AN OPTIMAL NETWORK-BASED APPROACH TO SCHEDULING AND RE-ROSTERING CONTINUOUS HETEROGENEOUS WORKFORCES

by

Shane A. Knighton

A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

ARIZONA STATE UNIVERSITY

August 2005

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **01 AUG 2005**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE **An Optimal Network-Based Approach To Scheduling And Re-Rostering Continuous Heterogeneous Workforces**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Arizona State University**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **The Department of the Air Force AFIT/CIA WPAFB, OH**	10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**197**	

AN OPTIMAL NETWORK-BASED APPROACH TO SCHEDULING AND

RE-ROSTERING CONTINUOUS HETEROGENEOUS WORKFORCES

by

Shane A. Knighton

has been approved

July 2005

APPROVED:

, Chair

Supervisory Committee

ACCEPTED:

Department Chair

Dean, Division of Graduate Studies

ABSTRACT

This dissertation presents a mathematical programming approach to the personnel tour scheduling problem based upon a minimum cost network flow formulation with specialized side constraints. The linear program optimally solves the tour scheduling problem for industries with continuous (24 hour) operations and a heterogeneous workforce such as fast food restaurants, hotels and resorts, university computer labs, nurses in large health care systems, and retail sales. The methodology allows for a workforce with varying availabilities, part and full-time employees, differing skill-sets and wage rates, and minimum and maximum shift requirements per week. Additionally, the formulation can quickly adjust an optimal schedule due to sickness, vacation, hiring or firing with the workforce, and minimize the number of deviations within the original schedule. A methodology that can optimally schedule both a continuous and a heterogeneous workforce in an insignificant amount of computational time is unique in the literature. In many instances the linear program generates integral solutions to the tour-scheduling problem without branching, bounding, or cutting techniques. An interior-point method solves the formulation in less than 3 seconds for large problem instances of 80 employees scheduled to 420 distinct shifts. The efficiency of the formulation is presented for many test sets of problems as well as an application to the tour-scheduling problem of computer lab staffing at Arizona State University and nurses with the Banner Health network of heath care providers.

DEDICATION

This accomplishment is a direct result of the unyielding love and support of my wife. You have made my life complete and are my motivation to succeed. It is all impossible without you. To my friends whose support was incredible and to my family whose love is unwavering, I love you and I thank you. I love you too Quila.

Thank you to the United States Air Force for the wonderful opportunity to pursue the degree.

ACKNOWLEDGMENTS

I want to express my extreme gratitude to my committee chair, Dr. Jeffery Cochran, for his insight, support and most importantly, his guidance. Thank you to the remaining members of my committee for their oversight and help. Additionally, I would like to thank Maj Tom Boushell for his assistance and friendship. Thank you to Tom and Jeff Laub for their help in sculpting the ASU Phase 1 computer lab problem and to James Broyles for his help is scoping the nurse scheduling problem at Banner Health.

Thank you to the faculty and staff of the Industrial Engineering Department at Arizona State University. I have learned a tremendous amount from all of you.

The views expressed in this article are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U.S. Government.

TABLE OF CONTENTS

CHAPTER Page

CHAPTER Page

LIST OF TABLES

TABLE Page

LIST OF FIGURES

FIGURE Page

FIGURE Page

Chapter 1

INTRODUCTION

1.1 Overview

Personnel scheduling is one of the most difficult, important, and often studied problems in operations research. The optimal choice of the number of employees required to meet customer demand, shift start and stop times, daily lunches and breaks, and the assignment of the adequately skilled employee to the best shift is a large combinatorial optimization problem. Managers who attempt to manually solve the personnel scheduling problem expend many valuable work hours to find even a feasible solution which has little probability of being optimal based on any objective function. Sub-optimal scheduling increases an industry's tangible costs, not only through the consumption of a manager's time, but also through the misallocation of shifts to meet customer demand and through employees staffing shifts for which they are not qualified. Furthermore, a sub-optimal schedule will increase intangible costs such as lower employee morale from dissatisfaction with a poor work schedule. The problem is further complicated by employee sicknesses, vacations, hiring, or firing. In this inevitable instance the sub-optimal problem must again be adjusted, costing more hours, to find another feasible schedule that will also likely be sub-optimal. Eliminating these costs is an easy way to increase the profitability of any business.

Personnel scheduling is a necessary chore associated with any organization employing people to get work accomplished. Some organizations that employ a professional workforce during traditionally scheduled business hours find the personnel scheduling problem a trivial concern. However, industries that rely on a large portion of part-time employees with highly constrained availabilities (work hours), varying skill sets within the workforce, and require staffing for the entire day and evening hours or even continuous (24-hour) operations have a much more challenging problem. A timely and optimal solution to the personnel scheduling problem will save each manager valuable time, meet customer demand, and increase worker morale by matching each employee to the best shift possible. This dissertation presents an extremely fast, computationally efficient, and optimal network-flow based mathematical programming solution for the extremely challenging problem of rostering and re-rostering a continuous heterogeneous workforce.

The large, complex problem of personnel scheduling is generally decomposed into more tractable sub-problems. The three primary sub-problems are demand modeling, shift selection, and employee tour-scheduling or rostering. Demand modeling determines the number of employees required during a given time interval to satisfy customer needs. Forecasting and queuing theory are the primary tools applied to demand modeling. A typical chart of the number of employees required, illustrating the results of demand modeling, is shown in Figure 1.1.

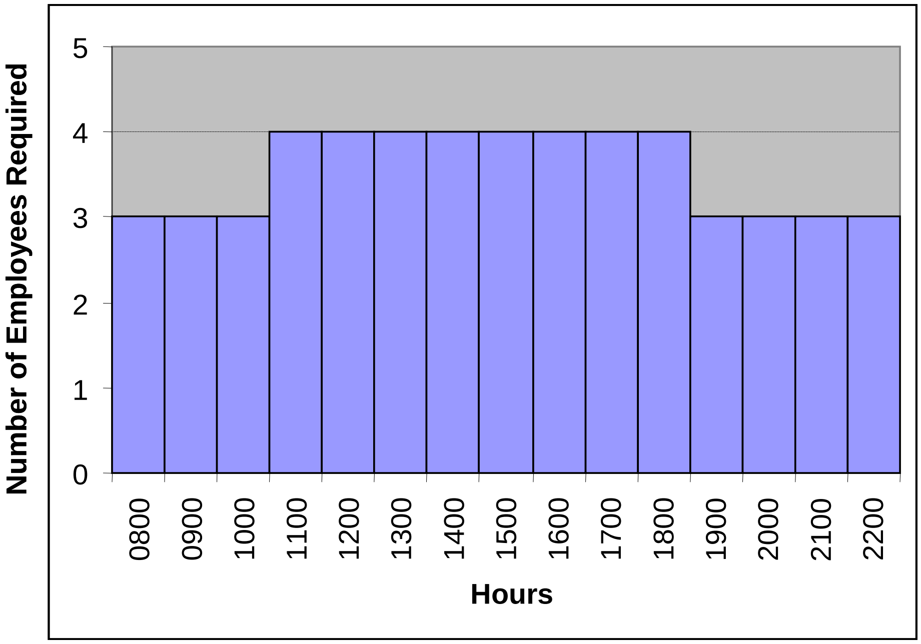


Figure 1.1: Number of employees required at hourly intervals is the result of demand modeling

The second primary personnel scheduling sub-problem is shift selection. Shift selection assigns consecutive hour shifts to satisfy the customer demand while meeting organizational and regulatory requirements for shift lengths, breaks, and mealtime allowances. The objective of shift selection is generally to minimize the number of hours scheduled that exceed customer demand. Figure 1.2 shows an example of a set of shifts that will optimally cover the number of employees required in Figure 1.1.

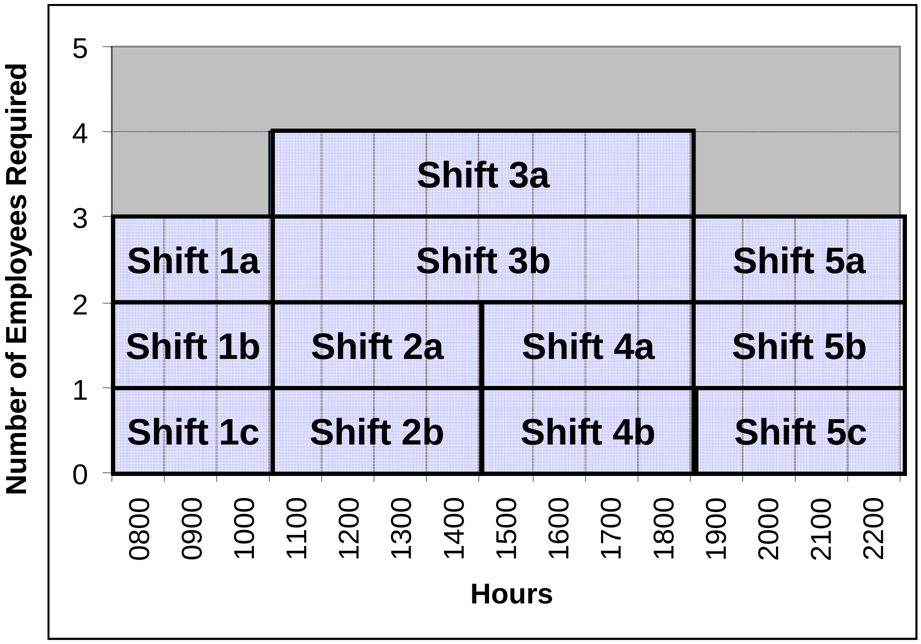


Figure 1.2: Set of shifts to optimally cover customer demand from Figure 1.1

Finally, the third primary personnel scheduling sub-problem is tour-scheduling also known as employee rostering. Tour-scheduling assigns individuals to specific shifts detailed during shift selection. Tour-scheduling problem complexity differs in the literature depending upon the nature of the workforce under consideration. This dissertation will look at one of the most complex tour-scheduling problems due to its consideration of a heterogeneous continuous workforce that gives consideration to employee shift preferences and managerial employee preferences. A heterogeneous workforce is a collection of personnel who have significantly variant availabilities, skill sets, and wage rates. Tour-scheduling a heterogeneous workforce must consider all of these differences and optimally match the personnel with the best shift possible and for which they are eligible. Additionally, considerations for individuals working a minimum and maximum number of shifts per week and insuring adequate rest between shifts for industries with continuous (24-hour) operations are critical to an optimal tour schedule. Employee shift preferences and management employee weighting comprise the objective function.

1.2 Statement of the Problem

This dissertation will optimally solve the tour-scheduling problem for a continuous heterogeneous workforce, meeting all minimum and maximum shift regulations, skill set requirements, employee availabilities, and rest periods between shifts. The formulation proposed will provide a solution to the tour-scheduling problem in an insignificant amount of computational time for large problem instances. A set of test problems are given to benchmark the formulation proposed in this dissertation. Optimally solving the tour-scheduling sub-problem does not guarantee a global optimal solution to the combined shift selection/tour-scheduling problem. An alternate set of shifts that minimizes excess employee hours, but increases the number of preferred shifts of personnel can only be found by solving the combined shift selection tour-scheduling problem. The proposed methodology is examined to solve the combined problem.

1.3 Original Contribution

The work in this dissertation will provide several original contributions to the personnel scheduling literature. The following list summarizes the advancements proposed by this research.

- Provide a solution methodology that can simultaneously account for variations in employee availabilities, skill sets, and wage rates
- Provide a methodology that can roster employees for continuous (24-hour) operations by allowing for rest hours between shifts
- Develop a network-flow based formulation that will rapidly solve the tour-scheduling problem, providing integral answers without branching, bounding, or cutting techniques
- Roster beyond the traditional weekly schedules to monthly, quarterly, and yearly schedules while restricting the amount of weekends worked when creating multi-week tour schedules
- Provide a framework that only requires a minimal adjustment to the formulation to re-roster the workforce when perturbations in an optimal schedule occur due to employee sickness, vacations, hiring, or firing
- Allow for each employee to provide preferences for more favorable shifts and each manager to provide preference for employees based on seniority or performance

- Explore the ability to expand the network-flow formulation to simultaneously solve both the shift selection and tour-scheduling problem for a continuous heterogeneous workforce

1.4 Organization

This dissertation is organized in the following manner. Chapter 2 contains a survey of the pertinent literature and important previous work accomplished in personnel scheduling. Chapters 3, 4, and 5 each contain a complete stand alone journal article which, when taken together, embody the research and original contributions set forth in this dissertation. Chapter 3 is a journal article entitled “A Network-based Mathematical Programming Approach to Optimal Rostering of Continuous Heterogeneous Workforces” detailing a mathematical programming formulation for the tour-scheduling problem. Chapter 4 expands the model in Chapter 3 to multi-weekly time horizons and restricts the number of weekends worked by each employee. The article is entitled “A Mathematical Programming Approach to Optimal Nurse Rostering over Continuous and Multi-Weekly Time Periods with Employee Weekend Restrictions and Shift Preferences”. Chapter 5 contains an article entitled “A Network-based Mathematical Programming Approach to Using Employee Preferences in Re-rostering Optimal Tour-Schedules”. This work modifies the models presented in Chapters 3 and 4 to provide a mechanism for re-rostering an optimal schedule that has been perturbed by employee absences. Chapter 6 presents the overall conclusions and recommendations. Appendices follow the references.

Chapter 2

LITERATURE SURVEY

2.1 Overview

A vast amount of scholastic work has been accomplished on the personnel scheduling problem. This work has been documented in a wide array of refereed journals, conference proceedings, and lecture notes. Personnel scheduling spans a wide range of problem instances, specific applications, and solution techniques and methodologies.

The journal *Annals of Operations Research* published two special issues in March and April 2004 dedicated to the personnel scheduling problem. The issues are comprised of 22 personnel scheduling papers of which "2 are survey papers, 18 are applications-oriented papers in specific application areas, and 2 talk to general methodological and theoretical advances" (Jiang et. al., 2004). These special issues indicate the continued interest in the personnel scheduling problem as well as the breadth of research topics being explored by the research community. This literature review will make extensive use of the articles in these special issues in addition to other prior pertinent work in personnel scheduling.

This literature review will begin by defining specific problem instances studied in the personnel scheduling literature. The next section will describe many of the application areas using personnel scheduling and the solution techniques and methodology applied. Finally, the literature review will concentrate on the prior work dedicated to the tour-scheduling problem using network flow problems for an industry characterized by flexible demand, a heterogeneous workforce, and continuous operations. The relatively small quantity of work in this area is contrasted with the proposed methodology of this dissertation.

2.2 Problem Instance Classification

Classification of the problem instances, application areas, and solution techniques is essential to understanding the focus of the work proposed in this dissertation. Ernst, et. al. (2004a) propose such definitions for classification of the literature in personnel scheduling. Of primary initial concern are the definitions of the problem instances currently under study. In their article, Ernst et. al. define six problem instances. They include: demand modeling, days off scheduling, shift scheduling, line of work construction, task assignment, and staff assignment.

2.2.1 Demand Modeling

Demand modeling is the first step in the process of personnel scheduling. Demand modeling concentrates on the determination of “how many staff are needed at different times over some planning period. Demand modeling is the process of translating a predicted pattern of incidents into associated duties and then using the duty requirements to ascertain a demand for staff” (Ernst et. al., 2004a). Demand modeling is further broken down into three distinct types.

The first subcategory is task based demand. Task based demand “is obtained from lists of individual tasks to be performed” (Ernst et. al., 2004a). The tasks are of a specified length and may have preset start times at which the task must begin. Applications include airline flight schedules or public transportation. Scheduling to accommodate task based demand is often “associated with crew pairing generation and crew pairing optimization” (Ernst et. al., 2004a).

The second subcategory is flexible demand. Flexible demand is characterized by customer service requirements that are not fixed and cannot be exactly known. In general, this demand is measured using stochastic modeling or forecasting; the uncertain demand is predicted and staffing levels are determined. Application areas are associated with customer service driven industries such as call-centers, financial institutions, and restaurants.

Finally, the third subcategory defined by Ernst, et. al. (2004a) is shift based demand. Shift based demand is characterized by a minimum number of staff required at specific time intervals. “Shift based demand arises naturally in applications such as nurse scheduling and ambulance services where staff levels are determined from a need to meet service measures such as nurse/patient ratios or response times” (Ernst et. al., 2004a). It is pointed out that shift based demand can be considered a simplification of both task based and flexible demand problem instances.

2.2.2 Days off Scheduling

Once the demand has been determined, the next step in personnel scheduling may be to determine the associated rest periods for a specific service organization. Examples might be operating hours of a bank, not operating a garage on the weekends, or a fast-food restaurant remaining open later on the weekends. This analysis is often characterized by a cost analysis based upon the predicted demand and is predominantly used when scheduling “flexible or shift based demand rather than when rostering to task based demand” (Ernst et. al., 2004a).

2.2.3 Shift Scheduling

Shift scheduling chooses the appropriate start and finish times for a particular shift. The overall set of shifts will provide for the required number of personnel to meet the demand required or predicted through demand modeling. The overall goal is “to select a good set of feasible duties, shifts or (crew) pairings to cover all tasks” or demand (Ernst et. al., 2004a). Shift scheduling must incorporate such constraints as maximum and minimum shift lengths prescribed by organizational policy or by law and may specify break and meal periods.

2.2.4 Line of Work Construction

Line of work construction assigns personnel to the set of shifts found through days off scheduling and shift scheduling. Personnel availability, skill set, and preferences are considerations when assigning shifts. Additionally, shift conflictions and patterns are important, “for example it might not be possible to immediately follow a sequence of night shifts with a day shift” (Ernst et. al., 2004a). Ernst, et. al. point out that line of work construction is generally called “tour-scheduling” or “rostering” for flexible demand industries and “crew rostering” for task based demand. This dissertation will concentrate on the tour-scheduling problem.

2.2.5 Task and Shift Assignment

Task Assignment and Shift Assignment are considered by some in the literature to be redundant problem instances, but are presented here for the purpose of completeness. Task Assignment assigns personnel, previously scheduled to a particular shift, a set of specific tasks to be completed during that shift. Shift assignment “involves assignment of individual staff to the lines of work” and “is often done during the construction of the work lines” (Ernst et. al., 2004a).

2.3 Application Areas

Personnel scheduling has a broad range of application. In a separate article Ernst, Jiang, Krishnamoorthy, Owens, and Sier surveyed over 700 papers of the personnel scheduling literature (Ernst et. al., 2004b). They developed the following table to illustrate the prior work in each application area. The application area is listed along with the number of papers surveyed in that particular application area.

Application	Papers	Application	Papers
Buses	129	Civic Services and Utilities	22
Nurse Scheduling	103	Venue Management	19
Airlines	99	Protection/Emergency Systems	16
Railways	37	Other Applications	14
Call Centers	37	Transportation Systems	12
General	33	Hospitality and Tourism	7
Manufacturing	29	Financial Services	6
Mass Transit	28	Sales	3
Health Care Systems	23		

Table 2.1: Number of surveyed papers in each application area of personnel scheduling (Ernst et. al., 2004b)

Ernst, et. al. (2004b) define each application area in their survey paper. I refer the reader to this article for additional detail of the categorization. The articles from each application area may concentrate on all or part of the problem instances described above as it pertains to the specific area.

The application areas pertinent to the research in this dissertation are characterized by flexible demand, a heterogeneous or mixed workforce, possible continuous operations, and personnel preference consideration. Alfares (2004) defines a mixed workforce as employees having differing “skill level, learning rate, wage, availability, and work hours.” Continuous operations are characterized by demand requirements 24 hours a day. Of the application areas defined by Ernst, et. al. (2004b), Hospitality and Tourism, Financial Services, and Sales are of particular relevance here. These areas represent a small portion of the personnel scheduling literature. They encompass only 16 of the over 700 papers surveyed. Furthermore, the subset of these papers examining the tour-scheduling problem is a further reduction of the papers surveyed.

2.3.1 Hospitality and Tourism Tour-scheduling

Hospitality and tourism industries as defined include “hotels, tourist resorts and restaurants” (Ernst et. al., 2004b). These industries have flexible demands and night and weekend shift requirements as well as employees with varying skill sets. Eveborn and Ronnqvist (2004) propose an elastic set-partitioning model and a branch and price algorithm to solve the tour-scheduling problem. The methodology is imbedded in a general scheduling software package called SCHEDULER. Glover and McMillan (1986) employ a tabu search and Thompson (1996) makes use of simulated annealing. Loucks and Jacobs (1991) begin by using a goal programming approach and then solve the corresponding integer program using a two-phase heuristic. A heuristic approach combined with a tabu search is developed by Litchfield, Ingolfsson, and Cheng (2003) to roster a restaurant. Finally, Love and Hoey (1990) define a mixed-integer program and solve the tour-scheduling problem using a minimum cost network flow simplex algorithm. This paper is of particular interest and will be discussed in further detail.

2.3.2 Financial Services Tour-scheduling

Financial services tour-scheduling is applied to staffing “clerical workers in service industries such as banking and insurance.” Again, the industry is characterized by flexible demand and by full and part-time workers. Li, Robinson, and Mabert (1991) and Mabert and Raedels (1977) use heuristics to roster workers, the former considering differing skill sets. Mabert and Watts (1982) propose simulation to solve a set-covering formulation and Mould (1996) developed a spreadsheet decision support system (DSS) to allow employers to explore different tour schedules.

2.3.3 Sales Tour-scheduling

Retail sales tour-scheduling has received the least amount of attention in the literature (Ernst et. al., 2004b). Abboud et. al. (1998) propose a global search technique that is a hybrid of genetic algorithms and simulated annealing. Glover, McMillan, and Grover (1985) develop a DSS that uses a heuristic and Haase (1999) shows a column-generation technique to solve an integer-programming formulation.

2.4 Solution Techniques and Methodologies

The most important aspect of this literature review is to detail the prior work in methodology used to solve personnel scheduling problems. Ernst, et. al. (2004b) show a similar break down of solution techniques and methodologies as was shown for various application areas. The following table shows the techniques and the number of papers using that methodology. Papers employing more than one solution technique will appear in multiple categories.

Method	Papers	Method	Papers
Branch-and-Bound	14	Lagrangean Relaxation	32
Branch-and-Cut	9	Linear Programming	35
Branch-and-Price	30	Matching	36
Column Generation	48	Mathematical	27
Constraint Logic Programming	46	Network Flow	38
Constructive Heuristic	133	Other Meta-Heuristics	11
Dynamic Programming	17	Other Methods	35
Enumeration	13	Queuing Theory	32
Evolution	4	Set Covering	58
Expert Systems	15	Set Partitioning	72
Genetic Algorithms	28	Simple Local Search	39
Goal Programming	19	Simulated Annealing	20
Integer Programming	139	Simulation	31
Iterated Randomized Construction	5	Tabu Search	16

Table 2.2: Number of surveyed papers using each methodology for personnel scheduling (Ernst et. al., 2004b)

Table 2.2 shows the disparate number of solution techniques and methodologies used to solve personnel scheduling problems. The methodology of particular interest to the research in this dissertation is network flow modeling. The following sections will show the application of the network flow modeling literature to the personnel scheduling problem instances with concentration on the tour-scheduling problem.

2.4.1 Network Flow Tour-scheduling (Airline, Mass Transit, Nurse Applications)

Network flow modeling has been used extensively in the airline industry. Yan and Chang (2002) and Barnhart, et. al. (1994) solve shortest-path network problems in a column generation formulation to pair airline crews to flight schedules. An additional column generation approach using network models to price new columns is proposed by Mason and Smith (1998). Mellouli (2001) uses a “state-expanded aggregated time-space network” to solve airline and rail crew scheduling. Nicoletti (1975) uses assignment sub-problems within a constructive heuristic and Tingley (1979) sequentially solves assignment and matching problems to roster airlines.

Public and mass transit systems are also well addressed in the tour-scheduling literature. Patrikalakis and Xerocostas (1992) develop an approach that uses a network flow problem with side constraints to schedule vehicles for the Athens Area Urban Transport Organization. Carraresi and Gallo (1984) combine a matching approach and Lagrangean relaxation for crew rostering in mass transit. Borndorfer et. al. (2001) apply a column generation technique using network flow sub-problems for duty scheduling in European public transit. Ball and Benoit-Thompson (1988) sequentially solve shortest path and matching problems within a Lagrangean relaxation based heuristic. Banihashemi and Haghani (1991) propose to solve mass transit crew scheduling using a multi-commodity flow model.

One of the most studied applications in personnel scheduling is nurse scheduling. Nurse tour-scheduling or rostering is a subset of the literature for this application. The literature on nurse tour-scheduling is characterized by a known demand for nurses and a given set of nurse shifts. The heterogeneous workforce is characterized by varying nurse grades or skill sets and continuous or 24-hour operations. Ernst, et. al. (2004) give a breakdown of the various algorithmic approaches to nurse tour-scheduling. These mathematical approaches include simulation, heuristics, global search techniques, constraint programming, and mathematical programming. The scope of this literature survey will be limited to the mathematical programming approaches, the differences between these approaches, and the proposed model in this research which I term the network-based mathematical program (NBMP).

Jumard, et. al. (1998) propose a 0 - 1 mixed-integer linear program (MIP) master problem to solve the nurse tour-scheduling problem. They incorporate a heterogeneous workforce, based upon nursing specialties or skills, overlapping shifts, and nurse preferences. The master problem contains a set of feasible shifts for the planning horizon that meet all the constraints. The goal of the master problem is to match nurses with each shift pattern. Other constraints such as minimum number of nurses with skill level ℓ scheduled at any time and the ratio of less experienced to more experienced nurses, are contained in the master problem. A branch and bound approach is taken to solve the master problem and maximize the preferences of the nurses. After the initial linear programming (LP) relaxation, an auxiliary problem is then solved using a resource constrained shortest path. The auxiliary problem determines where to branch and which column to generate into the basis.

Similarly, Millar and Kiragu (1998) develop patterns of shifts and days-off, which they term stints, where “all possible work-stretches can be pre-specified”. They attempt to find a set of stints that cover the demand for nurses using a mixed-integer branch and bound strategy. Warner (1976) also proposes an approach where all feasible patterns of shifts and days off were enumerated and then allocated to a nurse.

One additional application of the day/shift and day-off pattern is found in “Solving the Problem of Re-rostering Nurse Schedules with Hard Constraints: New MultiComodity Flow Models” by Margarida Moz and Margarida Vaz Pato. Published in the *Annals of Operations Research* in April 2004, the article proposes a method to re-roster a tour-schedule once changes in the given schedule no longer meet demand requirements. Sickness, termination, or hiring may be reasons for the need to change a given roster once it has been published.

Moz and Pato (2004) look at a continuous workforce that is heterogeneous. Differing nurse seniorities or specialties are taken into account by giving each shift in their model a separate task designation. The continuous workday is separated into 3 distinct shifts that start and end at the same time each day. These shifts are labeled Morning, Evening, and Night. A feasible schedule will meet hard constraints such as one shift per day, elimination of non-consecutive shifts, each shift/task combination assigned to a single nurse, the nurse assigned to a task must be eligible to perform the task, and a minimum number of days off per week for each nurse.

The algorithm approach proposed by Moz and Pato is an “integer multicommodity flow problem in a directed multi-level acyclic network” (2004). Each nurse is a commodity and is represented by an origin node in the network and by a destination node. The number of days to be scheduled is the number of levels between the origin nodes and the destination node. Each non-origin/non-destination node in the network represents a task for a nurse to perform, including being off that day. The goal is to find a feasible path for each nurse through the network that meets all the hard constraints mentioned above. Below is a figure illustrating the network.

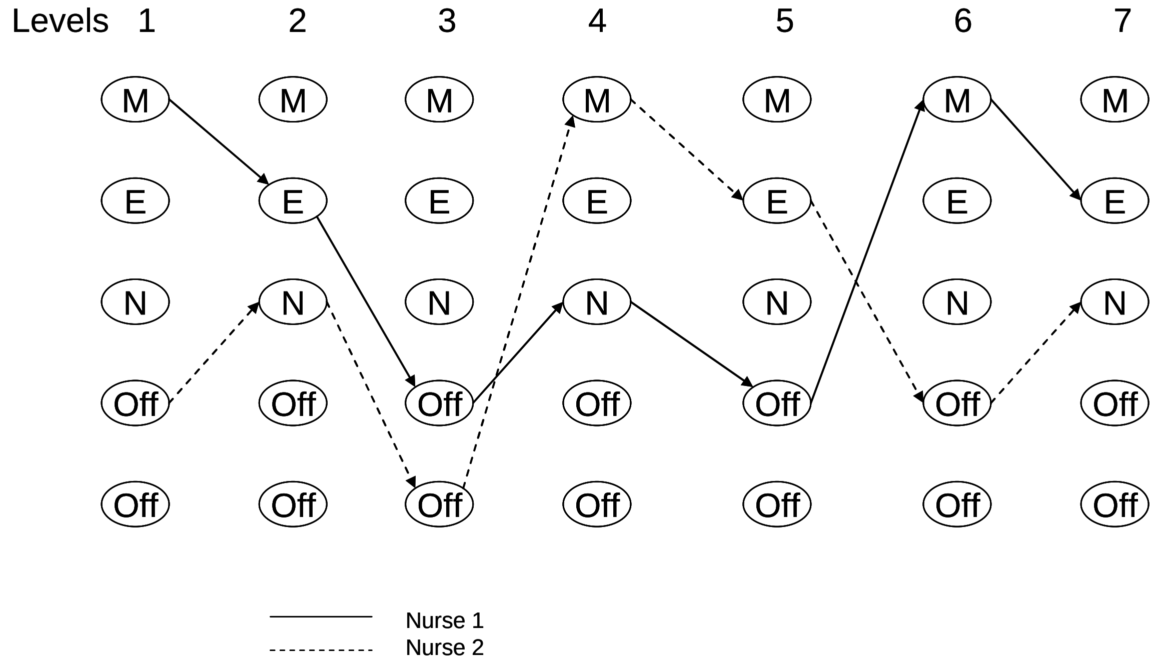


Figure 2.1: Multicommodity flow problem in a directed multi-level acyclic network (Moz & Pato, 2004)

Moz and Pato use this network formulation to re-roster a completed 28 day schedule that has been perturbed at some day *d*, which is considered the origin of the network. “The re-rostering problem amounts to finding an optimal integer flow of *n* commodities, from the nodes of level d-1 to those of the last level” (Moz & Pato, 2004). The formulation is a mixed integer linear program with binary constraints on the decision variables; they term this formulation as a binary linear program (BLP). The objective function of the algorithm is to find a new schedule that will minimize the number of nurses who must swap shifts.

The BLP does not use the network structure to solve the problem. The network structure is for illustrative purposes, as noted by Moz and Pato "this flow approach provides a suggestive network model for RSP (re-rostering problem), it leads to large scale BLP formulation, with many variables and constrains, some of which are additional to the flow model" (2004). Due to the low number of shifts, the formulation in effect enumerates all possible paths through a network that will satisfy the constraints described above. The nodes of the network represent shift/skill combinations and the arcs represent links between shifts that are feasible. Moz and Pato acknowledge this. "It is worth noting that… the flow of each commodity can only use one path from its origin to its destination, thus resulting in binary flows" (Moz & Pato, 2004). The commodities are the various nurses working a specific shift performing a given task. The origin is day *d*, while the destination is the 28th day.

The Moz and Pato models, as well as Jumard, et. al. (1998), Millar and Kiragu (1998), and Warner (1976), vary significantly from the research proposed by the network-based mathematical program formulation; although, both models use the network formulation for mostly illustrative purposes. The prime difference is the models presented by Moz and Pato are mixed integer linear programs with binary constraints. This requires a branch and bound approach to solve the problem. In many instances the network-based mathematical program (NBMP) requires no branching because the solution to the linear program provides binary decision variables. The proposed NBMP research can accommodate a large number of shifts with varied start times and of various lengths as well. The shifts in the NBMP research can also overlap. Moz and Pato describe a model where the shifts start and end at the same times each day and do not overlap. It is not readily apparent that their formulation could account for overlapping or varied length shifts. The enumeration of each feasible shift to shift to day off path through the network would make the problem grow rapidly if such variations in shift pattern were present. Next, although the workforce of nurses are heterogeneous in the aspect that they have varying skill sets, they do not have constraints on their availability apart from restrictions on consecutive shifts. The network-based mathematical program accounts for employee restrictions on availability as well as varying skill sets. Finally, the objective function of the Moz and Pato model is a deviation from a previously published tour-schedule. The objective of the proposed NBMP research is employee's weighted preference.

Although the use of network modeling has been well explored in the literature for airline, mass transit, and nurse tour-scheduling, these problems differ in nature from the problems of interest in this dissertation. In general, airlines, mass transit, and nursing applications are not characterized by flexible demand. They also employ a homogeneous workforce where each employee has similar availabilities, skill-sets, and work hours. These differences dramatically affect the formulation of the tour-scheduling problem. Additional constraints needed to model employee availability, minimum and maximum work hours, consecutive shift restrictions, and skill-set eligibility are examples of the dissimilarities.

