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Optimal Restraint Characteristics for Minimization of Peak Occupant Deceleration in Frontal Impact

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ABSTRACT

In automobile frontal impact, given the vehicle motion and the interior free space for the occupant's excursion, what are the optimal characteristics of restraint systems for the minimization of the peak occupant deceleration? In this paper, the problem is treated as the optimal protection from impact based on a lumped-parameter model of the occupant-vehicle system. The optimal kinematics of the occupant in frontal impact is studied. The optimal characteristics of passive restraint systems are investigated in detail for three types of vehicle crash pulses: optimal pulse, constant deceleration pulse, and half-sine pulse. Optimization of the characteristics of active and pre-acting restraint systems is addressed. It is found that the optimal kinematics of the occupant in frontal impact is such that the occupant moves at a constant deceleration. Passive restraint systems are not able to provide required protection for the occupant to attain optimal kinematics, but active and pre-acting restraint systems can achieve that if optimally designed.

INTRODUCTION

The use of occupant restraint systems, such as seatbelts and airbags, has helped to reduce the severity of injuries to the head, chest, and extremities, and has led to an increase in the survival rate of occupants in automobile crashes. As restraint systems play crucial roles in the prevention and reduction of occupant injuries in automobile crashes, their performance and effectiveness for injury prevention and reduction becomes an important issue in automobile design.

Engineering design of restraint systems is generally executed via integrated testing with analytical modeling and analysis. Optimization is an effective tool for improving the effectiveness of restraint systems. The optimization problem can be dealt with analytically with two approaches, direct and indirect [1]. The direct approach uses physical parameters of a restraint system as the design variables in optimization. As the objective function in optimization is to minimize certain injury criteria, which in turn involve respective occupant responses, it is necessary to establish a direct relationship between these physical parameters and occupant responses. This requires efficient and accurate modeling and simulation of the problem, which, at the present time and for given complexity of automobile structures/restraint systems, occupants, and the interaction between them, still represents a great challenge. The indirect approach handles the issue by a two-step strategy: first, for the minimization of the occupant injuries, find the optimal characteristics for a restraint system; then, either solve an inverse design problem for the system according to its optimal characteristics or use the optimal characteristics as a guideline for the restraint system design.

The study in this paper will use the indirect approach and deal with the problem in the first step for frontal impact, i.e., the optimal restraint characteristics. The question can be expressed as: for prescribed vehicle motion in frontal impact (including impact speed, crash deformation, and crash pulse), what are the optimal characteristics of restraint systems that provide the best possible protection for the occupant?

Optimal characteristics of restraint systems have produced extensive research, along with the development of seat belts, airbags, and vehicle interior padding. For example, two studies were performed recently by the authors on the limiting or best possible performance of the seat belt and toepan padding. In [2], the theoretical optimal performance of seat belt systems in the prevention and reduction of thoracic injuries of occupants in automobile frontal crashes was investigated based on a two-mass injury model of the thorax. The results indicated that the optimal seat belt force should keep the major mass of the thorax decelerating at a constant rate, and the optimal seat belt force should have an initial impulse and remain constant afterwards. In [3], potential use of toepan padding for mitigating lower limb injuries was studied computationally with a two-mass lower limb injury model developed from test data. The control force was used to represent the characteristics of passive and active padding. The results showed that the optimal control force that results in the minimum tibia force should have an impulse initially and then remain constant throughout the impact duration; correspondingly, the major mass of the lower limb would move at a constant deceleration during impact.

In this paper, the problem of the optimal characteristics of restraint systems is treated as a problem of optimal protection from impact.

SYSTEM MODELING AND PROBLEM STATEMENT

Whereas automobile frontal impact can now be well described by a rigid multi-body (RMB) model, a finite element (FE) model, or an integrated model of RMB and FE [4], with different levels of abstraction, the description of the problem with a simple lumped-parameter model is still advantageous and desirable, as it could have an analytical solution and provide insight into the problem. In frontal impact, the occupant can be subject to injuries to various regions of the body, such as the head, thorax, upper extremities, and lower extremities. These injuries are the results of excessive stresses in respective regions induced by impact. Injury criteria, which are used to measure these injuries, are usually expressed by impact responses (such as acceleration) in respective regions. When the prevention and reduction of injuries to the occupant is considered in general, the occupant can be reasonably treated as a point mass and its peak acceleration (deceleration) can be used as the injury criterion. The vehicle can then be treated as a whole body and only its gross motion is considered.

