-0.9679
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	-1.0401	0.2611	0.2626	0.2540	0.2712	-1.0325

Table 12. Analytical Markov model (A.4) and Exponential simulation model

Figure 25 and Figure 26 display the outputs of the analytical Markov model (A.4) and those of the Exponential simulation model.

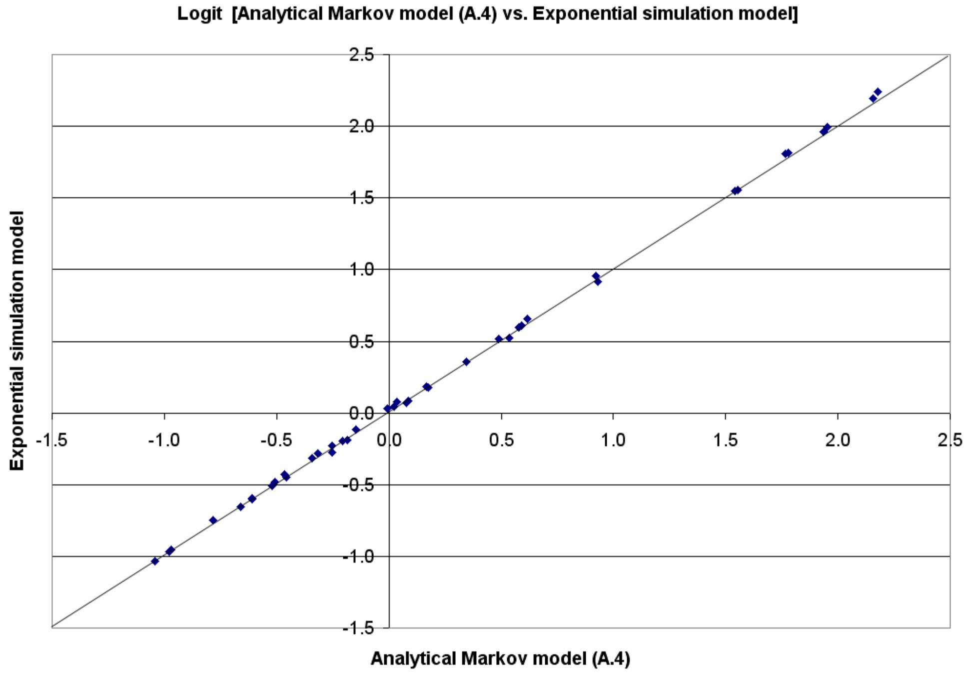

Figure 25. Logit [Analytical Markov model (A.4) vs. Exponential simulation model]

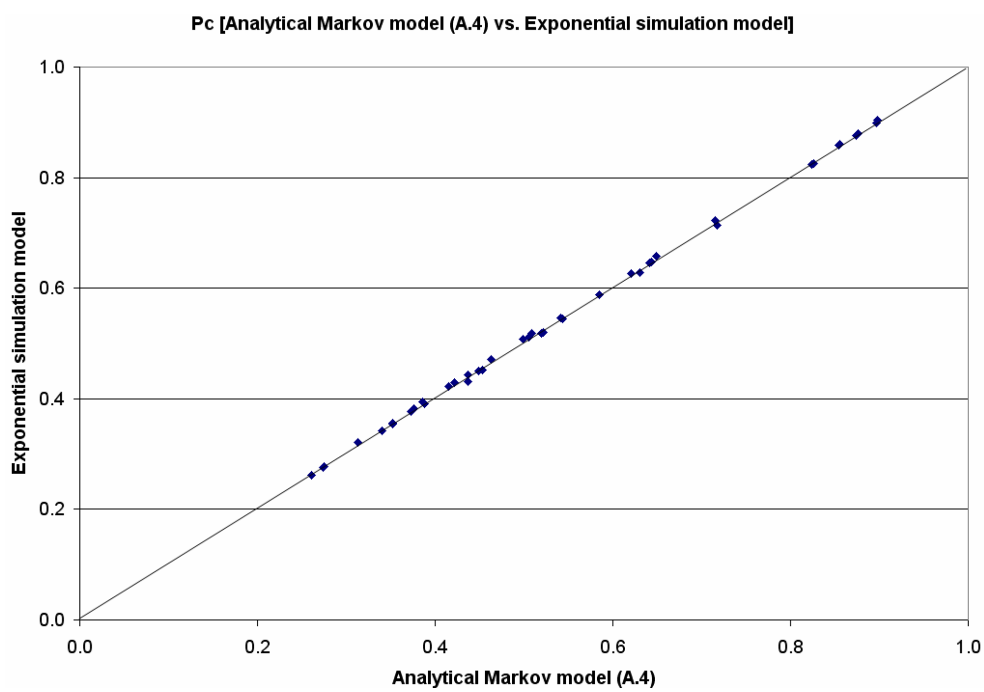

Figure 26. P_C [Analytical Markov model (A.4) vs. Exponential simulation model]

Logistic regression model:

We consider the following regression model based on (A.4)

$$\ln\left(\frac{P_C}{1-P_C}\right) = b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln\left(\frac{w(1-c_{ww})}{2v_I + 3u} + \frac{M_x}{f \cdot v} + \frac{(w+1)\tau}{M_y}\right) \tag{A.6}$$

- When $b_0 \neq 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	1.000
Adjusted R Square	1.000
Standard Error	0.018
Observations	42

ANOVA

	df	SS	MS	F	Significance F
Regression	3	35.681	11.894	38590	0.000
Residual	38	0.012	0.000		
Total	41	35.693			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.040	0.032	1.234	0.225	-0.026	0.106
b1	1.033	0.041	25.268	0.000	0.950	1.116
b2	-1.009	0.008	-122.202	0.000	-1.026	-0.993
b3	-1.003	0.005	-208.788	0.000	-1.012	-0.993

Since the p-value of b_0 is 0.225>0.05 and the 90% confidence interval contains 0. We next treat b_0 as 0.

- When $b_0 = 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	1.000
Adjusted R Square	0.974
Standard Error	0.018
Observations	42

ANOVA

	df	SS	MS	F	Significance F
Regression	3	38.775	12.925	41383	0.000
Residual	39	0.012	0.000		
Total	42	38.787			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.000	-	-	-	-	-
b1	1.025	0.041	25.224	0.000	0.943	1.107
b2	-1.001	0.004	-237.084	0.000	-1.009	-0.992
b3	-1.007	0.003	-313.484	0.000	-1.013	-1.000

Thus, the estimated logistic regression model is

$$\text{Estimated } \ln\left(\frac{P_C}{1-P_C}\right) = 1.025\ln c_{rr} - 1.001\ln u - 1.007\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\cdot\tau}{M_y}\right) \quad \text{(A.7)}$$

Since the estimated coefficients of the independent random variables are not statistically different than those of the Analytical Markov model (A.4), the estimated logistic regression model (A.7) is not statistically different from the Analytical Markov model (A.4). Table 13 displays the outputs of the estimated regression model (A.7) and those of the Exponential simulation model. Table 13 is sorted in descending order based on the outputs of the Exponential simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Estimated regresion model		Exponential simulation model			
													Pc			Logit
											E.Logit	E.Pc	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	2.2070	0.9009	0.9038	0.8980	0.9096	2.2402
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	2.1875	0.8991	0.8995	0.8936	0.9054	2.1917
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	1.9837	0.8791	0.8799	0.8735	0.8863	1.9915
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	1.9642	0.8770	0.8763	0.8698	0.8828	1.9578
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	1.8054	0.8588	0.8598	0.8530	0.8666	1.8136
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	1.7922	0.8572	0.8591	0.8523	0.8659	1.8078
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	1.5821	0.8295	0.8259	0.8185	0.8333	1.5568
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	1.5689	0.8276	0.8242	0.8167	0.8317	1.5451
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	0.9438	0.7199	0.7226	0.7138	0.7314	0.9574
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	0.9486	0.7208	0.7136	0.7047	0.7225	0.9129
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	0.6383	0.6544	0.6584	0.6491	0.6677	0.6562
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	0.6070	0.6473	0.6477	0.6383	0.6571	0.6089
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	0.6005	0.6458	0.6449	0.6355	0.6543	0.5967
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	0.5577	0.6359	0.6286	0.6191	0.6381	0.5262
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	0.5115	0.6252	0.6270	0.6175	0.6365	0.5194
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	0.3615	0.5894	0.5888	0.5792	0.5984	0.3590
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	0.1804	0.5450	0.5464	0.5366	0.5562	0.1861
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	0.1947	0.5485	0.5438	0.5340	0.5536	0.1757
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	0.1050	0.5262	0.5209	0.5111	0.5307	0.0836
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	0.0561	0.5140	0.5187	0.5089	0.5285	0.0748
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	0.0914	0.5228	0.5176	0.5078	0.5274	0.0704
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	0.0367	0.5092	0.5117	0.5019	0.5215	0.0468
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	0.0084	0.5021	0.5084	0.4986	0.5182	0.0336
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	-0.1306	0.4674	0.4711	0.4613	0.4809	-0.1157
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	-0.1715	0.4572	0.4527	0.4429	0.4625	-0.1898
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	-0.1917	0.4522	0.4509	0.4411	0.4607	-0.1970
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.2381	0.4408	0.4425	0.4328	0.4522	-0.2310
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.2381	0.4408	0.4314	0.4217	0.4411	-0.2761
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	-0.3016	0.4252	0.4301	0.4204	0.4398	-0.2814
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	-0.3303	0.4182	0.4218	0.4121	0.4315	-0.3154
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	-0.4504	0.3893	0.3939	0.3843	0.4035	-0.4309
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	-0.4420	0.3913	0.3904	0.3808	0.4000	-0.4456
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	-0.4937	0.3790	0.3819	0.3724	0.3914	-0.4815
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	-0.4925	0.3793	0.3818	0.3723	0.3913	-0.4819
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	-0.5052	0.3763	0.3763	0.3668	0.3858	-0.5053
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	-0.5930	0.3559	0.3552	0.3458	0.3646	-0.5963
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	-0.5952	0.3554	0.3541	0.3447	0.3635	-0.6011
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	-0.6450	0.3441	0.3413	0.3320	0.3506	-0.6575
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	-0.7744	0.3155	0.3210	0.3118	0.3302	-0.7492
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	-0.9552	0.2778	0.2776	0.2688	0.2864	-0.9564
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	-0.9669	0.2755	0.2753	0.2665	0.2841	-0.9679
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	-1.0293	0.2632	0.2626	0.2540	0.2712	-1.0325

Table 13. Estimated model (A.7) and Exponential simulation model

Figures 27 and Figure 28 display the outputs of the estimated regression model (A.7) and those of the Exponential simulation model. The estimated regression model (A.7) is not statistically different from the Exponential simulation model.

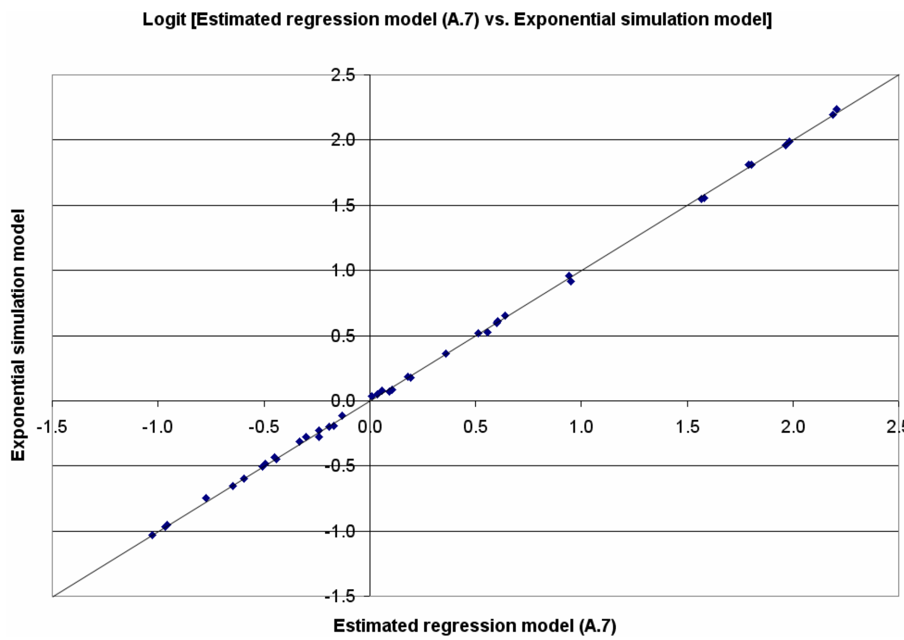

Figure 27. Logit [Estimated model (A.7) vs. Exponential simulation model]

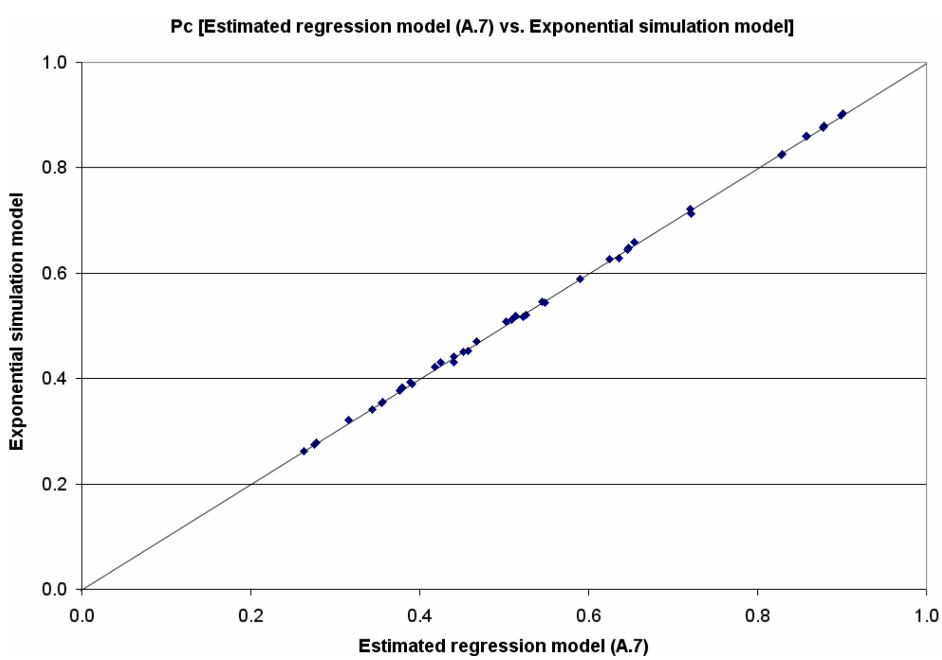

Figure 28. P_C [Estimated model (A.7) vs. Exponential simulation model]

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

Output data:

The probability, P_E, is estimated as the fraction of replications for which an R is detected, correctly classified, and escorted before leaving the AOI. Table 14 displays the outputs of the analytical Markov model (A.5) and those of the Exponential simulation model (all random times in the simulation are independent and have exponential distributions). Table 14 is sorted in descending order based on the outputs of the Exponential simulation model. The maximum value of P_E is estimated as about 0.8.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Analytical Markov model		Exponential simulation model			
													P_E			Logit
											Logit	P_E	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	1.3763	0.7984	0.8056	0.7978	0.8134	1.4217
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	1.3665	0.7968	0.8048	0.7970	0.8126	1.4166
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	1.1726	0.7636	0.7708	0.7626	0.7790	1.2128
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	1.1552	0.7605	0.7660	0.7577	0.7743	1.1859
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	1.1473	0.7590	0.7657	0.7574	0.7740	1.1842
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	1.1624	0.7618	0.7637	0.7554	0.7720	1.1731
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	0.9436	0.7198	0.7260	0.7173	0.7347	0.9744
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	0.9355	0.7182	0.7192	0.7104	0.7280	0.9405
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	0.5301	0.6295	0.6336	0.6242	0.6430	0.5477
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	0.5353	0.6307	0.6302	0.6207	0.6397	0.5331
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	0.2906	0.5721	0.5755	0.5658	0.5852	0.3043
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	0.1981	0.5494	0.5570	0.5473	0.5667	0.2290
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	0.1395	0.5348	0.5447	0.5349	0.5545	0.1793
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	0.0888	0.5222	0.5251	0.5153	0.5349	0.1005
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	0.0438	0.5109	0.5148	0.5050	0.5246	0.0592
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	0.0065	0.5016	0.5086	0.4988	0.5184	0.0344
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	-0.1289	0.4678	0.4726	0.4628	0.4824	-0.1097
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	-0.2018	0.4497	0.4526	0.4428	0.4624	-0.1902
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	-0.2457	0.4389	0.4412	0.4315	0.4509	-0.2363
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	-0.2634	0.4345	0.4408	0.4311	0.4505	-0.2379
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	-0.2353	0.4414	0.4358	0.4261	0.4455	-0.2582
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	-0.3191	0.4209	0.4253	0.4156	0.4350	-0.3011
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	-0.3534	0.4125	0.4190	0.4093	0.4287	-0.3269
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	-0.4307	0.3940	0.4010	0.3914	0.4106	-0.4013
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	-0.4238	0.3956	0.3943	0.3847	0.4039	-0.4293
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	-0.5061	0.3761	0.3793	0.3698	0.3888	-0.4925
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.5573	0.3642	0.3736	0.3641	0.3831	-0.5168
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	-0.5798	0.3590	0.3649	0.3555	0.3743	-0.5542
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.5573	0.3642	0.3614	0.3520	0.3708	-0.5693
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	-0.6239	0.3489	0.3568	0.3474	0.3662	-0.5893
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	-0.7266	0.3259	0.3333	0.3241	0.3425	-0.6933
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	-0.7184	0.3277	0.3309	0.3217	0.3401	-0.7041
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	-0.8098	0.3079	0.3179	0.3088	0.3270	-0.7634
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	-0.8232	0.3051	0.3072	0.2982	0.3162	-0.8132
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	-0.8451	0.3005	0.3061	0.2971	0.3151	-0.8184
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	-0.9107	0.2869	0.2915	0.2826	0.3004	-0.8881
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	-0.9171	0.2855	0.2874	0.2785	0.2963	-0.9080
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	-0.9824	0.2724	0.2763	0.2675	0.2851	-0.9629
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	-1.0308	0.2629	0.2663	0.2576	0.2750	-1.0135
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	-1.2597	0.2210	0.2232	0.2150	0.2314	-1.2471
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	-1.2487	0.2229	0.2232	0.2150	0.2314	-1.2471
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	-1.3264	0.2098	0.2123	0.2043	0.2203	-1.3111

Table 14. Analytical Markov model (A.5) and Exponential simulation model

Figure 29 and Figure 30 display the outputs of the analytical Markov model (A.5) and those of the Exponential simulation model.

