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The main objective of this proposal was to develop statistical control theory. Statistical control is a generalization of Kalman's linear-quadratic-Gaussian control, where we optimize the probability density of the performance index by controlling the cumulants. A statistical controller will have better performance and stability margin than the linear-quadratic-Gaussian controller. In this project, we investigated the characteristics of linear statistical controllers, developed nonlinear statistical control theory, and utilized the statistical control paradigm in dynamic game theory.

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Enclosure 1

## Forward

The PI was awarded a short term innovative research grant, number W911NF-05-1-0212, through the University of North Dakota. The grant period was 15 April 2005 to 14 January 2006. However, during the grant period the PI transferred to Temple University in August 2005. So, the PI is submitting the final progress report with the results that the PI obtained during 15 April 2005 to 31 August 2005.

The main objective of this proposal was to develop statistical control theory. Statistical control is a generalization of Kalman's linear-quadratic-Gaussian control, where we optimize the probability density of the performance index by controlling the cumulants. A statistical controller will have better performance and stability margin than the linear-quadratic-Gaussian controller. In this project, we investigated the characteristics of linear statistical controllers, developed nonlinear statistical control theory, and utilized the statistical control paradigm in dynamic game theory.

Although the linear version of the statistical control theory is almost completely developed, the nonlinear version is still at its infancy. However, almost all real systems are nonlinear and nonlinear control theory is becoming more and more important for the real world applications. This is especially true for autonomous systems such as satellites or unmanned aerial vehicles with wide operating conditions. So, the PI and his collaborators investigated statistical control for nonlinear systems. We formulated and solved a nonlinear statistical control problem using dynamic programming. Dynamic programming is a powerful method that leads to Hamilton-Jacobi-Bellman partial differential equations. We derived the Hamilton-Jacobi-Bellman equation for a nonlinear system, and proposed a method to solve this partial differential equation.

Another interesting research area related to statistical control is dynamic game theory. Dynamic game theory is currently an active and important research area. In dynamic game theory, we introduced multiple players with conflicting objectives and optimize cost functions with respect to their objectives. So, this is an ideal setting for many military applications. In this project, we formulated stochastic game problems and solved them using statistical control paradigms. In other words, we optimized any function of the statistical parameters of the cost function instead of the average cost as traditionally done in game theory.

The development of statistical control theory will further advance the existing knowledge in stochastic optimal control.

## Statement of the Problem Studied

In this project, we studied characteristics of linear statistical control and nonlinear statistical control. We also studied the relationship between statistical control and dynamic game theory. In statistical control, we view the cost function as a random variable and optimize any of the cost cumulants rather than optimizing just the mean as in the traditional linear-quadratic-Gaussian optimal control. The summary of the main problems studied in this project is given as follows.

LQG control optimizes the average of a quadratic cost function which intuitively corresponds to the energy of the system. In statistical control, we can optimize any cumulant of the quadratic cost function. Consequently the natural question to ask is "what is the intuitive meaning behind this optimization?" We answer this question using entropy, distribution shaping, and Bode integral. Also, we investigate the relations between cost cumulants and statistical density.

Even though many systems can be approximated as a linear system, there are many systems such as satellite attitude dynamics, which may benefit from a nonlinear model and control. Thus, we proposed to formulate and solve nonlinear statistical control problems using dynamic programming methods. We derived the Hamilton-Jacobi-Bellman (HJB) equation for *n*-th cost moment case. Then utilizing the relationship between moments and cumulants, we found the HJB equation for *n*-th cost cumulant case. Then we determined the optimal controller for first, second, and third cost cumulant optimization problem using newly derived HJB equations. This theory enabled us to determine the optimal statistical controller for a nonlinear system.

Another theoretical research of this proposal is in examining the relations between statistical control and dynamic game theory. We can view statistical control in terms of dynamic game theory with player one being the controller and player two being the model error. This is an interesting extension that is being investigated. On the other hand, instead of viewing statistical control as a part of dynamic game theory, we can formulate the stochastic game problem and solve this problem using statistical control concepts, which means that we can use any statistical parameters (cumulants) instead of using just the average cost as in traditional stochastic game theory. This project investigated statistical control from this perspective. By applying dynamic games concepts to statistical control and vice versa, we hope to deepen the understanding of both areas.

## Summary of the Most Important Results

**Characteristics of Linear Statistical Control.** The cost function in stochastic optimal control is viewed as a random variable. Then the classical linear-quadratic-Gaussian control, entropy control, risk-sensitive control, and statistical control can be viewed as the cost distribution shaping methods. We determined the relations between entropy, Bode integral, risk-sensitive cost function, and statistical control cost function. Furthermore, we related the cost cumulants with information theoretic entropy, and Bode integral. The interpretation of statistical control is given in terms of the control entropy minimization. We also related information theoretic entropy with exponential-of-integral cost function using a Lagrange multiplier and calculus of variations. Finally, the logarithmic-exponential-of-integral cost function is related to the information theoretic entropy using large deviation theory.