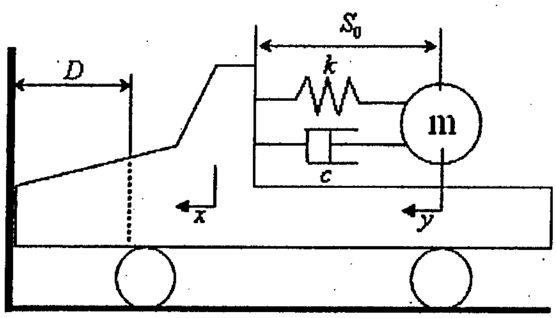


Figure 1. A lumped-parameter model of the system

Therefore, automobile frontal impact is described by a lumped-parameter model shown in Fig. 1, where the occupant is represented by a point mass m, $x(t)$ describes the gross motion of the vehicle, and $y(t)$ is the motion of the occupant relative to the vehicle. The interaction between the occupant and the restraint systems such as seat belt and airbag is simply represented by a spring and a damper in the figure, but in general, it can be described by a control force $u(y,\dot{y},t)$. For the vehicle motion,

$$a(t) = -\ddot{x}(t), \tag{1}$$

usually referred to as the vehicle crash pulse, and

$$D_v = \max_t\{x(t)\}, \tag{2}$$

defined as the vehicle crash deformation. The free space between the occupant and the vehicle interior is called the rattlespace S_0, which is the maximum allowable forward excursion of the occupant with respect to the vehicle, i.e.,

$$\max_t\{y(t)\} \le S_0, \tag{3}$$

The equation of motion of the system is

$$m(\ddot{x}+\ddot{y}) + u(y,\dot{y},t) = 0, \tag{4}$$

with the initial conditions

$$x(0)=0,\ \dot{x}(0)=v_0, \tag{5}$$

and

$$y(0)=0,\ \dot{y}(0)=0, \tag{6}$$

where v_0 is the impact velocity.

The problem to be addressed in this paper can be expressed as:

Problem-1: For prescribed crash pulse and rattlespace, find the optimal characteristics for restraint systems such that the peak occupant deceleration is minimized. The problem can be formulated as: Find the optimal restraint characteristics $u_0(y,\dot{y},t)$, such that

$$J_1(u_0) = \min_u\{J_1(u) \mid J_2(u) \le S_0\}. \tag{7}$$

where

$$J_1 = \max_t\{-[\ddot{x}(t)+\ddot{y}(t)]\}, \tag{8}$$

the peak occupant deceleration, and

$$J_2 = \max_t\{y(t)\}, \tag{9}$$

the maximum forward excursion of the occupant in the vehicle.

In view of the kinematics of the occupant, *Problem-1* can be reduced to a more generic problem:

Problem-2: For given impact velocity, crash deformation, and rattlespace, find the optimal kinematics of the occupant such that the peak occupant deceleration is minimized while the maximum occupant forward displacement is bounded. The problem can be formulated as: Find the optimal kinematics of the occupant $w_0(t)$ such that

$$J_1(w_0) = \min_w\{J_1(w) \mid J_3(w) \le D_o\}, \tag{10}$$

where

$$J_3 = \max_t\{x(t)+y(t)\}, \tag{11}$$

which is the maximum forward displacement of the occupant, and

$$D_o = D_v + S_0, \quad (12)$$

the allowable maximum forward excursion for the occupant. Unlike $u(y,\dot{y},t)$ in *Problem-1*, $w(t)$ is not a control function of the system. Instead, it is controlled or produced by $u(y,\dot{y},t)$.

OPTIMAL OCCUPANT KINEMATICS

To find the optimal kinematics of the occupant for *Problem-2*, according to duality or reciprocity of optimization [5], the dual or reciprocal problem of *Problem-2* can be formulated as follows.

Problem-3: For given impact velocity, vehicle crash deformation, and rattlespace, find the optimal kinematics of the occupant such that the maximum occupant displacement is minimized while the peak occupant deceleration is bounded. The problem can be formulated as: Find the optimal kinematics of the occupant $w_0(t)$, such that

$$J_3(w_0) = \min_w \{J_3(w) \mid J_1(w) \le A_m\}, \quad (13)$$

where A_m is the upper bound on the peak occupant deceleration.

To find the theoretical solution of *Problem-3*, denote

$$z(t) = x(t) + y(t), \quad (14)$$

which is the absolute motion of the occupant with respect to an inertial frame during impact. Consider the motion of the system from the initial $t = 0$ to the instant $t = T_0$ when the occupant (m) comes to a rest for the first time. That is,

$$v_0 + \int_0^{T_0} \ddot{z}(\tau)d\tau = 0. \quad (15)$$

The velocity of the occupant $\dot{z}(t)$ starts with v_0 at $t = 0$ and decreases to 0 at $t = T_0$, i.e.,

$$\dot{z}(t) \ge 0, \; 0 \le t \le T_0. \quad (16)$$

Therefore, for $0 \le t \le T_0$, the displacement of the occupant $z(t)$, which is given by

$$z(t) = \int_0^t \dot{z}(\tau)d\tau, \quad (17)$$

increases monotonically with respect to time and reaches its maximum value at $t = T_0$, that is,

$$\max_t\{z(t)\} = z(T_0). \quad (18)$$

To minimize the maximum occupant forward displacement J_3

$$J_3 = \max_t\{x(t) + y(t)\} = \max_{t\in[0,T_0]}\{z(t)\} = z(T_0), \quad (19)$$

T_0 should be minimized. According to the definition of T_0 (Eq. (15)), if the deceleration of the occupant $-\ddot{z}(t)$ takes the maximum allowable value A_m, that is,

$$\ddot{z}(t) = -A_m, \quad (20)$$

T_0 is minimized, and

$$T_0 = \frac{v_0}{A_m}. \quad (21)$$

This means that in order to minimize the peak displacement of the occupant, the deceleration of the occupant should remain constant at the value of A_m. This represents the optimal kinematics of the occupant for *Problem-3*.