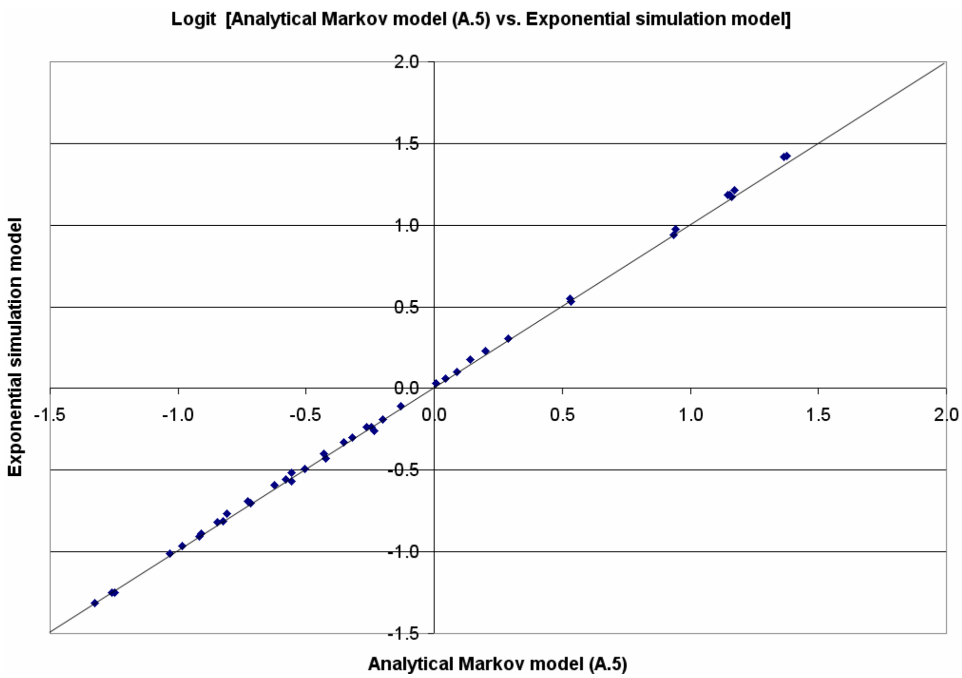

Figure 29. Logit [Analytical Markov model (A.5) vs. Exponential simulation model]

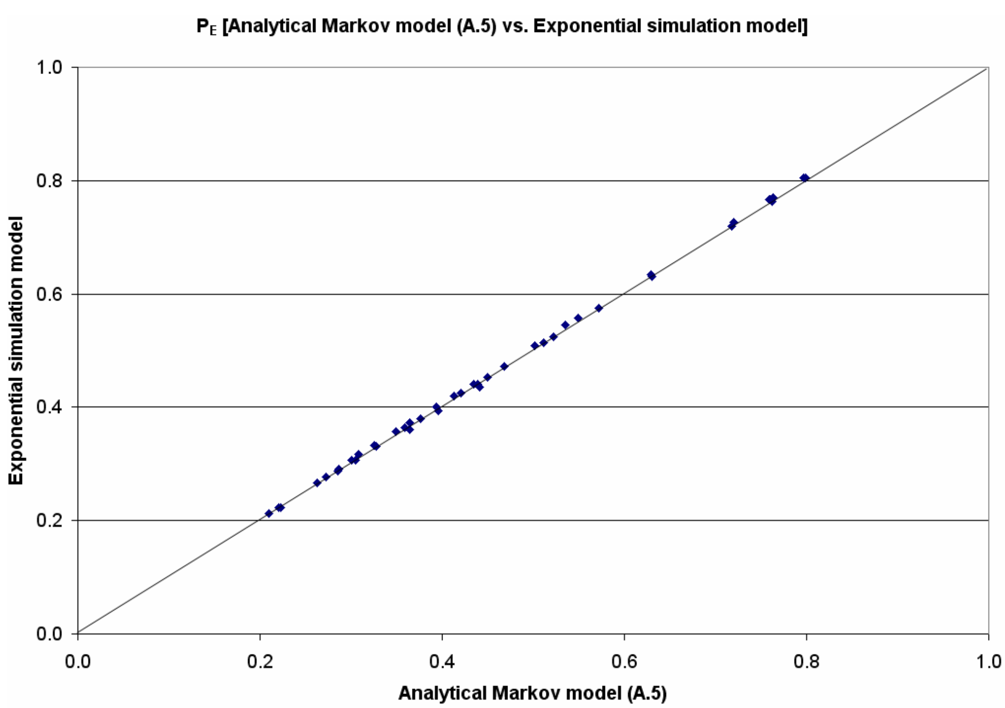

Figure 30. P_E [Analytical Markov model (A.5) vs. Exponential simulation model]

Logistic regression model:

We consider the following regression model based on (A.5)

$$\boxed{\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln\left(2v_I + 2u\right) + \\ & b_4 \ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f \cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)\end{aligned}} \quad (A.8)$$

- When $b_0 \neq 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	1.000
Adjusted R Square	1.000
Standard Error	0.015
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	25.215	6.304	26306	0.000
Residual	37	0.009	0.000		
Total	41	25.223			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	-0.240	0.713	-0.337	0.738	-1.684	1.204
b1	1.039	0.036	29.222	0.000	0.967	1.111
b2	-1.025	0.075	-13.621	0.000	-1.177	-0.872
b3	1.076	0.202	5.317	0.000	0.666	1.486
b4	-1.007	0.005	-183.835	0.000	-1.018	-0.996

Since the p-value of b_0 is 0.738>0.05 and the 90% confidence interval contains 0. We next treat b_0 as 0.

- When $b_0 = 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	1.000
Adjusted R Square	0.973
Standard Error	0.015
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	25.831	6.458	27592	0.000
Residual	38	0.009	0.000		
Total	42	25.839			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	0.000	-	-	-	-	-
b1	1.040	0.035	29.719	0.000	0.970	1.111
b2	-0.999	0.010	-101.175	0.000	-1.019	-0.979
b3	1.008	0.005	185.661	0.000	0.997	1.019
b4	-1.007	0.005	-188.524	0.000	-1.018	-0.996

Thus, the estimated logistic regression model is

$$
\begin{aligned}
&\text{Estimated } \ln\left(\frac{P_E}{1-P_E}\right) \\
&\quad = 1.040\ln c_{rr} - 0.999\ln u + 1.008\ln\left(2v_I + 2u\right) \\
&\qquad - 1.007\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)
\end{aligned}
\tag{A.9}
$$

Since the estimated coefficients of the independent variables are not statistically different from those of the Analytical Markov model (A.5), the estimated regression model (A.9) is not statistically different from the Analytical Markov model (A.5). Table 15 displays the outputs of the estimated regression model (A.9) and those of the Exponential simulation model. Table 15 is sorted in descending order based on the outputs of the Exponential simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Estimated regresion model E.Logit	E.P_E	Exponential simulation model P_E Mean	95%LB	95%UB	Logit
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	1.4060	0.8031	0.8056	0.7978	0.8134	1.4217
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	1.3961	0.8016	0.8048	0.7970	0.8126	1.4166
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	1.2025	0.7690	0.7708	0.7626	0.7790	1.2128
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	1.1833	0.7655	0.7660	0.7577	0.7743	1.1859
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	1.1753	0.7641	0.7657	0.7574	0.7740	1.1842
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	1.1922	0.7671	0.7637	0.7554	0.7720	1.1731
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	0.9719	0.7255	0.7260	0.7173	0.7347	0.9744
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	0.9637	0.7239	0.7192	0.7104	0.7280	0.9405
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	0.5533	0.6349	0.6336	0.6242	0.6430	0.5477
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	0.5548	0.6353	0.6302	0.6207	0.6397	0.5331
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	0.3071	0.5762	0.5755	0.5658	0.5852	0.3043
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	0.2225	0.5554	0.5570	0.5473	0.5667	0.2290
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	0.1628	0.5406	0.5447	0.5349	0.5545	0.1793
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	0.1151	0.5287	0.5251	0.5153	0.5349	0.1005
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	0.0649	0.5162	0.5148	0.5050	0.5246	0.0592
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	0.0253	0.5063	0.5086	0.4988	0.5184	0.0344
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	-0.1145	0.4714	0.4726	0.4628	0.4824	-0.1097
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	-0.1866	0.4535	0.4526	0.4428	0.4624	-0.1902
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	-0.2255	0.4439	0.4412	0.4315	0.4509	-0.2363
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	-0.2458	0.4388	0.4408	0.4311	0.4505	-0.2379
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	-0.2196	0.4453	0.4358	0.4261	0.4455	-0.2582
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	-0.2987	0.4259	0.4253	0.4156	0.4350	-0.3011
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	-0.3293	0.4184	0.4190	0.4093	0.4287	-0.3269
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	-0.4105	0.3988	0.4010	0.3914	0.4106	-0.4013
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	-0.4067	0.3997	0.3943	0.3847	0.4039	-0.4293
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	-0.4893	0.3801	0.3793	0.3698	0.3888	-0.4925
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.5396	0.3683	0.3736	0.3641	0.3831	-0.5168
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	-0.5679	0.3617	0.3649	0.3555	0.3743	-0.5542
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	-0.5396	0.3683	0.3614	0.3520	0.3708	-0.5693
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	-0.6051	0.3532	0.3568	0.3474	0.3662	-0.5893
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	-0.7094	0.3297	0.3333	0.3241	0.3425	-0.6933
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	-0.7025	0.3313	0.3309	0.3217	0.3401	-0.7041
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	-0.7960	0.3109	0.3179	0.3088	0.3270	-0.7634
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	-0.8031	0.3094	0.3072	0.2982	0.3162	-0.8132
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	-0.8266	0.3044	0.3061	0.2971	0.3151	-0.8184
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	-0.8926	0.2906	0.2915	0.2826	0.3004	-0.8881
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	-0.8967	0.2897	0.2874	0.2785	0.2963	-0.9080
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	-0.9627	0.2763	0.2763	0.2675	0.2851	-0.9629
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	-1.0209	0.2649	0.2663	0.2576	0.2750	-1.0135
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	-1.2426	0.2240	0.2232	0.2150	0.2314	-1.2471
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	-1.2381	0.2248	0.2232	0.2150	0.2314	-1.2471
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	-1.3136	0.2119	0.2123	0.2043	0.2203	-1.3111

Table 15. Estimated model (A.9) and Exponential simulation model

Figures 31 and Figure 32 display the outputs of the estimated regression model (A.9) and those of the Exponential simulation model.

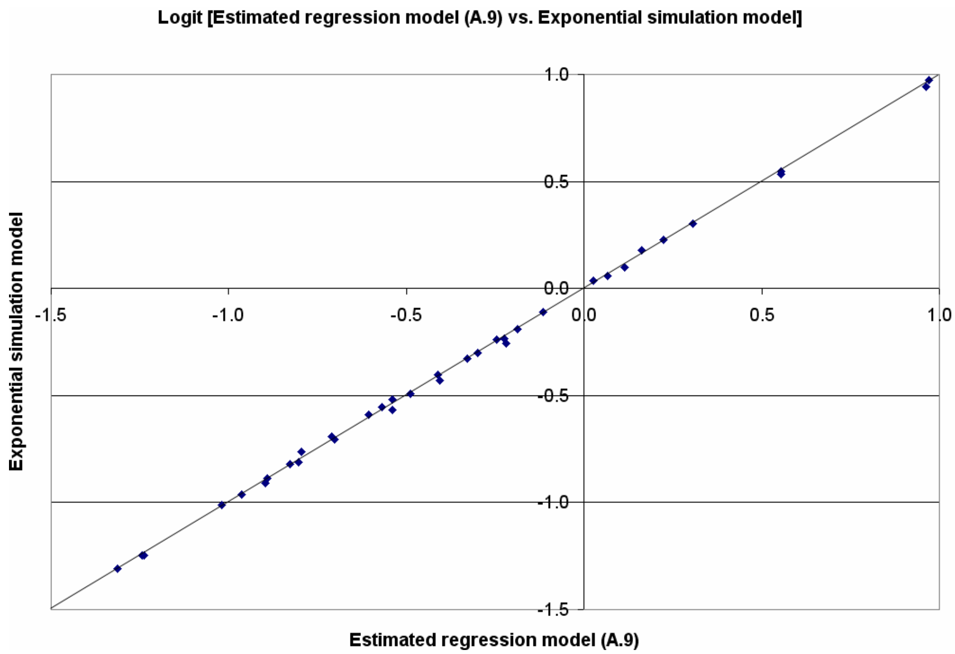

Figure 31. Logit [Estimated model (A.9) vs. Exponential simulation model]

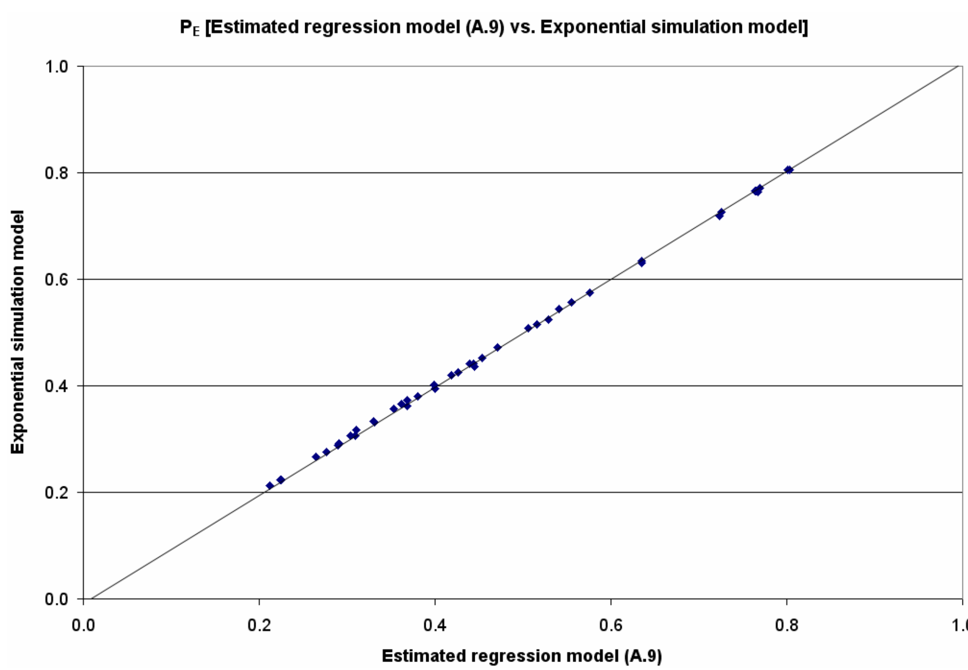

Figure 32. P_E [Estimated model (A.9) vs. Exponential simulation model]

CASE-2:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape β (=5 to 50)
- F_D is a Gamma distribution with mean $1/\phi$ and shape β which is same as that of F_U. (U and D are statistically independent.)
- The MPA has no memory of its previous classification information (pessimistic assumption)

Input parameters:

- Constant parameters

  $M_y = 300NM,\ \ v = 300kt,\ \ v_I = 30kt,\ \ f = 15NM$

- Multi-level parameters

Table 16 shows the specified range of the seven multi-level parameters/factors respectively. β is the common shape parameter for the Gamma distributions F_U (distribution of the time an un-encountered R spends in the AOI) and F_D (distribution of the time for the MPA to track a suspicious vessel).

	Low	High
M_x	50NM	300NM
w	0	50
u	10kt	30kt
τ	4min	8min
c_{ww}	0.9	1.0
c_{rr}	0.8	1.0
β	5	50

Table 16. Multi-level parameters

1. Probability that the one R is detected and correctly classified before leaving the AOI, P_C

Output data:

The probability, P_C, is estimated as the fraction of replications for which an R is detected and correctly classified before leaving the AOI. Table 17 displays the outputs of the Gamma simulation model. As a reference, the outputs of the analytical Markov model (A.4) (F_U and F_D follow exponential distribution) are also shown. Table 17 is sorted in descending order based on the outputs of the Gamma simulation model. The maximum value of P_C is estimated as almost 1.0.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Beta	Analytical Markov model		Gamma simulation model			
														Pc			Logit
												Logit	Pc	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	47	2.1774	0.8982	0.9998	0.9995	1.0001	8.5170
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	19	2.1580	0.8964	0.9994	0.9989	0.9999	7.4180
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	42	1.9543	0.8759	0.9992	0.9986	0.9998	7.1301
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	39	1.7654	0.8539	0.9968	0.9957	0.9979	5.7414
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	11	1.9349	0.8738	0.9954	0.9941	0.9967	5.3771
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	25	1.5554	0.8257	0.9886	0.9865	0.9907	4.4627
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	33	1.5423	0.8238	0.9882	0.9861	0.9903	4.4278
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	5	1.7785	0.8555	0.9809	0.9782	0.9836	3.9388
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	50	0.9228	0.7156	0.9020	0.8962	0.9078	2.2196
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	25	0.9296	0.7170	0.8996	0.8937	0.9055	2.1928
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	42	0.5913	0.6437	0.8274	0.8200	0.8348	1.5673
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	36	0.6165	0.6494	0.8164	0.8088	0.8240	1.4921
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	11	0.5794	0.6409	0.8110	0.8033	0.8187	1.4565
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	13	0.5347	0.6306	0.7984	0.7905	0.8063	1.3763
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	11	0.4914	0.6204	0.7713	0.7631	0.7795	1.2157
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	19	0.3443	0.5852	0.7392	0.7306	0.7478	1.0418
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	25	0.1665	0.5415	0.6934	0.6844	0.7024	0.8161
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	16	0.1758	0.5438	0.6896	0.6805	0.6987	0.7982
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	39	0.0766	0.5191	0.6385	0.6291	0.6479	0.5689
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	33	0.0862	0.5215	0.6328	0.6234	0.6422	0.5442
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	50	0.0356	0.5089	0.6299	0.6204	0.6394	0.5318
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	22	-0.0063	0.4984	0.6189	0.6094	0.6284	0.4849
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	8	0.0237	0.5059	0.6126	0.6031	0.6221	0.4583
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	47	-0.2065	0.4486	0.5578	0.5481	0.5675	0.2322
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	39	-0.1854	0.4538	0.5551	0.5454	0.5648	0.2213
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	33	-0.1470	0.4633	0.5514	0.5417	0.5611	0.2063
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.2533	0.4370	0.5216	0.5118	0.5314	0.0865
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.2533	0.4370	0.5201	0.5103	0.5299	0.0804
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	8	-0.3174	0.4213	0.4884	0.4786	0.4982	-0.0464
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	5	-0.3412	0.4155	0.4804	0.4706	0.4902	-0.0784
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	19	-0.4552	0.3881	0.4604	0.4506	0.4702	-0.1587
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	44	-0.4643	0.3860	0.4587	0.4489	0.4685	-0.1656
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	47	-0.5084	0.3756	0.4520	0.4422	0.4618	-0.1926
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	22	-0.5064	0.3760	0.4505	0.4407	0.4603	-0.1987
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	16	-0.5212	0.3726	0.4378	0.4281	0.4475	-0.2501
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	36	-0.6100	0.3521	0.4170	0.4073	0.4267	-0.3351
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	30	-0.6092	0.3522	0.3973	0.3877	0.4069	-0.4167
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	13	-0.6610	0.3405	0.3932	0.3836	0.4028	-0.4339
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	42	-0.7835	0.3136	0.3482	0.3389	0.3575	-0.6270
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	30	-0.9686	0.2752	0.3114	0.3023	0.3205	-0.7936
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	44	-0.9765	0.2736	0.3048	0.2958	0.3138	-0.8245
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	5	-1.0401	0.2611	0.2900	0.2811	0.2989	-0.8954

Table 17. Analytical Markov model (A.4) and Gamma simulation model

Figure 33 and Figure 34 display the outputs of the analytical Markov model (A.4) and those of the Gamma simulation model.