2.4.2 Network Flow Tour-scheduling with a Heterogeneous Workforce

Tour-scheduling a work force with varying daily and weekly availabilities, skill-sets, minimum and maximum work hours, and wage rates has not been given a proportionate amount of attention in the personnel scheduling literature. Furthermore, only Love and Hoey (1990) attack this problem from a network flow approach. Love and Hoey define a mixed-integer linear program (MILP) for shift scheduling and tour-scheduling for a fast food restaurant. The MILP is consistent with other formulations in the literature of the heterogeneous workforce problem and is shown below.

b_{hl} = number of employees needed to work in time period *h* at work area *l*;
x_{jkl} = number of employees to work shift *j* on day *k* in work area *l*;
a_{hjk} = 1 if time period *h* is part of shift *j* on day *k*, 0 otherwise;
s_{hl} = surplus employees for time period *h* in work area *l*;
w_{ijkl} = 1 if employee *i* is to work shift *j* on day *k* in work area *l*, 0 otherwise;
r_{jkl} = shortage of employees to work shift *j* on day *k* in work area *l*;
y_{ik} = 1 if employee *i* is scheduled to work on day *k*, 0 otherwise;
z_i = number of days employee *i* is scheduled to work; and
ν_k = 1 if employee *i* is to work at least *k* days in the week, 0 otherwise.

minimize:

$$\sum_{hl} f_{hl} s_{hl} + \sum_{ijkl} c_{ijkl} w_{ijkl} + \sum_{jkl} g_{jkl} r_{jkl} + \sum_{ik} d_{ik} v_{ik} \quad (2.1)$$

subject to:

$$\sum_{jk} a_{hjk} x_{jkl} - s_{hl} = b_{hl} \quad \textit{all h,l} \quad (2.2)$$

$$\sum_{i} w_{ijkl} + r_{jkl} - x_{jkl} = 0 \quad \textit{all j,k,l} \quad (2.3)$$

$$\sum_{jl} w_{ijkl} - y_{ik} = 0 \quad \textit{all i,k} \quad (2.4)$$

$$\sum_{k} y_{ik} - z_i = 0 \quad \textit{all I} \quad (2.5)$$

$$\sum_{k} v_{ik} - z_i = 0 \quad \textit{all I} \quad (2.6)$$

$$y_{ik} \leq 1 \quad \textit{all i,k} \quad (2.7)$$

$$z_{ik} \leq 1 \quad \textit{all i,k} \quad (2.8)$$

Love and Hoey then propose the decomposition of this MILP into two sub-problems. Minimizing

$$\sum_{hl} f_{hl} s_{hl} \quad (2.9)$$

subject to constraint 2.2 will solve the shift assignment problem. This first sub-problem will define the shifts needed to cover demand. The second sub-problem minimizes the rest of the objective function subject to equations 2.3 – 2.8. Love and Hoey (1990) state that since “each column in equations (2.3) – (2.6) has at most two nonzero entries and these equations can be constructed such that each column with the two nonzero entries has one +1 and one -1, this second sub-problem can also be solved as a minimum cost network flow problem.” The authors propose solving these sub-problems using network simplex algorithms.

Although Love and Hoey (1990) show that a tour-scheduling problem can be solved with a minimum cost network flow, there are limitations to the complexity of the heterogeneous problem that can be solved with this formulation. As acknowledged by the authors, this formulation does not perform well for continuous operations. Equation 2.4 insures that each employee works only one shift per day. However, this constraint does not preclude an employee that just worked an 8 hour evening shift from being scheduled for the consecutive late night shift which is technically on a different day. Additionally, there is no discussion of employees who have differing skill sets or the more complex case of employees who have overlapping and differing skill sets. An example would be employee 1 who can run the cash register and drive-thru, while employee 2 can run the cash register and the fry machine. A more robust tour schedule must take into account the specific skill-sets of the employees and the skills required per shift.

The research detailed in this dissertation uses an expanded network model to allow for varying skill sets and specialized side constraints to manage continuous operations. The expanded network formulation with specialized side constraints forms a linear program whose solutions are typically integer.

2.5 Linear Programming Approach to Tour-scheduling

Alfares (2004), in his survey paper, notes that most of the linear programming approaches in the literature are heuristic or are approximations. Many, such as Brusco and Johns (1995), Mabert and Showalter (1990), Ashley

(1995), and Lauer et. al. (1994) use LP-based heuristics to solve the tour-scheduling problem. Ashley (1995) in particular uses an iterative LP approach to solve a heterogeneous tour-scheduling problem for the library staff at the University of Kansas. The implementation of the methodology requires human interaction with a spreadsheet optimizer. In each iteration, the operator must manually set fractional variables to be integer until a feasible schedule is created. Although Ashley (1995) reports success in a relatively few number of iterations, with only 14 employees the problem size discussed is small. There is no guarantee of optimality for large problems.

Lauer et. al. (1994) describe the heterogeneous nature of scheduling university students to a computer lab. Students have restricted availability due to class participation requirements, differing skill-sets, and many labs are open and must be staffed 24 hours. Lauer et. al. (1994) allow each employee to choose their own schedule from a set of possible shifts which have been determined using an LP. This approach may optimize preferences based upon seniority or the sequence in which the employees chose available shifts, but will not guarantee a feasible solution. If the final employees to be scheduled are not available or do not possess the required skills for the remaining shifts, the schedule will have to be iterated. The research presented in this dissertation will show application by solving a similar university computer lab staffing problem to optimality.

2.6 Literature Summary

The amount of work previously done and currently being pursued to solve personnel scheduling problems is extensive. Obviously, the area of research continues to be important to both the academic and business communities. Although much work has been done, the concentration on tour-scheduling or crew rostering of a flexible demand industry with a heterogeneous workforce and continuous operations using network flow or linear programming approaches is much more limited. Table 2.3 shows the void in the literature this dissertation will address. The table uses the previously proposed categorization by Ernst et. al. (2004b) of the tour-scheduling papers reviewed in this chapter.

Tour Scheduling/Rostering Problem Instance	Buses	Nurse Scheduling	Airlines	Railways	Call Centers	General	Manufacturing	Mass Transit	Health Care Systems	Civic Services and Utilities	Venue Management	Protection/Emergency Systems	Other Applications	Transportation Systems	Hospitality and Tourism	Financial Services	Sales
Branch-and-Bound																	
Branch-and-Cut																	
Branch-and-Price															1		
Column Generation																	1
Constraint Logic Programming																	
Constructive Heuristic															1	2	1
Dynamic Programming																	
Enumeration																	
Evolution																	
Expert Systems																	
Genetic Algorithms																	1
Goal Programming															1		
Integer Programming																	
Iterated Randomized Construction																	
Lagrangean Relaxation																	
Linear Programming																	
Matching																	
Mathematical																	
Network Flow		4	6					5							1		
Other Meta-Heuristics																	
Other Methods																	
Queuing Theory																	
Set Covering																	1
Set Partitioning															1		
Simple Local Search																	
Simulated Annealing															1	1	
Simulation																1	
Tabu Search															2		

Table 2.3: Number of papers reviewed that specifically addressed the tour-scheduling problem instance using the methodology breakdown and application areas proposed by Ernst, et. al. (2004b)

Chapter 3

A NETWORK-FLOW BASED MATHEMATICAL PROGRAMMING APPROACH TO OPTIMAL ROSTERING OF CONTINUOUS HETEROGENEOUS WORKFORCES

Abstract

We present a linear programming (LP) approach to a generalized personnel tour scheduling integer programming (IP) problem based upon a minimum cost network-flow formulation with specialized side constraints. The formulation simultaneously incorporates many realistic constraint types and optimally solves the IP tour scheduling problem for industries with continuous (24 hour) operations and a heterogeneous workforce with varying availabilities, shift preferences, restrictions on working consecutive shifts, differing skill-sets, and minimum and maximum shift requirements per week. Our work is the first to solve this generalized class of IP problems optimally. Problems of this complexity routinely occur in industry but are computationally prohibitive to solve. In many instances integer solutions to our tour scheduling problem are found without branching, bounding, or cutting schemes. This allows the CPLEX interior-point method to generate solutions orders of magnitude faster than the corresponding integer program does. Our methodology is demonstrated by finding an optimal tour schedule for staffing the four main Computer Commons at Arizona State University, a problem of order 10,000 binary variables and 1,000 constraints in 0.12 CPU seconds. It is also used to perform the calculations for a generalized staff sizing problem.

3.1 Introduction

Personnel scheduling is one of the most difficult, important, and studied problems in operations research. The optimal choice of the number of employees required to meet customer demand, shift start and stop times, daily lunches and breaks, and the assignment of the adequately skilled employee to the best shift is a classic combinatorial optimization problem. Managers who attempt to manually solve the personnel scheduling problem expend many valuable work hours to find even a feasible solution which has little probability of being optimal based on any objective function. Sub-optimal scheduling increases an industry's tangible costs, not only through the consumption of a manager's time, but also through the misallocation of shifts to meet customer demand and through employees staffing shifts for which they are not qualified. Furthermore, a sub-optimal schedule will increase intangible costs such as lower employee morale from dissatisfaction with a poor work schedule. Eliminating these costs is an easy way to increase the profitability of any business.

Personnel scheduling is a necessary chore associated with any organization employing people to get work accomplished. Some organizations that employ a professional workforce during traditionally scheduled business hours find the personnel scheduling problem a trivial concern. However, industries that rely on a large portion of part-time employees with highly constrained availabilities (work hours) and require staffing for the entire day and evening hours or, in particular, continuous (24-hour) operations have a much more challenging problem. Such industries include fast-food restaurants, hotels and resorts, grocery stores, customer assistance telephone services, hospitals, and others. The combined salary cost of this segment of the economy is quite large. A timely and optimal solution to their personnel scheduling problem will save managers valuable time, meet customer demand at minimum labor cost, and increase worker morale by matching each employee to the best shift possible. This article presents an extremely fast, computationally efficient, and optimal network-flow based linear programming solution for the extremely challenging integer programming problem of rostering a continuous heterogeneous workforce with realistic constraints.

Personnel scheduling is generally decomposed into more tractable sub-problems. The three primary sub-problems are demand modeling, shift selection, and employee tour scheduling or rostering. Demand modeling determines the number of employees required on duty during a given time interval to satisfy customer needs. Forecasting and queuing theory are the primary tools applied to demand modeling. A typical chart of the number of employees required, illustrating the results of demand modeling, is shown in Figure 3.1.

The second primary personnel scheduling sub-problem is shift selection. Shift selection assigns consecutive hour shifts to satisfy the customer demand while meeting organizational and regulatory requirements for shift lengths, breaks, and mealtime allowances. The objective of shift selection is generally to minimize the number of hours scheduled that exceed customer demand. Figure

3.2 shows an example of a set of shifts that will optimally cover the number of employees required in Figure 3.1.

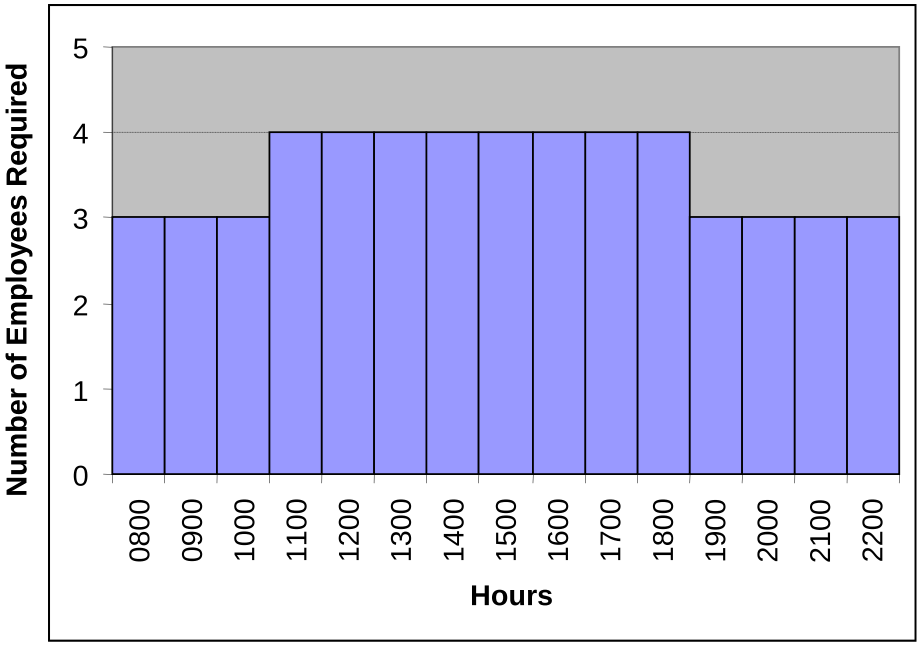


Figure 3.1: Employees requirement by hour from demand modeling

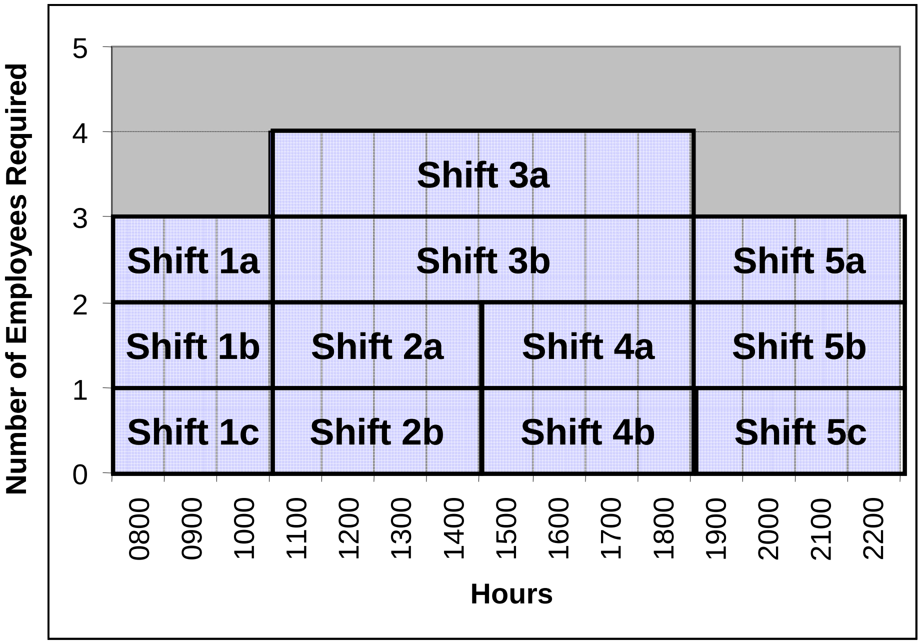


Figure 3.2: Example set of shifts that cover the demand in Fig. 3.1

The third personnel scheduling sub-problem is tour scheduling also know as employee rostering. Tour scheduling assigns individuals to specific shifts detailed during shift selection. A heterogeneous workforce is a collection of personnel who have significantly different availabilities, skill sets, and seniorities. Tour scheduling a heterogeneous workforce must consider all of these differences and optimally match the personnel with the best shift possible for which they are eligible. Additionally, considerations for individuals working a minimum and maximum number of shifts per week and insuring adequate rest between shifts for industries with continuous (24-hour) operations are critical to an optimal tour schedule. Heterogeneous and continuous workforces are realistic considerations for many industries, as previously noted. Employee shift preferences and management employee weighting comprise the objective function.

This methodology provides several original contributions to the personnel scheduling literature. The following list summarizes the advancements proposed by this research.

- Provide a solution methodology that simultaneously accounts for variations in employee availabilities, skill sets, and preferences.
- Provide a methodology that rosters employees for continuous operations by allowing for rest hours between shifts.
- Provide a network-flow based formulation that rapidly solves the tour scheduling problem, many times providing integer answers without any need for branching, bounding, or cutting schemes.

This article is organized in the following manner. Section 2 contains a survey of the pertinent literature and important previous work accomplished in personnel tour scheduling. Section 3 details the tour scheduling problem at Arizona State University for computer lab technicians. Our methodology is general, but we feel that for realism and clarity we will introduce our case study early. Section 4 provides the proposed solution method and model. Section 5 shows the results of the methodology as applied to the case study as well as applied to larger instances of the IP tour scheduling problem. Section 6 provides conclusions and recommendations.

3.2 Literature Survey

A vast amount of scholastic work has been accomplished on the personnel scheduling problem. This work has been documented in a wide array of refereed journals, conference proceedings, and lecture notes. Personnel scheduling spans a wide range of problem instances, specific applications, and solution techniques and methodologies. Many survey papers of the pertinent literature in the area have been written. Some of the most extensive and most recent include Ernst, et. al. (2004a), Ernst, et. al. (2004b), and Alfares (2004).

3.2.1 Application Areas

Personnel scheduling has a broad range of application. In their article, Ernst, et. al. (2004b) survey over 700 papers in the personnel scheduling literature. They define application areas and catalog prior work in each application area. We refer the reader to this article for additional detail of the categorization. The articles from each application area may concentrate on all or part of the problem instances described as it pertains to the specific area.

The application areas pertinent to our research are characterized by flexible demand, a heterogeneous or mixed workforce, possible continuous operations, and personnel preference consideration. Alfares (2004) defines a mixed workforce as employees having differing “skill level, learning rate, wage, availability, and work hours.” Continuous operations are characterized by demand requirements 24 hours a day. Important application areas of interest as defined by Ernst, et. al. (2004b) are Hospitality and Tourism, Financial Services, and Sales. Although these areas represent a significant proportion of economic activity, they form only a small portion of the personnel scheduling literature, only 16 of the over 700 papers surveyed. Furthermore, the subset of these papers examining the tour scheduling problem is a further reduction of the papers surveyed.

3.2.1.1 Hospitality and Tourism Tour Scheduling

Hospitality and tourism industries as defined include “hotels, tourist resorts and restaurants” (Ernst et. al., 2004b). These industries have flexible demands and night and weekend shift requirements as well as employees with varying skill sets. Eveborn and Ronnqvist (2004) propose an elastic set-partitioning model and a branch-and price algorithm to solve the tour scheduling problem. The methodology is imbedded in a general scheduling software package called SCHEDULER. Glover and McMillan (1986) employ a tabu search and Thompson (1996) makes use of simulated annealing. Loucks and Jacobs (1991) begin by using a goal programming approach and then solve the corresponding integer program using a two-phase heuristic. A heuristic approach combined with a tabu search is developed by Litchfield, Ingolfsson, and Cheng (2003) to roster a restaurant. Finally, Love and Hoey (1990) define a mixed-integer program and solve the tour scheduling problem using a minimum cost network flow simplex algorithm. This paper is of particular interest and will be discussed in further detail.

3.2.1.2 Financial Services Tour Scheduling

Financial services tour scheduling is applied to staffing “clerical workers in service industries such as banking and insurance.” Again, the industry is characterized by flexible demand and by full and part-time workers. Li, Robinson, and Mabert (1991) and Mabert and Raedels (1977) use heuristics to roster workers, the former considering differing skill sets. Mabert and Watts (1982) propose simulation to solve a set-covering formulation and Mould (1996) develops a spreadsheet decision support system (DSS) to allow employers to explore different tour schedules.

3.2.1.3 Sales Tour Scheduling

Retail sales tour scheduling has received the least amount of attention in the literature. Glover, McMillan, and Grover (1985) develop a DSS that uses a heuristic and Haase (1999) shows a column-generation technique to solve an integer-programming formulation.

3.2.2 Solution Techniques and Methodologies

The most important aspect of this literature review is to detail the prior work in methodology to solve personnel scheduling problems. Ernst, et. al. (2004b) show a break down of solution techniques and methodologies as was shown for various application areas. Papers employing more than one solution technique appear in multiple categories. The methodology of particular interest to our research in this article is network flow modeling. The following sections will show the application of the network flow modeling literature to the personnel scheduling problem instances with a concentration on the tour scheduling problem.

3.2.2.1 Network Flow Tour Scheduling (Airline, Mass Transit, Nurse Applications)

Network flow models have been used extensively in the airline industry. Yan and Chang (2002) and Barnhart, et. al. (1994) solve shortest-path network problems in a column generation formulation to pair airline crews to flight schedules. Column generation using network models to price new columns is proposed by Mason and Smith (1998). Mellouli (2001) uses a “state-expanded aggregated time-space network” to solve airline and rail crew scheduling. Nicoletti (1975) uses assignment sub-problems within a constructive heuristic, and Tingley (1979) sequentially solves assignment and matching problems to roster airlines.

Public and mass transit systems are also well addressed in the tour scheduling literature. Patrikalakis and Xerocostas (1992) develop an approach that uses a network flow problem with side constraints to schedule vehicles for the Athens Area Urban Transport Organization. Carraresi and Gallo (1984) combine a matching approach and Lagrangean relaxation for crew rostering in mass transit. Borndorfer et. al. (2001) apply a column generation technique using network flow sub-problems for duty scheduling in European public transit. Ball and Benoit-Thompson (1988) sequentially solve shortest path and matching problems within a Lagrangean relaxation based heuristic. Banihashemi and Haghani (1991) propose to solve mass transit crew scheduling using a multi-commodity flow model.

Another popular application area for network flow modeling is nurse scheduling. Dowsland and Thompson (2000) mix knapsack, network flow, and tabu search techniques to develop nurse rosters. Jaumard, Semet, and Vovor (1998) solve shortest-path network sub-problems within a column generation integer program. Millar and Kiragu (1998) develop cyclic and acyclic work schedules for nurses using a shortest-path problem with side constraints. Finally, Moz and Pato (2004) solve a nurse re-rostering problem using multi-commodity flow models.

Although the use of network modeling has been well explored in the literature for airline, mass transit, and nurse tour scheduling, these problems differ in nature from our problems of interest. In general, airlines, mass transit, and nursing applications are not characterized by flexible demand and additionally employ a homogeneous workforce where each employee has similar availabilities, skill-sets, and work hours. These differences dramatically affect the formulation of the tour scheduling problem. Additional constraints needed to model employee availability, minimum and maximum work hours, consecutive shift restrictions, and skill-set eligibility are examples of the dissimilarities.

3.2.2.2 Network Flow Tour Scheduling with a Heterogeneous Workforce

Tour scheduling a work force with varying daily and weekly availabilities, skill-sets, minimum and maximum work hours, and wage rates has not been given a proportionate amount of attention in the personnel scheduling literature. Furthermore, only Love and Hoey (1990) attack this problem from a network flow approach. Love and Hoey define a mixed-integer linear program (MILP) for shift scheduling and tour scheduling for a fast-food restaurant. They then decompose the MILP into two sub-problems. This is the same approach that we will take.

The first sub-problem defines the shifts needed to cover demand. The second sub-problem minimizes the rest of the objective function. As Love and Hoey (1990) state: “each column has at most two nonzero entries and these equations can be constructed such that each column with the two nonzero entries has one +1 and one -1, so this second sub-problem can be solved as a minimum cost network flow problem.” They propose solving these sub-problems using network simplex algorithms.

Although Love and Hoey (1990) show that a tour scheduling problem can be solved with a minimum cost network flow, there are limitations to the complexity of the heterogeneous problem that can be solved with their formulation. As acknowledged by the authors, this formulation does not perform well for continuous operations. Additionally, there is no discussion of employees who have differing skill sets or the more complex case of employees with overlapping and differing skill sets. A more robust tour schedule must take into account the specific skill-sets of the employees and the skills required per shift.

Our methodology uses an expanded network model to allow for varying skill sets and specialized side constraints to manage continuous operations. As we will see, the expanded network formulation with specialized side constraints forms a linear program (LP) whose solutions are in many cases integer.

3.3 A Case Study - ASU Computer Lab Technician Tour Scheduling

There are four computer labs at Arizona State University (ASU): the Atrium, BAC, GWC, and ECG. Each lab is staffed by technical experts (typically enrolled ASU science and engineering students) who assist other students using the computers in the lab. Currently, the Technology Support Analyst Principal (TSAP) is tasked with scheduling the technicians to meet the historic demand for each lab based upon the time of day. Each technician has a subset of the four labs in which they are qualified to work. The TSAP currently schedules over fifty technicians to the various labs by hand. He must take into consideration the demand for technicians as a function of day and time, availability and non-availability of the technicians, shift restrictions and maximum hours per week, minimum hours per week, as well as the preferences and qualifications of the technicians for particular shifts. Each such effort requires 2-3 days and a satisfactory solution is the first feasible one found.

Each lab has its own operating hours and staffing requirements. For example, Table 3.1 depicts the operating hours and staff requirements for the Atrium. Although there must be no staff during closed hours, and at least one technician working each operating hour, there is some flexibility regarding the staffing levels during operating hours. Staffing the lab in excess of the requirements in the table will result in excess costs to the university. Staffing the labs at a lower level is not allowed.

	7a	8a	9a	10a	11a	12p	1p	2p	3p	4p	5p	6p	7p	8p	9p	10p	11p
Sunday	▨	▨	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Monday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Tuesday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Wednesday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Thursday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Friday	3	3	3	4	4	5	5	5	4	4	3	3	4	▨	▨	▨	▨
Saturday	3	3	3	4	4	4	4	4	4	4	4	3	3	3	3	▨	▨

Table 3.1: Lab technician requirements (Atrium) based on hourly demand

Each technician has his/her own availability profile and preferences among those hours. The minimum and maximum shifts per week are developed by the TASP after his consultation with each technician.

A two-phase approach to the tour scheduling problem for ASU computer labs is proposed as depicted in Figure 3.3. In Phase 1, we select a set of shifts to cover the staffing requirement for each of the four labs. The scheduler can indicate shift length priorities in order to bias the model in favor of specific shift lengths. For example the scheduler can specify a preference for 6-hour shifts as opposed to 3-hour shifts by assigning them different weights. Additionally, benefits arise from the fact that selecting the shifts for a single lab has no effect on which shifts should be selected for the other three. This independence allows for the selection of shifts for each of the four labs to be done individually, resulting in four smaller Phase 1 models.

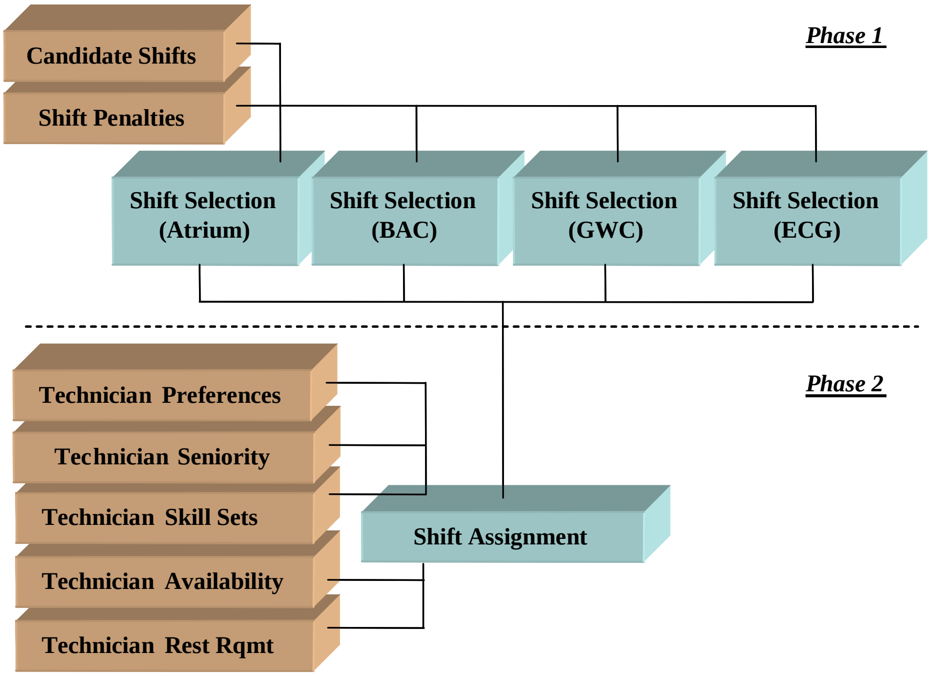


Figure 3.3: Two-phase methodology for shift selection and rostering

In Phase 1, we consider all reasonable shift lengths over the planning horizon. The objective of this phase is to select a set of shifts that covers the lab requirements, as well as minimizes the sum of “shift penalties”. Shift penalties are assigned based on the scheduler’s preference for a particular shift length. A penalty of 1.0 signifies that no extra weight is given to the hours in a given shift length. Phase 1 then minimizes the total cost of selecting individual shifts, where the cost of each shift is the product of the number of hours and the scheduler's penalty factor for the shift. The penalties for shift lengths solicited from the TSAP for our case study are contained in Table 3.2.

Shift Length	**Penalty Factor**	**Cost**
3	1.05	3.15
4	1.00	4.00
5	1.00	5.00
6	1.00	6.00
7	1.10	7.70
8	1.15	9.20

Table 3.2: Shift penalties used in Phase 1 set covering problem

For example, consider the two possible shift selections with regard to Saturday mid-day lab requirements in Figure 3.4 below. The solutions show the shift lengths chosen as well as the number of employees on each shift. The value of solution 1 is 53.45, whereas the value of solution 2 is 59.9. Solution 1 happens to be the optimal solution. Both solutions cover the demand profile exactly, but Solution 1 uses shift lengths preferred by the manager.

<table>
<tr><td>Saturday
Employee
Demands</td><td>3</td><td>3</td><td>3</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>3</td><td>3</td><td>3</td><td>3</td><td></td><td></td></tr>
<tr><td>solution 1</td><td colspan="3">shift 1 (3 emp)</td><td colspan="4">shift 2 (4 emp)</td><td colspan="4">shift 3 (4 emp)</td><td colspan="4">shift 4 (3 emp)</td><td></td><td></td></tr>
<tr><td rowspan="2">solution 2</td><td colspan="7">shift 1 (3 emp)</td><td colspan="8">shift 2 (3 emp)</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td colspan="8">shift 3 (1 emp)</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 3.4: Example of effect of preference on two distinct shift definitions

Though NP-complete, the Phase 1 problem is not computationally challenging. A set-covering integer program (IP) is directly used to solve Phase 1. It selects the optimal set of shifts to cover customer demand based on shift length preferences. In Phase 2, we must assign employees to these shifts, subject to employee preferences by hour, availabilities, skills, rest periods as well as some notion of employee preference by management. This is a very difficult problem to solve efficiently and is the focus of this article.

3.4 Proposed Tour-Schedule Solution Method and Model

The tour-scheduling problem is inherently a binary set-covering problem. Therefore, as problem size increases, the efficiency of the associated IP decreases rapidly. However, we have developed a formulation of the problem as a minimum cost network-flow, using an arc capacity method, so that the resulting network structure many times provides integer binary answers. The network structure can easily be written as an LP and solved using fast solution algorithms such as the CPLEX interior point method. Solutions of this formulation do not require any branching, bounding, or cutting schemes to find integer solutions – the frequency of the integer solutions are a consequence of the formulation method. Therefore, very large problem instances can be solved in a computationally insignificant amount of time compared to the corresponding IP solution. Figure 3.5 shows our formulation of our generalized tour-scheduling problem as a minimum cost network-flow.

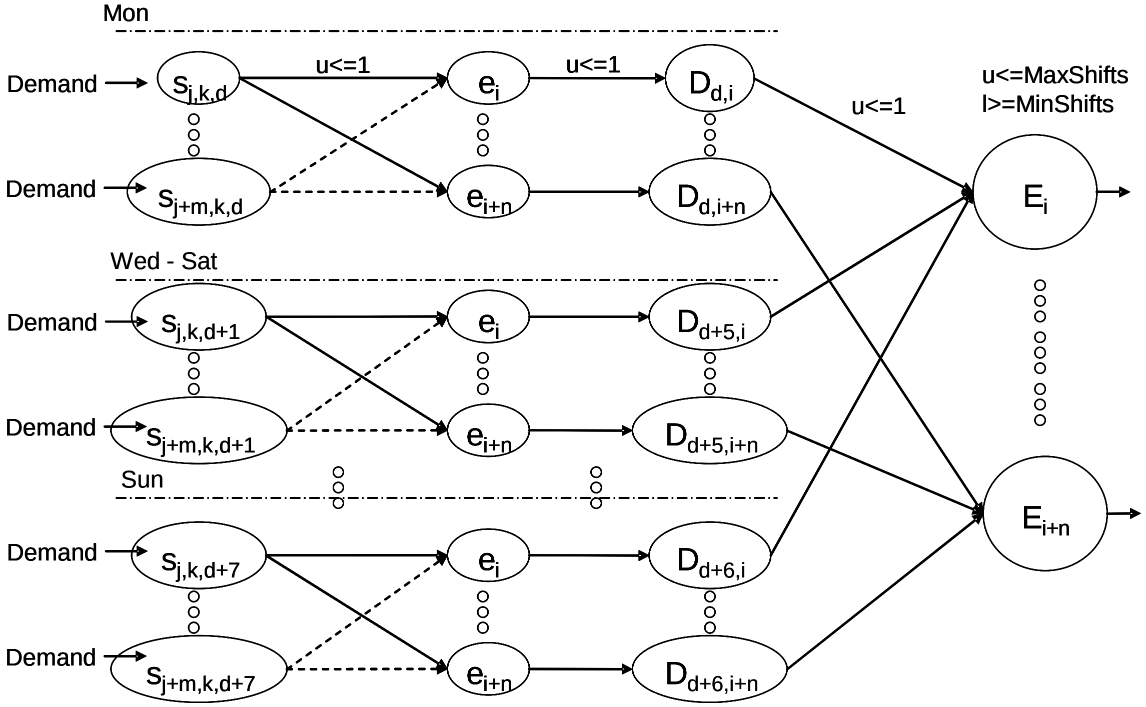


Figure 3.5: Network representation of the Phase 2 tour-scheduling problem

Here, $s_{j,k,d}$ denotes shift number *j* requiring skill set *k* and on day *d*, e_i is employee number *i*, $D_{d,i}$ is the total number of shifts, *D*, on day d for employee *i*, and E_i is the total number of shifts per week, *E*, for employee *i*.

The number of employees needed for each shift node flows from the Demand arc. Each shift node has an edge to each qualified and available employee node. The shift-to-employee arc is capacitated at 1, meaning only one of the required staffing for a shift can be assigned to a single employee. Each employee then flows their daily work assignment to the employees' daily-total-node, D. This arc is capacitated at 1, meaning each employee can work only one shift per day. Finally, the employees' daily-total-nodes channel the flow to the employees' weekly-total-node which contains the capacities to enforce min and max shifts per week.