Based on the duality or reciprocity between *Problem-2* and *Problem-3*, the optimal kinematics of the occupant for *Problem-2* can be stated as: In order to minimize the peak occupant deceleration, the deceleration of the occupant should remain constant at a value, which is denoted as A_m and is given by

$$A_m = v_0^2/(2D_o). \quad (22)$$

This is also the optimal kinematics of the occupant for *Problem-1*.

OPTIMAL RESTRAINT CHARACTERISTICS

In terms of the manner in which a restraint system exerts its action on the occupant, restraint systems can be categorized into three types: passive, active, and pre-acting [6]. In this paper, a safety device is considered to be passive if it generates a force only when it is being externally excited. It is active if it can generate the force not only from the external action exerted on it, but also from the internal power source it has.

An example of passive systems is an ordinary knee bolster that provides a cushion between the knee and the front interior of the vehicle. The cushion generates a dynamic reaction force on the occupant's knee only when it is compressed. If the knee separates from the cushion, the cushion will apply no action on it. The reaction force of a cushion depends on its deformation and deformation rate. The action of a passive restraint system on the occupant can be expressed as

$$u = u(y,\dot{y}). \quad (23)$$

An airbag is an active restraint system. The deployment of an airbag is controlled by an inflator that is an internal power source. Therefore, the reaction force of a deploying airbag on the occupant is time-dependent, although it also depends on the interaction with the occupant. The action of an active restraint system on the occupant can be expressed as

$$u = u(y,\dot{y},t). \quad (24)$$

A safety device, such as a seatbelt pre-tensioner, can start to act before the onset of impact and will be referred to as a pre-acting system. A seatbelt pre-tensioner removes the slack in the belt and applies an initial load to the occupant before the occupant moves forward and stretches the belt. A pre-acting mechanism can be incorporated into a passive or an active restraint system. Its action on the occupant can be described as

$$u = u(y, \dot{y}, t, t_0), \qquad (25)$$

where $t_0 < 0$ denotes the time instant at which the device starts to exert actions.

According to Eqs. (4) and (14), when the occupant deceleration remains constant at the value of A_m,

$$u = -m\ddot{z} = mA_m. \qquad (26)$$

This means that restraint systems have to exert a constant force on the occupant. How to achieve this? The optimal characteristics of each type of restraint systems will be investigated or discussed in the following analyses.

Passive Restraint Systems

As the vehicle motion (crash pulse) is prescribed and the optimal kinematics of the occupant is determined, the relative motion between the occupant and the vehicle is determined too, that is,

$$\begin{aligned} \dot{y}(t) &= \dot{z}(t) - \dot{x}(t) \\ y(t) &= z(t) - x(t) \end{aligned} \qquad (27)$$

According to Eqs. (23) and (26), the restraint characteristics need to meet the following relation:

$$u(y, \dot{y}) = mA_m. \qquad (28)$$

However, since only a limited number of parameters can be chosen for a restraint system, for a given crash pulse in a general form, it is impossible for Eq. (28) to be satisfied every time for the entire impact duration, regardless whether a system is linear or nonlinear.

If the characteristics of restraint systems are parameterized and represented by a set of parameters $\{u_1 u_2 \cdots u_N\}$, parametric optimization can be utilized to find the optimal parameters of the systems. Denote

$$\mathbf{u} = [u_1 u_2 \cdots u_N]^T, \qquad (29)$$

$$J_1(\mathbf{u}) = \max_t\{-\ddot{z}(t)\}, \qquad (30)$$

$$J_2(\mathbf{u}) = \max_t\{y(t)\}. \qquad (31)$$

Then this parametric optimization problem can be formulated as

Design Variables: $\mathbf{u}$;

Objective Function: $\min\{J_1(\mathbf{u})\}$; (32)

Constraints: $J_2(\mathbf{u}) \le S_0$, and $\mathbf{u}_L \le \mathbf{u} \le \mathbf{u}_U$;

where $\mathbf{u}_L$ and $\mathbf{u}_U$ are the lower and upper bounds on the parameters of restraint characteristics.

Suppose the characteristics of passive restraint systems are linear and expressed by a linear spring and a linear damper, that is,

$$u(y, \dot{y}) = ky + c\dot{y}. \qquad (33)$$

Then, analytical solutions of the occupant responses can be found for several particular types of crash pulses that are described below. It can be reasonably assumed that the peak occupant deceleration occurs within the duration of the vehicle crash pulse. Therefore, the occupant responses will be considered within the time span of the crash pulse in the following analyses. In the following computations, choose impact velocity $v_0 = 15.56$ m/s (35 mph) and crash deformation $D_v = 0.71$ m (28 in), to be representative of automobile frontal impact [1].