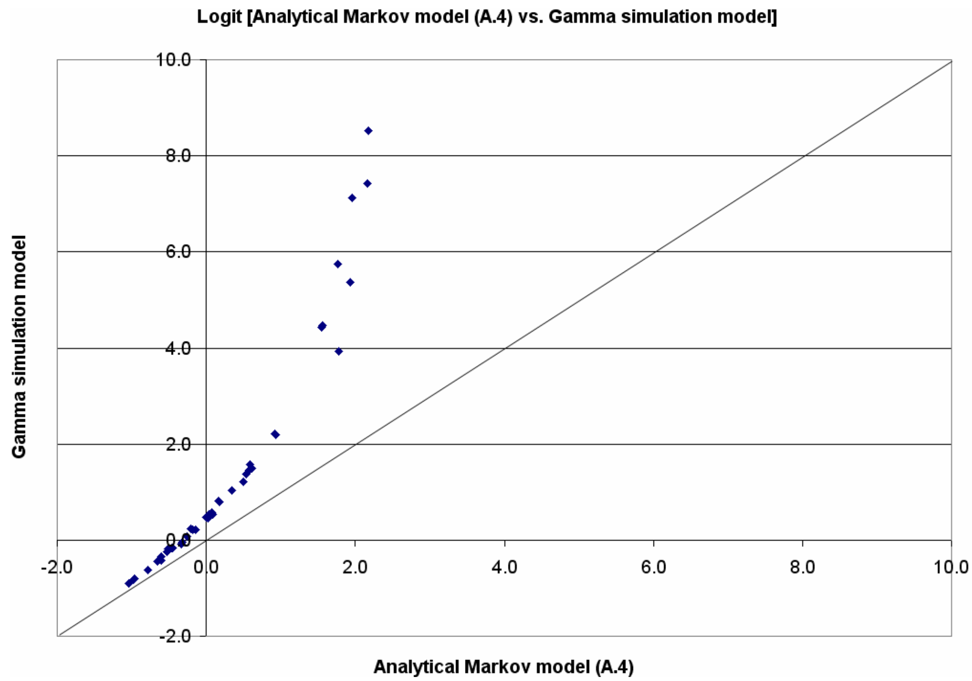

Figure 33. Logit [Analytical Markov model (A.4) vs. Gamma simulation model]

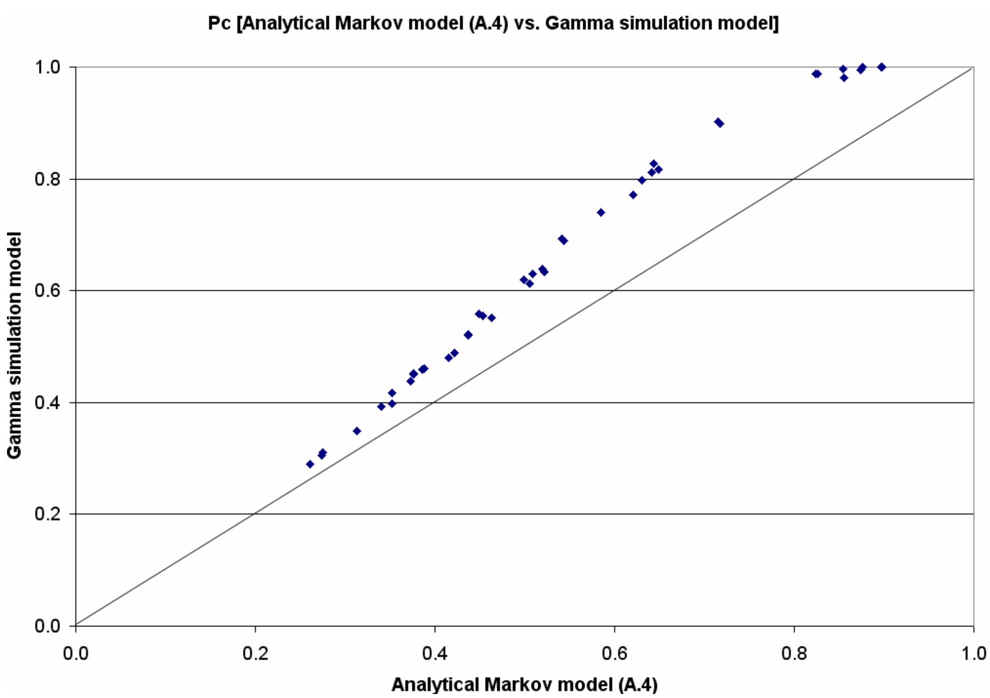

Figure 34. P_C [Analytical Markov model (A.4) vs. Gamma simulation model]

Logistic regression model:

We consider the following regression model based on (A.4).

$$\boxed{\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln \beta \\ &\quad + b_4 \ln\left(\frac{w(1-c_{ww})}{2v_I + 3u} + \frac{M_x}{f \cdot v} + \frac{(w+1)\tau}{M_y}\right)\end{aligned}} \quad \text{(A.10)}$$

- When $b_0 \neq 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.960
R Square	0.922
Adjusted R Square	0.913
Standard Error	0.711
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	220.163	55.041	109	0.000
Residual	37	18.730	0.506		
Total	41	238.893			

	Coefficients	*Standard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	-0.575	1.396	-0.412	0.683	-3.403	2.254
b1	3.932	1.657	2.374	0.023	0.576	7.289
b2	-2.244	0.335	-6.700	0.000	-2.923	-1.565
b3	0.237	0.166	1.431	0.161	-0.099	0.574
b4	-2.501	0.195	-12.843	0.000	-2.895	-2.106

Since the p-value of b_0 is 0.683>0.05 and the 90% confidence interval contains 0, we next treat b_0 as 0.

- When $b_0 = 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.970
R Square	0.942
Adjusted R Square	0.911
Standard Error	0.704
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	303.486	75.871	153	0.000
Residual	38	18.816	0.495		
Total	42	322.301			

	Coefficients	*Standard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	0.000	-	-	-	-	-
b1	4.035	1.620	2.492	0.017	0.757	7.314
b2	-2.354	0.199	-11.841	0.000	-2.757	-1.952
b3	0.215	0.155	1.388	0.173	-0.098	0.528
b4	-2.445	0.139	-17.594	0.000	-2.726	-2.164

Although the p-value of b_3 is 0.173>0.05 and the 90% confidence interval contains 0, here we hold the variable $\ln\beta$ in the regression model. The estimated logistic regression model is

$$\boxed{\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= 4.035\ln c_{rr} - 2.354\ln u + 0.215\ln\beta \\ &\quad - 2.445\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\end{aligned}} \tag{A.11}$$

Table 18 displays the outputs of the estimated regression model (A.11) and those of the Gamma simulation model. Table 18 is sorted in descending order based on the outputs of the Gamma simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Beta	Estimated regression model		Gamma simulation model			
														Pc			Logit
												E.Logit	E.Pc	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	47	6.3588	0.9983	0.9998	0.9995	1.0001	8.5170
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	19	6.1170	0.9978	0.9994	0.9989	0.9999	7.4180
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	42	5.8093	0.9970	0.9992	0.9986	0.9998	7.1301
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	39	5.3115	0.9951	0.9968	0.9957	0.9979	5.7414
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	11	5.4744	0.9958	0.9954	0.9941	0.9967	5.3771
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	25	4.7226	0.9912	0.9886	0.9865	0.9907	4.4627
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	33	4.7502	0.9914	0.9882	0.9861	0.9903	4.4278
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	5	4.9027	0.9926	0.9809	0.9782	0.9836	3.9388
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	50	3.2507	0.9627	0.9020	0.8962	0.9078	2.2196
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	25	2.9436	0.9500	0.8996	0.8937	0.9055	2.1928
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	42	2.1981	0.9001	0.8274	0.8200	0.8348	1.5673
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	36	2.3732	0.9148	0.8164	0.8088	0.8240	1.4921
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	11	2.1767	0.8981	0.8110	0.8033	0.8187	1.4565
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	13	2.1350	0.8943	0.7984	0.7905	0.8063	1.3763
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	11	1.7398	0.8507	0.7713	0.7631	0.7795	1.2157
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	19	1.5159	0.8199	0.7392	0.7306	0.7478	1.0418
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	25	1.0005	0.7312	0.6934	0.6844	0.7024	0.8161
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	16	1.0478	0.7404	0.6896	0.6805	0.6987	0.7982
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	39	0.9301	0.7171	0.6385	0.6291	0.6479	0.5689
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	33	1.0216	0.7353	0.6328	0.6234	0.6422	0.5442
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	50	1.1907	0.7669	0.6299	0.6204	0.6394	0.5318
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	22	0.7783	0.6853	0.6189	0.6094	0.6284	0.4849
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	8	0.5629	0.6371	0.6126	0.6031	0.6221	0.4583
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	47	0.3838	0.5948	0.5578	0.5481	0.5675	0.2322
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	39	0.4967	0.6217	0.5551	0.5454	0.5648	0.2213
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	33	0.6278	0.6520	0.5514	0.5417	0.5611	0.2063
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	0.1996	0.5497	0.5216	0.5118	0.5314	0.0865
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	0.1996	0.5497	0.5201	0.5103	0.5299	0.0804
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	8	-0.1679	0.4581	0.4884	0.4786	0.4982	-0.0464
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	5	-0.5473	0.3665	0.4804	0.4706	0.4902	-0.0784
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	19	-0.3718	0.4081	0.4604	0.4506	0.4702	-0.1587
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	44	-0.1454	0.4637	0.4587	0.4489	0.4685	-0.1656
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	47	-0.2939	0.4271	0.4520	0.4422	0.4618	-0.1926
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	22	-0.6227	0.3492	0.4505	0.4407	0.4603	-0.1987
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	16	-0.4243	0.3955	0.4378	0.4281	0.4475	-0.2501
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	36	-0.5462	0.3667	0.4170	0.4073	0.4267	-0.3351
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	30	-0.4795	0.3824	0.3973	0.3877	0.4069	-0.4167
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	13	-0.8390	0.3017	0.3932	0.3836	0.4028	-0.4339
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	42	-1.2027	0.2310	0.3482	0.3389	0.3575	-0.6270
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	30	-1.4323	0.1927	0.3114	0.3023	0.3205	-0.7936
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	44	-1.6387	0.1626	0.3048	0.2958	0.3138	-0.8245
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	5	-2.1706	0.1024	0.2900	0.2811	0.2989	-0.8954

Table 18. Estimated regression model (A.11) and Gamma simulation model

Figure 35 and Figure 36 display the outputs of the estimated regression model (A.11) and those of the Gamma simulation model.

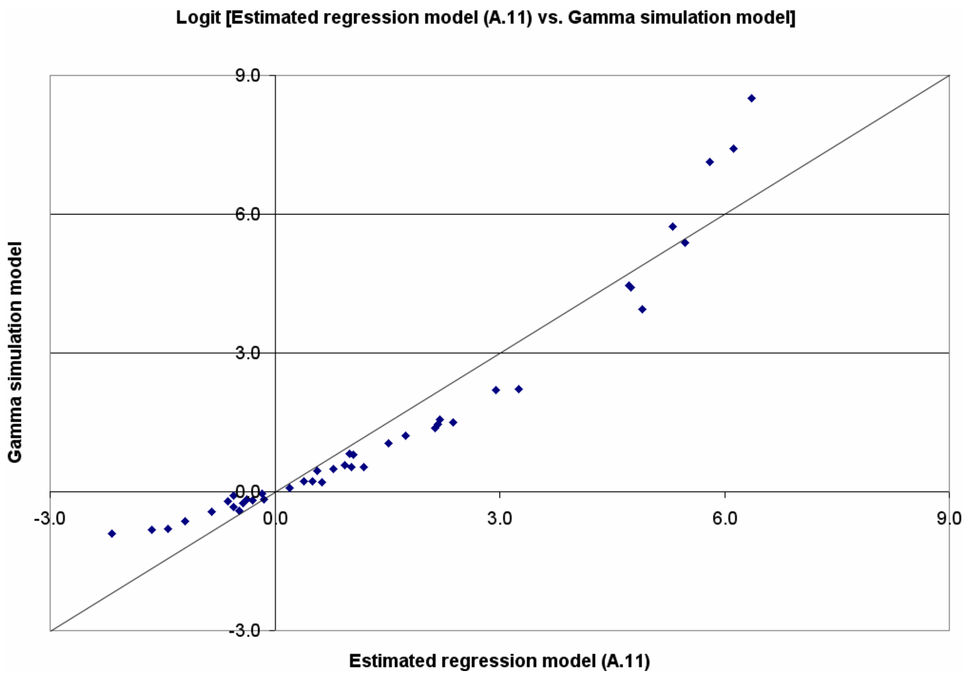

Figure 35. Logit [Estimated regression model (A.11) vs. Gamma simulation model]

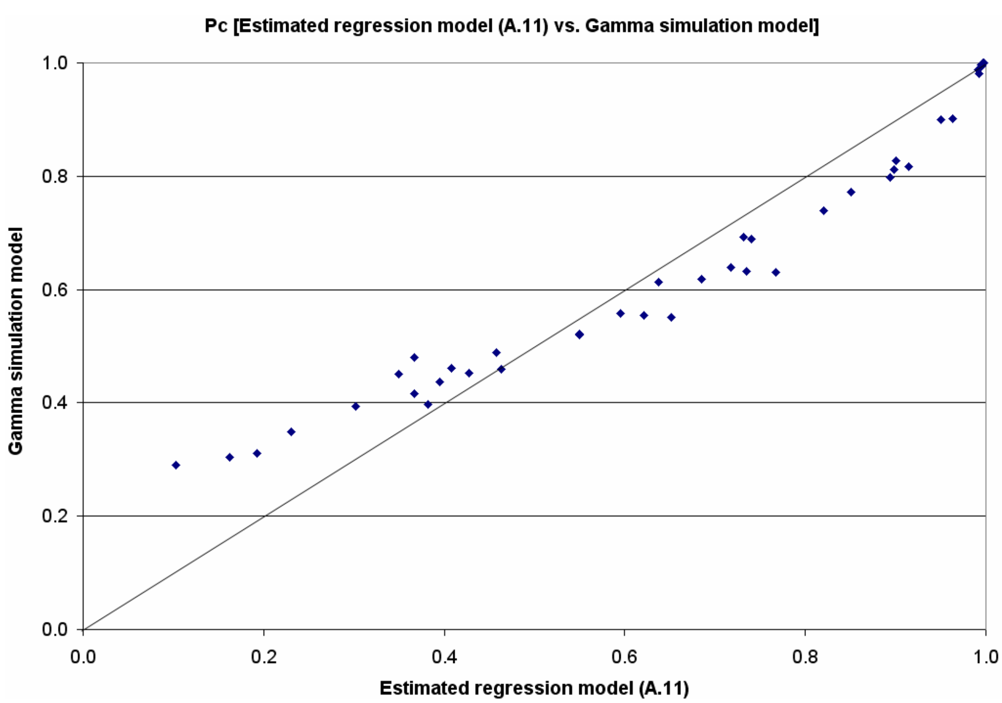

Figure 36. P_C [Estimated regression model (A.11) vs. Gamma simulation model]

Here we still consider the regression model which includes quadratic terms.

$$\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln \beta \\ &+ b_4 \ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right) + b_{11}(\ln c_{rr})^2 + b_{22}(\ln u)^2 \\ &+ b_{33}(\ln\beta)^2 + b_{44}\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\right)^2\end{aligned} \tag{A.12}$$

The statistically fitted quadratic regression model, which includes linear and quadratic terms is

$$\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= 13.737 + 2.119\ln c_{rr} - 1.773\ln u + 0.287\ln\beta \\ &+ 7.530\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right) \\ &+ 1.510\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\right)^2\end{aligned} \tag{A.13}$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.985
R Square	0.971
Adjusted R Square	0.967
Standard Error	0.438
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	231.991	46.398	242	0.000
Residual	36	6.902	0.192		
Total	41	238.893			

	Coefficients	*Standard Erroi*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	13.737	2.015	6.819	0.000	9.651	17.823
b1	2.119	1.045	2.027	0.050	-0.001	4.239
b2	-1.773	0.215	-8.260	0.000	-2.209	-1.338
b3	0.287	0.102	2.801	0.008	0.079	0.494
b4	7.530	1.283	5.871	0.000	4.929	10.132
b5	1.510	0.192	7.854	0.000	1.120	1.901

Although the p-value of b_1 is 0.05 and the 90% confidence interval barely contains 0, we treat the variable $\ln c_{rr}$ as a significant factor in the regression model.

Table 19 displays the outputs of the estimated quadratic regression model (A.13) and those of the Gamma simulation model. Table 19 is sorted in descending order based on the outputs of the Gamma simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Beta	Estimated regression model		Gamma simulation model			
														Pc			Logit
												E.Logit	E.Pc	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	47	7.3378	0.9993	0.9998	0.9995	1.0001	8.5170
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	19	6.9622	0.9991	0.9994	0.9989	0.9999	7.4180
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	42	6.9099	0.9990	0.9992	0.9986	0.9998	7.1301
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	39	5.0673	0.9937	0.9968	0.9957	0.9979	5.7414
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	11	6.4099	0.9984	0.9954	0.9941	0.9967	5.3771
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	25	4.6067	0.9901	0.9886	0.9865	0.9907	4.4627
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	33	4.6238	0.9903	0.9882	0.9861	0.9903	4.4278
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	5	4.5411	0.9895	0.9809	0.9782	0.9836	3.9388
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	50	2.2988	0.9088	0.9020	0.8962	0.9078	2.2196
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	25	2.2036	0.9006	0.8996	0.8937	0.9055	2.1928
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	42	1.4757	0.8139	0.8274	0.8200	0.8348	1.5673
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	36	2.3796	0.9153	0.8164	0.8088	0.8240	1.4921
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	11	1.1911	0.7669	0.8110	0.8033	0.8187	1.4565
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	13	1.4932	0.8166	0.7984	0.7905	0.8063	1.3763
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	11	1.6282	0.8359	0.7713	0.7631	0.7795	1.2157
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	19	0.8368	0.6978	0.7392	0.7306	0.7478	1.0418
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	25	0.5653	0.6377	0.6934	0.6844	0.7024	0.8161
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	16	0.9736	0.7258	0.6896	0.6805	0.6987	0.7982
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	39	0.5337	0.6303	0.6385	0.6291	0.6479	0.5689
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	33	0.8500	0.7006	0.6328	0.6234	0.6422	0.5442
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	50	0.6798	0.6637	0.6299	0.6204	0.6394	0.5318
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	22	0.6397	0.6547	0.6189	0.6094	0.6284	0.4849
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	8	0.6727	0.6621	0.6126	0.6031	0.6221	0.4583
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	47	0.1804	0.5450	0.5578	0.5481	0.5675	0.2322
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	39	0.8553	0.7017	0.5551	0.5454	0.5648	0.2213
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	33	0.4226	0.6041	0.5514	0.5417	0.5611	0.2063
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.0381	0.4905	0.5216	0.5118	0.5314	0.0865
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.0381	0.4905	0.5201	0.5103	0.5299	0.0804
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	8	-0.4740	0.3837	0.4884	0.4786	0.4982	-0.0464
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	5	-0.2351	0.4415	0.4804	0.4706	0.4902	-0.0784
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	19	-0.0794	0.4801	0.4604	0.4506	0.4702	-0.1587
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	44	0.2551	0.5634	0.4587	0.4489	0.4685	-0.1656
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	47	-0.4196	0.3966	0.4520	0.4422	0.4618	-0.1926
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	22	-0.6098	0.3521	0.4505	0.4407	0.4603	-0.1987
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	16	-0.4702	0.3846	0.4378	0.4281	0.4475	-0.2501
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	36	-0.4290	0.3944	0.4170	0.4073	0.4267	-0.3351
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	30	-0.4448	0.3906	0.3973	0.3877	0.4069	-0.4167
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	13	-0.9180	0.2854	0.3932	0.3836	0.4028	-0.4339
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	42	-0.2308	0.4426	0.3482	0.3389	0.3575	-0.6270
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	30	-0.7610	0.3184	0.3114	0.3023	0.3205	-0.7936
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	44	-0.7432	0.3223	0.3048	0.2958	0.3138	-0.8245
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	5	-1.5151	0.1802	0.2900	0.2811	0.2989	-0.8954

Table 19. Estimated regression model (A.13) and Gamma simulation model

Figure 37 and Figure 38 display the outputs of the estimated quadratic regression model (A.13) and those of the Gamma simulation model. Although, at each design point, the difference between the output of the estimated regression model (A.13) and that of the Gamma simulation model is moderate, the estimated regression model (A.13) summarizes the output of the Gamma simulation model well.