The balance equations for the above network flow formulation are:

$$\sum_{i} s_{j,k,d} e_i = demand \qquad \text{for all } j,k,d \tag{3.1}$$

$$\sum_{j,k} s_{j,k,d} e_i = D_{d,i} \qquad \text{for all } i,d \tag{3.2}$$

$$\sum_{d} D_{d,i} \leq \max number\_of\_shifts \qquad \text{for all } j \tag{3.3}$$

$$\sum_{d} D_{d,i} \geq \min number\_of\_shifts \qquad \text{for all } j \tag{3.4}$$

$$D_{d,i} \leq 1 \qquad \text{for all } i,d \tag{3.5}$$

Constraint set 1 requires that the demand for a number of employees for each type of shift is met. Set 2 totals the shifts per employee for each day and requires that the total be less than or equal to 1 in constraint set 5 (no employee works more than one shift per day, but remember that the shift length is a variable). Constraint sets 3 and 4 require that the minimum and maximum numbers of shifts per week are met for each employee. The following three sub-sections describe how our model incorporates all the constraints needed to create a tour-schedule for a real world business, including a heterogeneous work force and incorporate consecutive shift restrictions.

3.4.1 Consecutive Shift Restrictions

Adequate rest between consecutive shifts is a requirement for any realistic tour-schedule of continuous operations. Industries that are only open for an 8-hour day have tour-schedules that implicitly contain adequate rest. However, when scheduling an industry with continuous or 24-hour operations, restrictions on consecutive shifts is a requirement for any realistic tour-scheduling formulation. Love and Hoey (1990) and Loucks and Jacobs (1991) note the need for consecutive shift restrictions and state the lack of such restrictions as limitations to their tour-scheduling models. Our formulation allows for specialized side constraints to accommodate continuous operations.

As input in any specific application, we use the appropriate industry standard for how much time is required between the start of shift *j* and the start of the next shift. This leads to a matrix of shifts, *J*, that conflict with each other in the sense that an employee assigned to work shift *j* cannot be scheduled to work any shift conflicted with shift *j*. The following constraint is added to constraints 1-5 above to accommodate consecutive shift restrictions.

$$\sum_{j \in J} s_{j,k,d} e_i \leq 1 \qquad \text{for all conflicted shifts} \qquad (3.6)$$

Surprisingly, our empirical experience indicates that these side constraints do not destroy the integrality of the solution to the minimum cost network flow in many cases. However, in theory non-integer answers are possible.

3.4.2 Employee Availabilities and Varying Skill Sets

Many industries have employees with heterogeneous availabilities. For example, service industries often employ younger, part-time employees who have restrictions on their time due to schooling or other activities. Our model can be modified to accommodate such heterogeneous availabilities. We simply remove the network arc from a shift to an employee who is not available during the hours encompassing that shift. No flow units can travel from that shift to the employee and consequently the employee is not assigned to that shift.

Similarly, it is often the case that many service industries have different skill requirements needed during a particular shift. If an employee does not possess the required skill for a particular shift, the arc from the shift to the employee is removed from the network. If the employee has multiple skill sets, the arc from the employee to all shifts within their set is contained within the network.

This is an important aspect to the efficiency of the tour-schedule. Many examples within the tour-scheduling literature schedule only one department within the industry at a time. Therefore, the employees assigned to that department can be scheduled only within that department and there is no mechanism to schedule that employee across departments. This could lead a department to over-staffing costing a business extra payroll. We have no such restriction.

Removal of the two types of arcs described above are easy to program. They have no effect on the integrality of solutions from the LP-network solution. This is because the resulting network is still of minimum cost flow form.

3.4.3 The Objective Function - Employee and Manager Preferences and a Perturbation

The objective function of the constraint set described above can now be defined. First, each employee scores each shift, on a preference scale, based upon their desire to work at that time. Any scale could be used, but we have found in practice that one based on a scale of (0,100) allows for adequate granularity. The preliminary objective function is then:

MAX $$\sum_{j,k,d,i} s_{j,k,d} e_i \bullet pref(s_{j,k,d} e_i) \qquad (3.7)$$

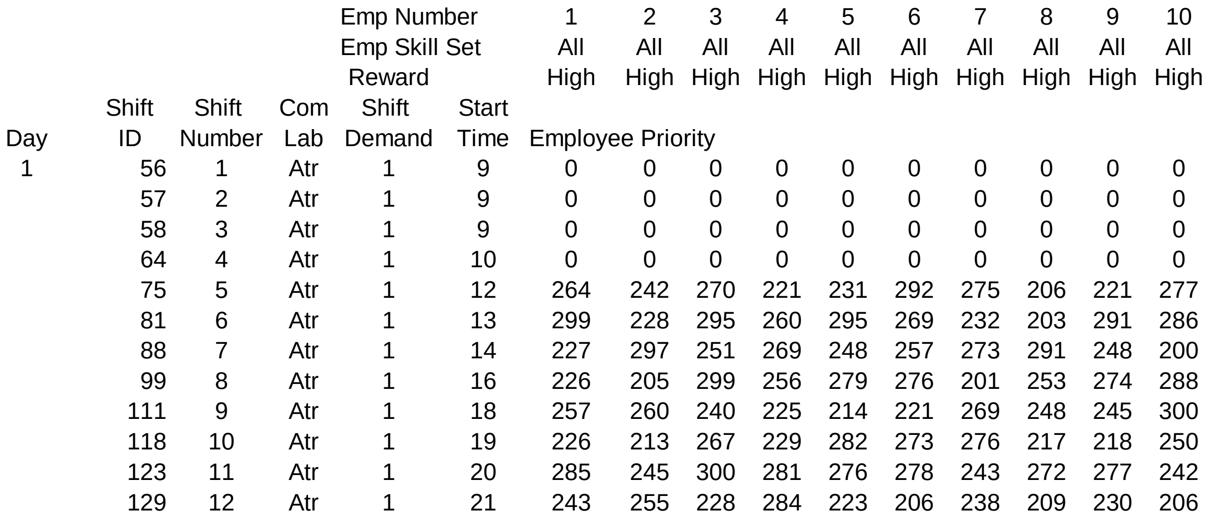
Additionally, we allow the manager to express their preferences of employees on shifts taking such factors as seniority or job performance into account. We call these preferences “rewards”. The larger the reward on a scale of (0,100), the more likely the model will select that employee for desired shifts (and it certainly helps to break ties). The objective function becomes:

MAX $$\sum_{j,k,d,i} s_{j,k,d} e_i \bullet \left[pref(s_{j,k,d} e_i) + \mathrm{Re}\, ward(s_{j,k,d}, e_i)\right] \qquad (3.8)$$

Finally, it is possible for the algorithm to find an alternate optimal solution that contains fractional employee assignments. This occurs when the preferences of two employees for a particular shift are equal and neither is violating a consecutive shift restriction or a maximum number of shifts per week constraint. To remove this non-integer alternate optimal solution effect from the view of manager we add to the objective function a small random value, ε, whose magnitude is chosen to range from (0,1) based on the magnitude of the preference and reward scales of (0,100). The random value is not reported as part of the final objective function value, merely used to remove the possibility of non-integer alternative optimal solutions

MAX $$\sum_{j,k,d,i} s_{j,k,d} e_i \bullet \left[pref(s_{j,k,d} e_i) + \mathrm{Re}\, ward(e_i)\right] + \varepsilon \qquad (3.9)$$

Data organization to support these calculations are straightforward. A spreadsheet is used as the input data file. A Visual Basic macro then writes the linear program in CPLEX .lp input format. This macro runs almost instantaneously. The output file is then given to CPLEX for optimization. The ease of changing data in the input data file, the speed with which the output file is written, and the speed with which the problem is optimized (more on this in the next section) allows the user to study a wide variety of problem instances and conduct sensitivity analysis. Figure 3.6 shows an example of the input data file format. The data file for the ASU computer lab problem has 195 rows of shifts with 50 columns of preferences (the sum of employee and manager values).



				Emp Number		1	2	3	4	5	6	7	8	9	10
				Emp Skill Set		All	All	All	All	All	All	All	All	All	All
				Reward		High	High	High	High	High	High	High	High	High	High
	Shift	Shift	Com	Shift	Start										
Day	ID	Number	Lab	Demand	Time	Employee Priority									
1	56	1	Atr	1	9	0	0	0	0	0	0	0	0	0	0
	57	2	Atr	1	9	0	0	0	0	0	0	0	0	0	0
	58	3	Atr	1	9	0	0	0	0	0	0	0	0	0	0
	64	4	Atr	1	10	0	0	0	0	0	0	0	0	0	0
	75	5	Atr	1	12	264	242	270	221	231	292	275	206	221	277
	81	6	Atr	1	13	299	228	295	260	295	269	232	203	291	286
	88	7	Atr	1	14	227	297	251	269	248	257	273	291	248	200
	99	8	Atr	1	16	226	205	299	256	279	276	201	253	274	288
	111	9	Atr	1	18	257	260	240	225	214	221	269	248	245	300
	118	10	Atr	1	19	226	213	267	229	282	273	276	217	218	250
	123	11	Atr	1	20	285	245	300	281	276	278	243	272	277	242
	129	12	Atr	1	21	243	255	228	284	223	206	238	209	230	206

Figure 3.6: Input data file

In the next section, our model of the ASU Computer Labs Phase 2 tour-schedule as well as other larger problem instances are solved – in orders of magnitude less time than the corresponding IP.

3.5 Computational Results

3.5.1 Tour-Schedule for ASU Computer Labs

The model described in Section 4 was applied to the real world example of scheduling student technicians at ASU Computer Labs. We worked closely with a scheduler using the data for shift requirements for the spring semester of 2003. The four labs combined have 195 distinct shifts that must be covered. In order to protect student privacy, student technician preferences and availabilities are generated from typical behaviors. We generated a class schedule for 50 students, which is the current number of technicians employed. The class schedules consist of either morning classes (unavailable from 0800-1200), afternoon classes (unavailable from 1200 – 1600), or evening classes (unavailable from 1600-2000). Each student technician is assumed to attend a typical student-worker load of three different classes per week, with each class meeting twice a week. Therefore, each student is unavailable for six time periods per week and available all weekend. We ran five distinct models, where each model adds one more of the real world constraints described in Section 4. Solutions used CPLEX 8.1 on a 2.4 GHz PC with parallel processors and 1 Mb of RAM.

Model	CPU Time (sec)	Optimal Objective
Complete Availability, Technician Preferences	0.03	38625
Restricted Availability, Technician Preferences	0.04	38495
Restricted Availability, Technician Preferences, Consecutive Shift Restrictions	0.29	38452
Restricted Availability, Technician Preferences, Consecutive Shift Restrictions, Varying Skill Sets	0.70	38091
Restricted Availability, Technician Preferences, Consecutive Shift Restrictions, Varying Skill Sets, Preference + Reward	0.93	43088

Table 3.3: Results of the tour-schedule model for ASU computer labs

In each case, a feasible solution to the tour-scheduling problem is solved in a fraction of a second, rather than seconds or minutes. Why is this computational improvement important? In general, it means that variations on optimization problems may be run in real-time that allow the manager to see the effect of changes of interest. In the ASU Computer Lab case study, the following are examples of such uses:

- Re-rostering when someone is unavailable, or whose schedule changes (dropping and adding classes after the initial schedule, for example).
- Introduction of the student confidential information, after an initial basic schedule is built by developers.
- Determination of which types of employees are needed to improve coverage and even staff sizing (which we cover in detail in Section 5.3).

- Study the effects of rewards chosen by the manager and preferences selected by the students.

These types of analyses are essentially impossible in the manual system, yet require only seconds for a re-roster up to hours for a sensitivity analysis. These time periods are a small fraction of the 2-3 days spent just to get one feasible schedule manually using the current practice. They are also a fraction of the time needed if solutions were obtained by a brute-form IP as we can see from Table 3.4. The benefits of our method increase as problem size grows larger. The ASU example has about 10,000 binary variables and 1,000 constraints, but is in no way a challenge to our method. In the next sub-section we look at the computational behavior of larger problem instances. Those larger problems are randomly generated problems, not from our case study, but containing the same types of features.

3.5.2 Computational Efficiency for Larger Problem Instances

Four larger problem instances were solved using realistic data. The computational effort is summarized in Table 3.4 below. The ASU computer lab problem is on the order of the third problem listed. Our formulation continues to solve the tour-scheduling problem in an insignificant amount of computer time even for very large problem instances of 420 distinct shifts (about the number of shifts in three months with a granularity of 4 hours on shift definition) and 80 employees. Our formulation is completely adaptable to scheduling additional time periods and, as we can see below, the formulation will solve these larger problems very quickly.

		Network-Based LP			Integer Program		
Number of Shifts	Number of Employees	Variables (continuous)	Constraints	CPU Time (sec)	Variables (binary)	Constraints	CPU Time (sec)
35	10	420	205	0	350	90	0.02
70	20	1540	430	0	1400	180	0.07
175	50	9100	1225	0.06	8750	450	1.53
420	100	42700	3020	0.5	42000	1040	135.98

Table 3.4: Computational results for larger problem instances

Figures 3.7 and 3.8 illustrate the emerging gap in computational time required to get a solution between our formulation, Network-based Mathematical Program (NBMP), and a brute-force IP. Our formulation will be able to solve much larger problems before the computational time becomes excessive. This will allow businesses to schedule months, quarters, or years at a time.

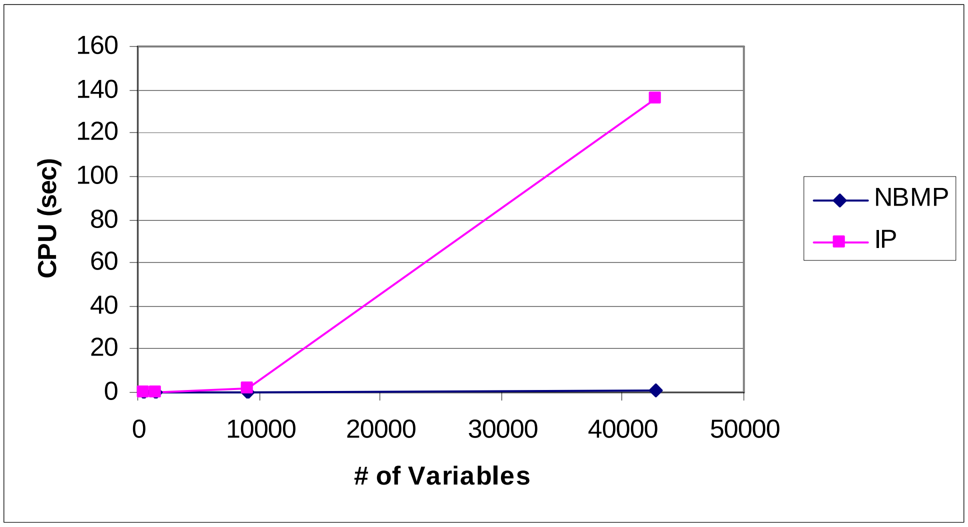


Figure 3.7: Computational time comparison versus the number of variables

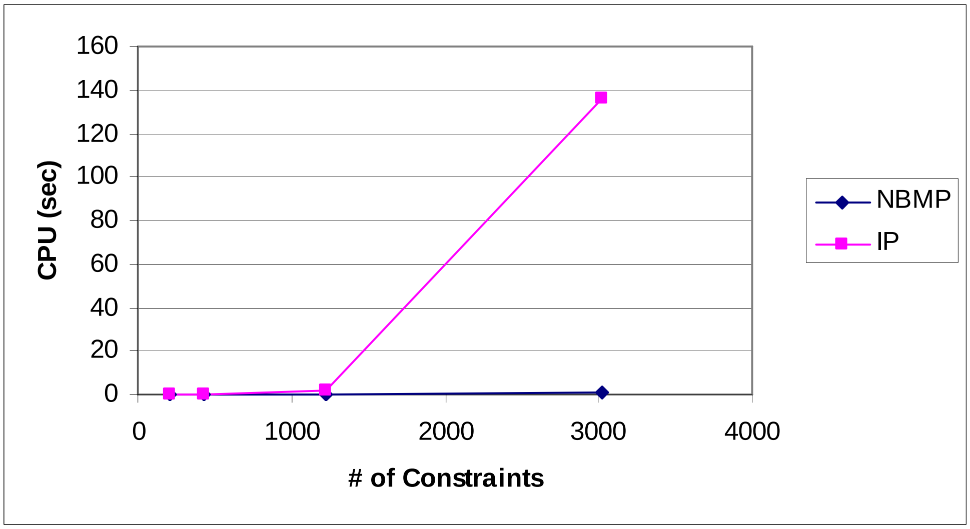


Figure 3.8: Computational time comparison versus the number of constraints

3.5.3 Staff Sizing Application

The speed at which the formulation can be solved and the ease with which real world constraints such as employee availability, consecutive shift restrictions and varying skills sets are incorporated, allows for an additional important result for industries with heterogeneous workforces. Industries that are looking to streamline their employee payrolls or are opening a new facility can use the algorithm to predict the minimum number of staff required to fill shifts based on customer demand.

Given a level of employee availability, the industry standard for rest between consecutive shifts, and the number of skills required, a set of employee data can be generated. The formulation can then be solved for this case and the minimum number of employees that covers that definition of demand can be found by running the model until a feasible solution. Repeating this for several sets of employee data, an actual minimum number of employees needed to staff the business will become apparent.

Below is an example of how our formulation can be used to generate the minimum number of employees required. The example problem instance contains 49 shifts (7 shifts per day for a week) and 15 employees. The table shows employee requirements for a typical industry as employee availability mix reduces from full (100%) availability to half time (50%) availability. The availabilities are generated randomly for this example, so 50% availability would mean each employee is randomly unavailable for half of the hours each week. In Table 3.5, below each availability is the minimum number of employees required for that problem instance. An average, standard deviation, and a minimum employee to shift ratio is calculated.

Availability		100%	90%	80%	70%	60%	50%
Minimum	Trial 1	10	11	12	13	15	15
Number	Trial 2		12	13	14	14	15
of	Trial 3		11	12	14	14	16
Employees	Trial 4		11	13	14	15	15
Average Minimum		10	11.25	12.5	13.75	14.5	15.25
Standard Deviation			0.500	0.577	0.500	0.577	0.500
Employee/Shift Ratio		0.204	0.230	0.255	0.281	0.296	0.311

Table 3.5: Employee requirements as a function of employee availability

As expected, as the time each employee has available decreases the minimum number of employees required to staff the demanded number of shifts increases. This formulation leads to an employee/shift ratio. Given the parameters of the proposed business, the entrepreneur can look at varying scenarios, chose as conservative an estimate as desired and find the ratio of employees to shifts from a graph like the one shown below. If circumstances change as the business moves closer to opening or in streamlining processes, the graph can be consulted to determine a new level of staffing ratio based upon any new level of employee availability. Similar graphs could be constructed varying consecutive shift restrictions or skill sets.

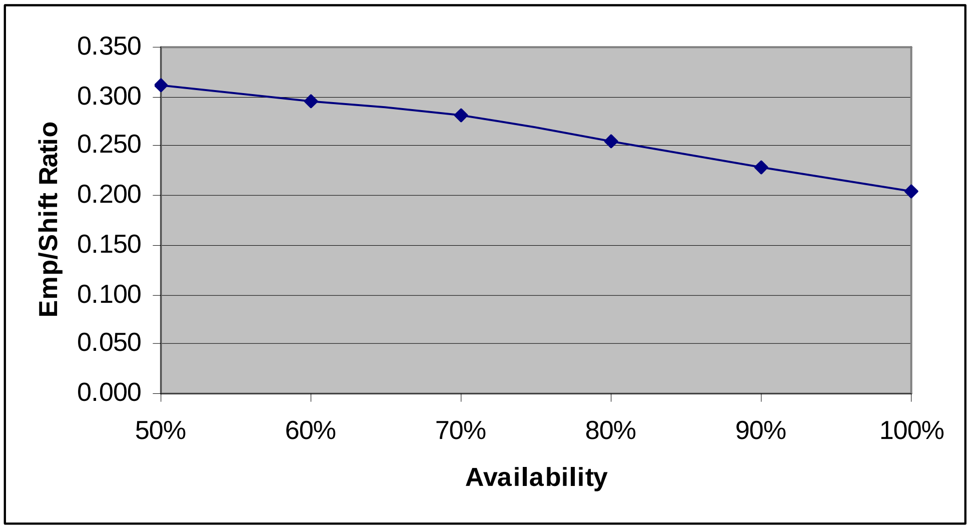


Figure 3.9: Employee to shift ratio based upon employee availability

3.6 Conclusions and Recommendations

Our formulation of the tour-scheduling problem allows a scheduler to incorporate many of the real world constraints inherent in an implementable tour-schedule. The scheduler can consider heterogeneous employee availabilities, varying skill sets, consecutive shift restrictions, and seniority or job performance incentives. Additionally, the formulation solves very large problem instances in a computationally insignificant amount of time compared to the corresponding IP solution. This allows the scheduler to reschedule often or consider new information that would require additional tweaking of the tour-schedule.

The formulation was applied successfully to a real world example involving a very heterogeneous workforce and continuous operations. The Technology Support Analyst Principal (TSAP) for the ASU computer labs indicated that it took him 2 to 3 days per semester to schedule the computer labs. Due to the high constraint on the availabilities of the technicians it is a very laborious process that the TSAP does manually with the aid of a spreadsheet for recording the tour schedule. By hand, it is very difficult to ensure that students are given a proper amount of rest between shifts. Shifts that occur consecutively or within a minimum number of hours are deemed to be conflicted and not scheduled to the same technician. The proposed model can accommodate any minimum window of rest required by the scheduling organization and can, therefore, schedule businesses with continuous operations.

Finally, we took advantage of the computational efficiency of the model and solved many problem instances to generate a minimum employee to shift ratio. Such an analysis would be very valuable to any business seeking to cut payroll or to an entrepreneur who is looking to open a new business and needs to determine appropriate staffing levels dynamically as conditions change, or in the planning stages of a business that does not exist yet.

In all cases we considered, the inclusion of heterogeneous workers and mandated rest breaks in a continuous schedule add realism to the tour scheduling problem. Appendices A and B contain the code and CPLEX output.

Chapter 4

A MATHEMATICAL PROGRAMMING APPROACH TO OPTIMAL NURSE ROSTERING OVER CONTINUOUS AND MULTI-WEEKLY TIME PERIODS WITH EMPLOYEE WEEKEND RESTRICTIONS AND SHIFT PREFERENCES

Abstract

This paper demonstrates an efficient mathematical programming approach to optimally solving the nursing tour-scheduling problem for continuous (24-hour) hospital operations and across multi-weekly time periods. The tour-scheduling problem is extended from the traditional weekly schedule to monthly, quarterly, and yearly schedules. The network model is amended to contain a nodal structure that will limit the number of weekends each employee is required to work per month. Employee shift preferences and managerial employee preferences are used as the objective function and are maximized to optimality with this approach. The framework is very flexible, it allows for a heterogeneous workforce, varying wage rates and skill levels, employee and management preferences, and continuous (24-hour) operations. The approach is validated using a practical nurse scheduling problem instance for Banner Health Hospitals, a leading health care provider in Phoenix, Arizona and the surrounding areas.

4.1 Introduction

Personnel scheduling is one of the most difficult, important, and often studied problems in operations research. The optimal choice of the number of employees required to meet customer demand, shift start and stop times, daily lunches and breaks, and the assignment of the adequately skilled employee to the best shift is a large combinatorial optimization problem. Managers who attempt to manually solve the personnel scheduling problem expend many valuable work hours to find even a feasible solution which has little probability of being optimal based on any objective function. Sub-optimal scheduling increases an industry's tangible costs, not only through the consumption of a manager's time, but also through the misallocation of shifts to meet customer demand and through employees staffing shifts for which they are not qualified. Furthermore, a sub-optimal schedule will increase intangible costs such as lower employee morale from dissatisfaction with a poor work schedule. Eliminating these costs is an easy way to increase the profitability of any business.

Personnel scheduling is a necessary chore associated with any organization employing people to get work accomplished. Some organizations that employ a professional workforce during traditionally scheduled business hours find the personnel scheduling problem of trivial concern. However, industries that rely on a large workforce with differing skill sets, highly constrained availabilities (work hours), and require staffing for the entire day and evening hours or even continuous (24-hour) operations have a much more challenging problem. One such industry is the health care industry, specifically the personnel tour-scheduling problem associated with nurses.

A timely and optimal solution to the personnel scheduling problem will save each manager valuable time, meet customer demand, and increase worker morale by matching each employee to the best shift possible. This article presents an extremely fast, computationally efficient, and optimal network-flow based mathematical programming solution for the extremely challenging problem of rostering a nursing workforce. We will demonstrate that the formulation can find integer solutions without branching, bounding, or cutting schemes.

4.1.1 Problem Overview

The large, complex problem of personnel scheduling is generally decomposed into more tractable sub-problems. The three primary sub-problems are demand modeling, shift selection, and employee tour-scheduling or rostering. Demand modeling determines the number of employees required during a given time interval to satisfy customer needs. Forecasting and queuing theory are the primary tools applied to demand modeling.

The second primary personnel scheduling sub-problem is shift selection. Shift selection assigns consecutive hour shifts to satisfy the customer demand while meeting organizational and regulatory requirements for shift lengths, breaks, and mealtime allowances. The objective of shift selection is generally to minimize the number of hours scheduled that exceed customer demand.

Finally, the third primary personnel scheduling sub-problem is tour-scheduling also know as employee rostering. Tour-scheduling assigns individuals to specific shifts detailed during shift selection. A heterogeneous workforce is a collection of personnel who have significantly variant availabilities, skill sets, and wage rates. Tour-scheduling a heterogeneous workforce must consider all of these differences and optimally match the personnel with the best shift possible and for which they are eligible. Additionally, considerations for individuals working a minimum and maximum number of shifts per week and insuring adequate rest between shifts for industries with continuous (24-hour) operations are critical to an optimal tour-schedule.

Additionally, a tour-schedule that considers employee shift preferences and management employee weighting is very advantageous to an organization. If employees are matched to shifts that fit well with their home life, availabilities, work habits, or other personal considerations, they will be motivated to work harder and more efficiently. The framework to collect and represent these preferences must be repeatable and comprehendible to the workforce.

The formulation and methodology presented in this article provide several contributions to the nurse scheduling literature. The following list summarizes the advancements proposed by this research.

- Provide a formulation that can account for variations in nursing skill sets and assign them to appropriate shifts for which they have the proper training
- Insure adequate rest between shifts and limit the number of weekends assigned per month
- Efficiently roster for monthly, quarterly, or yearly schedules

- Describe a structured framework for obtaining employee shift preferences and managerial preferences for each employee

This article is organized in the following manner. Section 2 contains a survey of the pertinent literature and important previous work accomplished in nursing tour-scheduling. Section 3 details the nurse tour-scheduling problem at Banner Health. Our methodology is general, but we feel that for realism and clarity we will introduce our case study early. Section 4 provides the proposed solution method and model. Section 5 shows the results of the methodology as applied to the case study. Section 6 finishes with conclusions and recommendations.

4.2 Literature Survey

One of the most studied applications in personnel scheduling is nurse scheduling. Nurse tour-scheduling or rostering is a subset of the literature for this application. The literature on nurse tour-scheduling is characterized by a known demand for nurses and a given set of nurse shifts. The heterogeneous workforce is characterized by varying nurse seniorities or skill sets and continuous or 24-hour operations. Ernst, et. al. (2004) give a breakdown of the various algorithmic approaches to nurse tour-scheduling. These mathematical approaches include simulation, heuristics, global search techniques, constraint programming, and mathematical programming. The scope of this literature survey will be limited to the mathematical programming approaches, the differences between these approaches, and the proposed model in this research which I term the network-based mathematical program (NBMP).

Jumard, et. al. (1998) propose a 0 - 1 mixed integer linear program (MIP) master problem to solve the nurse tour-scheduling problem. They incorporate a heterogeneous workforce, based upon nursing specialties or skills, overlapping shifts, and nurse preferences. The master problem contains a set of feasible shifts for the planning horizon that meet all the constraints. The goal of the master problem is to match a nurse with each shift pattern. Other constraints such as the minimum number of nurse with skill level ℓ scheduled at any time and the ratio of less experienced to more experienced nurses are contained in the master problem. A branch and bound approach is taken to solve the master problem and maximize the preferences of the nurses. After the initial linear programming relaxation, an auxiliary problem is then solved using a resource constrained shortest path. The auxiliary problem determines where to branch and which column to generate into the basis.

Similarly, Millar and Kiragu (1998) develop patterns of shifts and days-off, which they term stints, where “all possible work-stretches can be pre-specified”. They attempt to find a set of stints that cover the demand for nurses using a mixed-integer branch and bound strategy. Warner (1976) also proposes an approach where all feasible patterns of shifts and days off were enumerated and then allocated to a nurse.

4.3 Nurse Tour-Scheduling Problem

Banner Health is a large health care provider in Phoenix, Arizona and the surrounding areas. Banner runs many different hospitals that specialize in many different aspects of health care. One commonality from hospital to hospital is the need for tour-scheduling its cadre of nurses. The cadre is made of many different specializations, different daily staffing levels, and hours of operation. We have chosen a representative case study on which to apply our methodology. This case study will be used to illustrate our proposed solution technique. We do this without the loss of generality to other problem instances within the health care field.

4.3.1 A Case Study: Banner Health Baywood Hospital

We relied on the expertise of James Broyles, an industrial engineer at one of Banner Health hospitals, to help us frame the problem as realistically as possible. Actual nurse preferences were not obtained; however, the production questionnaires or surveys that would have been needed to acquire such preferences are not important to the illustration of the methodology. Representative preferences were generated using the framework proposed in Section 4.4.

The Intensive Care Unit at Banner Health Mesa is staffed with qualified nurses 24 hours a day due to the nature of care required in a critical care situation. The number of nurses required each day varies with day of the week and month of the year. We will pick a representative number of 3 distinct shifts per day. Each distinct shift will be staffed by multiple nurses. The shifts will have staggering lunch hours and break times to insure adequate staffing. The determination of these shifts (shift selection sub-problem) has been done by Banner Health Baywood and is not part of the scope of this article. We will staff a cadre of both 20 and 40 nurses which will illustrate both a slow and heavy workload and develop a weekly schedule. Each nurse will not work more than 48 hours per week or fewer than 24. This will represent the part-time and full-time mixture of nurses at Banner Health Baywood.

Finally, we will expand our model to efficiently schedule multi-weekly time periods. We will demonstrate the model's ability to schedule bi-weekly, monthly, quarterly, and even yearly tour schedules. An important consideration when scheduling nurses over multi-weekly schedules is the number of weekends each nurse will work per month. We can easily amend our model with a nodal structure that can insure the restriction on weekend shifts is met.

4.4 Proposed Tour-Schedule Solution Method and Model

The tour-scheduling problem is inherently a binary set-covering problem. Therefore, as problem size increases, the efficiency of the associated integer program decreases rapidly. However, we have developed a formulation of the problem as a minimum cost network-flow, using an arc capacity method, so that the resulting network structure generally provides integer binary answers. The pseudo-network structure can easily be written as an LP and solved using fast solution algorithms such as the CPLEX interior point method. Many solutions of this formulation do not require any branching, bounding, or cutting schemes to find integer solutions – the integer solutions are a consequence of the formulation method. Therefore, very large problem instances can be solved in a computationally insignificant amount of time compared to the corresponding IP solution. The data will empirically show that even when solutions to the problem using our formulation are not entirely integer, they are high quality solutions that provide all but a small percentage of fractional values. The convex hull of the NMBL formulation closely coincides with the optimal solution from a pure integer mathematical program. Only cutting techniques automatically implemented within CPLEX are needed to find the integer solution extremely efficiently, with little to no extra computational time. No branching is needed due to the nearly optimal characteristic of the solution to the network-based mathematical program. Figure 4.1 shows the formulation of our generalized weekly tour-scheduling problem as a minimum cost network-flow.

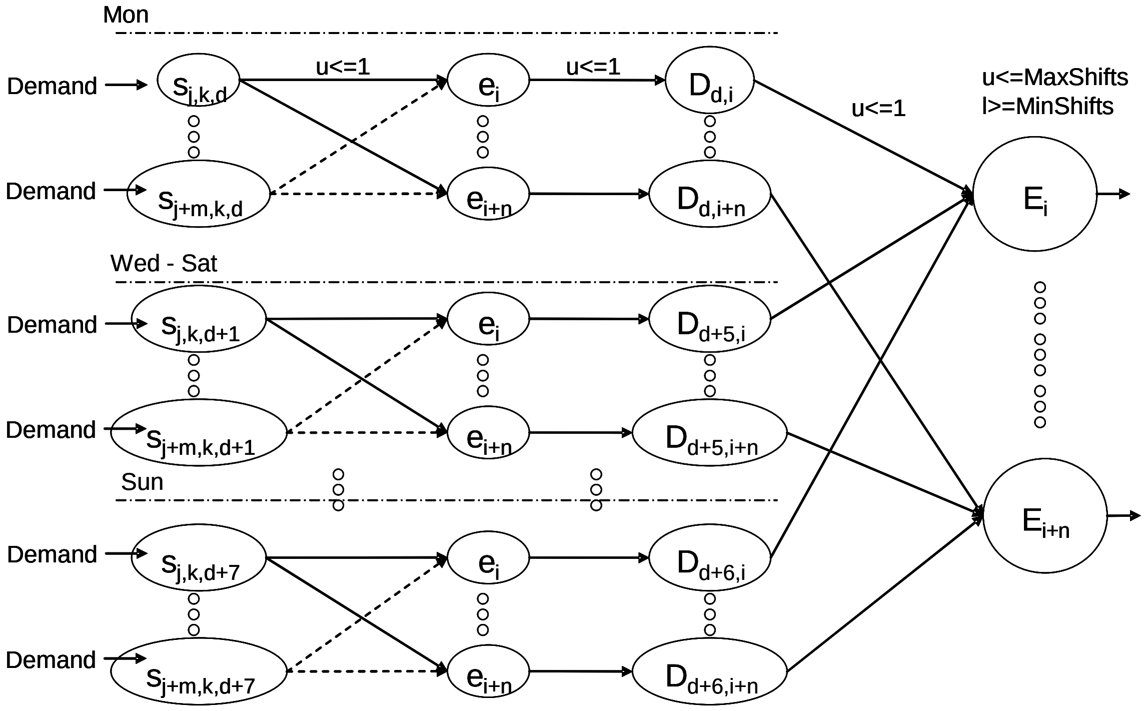


Figure 4.1: Network representation of the Phase 2 tour-scheduling problem

Here, $s_{j,k,d}$ denotes shift number *j* requiring skill set *k* and on day *d*, e_i is employee number *i*, $D_{d,i}$ is the total number of shifts, *D*, on day d for employee *i*, and E_i is the total number of shifts per week, *E*, for employee *i*.