Optimal Pulse

For a linear passive system whose characteristics are represented by a spring and a damper, the theoretically optimal crash pulse is given by [7]

$$\ddot{x}(t) = -A_m[1 + \frac{m}{c}\delta(t) - \frac{km}{c^2}e^{-\frac{k}{c}t}],\ 0 \le t \le T_v, \qquad (34)$$

where T_v is the duration of the crash pulse, and $\delta(t)$ is the Dirac delta function which is given by

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \ne 0 \end{cases}, \qquad (35)$$

and

$$\int_{-\infty}^{\infty} \delta(t)dt = 1. \qquad (36)$$

Since the impulse at the initial of impact, which is represented by $\delta(t)$, cannot be produced in practice, Eq. (34) represents a theoretically optimal crash pulse. Under this pulse, the occupant moves at a constant deceleration of A_m, which is determined by

$$\begin{aligned} A_m &= \frac{kD_v}{2m}\left(\sqrt{1 + \frac{2mv_0^2}{kD_v^2}} - 1\right) \\ &= \frac{\omega^2 D_v}{2}\left(\sqrt{1 + \frac{2v_0^2}{\omega^2 D_v^2}} - 1\right) \end{aligned}, \qquad (37)$$

where

$$\omega^2 = \frac{k}{m}, \qquad (38)$$

which can be considered as the natural frequency of the restraint-occupant system.

In order to meet the requirement of Eq. (3) ($\max_t\{y(t)\} \le S_0$), it is required that

$$\omega^2 \ge \frac{v_0^2}{2S_0(D_v + S_0)}. \qquad (39)$$

If

$$\omega^2 = \frac{v_0^2}{2S_0(D_v + S_0)}, \tag{40}$$

the rattlespace will be fully used for the occupant's excursion, i.e., $\max_t\{y(t)\} = S_0$, and then the maximum deceleration of the occupant will be minimized and is given by

$$\max_t\{-\ddot{z}(t)\} = A_m = \frac{v_0^2}{2(D_v + S_0)}. \tag{41}$$

Equation (41) describes the relationship between the peak occupant deceleration A_m and the rattlespace S_0, which is displayed by the blue curve in Fig. 2 for a range of S_0. The curve indicates that as the rattlespace S_0 increases, the peak occupant deceleration A_m decreases. Based on Eq. (40), the relationship between the natural frequency ω and the rattlespace S_0 is displayed by the blue curve in Fig. 3, which indicates that the optimal natural frequency decreases with the increase of the rattlespace. In other words, a restraint system needs to be softer in order for a larger rattlespace to be fully used. Note that under the optimal vehicle crash pulse, the peak occupant deceleration is independent of the damping of restraint characteristics. However, certain damping in restraint systems is required, as indicated by Eq. (35).

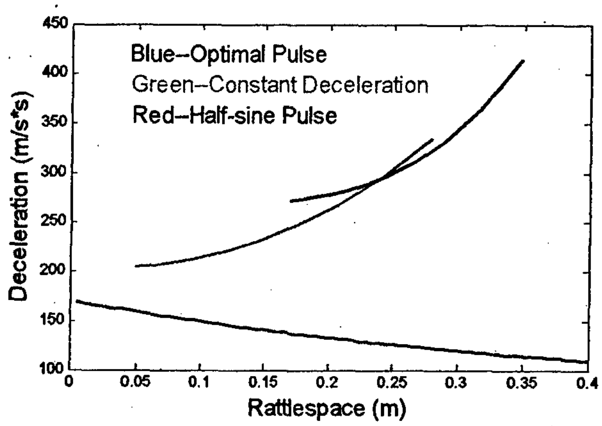


Figure 2. Relationship between the peak occupant deceleration and rattlespace

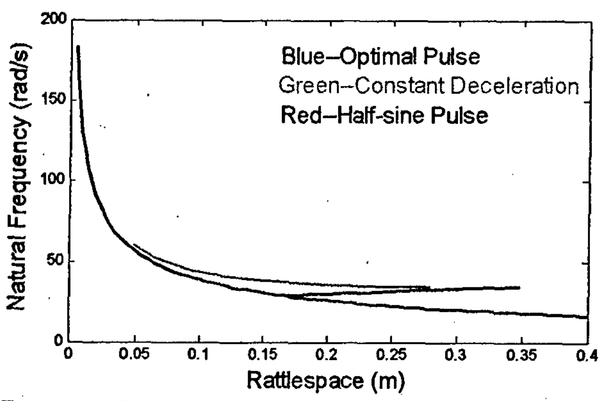


Figure 3. Relationship between the natural frequency and rattlespace

Constant Deceleration Pulse

Suppose the vehicle deceleration is constant, that is

$$\ddot{x}(t) = -A_v,\ 0 \le t \le T_v, \tag{42}$$

where

$$A_v = \frac{v_0^2}{2D_v}, \tag{43}$$

and

$$T_v = \frac{2D_v}{v_0}. \tag{44}$$

The velocity and displacement of the vehicle are

$$\dot{x}(t) = v_0 - A_v t, \tag{45}$$

and

$$x(t) = v_0 t - \frac{1}{2}A_v t^2. \tag{46}$$

The motion of the occupant under constant deceleration pulse can be expressed as [7]:

$$z(t) = x(t) + y(t) = x(t) + y_c(t) + y_p(t), \tag{47}$$

where $x(t)$ is given by Eq. (46),

$$y_c(t) = Y_c e^{-\zeta\omega t}\sin(\sqrt{1-\zeta^2}\,\omega t + \beta_y), \tag{48}$$

and

$$y_p(t) = \frac{A_v}{\omega^2}[1 - \frac{e^{-\zeta\omega t}}{\sqrt{1-\zeta^2}}\cos(\sqrt{1-\zeta^2}\,\omega t - \alpha_y)]. \tag{49}$$

