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Report Title

Diamond/Sapphire gain element for Modelocked Backbone Laser

ABSTRACT

An integrated diamond sapphire laser design was explored using analytical expressions, computer based simulation, and experimentation. The analytical expressions and computer based simulation predicted a solid state laser gain medium composed of diamond for removing heat and Ti:sapphire for gain could access the high average power densities required for high energy lasers satisfying requirements of the JTO program. The experimental efforts indicated that single crystal diamond could be fabricated with the required surface properties for implementation of the basic diamond/Ti:sapphire component. The task of experimentally measuring thermally induced refractive index change was found to call for parallelepiped shaped samples. Given the expense of procuring diamond in the required shape and size the experimental investigation was performed with Ti:sapphire as the gain material and undoped sapphire as the cooling material. The experimental study demonstrated that the most important physical requirement, heat removal across the interface between the gain sample and the cooling sample with a negligible temperature change across the interface could be obtained by proper preparation of the interface surfaces. A means of optically measuring the dynamics of the thermally induced temperature change, including the change across the interface was developed.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

1. R. L. Fork, W. W. Walker, R. L. Laycock, J. J. A. Green, and S. T. Cole, "Integrated diamond sapphire laser," Opt. Express 11, 2532-2548 (2003), <http://www.opticsexpress.org/abstract.cfm?URI=OPEX-11-20-2532>.
2. R.L. Fork, W.W. Walker, S.T. Cole, S.D. Moultrie, D.J. Phillips, J.C. Reinhardt, "Surface High-Energy Laser," Proceedings of the IEEE, Vol.93, No. 10, October 2005.

Number of Papers published in peer-reviewed journals: 2.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

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(c) Papers presented at meetings, but not published in conference proceedings (N/A for none)

Richard L. Fork, Wesley W. Walker, Rustin L. Laycock, Jason J.A. Green, Spencer T. Cole, "Integrated diamond sapphire laser", Invited talk at Willis Lamb 90th birthday session, Optical Society Annual Meeting, Tucson, Arizona, October 2003.

Number of Papers not Published: 1.00

(d) Manuscripts

Wesley W. Walker, "ANALYSIS AND EVALUATION OF THERMAL PROPERTIES OF A MULTILAYER LASER GAIN ELEMENT USING TIME-RESOLVED INTERFEROMETRIC METHODS," Doctoral Dissertation, University of Alabama in Huntsville, 2005.

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Spencer Cole, 0.5
Dane Phillips, 0.5
Wesley Walker, 1.0

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Richard L. Fork, Professor Electrical & Computer Engineering

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Rustin Laycock

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5 SOLID STATE LASER MEDIUM AND LASER MEDIUM HEAT TRANSFER METHOD

Patent Filed in US? (5d-1) Y

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Final Report
Proposal Number: 45620EL
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Diamond/Sapphire gain element for Modelocked Backbone Laser

Report Period Begin Date: Sep 1, 2003

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Statement of the Problem Studied

The problem studied was that of using integrated gain and cooling materials to remove waste heat from a solid state laser gain medium so as limit optical distortion and stress to the degree necessary to allow scaling of solid state lasers to the average power, e.g., > 100 kW, needed for the Joint Technical Office program. Diamond and Ti:sapphire were chosen respectively, as the candidate cooling and gain materials. The focus area was the interface between the gain material and cooling material and the degree to which heat flow across that interface could approach the rate of heat flow within intrinsic material. Computer simulations and analytical expressions were developed demonstrating that this quality of heat flow across the interface was of major importance in achieving the goal of high average power and acceptably small optical distortion and stress.

Summary of Most Important Results

1.0 Experiments addressing heat flow across the interface between amplifying and cooling elements

We developed a solid state laser design based on the combination of gain and cooling elements designed to achieve high average power >100 kW from a solid state laser and also a method for measuring the performance of rate of heat flow across the interface between such solid state gain and cooling elements. We found that, provided the surfaces of the materials at the interface are prepared with maximum achievable precision, i.e. flat to tenth wave (optical contact quality), the heat flow across the interface is comparable to heat flow in continuous material, i.e. no discernable abrupt change in temperature is observable at the interface.

We included in this investigation diamond and sapphire materials and operation of the materials at temperatures as low as 40k where sapphire has thermal properties similar to those of diamond. In particular, we have calculated that composite structures of sapphire and diamond, or doped gain material and a cooling material such as sapphire at low temperatures can access the performance required to achieve >100 kW of average power from a solid state laser given the interface performance which we have observed.