The number of employees needed for each shift node flows from the Demand arc. Each shift node has an edge to each qualified and available employee node. The shift-to-employee arc is capacitated at 1, meaning only one of the required staffing for a shift can be assigned to a single employee. Each employee then flows their daily work assignment to the employees’ daily-total-node, D. This new arc is capacitated at 1, meaning each employee can work only one shift per day. Finally, the employees’ daily-total-nodes channel the flow to the employees’ weekly-total-node which contains the capacities to enforce minimum and maximum shifts per week.

The weekly tour-scheduling model above is then stacked upon itself to simultaneously schedule longer time horizons, such as months, quarter, or years depending upon the requirements of the industry. Figure 4.2 illustrates this extended problem.

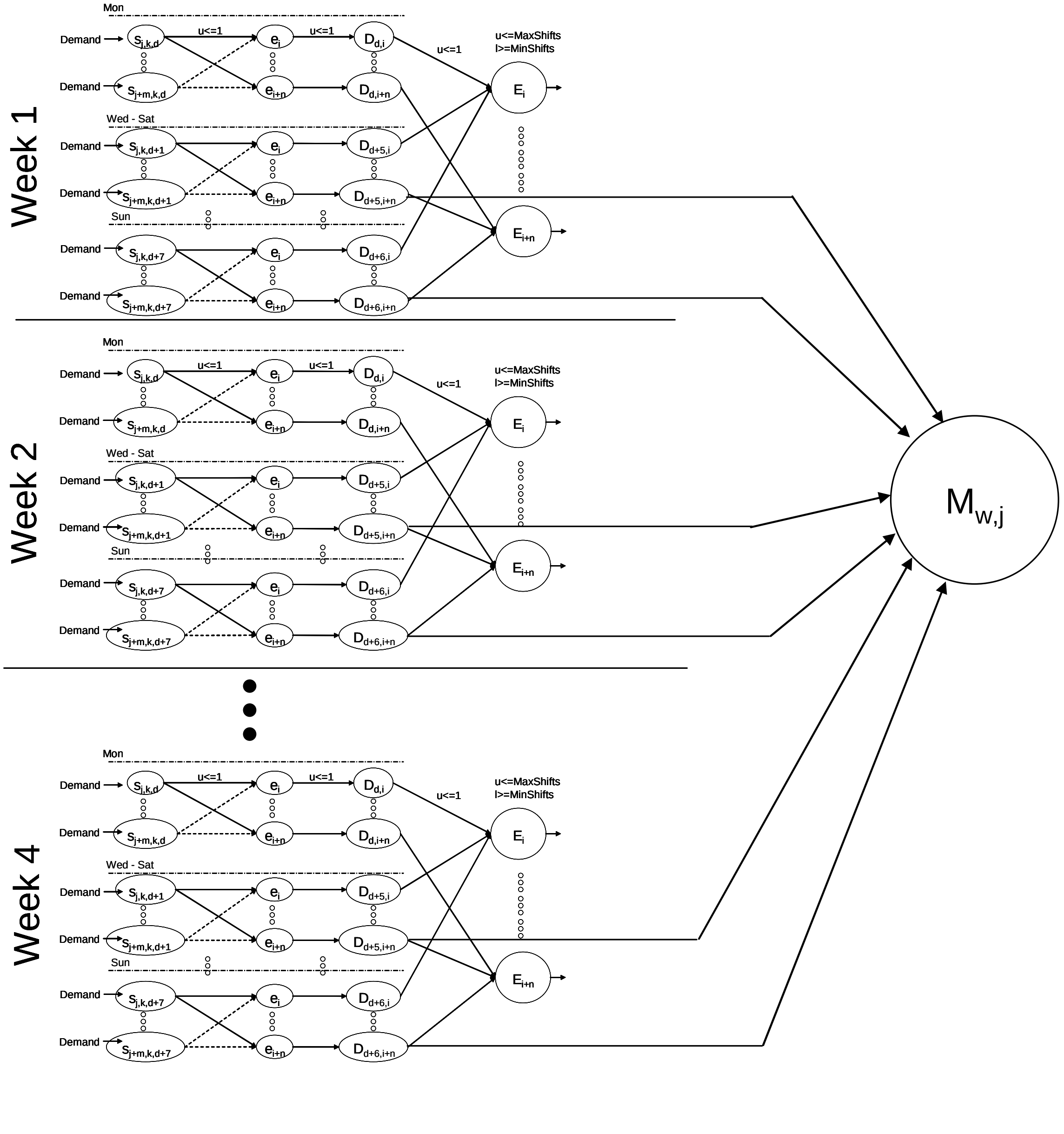


Figure 4.2: Multi-weekly network representation of tour-scheduling problem

Here $M_{w,i}$ is the total number of weekend days, *w*, worked in a month for each employee *j*. The node is capacitated at the number of weekend days each nurse is allowed to work each month. Therefore, we must add a set of constraints, equation 4.6, to the mathematical formulation given in Chapter 3's article, "A

Network-based Mathematical Programming Approach to Optimal Rostering of Continuous Heterogeneous Workforces".

The balance equations for the multi-weekly network flow formulation are:

$$\sum_{i} s_{j,k,d} e_i = demand \qquad \text{for all } j,k,d \tag{4.1}$$

$$\sum_{j,k} s_{j,k,d} e_i = D_{d,i} \qquad \text{for all } i,d \tag{4.2}$$

$$\sum_{d} D_{d,i} \leq \max number\_of\_shifts \qquad \text{for all } j \tag{4.3}$$

$$\sum_{d} D_{d,i} \geq \min number\_of\_shifts \qquad \text{for all } j \tag{4.4}$$

$$D_{d,i} \leq 1 \qquad \text{for all } i,d \tag{4.5}$$

$$\sum_{w} W_{m,j} \leq \max weedend\_shifts \qquad \text{for all } j \tag{4.6}$$

Constraint set 1 requires that the demand for a number of employees for each type of shift is met. Set 2 totals the shifts per employee for each day and requires that the total be less than or equal to 1 in constraint set 5 (no employee works more than one shift per day, but remember that the shift length is a variable). Constraint sets 3 and 4 require that the minimum and maximum numbers of shifts per week are met for each employee. Finally, set 6 restricts the number of weekend days worked by each employee. The following three sub-sections describe how our model incorporates all the constraints needed to create a tour-schedule for a real world heath care organizations, including a heterogeneous work force and consecutive shift restrictions.

4.4.1 Consecutive Shift Restrictions

Adequate rest between consecutive shifts is a requirement for any realistic tour-schedule of continuous operations. Industries that are only open for an 8-

hour day have tour-schedules that implicitly contain adequate rest. However, when scheduling an industry, such as ICU nurses, with continuous or 24-hour operations, restrictions on consecutive shifts is a requirement for any realistic tour-scheduling formulation. Love and Hoey (1990) and Loucks and Jacobs (1991) note the need for consecutive shift restrictions and state the lack of such restrictions as limitations to their tour-scheduling models. Our formulation allows for specialized side constraints to accommodate continuous operations.

As input in any specific application, we use the appropriate industry standard for how much time is required between the start of shift *j* and the start of the next shift. This leads to a matrix of shifts, *J*, that conflict with each other in the sense that an employee assigned to work shift *j* cannot be scheduled to work any shift conflicted with shift *j*. The following constraint is added to constraints 1-5 above to accommodate consecutive shift restrictions.

$$\sum_{j \in J} s_{j,k,d} e_i \leq 1 \qquad \text{for all conflicted shifts} \qquad (4.7)$$

Our empirical experience indicates that in many cases these side constraints do not destroy the integrality of the solution to the minimum cost network flow.

4.4.2 Employee Availabilities and Varying Skill Sets

Many industries have employees with heterogeneous availabilities. For example, service industries often employ younger, part-time employees who have restrictions on their time due to schooling or other activities. Our model can be modified to accommodate such heterogeneous availabilities. We simply remove the network arc from a shift to an employee who is not available during the hours encompassing that shift. No flow units can travel from that shift to the employee and consequently the employee is not assigned to that shift.

Similarly, it is often the case that many service industries have different skill requirements needed during a particular shift. This is especially true of the nursing industry where nurse often specialize in different level of care or surgical-assistance expertise. If a nurse does not possess the required skill for a particular shift, the arc from the shift to the employee is removed from the network. If the nurse has multiple skill sets, the arc from the employee to all shifts within their set is contained within the network.

This is an important aspect to the efficiency of the tour-schedule. Many examples within the tour-scheduling literature schedule only one department within the industry at a time. Therefore, the employees assigned to that department can be scheduled only within that department and there is no mechanism to schedule that employee across departments. This could lead a department to over-staffing costing a business extra payroll. We have no such restriction.

Removal of the two types of arcs described above are easy to program. They have no effect on the integrality of solutions from the LP-network solution. This is because the resulting network is still of minimum cost flow form.

4.4.3 The Objective Function - Employee and Manager Preferences

The objective function of the constraint set described above can now be defined. The objective function is made up of the preferences for each shift from the employees and by the ranking of the employees from the manager. It should be noted that there are two distinct situations when the formulation presented may cause the solver to find an alternate optimal solution that contains fractional employee assignments (there is always an integer solution). The first situation can be subverted through the intelligent use of preferences. This is discussed next and the second is discussed in the section 4.5 Computation Results.

Fractional solutions may occur when the preferences of a set of employees for a particular shift are equal and none is violating a consecutive shift restriction, maximum number of shifts per week constraint, or have more than the allotted weekend shifts. The solution is obviously an alternate optima, since the entire shift can be given to any of the fractional employees and the objective function will not change, nor is any constraint violated. This phenomenon can be combated by using relative rankings for both the shifts and the employees.

Relative rankings will break ties between the shift preferences of employees. Each employee, *j*, ranks all the shifts and each shift is given a score, $emppref_{i,j}$, on a relative scale, 1 through the number of shifts, ensuring no employee has the exact same preference for each shift, *i*. Next, the manager ranks all employees and then each employee is given the appropriate relative score, $manpref_i$, ranging from 1 through number of employees. This number is then given an order of magnitude increase, *O*, over the scale for employee shift preferences and the two preferences are added together. The preferences of each shift/employee combination are the aggregate of the employee's and manager's preferences and are distinct from one another.

$$pref(s_{j,k,d}e_i) = emppref_{i,j} + (O \bullet manprep_i) \qquad (4.8)$$

The objective function for the constraint set described in equations 4.1 – 4.6 then becomes

$$\text{MAX} \qquad \sum_{j,k,d,i} s_{j,k,d} e_i \bullet pref(s_{j,k,d} e_i) \qquad (4.9)$$

Figure 4.3 graphically illustrates the employee/shift preferences for the first four shifts and four employees after applying the above approach. The most important employee, as ranked by the manager, is on the top of the graph.

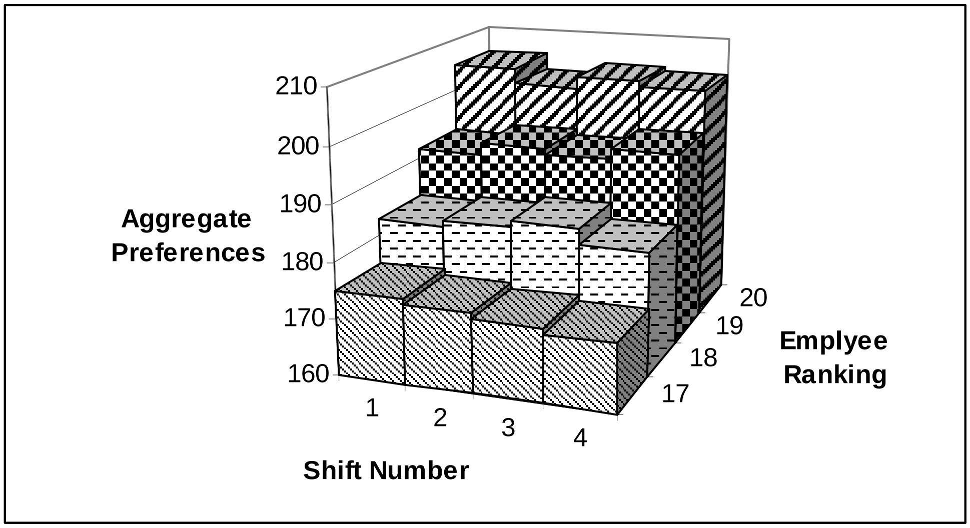


Figure 4.3: Illustration of aggregate preferences for employ/shift combination

Data organization to support these calculations is straightforward. A spreadsheet is used as the input data file. A Visual Basic macro then writes the linear program in CPLEX .lp input format. This macro runs almost instantaneously. The output file is then given to CPLEX for optimization. The ease of changing data in the input data file, the speed with which the output file is written, and the speed with which the problem is optimized (more on this in the next section) allows the user to study a wide variety of problem instances and conduct sensitivity analysis. Figures 4.4 and 4.5 show examples of the input data files.

Employee	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Shifts	Preferences																			
1	2175	2022	1875	1933	1824	1656		1489	1430	1265	1221	953	935	923	815				205	280.5
2	2206			1917			1632	1472		1314		989.5	982.5	783.5	615		575.5		439.5	233
3		2094				1590				1259		984		742	837.5			356.5	354.5	
4		1926	2006		1741	1560	1457	1445		1330	1237	1006	949	900.5			624.5	492	320.5	180
5	2210						1549	1425	1358	1177		955.5		916.5	636.5	707	464.5	432.5	404.5	316
6		2044	1909				1612					994.5	1025		751.5		587	313	375	125
7	2246	1925	1979	1840	1746	1629	1547	1304	1412	1290		922	939.5				574	528.5	351.5	297
8	2230		1932		1653		1638	1372		1185	1238		850		759.5	703.5	642.5	350	429.5	
9	2059	1907	1901		1847		1508	1420	1398		1106	1121	1048	935		553	610.5	498	398.5	166.5
10					1766		1536		1260	1264		1022	993.5	944	842		649.5	452	260	330.5
										● ● ●										
200	2191	2105	1816				1588	1527	1341		1034	1109		839.5	660.5	668.5	612.5	533	408	
201	2233			1809	1672	1703	1463	1331	1217		1214	1150	811		802.5	622.5	603.5			335.5
202	2237	2091		1791	1750	1695			1273		1083	1125	1027	826.5		525		376.5		311.5
203							1465	1344	1247	1307			966.5		735				305	300
204	2239	2125	1986	1882	1675		1554	1477		1295	1250	933.5	852.5	829	713.5	641	583.5	491	399	172
205		2056	1835		1619	1579			1448	1230	1088	1136	1001	878		568	620			
										● ● ●										

Figure 4.4: Employee/Manager shift preferences input data file

Shift ID	Conflicted Shifts											
1												
2												
3												
4												
5												
6												
7	19	20	21	22	23	24						
8	19	20	21	22	23	24						
9	19	20	21	22	23	24						
10	19	20	21	22	23	24						
11	19	20	21	22	23	24						
12	19	20	21	22	23	24						
13	19	20	21	22	23	24	25	26	27	28	29	30
14	19	20	21	22	23	24	25	26	27	28	29	30
15	19	20	21	22	23	24	25	26	27	28	29	30
16	19	20	21	22	23	24	25	26	27	28	29	30
17	19	20	21	22	23	24	25	26	27	28	29	30
18	19	20	21	22	23	24	25	26	27	28	29	30

Figure 4.5: List of conflicted shifts input file

In the next section, our representative model of the Banner Health nurse tour-schedule as well as other larger problem instances is solved.

4.5 Computational Results

Before presenting the computational results, the second situation where fractional decision variables may occur must be discussed. The second phenomenon where fractional solutions are returned by the solver is due to a small mismatch in the convex hull of the optimal solution to the NBMP and that of the optimal integer solution. The matrices for the larger of the problem instances are very expansive. The models being solved are on the order of tens-of-thousands of constraints and ten-of-thousands of variables. However, because the NBMP solution is so close to the integer optimal, minimal additional computational time is required to find the integer solution. A comparison of each solution and the differences in optimal values is presented below. Each problem was solved with and without the requirement of an integer constraint on the decision variables.

Number of Weeks	Number of Employees	Number of Shifts	Optimal Obj. NBMP	Number of Variables	Number of non-IP variables	Optimal Obj. NBMP/ w IP	Difference
1	20	63	7.96E+04	1400	0	7.96E+04	0.000%
2	20	126	1.83E+05	2800	71	1.83E+05	0.025%
4	20	252	5.03E+06	5600	327	5.03E+06	0.107%
12	20	756	4.71E+07	16800	960	4.71E+07	0.088%
52	20	3276	8.94E+08	72800	4723	8.85E+08	1.019%

Number of Weeks	Number of Employees	Number of Shifts	Optimal Obj. NBMP	Number of Variables	Number of non-IP variables	Optimal Obj. NBMP/ w IP	Difference
1	40	126	5.07E+07	5320	0	5.07E+07	0.000%
2	40	252	1.17E+08	10640	238	1.17E+08	0.029%
4	40	504	2.51E+08	21280	574	2.51E+08	0.040%
12	40	1512	7.85E+08	63840	1802	7.85E+08	0.039%
52	40	6552	3.44E+09	276640	7474	3.39E+09	1.490%

Table 4.1: Summary of results for formulation with and without IP variables

It can be seen that when developing a weekly schedule comprised of 20 employees and 63 shifts and 40 employees and 126 shifts, the NBMP formulation does not affect the ability of the model to find integer solutions without integer constraints in the constraint set. As the model grows in size a relatively small number of variables return as fractions. When scheduling 20 nurses for two weeks, only 71 of 2,800 decision variables are fractional and at four weeks only 327 of 5,600. Additionally, the last column shows the difference in the objective function value, in both the smaller cases as less than 0.107%.

Next it is shown that the model approaches the integer optimal. Therefore, even when all variables are restricted to be integer, the solution times are in hundredths of seconds using CPLEX 8.1 on a 2.4 GHz PC with parallel processors and 1 Mb of RAM. In fact, CPLEX does not have to use any branching techniques. Figure 4.6 shows the CPLEX header output for the 4 week model. No branching is required; the only technique needed is a Gomory fractional cut.

```
Problem 'w4e20ip.lp' read.
Read time =    0.07 sec.
Tried aggregator 1 time.
MIP Presolve eliminated 21926 rows and 7009 columns.
MIP Presolve modified 319 coefficients.
Aggregator did 12 substitutions.
Reduced MIP has 1857 rows, 2035 columns, and 7354 nonzeros.
Presolve time =    0.06 sec.
Clique table members: 1817
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    0.03 sec.

        Nodes                                Cuts/
   Node  Left     Objective  IInf  Best Integer     Best Node    ItCnt     Gap         Variable B Parent  Depth

      0     0   299531.8750    58              299531.8750      520
*             299458.0000     0   299458.0000     Cuts:  33      527    0.00%

Clique cuts applied:  5
Gomory fractional cuts applied: 2

Integer optimal solution:  Objective =    2.9945800000e+05
Solution time =    0.13 sec.  Iterations = 527  Nodes = 0
```

Figure 4.6: CPLEX header output

Table 4.2 shows that the computational time for the network-based mathematical program approach with integer constraints on the decision variables are close to or better than the pure network flow model.

Number of Weeks	Number of Employees	Number of Shifts	Number of Constraints	Number of Variables	Solution (sec) NBMP	Solution (sec) NBMP/ w IP
1	20	63	1805	1400	0.02	0.03
2	20	126	3568	2800	0.05	0.17
4	20	252	7172	5600	0.13	2.14
12	20	756	20835	16800	1.56	10.87
52	20	3276	92600	72800	7.42	204.15

Number of Weeks	Number of Employees	Number of Shifts	Number of Constraints	Number of Variables	Solution (sec) NBMP	Solution (sec) NBMP/ w IP
1	40	126	3522	5320	0.1	0.09
2	40	252	6988	10640	0.35	0.9
4	40	504	13676	21280	0.9	3.38
12	40	1512	40972	63840	8.34	31.26
52	40	6552	176781	276640	20.65	430.51

Table 4.2: Computational time results

Finally, we compare our results with those from a traditional MIP originally computed in the article in Chapter 3 entitled “A Network-based Mathematical Programming Approach to Optimal Rostering of Continuous Heterogeneous Workforces”. Figure 4.7 compares the computational time of a traditional tour-scheduling mixed-integer programming formulation with the network-based mathematical programming approach both with and without integer constraints. We can see that the NBMP out-performs the more traditional approach as the problem size grows larger.

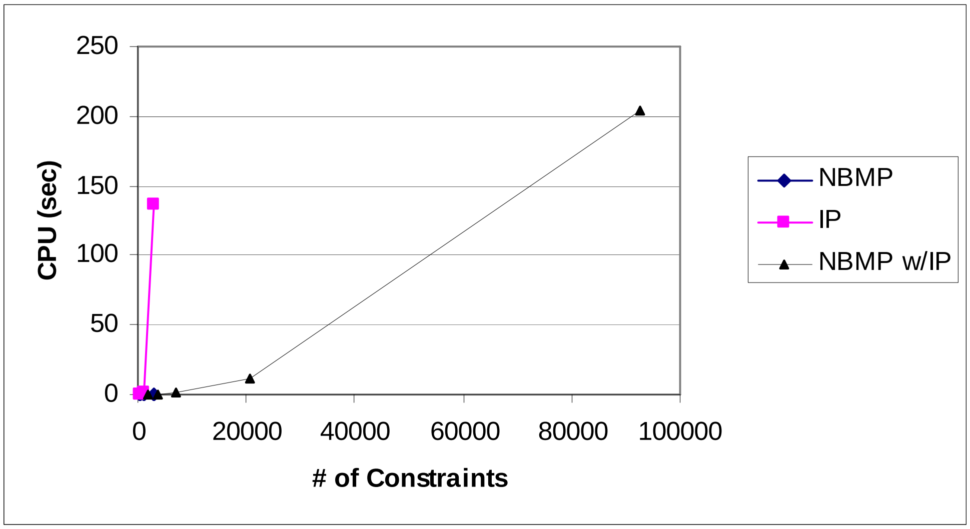


Figure 4.7: Comparison of computational times and number of constraints

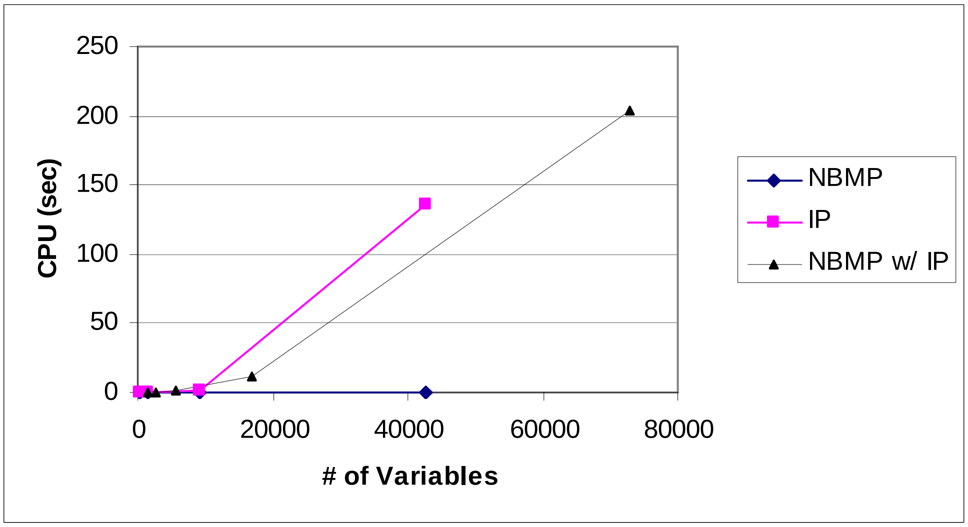


Figure 4.8: Comparison of computational times and number of variables

Finally, illustrated in Figure 4.8, similar efficiency of the NBMP as the variables of the model are increased is shown.

4.6 Conclusions

In this article we demonstrated a network-based mathematical programming approach to solving the tour-scheduling problem. We validated our approach by solving a realistic nurse rostering problem instance at Banner Health Baywood Hospital in Phoenix, AZ. The case study is scheduled at two different staffing levels and across multiple weeks, including up to one year. Next, we present a repeatable and easily understood framework for collecting employee shift preferences and managerial employee rankings. These preferences are then maximized to optimality. Finally, we compared the efficiency of the network-based mathematical approach to a traditional tour-scheduling mixed-integer program. This comparison illustrates the minimal computational time NBMP requires as the problem grows larger. The network-based mathematical programming approach proposed in this article is a valid, efficient, and optimal approach to solving the personnel tour-scheduling problem.

Appendices C contains the Visual Basic code used to write the model into a CPLEX input file. Appendix D contains all the CPLEX output for the nursing models solved for this article.

Chapter 5

A NETWORK-BASED MATHEMATICAL PROGRAMMING APPROACH TO USING EMPLOYEE PREFERENCES IN RE-ROSTERING OPTIMAL TOUR-SCHEDULES

Abstract

This article presents a methodology that will efficiently re-roster an optimal schedule when employee absenteeism causes a shift to be re-staffed. The goal and the success of the methodology presented are to minimize the disturbance to the original schedule while covering all unfilled shifts. Employee preferences, needed to find the original optimal schedule, are deftly modified to ensure the minimization of the disturbances. The manger is required to gain no new information and can quickly publish a new schedule, thus saving valuable time and notifying the changes to the workforce as soon as possible. The re-rostering technique is then validated by re-rostering two optimal schedules produced for two real-life case studies: Arizona State University Computer Labs and Banner Health Baywood Hospital. These case studies are large personnel tour-scheduling problem instances and each are scheduled under three distinct scenarios. The scenarios show the performance of the methodology when re-rostering is required in benign, general, and difficult conditions. The methodology requires hundredths of a second of computational time and minimizes the deviation to the original schedule.

5.1 Introduction

Personnel scheduling is one of the most difficult, important, and often studied problems in operations research. The optimal choice of the number of employees required to meet customer demand, shift start and stop times, daily lunches and breaks, and the assignment of the adequately skilled employee to the best shift is a large combinatorial optimization problem. Managers who attempt to manually solve the personnel scheduling problem expend many valuable work hours to find even a feasible solution which has little probability of being optimal based on any objective function. Sub-optimal scheduling increases an industry’s tangible costs, not only through the consumption of a manager’s time, but also through the misallocation of shifts to meet customer demand and through employees staffing shifts for which they are not qualified. Furthermore, a sub-optimal schedule will increase intangible costs such as lower employee morale from dissatisfaction with a poor work schedule. The problem is further complicated by employee sicknesses, vacations, hiring, or firing. In this inevitable instance the sub-optimal problem must again be adjusted, costing more hours, to find another feasible schedule that will also likely be sub-optimal. This article presents a methodology to solve the problem of re-rostering when this employee absenteeism occurs.

5.1.1 Problem Overview

A primary personnel scheduling sub-problem is tour-scheduling also known as employee rostering. Tour-scheduling assigns individuals to specific shifts detailed during shift selection. A heterogeneous workforce is a collection of personnel who have significantly variant availabilities, skill sets, and wage rates. Tour-scheduling a heterogeneous workforce must consider all of these differences and optimally match the personnel with the best shift possible and for which they are eligible. Moreover, considerations for individuals working a minimum and maximum number of shifts per week and insuring adequate rest between shifts for industries with continuous (24-hour) operations are critical to an optimal tour-schedule.

A tour-schedule that considers employee shift preferences and management employee weighting is very advantageous to an organization. If employees are matched to shifts that fit well with their home life, availabilities, work habits, or other personal considerations, they will be motivated to work harder and more efficiently. The framework to collect and represent these preferences must be repeatable and comprehendible to the workforce.

The original tour-schedule often can not be implemented because of employee sickness, vacations, hiring, or firing that cause shifts originally assigned to go unstaffed. The new schedule will have to meet all the constraints of the original tour-schedule with the added goal of minimizing deviations. Minimizing deviations is an important goal of the new tour-schedule. The stability of the schedule is crucial for employee morale and the ability of employees to schedule their non-working lives.

The formulation and methodology presented in this article provide several contributions to the re-rostering literature. The following list summarizes the advancements proposed by this research.

- Provide a formulation that can account for heterogeneous workforce
- Insure adequate rest between shifts and limit the number of weekends assigned per month
- Describe a structured framework for obtaining employee shift preferences and managerial preferences for each employee
- Efficiently re-roster a tour-schedule while minimizing deviations

This article is organized in the following manner. Section 2 contains a survey of the pertinent literature and important previous work accomplished in nursing tour-scheduling. Section 3 details the re-rostering problem at Banner Health Baywood Hospital. Our methodology is general, but we feel that for realism and clarity we will introduce our case study early. Section 4 provides the proposed solution methodology. Section 5 shows the results of the methodology as applied to the case study. Section 6 finishes with conclusions and recommendations.

5.2 Literature Summary

A vast amount of scholastic work has been accomplished on the personnel scheduling problem. This work has been documented in a wide array of refereed journals, conference proceedings, and lecture notes. Personnel scheduling spans a wide range of problem instances, specific applications, solution techniques, and methodologies. Many survey papers of the pertinent literature in the area have been written. Some of the most extensive and most recent include Ernst, et. al. (2004a), Ernst, et. al. (2004b), and Alfares (2004).

A primary application of the personal tour scheduling problem is found in "Solving the Problem of Re-rostering Nurse Schedules with Hard Constraints: New MultiComodity Flow Models" by Margarida Moz and Margarida Vaz Pato. Published in the *Annals of Operations Research* in April 2004, the article proposes a method to re-roster a tour-schedule once changes in the given schedule no longer meet demand requirements. Sickness, termination, or hiring may be reasons for the need to change a given roster once it has been published.

Moz and Pato (2004) look at a continuous workforce that is heterogeneous. Differing nurse seniorities or specialties are taken into account by giving each shift in their model a separate task designation. The continuous workday is separated into 3 distinct shifts that start and end at the same time each day. These shifts are labeled Morning, Evening, and Night. A feasible schedule will meet hard constraints such as one shift per day, elimination of non-consecutive shifts, each shift/task combination assigned to a single nurse, the nurse assigned to a task must be eligible to perform the task, and a minimum number of days off per week for each nurse.

The algorithm approach proposed by Moz and Pato is an "integer multicommodity flow problem in a directed multi-level acyclic network" (2004). Each nurse is a commodity and is represented by an origin node in the network and by a destination node. The number of days to be scheduled is the number of levels between the origin nodes and the destination node. Each non-origin/non-destination node in the network represents a task for a nurse to perform, including being off that day. The goal is to find a feasible path for each nurse through the network that meets all the hard constraints mentioned above. Below is a figure illustrating the network.

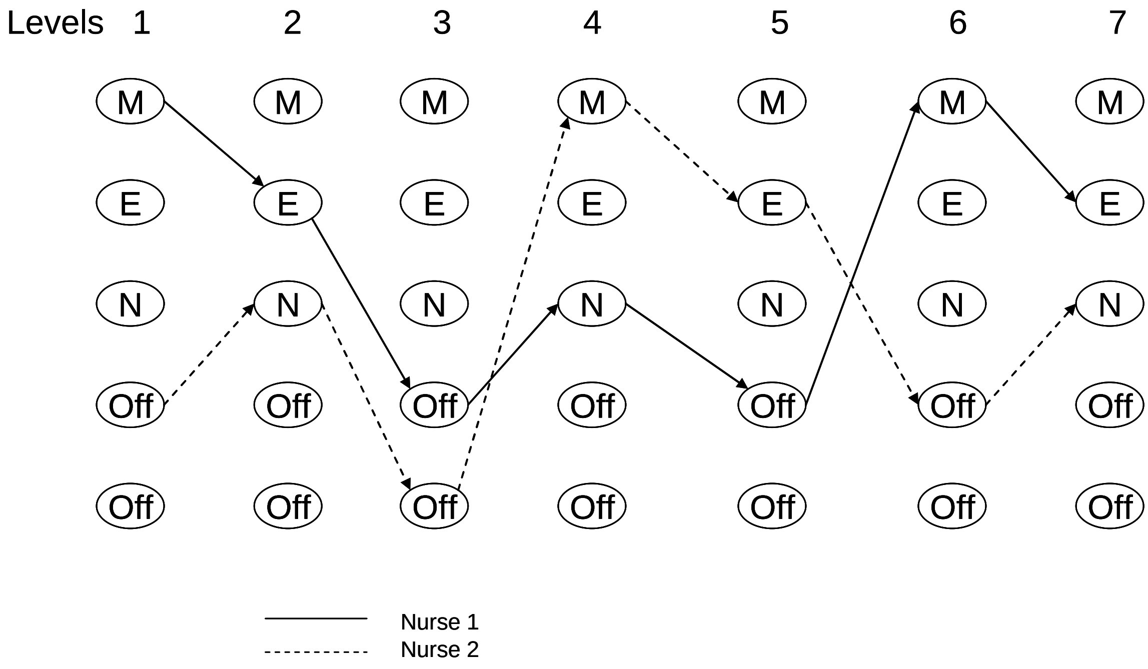


Figure 5.1: Multicommodity flow problem in a directed multi-level acyclic network (Moz and Pato, 2004)

Moz and Pato use this network formulation to re-roster a completed 28 day schedule that has been perturbed at some day *d*, which is considered the origin of the network. “The re-rostering problem amounts to finding an optimal integer flow of *n* commodities, from the nodes of level d-1 to those of the last level” (Moz and Pato, 2004). The formulation is a mixed integer linear program with binary constraints on the decision variables; they term this formulation as a binary linear program (BLP). The objective function of the algorithm is to find a new schedule that will minimize the number of nurses who must swap shifts.