In Eq. (48),

$$Y_c = \sqrt{\frac{C_{1y}^2 + C_{2y}^2 + 2\zeta C_{1y}C_{2y}}{1-\zeta^2}}, \tag{50}$$

and

$$\beta_y = \tan^{-1}\frac{\sqrt{1-\zeta^2}\,C_{1y}}{\zeta C_{1y} + C_{2y}}, \tag{51}$$

where

$$\zeta = \frac{c}{2\omega m} = \frac{c}{2\sqrt{km}}, \tag{52}$$

which is referred to as the damping ratio of the restraint-occupant system, and

$$C_{1y} = \frac{A_v}{\omega^2}(\frac{1}{\sqrt{1-\zeta^2}}\cos\alpha_y - 1)$$
$$C_{2y} = \frac{A_v}{\omega^2}(\sin\alpha_y - \frac{\zeta}{\sqrt{1-\zeta^2}}\cos\alpha_y) \tag{53}$$

In Eqs. (49) and (53)

$$\alpha_y = \tan^{-1}(\frac{\zeta}{\sqrt{1-\zeta^2}}). \tag{54}$$

While the velocity and acceleration of the occupant can be readily obtained from Eqs. (47)—(49), the maximum deceleration of the occupant cannot be explicitly expressed by an equation, and thus the relationship between the peak occupant deceleration and the restraint characteristics (ω, ζ) cannot be explicitly described either. Therefore, numerical optimization is used to find the optimal values of the restraint characteristics (ω, ζ).

It is found that when the damping ratio ζ is bounded within [0.01, 0.99], i.e., $\zeta \in [0.01, 0.99]$, the optimal damping ratio is $\zeta_0 = 0.728$; but when $\zeta \in [0.01, \Omega]$, where $\Omega \leq 0.728$, the optimal damping ratio tends to reach the upper bound, i.e., $\zeta_0 = \Omega$. Therefore, in order to seek the optimal characteristics of restraint systems with different damping, the damping ratio is set to be at different levels, as shown in Table 1. The optimal natural frequency corresponding to each damping level is found and given in Table 1. The relationship between A_m and ζ is shown in Fig. 4 by the blue curve based on the values given in Table 1. Also, based on the results given in Table 1, the relationship between the peak occupant deceleration and the rattlespace is displayed in Fig. 2 and the relationship between the optimal natural frequency and the rattlespace is shown in Fig. 3, both by green curves.

The motions of the vehicle and the occupant and the relative motion of the occupant with respect to the vehicle are displayed in Figs. 5 (a) and (b) for two sets of optimal parameters, respectively.

Table 1. Optimization results for constant deceleration pulse

ζ	ω (rad/s)	A_m (m/s^2)	S_0 (m)
0.01	34.63	335.76	0.280
0.10	34.83	297.37	0.243
0.20	35.43	267.99	0.207
0.30	36.47	247.39	0.176
0.40	38.10	232.48	0.147
0.50	40.60	221.38	0.120
0.60	44.61	212.93	0.094
0.70	52.73	206.35	0.064
0.728	60.39	204.77	0.048