1.1 Experimental measurement techniques

A necessary tool was an experimental means of measuring the heat flow across the interface between the gain and cooling elements. No such experimental device appeared to be available. We consequently developed a dynamical interferometric method of measuring the time dependant heat flow in the materials and across the interface. We used both Mach-Zehnder and Michelson configuration interferometers to record the dynamics of heat flow in terms of the thermo-optically induced change in optical delay in the materials following excitation of the gain medium by an energetic short optical pulse.

1.2 Sample configuration

We found in the course of the experimental work that it was necessary, given the samples and laser intensities available, to orient the pair of elements being studied, one for gain and one for cooling, so that the probe optical field propagated parallel to the interface between the two elements. This permitted accumulation of a net induced optical phase shift in the optical fields sufficient to read out the temperature variation in the samples and also across the interface with adequate precision at all successive delays of the probe measurement relative to the exciting pulse. These measurements provided data on the dynamics of the heat flow through sequences of observations at successive time delays after the initial excitation.

1.3 Influence of sample configuration on the choice of materials

The need to propagate the optical probe field parallel to the interface between samples strongly favors the use of materials that do not alter the optical field wavefront of curvature as the optical field propagates through the pair of samples. This calls for flat parallel surfaces on the entrance and exit surfaces that intersect, and are normal to, the surfaces used for the interface. This has the consequence that the preferred samples have the form of parallelepipeds.

The two crystalline diamond samples which we had available were in the form of circular disks. While these samples, on loan from the organization C6 through Dr. Butler, were high quality and had adequately smooth surfaces, the round nature of disk boundary precluded the use of our interferometric method for measuring the properties of the diamond samples. We also investigated polycrystalline diamond samples which might have been configured in parallelepiped form, but found the optical scattering to be unacceptably large.

We did find that we could obtain at reasonable cost samples of Ti:sapphire and undoped sapphire that had the required dimensions, intrinsic optical quality and surface quality to perform the experimental measurements. Since undoped sapphire has comparable heat removal properties to diamond at low temperature, and the obtaining of a diamond sample of adequate dimensions and shape appeared outside the scope of this particular effort, we proceeded with the Ti:sapphire sample as the gain material and a cooling element composed of undoped sapphire.

1.4 Cryogenic measurements

We included in this study, an examination of the performance of the gain and cooling elements as a function of temperature. We accessed the temperature regime, 40K, where the undoped sapphire has thermal properties similar to those of diamond. Data was recorded for the gain/cool samples down to a temperature of 40K. An extensive description of the resulting data is given in the doctoral thesis of Wesley Walker which is currently in preparation. The essence of the findings was that the rate of removal of heat from the gain material into the cooling material improves as predicted by theory given the improved thermal transport properties of sapphire at lower temperature.

1.5 Influence of surface quality of experimental samples

The first pair of Ti:sapphire and undoped sapphire samples that we investigated were given a “good” surface polish. We found a significant abrupt change in temperature across the interface during heat flow. This indicated thermal impedance at the interface that was significantly larger than the heat flow across a comparable region of continuous material having no interface. A second pair of Ti:sapphire and undoped sapphire samples was investigated. This second pair was prepared with the surfaces having the best obtainable surface flatness, essentially “laser quality” surfaces of tenth wave flatness from Crystal Systems, Inc.

No abrupt change in temperature of the media was observed in this second pair of samples that had the superior surface quality. These samples also exhibited good optical contact. That is, the transmission through the sample pair changed noticeably on placing the samples in contact and exerting a slight pressure forcing the two materials in close contact. This indicated that “adhesive free bonding”, as through introduction of Van der Waals forces was present.

1.6 Dynamics of the heat flow

The dynamics of the heat flow were measured following excitation of the sample with a “burst” of laser pulses during a time short compared to the millisecond time constant that characterized the rate of heat removal from the samples. The rate of heat removal was found to be that expected given the thermal properties of the intrinsic material at a given temperature and the absence of significant thermal impedance at the interface between the two materials.

2.0 Calculational and analytical characterization of heat flow in the gain/cool element

We also performed analytical calculations and computer simulations characterizing the heat flow in the gain/cool element. In general, where comparisons could be made between experiment and theory, for the pair of gain/cool samples having the “laser quality” interface described above, there was good agreement between the analytical and computer based simulation predictions and the experimental results.