The BLP does not use the network structure to solve the problem. The network structure is for illustrative purposes, as noted by Moz and Pato "this flow approach provides a suggestive network model for RSP (re-rostering problem), it leads to large scale BLP formulation, with many variables and constrains, some of which are additional to the flow model" (2004). Due to the low number of shifts, the formulation in effect enumerates all possible paths through a network that will satisfy the constraints described above. The nodes of the network represent shift/skill combinations and the arcs represent links between shifts that are feasible. Moz and Pato acknowledge this. "It is worth noting that… the flow of each commodity can only use one path from its origin to its destination, thus resulting in binary flows" (Moz and Pato, 2004). The commodities are the various nurses working a specific shift performing a given task. The origin is day *d*, while the destination is the 28th day.

The Moz and Pato models, as well as Jumard, et. al. (1998), Millar and Kiragu (1998), and Warner (1976), vary significantly from the research proposed by the network-based linear program formulation; although, both models use the network formulation for mostly illustrative purposes. The prime difference is the models presented by Moz and Pato are mixed integer linear programs with binary constraints. This requires a branch and bound approach to solve the problem. The network-based linear program requires no branching because the solution to the linear program provides binary decision variables. The proposed network-based mathematical program (NBMP) can accommodate a large number of shifts with varied start times and of various lengths as well. The shifts in the NBMP research can also overlap. Moz and Pato describe a model where the shifts start and end at the same times each day and do not overlap. It is not readily apparent that their formulation could account for overlapping or varied length shifts. The enumeration of each feasible shift to shift to day off path through the network would make the problem grow rapidly if such variations in shift pattern were present. Next, although the workforce of nurses are heterogeneous in the aspect that they have varying skill sets, they do not have constraints on their availability apart from restrictions on consecutive shifts. The network-based mathematical program accounts for employee restrictions on availability as well as varying skill sets. Finally, the objective function of the Moz and Pato model is a deviation from a previously published tour-schedule. The objective of the proposed NBMP research is employee's weighted preference.

5.3 Re-rostering Case Study

The proposed re-rostering methodology will be demonstrated on an important real-world case study. The case study examined is a nurse scheduling problem instance at Banner Health Baywood Hospital in Phoenix, Arizona. The case study were chosen to illustrate the re-rostering approach and is presented without the loss of generality to other problem instances of personnel tour-scheduling. A brief description follows.

5.3.1 Banner Health Baywood Hospital

The second case study to re-roster is a nursing schedule at Banner Health Baywood Hospital. Banner Health is a large health care provider in Phoenix, Arizona and the surrounding areas. Banner runs many different hospitals that specialize in many different aspects of health care. One commonality from hospital to hospital is the need for tour-scheduling and then re-rostering its cadre of nurses. The cadre is made of many different specializations, different daily staffing levels, and hours of operation.

We relied on the expertise of James Broyles, an industrial engineer at one of Banner Health hospitals, to help us frame the problem as realistically as possible. Actual nurse preferences were not obtained; however, the production questionnaires or surveys that would have been needed to acquire such preferences are not important to the illustration of the methodology. Representative preferences were generated using the framework proposed in Section 5.4.

The Intensive Care Unit at Banner Health Mesa is staffed with qualified nurses 24 hours a day due to the nature of care required in a critical care situation. The number of nurses required each day varies with day of the week and month of the year. We will pick a representative number of 3 distinct shifts per day. Each distinct shift will be staffed by multiple nurses. The shifts will have staggering lunch hours and break times to insure adequate staffing. We will re-roster a cadre 40 nurses which will illustrate both a heavy workload and develop a new weekly schedule. Each nurse will not work more than 48 hours per week or fewer than 24. This will represent the part-time and full-time mixture of nurses at Banner Health Baywood.

5.4 Proposed Tour-Schedule Solution Method and Model

The tour-scheduling problem is inherently a binary set-covering problem. Therefore, as problem size increases, the efficiency of the associated integer program decreases rapidly. However, we have developed a formulation of the problem as a minimum cost network-flow, using an arc capacity method, so that the resulting network structure generally provides integer binary answers. The network structure can easily be written as a linear program and solved using fast solution algorithms such as the CPLEX interior point method. Many solutions of this formulation do not require any branching, bounding, or cutting schemes to find integer solutions – the integer solutions are a consequence of the formulation method. The result being, very large problem instances can be solved in a computationally insignificant amount of time compared to the corresponding integer programming solution.

The model is detailed in Chapters 3 and 4. This model is named the network-based mathematical program (NBMP). The constraint set of this model requires all employees are scheduled only when available, have adequate skill level, meet minimum and maximum shifts per week, and have adequate rest between shifts. This constraint set will not change when re-rostering an optimal tour-schedule. The model’s objective function will provide the mechanism for re-rostering and is presented below.

5.4.1 The Objective Function - Employee and Manager Preferences

The objective function of the tour-scheduling formulation, NBMP, is very important in the ability of the model to efficiently re-roster an optimal schedule. The objective function is made up of the preferences for each shift from the employees and by the ranking of the employees from the manager. It should be noted that there are two distinct situations when the formulation presented may cause the solver to find an alternate optimal solution that contains fractional employee assignments (there is always an integer solution). The first situation can be subverted through the intelligent use of preferences. This is discussed next and the reader is referred to the article in Chapter 4 for a discussion of the second.

Fractional solutions may occur when the preferences of a set of employees for a particular shift are equal and none is violating a consecutive shift restriction, maximum number of shifts per week constraint, or have more than the allotted weekend shifts. The solution is obviously an alternate optima, since the entire shift can be given to any of the fractional employees and the objective function will not change, nor is any constraint violated. This phenomenon can be combated by using relative rankings for both the shifts and the employees.

Relative rankings will break ties between the shift preferences of employees. Each employee, j, ranks all the shifts and each shift is given a score, $emppref_{i,j}$, on a relative scale, 1 through the number of shifts, ensuring no employee has the exact same preference for each shift, i. Next, the manager ranks all employees and then each employee is given the appropriate relative score, *manpref_i*, ranging from 1 through number of employees. This number is then given an order of magnitude increase, *O*, over the scale for employee shift preferences and the two preferences are added together. The preferences of each shift/employee combination are the aggregate of the employee's and manager's preferences and are distinct from one another.

$$pref(s_{j,k,d}e_i) = emppref_{i,j} + (O \bullet manprep_i) \qquad (4.8)$$

The objective function for the constraint set described in equations 4.1 – 4.6 then becomes

$$\text{MAX} \qquad \sum_{j,k,d,i} s_{j,k,d}e_i \bullet pref(s_{j,k,d}e_i) \qquad (4.9)$$

Figure 5.2 graphically illustrates the employee/shift preferences for the first eight shifts and ten employees after applying the above approach. The most important employee, as ranked by the manager, is on the top of the graph.

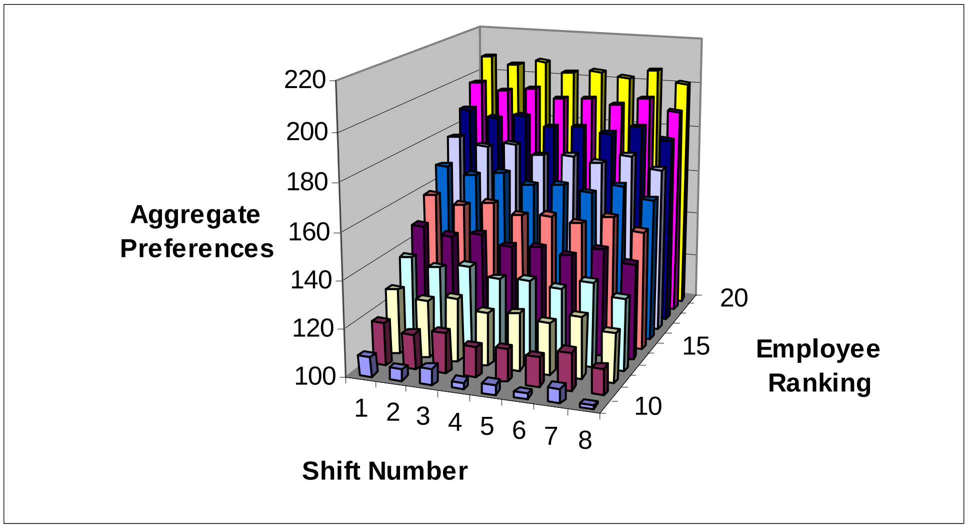


Figure 5.2: Illustration of aggregate preferences for employ/shift combination

This set of preferences is then modified to re-roster an optimal schedule. The goal of the re-rostered schedule is to have a minimal amount of deviations to the original schedule while covering the shift that is no longer staffed. The methodology is illustrated on a representative tour-scheduling problem instance. Figure 5.3 depicts the set of preferences selected in the original optimal schedule.

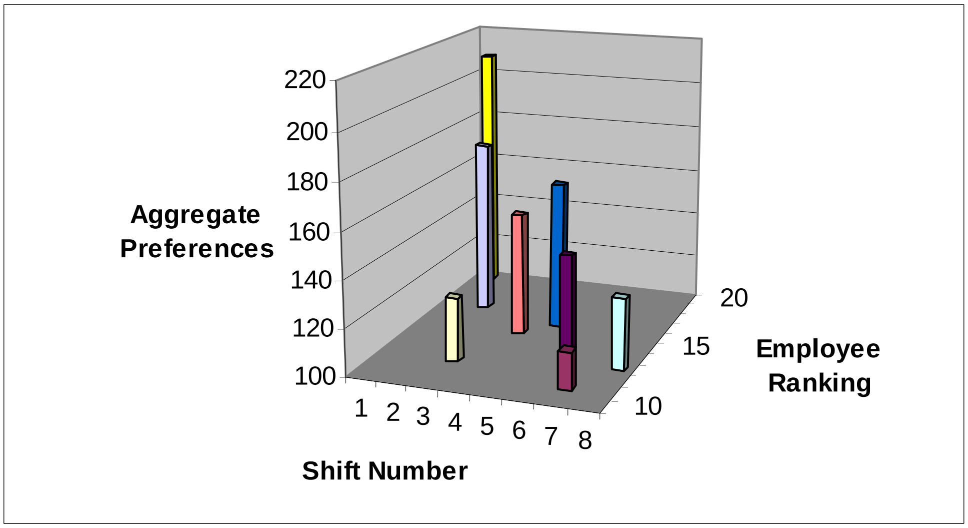


Figure 5.3: Preferences selected in original optimal schedule

The optimal schedule required all 8 shifts be staffed by only one of the 10 employees. These 8 employee/shift combinations constitute an optimal tour-schedule for a problem instance. Figure 5.4 then shows a perturbation in the schedule due to an employee’s absence. In this illustration, employee 12 is unable to staff the shift for which they have been scheduled.

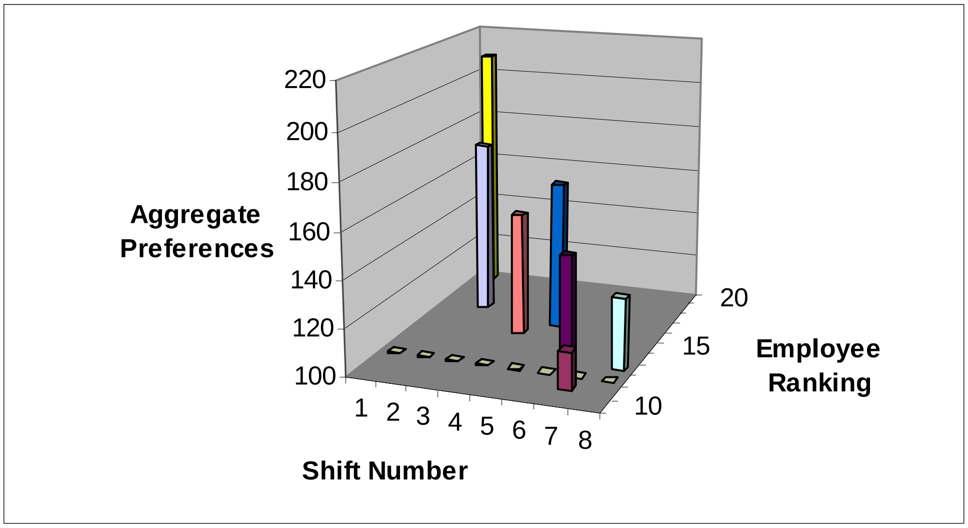


Figure 5.4: Absence of employee 12

We now apply our re-rostering methodology. In order to re-roster with minimal deviations to the original schedule, we modify the set of original preferences. Each preference for an employee who was assigned in the original schedule is increased by a factor M. The magnitude of factor M is problem dependent; however, a factor of three to five has worked well. The original preference set is then reentered into the model with the exception of employee twelve's preferences and the modified preferences of the currently scheduled employees. Figure 5.5 illustrates the modified preference set that will now be resolved to find a new schedule that will meet the shift requirement vacated by employee twelve while minimizing the changes to the original schedule.

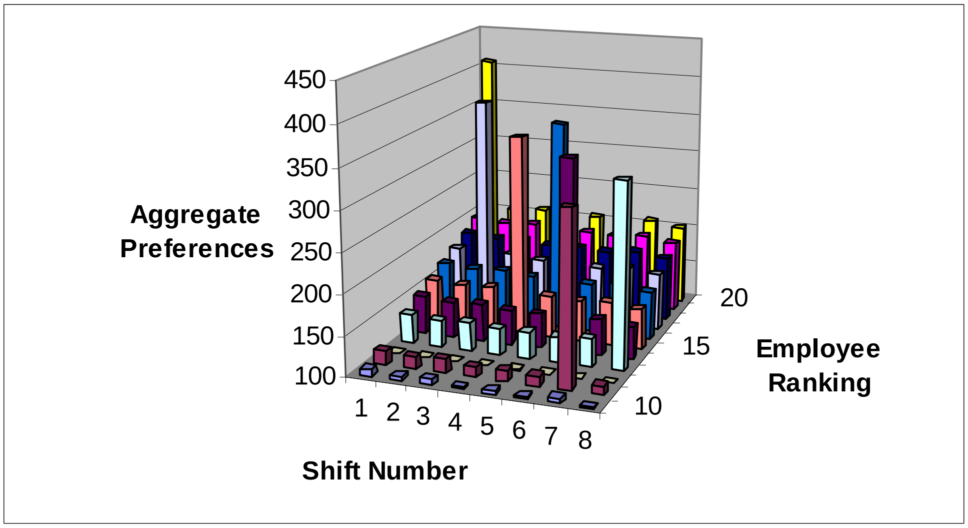


Figure 5.5: Modified preference set used in re-rostering

This methodology is now applied to the case study described above.

5.5 Re-rostering results

The re-rostering methodology shown in Section 5.4 is applied to the Banner Health case study. The results are presented below. The case study was re-rostered three times, using differing employee absentee scenarios. The first scenario is a relatively benign case of a single employee being absent from a single shift. The second scenario is more difficult, one employee absent for an entire week. Finally, the model will re-roster a schedule when five employees are absent for the entire week. The size of the problem instance for each case study, the number of deviations for each scenario, and the computational time to re-roster is shown in Table 5.1.

Case Study	Number of	Number of	Scenario	Number of	Computational
	Employees	Shifts		Deviations	Time (s)
Banner Health	20	63	Single Shift	0	0.01
			Single Employee	2	0.01
			Five Employees	3	0.03
Banner Health	40	126	Single Shift	0	0.04
			Single Employee	2	0.04
			Five Employees	10	0.06

Table 5.1: Re-rostering results

The results are very encouraging. Each scenario is re-rostered in a minimal amount of computational time and the deviations from the original schedule are minimal. The methodology works well for both case studies and finds a new optimal schedule for each instance.

5.6 Conclusion

The article presented a unique methodology to the very real problem of re-rostering a personnel tour-schedule. Employee schedules are inevitably in need of modification due to employee absenteeism. The chore of re-rostering a feasible schedule is not an easy one for a manager especially with the goal of minimizing the disturbances to the rest of the original schedule. The methodology here uses already obtained employee preferences to efficiently and with minimum deviations re-roster a published schedule. Minimizing deviations is an important goal of the new tour-schedule. The stability of the schedule is crucial for employee morale and the ability of employees to schedule their non-working lives. Appendices E and F contain the Visual Basic Code used to formulate to modify the preferences for re-rostering and the CPLEX output respectively.

Chapter 6

CONTRIBUTIONS AND FUTURE RESEARCH

6.1 Contributions

This dissertation concentrated on developing a new representation of the personnel tour-scheduling problem. Personnel scheduling is one of the most difficult, important, and often studied problems in operations research. The optimal choice of the number of employees required to meet customer demand, shift start and stop times, daily lunches and breaks, and the assignment of the adequately skilled employee to the best shift is a large combinatorial optimization problem. Managers who attempt to manually solve the personnel scheduling problem expend many valuable work hours to find even a feasible solution which has little probability of being optimal based on any objective function.

Tour-scheduling, also known as the rostering problem in the literature, this dissertation presents a network-based mathematical programming model that can solve the tour-scheduling problem optimally using completely linear programming algorithms in many cases. In these cases, there is no need for branching, bounding, or cutting techniques, resulting in a savings of valuable computational time. In the cases where integer solutions are not found by the solver, the dissertation shows that the model produces solutions with a very small amount of fractional variables. Further, the fractional variables that are found are quickly converted to integer solutions via highly proven techniques such as Gomory fractional cuts.

The next fundamental contribution of this dissertation is the ability of the formulation to account for industries that have continuous (24-hour) operations. Modeling the rest periods between shifts during continuous operations has long been a limitation of tour-scheduling models. The network structure of the formulation proposed in this dissertation allows for specialized side constraints that restrict consecutive shifts for which an employee can be assigned. It is shown that these side constraints, in many cases, do not destroy the ability of the model to produce integer answers.

The ability to model heterogeneous workforces is another contribution of this dissertation. This collection of personnel who have significantly variant availabilities, skill sets, and wage rates is a very realistic makeup of today's workforces. Tour-scheduling a heterogeneous workforce must consider all of these differences and optimally match the personnel with the best shift possible and for which they are eligible. The model in this dissertation can be easily adapted to incorporate all the factors that are inherent in a heterogeneous workforce.

The formulation further allows for each employee to designate a preference for each shift in the schedule and for managers to rank the employees based on seniority or workplace performance. A tour-schedule that considers employee shift preferences and management employee weighting is very advantageous to an organization. If employees are matched to shifts that fit well with their home life, availabilities, work habits, or other personal considerations, they will be motivated to work harder and more efficiently. A framework to collect and represent these preferences that is repeatable and comprehendible to the workforce is presented.

The formulation is then modified to account for two very important aspects of realistic employee rostering. The original model is stacked upon itself to allow for the management to schedule beyond the traditional weekly time horizon. The dissertation presents various multi-weekly models, including monthly, quarterly, and yearly. An additional network arc and node structure is then added to the model to insure that organization whose work week include all 7 days do not schedule employees for too many weekend shifts.

Finally, the model is once again modified to account for the inevitable perturbations that will occur in a tour-schedule, especially those of a multi-week length. These fluctuations will be the result of employee sicknesses, vacations, hiring, or firing. This dissertation demonstrates how the model can be used to re-roster a previous optimal schedule. The modification is powerful, but causes little complication in the way of reformulation or expansion of the computational time of the solver.

A timely and optimal solution to the personnel scheduling problem will save each manager valuable time, meet customer demand, and increase worker morale by matching each employee to the best shift possible. This dissertation presents an extremely fast, computationally efficient, and optimal network-flow based mathematical programming solution for the extremely challenging problem of rostering and re-rostering a continuous heterogeneous workforce.

6.2 Future Research

There are many areas of potential for future research. Three of these areas involve refining the current model, applying the network-based model to a different scheduling problem, developing robust tour-schedules against employee absenteeism, and combining the shift selection and tour-scheduling problems.

The current model can be refined by defining a set of rules for the preferences in the objective function. A real advancement would be a set of rules that gave the model the best chance to find an integer solution without the need for an integer constraint set. The set of rules must incorporate the number of shifts and employees, and the rest required between consecutive shifts.

The model is flexible enough that it could be applied to another scheduling problem. Academic scheduling is a classic operations research problem that might be solved using the model and formulation proposed here. The model would have to be modified to account for room scheduling in addition to the students and faculty. However, many of the same constraints associated with the tour-scheduling model would also be found in the academic scheduling problem. The model proposed in this dissertation could efficiently handle the academic scheduling constraints just as well.

Employee absenteeism is an unavoidable consequence of employee tour-scheduling. A modification of the re-rostering objective function to find schedules that may not be optimal in terms of preferences, but are more robust to absent employees would be a valuable extension to the model. Furthermore, a design of experiments approach to quantifying the size of the Big M reward to the objective function for optimally scheduled employees would provide the manager with important information regarding the number of deviations that may be tolerated.

Finally, using a network-based model to solve the combined shift selection and tour-scheduling problem would be a powerful tool. The model would have to be modified considerably to account for the non-linearity aspects of the combined model. A network-based model with an accompanying heuristic may solve this problem. A more detailed of this extension is discussion is presented next.

6.2.1 Combined Shift Selection/Tour-Scheduling

Optimally solving the tour scheduling sub-problem does not guarantee a global optimal solution to the combined shift selection/tour scheduling problem. An alternate set of shifts that minimizes excess employee hours but increases the number of preferred shifts of personnel can only be found by solving the combined shift selection/tour-scheduling problem. The advantage of a global optimal, and the reason to pursue a solution, is the possibility of bettering the tour-scheduling preferences while still minimizing the number of excess hours devoted to shifts covering the required demand.

The proposed solution method presented in this dissertation was expanded to solve the combined problem. However, a pure network-flow structure is problematic for the combined problem. In short, the balance equations associated with the network-flow representation of the shift selection/tour-scheduling problem are non-linear. Figure 6.1 shows an illustration of the network framework for the combined problem. The extreme right hand side of the figure is not shown, but will look identical to the network representations presented earlier.

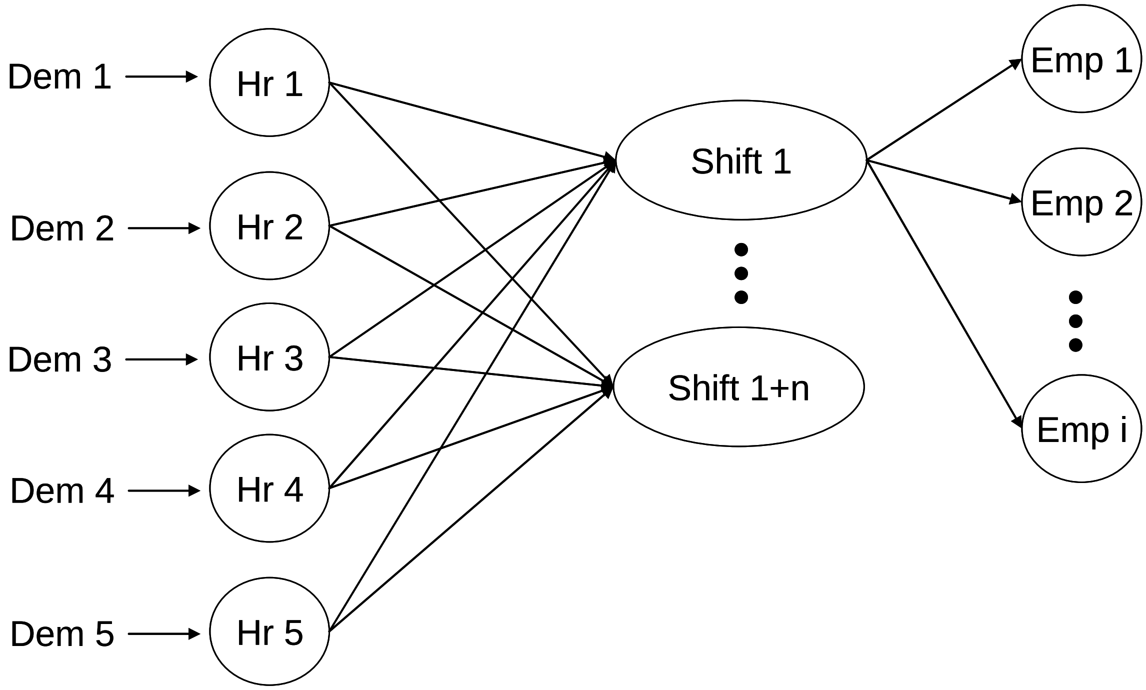


Figure 6.1: Network-flow representation of combined problem

Here, the nodes HR represent each individual hour of the day. The increment could easily be increase to multiple hours, or decrease to fractions of an hour. The increment should correspond to the profile produced by the forecasting models during the demand modeling sub-problem. The demand flows from each hour to a corresponding shift whose start and stop times encompass the particular hourly node. The shift is either then selected to fulfill the demand associated with that hour of the day, or a different shift encompassing that hour is selected. Once a shift is selected, the model executes just as described in this dissertation. However, this network representation of the combined problem leads to a constraint set that is non-linear.

The non-linearity is obvious when the balance equations for examples from the hourly and shift nodes are written. Equation 6.1 is the balance equation for node HR1. Theses equations are representative of all the balance equations for all the hourly nodes.

$$Hr1s1 + Hr1s2 + \ldots + Hr1s_j = Dem1 \tag{6.1}$$

Equation 6.1 is perfectly linear, however, the balance equations for the shift nodes are not. Equation 6.2 shows the non-linearity. The RHS must be divided by the number of hourly nodes flowing into the shift node, in order for the rest of the network to schedule only one shift. Without the normalization, the network would attempt to schedule the shift as many times as hourly demand was met with that shift.

$$Hr1s1 + Hr2s1 + \ldots + Hr5s1 = \left[\frac{1}{Hr1s1 + Hr2s1 + \ldots + Hr5s1}\right] \bullet \left(s1e1 + s1e2 + \ldots + s1e_i\right)$$

This is the simplest case. A more complicated case involving multiple demands for an hour (a likely case) or varying shift lengths make the problem even more difficult. Figure 6.2 shows a more realistic flow diagram for the first half of the network representation of the combined problem.

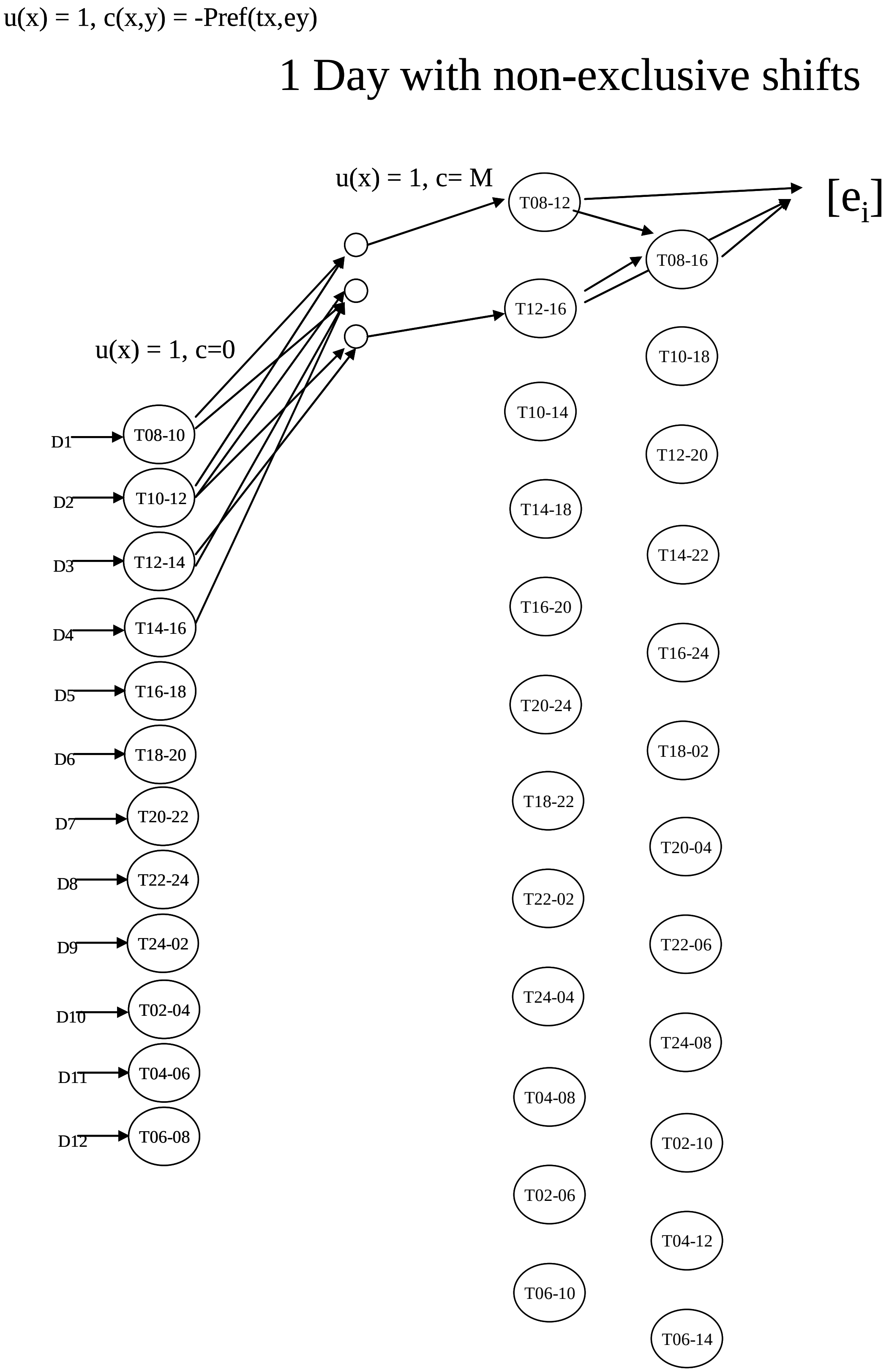


Figure 6.2: Combined problem with varied shift lengths

The non-linearity requires different solution techniques than those presented in this dissertation. Conjugate gradient methods or heuristics may be a viable option for solving the combined problem.   Another solution approach may be an iterative method. The approach would solve the two sub-problems once again separately, but return to each sub-problem with information from the solution of the other that can better the global optima.

REFERENCES

Alfares, H. (2004). Survey, categorization, and comparison of recent tour scheduling literature. *Annals of Operations Research*, *Special Issue on Staff Scheduling and Rostering*, *127,* 145-175.

Ashley, D.W. (1995). Spreadsheet optimization system for library staff scheduling. *Computers and Operations Research, 22(6)*, 615–624.

Ball, M. & Benoit-Thompson, H. (1988). “A lagrangian relaxation based heuristic for the urban transit crew scheduling problem. In J. Daduna and I. Wren (Eds.). *Computer-Aided Transit Scheduling*, *Lecture Notes in Economics and Mathematical Systems, 308,* 54–67.

Banihashemi, M. & Haghani, A. (2001). A new model for the mass transit crew scheduling problem. In S. Voss and J. Daduna (Eds.). *Computer-Aided Scheduling of Public Transport*, *Lecture Notes in Economics and Mathematical Systems, 505*, 1–16.

Barnhart, C., E. Johnson, R. Anbil, & Hatay, L. (1994). A column generation technique for the long-haul crew assignment problem. In T. Ciriani and R. Leachman (Eds.), *Optimization in Industry, 2*, 7–24. Chichester: Wiley.

Borndörfer, R.,M. Grötschel, & Löbel, A. (2001). Duty scheduling in public transit. *Technical Report, ZIB*.

Brusco, M. & Johns, T. (1995). Improving the dispersion of surplus labor in personnel scheduling solutions. *Computers and Industrial Engineering, 28(4),* 745-754.

Carraresi, P. & Gallo, G. (1984b). Network models for vehicle and crew scheduling. *European Journal of Operational Research, 16(2),* 139–151.

Dowsland, K. & Thompson, J. (2000). Solving a nurse scheduling problem with knapsacks, networks and tabu Search. *Journal of the Operational Research Society, 51*, 825–833.

Ernst, A., H. Jiang, M. Krishnamoorthy, & Sier, D. (2004a). Staff scheduling and rostering: A review of applications, methods and models. *European Journal of Operational Research, 153*, 3-27.

Ernst, A., H. Jiang, M. Krishnamoorthy, B. Owens , & Sier, D. (2004b). An annotated bibliography of personnel scheduling and rostering. *Annals of Operations Research, Special Issue on Staff Scheduling and Rostering*, *127,* 21-141.

Eveborn, P. & Ronnqvist, M. (2004). Scheduler – a system for staff planning. *Annals of Operations Research, Special Issue on Staff Scheduling and Rostering*, *128,* 21–45.

Glover, F. & McMillan, C. (1986). The general employee scheduling problem: an integration of MS and AI. *Computers and Operations Research, 13(5),* 563–573.

Glover, F., C. McMillan, & Grover, R. (1985). A heuristic programming approach to the employee scheduling problem and some thoughts on ‘managerial robots’. *Journal of Operations Management, 4*, 113–128.

Haase, K. (1999). Advanced column generation techniques with applications to marketing, retail and logistics management. *Ph.D. Thesis, Habilitation Thesis, University of Kiel.*

Jiang, H., M. Krishnamoorthy, & Sier, D. (2004). Preface: special issue: staff scheduling and rostering: theory and applications, part I. *Annals of Operations Research, Special Issue on Staff Scheduling and Rostering, 127,* 17-20.