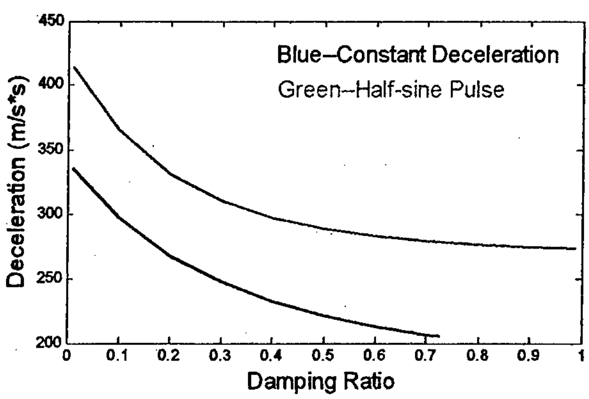
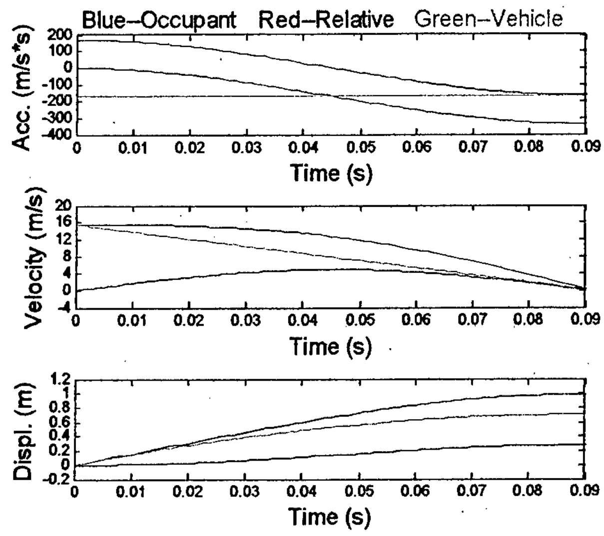
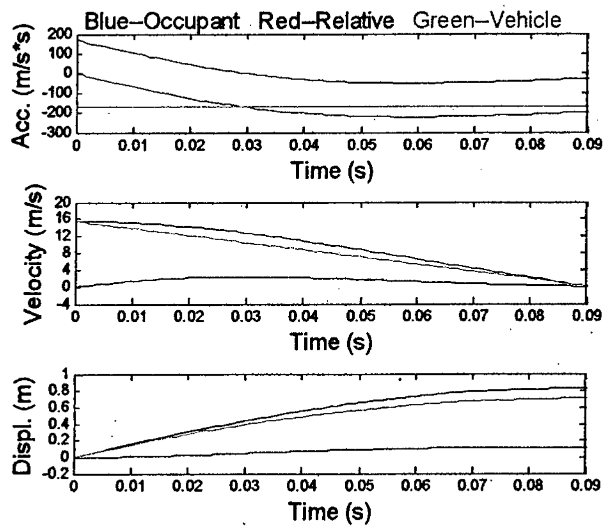


Figure 4. The relationship between peak occupant deceleration and damping ratio



(a) $\zeta = 0.01$ $\omega = 34.63$



(b) $\zeta = 0.50$ $\omega = 40.60$

Figure 5. The motions of a system under constant deceleration pulse

Half-sine Pulse

The vehicle impact deceleration is often expressed or approximated by a half-sine pulse,

$$\ddot{x}(t) = -A_v \sin(\frac{\pi t}{T_v}),\ 0 \leq t \leq T_v, \tag{55}$$

where

$$A_v = \frac{\pi v_0^2}{4D_v}, \tag{56}$$

and T_v is given by Eq. (44). The velocity and displacement of the vehicle are

$$\dot{x}(t) = \left\{ \frac{v_0}{2} + \frac{A_v T_v}{\pi} \cos\frac{\pi t}{T_v}, \right. \tag{57}$$

and

$$x(t) = \left\{ \frac{v_0}{2} t + \frac{A_v T_v^2}{\pi^2} \sin \frac{\pi t}{T_v} \right. . \quad (58)$$

The motion of the occupant under half-sine pulse is found to be [7]:

$$z(t) = z_c(t) + z_p(t). \quad (59)$$

where

$$z_c(t) = Z_c e^{-\zeta \omega t} \sin(\sqrt{1-\zeta^2}\,\omega t + \beta_z), \quad (60)$$

and

$$z_p(t) = \frac{v_0}{2} t + \frac{A_v}{\omega_v^2} \sqrt{\frac{1 + (2\zeta \frac{\omega_v}{\omega})^2}{[1 - (\frac{\omega_v}{\omega})^2]^2 + (\frac{2\zeta\omega_v}{\omega})^2}} \sin(\omega_v t + \alpha_z - \varphi), \quad (61)$$

where

$$\omega_v = \frac{\pi}{T_v}. \quad (62)$$

In Eq. (60),

$$Z_c = \sqrt{\frac{C_{1z}^2 \omega^2 + C_{2z}^2 + 2C_{1z}C_{2z}\zeta\omega}{(1-\zeta^2)\omega^2}}, \quad (63)$$

and

$$\beta_z = \tan^{-1}\left(\frac{C_{1z}\sqrt{1-\zeta^2}\,\omega}{C_{1z}\zeta\omega + C_{2z}}\right). \quad (64)$$

Here

$$C_{1z} = -\frac{A_v}{\omega_v^2} \sqrt{\frac{1 + (2\zeta \frac{\omega_v}{\omega})^2}{[1 - (\frac{\omega_v}{\omega})^2]^2 + (\frac{2\zeta\omega_v}{\omega})^2}} \sin(\alpha_z - \varphi),$$

$$C_{2z} = \frac{v_0}{2} - \frac{A_v}{\omega_v^2} \sqrt{\frac{1 + (2\zeta \frac{\omega_v}{\omega})^2}{[1 - (\frac{\omega_v}{\omega})^2]^2 + (\frac{2\zeta\omega_v}{\omega})^2}} \cos(\alpha_z - \varphi) \quad (65)$$