2.1 Analytical equations describing heat flow and temperature variation in gain/cool element

Analytical equations were developed that describe the heat flow and temperature variation with time and position in the gain/cool sample. One system of equations that characterized a Ti:sapphire/diamond interface with a thermal impedance that was radially independent was published in the reported peer reviewed publication 1. A second system of equations that addressed the case of an interface thermal impedance that was radially dependent was published in a second reported peer reviewed publication 2. (some of the material for these two publications was also supported under a separate ARO grant for the Surface High Energy Laser). The later radially dependent thermal impedance offers more options in minimizing the net thermally induced optical distortion, including in particular, undoped sapphire as the cooling element.

2.2 Computer based simulations describing heat flow and temperature variation in gain/cool element

Computer based simulations were also performed that described the heat flow and temperature variations in the gain/cool element. These simulations provided a degree of detail that is difficult to describe analytically. Where comparisons could be made between computer based simulations, the analytical expressions, and the experimental results, agreement was found. The computer based simulations included simulation of a composite multi-element gain/cool element that could not be easily represented analytically and could not be experimentally addressed within the scope of this effort. The prediction of this computer based simulation was that a

composite multi-element gain/cool material could be realized that would produce the average power of 100 kW or greater, as sought for JTO purposes.

3.0 Conclusions

We conclude from our analytical expressions, computer simulations, and experimental observations that the gain/cool strategy developed in this and a related effort supported by ARO, offers a means of constructing a solid state laser that can access the high average power, e.g., > 100 kW, sought for JTO applications. We give here some additional observations.

3.1 Cost and uniformity issues related to diamond

We greatly appreciate the polycrystalline and single crystal diamond samples made available by Dr. Butler of NRL. Use of diamond will need to address the need for lower optical scattering from the polycrystalline samples and, preferably, reduced position dependent birefringence in the single crystal diamond. The single crystal diamond samples we received appeared adequate for experimental measurements. The key problem in using these disks was the round geometry and thin dimension normal to the plane of the disk. The minimum needed thickness, for this measurement technique that required an optical probe field propagating order of half a centimeter parallel to the interface, was ~ 4 millimeters. The parallelepiped geometry and additional thickness is simple and comparatively inexpensive to achieve with sapphire, but tends to be costly for diamond.

3.2 Cost, internal uniformity, flatness and effect of compression of sapphire

The cost of sapphire is well known and appears acceptable for design of larger systems. The flatness of the interface surfaces needed to be tenth wave of visible light to achieve a thermal impedance so small as to show no measurable temperature change at the interface. This flatness is achievable in the best surface quality sapphire. The internal uniformity of commercial sapphire was excellent; however, under compression some of the sapphire samples exhibited a position dependent variation in optical delay path. We did not extensively explore this position and compression dependent optical distortion; however, further studies of the suitability of sapphire for a laser medium, where the sapphire experiences significant compressive force, a torque of a few foot-pounds applied to the clamping screws, should be considered.

3.3 Role of material temperature

As predicted from material properties operation of the gain/cool element at reduced temperature resulted in more rapid removal of heat with less thermally induced distortion. Reduction of temperature of the laser medium offers a desirable and valuable improvement in removal of waste heat while minimizing the thermally induced stress and optical distortion. In particular, many of the advantages of diamond appear to be susceptible of exploration by using sapphire at reduced temperature.

3.4 Method for measuring the dynamics of heat flow in laser materials

A method for measuring the dynamics of heat flow and the related optical changes in laser materials used both for gain and for cooling was developed. This method should be valuable for investigation of heat flow dynamics and the associated optical distortion which are critical to development of high power lasers. To the best of our knowledge this method is original. We have measured heat flow and the associated optical changes with the precision needed for evaluating application of this technology to laser systems at temperatures ranging from 40K to 300K.

3.5 Role of material interfaces

We have found that material interfaces can be made sufficiently uniform, tenth wave flatness for our sapphire materials, to provide thermal impedance comparable to a similar thickness of continuous material. In our samples temperature cycling does not appear to be a problem. We did not investigate possible changes in materials that are in optical contact over extended periods of time, as, e.g., many months.

3.6 Feasibility of gain/cool element based solid state laser operating at >100 kW.

Within the framework of the investigations carried out here a high average power solid state laser based on the gain/cool strategy we have outlined in the peer reviewed publication referenced here appears feasible. Some issues that should be addressed in further work are the effects of compression on the optical quality of the laser materials, the ability to produce a prescribed radial variation of the heat transfer coefficient at an interface, and the tolerance for long term temperature cycling of the structures.

3.7 Additional reference material:

Considerable additional material will be generally available within order of a month or so in the doctoral dissertation of Wesley Walker.

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