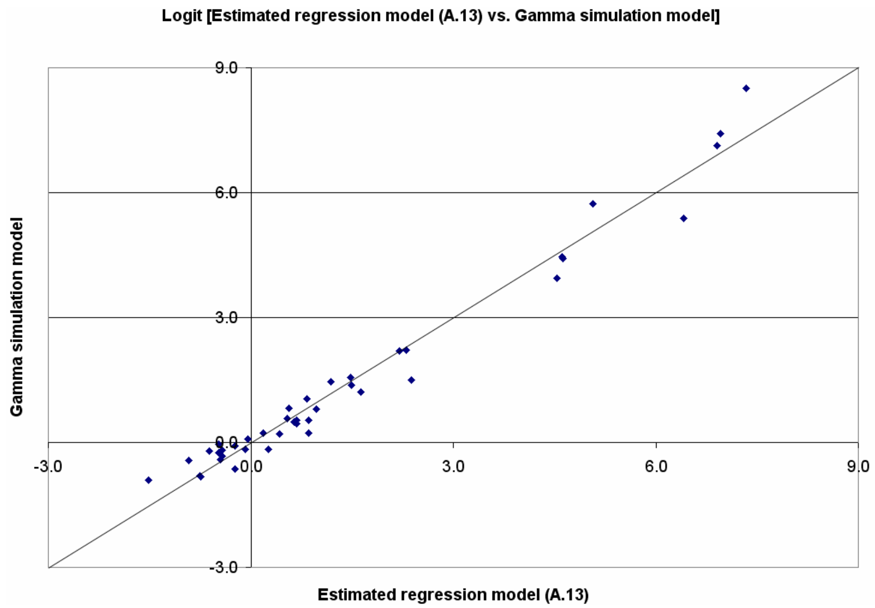

Figure 37. Logit [Estimated regression model (A.13) vs. Gamma simulation model]

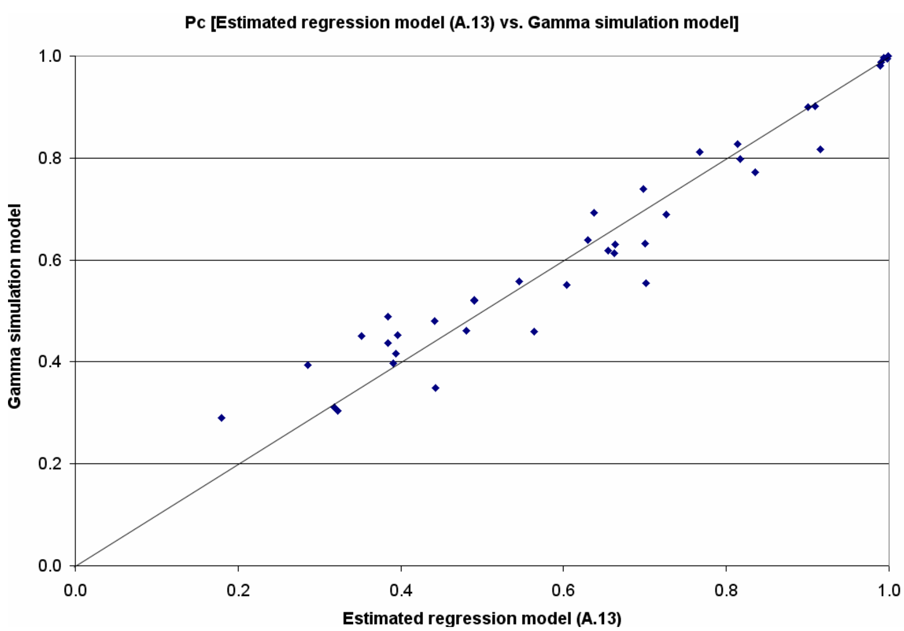

Figure 38. P_C [Estimated regression model (A.13) vs. Gamma simulation model]

2. Probability that the one R is detected, correctly classified, and escorted before leaving the AOI, P_E

<u>Output data</u>:

The probability, P_E, is estimated as the fraction of replications for which an R is detected, correctly classified, and escorted before leaving the AOI. Table 20 displays the outputs of the Gamma simulation model. As a reference, the outputs of the analytical Markov model (A.5) (F_U and F_D follow exponential distribution) are also shown. Table 20 is sorted in descending order based on the outputs of the Gamma simulation model. The maximum value of P_E is estimated as almost 1.0.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Beta	Analytical Markov model		Gamma simulation model			
														P_E			Logit
												Logit	P_E	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	47	1.3763	0.7984	0.9996	0.9992	1.0000	7.8236
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	19	1.3665	0.7968	0.9983	0.9975	0.9991	6.3754
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	42	1.1726	0.7636	0.9970	0.9959	0.9981	5.8061
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	39	1.1473	0.7590	0.9935	0.9919	0.9951	5.0294
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	11	1.1624	0.7618	0.9868	0.9846	0.9890	4.3143
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	25	0.9436	0.7198	0.9767	0.9737	0.9797	3.7357
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	33	0.9355	0.7182	0.9750	0.9719	0.9781	3.6636
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	5	1.1552	0.7605	0.9583	0.9544	0.9622	3.1347
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	50	0.5301	0.6295	0.8651	0.8584	0.8718	1.8583
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	25	0.5353	0.6307	0.8591	0.8523	0.8659	1.8078
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	42	0.2906	0.5721	0.7845	0.7764	0.7926	1.2921
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	11	0.1981	0.5494	0.7463	0.7378	0.7548	1.0790
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	36	0.1395	0.5348	0.7366	0.7280	0.7452	1.0284
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	13	0.0888	0.5222	0.7079	0.6990	0.7168	0.8852
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	11	0.0438	0.5109	0.6821	0.6730	0.6912	0.7634
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	19	0.0065	0.5016	0.6692	0.6600	0.6784	0.7046
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	25	-0.1289	0.4678	0.6267	0.6172	0.6362	0.5181
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	16	-0.2457	0.4389	0.5881	0.5785	0.5977	0.3561
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	39	-0.2353	0.4414	0.5678	0.5581	0.5775	0.2729
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	8	-0.2018	0.4497	0.5611	0.5514	0.5708	0.2456
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	22	-0.2634	0.4345	0.5608	0.5511	0.5705	0.2444
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	33	-0.3191	0.4209	0.5400	0.5302	0.5498	0.1603
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	50	-0.3534	0.4125	0.5333	0.5235	0.5431	0.1334
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	39	-0.4238	0.3956	0.4997	0.4899	0.5095	-0.0012
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	33	-0.4307	0.3940	0.4854	0.4756	0.4952	-0.0584
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	47	-0.5061	0.3761	0.4820	0.4722	0.4918	-0.0720
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.5573	0.3642	0.4460	0.4363	0.4557	-0.2168
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.5573	0.3642	0.4432	0.4335	0.4529	-0.2282
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	5	-0.5798	0.3590	0.4244	0.4147	0.4341	-0.3047
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	8	-0.6239	0.3489	0.4086	0.3990	0.4182	-0.3698
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	19	-0.7184	0.3277	0.3980	0.3884	0.4076	-0.4138
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	44	-0.7266	0.3259	0.3954	0.3858	0.4050	-0.4247
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	22	-0.8098	0.3079	0.3656	0.3562	0.3750	-0.5511
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	16	-0.8232	0.3051	0.3628	0.3534	0.3722	-0.5632
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	47	-0.8451	0.3005	0.3619	0.3525	0.3713	-0.5671
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	36	-0.9107	0.2869	0.3447	0.3354	0.3540	-0.6424
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	30	-0.9171	0.2855	0.3231	0.3139	0.3323	-0.7396
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	13	-0.9824	0.2724	0.3149	0.3058	0.3240	-0.7773
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	42	-1.0308	0.2629	0.2929	0.2840	0.3018	-0.8813
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	44	-1.2487	0.2229	0.2452	0.2368	0.2536	-1.1244
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	30	-1.2597	0.2210	0.2422	0.2338	0.2506	-1.1407
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	5	-1.3264	0.2098	0.2256	0.2174	0.2338	-1.2333

Table 20. Analytical Markov model (A.5) and Gamma simulation model

Figure 39 and Figure 40 display the outputs of the analytical Markov model (A.5) and those of the Gamma simulation model.

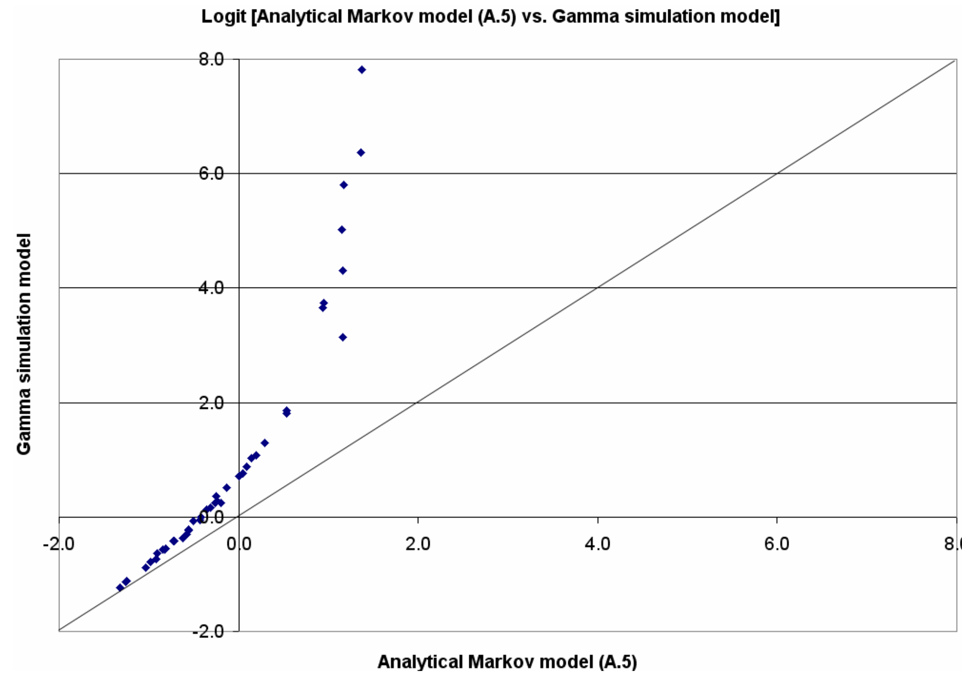

Figure 39. Logit [Analytical Markov model (A.5) vs. Gamma simulation model]

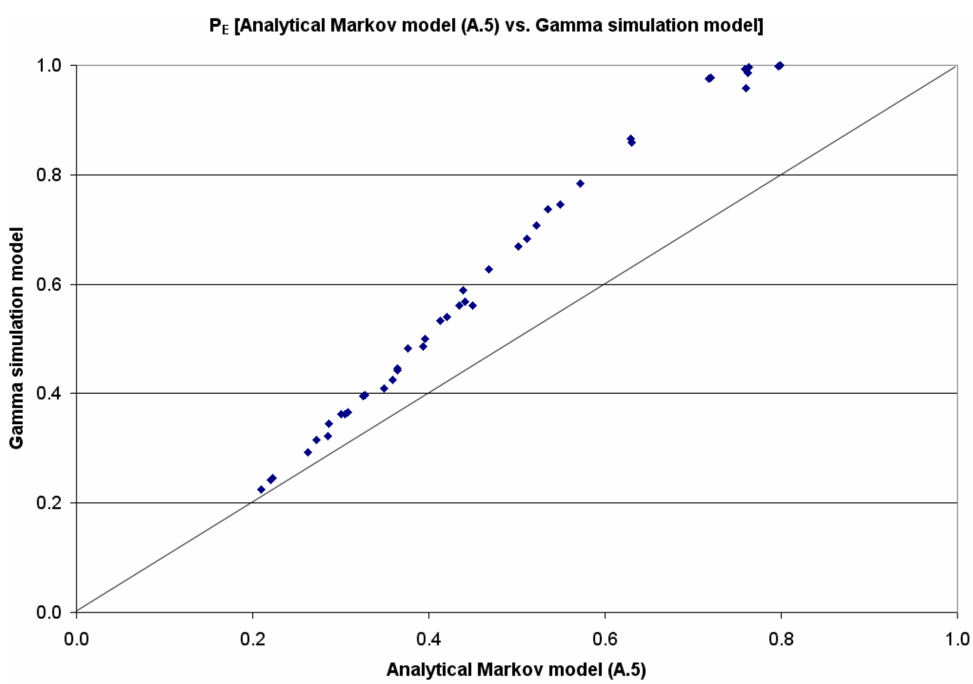

Figure 40. P_E [Analytical Markov model (A.5) vs. Gamma simulation model]

Logistic regression model:

We consider the following regression model based on (A.5)

$$\ln\left(\frac{P_E}{1-P_E}\right)=b_0+b_1\ln c_{rr}+b_2\ln u+b_3\ln\left(2v_I+2u\right)+b_4\ln\beta$$
$$+b_5\ln\left(c_{rr}+w(1-c_{ww})+\left(\frac{M_x}{f\cdot v}+\frac{w+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right) \tag{A.14}$$

- When $b_0 \neq 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.948
R Square	0.899
Adjusted R Square	0.885
Standard Error	0.819
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	214.727	42.945	64	0.000
Residual	36	24.166	0.671		
Total	41	238.893			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	-68.608	37.889	-1.811	0.079	-145.449	8.234
b1	5.020	1.883	2.666	0.011	1.202	8.839
b2	-9.508	3.996	-2.380	0.023	-17.612	-1.405
b3	22.324	10.743	2.078	0.045	0.535	44.113
b4	0.263	0.192	1.372	0.179	-0.126	0.652
b5	-2.925	0.290	-10.078	0.000	-3.514	-2.337

Since the p-value of b_0 is 0.079>0.05 and the 90% confidence interval contains 0, we next treat b_0 as 0.

- When $b_0 = 0$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.958
R Square	0.918
Adjusted R Square	0.882
Standard Error	0.844
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	295.934	59.187	83	0.000
Residual	37	26.368	0.713		
Total	42	322.301			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	0.000	-	-	-	-	-
b1	5.328	1.932	2.758	0.009	1.414	9.243
b2	-2.337	0.547	-4.275	0.000	-3.445	-1.229
b3	2.879	0.342	8.409	0.000	2.185	3.573
b4	0.230	0.197	1.169	0.250	-0.169	0.628
b5	-3.012	0.295	-10.210	0.000	-3.610	-2.414

Although the p-value of b_4 is 0.25>0.05 and the 90% confidence interval contains 0, here we hold the variable $\ln\beta$ in the regression model. The estimated logistic regression model is

$$\boxed{\begin{aligned}&\ln\left(\frac{P_E}{1-P_E}\right)=5.328\ln c_{rr}-2.337\ln u+2.879\ln\left(2v_I+2u\right)+0.230\ln\beta\\&-3.012\ln\left(c_{rr}+w(1-c_{ww})+\left(\frac{M_x}{f\cdot v}+\frac{w+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right)\end{aligned}} \tag{A.15}$$

Table 21 displays the outputs of the estimated regression model (A.15) and those of the Gamma simulation model. Table 21 is sorted in descending order based on the outputs of the Gamma simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Beta	Estimated regression model		Gamma simulation model			
														P_E			Logit
												E.Logit	E.P_E	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	47	6.0025	0.9975	0.9996	0.9992	1.0000	7.8236
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	19	5.7646	0.9969	0.9983	0.9975	0.9991	6.3754
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	42	5.5055	0.9960	0.9970	0.9959	0.9981	5.8061
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	39	5.2697	0.9949	0.9935	0.9919	0.9951	5.0294
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	11	5.1667	0.9943	0.9868	0.9846	0.9890	4.3143
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	25	4.6966	0.9910	0.9767	0.9737	0.9797	3.7357
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	33	4.7359	0.9913	0.9750	0.9719	0.9781	3.6636
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	5	4.8213	0.9920	0.9583	0.9544	0.9622	3.1347
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	50	3.4516	0.9693	0.8651	0.8584	0.8718	1.8583
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	25	3.0530	0.9549	0.8591	0.8523	0.8659	1.8078
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	42	2.3302	0.9113	0.7845	0.7764	0.7926	1.2921
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	11	2.3779	0.9151	0.7463	0.7378	0.7548	1.0790
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	36	2.4571	0.9211	0.7366	0.7280	0.7452	1.0284
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	13	2.3062	0.9094	0.7079	0.6990	0.7168	0.8852
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	11	1.7898	0.8569	0.6821	0.6730	0.6912	0.7634
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	19	1.6305	0.8362	0.6692	0.6600	0.6784	0.7046
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	25	1.0410	0.7390	0.6267	0.6172	0.6362	0.5181
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	16	1.0998	0.7502	0.5881	0.5785	0.5977	0.3561
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	39	0.9841	0.7279	0.5678	0.5581	0.5775	0.2729
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	8	0.6238	0.6511	0.5611	0.5514	0.5708	0.2456
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	22	0.8884	0.7086	0.5608	0.5511	0.5705	0.2444
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	33	1.0997	0.7502	0.5400	0.5302	0.5498	0.1603
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	50	1.3671	0.7969	0.5333	0.5235	0.5431	0.1334
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	39	0.5857	0.6424	0.4997	0.4899	0.5095	-0.0012
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	33	0.7657	0.6826	0.4854	0.4756	0.4952	-0.0584
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	47	0.4362	0.6074	0.4820	0.4722	0.4918	-0.0720
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	0.2536	0.5631	0.4460	0.4363	0.4557	-0.2168
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	0.2536	0.5631	0.4432	0.4335	0.4529	-0.2282
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	5	-0.6307	0.3474	0.4244	0.4147	0.4341	-0.3047
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	8	-0.1201	0.4700	0.4086	0.3990	0.4182	-0.3698
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	19	-0.3784	0.4065	0.3980	0.3884	0.4076	-0.4138
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	44	-0.1106	0.4724	0.3954	0.3858	0.4050	-0.4247
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	22	-0.6989	0.3321	0.3656	0.3562	0.3750	-0.5511
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	16	-0.3710	0.4083	0.3628	0.3534	0.3722	-0.5632
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	47	-0.2707	0.4327	0.3619	0.3525	0.3713	-0.5671
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	36	-0.5391	0.3684	0.3447	0.3354	0.3540	-0.6424
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	30	-0.4246	0.3954	0.3231	0.3139	0.3323	-0.7396
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	13	-0.8285	0.3040	0.3149	0.3058	0.3240	-0.7773
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	42	-1.3908	0.1993	0.2929	0.2840	0.3018	-0.8813
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	44	-1.8599	0.1347	0.2452	0.2368	0.2536	-1.1244
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	30	-1.5207	0.1794	0.2422	0.2338	0.2506	-1.1407
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	5	-2.3943	0.0836	0.2256	0.2174	0.2338	-1.2333

Table 21. Estimated regression model (A.15) and Gamma simulation model

Figure 41 and Figure 42 display the outputs of the estimated regression model (A.15) and those of the Gamma simulation model.