where

$$\alpha_z = \tan^{-1}\left(\frac{2\zeta\omega_v}{\omega}\right), \quad (66)$$

and

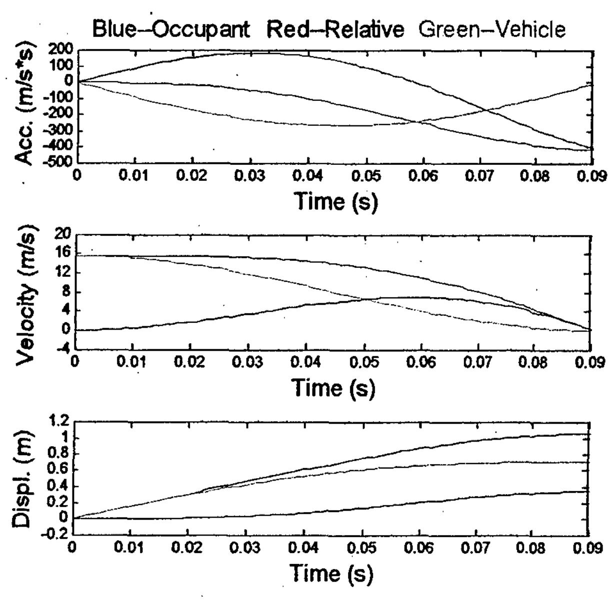
$$\varphi = \tan^{-1} \frac{2\zeta\omega\omega_v}{\omega^2 - \omega_v^2}. \quad (67)$$

While Eqs. (59)—(61) provide an analytical solution of the occupant motion, the relationship between the peak occupant deceleration J_1 and the restraint characteristics ω and ζ is not apparent. Therefore, numerical optimization is implemented to find the optimal restraint characteristics.

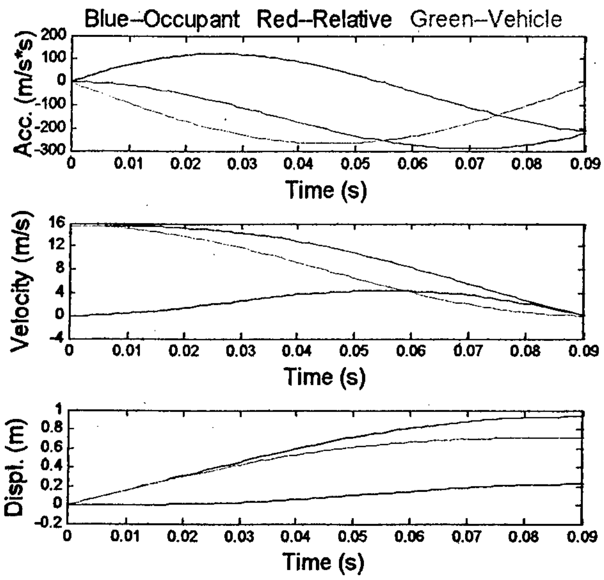
Trial computations indicated that when $\zeta \in [0.01, \Omega]$, where $\Omega \le 0.99$, the optimal damping ratio tends to reach the upper bound, i.e., $\zeta_0 = \Omega$. Therefore, the damping ratio is set to be at different levels within the range of [0.01, 0.99], as shown in Table 2, so that the optimal characteristics of restraint systems with different damping can be explored. Corresponding to each damping level, the optimal natural frequency, the required rattlespace, and the minimum peak occupant deceleration were found and are given in Table 2. The relationship between A_m and ζ is shown in Fig. 4 by the green curve based on the values given in Table 2. Also, the relationship between the peak occupant deceleration and the rattlespace is displayed in Fig. 2 and the relationship between the optimal natural frequency and the rattlespace is shown in Fig. 3, both by red curves. The motions of the system under half-sine pulse are illustrated in Fig. 6 (a) and (b) for two sets of optimal parameters, respectively.

Table 2. Optimization results for half-sine pulse

ζ	ω (rad/s)	A_m (m/s^2)	S_0 (m)
0.01	34.51	414.76	0.348
0.10	33.62	365.16	0.317
0.20	32.78	331.12	0.289
0.30	32.06	310.20	0.265
0.40	31.44	297.02	0.245
0.50	30.91	288.46	0.228
0.60	30.44	282.75	0.213
0.70	30.04	278.87	0.20
0.80	29.68	276.18	0.188
0.90	29.37	274.24	0.178
0.99	29.11	272.96	0.169



(a) $\zeta = 0.01$ $\omega = 34.51$



(b) $\zeta = 0.50 \ \omega = 30.91$

Figure 6. The motions of a system under half-sine pulse

Discussion

As shown in Figs. 5 and 6, when the vehicle crash pulse is in constant-deceleration form or in half-sine form, the occupant deceleration cannot remain constant, even with the optimized restraint characteristics. The influence of different crash pulses on the peak occupant deceleration can be seen from Fig. 2. For the same rattlespace, the peak occupant deceleration resulting from the optimal crash pulse is much lower than those produced by the other two pulses.

In general, the damping in restraint systems is beneficial. For constant-deceleration pulse and half-sine pulse, the peak occupant deceleration decreases with the increase of the damping in a certain range, as shown in Fig. 4. For the optimal crash pulse, certain damping is necessary, and large damping can reduce the amplitude of the initial impulse.