Jaumard, B., F. Semet, & Vovor, T. (1998). A generalized linear programming model for nurse scheduling. *European Journal of Operational Research, 107(1),* 1–18.

Lauer, J., L.W. Jacobs, M.J. Brusco, & Bechtold, S.E. (1994). An interactive optimization-based decision support system for scheduling part-time computer lab attendants. *Omega, 22(6),* 613–626.

Li, C., E. Robinson, & Mabert, V. (1991). An evaluation of tour scheduling heuristics with differences in employee productivity and cost. *Decision Sciences, 22*, 700–718.

Litchfield, J., A. Ingolfsson, & Cheng, K. (2003). Rostering for a restaurant. *INFOR, 41(3),* 287-300.

Loucks, J. & Jacobs, F. (1991). Tour scheduling and task assignment of a heterogeneous work force: a heuristic approach. *Decision Sciences, 22(4),* 719–739.

Love, R. & Hoey, J. (1990). Management science improves fast food operations. *Interfaces, 20(2),* 21–29.

Mabert, V. & Raedels, A. (1977). The detail scheduling of a part-time work force: a case study of teller staffing. *Decision Sciences, 8*, 109–120.

Mabert, V. & Watts, C. (1982). A simulation analysis of tour-shift construction procedures. *Management Science, 28(5),* 520–532.

Mabert, V. & Showalter, M. (1990). Measuring the impact of part-time workers in service organizations. *Journal of Operations Management, 9(2),* 209–229.

Mason, A. & Smith, M. (1998). A nested column generator for solving rostering problems with integer programming. In L. Caccetta, K. Teo, P. Siew, Y. Leung, L. Jennings, and V. Rehbock (Eds.), *International Conference on Optimization: Techniques and Applications.*

Mellouli, T. (2001). A network flow approach to crew scheduling based on an analogy to a vehicle maintenance routing problem. In S. Voss and J. Daduna (eds.), *Computer-Aided Scheduling of Public Transport*, *Lecture Notes in Economics and Mathematical Systems*, *505*, 91–120. Springer.

Millar, H. & Kiragu, M. (1998). Cyclic and non-cyclic scheduling of 12 h shift nurses by network programming. *European Journal of Operational Research, 104(3),* 582–592.

Mould, G. (1996). Case study of manpower planning for clerical operations. *Journal of the Operational Research Society, 47(3),* 358–368.

Moz, M. & Pato, M.V. (2004). Solving the problem of rerostering nurse schedules with hard constraints: new multicommodity flow models. *Annals of Operations Research, Special Issue on Staff Scheduling and Rostering*, *128,* 179–197.

Nicoletti, B. (1975). Automatic crew rostering. *Transportation Science 9(1),* 33–42.

Patrikalakis, I. & Xerocostas, D. (1992). A new decomposition scheme of the urban public transport scheduling problem. In M. Desrochers and J. Rousseau (eds.), *Computer-Aided Transit Scheduling*, *Lecture Notes in Economics and Mathematical Systems, 386*, 407–425.

Thompson, G. (1996b). A simulated-annealing heuristic for shift scheduling using non-continuously available employees. *Computers and Operations Research 23(3),* 275–288.

Tingley, G. (1979). Still another solution method for the monthly aircrew assignment problem. *In 1979 AGIFORS Symposium Proceedings, 19*, 143–203.

Warner, M.D. (1976). Scheduling nursing personnel according to nursing preference: a mathematical programming approach. *Operations Research, 24(5),* 842-856.

Yan, S. & Chang, J. (2002). Airline cockpit crew scheduling. *European Journal of Operational Research, 136(3)*, 501–511.

APPENDIX A

CHAPTER THREE VISUAL BASIC CODE

```
Private Sub WriteNetLP()

'This subroutine will write out in CPLEX .LP format
'a LP for solving large crew scheduling problems
'using a network-based linear program
'Shane Knighton  6-20-04

'Rev 2 9-22-04 - added Shift Demand Matrix allowing for
'variable number of employees needed per shift


'Dimensionalize variables
Dim NumShiftsM, NumShiftsT, NumShiftsW, NumShiftsR, NumShiftsF,
NumShiftsS, NumShiftsN As Integer
Dim NumShifts, dayShifts, NumDays, NumEmp As Integer
Dim MaxNumShifts, MinNumShifts, ShiftDemand As Integer
Dim i, j, k As Integer
Dim Name, filepath, text2write As String
Dim varName As String

'ConflictMatrix is a NumShift by maximum number of conflicts
'matrix that has a 1 in the i,j column if shift i and j
'are conflicted
Dim ConflictMatrix() As Integer
Dim MaxNumConfl As Integer

'PriorityMatrix is a shift by employee matrix
'containing the priority of each shift for each employee
Dim PriorityMatrix() As Single

'ShiftDemMatrix is the number of employees needed for each shift
Dim ShiftDemMatrix() As Integer

'Problem size data
NumDays = 7
NumEmp = 50
'Need to get the number of shifts on each day
NumShiftsM = 21
NumShiftsT = 34
NumShiftsW = 34
NumShiftsR = 34
NumShiftsF = 35
NumShiftsS = 24
NumShiftsN = 13
'Total all the shifts
```

```
NumShifts = NumShiftsM + NumShiftsT + NumShiftsW + NumShiftsR +
NumShiftsF + NumShiftsS + NumShiftsN
'Min and Max shifts per week per employee
MaxNumShifts = 5
MinNumShifts = 0 '2
'Maximum number of conflicted shifts
MaxNumConfl = 30

'Get Name of Problem
Name = "run5ahp" 'ActiveSheet.Name

'Size ConfliftMatrix and ConflictPointer
ReDim ConflictMatrix(1 To NumShifts, 1 To MaxNumConfl) As Integer

'Populate ConflictMatrix
Dim startRow, startCol As Integer
startRow = 10 + NumShifts
startCol = 3

For i = 1 To NumShifts
   For j = 1 To MaxNumConfl
      If Cells(startRow + i, startCol + j) <> 0 Then
         ConflictMatrix(i, j) = Cells(startRow + i, startCol + j)
      End If
   Next j
Next i

'Size PriorityMatrix
ReDim PriorityMatrix(1 To NumShifts, 1 To NumEmp) As Single

'Populate PriorityMatirix
startRow = 5
startCol = 6

For i = 1 To NumShifts
   For j = 1 To NumEmp
      PriorityMatrix(i, j) = Cells(startRow + i, startCol + j) ' + Round(Rnd(), 2)
   Next j
Next i

'Size ShiftDemMatrix
ReDim ShiftDemMatrix(1 To NumShifts) As Integer

'Populate ShiftDemMatrix
startRow = 5
```

```
startCol = 5

For i = 1 To NumShifts
    ShiftDemMatrix(i) = Cells(startRow + i, startCol)
Next i


'Open a file to write to
filepath = "C:\Documents and Settings\Shane\My Documents\Dissertation\Phase
1\" & Name & ".lp"
Open filepath For Output As #1

'Write header and objective function
Print #1, "MAXIMIZE"
Print #1, "Z:"
'i is the loop for shifts
For i = 1 To NumShifts
    'j is the loop for employees
    For j = 1 To NumEmp
        'Concatenate variable name
        varName = "s" & i & "e" & j
        'Write variables and cost coefficients to file
        'Write final variable and cost to file, leave off the "+"
        If i = NumShifts And j = NumEmp Then
            Print #1, PriorityMatrix(i, j) & " " & varName
        Else
            Print #1, PriorityMatrix(i, j) & " " & varName & " +"
        End If
    Next j
Next i

'Write constraints
'Header
Print #1, "SUBJECT TO"
'Constraint number counter
Dim ConstNum As Double
ConstNum = 1

'Define the 6 sets of constraints
'Set1: shifts = demand equality constraints
'       s001e01 + ... + s001e50 = demand
'Set2: total shifts per employee for each day
'       will be required to be less than one
'       by set 5, needed to total number
'       of shifts per week
'       s001e01 + ... + s025e01 = me01
'Set3: each employee has less than max number of shifts per week
'       me01 + ... + ne01 <= max number of shifts
'Set4: each employee has more than min number of shifts per week
'       me01 + ... + ne01 >= min number of shifts
'Set5: only one shift per day per employee
'       me01 <= 1
'Set6: shift confliction for each employee
'       example if shifts 25 and 26 are conflicted
'       s025e01 + s026e01 <= 1
'Write Set1 constraints
'i is the loop for shifts
For i = 1 To NumShifts
    'Write constraint lable
    Print #1, "c" & ConstNum & ":"
    'j is the loop for employees
    For j = 1 To NumEmp
        'Concatenate variable name
        varName = "s" & i & "e" & j
        'Write variables and coefficients to file, leave off "+" for final emp
        If j = NumEmp Then
            Print #1, varName & " = " & ShiftDemMatrix(i)
        Else
            Print #1, varName & " +"
        End If
    Next j
    ConstNum = ConstNum + 1
Next i

'Write Set2 constraints
'i is the loop for shifts
i = 1
Do While i < NumShifts

    'Determine what day this shift is on
    If i < NumShiftsM Then dayNum = 1
    If i > NumShiftsM Then dayNum = 2
    If i > NumShiftsM + NumShiftsT Then dayNum = 3
    If i > NumShiftsM + NumShiftsT + NumShiftsW Then dayNum = 4
    If i > NumShiftsM + NumShiftsT + NumShiftsW + NumShiftsR Then dayNum =
5
    If i > NumShiftsM + NumShiftsT + NumShiftsW + NumShiftsR + NumShiftsF
Then dayNum = 6
    If i > NumShiftsM + NumShiftsT + NumShiftsW + NumShiftsR + NumShiftsF +
NumShiftsS Then dayNum = 7
```

```
    'Find the right day
    If dayNum = 1 Then
        dayName = "m"
        dayShifts = NumShiftsM
    ElseIf dayNum = 2 Then
        dayName = "t"
        dayShifts = NumShiftsT
    ElseIf dayNum = 3 Then
        dayName = "w"
        dayShifts = NumShiftsW
    ElseIf dayNum = 4 Then
        dayName = "r"
        dayShifts = NumShiftsR
    ElseIf dayNum = 5 Then
        dayName = "f"
        dayShifts = NumShiftsF
    ElseIf dayNum = 6 Then
        dayName = "s"
        dayShifts = NumShiftsS
    ElseIf dayNum = 7 Then
        dayName = "n"
        dayShifts = NumShiftsN
    Else
        GoTo ErrorHandler
    End If

    'j is the loop for employees
    For j = 1 To NumEmp
        'Set2 constraints
        Print #1, "c" & ConstNum & ":"
        For k = i To i - 1 + dayShifts
            'Concatenate variable name
            varName = "s" & k & "e" & j
            'Write variables and coefficients to file
            If k = i - 1 + dayShifts Then
                Print #1, varName & " - " & dayName & j & " = 0"
            Else
                Print #1, varName & " +"
            End If
        Next k
        ConstNum = ConstNum + 1
    Next j
    i = i + dayShifts
Loop
```

```
'Write Set3 constraints
For j = 1 To NumEmp
   Print #1, "c" & ConstNum & ":"
   Print #1, "m" & j & " + t" & j & " + w" & j & " + r" & j & " + f" & j & " + s" & j & " +
n" & j & " <= " & MaxNumShifts
   ConstNum = ConstNum + 1
Next j

'Write Set4 constraints
For j = 1 To NumEmp
   Print #1, "c" & ConstNum & ":"
   Print #1, "m" & j & " + t" & j & " + w" & j & " + r" & j & " + f" & j & " + s" & j & " +
n" & j & " >= " & MinNumShifts
   ConstNum = ConstNum + 1
Next j

'Write Set5 constraints
For j = 1 To NumEmp
   Print #1, "c" & ConstNum & ":"
   Print #1, "m" & j & " <= 1"
   ConstNum = ConstNum + 1
   Print #1, "c" & ConstNum & ":"
   Print #1, "t" & j & " <= 1"
   ConstNum = ConstNum + 1
   Print #1, "c" & ConstNum & ":"
   Print #1, "w" & j & " <= 1"
   ConstNum = ConstNum + 1
   Print #1, "c" & ConstNum & ":"
   Print #1, "r" & j & " <= 1"
   ConstNum = ConstNum + 1
   Print #1, "c" & ConstNum & ":"
   Print #1, "f" & j & " <= 1"
   ConstNum = ConstNum + 1
   Print #1, "c" & ConstNum & ":"
   Print #1, "s" & j & " <= 1"
   ConstNum = ConstNum + 1
   Print #1, "c" & ConstNum & ":"
   Print #1, "n" & j & " <= 1"
   ConstNum = ConstNum + 1
Next j

'Write Set6 constraints
For i = 1 To NumShifts
   For k = 1 To MaxNumConfl
```

```
        If ConflictMatrix(i, k) <> 0 Then
            For j = 1 To NumEmp
                Print #1, "c" & ConstNum & ":"
                Print #1, "s" & i & "e" & j & " + s" & ConflictMatrix(i, k) & "e" & j & " <= 1"
                ConstNum = ConstNum + 1
            Next j
        End If
    Next k
Next i

'Write Integer requirements
'Print #1, "INTEGER"
'For i = 1 To NumShifts
'    For j = 1 To NumEmp
'        Print #1, "s" & i & "e" & j
'    Next j
'Next i

'Write END
Print #1, "END"
'Close mps file
Close #1

Exit Sub

ErrorHandler:
MsgBox ("Error")
'Close mps file
Close #1

End Sub

Private Sub WriteConflicts()

    rest = 12
    totshifts = 195

    For i = 1 To totshifts
        curshift = Cells(i + 5, 2).Value
        dy = Int(curshift / 144) + 1
        begin = Int(curshift / 6)
        dur = curshift - (begin * 6) + 2
        finish = begin + dur

        endrest = finish + rest
        k = 4
        For j = 1 To totshifts
            compareshift = Cells(j + 5, 2).Value
            begin2 = Int(compareshift / 6)
            If compareshift > dy * 144 And begin2 < endrest Then
                Cells(i + 10 + totshifts, k).Value = Cells(j + 5, 3).Value
                k = k + 1
            End If

        Next j
    Next i

End Sub

Private Sub Priorities()

End Sub

startRow = 5
startCol = 76

For i = 1 To 285
    shift = Cells(startRow + i, startCol).Value
    emp = Cells(startRow + i, startCol + 2).Value
    Cells(startRow + i, startCol + 5).Value = Cells(shift + 5, emp + 6).Value
Next i

Private Sub Values()

startRow = 5
startCol = 76

For i = 1 To 285
    shift = Cells(startRow + i, startCol).Value
    emp = Cells(startRow + i, startCol + 2).Value
    Cells(5 + shift, 87 + emp).Value = Cells(startRow + i, startCol + 4).Value
Next i

End Sub
```

APPENDIX B

CHAPTER THREE CPLEX OUTPUT

```
Log started (V8.1.0) Thu Oct 28 16:46:53 2004


Problem 'Run1nflp.lp' read.
Read time =    0.03 sec.
Tried aggregator 1 time.
LP Presolve eliminated 350 rows and 0 columns.
Reduced LP has 645 rows, 10100 columns, and 20550 nonzeros.
Presolve time =    0.02 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =         38722.250000

Dual simplex - Optimal:  Objective =    3.8625380000e+04
Solution time =    0.03 sec.  Iterations = 125 (0)


Variable Name           Solution Value
s1e20                         1.000000
s2e5                          1.000000
s3e6                          1.000000
s4e3                          1.000000
s5e50                         1.000000
s6e13                         1.000000
s7e25                         1.000000
s8e39                         1.000000
s9e10                         1.000000
s10e41                        1.000000
s11e49                        1.000000
s12e48                        1.000000
s13e35                        1.000000
s14e29                        1.000000
s15e11                        1.000000
s16e40                        1.000000
s17e28                        1.000000
s18e21                        1.000000
s19e19                        1.000000
s20e14                        1.000000
s21e47                        1.000000
s22e8                         1.000000
s23e32                        1.000000
s24e48                        1.000000
s25e2                         1.000000
s26e19                        1.000000
s27e50                        1.000000
s28e7                         1.000000
s29e13                        1.000000
s30e40                        1.000000
s31e20                        1.000000
s32e39                        1.000000
s33e49                        1.000000
s34e10                        1.000000
s35e21                        1.000000
s36e25                        1.000000
```

```
s37e6                         1.000000
s38e45                        1.000000
s39e17                        1.000000
s40e31                        1.000000
s41e18                        1.000000
s42e46                        1.000000
s43e42                        1.000000
s44e26                        1.000000
s45e27                        1.000000
s46e14                        1.000000
s47e47                        1.000000
s48e23                        1.000000
s49e16                        1.000000
s50e28                        1.000000
s51e4                         1.000000
s52e3                         1.000000
s53e11                        1.000000
s54e37                        1.000000
s55e34                        1.000000
s56e42                        1.000000
s57e30                        1.000000
s58e49                        1.000000
s59e19                        1.000000
s60e14                        1.000000
s61e18                        1.000000
s62e7                         1.000000
s63e41                        1.000000
s64e32                        1.000000
s65e46                        1.000000
s66e6                         1.000000
s67e1                         1.000000
s68e27                        1.000000
s69e36                        1.000000
s70e24                        1.000000
s71e17                        1.000000
s72e15                        1.000000
s73e44                        1.000000
s74e39                        1.000000
s75e16                        1.000000
s76e45                        1.000000
s77e8                         1.000000
s78e37                        1.000000
s79e9                         1.000000
s80e28                        1.000000
s81e33                        1.000000
s82e13                        1.000000
s83e40                        1.000000
s84e12                        1.000000
s85e10                        1.000000
s86e26                        1.000000
s87e25                        1.000000
s88e34                        1.000000
s89e43                        1.000000
s90e13                        1.000000
s91e39                        1.000000
```

```
s92e41                        1.000000
s93e19                        1.000000
s94e10                        1.000000
s95e40                        1.000000
s96e42                        1.000000
s97e45                        1.000000
s98e30                        1.000000
s99e36                        1.000000
s100e33                       1.000000
s101e23                       1.000000
s102e1                        1.000000
s103e38                       1.000000
s104e2                        1.000000
s105e26                       1.000000
s106e44                       1.000000
s107e28                       1.000000
s108e29                       1.000000
s109e35                       1.000000
s110e7                        1.000000
s111e47                       1.000000
s112e31                       1.000000
s113e46                       1.000000
s114e12                       1.000000
s115e21                       1.000000
s116e50                       1.000000
s117e9                        1.000000
s118e34                       1.000000
s119e15                       1.000000
s120e8                        1.000000
s121e27                       1.000000
s122e37                       1.000000
s123e22                       1.000000
s124e34                       1.000000
s125e44                       1.000000
s126e28                       1.000000
s127e31                       1.000000
s128e38                       1.000000
s129e35                       1.000000
s130e50                       1.000000
s131e30                       1.000000
s132e11                       1.000000
s133e9                        1.000000
s134e19                       1.000000
s135e4                        1.000000
s136e37                       1.000000
s137e8                        1.000000
s138e16                       1.000000
s139e41                       1.000000
s140e3                        1.000000
s141e1                        1.000000
s142e23                       1.000000
s143e48                       1.000000
s144e22                       1.000000
s145e21                       1.000000
s146e10                       1.000000
```

```
s147e14                       1.000000
s148e2                        1.000000
s149e18                       1.000000
s150e15                       1.000000
s151e36                       1.000000
s152e12                       1.000000
s153e24                       1.000000
s154e46                       1.000000
s155e13                       1.000000
s156e17                       1.000000
s157e47                       1.000000
s158e5                        1.000000
s159e15                       1.000000
s160e40                       1.000000
s161e5                        1.000000
s162e2                        1.000000
s163e34                       1.000000
s164e35                       1.000000
s165e9                        1.000000
s166e18                       1.000000
s167e11                       1.000000
s168e41                       1.000000
s169e32                       1.000000
s170e37                       1.000000
s171e43                       1.000000
s172e25                       1.000000
s173e7                        1.000000
s174e49                       1.000000
s175e50                       1.000000
s176e8                        1.000000
s177e31                       1.000000
s178e14                       1.000000
s179e3                        1.000000
s180e20                       1.000000
s181e27                       1.000000
s182e24                       1.000000
s183e33                       1.000000
s184e3                        1.000000
s185e9                        1.000000
s186e11                       1.000000
s187e15                       1.000000
s188e16                       1.000000
s189e17                       1.000000
s190e44                       1.000000
s191e39                       1.000000
s192e7                        1.000000
s193e26                       1.000000
s194e42                       1.000000
s195e49                       1.000000
m3                            1.000000
m5                            1.000000
m6                            1.000000
m10                           1.000000
m11                           1.000000
m13                           1.000000
```

```
m14                           1.000000
m19                           1.000000
m20                           1.000000
m21                           1.000000
m25                           1.000000
m28                           1.000000
m29                           1.000000
m35                           1.000000
m39                           1.000000
m40                           1.000000
m41                           1.000000
m47                           1.000000
m48                           1.000000
m49                           1.000000
m50                           1.000000
t2                            1.000000
t3                            1.000000
t4                            1.000000
t6                            1.000000
t7                            1.000000
t8                            1.000000
t10                           1.000000
t11                           1.000000
t13                           1.000000
t14                           1.000000
t16                           1.000000
t17                           1.000000
t18                           1.000000
t19                           1.000000
t20                           1.000000
t21                           1.000000
t23                           1.000000
t25                           1.000000
t26                           1.000000
t27                           1.000000
t28                           1.000000
t31                           1.000000
t32                           1.000000
t34                           1.000000
t37                           1.000000
t39                           1.000000
t40                           1.000000
t42                           1.000000
t45                           1.000000
t46                           1.000000
t47                           1.000000
t48                           1.000000
t49                           1.000000
t50                           1.000000
w1                            1.000000
w6                            1.000000
w7                            1.000000
w8                            1.000000
w9                            1.000000
w10                           1.000000
```

```
w12                           1.000000
w13                           1.000000
w14                           1.000000
w15                           1.000000
w16                           1.000000
w17                           1.000000
w18                           1.000000
w19                           1.000000
w24                           1.000000
w25                           1.000000
w26                           1.000000
w27                           1.000000
w28                           1.000000
w30                           1.000000
w32                           1.000000
w33                           1.000000
w34                           1.000000
w36                           1.000000
w37                           1.000000
w39                           1.000000
w40                           1.000000
w41                           1.000000
w42                           1.000000
w43                           1.000000
w44                           1.000000
w45                           1.000000
w46                           1.000000
w49                           1.000000
r1                            1.000000
r2                            1.000000
r7                            1.000000
r8                            1.000000
r9                            1.000000
r10                           1.000000
r12                           1.000000
r13                           1.000000
r15                           1.000000
r19                           1.000000
r21                           1.000000
r22                           1.000000
r23                           1.000000
r26                           1.000000
r27                           1.000000
r28                           1.000000
r29                           1.000000
r30                           1.000000
r31                           1.000000
r33                           1.000000
r34                           1.000000
r35                           1.000000
r36                           1.000000
r37                           1.000000
r38                           1.000000
r39                           1.000000
r40                           1.000000
```

```
r41                           1.000000
r42                           1.000000
r44                           1.000000
r45                           1.000000
r46                           1.000000
r47                           1.000000
r50                           1.000000
f1                            1.000000
f2                            1.000000
f3                            1.000000
f4                            1.000000
f5                            1.000000
f8                            1.000000
f9                            1.000000
f10                           1.000000
f11                           1.000000
f12                           1.000000
f13                           1.000000
f14                           1.000000
f15                           1.000000
f16                           1.000000
f17                           1.000000
f18                           1.000000
f19                           1.000000
f21                           1.000000
f22                           1.000000
f23                           1.000000
f24                           1.000000
f28                           1.000000
f30                           1.000000
f31                           1.000000
f34                           1.000000
f35                           1.000000
f36                           1.000000
f37                           1.000000
f38                           1.000000
f41                           1.000000
f44                           1.000000
f46                           1.000000
f47                           1.000000
f48                           1.000000
f50                           1.000000
s2                            1.000000
s3                            1.000000
s5                            1.000000
s7                            1.000000
s8                            1.000000
s9                            1.000000
s11                           1.000000
s14                           1.000000
s15                           1.000000
s18                           1.000000
s20                           1.000000
s24                           1.000000
s25                           1.000000
```

```
s27                           1.000000
s31                           1.000000
s32                           1.000000
s34                           1.000000
s35                           1.000000
s37                           1.000000
s40                           1.000000
s41                           1.000000
s43                           1.000000
s49                           1.000000
s50                           1.000000
n3                            1.000000
n7                            1.000000
n9                            1.000000
n11                           1.000000
n15                           1.000000
n16                           1.000000
n17                           1.000000
n26                           1.000000
n33                           1.000000
n39                           1.000000
n42                           1.000000
n44                           1.000000
n49                           1.000000
All other variables in the range 1-10100 are zero.

Problem 'Run2nflp.lp' read.
Read time =    0.04 sec.
Tried aggregator 1 time.
LP Presolve eliminated 350 rows and 0 columns.
Reduced LP has 645 rows, 10100 columns, and 20550 nonzeros.
Presolve time =    0.02 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =         38609.380000

Dual simplex - Optimal:  Objective =    3.8495660000e+04
Solution time =    0.04 sec.  Iterations = 129 (0)


Variable Name           Solution Value
s1e26                         1.000000
s2e47                         1.000000
s3e37                         1.000000
s4e29                         1.000000
s5e50                         1.000000
s6e31                         1.000000
s7e2                          1.000000
s8e39                         1.000000
s9e10                         1.000000
s10e11                        1.000000
s11e3                         1.000000
s12e24                        1.000000
s13e35                        1.000000
```

```
s14e18                        1.000000
s15e5                         1.000000
s16e25                        1.000000
s17e14                        1.000000
s18e21                        1.000000
s19e19                        1.000000
s20e8                         1.000000
s21e43                        1.000000
s22e43                        1.000000
s23e46                        1.000000
s24e48                        1.000000
s25e31                        1.000000
s26e19                        1.000000
s27e33                        1.000000
s28e45                        1.000000
s29e41                        1.000000
s30e40                        1.000000
s31e30                        1.000000
s32e25                        1.000000
s33e32                        1.000000
s34e10                        1.000000
s35e2                         1.000000
s36e23                        1.000000
s37e6                         1.000000
s38e21                        1.000000
s39e14                        1.000000
s40e9                         1.000000
s41e18                        1.000000
s42e5                         1.000000
s43e42                        1.000000
s44e26                        1.000000
s45e27                        1.000000
s46e28                        1.000000
s47e47                        1.000000
s48e50                        1.000000
s49e49                        1.000000
s50e39                        1.000000
s51e4                         1.000000
s52e3                         1.000000
s53e11                        1.000000
s54e37                        1.000000
s55e16                        1.000000
s56e42                        1.000000
s57e30                        1.000000
s58e49                        1.000000
s59e48                        1.000000
s60e12                        1.000000
s61e18                        1.000000
s62e7                         1.000000
s63e41                        1.000000
s64e32                        1.000000
s65e46                        1.000000
s66e6                         1.000000
s67e3                         1.000000
s68e1                         1.000000
```

```
s69e10                        1.000000
s70e11                        1.000000
s71e33                        1.000000
s72e15                        1.000000
s73e44                        1.000000
s74e39                        1.000000
s75e36                        1.000000
s76e28                        1.000000
s77e8                         1.000000
s78e37                        1.000000
s79e9                         1.000000
s80e17                        1.000000
s81e22                        1.000000
s82e13                        1.000000
s83e40                        1.000000
s84e43                        1.000000
s85e14                        1.000000
s86e26                        1.000000
s87e25                        1.000000
s88e34                        1.000000
s89e35                        1.000000
s90e13                        1.000000
s91e39                        1.000000
s92e41                        1.000000
s93e19                        1.000000
s94e10                        1.000000
s95e40                        1.000000
s96e42                        1.000000
s97e45                        1.000000
s98e30                        1.000000
s99e36                        1.000000
s100e33                       1.000000
s101e43                       1.000000
s102e44                       1.000000
s103e4                        1.000000
s104e2                        1.000000
s105e17                       1.000000
s106e7                        1.000000
s107e6                        1.000000
s108e12                       1.000000
s109e35                       1.000000
s110e23                       1.000000
s111e29                       1.000000
s112e31                       1.000000
s113e46                       1.000000
s114e38                       1.000000
s115e28                       1.000000
s116e50                       1.000000
s117e26                       1.000000
s118e34                       1.000000
s119e15                       1.000000
s120e8                        1.000000
s121e20                       1.000000
s122e37                       1.000000
s123e22                       1.000000
```

```
s124e34                       1.000000
s125e44                       1.000000
s126e28                       1.000000
s127e31                       1.000000
s128e38                       1.000000
s129e35                       1.000000
s130e50                       1.000000
s131e30                       1.000000
s132e11                       1.000000
s133e9                        1.000000
s134e19                       1.000000
s135e20                       1.000000
s136e37                       1.000000
s137e8                        1.000000
s138e49                       1.000000
s139e41                       1.000000
s140e3                        1.000000
s141e1                        1.000000
s142e23                       1.000000
s143e48                       1.000000
s144e22                       1.000000
s145e21                       1.000000
s146e10                       1.000000
s147e14                       1.000000
s148e2                        1.000000
s149e18                       1.000000
s150e15                       1.000000
s151e36                       1.000000
s152e12                       1.000000
s153e24                       1.000000
s154e46                       1.000000
s155e13                       1.000000
s156e6                        1.000000
s157e47                       1.000000
s158e5                        1.000000
s159e15                       1.000000
s160e40                       1.000000
s161e5                        1.000000
s162e2                        1.000000
s163e34                       1.000000
s164e35                       1.000000
s165e9                        1.000000
s166e50                       1.000000
s167e11                       1.000000
s168e41                       1.000000
s169e32                       1.000000
s170e19                       1.000000
s171e43                       1.000000
s172e25                       1.000000
s173e7                        1.000000
s174e49                       1.000000
s175e30                       1.000000
s176e8                        1.000000
s177e31                       1.000000
s178e14                       1.000000
```

```
s179e33                       1.000000
s180e20                       1.000000
s181e27                       1.000000
s182e24                       1.000000
s183e33                       1.000000
s184e3                        1.000000
s185e9                        1.000000
s186e34                       1.000000
s187e15                       1.000000
s188e16                       1.000000
s189e17                       1.000000
s190e44                       1.000000
s191e39                       1.000000
s192e7                        1.000000
s193e26                       1.000000
s194e42                       1.000000
s195e49                       1.000000
m2                            1.000000
m3                            1.000000
m5                            1.000000
m8                            1.000000
m10                           1.000000
m11                           1.000000
m14                           1.000000
m18                           1.000000
m19                           1.000000
m21                           1.000000
m24                           1.000000
m25                           1.000000
m26                           1.000000
m29                           1.000000
m31                           1.000000
m35                           1.000000
m37                           1.000000
m39                           1.000000
m43                           1.000000
m47                           1.000000
m50                           1.000000
t2                            1.000000
t3                            1.000000
t4                            1.000000
t5                            1.000000
t6                            1.000000
t9                            1.000000
t10                           1.000000
t11                           1.000000
t14                           1.000000
t16                           1.000000
t18                           1.000000
t19                           1.000000
t21                           1.000000
t23                           1.000000
t25                           1.000000
t26                           1.000000
t27                           1.000000
```

```
t28                           1.000000
t30                           1.000000
t31                           1.000000
t32                           1.000000
t33                           1.000000
t37                           1.000000
t39                           1.000000
t40                           1.000000
t41                           1.000000
t42                           1.000000
t43                           1.000000
t45                           1.000000
t46                           1.000000
t47                           1.000000
t48                           1.000000
t49                           1.000000
t50                           1.000000
w1                            1.000000
w3                            1.000000
w6                            1.000000
w7                            1.000000
w8                            1.000000
w9                            1.000000
w10                           1.000000
w11                           1.000000
w12                           1.000000
w13                           1.000000
w14                           1.000000
w15                           1.000000
w17                           1.000000
w18                           1.000000
w22                           1.000000
w25                           1.000000
w26                           1.000000
w28                           1.000000
w30                           1.000000
w32                           1.000000
w33                           1.000000
w34                           1.000000
w35                           1.000000
w36                           1.000000
w37                           1.000000
w39                           1.000000
w40                           1.000000
w41                           1.000000
w42                           1.000000
w43                           1.000000
w44                           1.000000
w46                           1.000000
w48                           1.000000
w49                           1.000000
r2                            1.000000
r4                            1.000000
r6                            1.000000
r7                            1.000000
```

```
r8                            1.000000
r10                           1.000000
r12                           1.000000
r13                           1.000000
r15                           1.000000
r17                           1.000000
r19                           1.000000
r20                           1.000000
r22                           1.000000
r23                           1.000000
r26                           1.000000
r28                           1.000000
r29                           1.000000
r30                           1.000000
r31                           1.000000
r33                           1.000000
r34                           1.000000
r35                           1.000000
r36                           1.000000
r37                           1.000000
r38                           1.000000
r39                           1.000000
r40                           1.000000
r41                           1.000000
r42                           1.000000
r43                           1.000000
r44                           1.000000
r45                           1.000000
r46                           1.000000
r50                           1.000000
f1                            1.000000
f2                            1.000000
f3                            1.000000
f5                            1.000000
f6                            1.000000
f8                            1.000000
f9                            1.000000
f10                           1.000000
f11                           1.000000
f12                           1.000000
f13                           1.000000
f14                           1.000000
f15                           1.000000
f18                           1.000000
f19                           1.000000
f20                           1.000000
f21                           1.000000
f22                           1.000000
f23                           1.000000
f24                           1.000000
f28                           1.000000
f30                           1.000000
f31                           1.000000
f34                           1.000000
f35                           1.000000
```

```
f36                           1.000000
f37                           1.000000
f38                           1.000000
f41                           1.000000
f44                           1.000000
f46                           1.000000
f47                           1.000000
f48                           1.000000
f49                           1.000000
f50                           1.000000
s2                            1.000000
s5                            1.000000
s7                            1.000000
s8                            1.000000
s9                            1.000000
s11                           1.000000
s14                           1.000000
s15                           1.000000
s19                           1.000000
s20                           1.000000
s24                           1.000000
s25                           1.000000
s27                           1.000000
s30                           1.000000
s31                           1.000000
s32                           1.000000
s33                           1.000000
s34                           1.000000
s35                           1.000000
s40                           1.000000
s41                           1.000000
s43                           1.000000
s49                           1.000000
s50                           1.000000
n3                            1.000000
n7                            1.000000
n9                            1.000000
n15                           1.000000
n16                           1.000000
n17                           1.000000
n26                           1.000000
n33                           1.000000
n34                           1.000000
n39                           1.000000
n42                           1.000000
n44                           1.000000
n49                           1.000000
All other variables in the range 1-10100 are zero.
-------------------------------------------------------------------------
```