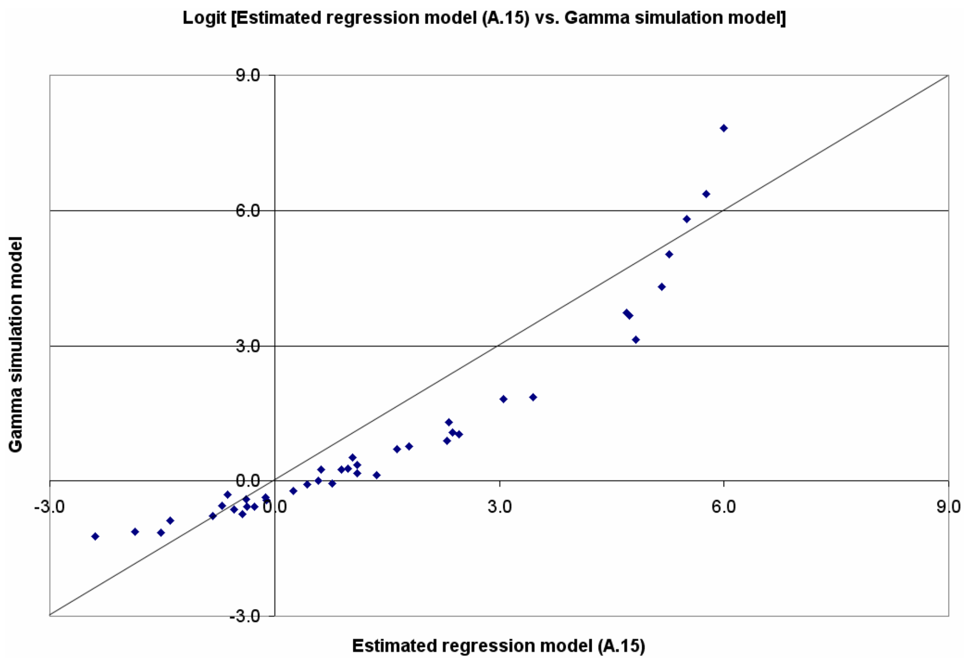

Figure 41. Logit [Estimated regression model (A.15) vs. Gamma simulation model]

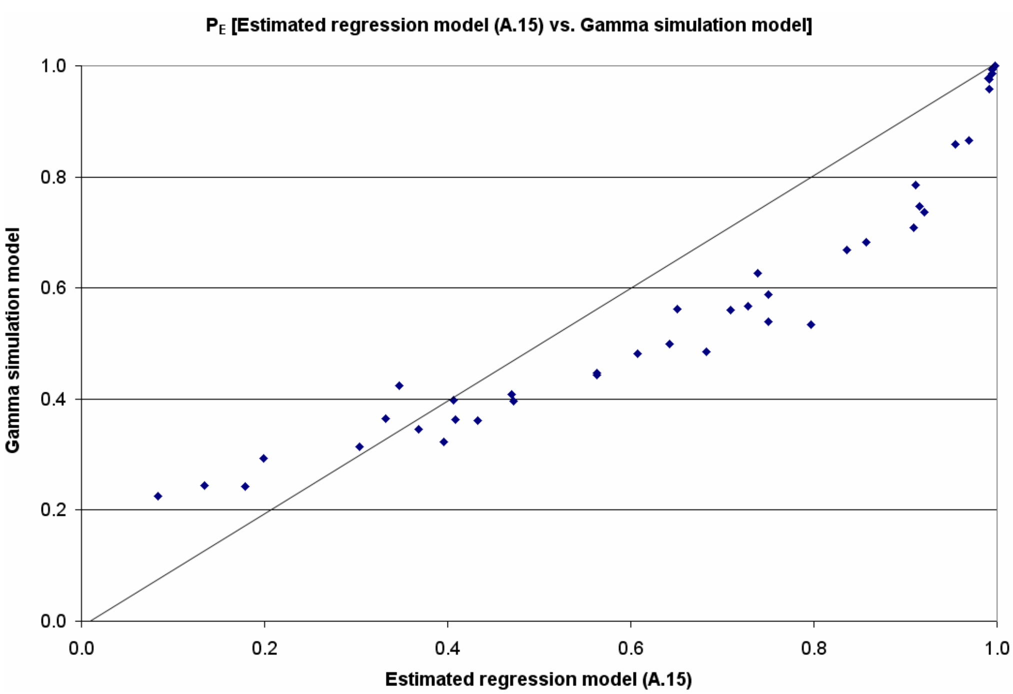

Figure 42. P_E [Estimated regression model (A.15) vs. Gamma simulation model]

Here we still consider the regression model which includes quadratic terms.

$$\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln\left(2v_I + 2u\right) + b_4 \ln \beta \\ &+ b_5 \ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right) \\ &+ b_{11}\left(\ln c_{rr}\right)^2 + b_{22}\left(\ln u\right)^2 + b_{33}\left(\ln\left(2v_I + 2u\right)\right)^2 + b_{44}\left(\ln\beta\right)^2 \\ &+ b_{55}\left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)\right)^2\end{aligned} \qquad (A.16)$$

The statistically fitted quadratic regression model, which includes linear and quadratic terms is

$$\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= 1.871 \ln c_{rr} - 2.689 \ln u + 3.911 \ln\left(2v_I + 2u\right) + 0.302 \ln \beta \\ &- 9.810 \ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right) \\ &+ 2.128\left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)\right)^2\end{aligned} \qquad (A.17)$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.985
R Square	0.970
Adjusted R Square	0.938
Standard Error	0.447
Observations	42

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	230.656	38.443	192	0.000
Residual	36	7.189	0.200		
Total	42	237.846			

	Coefficients	*Standard Erroi*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	0.000	-	-	-	-	-
b1	1.871	1.073	1.745	0.090	-0.304	4.047
b2	-2.689	0.292	-9.204	0.000	-3.282	-2.097
b3	3.911	0.237	16.503	0.000	3.430	4.391
b4	0.302	0.104	2.894	0.006	0.090	0.513
b5	-9.810	0.865	-11.347	0.000	-11.563	-8.056
b6	2.128	0.253	8.413	0.000	1.615	2.641

Although the p-value of b_1 is 0.09>0.05 and the 90% confidence interval contains 0, we treat the variable $\ln c_{rr}$ as a significant factor in the regression model.

Table 22 displays the outputs of the estimated quadratic regression model (A.17) and those of the Gamma simulation model. Table 22 is sorted in descending order based on the outputs of the Gamma simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	Beta	Estimated regression model		Gamma simulation model			
														P_E			Logit
												E.Logit	E.P_E	Mean	95%LB	95%UB	
35	50	300	0	300	30	10.0	15	4.0	1.00	1.00	47	6.2606	0.9981	0.9996	0.9992	1.0000	7.8236
36	50	300	1	300	30	10.0	15	4.0	1.00	1.00	19	5.9202	0.9973	0.9983	0.9975	0.9991	6.3754
37	50	300	0	300	30	12.5	15	4.0	1.00	1.00	42	5.5862	0.9963	0.9970	0.9959	0.9981	5.8061
40	75	300	1	300	30	10.0	15	4.0	1.00	1.00	39	4.7544	0.9915	0.9935	0.9919	0.9951	5.0294
38	50	300	1	300	30	12.5	15	4.0	1.00	1.00	11	5.1140	0.9940	0.9868	0.9846	0.9890	4.3143
41	75	300	0	300	30	12.5	15	4.0	1.00	1.00	25	4.0203	0.9824	0.9767	0.9737	0.9797	3.7357
42	75	300	1	300	30	12.5	15	4.0	1.00	1.00	33	4.0581	0.9830	0.9750	0.9719	0.9781	3.6636
39	75	300	0	300	30	10.0	15	4.0	1.00	1.00	5	4.1806	0.9849	0.9583	0.9544	0.9622	3.1347
3	81	300	22	300	30	11.3	15	5.0	0.96	0.96	50	2.2122	0.9013	0.8651	0.8584	0.8718	1.8583
34	81	300	13	300	30	11.3	15	4.8	0.94	0.86	25	2.2599	0.9055	0.8591	0.8523	0.8659	1.8078
30	144	300	34	300	30	10.0	15	5.8	0.99	0.85	42	1.4428	0.8089	0.7845	0.7764	0.7926	1.2921
20	159	300	3	300	30	15.0	15	6.5	0.98	1.00	11	0.8091	0.6919	0.7463	0.7378	0.7548	1.0790
27	50	300	9	300	30	22.5	15	7.0	0.91	0.89	36	1.6008	0.8321	0.7366	0.7280	0.7452	1.0284
14	113	300	3	300	30	21.3	15	7.5	0.97	1.00	13	0.7886	0.6875	0.7079	0.6990	0.7168	0.8852
15	50	300	34	300	30	22.5	15	4.8	0.99	0.85	11	0.9462	0.7203	0.6821	0.6730	0.6912	0.7634
17	159	300	6	300	30	15.0	15	4.3	0.92	0.88	19	0.5506	0.6343	0.6692	0.6600	0.6784	0.7046
10	222	300	0	300	30	13.8	15	6.5	0.98	0.81	25	0.3221	0.5798	0.6267	0.6172	0.6362	0.5181
31	66	300	28	300	30	27.5	15	6.8	1.00	0.84	16	0.2193	0.5546	0.5881	0.5785	0.5977	0.3561
4	97	300	31	300	30	16.3	15	8.0	0.96	0.83	39	0.2267	0.5564	0.5678	0.5581	0.5775	0.2729
16	144	300	41	300	30	10.0	15	6.8	0.91	0.91	8	0.1198	0.5299	0.5611	0.5514	0.5708	0.2456
11	284	300	38	300	30	12.5	15	5.8	1.00	0.94	22	0.1671	0.5417	0.5608	0.5511	0.5705	0.2444
2	66	300	13	300	30	27.5	15	6.3	0.90	0.86	33	0.1707	0.5426	0.5400	0.5302	0.5498	0.1603
23	128	300	19	300	30	26.3	15	4.3	0.98	0.98	50	0.0875	0.5219	0.5333	0.5235	0.5431	0.1334
22	284	300	22	300	30	12.5	15	5.3	0.90	0.96	39	0.3055	0.5758	0.4997	0.4899	0.5095	-0.0012
24	97	300	50	300	30	16.3	15	7.5	0.94	0.99	33	-0.0204	0.4949	0.4854	0.4756	0.4952	-0.0584
28	238	300	6	300	30	18.8	15	8.0	0.97	0.88	47	-0.1919	0.4522	0.4820	0.4722	0.4918	-0.0720
26	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.4178	0.3971	0.4460	0.4363	0.4557	-0.2168
9	175	300	25	300	30	20.0	15	6.0	0.95	0.90	28	-0.4178	0.3971	0.4432	0.4335	0.4529	-0.2282
32	222	300	31	300	30	13.8	15	7.8	0.93	0.83	5	-0.7267	0.3259	0.4244	0.4147	0.4341	-0.3047
19	113	300	44	300	30	21.3	15	4.0	0.93	0.93	8	-0.8597	0.2974	0.4086	0.3990	0.4182	-0.3698
18	300	300	41	300	30	17.5	15	5.0	0.99	0.91	19	-0.4814	0.3819	0.3980	0.3884	0.4076	-0.4138
6	300	300	16	300	30	17.5	15	7.3	0.91	0.95	44	-0.1395	0.4652	0.3954	0.3858	0.4050	-0.4247
33	253	300	0	300	30	23.8	15	4.5	0.96	0.81	22	-0.9127	0.2865	0.3656	0.3562	0.3750	-0.5511
13	253	300	19	300	30	23.8	15	4.0	0.94	0.98	16	-0.7347	0.3242	0.3628	0.3534	0.3722	-0.5632
7	206	300	9	300	30	30.0	15	5.3	0.99	0.89	47	-0.7092	0.3298	0.3619	0.3525	0.3713	-0.5671
8	191	300	44	300	30	25.0	15	7.8	0.98	0.93	36	-0.6336	0.3467	0.3447	0.3354	0.3540	-0.6424
1	128	300	50	300	30	26.3	15	5.5	0.93	0.99	30	-0.6103	0.3520	0.3231	0.3139	0.3323	-0.7396
21	206	300	16	300	30	30.0	15	6.3	0.91	0.95	13	-1.0514	0.2589	0.3149	0.3058	0.3240	-0.7773
5	238	300	47	300	30	18.8	15	4.5	0.93	0.80	42	-0.5226	0.3722	0.2929	0.2840	0.3018	-0.8813
29	191	300	47	300	30	25.0	15	5.5	0.92	0.80	44	-0.7681	0.3169	0.2452	0.2368	0.2536	-1.1244
25	269	300	38	300	30	28.8	15	7.3	0.96	0.94	30	-0.5970	0.3550	0.2422	0.2338	0.2506	-1.1407
12	269	300	28	300	30	28.8	15	7.0	0.94	0.84	5	-1.4022	0.1975	0.2256	0.2174	0.2338	-1.2333

Table 22. Estimated regression model (A.17) and Gamma simulation model

Figure 43 and Figure 44 display the outputs of the estimated quadratic regression model (A.17) and those of the Gamma simulation model. Although, at each design point, the difference between the output of the estimated regression model (A.17) and that of the Gamma simulation model is moderate, the estimated regression model (A.17) summarizes the output of the Gamma simulation model well.

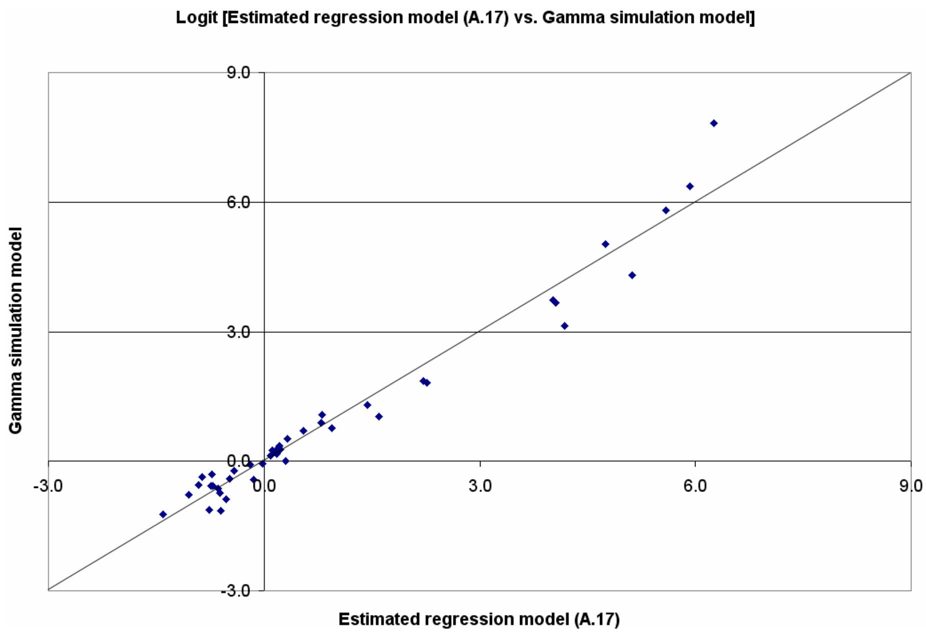

Figure 43. Logit [Estimated regression model (A.17) vs. Gamma simulation model]

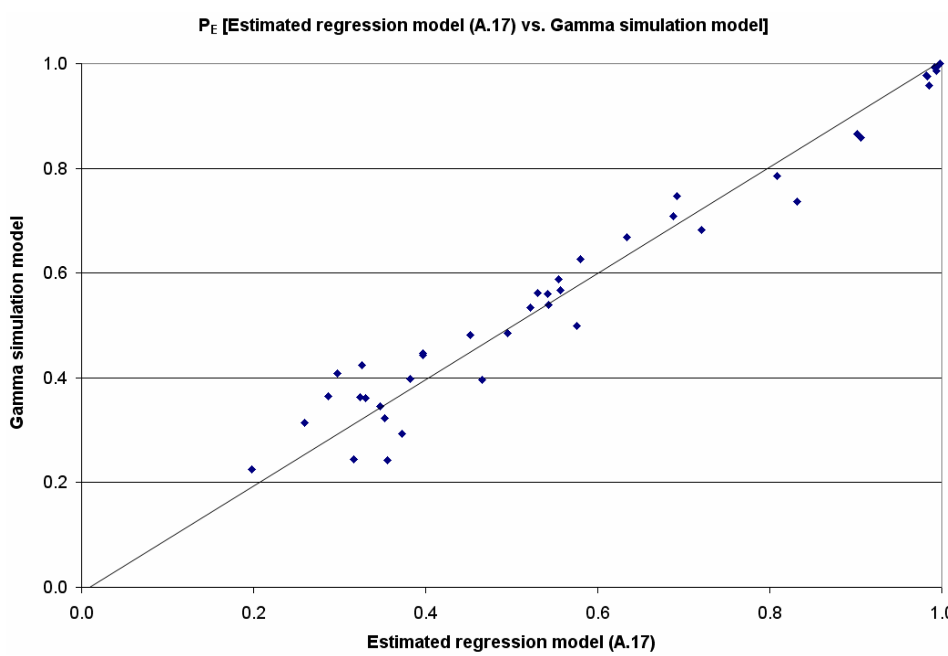

Figure 44. P_E [Estimated regression model (A.17) vs. Gamma simulation model]

G. DISCUSSION AND CONCLUSION

CASE-1:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape 1 (i.e. Exponential distribution)
- F_D is a Gamma distribution with mean $1/\phi$ and shape 1 (i.e. Exponential distribution)
- The MPA has no memory of its previous classification information (pessimistic assumption)

Tables 12 – 15 and the corresponding graphs show that if an all Markov model (in which all random variables are independent and exponentially distributed) is simulated, compared numerically to a corresponding analytical/numerical-calculated Markov model, and compared to a statistically fitted logistic regression model based on the explanatory variables/parameters suggested by the Markov model, ***the models' results for both P_C and P_E are very close for the parameter values selected. However, the highest P_E values are approximately 0.8, and these are achieved only when there are a few ($\approx$0 or 1) Whites in the area of interest (AOI) and $c_{ww} \approx$ 1.0 and $c_{rr} \approx$ 1.0. A large number of Whites ($w \approx$ 25–50) and only slightly less high classification capabilities c_{ww} and c_{rr} ($\approx$0.90–0.95) reduces P_C and P_E quickly, to approximately 0.25–0.50.*** This would be operationally unacceptable; so if the all Markov model were "reasonable," it would imply that a single Maritime Patrol Aircraft (MPA) is operationally inadequate, especially if it has no effective memory of previous identifications, and/or is not effectively supported (by an overhead/satellite sensor system), and does not have warning intelligence. The number of unidentified vessels in the AOI may be decreased by a highly effective Automatic Identification System (AIS)[27] coverage.

[27] United States, Government Accountability Office, "Maritime Security: Partnering could reduce federal costs and facilitate implementation of automatic vessel identification system " (2004)

CASE-2:

- F_L is an Exponential distribution with rate $\delta(t_j)$
- F_U is a Gamma distribution with mean $1/\mu$ and shape β (=5 to 50)
- F_D is a Gamma distribution with mean $1/\phi$ and shape β which is the same as that of F_U. (U and D are statistically independent.)
- The MPA has no memory of its previous classification information (pessimistic assumption)

Examination of Tables 18, 19, 21, and 22 along with the corresponding graphs (Simulation model, Analytical/numerical-calculated Markov model, and statistically fitted linear logistic regression applied to simulated data) show that the results for P_C and P_E differ considerably between the models. The Markov model (in which all random variables are independent and exponentially distributed) is pessimistic when compared to the more physically plausible gamma model with shape parameter β (approximately 25 or more). For example, for design point #39 with shape parameter β =5 (not large), the simple Markov model gives $P_E \approx 0.76$, while the simulated gamma model gives $P_E \approx 0.96$ (which is possibly unrealistically high). However, when a statistically fitted quadratic regression model is considered, the results for P_C and P_E for the estimated logistic regression model are close to those of the gamma simulation model for design point #39; the estimated regression model gives $P_E \approx 0.98$, while the simulated gamma model gives $P_E \approx 0.96$), as shown in Tables 19 and 22, and the corresponding graphical figures.