**Deterministic Nonlinear Statistical Control Theory.** A full-state feedback optimal control problem is solved for a deterministic quasi-linear system with a quasi-quadratic cost function. The solution method is based on algebraically solving Hamilton-Jacobi equation in terms of the gradient of the value function. Then we interpreted the value function in terms of the control Lyapunov function and provided the stabilizing controller and the stability margins. We derived an optimal controller for a nonlinear system which requires a solution of the state dependent Riccati equation. Moreover, we found the optimal controller when the cost function is the exponential-of-integral case, which is risk sensitive control. Then we showed that these two methods give equivalent optimal controllers for nonlinear deterministic systems.

**Stochastic Nonlinear Statistical Control Theory**. A nonlinear stochastic system with non-quadratic cost function with respect to the state is considered for the statistical control problem. The optimal controller is determined via the Hamilton-Jacobi-Bellman equation. The Hamilton-Jacobi-Bellman equation for the n-th cost moment case is derived as a necessary condition for optimality using dynamic programming. Then the sufficient condition theorem for the optimality is presented. The n-th cost cumulant Hamilton-Jacobi-Bellman equation derivation procedure is given. Second, third, and fourth cost cumulant Hamilton-Jacobi-Bellman equations were derived using the proposed procedure. Then the optimizing controller for the first and second cost cumulants is found using the Hamilton-Jacobi-Bellman equation. We transformed the HJB equation to a first order partial differential equation using the inversion method. Both time varying and time invariant systems are considered.

**Dynamic Game Theory and Statistical Control.** There is a close relationship between statistical control and dynamic game theory. In this project, we investigated the possibilities of using statistical control paradigm in stochastic dynamic game theory. It is assumed that the players will try to minimize a linear combination of cumulants of their own cost function. More specifically, player one will seek to minimize a performance index made up of a linear combination of cumulants of the first cost function and the player two will minimize a performance index consisting of a linear combination of cumulants of the second cost function.

This linear combination of cumulants could be different for each player. We investigated the performance indices involving up to *k-th* cost cumulants and games with more than two players.

**Statistical Control Meeting.** The PI also held the meeting with Dr. Michael Sain and Ronald Diersing at the University of North Dakota from 18 July 2005 to 22 July 2005. Figure 1 shows the two researchers in front of the Engineering building at the University of North Dakota. This was proposed in Section 2 of the proposal. During the meeting we discussed statistical control issues and its relations to the dynamic game theory. We discussed the following issues: Nonlinear Statistical Control, Characteristics of Statistical Control, Discrete time Statistical Control, Game Theoretic Output Feedback Control, Long Term Research, Distribution Shaping Problem, and Applications.


Figure 1: Statistical Control Researchers Ronald Diersing and Michael Sain Visiting the University of North Dakota

## Publications Listing

The PI has worked on the statistical control theory throughout the grant period, he especially concentrated on the nonlinear statistical control theory. His paper titled, "Nonlinear n-th Cumulant Control and Hamilton-Jacobi-Bellman Equations for Markov Diffusion Process," has been accepted for publication in *44th IEEE Conference on Decision and Control*. Because the grant was delayed for a few months, the part of the work was published without acknowledgement with the grant number. In particular, the paper titled, "State-Feedback Optimal Controllers for Deterministic Nonlinear Systems" was published in *2005 American Control Conference*. Also, his coworkers—Michael Sain and Ron Diersing—published a paper titled, "A Multiobjective Cost Cumulant Control Problem: The Nash Game Solution," at the same *2005 American Control Conference* without acknowledging the grant.

(a) Papers Published in Peer-Reviewed Conference Proceedings

- Chang-Hee Won, "Nonlinear n-th Cost Cumulant Control and Hamilton-Jacobi-Bellman Equations for Markov Diffusion Process," *Proceedings of 40th IEEE Conference on Decision and Control*, Seville, Spain, December 2005, accepted for publication.

- Chang-Hee Won, "State-Feedback Optimal Controllers for Deterministic Nonlinear Systems," *Proceedings American Control Conference*, Portland, Oregon, pp. 858-863, June 8-10, 2005.

- Ron Diersing and Michael Sain, "A Multiobjective Cost Cumulant Control Problem: The Nash Game Solution," *Proceedings American Control Conference*, Portland, Oregon, June 8-10, 2005.

(b) Manuscripts submitted to Peer-Reviewed Journals

- Chang-Hee Won, "State Feedback Statistical Control and Hamilton-Jacobi-Bellman Equations for a Quasi-Linear Stochastic System," *IEEE Transactions on Automatic Control*, submitted.

(c) Manuscripts in Preparation for Peer-Reviewed Journals

- Chang-Hee Won, "Characteristics of Linear Statistical Control: Bode Integral, Entropy, and Risk-Sensitivity."

- Chang-Hee Won, Saroj Biswas, and Randal Beard, "An Algebraic Method of Solving State-Feedback Optimal Controllers for a Quasi-Linear System with Quasi-Quadratic Cost."