APPENDIX C

CHAPTER FOUR VISUAL BASIC CODE

```
Private Sub CommandButton1_Click()

'This subroutine finds and writes the preference for the employees
'on the Employees page

'Shane Knighton 06-09-05

'Revised 07-13-05 to contain weekend shift restrictions

'Dimension counting varialbes for looping
Dim i, j, k As Integer

'PROBLEM DATA:  RETRIEVED FROM THE WORKSHEETS

'SHIFTS WORKSHEET
'Get number of shifts, days, and weeks to schedule
Dim NumShifts As Integer
NumShifts = Sheets("Shifts").Cells(2, 6).Value

'EMPLOYEE WORKSHEET
'Get number of employees
Dim NumEmp As Integer
NumEmp = Sheets("Employees").Cells(4, 2)
'Get the employee ranking deviation and the deviation within each
'employee's shifts
Dim EmpDevPer, EmpShiftDevPer As Single
EmpDevPer = Sheets("Employees").Cells(4, 19)
EmpShiftDevPer = Sheets("Employees").Cells(5, 23)

'Calculate the rankings
Dim HighRank, EmpDev, EmpShiftDev As Single
HighRank = NumEmp * 100
EmpDev = HighRank * EmpDevPer
EmpShiftDev = EmpDev * EmpShiftDevPer

'Fill in the employee preferences on the "Employee" sheet
Dim rank As Single
Dim putrow As Integer
Range(Cells(10, 4), Cells(9 + NumShifts, 4 + NumEmp)).Clear
For j = 1 To NumEmp
   For i = 1 To NumShifts
      rank = HighRank - EmpDev * (j - 1)
      putrow = Int((NumShifts * Rnd()))
      If putrow < 1 Then putrow = 126
      Sheets("Employees").Cells(9 + putrow, 3 + j).Value = rank + EmpShiftDev *
(i - 1)
   Next i
Next j

End Sub

Private Sub CommandButton2_Click()
'This subroutine will write out in CPLEX .LP format
'a LP for solving large crew/nurse scheduling problems
'using a network-based linear program.  The network is designed
'to do tour scheduling problems.
'Shane Knighton  6-09-05
'Rev 06-10-5


'Dimension counting varialbes for looping
Dim i, j, k As Integer

'PROBLEM DATA:  RETRIEVED FROM THE WORKSHEETS

'SHIFTS WORKSHEET
'Get number of shifts, days, and weeks to schedule
Dim NumShifts, NumShiftsPerDay, NumDaysPerWeek, NumWeeks As Integer
NumShiftsPerDay = Sheets("Shifts").Cells(3, 2).Value
NumDaysPerWeek = Sheets("Shifts").Cells(5, 2).Value
NumWeeks = Sheets("Shifts").Cells(2, 4).Value
NumShifts = Sheets("Shifts").Cells(2, 6).Value
'Min and Max shifts per week per employee
Dim MaxNumShifts, MinNumShifts As Integer
MaxNumShifts = Sheets("Shifts").Cells(4, 4).Value
MinNumShifts = Sheets("Shifts").Cells(5, 4).Value
'ShiftPref is a NumShift by 1 matrix that stores
'the preference for each shift to be selected
'to cover the hourly demand from the HourDemMatrix
Dim ShiftPref() As Single
ReDim ShiftPref(1 To NumShifts)
For i = 1 To NumShifts
   ShiftPref(i) = Sheets("Shifts").Cells(9 + i, 7).Value
Next i
'Find the shift starting hour and length
Dim ShiftStartHour() As Integer
ReDim ShiftStartHour(1 To NumShifts, 1 To NumShifts) As Integer
For i = 1 To NumShifts
```

```
   ShiftStartHour(i, 1) = Sheets("Shifts").Cells(9 + i, 3).Value
   ShiftStartHour(i, 2) = Sheets("Shifts").Cells(9 + i, 5).Value
Next i
'Get the length of the lunch break
Dim LunchLen As Integer
LunchLen = Sheets("Shifts").Cells(5, 6).Value
'Get the demand for each shift
Dim ShiftDemand() As Integer
ReDim ShiftDemand(1 To NumShifts) As Integer
For i = 1 To NumShifts
   ShiftDemand(i) = Sheets("Shifts").Cells(9 + i, 8).Value
Next i


'CONFLICTS WORKSHEET
'Get the maximum number of conflicted shifts
Dim MaxNumConfl As Integer
MaxNumConfl = Sheets("Conflicts").Cells(5, 2).Value
'ConflictMatrix is a NumShift by maximum number of conflicts
'matrix that has the shift id of a conflicted shift in row of
'each shift that has any conflicted shifts.  The row of the matrix
'is empty if the shift id corresponding to that shift does not
'have any conflicted shifts.
Dim ConflictMatrix() As Integer
ReDim ConflictMatrix(1 To NumShifts, 1 To MaxNumConfl) As Integer
'Populate ConflictMatrix
For i = 1 To NumShifts
   If Sheets("Conflicts").Cells(9 + i, 2) <> 0 Then
      For j = 1 To MaxNumConfl
         ConflictMatrix(i, j) = Sheets("Conflicts").Cells(9 + i, 1 + j)
      Next j
   End If
Next i

'EMPLOYEE WORKSHEET
'Get number of employees
Dim NumEmp As Integer
NumEmp = Sheets("Employees").Cells(4, 2)
'PriorityMatrix is a NumShifts by NumEmployee matrix
'containing the priority of each shift for each employee
Dim PriorityMatrix() As Single
ReDim PriorityMatrix(1 To NumShifts, 1 To NumEmp) As Single
'Populate PriorityMatirix
For i = 1 To NumShifts
```

```
    For j = 1 To NumEmp
        PriorityMatrix(i, j) = Sheets("Employees").Cells(9 + i, 3 + j)
    Next j
Next i

'OPEN FILE TO WRITE .LP TO
'Get Name of Problem
Dim Name, filepath As String
Name = "w" & NumWeeks & "e" & NumEmp
'Open a file to write to
filepath = "C:\Documents and Settings\Shane\My Documents\Dissertation\Phase
4\" & Name & ".lp"
Open filepath For Output As #1


'*********************************

'BEGIN WRITE LP TO FILE

'WRITE HEADER AND OBJECTIVE FUNCTION
'VarName is the concatonized shift and employee variable
Print #1, "MAXIMIZE"
Print #1, "Z:"

'Write employee preferences into objective function
Dim varName As String
Dim EmpAvailable As Boolean
Dim dum1 As Integer
'i is the loop for shifts
For i = 1 To NumShifts
    'j is the loop for employees
    For j = 1 To NumEmp
        'Check if variable should be included
        'If preference is zero then employee is unavailable
        'and variable should not be in model
        If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
        'Skip writing the variable to the file if EmpAvailable is false
        If EmpAvailable = True Then
            'Concatenate variable name
            varName = "s" & i & "e" & j
            'Write variables and cost coefficients to file
            'Write final variable and cost to file, leave off the "+"
            If i = NumShifts And j = NumEmp Then
                Print #1, PriorityMatrix(i, j) & " " & varName
```

```
         Else
            Print #1, PriorityMatrix(i, j) & " " & varName & " + "
         End If
      End If
      'If the final employee is unavailable need to write the ending to file
      'use a dummy variable to keep formatting
      If EmpAvailable = False And i = NumShifts And j = NumEmp Then
         Print #1, " dum1"
      End If
      'Set availability to true
      EmpAvailable = True
   Next j
Next i

'WRITE CONSTRAINTS
'Header
Print #1, "SUBJECT TO"
'Constraint number counter, needs to be double because
'of the large number of constraints
Dim ConstNum As Double
ConstNum = 1

'Define the 9 sets of constraints
' i = loop for shifts
' j = loop for employees
' k = loop for weeks
' l = loop for days
'Set1: shifts = demand for that shift determined in Set 0
'      s(i)e(1) + ... + s(i)e(NumEmp) = ShiftDemand(i)
'Set2: total shifts per employee for each day
'      of each week
'      will be required to be less than one
'      by Set 5, needed to total number
'      of shifts per week
'      s(i)e(j) + ... + s(NumShiftsPerDay)e(j) = e(j)w(k)d(l)
'Set3: each employee has less than max number of shifts per week
'      e(j)w(k)d(l) + ... + e(j)w(k)d(NumDaysPerWeek) <= MaxNumShifts
'Set4: each employee has more than min number of shifts per week
'      e(j)w(k)d(l) + ... + e(j)w(k)d(NumDaysPerWeek) >= MinNumShifts
'Set5: only one shift per day per employee
'      e(j)w(k)d(l) <= 1
'Set6: shift confliction for each employee
'      example if shifts 25 and 26 are conflicted
'      s025e(j) + s026e(j) <= 1
```

```
'Set7: each employee has less than max number of weekend shifts
'      e(j)w(k)d(6)+ e(j)w(k)d(7) + ... + e(j)w(k+4)d(6) + e(j)w(k)d(7)
<=MaxWeekendShifts

'Write Set1 constraints
Dim dum2 As Integer
'i is the loop for shifts
For i = 1 To NumShifts
   'Write constraint lable
   Print #1, "c" & ConstNum & ":"
   'j is the loop for employees
   For j = 1 To NumEmp
      'Check if variable should be included
      'If preference is zero then employee is unavailable
      'and variable should not be in model
      If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
      'Skip writing the variable to the file if EmpAvailable is false
      If EmpAvailable = True Then
         'Concatenate variable name
         varName = "s" & i & "e" & j
         'Write variables and coefficients to file, leave off "+" for final emp
         If j = NumEmp Then
            Print #1, varName & " = " & ShiftDemand(i)
         Else
            Print #1, varName & " + "
         End If
      End If
      'If the final employee is unavailable need to write the = ShiftDemand to file
      'use a dummy variable to keep formatting
      If EmpAvailable = False And j = NumEmp Then
         Print #1, " dum2 = " & ShiftDemand(i)
      End If
      'Set availability to true
      EmpAvailable = True
   Next j
   ConstNum = ConstNum + 1
Next i

'Write Set2 constraints
Dim DayWeekVarName As String
Dim dum3 As Integer
'Initialize i, day and week
i = 1
k = 1
```

```
l = 1
'i is the loop for shifts
Do While i < NumShifts
    'j is the loop for employees
    For j = 1 To NumEmp
        'Write constraint lable
        Print #1, "c" & ConstNum & ":"
        'ii is the nested loop for shifts
        For ii = i To i + (NumShiftsPerDay - 1)
            'Check if variable should be included
            'If preference is zero then employee is unavailable
            'and variable should not be in model
            If PriorityMatrix(ii, j) = 0 Then EmpAvailable = False
            'Skip writing the variable to the file if EmpAvailable is false
            If EmpAvailable = True Then
                'Concatenate variable names
                varName = "s" & ii & "e" & j
                DayWeekVarName = "em" & j & "w" & k & "d" & l
                'Write variables and coefficients to file
                If ii = i + (NumShiftsPerDay - 1) Then
                    Print #1, varName & " - " & DayWeekVarName & " = 0"
                Else
                    Print #1, varName & " + "
                End If
            End If
            'If the final employee is unavailable need to write the = ShiftDemand to file
            'use a dummy variable to keep formatting
            If EmpAvailable = False And ii = i + (NumShiftsPerDay - 1) Then
                Print #1, " dum3 - " & DayWeekVarName & " = 0"
            End If
            'Set availability to true
            EmpAvailable = True
        Next ii
        ConstNum = ConstNum + 1
    Next j
    i = i + NumShiftsPerDay
    If l < NumDaysPerWeek Then
        l = l + 1
    ElseIf l = NumShiftsPerDay Then
        k = k + 1
        l = 1
    End If
Loop 'End loop for Set2 Constraints
```

```
'Write Set3 constraints
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
    For k = 1 To NumWeeks
        'l is loop for days
        For l = 1 To NumDaysPerWeek
            DayWeekVarName = "em" & j & "w" & k & "d" & l
            If l = NumDaysPerWeek Then
                Print #1, DayWeekVarName & " <= " & MaxNumShifts
            Else
                Print #1, DayWeekVarName & " + "
            End If
        Next l
    Next k
    ConstNum = ConstNum + 1
Next j

'Write Set4 constraints
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
    For k = 1 To NumWeeks
        'l is loop for days
        For l = 1 To NumDaysPerWeek
            DayWeekVarName = "em" & j & "w" & k & "d" & l
            If l = NumDaysPerWeek Then
                Print #1, DayWeekVarName & " >= " & MinNumShifts
            Else
                Print #1, DayWeekVarName & " + "
            End If
        Next l
    Next k
    ConstNum = ConstNum + 1
Next j

'Write Set5 constraints
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
```

```
   For k = 1 To NumWeeks
      'l is loop for days
      For l = 1 To NumDaysPerWeek
         DayWeekVarName = "em" & j & "w" & k & "d" & l
         Print #1, DayWeekVarName & " <= 1"
      Next l
   Next k
   ConstNum = ConstNum + 1
Next j

'Write Set6 constraints
For i = 1 To NumShifts
   For k = 1 To MaxNumConfl
      If ConflictMatrix(i, k) <> 0 Then
         For j = 1 To NumEmp
            'Check if variable should be included
            'If preference is zero for either shift then employee is unavailable
            'and variable should not be in model
            If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
            If PriorityMatrix(i, k) = 0 Then EmpAvailable = False
            'Skip writing the constraint to the file if EmpAvailable is false
            If EmpAvailable = True Then
                  Print #1, "c" & ConstNum & ":"
                  Print #1, "s" & i & "e" & j & " + s" & ConflictMatrix(i, k) & "e" & j & "
<= 1"
                  ConstNum = ConstNum + 1
            End If
            'Set availability to true
            EmpAvailable = True
         Next j
      End If
   Next k
Next i

'Write Set7 constraints
'Get maximum number of weekend shifts per month
Dim NumWeekendShifts, NumMonths, m As Integer
NumWeekendShifts = Sheets("Shifts").Cells(4, 8).Value
'j is loop for employees
For j = 1 To NumEmp
   Print #1, "c" & ConstNum & ":"
   'k is loop for weeks
   'm is loop for months
   NumMonths = Int(NumWeeks / 4)
   For m = 1 To NumMonths
   For k = 1 To 4
      'l is loop for days
      For l = 6 To 7
         If l = 6 Then DayWeekVarName = "em" & j & "w" & m * k & "d" & (m * k *
7) - 1
         If l = 7 Then DayWeekVarName = "em" & j & "w" & m * k & "d" & (m * k *
7)
         If k * l = 4 * 7 Then
            Print #1, DayWeekVarName & " <= " & NumWeekendShifts
         Else
            Print #1, DayWeekVarName & " + "
         End If
      Next l
   Next k
   Next m
   ConstNum = ConstNum + 1
Next j



'Set all dummy variables to 0
Print #1, "dum1 = 0"
Print #1, "dum2 = 0"
Print #1, "dum3 = 0"

'Write END
Print #1, "END"
'Close mps file
Close #1

Exit Sub

ErrorHandler:
MsgBox ("Error")
'Close mps file
Close #1

End Sub

Private Sub CommandButton3_Click()

'This subroutine will write out in CPLEX .LP format
'a LP for solving large crew/nurse scheduling problems
```

```
'using a network-based linear program.  The network is designed
'to do tour scheduling problems.
'Shane Knighton  6-09-05
'Rev 06-10-5 - Writes integer constraints


'Dimension counting varialbes for looping
Dim i, j, k As Integer

'PROBLEM DATA:  RETRIEVED FROM THE WORKSHEETS

'SHIFTS WORKSHEET
'Get number of shifts, days, and weeks to schedule
Dim NumShifts, NumShiftsPerDay, NumDaysPerWeek, NumWeeks As Integer
NumShiftsPerDay = Sheets("Shifts").Cells(3, 2).Value
NumDaysPerWeek = Sheets("Shifts").Cells(5, 2).Value
NumWeeks = Sheets("Shifts").Cells(2, 4).Value
NumShifts = Sheets("Shifts").Cells(2, 6).Value
'Min and Max shifts per week per employee
Dim MaxNumShifts, MinNumShifts As Integer
MaxNumShifts = Sheets("Shifts").Cells(4, 4).Value
MinNumShifts = Sheets("Shifts").Cells(5, 4).Value
'ShiftPref is a NumShift by 1 matrix that stores
'the preference for each shift to be selected
'to cover the hourly demand from the HourDemMatrix
Dim ShiftPref() As Single
ReDim ShiftPref(1 To NumShifts)
For i = 1 To NumShifts
   ShiftPref(i) = Sheets("Shifts").Cells(9 + i, 7).Value
Next i
'Find the shift starting hour and length
Dim ShiftStartHour() As Integer
ReDim ShiftStartHour(1 To NumShifts, 1 To NumShifts) As Integer
For i = 1 To NumShifts
   ShiftStartHour(i, 1) = Sheets("Shifts").Cells(9 + i, 3).Value
   ShiftStartHour(i, 2) = Sheets("Shifts").Cells(9 + i, 5).Value
Next i
'Get the length of the lunch break
Dim LunchLen As Integer
LunchLen = Sheets("Shifts").Cells(5, 6).Value
'Get the demand for each shift
Dim ShiftDemand() As Integer
ReDim ShiftDemand(1 To NumShifts) As Integer
For i = 1 To NumShifts
```

```
    ShiftDemand(i) = Sheets("Shifts").Cells(9 + i, 8).Value
Next i


'CONFLICTS WORKSHEET
'Get the maximum number of conflicted shifts
Dim MaxNumConfl As Integer
MaxNumConfl = Sheets("Conflicts").Cells(5, 2).Value
'ConflictMatrix is a NumShift by maximum number of conflicts
'matrix that has the shift id of a conflicted shift in row of
'each shift that has any conflicted shifts.  The row of the matrix
'is empty if the shift id corresponding to that shift does not
'have any conflicted shifts.
Dim ConflictMatrix() As Integer
ReDim ConflictMatrix(1 To NumShifts, 1 To MaxNumConfl) As Integer
'Populate ConflictMatrix
For i = 1 To NumShifts
    If Sheets("Conflicts").Cells(9 + i, 2) <> 0 Then
        For j = 1 To MaxNumConfl
            ConflictMatrix(i, j) = Sheets("Conflicts").Cells(9 + i, 1 + j)
        Next j
    End If
Next i

'EMPLOYEE WORKSHEET
'Get number of employees
Dim NumEmp As Integer
NumEmp = Sheets("Employees").Cells(4, 2)
'PriorityMatrix is a NumShifts by NumEmployee matrix
'containing the priority of each shift for each employee
Dim PriorityMatrix() As Single
ReDim PriorityMatrix(1 To NumShifts, 1 To NumEmp) As Single
'Populate PriorityMatirix
For i = 1 To NumShifts
    For j = 1 To NumEmp
        PriorityMatrix(i, j) = Sheets("Employees").Cells(9 + i, 3 + j)
    Next j
Next i

'OPEN FILE TO WRITE .LP TO
'Get Name of Problem
Dim Name, filepath As String
Name = "w" & NumWeeks & "e" & NumEmp & "ip"
'Open a file to write to
```

```
filepath = "C:\Documents and Settings\Shane\My Documents\Dissertation\Phase
1\New Coding\" & Name & ".lp"
Open filepath For Output As #1


'*************************************

'BEGIN WRITE LP TO FILE

'WRITE HEADER AND OBJECTIVE FUNCTION
'VarName is the concatonized shift and employee variable
Print #1, "MAXIMIZE"
Print #1, "Z:"

'Write employee preferences into objective function
Dim varName As String
Dim EmpAvailable As Boolean
Dim dum1 As Integer
'i is the loop for shifts
For i = 1 To NumShifts
   'j is the loop for employees
   For j = 1 To NumEmp
      'Check if variable should be included
      'If preference is zero then employee is unavailable
      'and variable should not be in model
      If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
      'Skip writing the variable to the file if EmpAvailable is false
      If EmpAvailable = True Then
         'Concatenate variable name
         varName = "s" & i & "e" & j
         'Write variables and cost coefficients to file
         'Write final variable and cost to file, leave off the "+"
         If i = NumShifts And j = NumEmp Then
            Print #1, PriorityMatrix(i, j) & " " & varName
         Else
            Print #1, PriorityMatrix(i, j) & " " & varName & " + "
         End If
      End If
      'If the final employee is unavailable need to write the ending to file
      'use a dummy variable to keep formatting
      If EmpAvailable = False And i = NumShifts And j = NumEmp Then
         Print #1, " dum1"
      End If
      'Set availability to true
```

```
        EmpAvailable = True
    Next j
Next i

'WRITE CONSTRAINTS
'Header
Print #1, "SUBJECT TO"
'Constraint number counter, needs to be double because
'of the large number of constraints
Dim ConstNum As Double
ConstNum = 1

'Define the 9 sets of constraints
' i = loop for shifts
' j = loop for employees
' k = loop for weeks
' l = loop for days
'Set1: shifts = demand for that shift determined in Set 0
'       s(i)e(1) + ... + s(i)e(NumEmp) = ShiftDemand(i)
'Set2: total shifts per employee for each day
'       of each week
'       will be required to be less than one
'       by Set 5, needed to total number
'       of shifts per week
'       s(i)e(j) + ... + s(NumShiftsPerDay)e(j) = e(j)w(k)d(l)
'Set3: each employee has less than max number of shifts per week
'       e(j)w(k)d(l) + ... + e(j)w(k)d(NumDaysPerWeek) <= MaxNumShifts
'Set4: each employee has more than min number of shifts per week
'       e(j)w(k)d(l) + ... + e(j)w(k)d(NumDaysPerWeek) >= MinNumShifts
'Set5: only one shift per day per employee
'       e(j)w(k)d(l) <= 1
'Set6: shift confliction for each employee
'       example if shifts 25 and 26 are conflicted
'       s025e(j) + s026e(j) <= 1
'Set7: each employee has less than max number of weekend shifts
'       e(j)w(k)d(6)+ e(j)w(k)d(7) + ... + e(j)w(k+4)d(6) + e(j)w(k)d(7)
<=MaxWeekendShifts

'Write Set1 constraints
Dim dum2 As Integer
'i is the loop for shifts
For i = 1 To NumShifts
    'Write constraint lable
    Print #1, "c" & ConstNum & ":"
    'j is the loop for employees
    For j = 1 To NumEmp
        'Check if variable should be included
        'If preference is zero then employee is unavailable
        'and variable should not be in model
        If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
        'Skip writing the variable to the file if EmpAvailable is false
        If EmpAvailable = True Then
            'Concatenate variable name
            varName = "s" & i & "e" & j
            'Write variables and coefficients to file, leave off "+" for final emp
            If j = NumEmp Then
                Print #1, varName & " = " & ShiftDemand(i)
            Else
                Print #1, varName & " + "
            End If
        End If
        'If the final employee is unavailable need to write the = ShiftDemand to file
        'use a dummy variable to keep formatting
        If EmpAvailable = False And j = NumEmp Then
            Print #1, " dum2 = " & ShiftDemand(i)
        End If
        'Set availability to true
        EmpAvailable = True
    Next j
    ConstNum = ConstNum + 1
Next i

'Write Set2 constraints
Dim DayWeekVarName As String
Dim dum3 As Integer
'Initialize i, day and week
i = 1
k = 1
l = 1
'i is the loop for shifts
Do While i < NumShifts
    'j is the loop for employees
    For j = 1 To NumEmp
        'Write constraint lable
        Print #1, "c" & ConstNum & ":"
        'ii is the nested loop for shifts
        For ii = i To i + (NumShiftsPerDay - 1)
            'Check if variable should be included
            'If preference is zero then employee is unavailable
            'and variable should not be in model
            If PriorityMatrix(ii, j) = 0 Then EmpAvailable = False
            'Skip writing the variable to the file if EmpAvailable is false
            If EmpAvailable = True Then
                'Concatenate variable names
                varName = "s" & ii & "e" & j
                DayWeekVarName = "em" & j & "w" & k & "d" & l
                'Write variables and coefficients to file
                If ii = i + (NumShiftsPerDay - 1) Then
                    Print #1, varName & " - " & DayWeekVarName & " = 0"
                Else
                    Print #1, varName & " + "
                End If
            End If
            'If the final employee is unavailable need to write the = ShiftDemand to
file
            'use a dummy variable to keep formatting
            If EmpAvailable = False And ii = i + (NumShiftsPerDay - 1) Then
                Print #1, " dum3 - " & DayWeekVarName & " = 0"
            End If
            'Set availability to true
            EmpAvailable = True
        Next ii
        ConstNum = ConstNum + 1
    Next j
    i = i + NumShiftsPerDay
    If l < NumDaysPerWeek Then
        l = l + 1
    ElseIf l = NumShiftsPerDay Then
        k = k + 1
        l = 1
    End If
Loop 'End loop for Set2 Constraints

'Write Set3 constraints
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
    For k = 1 To NumWeeks
        'l is loop for days
        For l = 1 To NumDaysPerWeek
            DayWeekVarName = "em" & j & "w" & k & "d" & l
            If l = NumDaysPerWeek Then
                Print #1, DayWeekVarName & " <= " & MaxNumShifts
            Else
                Print #1, DayWeekVarName & " + "
            End If
        Next l
    Next k
    ConstNum = ConstNum + 1
Next j

'Write Set4 constraints
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
    For k = 1 To NumWeeks
        'l is loop for days
        For l = 1 To NumDaysPerWeek
            DayWeekVarName = "em" & j & "w" & k & "d" & l
            If l = NumDaysPerWeek Then
                Print #1, DayWeekVarName & " >= " & MinNumShifts
            Else
                Print #1, DayWeekVarName & " + "
            End If
        Next l
    Next k
    ConstNum = ConstNum + 1
Next j

'Write Set5 constraints
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
    For k = 1 To NumWeeks
        'l is loop for days
        For l = 1 To NumDaysPerWeek
            DayWeekVarName = "em" & j & "w" & k & "d" & l
            Print #1, DayWeekVarName & " <= 1"
        Next l
    Next k
    ConstNum = ConstNum + 1
Next j
```

```
'Write Set6 constraints
For i = 1 To NumShifts
    For k = 1 To MaxNumConfl
        If ConflictMatrix(i, k) <> 0 Then
            For j = 1 To NumEmp
                'Check if variable should be included
                'If preference is zero for either shift then employee is unavailable
                'and variable should not be in model
                If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
                If PriorityMatrix(i, k) = 0 Then EmpAvailable = False
                'Skip writing the constraint to the file if EmpAvailable is false
                If EmpAvailable = True Then
                        Print #1, "c" & ConstNum & ":"
                        Print #1, "s" & i & "e" & j & " + s" & ConflictMatrix(i, k) & "e" & j & "
<= 1"
                        ConstNum = ConstNum + 1
                End If
                'Set availability to true
                EmpAvailable = True
            Next j
        End If
    Next k
Next i

'Write Set7 constraints
'Get maximum number of weekend shifts per month
Dim NumWeekendShifts, NumMonths, m As Integer
NumWeekendShifts = Sheets("Shifts").Cells(4, 8).Value
'j is loop for employees
For j = 1 To NumEmp
    Print #1, "c" & ConstNum & ":"
    'k is loop for weeks
    'm is loop for months
    NumMonths = Int(NumWeeks / 4)
    For m = 1 To NumMonths
    For k = 1 To 4
        'l is loop for days
        For l = 6 To 7
            If l = 6 Then DayWeekVarName = "em" & j & "w" & m * k & "d" & (m * k *
7) - 1
            If l = 7 Then DayWeekVarName = "em" & j & "w" & m * k & "d" & (m * k *
7)
            If k * l = 4 * 7 Then
                Print #1, DayWeekVarName & " <= " & NumWeekendShifts
            Else
               Print #1, DayWeekVarName & " + "
            End If
         Next l
      Next k
      Next m
      ConstNum = ConstNum + 1
   Next j

   'Set all dummy variables to 0
   Print #1, "dum1 = 0"
   Print #1, "dum2 = 0"
   Print #1, "dum3 = 0"


   'Write Integer requirements
   Print #1, "INTEGER"
   For i = 1 To NumShifts
      For j = 1 To NumEmp
         'Check if variable should be included
         'If preference is zero for then employee is unavailable
         'and variable should not be in model
         If PriorityMatrix(i, j) = 0 Then EmpAvailable = False
         'Skip writing the variable to the file if EmpAvailable is false
         If EmpAvailable = True Then
            Print #1, "s" & i & "e" & j
         End If
         'Set availability to true
         EmpAvailable = True
      Next j
   Next i


   'Write END
   Print #1, "END"
   'Close mps file
   Close #1

   Exit Sub

   ErrorHandler:
   MsgBox ("Error")
   'Close mps file
   Close #1
```

```
End Sub
```