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APPENDIX 2 LOGIT OF ANALYTICAL MARKOVIAN MODELS

1. Probability that the one R is detected and correctly classified before leaving the AOI, P_C

$$P_C = \frac{\delta(t_j)\frac{1}{w+1}c_{rr}}{\delta(t_j)\frac{1}{w+1}c_{rr} + \delta(t_j)\frac{w}{w+1}c_{wr}\frac{\mu}{\phi+\mu} + \mu}$$

$$\frac{P_C}{1-P_C} = \frac{\delta(t_j)\frac{1}{w+1}c_{rr}}{\delta(t_j)\frac{w}{w+1}c_{wr}\frac{\mu}{\phi+\mu} + \mu} = \frac{c_{rr}}{\mu\left(\frac{wc_{wr}}{\phi+\mu} + \frac{w+1}{\delta(t_j)}\right)}$$

Here,

$$\delta(t_j) = \frac{W_u(t_j)+R_u(t_j)}{\left(\frac{M_x \cdot M_y}{f^2}\right)\frac{f}{v} + \left(W_u(t_j)+R_u(t_j)\right)\cdot\tau} = \frac{w+1}{\frac{M_x \cdot M_y}{f \cdot v} + (w+1)\cdot\tau}$$

$$\mu = \frac{u}{M_y},\ \phi = \frac{2v_I + 2u}{M_y},\ \mu+\phi = \frac{2v_I+3u}{M_y}$$

Thus,

$$\frac{P_C}{1-P_C} = \frac{c_{rr}}{\frac{u}{M_y}\left(\frac{wc_{wr}}{\frac{2v_I+3u}{M_y}} + (w+1)\cdot\frac{\frac{M_x \cdot M_y}{f \cdot v} + (w+1)\cdot\tau}{w+1}\right)}$$

$$= \frac{c_{rr}}{u\left(\frac{w(1-c_{ww})}{2v_I+3u} + \cdot\frac{M_x}{f \cdot v} + \frac{w+1}{M_y}\cdot\tau\right)}$$

Thus, the analytical logistic regression model is

$$\boxed{\ln\left(\frac{P_C}{1-P_C}\right) = \ln c_{rr} - \ln u - \ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f \cdot v} + \frac{w+1}{M_y}\cdot\tau\right)}$$

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

$$P_E = \frac{\delta(t_j)\frac{1}{w+1}c_{rr}\frac{\phi}{\phi+\mu}}{\delta(t_j)\frac{1}{w+1}c_{rr}+\delta(t_j)\frac{w}{w+1}c_{wr}\frac{\mu}{\phi+\mu}+\mu}$$

$$= \frac{\delta(t_j)\frac{1}{w+1}c_{rr}\frac{\phi}{\phi+\mu}}{\delta(t_j)\frac{1}{w+1}c_{rr}+\delta(t_j)\frac{w}{w+1}c_{wr}\frac{\mu}{\phi+\mu}+\mu-\delta(t_j)\frac{1}{w+1}c_{rr}\frac{\phi}{\phi+\mu}}$$

$$= \frac{c_{rr}\frac{\phi}{\phi+\mu}}{c_{rr}\frac{\mu}{\phi+\mu}+wc_{wr}\frac{\mu}{\phi+\mu}+\frac{w+1}{\delta(t_j)}\mu} = \frac{c_{rr}\phi}{\mu\left(c_{rr}+wc_{wr}+\frac{w+1}{\delta(t_j)}(\phi+\mu)\right)}$$

Here,

$$\delta(t_j) = \frac{W_u(t_j)+R_u(t_j)}{\left(\frac{M_x\cdot M_y}{f^2}\right)\frac{f}{v}+\left(W_u(t_j)+R_u(t_j)\right)\cdot\tau} = \frac{w+1}{\frac{M_x\cdot M_y}{f\cdot v}+(w+1)\cdot\tau}$$

$$\mu = \frac{u}{M_y},\ \phi = \frac{2v_I+2u}{M_y},\ \mu+\phi = \frac{2v_I+3u}{M_y} \qquad \textit{Therefore,}$$

$$\frac{P_E}{1-P_E} = \frac{c_{rr}\frac{2v_I+2u}{M_y}}{\frac{u}{M_y}\left(c_{rr}+wc_{wr}+\left(\frac{M_x\cdot M_y}{f\cdot v}+(w+1)\cdot\tau\right)\left(\frac{2v_I+3u}{M_y}\right)\right)}$$

$$= \frac{c_{rr}\left(2v_I+2u\right)}{u\left(c_{rr}+w\left(1-c_{ww}\right)+\left(\frac{M_x}{f\cdot v}+\frac{w+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right)}$$

Thus, the analytical logistic regression model is

$$\boxed{\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= \ln c_{rr}-\ln u+\ln\left(2v_I+2u\right)\\ &\quad -\ln\left(c_{rr}+w(1-c_{ww})+\left(\frac{M_x}{f\cdot v}+\frac{w+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right)\end{aligned}}$$

APPENDIX 3 LOGISTIC REGRESSION MODELS (PART2)

A. OBJECTIVE

The objective of this appendix is to describe and explore appropriate logistic regression models for the Maritime Intercept Operations (MIO) capabilities which consider the effects of Maritime Patrol Aircraft (MPA)'s information retention.

B. MEASURES OF EFFECTIVENESS

1. Probability that the one R is detected and correctly classified before leaving the Area of Interest (AOI), P_C

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

C. PREVIOUS STUDIES

The statistically fitted logistic regression models which do not consider the effects of the MPA's information retention (do not include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$) are (A.18) and (A.19) introduced as (4.7) and (4.8) in the Chapter IV.

1. Probability that the one R is detected and correctly classified before leaving the Area of Interest (AOI), P_C

$$\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= 0.881+1.117\ln c_{rr}-1.134\ln u \\ &\quad -0.440\ln\left(\frac{w(1-c_{ww})}{2v_I+3u}+\frac{M_x}{f\cdot v}+\frac{(w+1)\tau}{M_y}\right) \\ &\quad +0.199\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u}+\frac{M_x}{f\cdot v}+\frac{(w+1)\tau}{M_y}\right)\right)^2\end{aligned} \tag{A.18}$$

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

$$\ln\left(\frac{P_E}{1-P_E}\right) = 1.122\ln c_{rr} - 1.474\ln u + 1.965\ln\left(2v_I + 2u\right)$$
$$-2.790\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right) \quad \text{(A.19)}$$
$$+0.269\left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)\right)^2$$

D. INPUT DATA

1. Constant parameters

$M_y = 200NM,\ \ v = 200kt,\ \ v_I = 30kt,\ \ f = 15NM$

2. Multi-level parameters

Table 23 shows the specified factor values of the multi-level factors.

	Low	High
M_x	200NM	400NM
w	0	100
u	15kt	30kt
τ	4min	8min
c_{ww}	0.8	1.0
c_{rr}	0.6	1.0
$1/\psi$	0hrs	4hrs

Table 23. Multi-level factor values

3. Design of experiment

Table 24 displays the 34 design points. These design points are used as input to the simulations. Design points (#1 – 17) are obtained from the Nearly-Orthogonal Latin Hypercube (NOLH) based on Table 23, and design points (#18 – 34) are intentionally added to obtain relatively higher probabilities for P_C and P_E by using the other NOLH based on Table 25.

Design Point ID	Mx (NM)	W	u (kt)	tau (min)	Cww	Crr	1/psi (hrs)
1	263	100	27.2	5.5	0.85	0.98	2.3
2	213	25	28.1	6.3	0.80	0.73	2.5
3	225	44	15.9	5.0	0.93	0.93	4.0
4	238	63	19.7	8.0	0.91	0.65	3.0
5	350	94	21.6	4.5	0.86	0.60	3.3
6	400	31	20.6	7.3	0.81	0.90	3.5
7	325	19	30.0	5.3	0.98	0.78	3.8
8	313	88	26.3	7.8	0.96	0.85	2.8
9	300	50	22.5	6.0	0.90	0.80	2.0
10	338	0	17.8	6.5	0.95	0.63	1.8
11	388	75	16.9	5.8	1.00	0.88	1.5
12	375	56	29.1	7.0	0.88	0.68	0.0
13	363	38	25.3	4.0	0.89	0.95	1.0
14	250	6	23.4	7.5	0.94	1.00	0.8
15	200	69	24.4	4.8	0.99	0.70	0.5
16	275	81	15.0	6.8	0.83	0.83	0.3
17	288	13	18.8	4.3	0.84	0.75	1.3
18	216	20	19.1	4.8	0.96	0.99	2.3
19	203	5	19.4	5.1	0.95	0.93	2.5
20	206	9	15.3	4.5	0.98	0.98	4.0
21	209	13	16.6	6.0	0.98	0.91	3.0
22	238	19	17.2	4.3	0.97	0.90	3.3
23	250	6	16.9	5.6	0.95	0.98	3.5
24	231	4	20.0	4.6	0.99	0.94	3.8
25	228	18	18.8	5.9	0.99	0.96	2.8
26	225	10	17.5	5.0	0.98	0.95	2.0
27	234	0	15.9	5.3	0.99	0.91	1.8
28	247	15	15.6	4.9	1.00	0.97	1.5
29	244	11	19.7	5.5	0.97	0.92	0.0
30	241	8	18.4	4.0	0.97	0.99	1.0
31	213	1	17.8	5.8	0.98	1.00	0.8
32	200	14	18.1	4.4	1.00	0.93	0.5
33	219	16	15.0	5.4	0.96	0.96	0.3
34	222	3	16.3	4.1	0.96	0.94	1.3

Table 24. Design of experiment

	Low	High
M_x	200NM	250NM
w	0	20
u	15kt	20kt
τ	4min	6min
c_{ww}	0.95	1.0
c_{rr}	0.90	1.0
$1/\psi$	0hrs	4hrs

Table 25. Multi-level factor values additionally considered

E. LOGISTIC REGRESSION MODELS

1. Probability that the one R is detected and correctly classified before leaving the Area of Interest (AOI), P_C

We consider the following two types of logistic regression models as alternatives.

- $1/\psi$ is considered as a single independent variable.

$$\boxed{\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u \\ &\quad + b_3 \ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right) \\ &\quad + b_4\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\right)^2 + b_5(1/\psi)\end{aligned}} \qquad \text{(A.20)}$$

When the fifth independent variable (the mean time until the MPA loses classification information of a classified vessel, $1/\psi$) is 0, the model (A.20) corresponds to the model (A.18) which does not consider $1/\psi$.

- $1/\psi$ influences the number of White vessels in the AOI, w.

$$\boxed{\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u \\ &\quad + b_3 \ln\left(\frac{\frac{w}{\sqrt[n]{1+1/\psi}}(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{\left(\frac{w}{\sqrt[n]{1+1/\psi}}+1\right)\tau}{M_y}\right) \\ &\quad + b_4\left(\ln\left(\frac{\frac{w}{\sqrt[n]{1+1/\psi}}(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{\left(\frac{w}{\sqrt[n]{1+1/\psi}}+1\right)\tau}{M_y}\right)\right)^2\end{aligned}} \qquad \text{(A.21)}$$

When the mean time until the MPA loses classification information of a classified vessel, $1/\psi$ is 0 or n is large, the model (A.21) corresponds to the model

(A.18) which does not include $1/\psi$. n is determined so that the four independent variables are statistically significant in this regression model.

The statistically fitted logistic regression models:

- $1/\psi$ is considered as a single independent variable

$$\ln\left(\frac{P_C}{1-P_C}\right) = 0.886 + 1.116\ln c_{rr} - 1.133\ln u - 0.420\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right) + 0.204\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\right)^2 + 0.005(1/\psi) \tag{A.22}$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999
R Square	0.998
Adjusted R Square	0.998
Standard Error	0.032
Observations	34

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	18.582	3.716	3631	0.000
Residual	28	0.029	0.001		
Total	33	18.611			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
b0	0.886	0.177	5.013	0.000	0.524	1.248
b1	1.116	0.046	24.063	0.000	1.021	1.211
b2	-1.133	0.033	-34.670	0.000	-1.200	-1.066
b3	-0.420	0.180	-2.335	0.027	-0.788	-0.051
b4	0.204	0.045	4.517	0.000	0.111	0.296
b5	0.005	0.005	1.012	0.320	-0.005	0.014

Since the p-value of b_5 is 0.32>0.05 and the 90% confidence interval contains 0, b_5 is not statistically different from 0. Figure 45 and Figure 46 display the outputs of the regression model (A.22) and those of the simulation model.

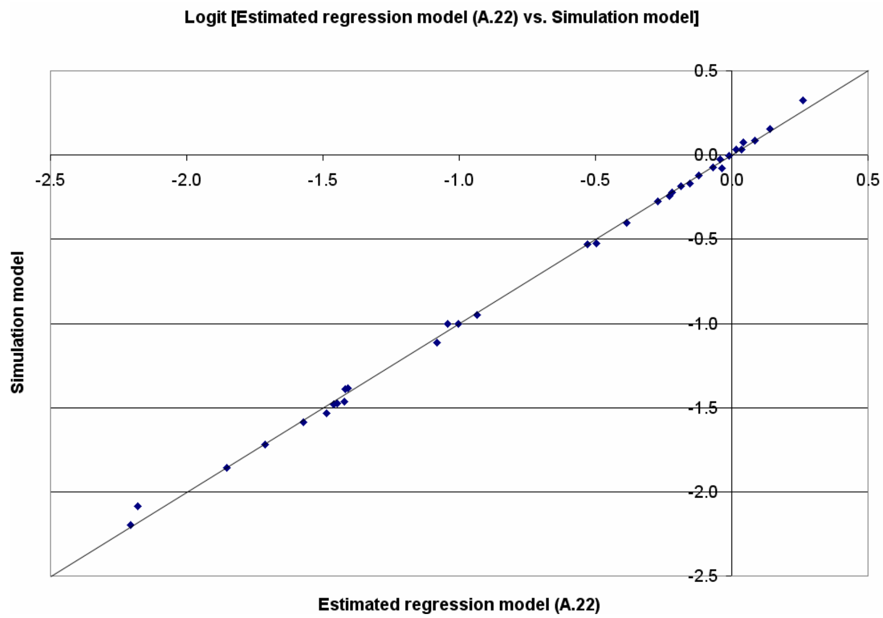

Figure 45. Logit [Estimated regression model (A.22) vs. Simulation model]

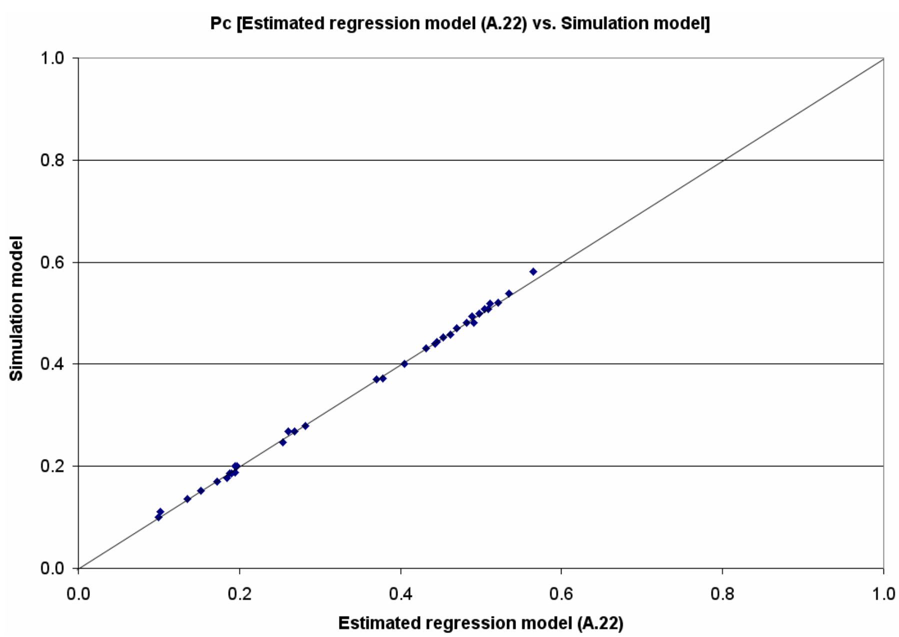

Figure 46. P_C [Estimated regression model (A.22) vs. Simulation model]

The estimated regression model (A.22) summarizes the output of the simulation model well. However, we employ the estimated model (A.18) rather than the estimated model (A.22) because the fifth independent variable of the model (A.22) is not statistically significant in the regression model.

• $1/\psi$ influences the number of White vessels in the AOI, w.

$$
\ln\left(\frac{P_C}{1-P_C}\right) = 0.890 + 1.116\ln c_{rr} - 1.135\ln u
- 0.407\ln\left(\frac{\frac{w}{\sqrt[30]{1+1/\psi}}(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{\left(\frac{w}{\sqrt[30]{1+1/\psi}}+1\right)\tau}{M_y}\right)
+0.210\left(\ln\left(\frac{\frac{w}{\sqrt[30]{1+1/\psi}}(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{\left(\frac{w}{\sqrt[30]{1+1/\psi}}+1\right)\tau}{M_y}\right)\right)^2 \tag{A.23}
$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999
R Square	0.998
Adjusted R Square	0.998
Standard Error	0.034
Observations	34

ANOVA

	df	SS	MS	F	Significance F
Regression	4	18.577	4.644	3980	0.000
Residual	29	0.034	0.001		
Total	33	18.611			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.890	0.193	4.607	0.000	0.495	1.285
b1	1.116	0.050	22.531	0.000	1.015	1.217
b2	-1.135	0.035	-32.495	0.000	-1.206	-1.064
b3	-0.407	0.197	-2.071	0.047	-0.809	-0.005
b4	0.210	0.049	4.276	0.000	0.110	0.310

Since the R-square value of the regression model (A.23) is 0.998, and the p-value of each independent variable is less than 0.05, the estimated model (A.23) summarizes the simulation model output well. When n is less than 30, we could not find an appropriate logistic regression model which satisfies a high R-square value and the significance of the four independent variables. Figure 47 and Figure 48 display the outputs of the regression model (A.23) and those of the simulation model.

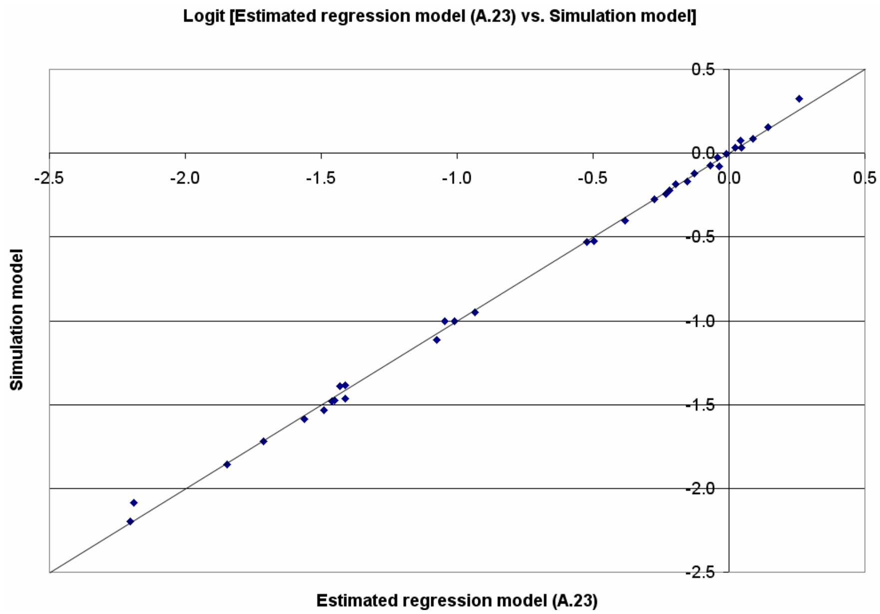

Figure 47. Logit [Estimated regression model (A.23) vs. Simulation model]

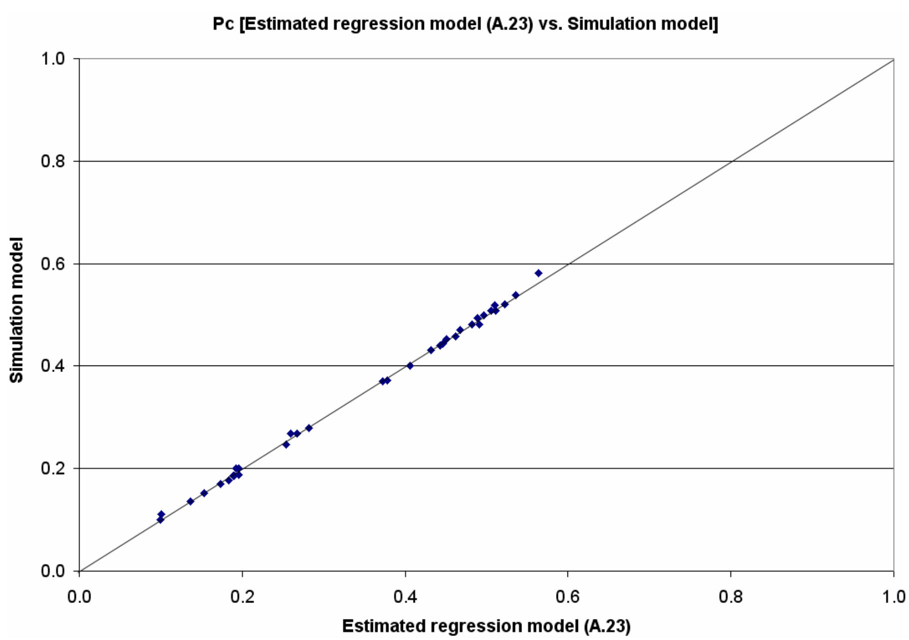

Figure 48. P_C [Estimated regression model (A.23) vs. Simulation model]

The estimated regression model (A.23) summarizes the simulation output well. However, we will use the simpler estimated model (A.18) rather than the estimated model (A.23).

Since we consider few alternative regression models which consider the effects of the MPA's information retention (include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$ as an independent variable), there may be an appropriate regression model in which $1/\psi$ is statistically significant to estimate the MIO capability (P_C) well. However, because the estimated regression model (A.18) summarizes the simulation output well, we consider the logistic regression model (A.18) to analyze the MIO capabilities (P_C).