Active Restraint Systems

As described above, when the vehicle crash pulse is in general forms, it is impossible for a passive restraint system to exert an action on the occupant so that the optimal occupant kinematics is attained. What about an active restraint system?

For active restraint systems, Eq. (24) becomes

$$u(y,\dot{y},t) = mA_m . \qquad (68)$$

Suppose the passive part and the active mechanism are separable, i.e.,

$$u(y,\dot{y},t) = g(y,\dot{y}) + h(t) , \qquad (69)$$

where $g(y,\dot{y})$ corresponds to the passive characteristics and $h(t)$ represents the active mechanism. Then,

$$h(t) = mA_m - g(y,\dot{y}) . \qquad (70)$$

This means that at least in a theoretical sense, by optimally designing the active mechanism, an active restraint system may be able to provide required protection to the occupant so that the optimal kinematics of the occupant is attained and the peak occupant deceleration is minimized.

Pre-acting Restraint Systems

A pre-acting mechanism exerts an action on the occupant before the onset of impact. It can be designed to move the occupant in the opposite direction of impact. If a free space is available for the occupant to move backward, after pre-action, the space for the occupant to move forward during impact will become larger. This means that additional rattlespace is created. This additional rattlespace will reduce the peak occupant deceleration for either passive or active restraint systems.

It has been shown that a pre-acting system is superior to an active system, which in turn is superior to a passive system, in terms of their limiting or optimal performance in the impact isolation or attenuation [9].

CONCLUDING REMARKS

In automobile frontal impact, the optimal kinematics of the occupant is such that the occupant moves at a constant deceleration, which is independent of crash pulses and restraint characteristics. In order to minimize the peak occupant deceleration, the restraint systems should be designed to provide such protection to the occupant that the optimal occupant kinematics is attained.

In general, passive restraint systems are not able to provide the required protection to the occupant for it to attain optimal kinematics, except when the vehicle deceleration follows the optimal crash pulse. However, the characteristics of passive restraint systems can still be optimized to minimize the peak occupant deceleration as much as possible. Theoretically, active and pre-acting restraint systems can be designed to provide required protection so that the optimal occupant kinematics is attained. In terms of their limiting or optimal performance, a pre-acting system is superior to an active system, which in turn is superior to a passive system.

Different crash pulses have different influences on the occupant motion and the restraint system design. Among the three crash pulses considered, the optimal crash pulse, which basically is a constant deceleration with an impulse at the initial of impact, is the best, followed by constant-deceleration pulse, whereas half-sine pulse is the worst, in terms of the resulting peak occupant deceleration.

The use of a lumped-parameter model for the occupant-vehicle modeling may impose certain limitations on the analyses in the paper. Also, linear characteristics may be a major assumption and simplification for most passive restraint systems. However, the results and conclusions derived can still be used as general guidelines for the restraint system design.

REFERENCES

1. Shi, Yibing, Wu, Jianping, and Nusholtz, Guy S., Optimal Frontal Vehicle Crash Pulses—A Numerical Methods for Design, EMV, 2002
2. Crandall, J.R., Cheng, Z.Q., and Pilkey, W.D., Limiting Performance Analysis of Seat Belt Systems for the Prevention of Thoracic Injuries, Journal of Automobile Engineering, Proceedings of Institution of Mechanical Engineers, Vol. 214 Part D, 127-139, 2000.
3. Cheng, Z.Q., Crandall, J.R., Darvish, K., and Pilkey, W.D., Limiting Performance Analysis of Toepan Padding for Mitigating Lower Limb Injuries, Journal of Automobile Engineering, Proceedings of Institution of Mechanical Engineers, Vol. 218 Part D, 619-628, 2004.
4. Cheng, Z.Q., Pilkey, W.D., Pellettiere, J.A., and Rizer, A.L., Limiting Performance Analysis of Biomechanical Systems for Optimal Injury Control, Part One—Theory and Methodology, International Journal of Crashworthiness, to appear in 2005.
5. E. Sevin and W.D. Pilkey, *Optimum Shock and Vibration Isolation*, Shock and Vibration Information Analysis Center, Washington D.C., 1971.
6. Zhiqing Cheng, Joseph A. Pellettiere, and Annette L. Rizer, Optimization of Biomechanical Systems for Crashworthiness and Safety, *Proceedings of 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, AIAA paper 2004-4394, August 2004, New York.
7. Zhiqing Cheng, Optimum Crash Pulse for Minimization of Peak Occupant Deceleration in Frontal Impact, submitted to 2006 SAE World Congress.
8. Balandin, D.V., Bolotnik, N.N., and Pileky, W.D., *Optimal Protection From Impact, Shock, and Vibration*, Gordon and Breach Science Publishers, Amsterdam, The Netherlands, 2001.
9. Z. Q. Cheng, W.D. Pilkey, N.N. Bolotnik, D.V. Balandin, J.R. Crandall, and C.G. Shaw, Optimal Control of Helicopter Seat Cushion for the Reduction of Spinal Injuries, *International Journal of Crashworthiness*, Vol. 6, No. 3, 2001.

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