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

We consider the following two types of logistic regression models as alternatives.

- $1/\psi$ is considered as a single independent variable.

$$\boxed{\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln\left(2v_I + 2u\right) \\ &+ b_4 \ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right) \\ &+ b_5\left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)\left(2v_I + 3u\right)\right)\right)^2 \\ &+ b_6\left(1/\psi\right)\end{aligned}} \quad \text{(A.24)}$$

When the sixth independent variable (the mean time until the MPA loses classification information of a classified vessel, $1/\psi$) is 0, the model (A.24) corresponds to the model (A.19) which does not consider $1/\psi$.

- $1/\psi$ influences the number of White vessels in the AOI, w.

$$
\begin{aligned}
\ln\left(\frac{P_E}{1-P_E}\right) &= b_0 + b_1 \ln c_{rr} + b_2 \ln u + b_3 \ln\left(2v_I + 2u\right) \\
&+ b_4 \ln\left(c_{rr} + \frac{w}{\sqrt[n]{1+1/\psi}}(1-c_{ww}) + \left(\frac{M_x}{f \cdot v} + \frac{\frac{w}{\sqrt[n]{1+1/\psi}}+1}{M_y} \cdot \tau \right)\left(2v_I + 3u\right) \right) \\
&+ b_5 \left(\ln\left(c_{rr} + \frac{w}{\sqrt[n]{1+1/\psi}}(1-c_{ww}) + \left(\frac{M_x}{f \cdot v} + \frac{\frac{w}{\sqrt[n]{1+1/\psi}}+1}{M_y} \cdot \tau \right)\left(2v_I + 3u\right) \right) \right)^2
\end{aligned}
\qquad \text{(A.25)}
$$

When the mean time until the MPA loses classification information of a classified vessel, $1/\psi$ is 0 or n is large, the model (A.25) corresponds to the model (A.19) which does not include $1/\psi$. n is determined so that the four independent variables are statistically significant in this regression model.

<u>The statistically fitted logistic regression models</u>:

- $1/\psi$ is considered as a single independent variable

$$
\begin{aligned}
\ln\left(\frac{P_E}{1-P_E}\right) &= 1.121 \ln c_{rr} - 1.474 \ln u + 1.966 \ln\left(2v_I + 2u\right) \\
&\quad - 2.797 \ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f \cdot v} + \frac{w+1}{M_y} \cdot \tau \right)\left(2v_I + 3u\right) \right) \\
&\quad + 0.271 \left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f \cdot v} + \frac{w+1}{M_y} \cdot \tau \right)\left(2v_I + 3u\right) \right) \right)^2 \\
&\quad + 0.002\left(1/\psi\right)
\end{aligned}
\qquad \text{(A.26)}
$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	1.000
Adjusted R Square	0.964
Standard Error	0.022
Observations	34

ANOVA

	df	SS	MS	F	Significance F
Regression	6	51.477	8.579	18264	0.000
Residual	28	0.013	0.000		
Total	34	51.490			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.000	-	-	-	-	-
b1	1.121	0.032	35.522	0.000	1.057	1.186
b2	-1.474	0.036	-40.484	0.000	-1.549	-1.399
b3	1.966	0.063	31.181	0.000	1.837	2.095
b4	-2.797	0.158	-17.711	0.000	-3.120	-2.473
b5	0.271	0.027	9.851	0.000	0.214	0.327
b6	0.002	0.003	0.758	0.455	-0.004	0.009

Since the p-value of b_6 is 0.455>0.05 and the 90% confidence interval contains 0, b_6 is not statistically different from 0. Figure 49 and Figure 50 display the outputs of the regression model (A.26) and those of the simulation model.

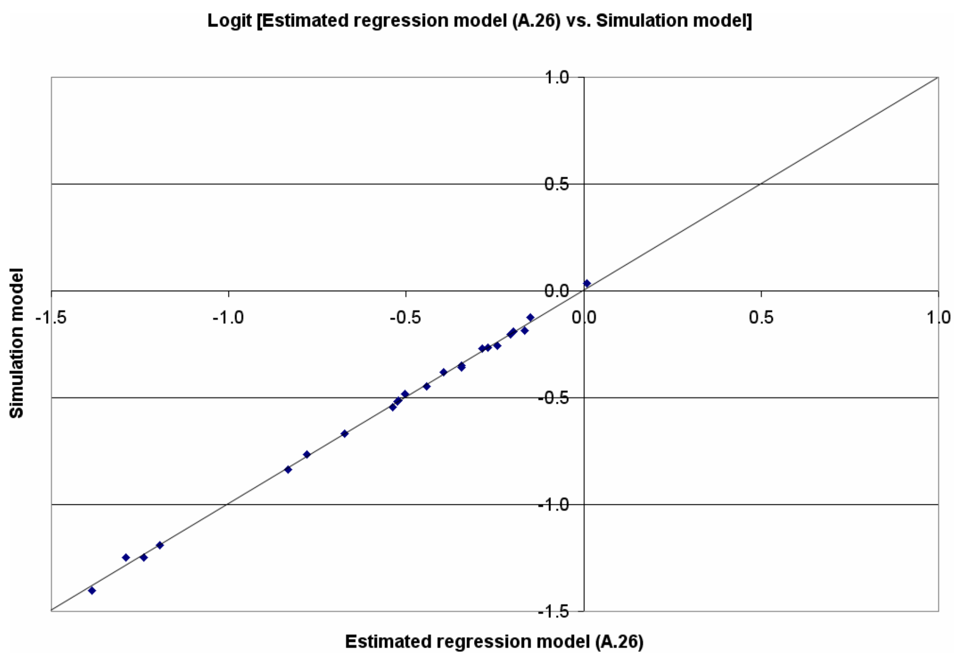

Figure 49. Logit [Estimated regression model (A.26) vs. Simulation model]

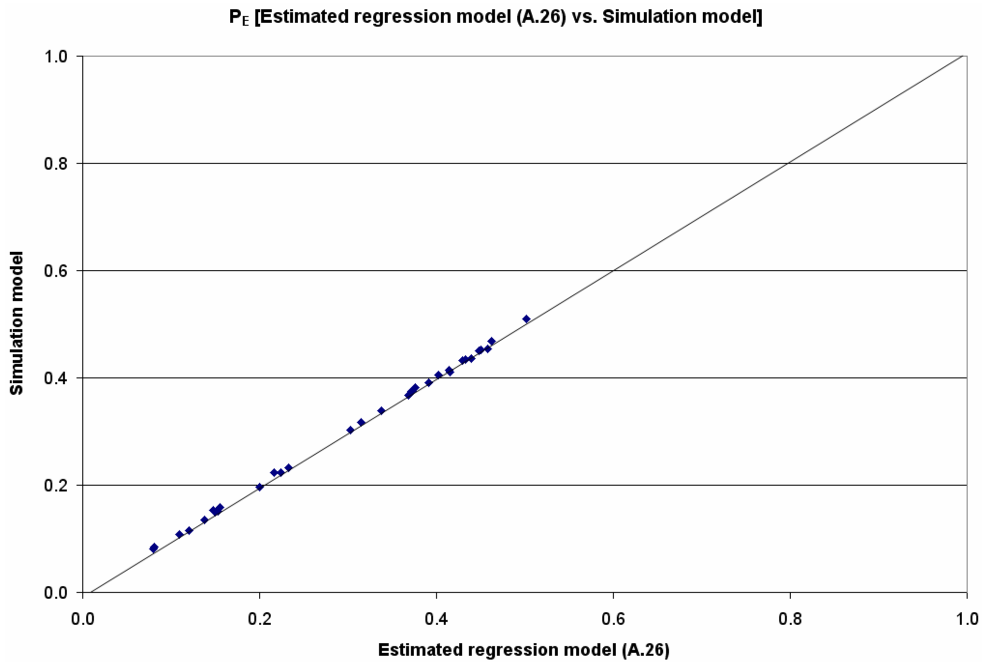

Figure 50. P_E [Estimated regression model (A.26) vs. Simulation model]

The estimated regression model (A.26) summarizes the output of the simulation model well. However, we employ the estimated model (A.19) rather than the estimated model (A.26) because the sixth independent variable of the model (A.26) is not statistically significant in the regression model.

- $1/\psi$ influences the number of White vessels in the AOI, w.

$$\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= 1.124\ln c_{rr} - 1.495\ln u + 2.055\ln\left(2v_I + 2u\right) \\ &-3.031\ln\left(c_{rr} + \frac{w}{\sqrt[10]{1+1/\psi}}(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{\frac{w}{\sqrt[10]{1+1/\psi}}+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right) \\ &+0.306\left(\ln\left(c_{rr} + \frac{w}{\sqrt[10]{1+1/\psi}}(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{\frac{w}{\sqrt[10]{1+1/\psi}}+1}{M_y}\cdot\tau\right)\left(2v_I+3u\right)\right)\right)^2\end{aligned} \tag{A.27}$$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1.000
R Square	0.999
Adjusted R Square	0.965
Standard Error	0.033
Observations	34

ANOVA

	df	SS	MS	F	Significance F
Regression	5	51.458	10.292	9403	0.000
Residual	29	0.032	0.001		
Total	34	51.490			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b0	0.000	-	-	-	-	-
b1	1.124	0.048	23.305	0.000	1.025	1.222
b2	-1.495	0.057	-26.406	0.000	-1.611	-1.379
b3	2.055	0.101	20.283	0.000	1.848	2.263
b4	-3.031	0.256	-11.831	0.000	-3.555	-2.507
b5	0.306	0.045	6.809	0.000	0.214	0.398

Since the R-square value of the regression model (A.27) is 0.999, and the p-value of each independent variable is 0, the estimated model (A.27) summarized the simulation model output well. When n is less than 10, we could not find an appropriate logistic regression model which satisfies a high R-square value and the significance of the four independent variables. Figure 51 and Figure 52 display the outputs of the regression model (A.27) and those of the simulation model.

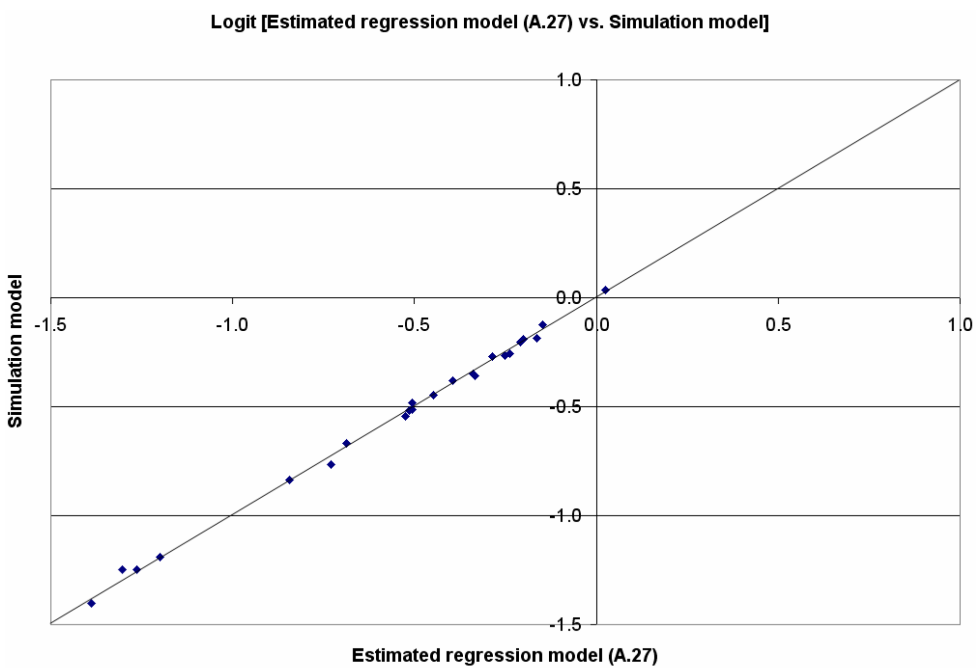

Figure 51. Logit [Estimated regression model (A.27) vs. Simulation model]

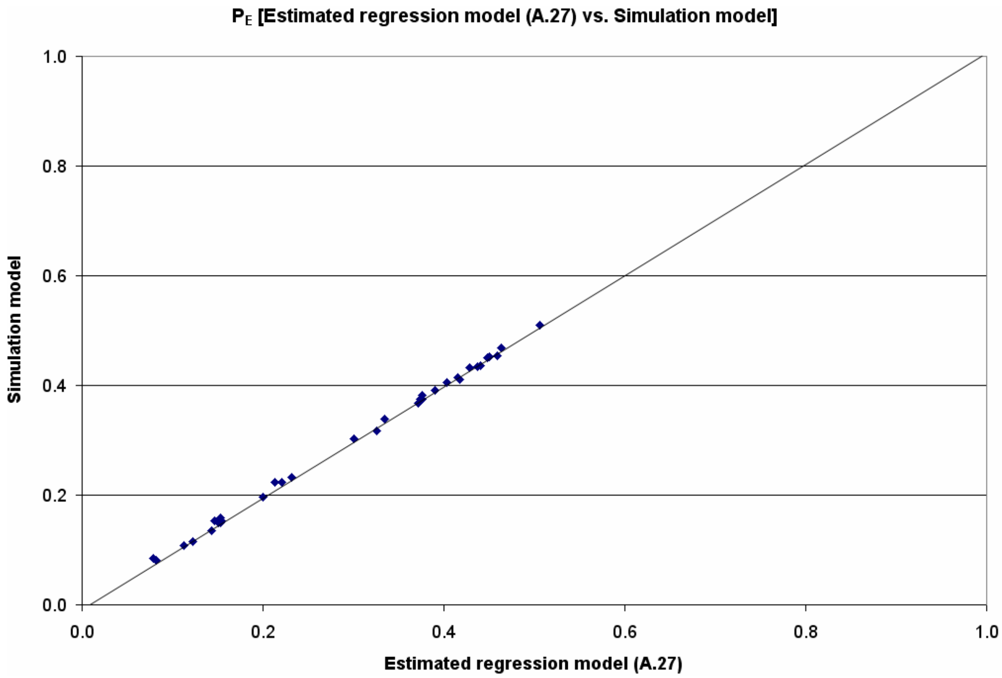

Figure 52. P_E [Estimated regression model (A.27) vs. Simulation model]

The estimated regression model (A.27) summarizes the simulation output well. However, we will use the simpler estimated model (A.19) rather than the estimated model (A.27).

Since we consider few alternative regression models which consider the effects of the MPA's information retention (include the mean time until the MPA loses classification information of a classified vessel, $1/\psi$ as an independent variable), there may be an appropriate regression model in which $1/\psi$ is statistically significant to estimate the MIO capability (P_E) well. However, because the estimated regression model (A.19) summarizes the simulation output well, we consider the logistic regression model (A.19) to analyze the MIO capabilities (P_E).

APPENDIX 4 VALIDATIONS OF THE ESTIMATED LOGISTIC REGRESSION MODELS

A. OBJECTIVE

The objective of this appendix is to study the ability of the estimated logistic regression models of Chapter IV for the Maritime Intercept Operations (MIO) capabilities to predict simulation output with new/different input values which lie in the parameter intervals given in Table 8 on page 47. The ability of the estimated regression models to predict simulation output using parameters outside of the intervals is not studied.

B. ESTIMATED LOGISTIC REGRESSION MODELS

1. Probability that the one R is detected and correctly classified before leaving the Area of Interest (AOI), P_C

$$\begin{aligned}\ln\left(\frac{P_C}{1-P_C}\right) &= 0.881 + 1.117\ln c_{rr} - 1.134\ln u \\ &\quad - 0.440\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right) \\ &\quad + 0.199\left(\ln\left(\frac{w(1-c_{ww})}{2v_I+3u} + \frac{M_x}{f\cdot v} + \frac{(w+1)\tau}{M_y}\right)\right)^2\end{aligned} \tag{A.28}$$

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

$$\begin{aligned}\ln\left(\frac{P_E}{1-P_E}\right) &= 1.122\ln c_{rr} - 1.474\ln u + 1.965\ln(2v_I+2u) \\ &\quad - 2.790\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)(2v_I+3u)\right) \\ &\quad + 0.269\left(\ln\left(c_{rr} + w(1-c_{ww}) + \left(\frac{M_x}{f\cdot v} + \frac{w+1}{M_y}\cdot\tau\right)(2v_I+3u)\right)\right)^2\end{aligned} \tag{A.29}$$

C. INPUT DATA

To design new input data set, we apply the similar approach by which we designed the original input data set. Design points (#1 – 17) are obtained from the NOLH based on Table 3 (page36), and design points (#18 – 34) are intentionally added to obtain relatively higher probabilities for P_C and P_E by using the other NOLH based on Table 5 (page38). The design points of Table 26 are completely different from those of Table 4 (page37); however they are chosen from the same parameter intervals given in Table 8 on page 47 as the original simulations.

Design Point ID	Mx (NM)	W	u (kt)	tau (min)	Cww	Crr	1/psi (hrs)
1	363	38	18.8	7.8	0.91	0.73	4.0
2	375	56	15.0	5.3	0.93	0.63	1.0
3	213	25	24.4	7.3	1.00	0.65	1.8
4	263	100	23.4	4.5	0.95	0.68	2.5
5	288	13	19.7	4.0	0.96	0.90	3.8
6	275	81	15.9	7.0	0.98	1.00	1.3
7	400	31	28.1	5.8	0.99	0.85	0.8
8	350	94	27.2	6.5	0.94	0.83	3.5
9	300	50	22.5	6.0	0.90	0.80	2.0
10	238	63	26.3	4.3	0.89	0.88	0.0
11	225	44	30.0	6.8	0.88	0.98	3.0
12	388	75	20.6	4.8	0.80	0.95	2.3
13	338	0	21.6	7.5	0.85	0.93	1.5
14	313	88	25.3	8.0	0.84	0.70	0.3
15	325	19	29.1	5.0	0.83	0.60	2.8
16	200	69	16.9	6.3	0.81	0.75	3.3
17	250	6	17.8	5.5	0.86	0.78	0.5
18	219	5	19.7	5.1	0.97	1.00	3.3
19	228	0	16.6	5.3	0.95	0.93	3.5
20	213	13	19.1	6.0	0.96	0.94	0.3
21	250	11	15.6	5.5	0.96	0.96	1.3
22	206	6	15.0	5.6	0.99	0.99	1.8
23	241	1	18.8	5.8	1.00	0.93	1.5
24	216	18	17.2	5.9	0.98	0.92	4.0
25	247	16	18.1	5.4	0.98	0.99	3.0
26	225	10	17.5	5.0	0.98	0.95	2.0
27	231	15	15.3	4.9	0.98	0.90	0.8
28	222	20	18.4	4.8	1.00	0.98	0.5
29	238	8	15.9	4.0	0.99	0.96	3.8
30	200	9	19.4	4.5	0.99	0.94	2.8
31	244	14	20.0	4.4	0.96	0.91	2.3
32	209	19	16.3	4.3	0.95	0.97	2.5
33	234	3	17.8	4.1	0.97	0.98	0.0
34	203	4	16.9	4.6	0.97	0.91	1.0

Table 26. Design of experiment

D. COMPARISON (LOGISTIC REGRESSION MODELS VS. SIMULATION MODELS)

1. Probability that the one R is detected and correctly classified before leaving the Area of Interest (AOI), P_C

Table 27 displays the outputs of the estimated logistic regression model (A.28) and those of the simulation model. Table 27 is sorted in descending order based on the outputs of the simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	1/psi (hrs)	Estimated Regression Model		Simulation model			
														Pc			Logit
												Logit	Pc	Mean	95%LB	95%UB	
22	206	200	6	200	30	15.0	15	5.6	0.99	0.99	1.8	0.3195	0.5792	0.5971	0.5875	0.6067	0.3934
34	203	200	4	200	30	16.9	15	4.6	0.97	0.91	1.0	0.1282	0.5320	0.5332	0.5234	0.5430	0.1330
19	228	200	0	200	30	16.6	15	5.3	0.95	0.93	3.5	0.0568	0.5142	0.5182	0.5084	0.5280	0.0728
29	238	200	8	200	30	15.9	15	4.0	0.99	0.96	3.8	0.0209	0.5052	0.5102	0.5004	0.5200	0.0408
32	209	200	19	200	30	16.3	15	4.3	0.95	0.97	2.5	-0.0375	0.4906	0.5001	0.4903	0.5099	0.0004
30	200	200	9	200	30	19.4	15	4.5	0.99	0.94	2.8	-0.0028	0.4993	0.4953	0.4855	0.5051	-0.0188
26	225	200	10	200	30	17.5	15	5.0	0.98	0.95	2.0	-0.0697	0.4826	0.4912	0.4814	0.5010	-0.0352
33	234	200	3	200	30	17.8	15	4.1	0.97	0.98	0.0	-0.0328	0.4918	0.4896	0.4798	0.4994	-0.0416
27	231	200	15	200	30	15.3	15	4.9	0.98	0.90	0.8	-0.0627	0.4843	0.4887	0.4789	0.4985	-0.0452
21	250	200	11	200	30	15.6	15	5.5	0.96	0.96	1.3	-0.1154	0.4712	0.4827	0.4729	0.4925	-0.0692
18	219	200	5	200	30	19.7	15	5.1	0.97	1.00	3.3	-0.0633	0.4842	0.4753	0.4655	0.4851	-0.0989
28	222	200	20	200	30	18.4	15	4.8	1.00	0.98	0.5	-0.1100	0.4725	0.4738	0.4640	0.4836	-0.1049
24	216	200	18	200	30	17.2	15	5.9	0.98	0.92	4.0	-0.1411	0.4648	0.4626	0.4528	0.4724	-0.1499
23	241	200	1	200	30	18.8	15	5.8	1.00	0.93	1.5	-0.1736	0.4567	0.4579	0.4481	0.4677	-0.1688
25	247	200	16	200	30	18.1	15	5.4	0.98	0.99	3.0	-0.2471	0.4385	0.4430	0.4333	0.4527	-0.2290
20	213	200	13	200	30	19.1	15	6.0	0.96	0.94	0.3	-0.1995	0.4503	0.4363	0.4266	0.4460	-0.2562
31	244	200	14	200	30	20.0	15	4.4	0.96	0.91	2.3	-0.4352	0.3929	0.3860	0.3765	0.3955	-0.4642
17	250	200	6	200	30	17.8	15	5.5	0.86	0.78	0.5	-0.5118	0.3748	0.3621	0.3527	0.3715	-0.5663
5	288	200	13	200	30	19.7	15	4.0	0.96	0.90	3.8	-0.6183	0.3502	0.3501	0.3408	0.3594	-0.6186
6	275	200	81	200	30	15.9	15	7.0	0.98	1.00	1.3	-0.7435	0.3222	0.3345	0.3253	0.3437	-0.6879
13	338	200	0	200	30	21.6	15	7.5	0.85	0.93	1.5	-0.7860	0.3130	0.3173	0.3082	0.3264	-0.7662
3	213	200	25	200	30	24.4	15	7.3	1.00	0.65	1.8	-0.9631	0.2763	0.2659	0.2572	0.2746	-1.0155
11	225	200	44	200	30	30.0	15	6.8	0.88	0.98	3.0	-1.3293	0.2093	0.2062	0.1983	0.2141	-1.3480
1	363	200	38	200	30	18.8	15	7.8	0.91	0.73	4.0	-1.4337	0.1925	0.2055	0.1976	0.2134	-1.3523
2	375	200	56	200	30	15.0	15	5.3	0.93	0.63	1.0	-1.4147	0.1955	0.1963	0.1885	0.2041	-1.4096
9	300	200	50	200	30	22.5	15	6.0	0.90	0.80	2.0	-1.4612	0.1883	0.1926	0.1849	0.2003	-1.4332
10	238	200	63	200	30	26.3	15	4.3	0.89	0.88	0.0	-1.4388	0.1917	0.1897	0.1820	0.1974	-1.4520
7	400	200	31	200	30	28.1	15	5.8	0.99	0.85	0.8	-1.5431	0.1761	0.1721	0.1647	0.1795	-1.5708
16	200	200	69	200	30	16.9	15	6.3	0.81	0.75	3.3	-1.5353	0.1772	0.1687	0.1614	0.1760	-1.5949
4	263	200	100	200	30	23.4	15	4.5	0.95	0.68	2.5	-1.6818	0.1569	0.1540	0.1469	0.1611	-1.7036
12	388	200	75	200	30	20.6	15	4.8	0.80	0.95	2.3	-1.7364	0.1498	0.1424	0.1356	0.1492	-1.7955
8	350	200	94	200	30	27.2	15	6.5	0.94	0.83	3.5	-1.8940	0.1308	0.1367	0.1300	0.1434	-1.8430
15	325	200	19	200	30	29.1	15	5.0	0.83	0.60	2.8	-1.8688	0.1337	0.1302	0.1236	0.1368	-1.8992
14	313	200	88	200	30	25.3	15	8.0	0.84	0.70	0.3	-2.2566	0.0948	0.0942	0.0885	0.0999	-2.2634

Table 27. Estimated regression model (A.28) and the simulation model

Figure 53 and Figure 54 display the outputs of the estimated logistic regression model (A.28) and those of the simulation model. The estimated regression model (A.28) summarizes the output of the simulation model well.

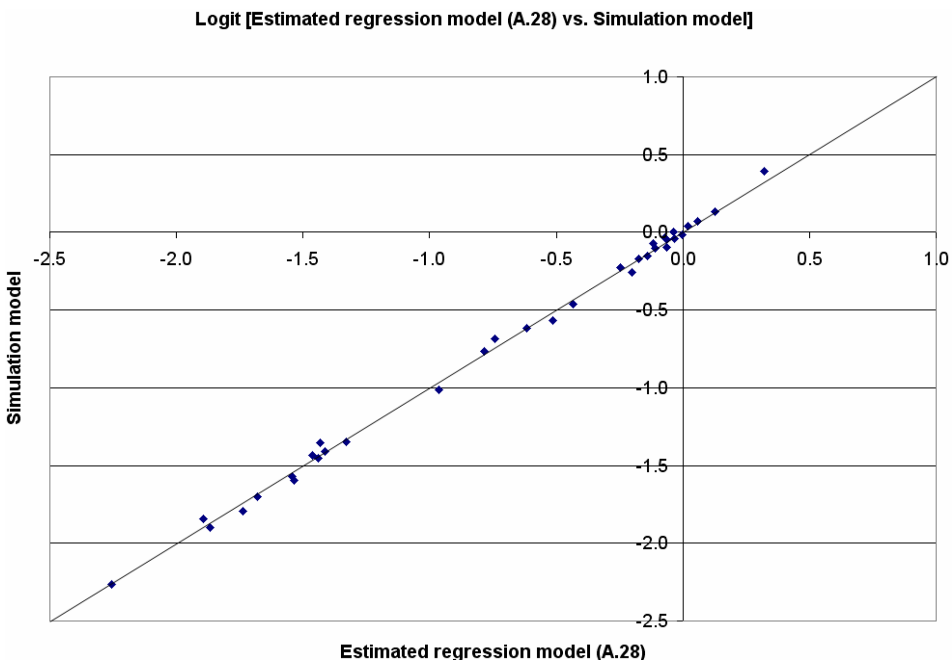

Figure 53. Logit [Estimated regression model (A.28) and the simulation model]

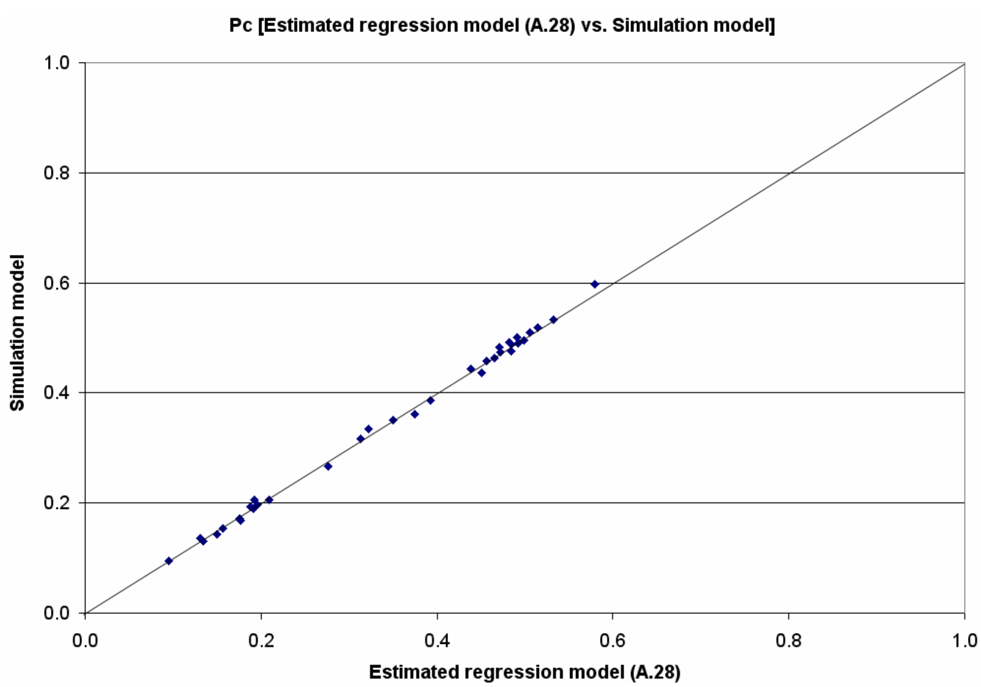

Figure 54. P_C [Estimated regression model (A.28) and the simulation model]

2. Probability that the one R is detected, correctly classified and escorted before leaving the AOI, P_E

Table 28 displays the outputs of the estimated logistic regression model (A.29) and those of the simulation model. Table 28 is sorted in descending order based on the outputs of the simulation model.

Design Point ID	Mx (NM)	My (NM)	W	V (kt)	Vi (kt)	u (kt)	f (NM)	tau (min)	Cww	Crr	1/psi (hrs)	Estimated Regression Model		Simulation model			
														P_E			Logit
												Logit	P_E	Mean	95%LB	95%UB	
22	206	200	6	200	30	15.0	15	5.6	0.99	0.99	1.8	0.0822	0.5205	0.5298	0.5200	0.5396	0.1193
34	203	200	4	200	30	16.9	15	4.6	0.97	0.91	1.0	-0.1313	0.4672	0.4648	0.4550	0.4746	-0.1410
19	228	200	0	200	30	16.6	15	5.3	0.95	0.93	3.5	-0.1981	0.4506	0.4481	0.4384	0.4578	-0.2084
29	238	200	8	200	30	15.9	15	4.0	0.99	0.96	3.8	-0.2241	0.4442	0.4405	0.4308	0.4502	-0.2391
32	209	200	19	200	30	16.3	15	4.3	0.95	0.97	2.5	-0.2902	0.4280	0.4330	0.4233	0.4427	-0.2696
27	231	200	15	200	30	15.3	15	4.9	0.98	0.90	0.8	-0.2873	0.4287	0.4281	0.4184	0.4378	-0.2896
30	200	200	9	200	30	19.4	15	4.5	0.99	0.94	2.8	-0.3062	0.4240	0.4251	0.4154	0.4348	-0.3019
33	234	200	3	200	30	17.8	15	4.1	0.97	0.98	0.0	-0.3130	0.4224	0.4231	0.4134	0.4328	-0.3101
26	225	200	10	200	30	17.5	15	5.0	0.98	0.95	2.0	-0.3402	0.4158	0.4216	0.4119	0.4313	-0.3162
21	250	200	11	200	30	15.6	15	5.5	0.96	0.96	1.3	-0.3532	0.4126	0.4166	0.4069	0.4263	-0.3367
18	219	200	5	200	30	19.7	15	5.1	0.97	1.00	3.3	-0.3751	0.4073	0.4066	0.3970	0.4162	-0.3780
28	222	200	20	200	30	18.4	15	4.8	1.00	0.98	0.5	-0.3975	0.4019	0.4047	0.3951	0.4143	-0.3859
24	216	200	18	200	30	17.2	15	5.9	0.98	0.92	4.0	-0.4014	0.4010	0.3990	0.3894	0.4086	-0.4096
23	241	200	1	200	30	18.8	15	5.8	1.00	0.93	1.5	-0.4601	0.3870	0.3882	0.3786	0.3978	-0.4549
25	247	200	16	200	30	18.1	15	5.4	0.98	0.99	3.0	-0.5265	0.3713	0.3796	0.3701	0.3891	-0.4912
20	213	200	13	200	30	19.1	15	6.0	0.96	0.94	0.3	-0.4908	0.3797	0.3666	0.3572	0.3760	-0.5468
31	244	200	14	200	30	20.0	15	4.4	0.96	0.91	2.3	-0.7288	0.3255	0.3241	0.3149	0.3333	-0.7350
17	250	200	6	200	30	17.8	15	5.5	0.86	0.78	0.5	-0.7610	0.3184	0.3080	0.2990	0.3170	-0.8095
5	288	200	13	200	30	19.7	15	4.0	0.96	0.90	3.8	-0.9020	0.2886	0.2916	0.2827	0.3005	-0.8876
6	275	200	81	200	30	15.9	15	7.0	0.98	1.00	1.3	-0.9859	0.2717	0.2889	0.2800	0.2978	-0.9007
13	338	200	0	200	30	21.6	15	7.5	0.85	0.93	1.5	-1.0859	0.2524	0.2573	0.2487	0.2659	-1.0600
3	213	200	25	200	30	24.4	15	7.3	1.00	0.65	1.8	-1.2726	0.2188	0.2117	0.2037	0.2197	-1.3147
1	363	200	38	200	30	18.8	15	7.8	0.91	0.73	4.0	-1.6840	0.1566	0.1697	0.1623	0.1771	-1.5878
2	375	200	56	200	30	15.0	15	5.3	0.93	0.63	1.0	-1.6274	0.1642	0.1655	0.1582	0.1728	-1.6179
11	225	200	44	200	30	30.0	15	6.8	0.88	0.98	3.0	-1.6445	0.1619	0.1628	0.1556	0.1700	-1.6375
9	300	200	50	200	30	22.5	15	6.0	0.90	0.80	2.0	-1.7351	0.1499	0.1562	0.1491	0.1633	-1.6868
10	238	200	63	200	30	26.3	15	4.3	0.89	0.88	0.0	-1.7318	0.1504	0.1494	0.1424	0.1564	-1.7393
16	200	200	69	200	30	16.9	15	6.3	0.81	0.75	3.3	-1.7698	0.1456	0.1406	0.1338	0.1474	-1.8103
7	400	200	31	200	30	28.1	15	5.8	0.99	0.85	0.8	-1.8373	0.1374	0.1365	0.1298	0.1432	-1.8447
4	263	200	100	200	30	23.4	15	4.5	0.95	0.68	2.5	-1.9533	0.1242	0.1226	0.1162	0.1290	-1.9680
12	388	200	75	200	30	20.6	15	4.8	0.80	0.95	2.3	-1.9692	0.1225	0.1161	0.1098	0.1224	-2.0299
8	350	200	94	200	30	27.2	15	6.5	0.94	0.83	3.5	-2.1469	0.1046	0.1085	0.1024	0.1146	-2.1062
15	325	200	19	200	30	29.1	15	5.0	0.83	0.60	2.8	-2.1624	0.1032	0.1004	0.0945	0.1063	-2.1928
14	313	200	88	200	30	25.3	15	8.0	0.84	0.70	0.3	-2.4785	0.0774	0.0749	0.0697	0.0801	-2.5137

Table 28. Estimated regression model (A.29) and the simulation model

Figure 55 and Figure 56 display the outputs of the estimated logistic regression model (A.29) and those of the simulation model. The estimated regression model (A.29) summarizes the output of the simulation model well.

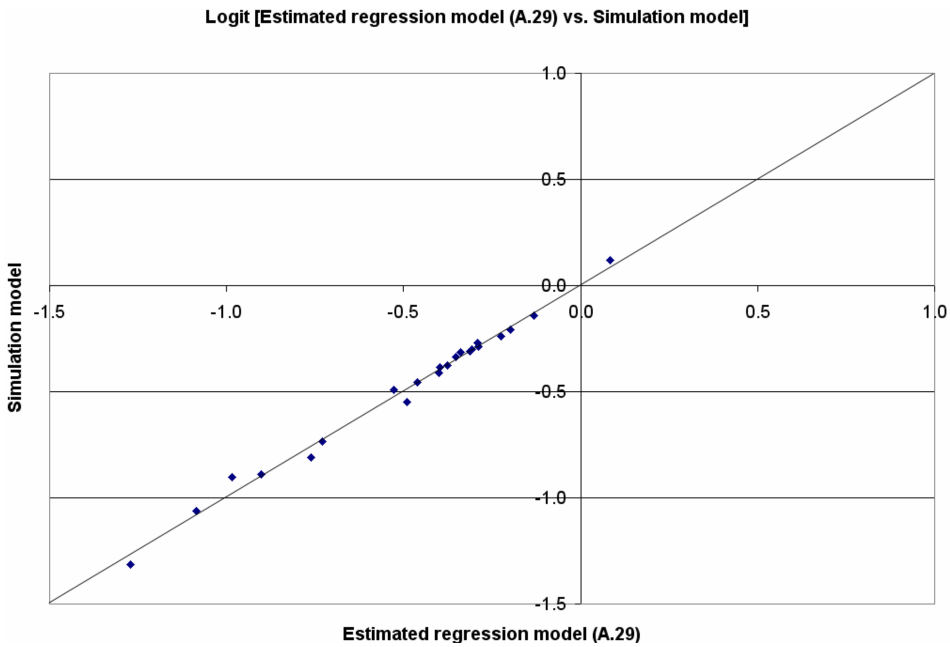

Figure 55. Logit [Estimated regression model (A.29) and the simulation model]

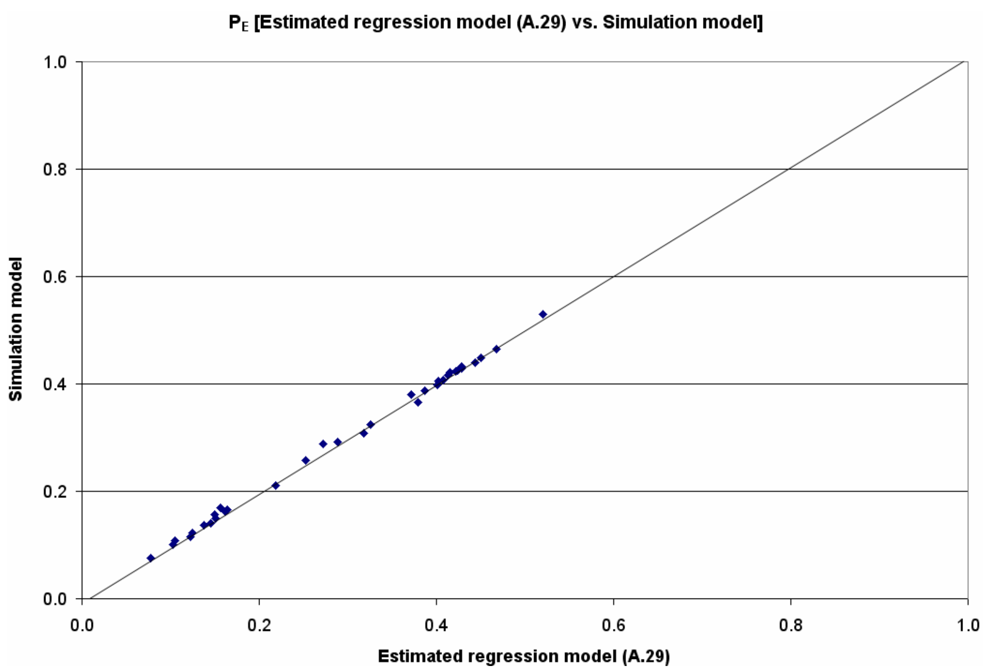

Figure 56. P_E [Estimated regression model (A.29) and the simulation model]

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