APPENDIX D

CHAPTER FOUR CPLEX OUTPUT

```
Log started (V9.1.0) Sun Jul 17 18:29:45 2005


Problem 'w1e20.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
LP Presolve eliminated 850 rows and 331 columns.
Aggregator did 1 substitutions.
Reduced LP has 1077 rows, 935 columns, and 3680 nonzeros.
Presolve time =    0.01 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =        122110.500000
Iteration:    62   Dual objective     =         91007.000000
Iteration:   127   Dual objective     =         85149.500000
Iteration:   193   Dual objective     =         79992.000000

Dual simplex - Optimal:  Objective =    7.9602500000e+04
Solution time =    0.02 sec.  Iterations = 237 (0)


Variable Name           Solution Value
s1e10                         1.000000
s2e6                          1.000000
s3e1                          1.000000
s4e19                         1.000000
s5e3                          1.000000
s6e11                         1.000000
s7e5                          1.000000
s8e14                         1.000000
s9e9                          1.000000
s10e6                         1.000000
s11e10                        1.000000
s12e2                         1.000000
s13e5                         1.000000
s14e12                        1.000000
s15e3                         1.000000
s16e8                         1.000000
s17e20                        1.000000
s18e7                         1.000000
s19e10                        1.000000
s20e12                        1.000000
s21e9                         1.000000
s22e1                         1.000000
s23e7                         1.000000
s24e11                        1.000000
s25e4                         1.000000
s26e6                         1.000000
s27e17                        1.000000
s28e12                        1.000000
s29e13                        1.000000
s30e20                        1.000000
s31e1                         1.000000
s32e9                         1.000000
```

```
s33e18                        1.000000
s34e14                        1.000000
s35e2                         1.000000
s36e15                        1.000000
s37e5                         1.000000
s38e11                        1.000000
s39e10                        1.000000
s40e7                         1.000000
s41e8                         1.000000
s42e15                        1.000000
s43e3                         1.000000
s44e4                         1.000000
s45e1                         1.000000
s46e5                         1.000000
s47e8                         1.000000
s48e4                         1.000000
s49e2                         1.000000
s50e7                         1.000000
s51e16                        1.000000
s52e6                         1.000000
s53e18                        1.000000
s54e11                        1.000000
s55e13                        1.000000
s56e19                        1.000000
s57e16                        1.000000
s58e2                         1.000000
s59e9                         1.000000
s60e3                         1.000000
s61e4                         1.000000
s62e17                        1.000000
s63e8                         1.000000
em1w1d1                       1.000000
em3w1d1                       1.000000
em5w1d1                       1.000000
em6w1d1                       1.000000
em9w1d1                       1.000000
em10w1d1                      1.000000
em11w1d1                      1.000000
em14w1d1                      1.000000
em19w1d1                      1.000000
em2w1d2                       1.000000
em3w1d2                       1.000000
em5w1d2                       1.000000
em6w1d2                       1.000000
em7w1d2                       1.000000
em8w1d2                       1.000000
em10w1d2                      1.000000
em12w1d2                      1.000000
em20w1d2                      1.000000
em1w1d3                       1.000000
em4w1d3                       1.000000
em6w1d3                       1.000000
em7w1d3                       1.000000
em9w1d3                       1.000000
em10w1d3                      1.000000
```

```
em11w1d3                      1.000000
em12w1d3                      1.000000
em17w1d3                      1.000000
em1w1d4                       1.000000
em2w1d4                       1.000000
em9w1d4                       1.000000
em12w1d4                      1.000000
em13w1d4                      1.000000
em14w1d4                      1.000000
em15w1d4                      1.000000
em18w1d4                      1.000000
em20w1d4                      1.000000
em1w1d5                       1.000000
em3w1d5                       1.000000
em4w1d5                       1.000000
em5w1d5                       1.000000
em7w1d5                       1.000000
em8w1d5                       1.000000
em10w1d5                      1.000000
em11w1d5                      1.000000
em15w1d5                      1.000000
em2w1d6                       1.000000
em4w1d6                       1.000000
em5w1d6                       1.000000
em6w1d6                       1.000000
em7w1d6                       1.000000
em8w1d6                       1.000000
em11w1d6                      1.000000
em16w1d6                      1.000000
em18w1d6                      1.000000
em2w1d7                       1.000000
em3w1d7                       1.000000
em4w1d7                       1.000000
em8w1d7                       1.000000
em9w1d7                       1.000000
em13w1d7                      1.000000
em16w1d7                      1.000000
em17w1d7                      1.000000
em19w1d7                      1.000000
All other variables in the range 1-1267 are zero.

Problem 'w1e20ip.lp' read.
Read time =    0.00 sec.
Tried aggregator 1 time.
MIP Presolve eliminated 850 rows and 331 columns.
Aggregator did 1 substitutions.
Reduced MIP has 1077 rows, 935 columns, and 3680 nonzeros.
Presolve time =    0.01 sec.
Clique table members: 1037
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    0.02 sec.

Integer optimal solution:  Objective =    7.9602500000e+04
Solution time =    0.03 sec.  Iterations = 322  Nodes = 0
```

```
Problem 'w2e20.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
LP Presolve eliminated 1601 rows and 744 columns.
Aggregator did 1 substitutions.
Reduced LP has 2269 rows, 1753 columns, and 7434 nonzeros.
Presolve time =    0.01 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =        246189.000000
Iteration:    63   Dual objective     =        214922.000000
Iteration:   163   Dual objective     =        196781.500000
Iteration:   262   Dual objective     =        190105.750000
Iteration:   361   Dual objective     =        184657.500000
Iteration:   449   Dual objective     =        183576.750000

Dual simplex - Optimal:  Objective =    1.8344200000e+05
Solution time =    0.05 sec.  Iterations = 513 (0)


Problem 'w2e20ip.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
MIP Presolve eliminated 1601 rows and 744 columns.
Aggregator did 1 substitutions.
Reduced MIP has 2269 rows, 1753 columns, and 7434 nonzeros.
Presolve time =    0.02 sec.
Clique table members: 2229
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    0.06 sec.

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer     Best Node    ItCnt
Gap         Variable B Parent  Depth

      0     0   183442.0000    71                183442.0000      632
                183409.5000    18                  Cuts:  89      696
                183399.7500    10                  Cuts:  21      746
*     0+    0                   0   183363.0000   183399.7500      746
0.02%
*               183396.5000     0   183396.5000   Cliques:  3      750
0.00%

Clique cuts applied:  24
Gomory fractional cuts applied: 11

Integer optimal solution:  Objective =    1.8339650000e+05
Solution time =    0.17 sec.  Iterations = 750  Nodes = 0



Log started (V9.1.0) Sun Jul 17 17:56:53 2005
```

```
Problem 'w12e20.lp' read.
Read time =    0.10 sec.
Tried aggregator 1 time.
LP Presolve eliminated 9196 rows and 5318 columns.
Aggregator did 4 substitutions.
Reduced LP has 13738 rows, 9634 columns, and 43480 nonzeros.
Presolve time =    0.06 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =      57069303.900000
Iteration:   358   Dual objective     =      52615722.200000
Iteration:   680   Dual objective     =      50621947.200000
Iteration:   993   Dual objective     =      49539532.850000
Iteration:  1306   Dual objective     =      48941165.000000
Iteration:  1585   Dual objective     =      48144221.100000
Iteration:  1892   Dual objective     =      47895102.950000
Iteration:  2152   Dual objective     =      47566514.575000
Iteration:  2412   Dual objective     =      47426345.412500
Iteration:  2659   Dual objective     =      47377069.943056
Iteration:  2882   Dual objective     =      47278573.700000
Iteration:  3101   Dual objective     =      47171073.833333
Iteration:  3328   Dual objective     =      47123625.433333
Iteration:  3519   Dual objective     =      47115675.873810
Iteration:  3705   Dual objective     =      47108769.452050

Dual simplex - Optimal:  Objective =    4.7108769452e+07
Solution time =    1.56 sec.  Iterations = 3705 (0)


Problem 'w12e20ip.lp' read.
Read time =    0.07 sec.
Tried aggregator 1 time.
MIP Presolve eliminated 10442 rows and 5339 columns.
MIP Presolve modified 868 coefficients.
Aggregator did 4 substitutions.
Reduced MIP has 12492 rows, 9613 columns, and 43401 nonzeros.
Presolve time =    0.16 sec.
Clique table members: 12452
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    1.21 sec.

        Nodes                                       Cuts/
   Node  Left     Objective  IInf  Best Integer     Best Node    ItCnt
Gap         Variable B Parent  Depth

      0     0    4.7099e+07   960                 4.7099e+07     4062
                 4.7076e+07   763                 Cuts:  363     4707
*     0+    0                   0    4.7008e+07    4.7076e+07     4707
0.14%
                 4.7073e+07   435    4.7008e+07    Cuts:  252     4947
0.14%
                 4.7072e+07   372    4.7008e+07  Cliques:  14     5007
0.14%
```

```
*     0+    0                   0    4.7018e+07    4.7072e+07     5007
0.12%
*     0+    0                   0    4.7054e+07    4.7072e+07     5007
0.04%
*     0+    0                   0    4.7067e+07    4.7072e+07     5007
0.01%
Elapsed time =  10.12 sec. (tree size =  0.00 MB)

Clique cuts applied:  208
Gomory fractional cuts applied: 45

Integer optimal, tolerance (0.0001/1e-06) :  Objective =
4.7067485000e+07
Current MIP best bound =    4.7072176250e+07 (gap = 4691.25, 0.01%)
Solution time =   10.87 sec.  Iterations = 5072  Nodes = 2 (3)


Problem 'w4e20.lp' read.
Read time =    0.03 sec.
Tried aggregator 1 time.
LP Presolve eliminated 3261 rows and 1705 columns.
Aggregator did 1 substitutions.
Reduced LP has 4573 rows, 3300 columns, and 14604 nonzeros.
Presolve time =    0.01 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =       6323675.000000
Iteration:    88   Dual objective     =       5879590.000000
Iteration:   219   Dual objective     =       5549335.000000
Iteration:   372   Dual objective     =       5385128.333333
Iteration:   507   Dual objective     =       5239922.500000
Iteration:   656   Dual objective     =       5121212.500000
Iteration:   773   Dual objective     =       5105440.833333
Iteration:   898   Dual objective     =       5060940.833333
Iteration:  1010   Dual objective     =       5055779.953704
Iteration:  1110   Dual objective     =       5040283.888889
Iteration:  1207   Dual objective     =       5032525.608974

Dual simplex - Optimal:  Objective =    5.0306182417e+06
Solution time =    0.22 sec.  Iterations = 1294 (0)


Problem 'w4e20ip.lp' read.
Read time =    0.02 sec.
Tried aggregator 1 time.
MIP Presolve eliminated 3261 rows and 1705 columns.
Aggregator did 1 substitutions.
Reduced MIP has 4573 rows, 3300 columns, and 14604 nonzeros.
Presolve time =    0.04 sec.
Clique table members: 4533
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    0.13 sec.

        Nodes                                             Cuts/
```

```
   Node  Left     Objective  IInf  Best Integer     Best Node    ItCnt
Gap         Variable B Parent  Depth

      0     0  5030618.2417   327               5030618.2417     1155
               5027005.3777   271                 Cuts:  269     1424
               5026072.3333   137                 Cuts:  232     1559
               5025847.6250   152               Cliques:  11     1604
               5025731.6667   143                Cliques:  7     1644
*     0+    0                 0  5022965.0000  5025731.6667     1644
0.06%
*     0+    0                 0  5023770.0000  5025731.6667     1644
0.04%
*     0+    0                 0  5024045.0000  5025731.6667     1644
0.03%
*     4     4                 0  5025100.0000  5025690.8333     1700
0.01%          s242e2 D      3      4
*     5     3                 0  5025225.0000  5025690.8333     1707
0.01%          s242e2 U      3      4

Clique cuts applied:  107
Gomory fractional cuts applied: 43

Integer optimal, tolerance (0.0001/1e-06) :  Objective =
5.0252250000e+06
Current MIP best bound =    5.0256908333e+06 (gap = 465.833, 0.01%)
Solution time =    2.14 sec.  Iterations = 1707  Nodes = 6 (3)


Problem 'w52e20.lp' read.
Read time =    0.39 sec.
Tried aggregator 1 time.
LP Presolve eliminated 42798 rows and 25459 columns.
Aggregator did 16 substitutions.
Reduced LP has 59089 rows, 39493 columns, and 183643 nonzeros.
Presolve time =    0.30 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =    1079294544.900000
Iteration:   732   Dual objective     =    1026452718.700001
Iteration:  1208   Dual objective     =    1007896717.350001
Iteration:  1875   Dual objective     =     986069005.450001
Iteration:  2526   Dual objective     =     964575678.600002
Iteration:  3201   Dual objective     =     954946214.700002
Iteration:  3809   Dual objective     =     946444003.000003
Iteration:  4488   Dual objective     =     932469531.550003
Iteration:  5423   Dual objective     =     921571715.100002
Iteration:  6138   Dual objective     =     916259411.625003
Iteration:  6853   Dual objective     =     908584913.466671
Iteration:  7526   Dual objective     =     906427589.795838
Iteration:  8233   Dual objective     =     904335752.500005
Iteration:  8917   Dual objective     =     903488376.490282
Iteration:  9521   Dual objective     =     902800464.221359
Iteration: 10139   Dual objective     =     902357825.588542
Iteration: 10733   Dual objective     =     900171320.041204
```

```
Iteration: 11365   Dual objective     =     897194064.482788
Iteration: 11992   Dual objective     =     896545640.482142
Iteration: 12612   Dual objective     =     896085408.583471
Iteration: 13229   Dual objective     =     895853503.492938
Iteration: 13786   Dual objective     =     895655075.838842
Iteration: 14333   Dual objective     =     895541668.713304
Iteration: 14859   Dual objective     =     895477957.820736
Iteration: 15402   Dual objective     =     895404683.273381
Iteration: 15821   Dual objective     =     894703935.102226
Iteration: 16361   Dual objective     =     894213914.280914
Iteration: 16894   Dual objective     =     893933458.040760

Dual simplex - Optimal:  Objective =    8.9392888657e+08
Solution time =    7.64 sec.  Iterations = 17054 (0)


Problem 'w52e20ip.lp' read.
Read time =    0.43 sec.
Tried aggregator 1 time.
Presolve has eliminated 61033 rows and 27649 columns...
Aggregator has done 16 substitutions...
MIP Presolve eliminated 61033 rows and 27649 columns.
MIP Presolve modified 10959 coefficients.
Aggregator did 16 substitutions.
Reduced MIP has 40854 rows, 37303 columns, and 172474 nonzeros.
Presolve time =    5.12 sec.
Clique table members: 40784
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    7.42 sec.

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer     Best Node    ItCnt
Gap         Variable B Parent  Depth

      0     0    8.8556e+08  4723                  8.8556e+08    14999
                 8.8510e+08  3317                  Cuts:  700    17496
                 8.8502e+08  2379                  Cuts:  395    18476
                 8.8500e+08  2457                Cliques:  66    18778
Heuristic still looking
Heuristic complete
Elapsed time =  73.59 sec. (tree size =  0.00 MB)
*    70+   70                   0    8.8479e+08    8.8500e+08    20269
0.02%
*    80+   80                   0    8.8481e+08    8.8500e+08    20581
0.02%
*    90+   90                   0    8.8482e+08    8.8500e+08    20800
0.02%
*    90+   90                   0    8.8484e+08    8.8500e+08    20800
0.02%
*    94    90                   0    8.8487e+08    8.8500e+08    20852
0.01%         s1958e5 U     93     74
    100    92    8.8498e+08  2378    8.8487e+08    8.8500e+08    21108
0.01%         s2117e4 U     99      4
    200   192    8.8489e+08    77    8.8487e+08    8.8500e+08    23251
0.01%          s503e2 U    199     83
```

```
*   206   197                   0    8.8487e+08    8.8500e+08    23418
0.01%         s532e10 U    205     88
    300   285    8.8491e+08   575    8.8487e+08    8.8500e+08    24822
0.01%         s1669e3 U    299     92
    400   381    8.8489e+08   589    8.8487e+08    8.8500e+08    26540
0.01%        s1759e10 U    399     87
    500   477    8.8490e+08   790    8.8487e+08    8.8500e+08    28618
0.01%         s1711e3 D    499     64
    600   573    8.8492e+08   838    8.8487e+08    8.8499e+08    30343
0.01%         s2559e2 U    599     56
    700   669    8.8494e+08  1199    8.8487e+08    8.8499e+08    32030
0.01%         s2545e1 U    699     58
    800   766    8.8495e+08  1454    8.8487e+08    8.8499e+08    33665
0.01%         s2337e6 D    799     47
*   860   801                   0    8.8488e+08    8.8499e+08    34397
0.01%         s2835e7 U    859    107
*   863   799                   0    8.8488e+08    8.8499e+08    34459
0.01%         s2835e4 D    861    108
    900   834    8.8495e+08  1680    8.8488e+08    8.8499e+08    35461
0.01%         s2529e3 U    899     33
*   960+  813                   0    8.8490e+08    8.8499e+08    36195
0.01%
   1000   848    8.8495e+08  1660    8.8490e+08    8.8499e+08    36945
0.01%         s1692e7 U    999     37
Elapsed time = 203.34 sec. (tree size = 15.30 MB)
*  1000+  722                   0    8.8491e+08    8.8499e+08    36945
0.01%

Clique cuts applied:  713
Gomory fractional cuts applied: 70

Integer optimal, tolerance (0.0001/1e-06) :  Objective =
8.8491464500e+08
Current MIP best bound =    8.8499398452e+08 (gap = 79339.5, 0.01%)
Solution time =  204.15 sec.  Iterations = 36945  Nodes = 1000 (723)
```

APPENDIX E

CHAPTER FIVE VISUAL BASIC CODE

```
Private Sub CommandButton4_Click()

'Will provide the prefernces for rerostering
'Shane Knighton  7-28-05

'Find optimal assignments and increase their preferences by BigM
Dim BigM, i As Integer
BigM = 10000
Dim NumShifts As Integer
NumShifts = Sheets("Shifts").Cells(2, 6).Value

'Get optimal shifts and employee combination
Dim Shift, Emp As Integer

'i is loop for shifts
For i = 1 To NumShifts
   Shift = Mid(Sheets("Employees").Cells(9 + i, 60).Value, 2, 1)
   EmpNumStart = 4
   If Mid(Sheets("Employees").Cells(9 + i, 60).Value, 3, 1) <> "e" Then
      Shift = Shift & Mid(Sheets("Employees").Cells(9 + i, 60).Value, 3, 1)
      EmpNumStart = 5
      If Mid(Sheets("Employees").Cells(9 + i, 60).Value, 4, 1) <> "e" Then
         Shift = Shift & Mid(Sheets("Employees").Cells(9 + i, 60).Value, 4, 1)
         EmpNumStart = 6
      End If
   End If
   Emp = Mid(Sheets("Employees").Cells(9 + i, 60).Value, EmpNumStart, 1)
   If Mid(Sheets("Employees").Cells(9 + i, 60).Value, EmpNumStart + 1, 1) <> ""
Then
      Emp = Emp & Mid(Sheets("Employees").Cells(9 + i, 60).Value,
EmpNumStart + 1, 1)
   End If

'Check
'Sheets("Employees").Cells(9 + i, 59).Value = Shift & " " & Emp

'Add Big M to preferences
Dim OldPref As Double

OldPref = Sheets("Employees").Cells(9 + Shift, 3 + Emp).Value
Sheets("Employees").Cells(9 + Shift, 3 + Emp).Value = OldPref + BigM
Sheets("Employees").Select
Range(Cells(9 + Shift, 3 + Emp), Cells(9 + Shift, 3 + Emp)).Select
   With Selection.Interior
      .ColorIndex = 4
```

```
        .Pattern = xlSolid
    End With

Next i

End Sub

Private Sub CommandButton5_Click()

'Will show the rerostered schedule rerostering
'Shane Knighton  7-28-05


'Get optimal shifts and employee combination
Dim Shift, Emp, i As Integer
Dim NumShifts As Integer
NumShifts = Sheets("Shifts").Cells(2, 6).Value

'i is loop for shifts
For i = 1 To NumShifts
    Shift = Mid(Sheets("Employees").Cells(9 + i, 65).Value, 2, 1)
    EmpNumStart = 4
    If Mid(Sheets("Employees").Cells(9 + i, 65).Value, 3, 1) <> "e" Then
        Shift = Shift & Mid(Sheets("Employees").Cells(9 + i, 65).Value, 3, 1)
        EmpNumStart = 5
        If Mid(Sheets("Employees").Cells(9 + i, 65).Value, 4, 1) <> "e" Then
            Shift = Shift & Mid(Sheets("Employees").Cells(9 + i, 65).Value, 4, 1)
            EmpNumStart = 6
        End If
    End If
    Emp = Mid(Sheets("Employees").Cells(9 + i, 65).Value, EmpNumStart, 1)
    If Mid(Sheets("Employees").Cells(9 + i, 65).Value, EmpNumStart + 1, 1) <> ""
Then
        Emp = Emp & Mid(Sheets("Employees").Cells(9 + i, 65).Value,
EmpNumStart + 1, 1)
    End If

'Check
'Sheets("Employees").Cells(9 + i, 59).Value = Shift & " " & Emp

'Show new optimal

Sheets("Employees").Select
Range(Cells(9 + Shift, 3 + Emp), Cells(9 + Shift, 3 + Emp)).Select
    Selection.Font.Bold = True
    Selection.Borders(xlDiagonalDown).LineStyle = xlNone
    Selection.Borders(xlDiagonalUp).LineStyle = xlNone
    With Selection.Borders(xlEdgeLeft)
        .LineStyle = xlContinuous
        .Weight = xlMedium
        .ColorIndex = xlAutomatic
    End With
    With Selection.Borders(xlEdgeTop)
        .LineStyle = xlContinuous
        .Weight = xlMedium
        .ColorIndex = xlAutomatic
    End With
    With Selection.Borders(xlEdgeBottom)
        .LineStyle = xlContinuous
        .Weight = xlMedium
        .ColorIndex = xlAutomatic
    End With
    With Selection.Borders(xlEdgeRight)
        .LineStyle = xlContinuous
        .Weight = xlMedium
        .ColorIndex = xlAutomatic
    End With

Next i

End Sub
```

APPENDIX F

CHAPTER FIVE CPLEX OUTPUT

```
Log started (V9.1.0) Thu Jul 28 13:04:04 2005


Problem 'w1e20.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
LP Presolve eliminated 905 rows and 343 columns.
Aggregator did 1 substitutions.
Reduced LP has 1079 rows, 942 columns, and 3698 nonzeros.
Presolve time =   -0.00 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =        234369.000000
Iteration:    62   Dual objective     =        176019.000000
Iteration:   125   Dual objective     =        163800.000000
Iteration:   193   Dual objective     =        155394.000000

Dual simplex - Optimal:  Objective =    1.5458100000e+05
Solution time =    0.02 sec.  Iterations = 247 (0)


Variable Name           Solution Value
s1e12                         1.000000
s2e5                          1.000000
s3e3                          1.000000
s4e1                          1.000000
s5e20                         1.000000
s6e4                          1.000000
s7e6                          1.000000
s8e11                         1.000000
s9e14                         1.000000
s10e18                        1.000000
s11e14                        1.000000
s12e8                         1.000000
s13e17                        1.000000
s14e12                        1.000000
s15e2                         1.000000
s16e7                         1.000000
s17e4                         1.000000
s18e1                         1.000000
s19e9                         1.000000
s20e7                         1.000000
s21e6                         1.000000
s22e2                         1.000000
s23e10                        1.000000
s24e4                         1.000000
s25e15                        1.000000
s26e1                         1.000000
s27e3                         1.000000
s28e11                        1.000000
s29e7                         1.000000
s30e16                        1.000000
s31e13                        1.000000
s32e4                         1.000000
```

```
s33e10                        1.000000
s34e3                         1.000000
s35e19                        1.000000
s36e15                        1.000000
s37e16                        1.000000
s38e11                        1.000000
s39e2                         1.000000
s40e5                         1.000000
s41e10                        1.000000
s42e9                         1.000000
s43e13                        1.000000
s44e6                         1.000000
s45e8                         1.000000
s46e3                         1.000000
s47e19                        1.000000
s48e8                         1.000000
s49e10                        1.000000
s50e9                         1.000000
s51e2                         1.000000
s52e20                        1.000000
s53e5                         1.000000
s54e17                        1.000000
s55e18                        1.000000
s56e8                         1.000000
s57e1                         1.000000
s58e9                         1.000000
s59e6                         1.000000
s60e12                        1.000000
s61e11                        1.000000
s62e7                         1.000000
s63e5                         1.000000
em1w1d1                       1.000000
em3w1d1                       1.000000
em4w1d1                       1.000000
em5w1d1                       1.000000
em6w1d1                       1.000000
em11w1d1                      1.000000
em12w1d1                      1.000000
em14w1d1                      1.000000
em20w1d1                      1.000000
em1w1d2                       1.000000
em2w1d2                       1.000000
em4w1d2                       1.000000
em7w1d2                       1.000000
em8w1d2                       1.000000
em12w1d2                      1.000000
em14w1d2                      1.000000
em17w1d2                      1.000000
em18w1d2                      1.000000
em1w1d3                       1.000000
em2w1d3                       1.000000
em3w1d3                       1.000000
em4w1d3                       1.000000
em6w1d3                       1.000000
em7w1d3                       1.000000
```

```
em9w1d3                       1.000000
em10w1d3                      1.000000
em15w1d3                      1.000000
em3w1d4                       1.000000
em4w1d4                       1.000000
em7w1d4                       1.000000
em10w1d4                      1.000000
em11w1d4                      1.000000
em13w1d4                      1.000000
em15w1d4                      1.000000
em16w1d4                      1.000000
em19w1d4                      1.000000
em2w1d5                       1.000000
em5w1d5                       1.000000
em6w1d5                       1.000000
em8w1d5                       1.000000
em9w1d5                       1.000000
em10w1d5                      1.000000
em11w1d5                      1.000000
em13w1d5                      1.000000
em16w1d5                      1.000000
em2w1d6                       1.000000
em3w1d6                       1.000000
em5w1d6                       1.000000
em8w1d6                       1.000000
em9w1d6                       1.000000
em10w1d6                      1.000000
em17w1d6                      1.000000
em19w1d6                      1.000000
em20w1d6                      1.000000
em1w1d7                       1.000000
em5w1d7                       1.000000
em6w1d7                       1.000000
em7w1d7                       1.000000
em8w1d7                       1.000000
em9w1d7                       1.000000
em11w1d7                      1.000000
em12w1d7                      1.000000
em18w1d7                      1.000000
All other variables in the range 1-1286 are zero.


Log started (V9.1.0) Thu Jul 28 13:59:56 2005


Problem 'w1e20s1.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
LP Presolve eliminated 905 rows and 343 columns.
Aggregator did 1 substitutions.
Reduced LP has 1079 rows, 941 columns, and 3696 nonzeros.
Presolve time =   -0.00 sec.
Initializing dual steep norms . . .

Iteration log . . .
```

```
Iteration:     1   Dual objective     =        775949.000000

Dual simplex - Optimal:  Objective =    7.7432900000e+05
Solution time =    0.01 sec.  Iterations = 5 (0)


Variable Name           Solution Value
s1e13                         1.000000
s2e5                          1.000000
s3e3                          1.000000
s4e1                          1.000000
s5e20                         1.000000
s6e4                          1.000000
s7e6                          1.000000
s8e11                         1.000000
s9e14                         1.000000
s10e18                        1.000000
s11e14                        1.000000
s12e8                         1.000000
s13e17                        1.000000
s14e12                        1.000000
s15e2                         1.000000
s16e7                         1.000000
s17e4                         1.000000
s18e1                         1.000000
s19e9                         1.000000
s20e7                         1.000000
s21e6                         1.000000
s22e2                         1.000000
s23e10                        1.000000
s24e4                         1.000000
s25e15                        1.000000
s26e1                         1.000000
s27e3                         1.000000
s28e11                        1.000000
s29e7                         1.000000
s30e16                        1.000000
s31e13                        1.000000
s32e4                         1.000000
s33e10                        1.000000
s34e3                         1.000000
s35e19                        1.000000
s36e15                        1.000000
s37e16                        1.000000
s38e11                        1.000000
s39e2                         1.000000
s40e5                         1.000000
s41e10                        1.000000
s42e9                         1.000000
s43e13                        1.000000
s44e6                         1.000000
s45e8                         1.000000
s46e3                         1.000000
s47e19                        1.000000
s48e8                         1.000000
```

```
s49e10                        1.000000
s50e9                         1.000000
s51e2                         1.000000
s52e20                        1.000000
s53e5                         1.000000
s54e17                        1.000000
s55e18                        1.000000
s56e8                         1.000000
s57e1                         1.000000
s58e9                         1.000000
s59e6                         1.000000
s60e12                        1.000000
s61e11                        1.000000
s62e7                         1.000000
s63e5                         1.000000
em1w1d1                       1.000000
em3w1d1                       1.000000
em4w1d1                       1.000000
em5w1d1                       1.000000
em6w1d1                       1.000000
em11w1d1                      1.000000
em13w1d1                      1.000000
em14w1d1                      1.000000
em20w1d1                      1.000000
em1w1d2                       1.000000
em2w1d2                       1.000000
em4w1d2                       1.000000
em7w1d2                       1.000000
em8w1d2                       1.000000
em12w1d2                      1.000000
em14w1d2                      1.000000
em17w1d2                      1.000000
em18w1d2                      1.000000
em1w1d3                       1.000000
em2w1d3                       1.000000
em3w1d3                       1.000000
em4w1d3                       1.000000
em6w1d3                       1.000000
em7w1d3                       1.000000
em9w1d3                       1.000000
em10w1d3                      1.000000
em15w1d3                      1.000000
em3w1d4                       1.000000
em4w1d4                       1.000000
em7w1d4                       1.000000
em10w1d4                      1.000000
em11w1d4                      1.000000
em13w1d4                      1.000000
em15w1d4                      1.000000
em16w1d4                      1.000000
em19w1d4                      1.000000
em2w1d5                       1.000000
em5w1d5                       1.000000
em6w1d5                       1.000000
em8w1d5                       1.000000
```

```
em9w1d5                       1.000000
em10w1d5                      1.000000
em11w1d5                      1.000000
em13w1d5                      1.000000
em16w1d5                      1.000000
em2w1d6                       1.000000
em3w1d6                       1.000000
em5w1d6                       1.000000
em8w1d6                       1.000000
em9w1d6                       1.000000
em10w1d6                      1.000000
em17w1d6                      1.000000
em19w1d6                      1.000000
em20w1d6                      1.000000
em1w1d7                       1.000000
em5w1d7                       1.000000
em6w1d7                       1.000000
em7w1d7                       1.000000
em8w1d7                       1.000000
em9w1d7                       1.000000
em11w1d7                      1.000000
em12w1d7                      1.000000
em18w1d7                      1.000000
All other variables in the range 1-1285 are zero.

Log started (V9.1.0) Thu Jul 28 14:28:25 2005


Problem 'w1e20s2.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
LP Presolve eliminated 914 rows and 365 columns.
Aggregator did 1 substitutions.
Reduced LP has 1004 rows, 871 columns, and 3417 nonzeros.
Presolve time =    0.01 sec.
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =        742375.000000

Dual simplex - Optimal:  Objective =    7.3161400000e+05
Solution time =    0.01 sec.  Iterations = 34 (0)


Variable Name           Solution Value
s1e15                         1.000000
s2e5                          1.000000
s3e3                          1.000000
s4e1                          1.000000
s5e20                         1.000000
s6e4                          1.000000
s7e6                          1.000000
s8e11                         1.000000
s9e14                         1.000000
s10e18                        1.000000
```

```
s11e14                        1.000000
s12e8                         1.000000
s13e17                        1.000000
s14e13                        1.000000
s15e2                         1.000000
s16e7                         1.000000
s17e4                         1.000000
s18e1                         1.000000
s19e9                         1.000000
s20e7                         1.000000
s21e6                         1.000000
s22e2                         1.000000
s23e10                        1.000000
s24e4                         1.000000
s25e15                        1.000000
s26e1                         1.000000
s27e3                         1.000000
s28e17                        1.000000
s29e7                         1.000000
s30e16                        1.000000
s31e13                        1.000000
s32e4                         1.000000
s33e10                        1.000000
s34e3                         1.000000
s35e19                        1.000000
s36e15                        1.000000
s37e16                        1.000000
s38e14                        1.000000
s39e2                         1.000000
s40e5                         1.000000
s41e10                        1.000000
s42e9                         1.000000
s43e13                        1.000000
s44e6                         1.000000
s45e8                         1.000000
s46e3                         1.000000
s47e19                        1.000000
s48e8                         1.000000
s49e10                        1.000000
s50e9                         1.000000
s51e2                         1.000000
s52e20                        1.000000
s53e5                         1.000000
s54e17                        1.000000
s55e18                        1.000000
s56e8                         1.000000
s57e1                         1.000000
s58e9                         1.000000
s59e6                         1.000000
s60e13                        1.000000
s61e11                        1.000000
s62e7                         1.000000
s63e5                         1.000000
em1w1d1                       1.000000
em3w1d1                       1.000000
```

```
em4w1d1                       1.000000
em5w1d1                       1.000000
em6w1d1                       1.000000
em11w1d1                      1.000000
em14w1d1                      1.000000
em15w1d1                      1.000000
em20w1d1                      1.000000
em1w1d2                       1.000000
em2w1d2                       1.000000
em4w1d2                       1.000000
em7w1d2                       1.000000
em8w1d2                       1.000000
em13w1d2                      1.000000
em14w1d2                      1.000000
em17w1d2                      1.000000
em18w1d2                      1.000000
em1w1d3                       1.000000
em2w1d3                       1.000000
em3w1d3                       1.000000
em4w1d3                       1.000000
em6w1d3                       1.000000
em7w1d3                       1.000000
em9w1d3                       1.000000
em10w1d3                      1.000000
em15w1d3                      1.000000
em3w1d4                       1.000000
em4w1d4                       1.000000
em7w1d4                       1.000000
em10w1d4                      1.000000
em13w1d4                      1.000000
em15w1d4                      1.000000
em16w1d4                      1.000000
em17w1d4                      1.000000
em19w1d4                      1.000000
em2w1d5                       1.000000
em5w1d5                       1.000000
em6w1d5                       1.000000
em8w1d5                       1.000000
em9w1d5                       1.000000
em10w1d5                      1.000000
em13w1d5                      1.000000
em14w1d5                      1.000000
em16w1d5                      1.000000
em2w1d6                       1.000000
em3w1d6                       1.000000
em5w1d6                       1.000000
em8w1d6                       1.000000
em9w1d6                       1.000000
em10w1d6                      1.000000
em17w1d6                      1.000000
em19w1d6                      1.000000
em20w1d6                      1.000000
em1w1d7                       1.000000
em5w1d7                       1.000000
em6w1d7                       1.000000
```

```
em7w1d7                       1.000000
em8w1d7                       1.000000
em9w1d7                       1.000000
em11w1d7                      1.000000
em13w1d7                      1.000000
em18w1d7                      1.000000
em12w1d3                      1.000000
em12w1d4                      1.000000
em12w1d5                      1.000000
em12w1d6                      1.000000
All other variables in the range 1-1237 are zero.

Log started (V9.1.0) Thu Jul 28 14:52:44 2005


Problem 'w1e20s3.lp' read.
Read time =    0.01 sec.
Tried aggregator 1 time.
MIP Presolve eliminated 801 rows and 346 columns.
Aggregator did 1 substitutions.
Reduced MIP has 810 rows, 687 columns, and 2700 nonzeros.
Presolve time =    0.01 sec.
Clique table members: 781
MIP emphasis: balance optimality and feasibility
Root relaxation solution time =    0.01 sec.

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer     Best Node    ItCnt
Gap         Variable B Parent  Depth

      0     0   604668.5000    16                 604668.5000      103
*     0+    0                   0   604598.0000   604668.5000      103
0.01%
*               604658.0000     0   604658.0000    Fract:  16      112
0.00%

Gomory fractional cuts applied: 1

Integer optimal solution:  Objective =    6.0465800003e+05
Solution time =    0.03 sec.  Iterations = 112  Nodes = 0


Variable Name           Solution Value
s1e16                         1.000000
s2e5                          1.000000
s3e3                          1.000000
s4e1                          1.000000
s5e20                         1.000000
s6e4                          1.000000
s7e6                          1.000000
s8e18                         1.000000
s9e14                         1.000000
s10e18                        1.000000
s11e14                        1.000000
s12e8                         1.000000
```

```
s13e17                        1.000000
s14e20                        1.000000
s15e2                         1.000000
s16e7                         1.000000
s17e4                         1.000000
s18e1                         1.000000
s19e9                         1.000000
s20e7                         1.000000
s21e6                         1.000000
s22e2                         1.000000
s23e19                        1.000000
s24e4                         1.000000
s25e15                        1.000000
s26e1                         1.000000
s27e3                         1.000000
s28e17                        1.000000
s29e7                         1.000000
s30e16                        1.000000
s31e9                         1.000000
s32e4                         1.000000
s33e10                        1.000000
s34e3                         1.000000
s35e19                        1.000000
s36e15                        1.000000
s37e16                        1.000000
s38e17                        1.000000
s39e2                         1.000000
s40e5                         1.000000
s41e18                        1.000000
s42e9                         1.000000
s43e19                        1.000000
s44e6                         1.000000
s45e8                         1.000000
s46e3                         1.000000
s47e19                        1.000000
s48e8                         1.000000
s49e15                        1.000000
s50e9                         1.000000
s51e2                         1.000000
s52e20                        1.000000
s53e5                         1.000000
s54e17                        1.000000
s55e18                        1.000000
s56e8                         1.000000
s57e1                         1.000000
s58e15                        1.000000
s59e6                         1.000000
s60e16                        1.000000
s61e20                        1.000000
s62e7                         1.000000
s63e5                         1.000000
em1w1d1                       1.000000
em3w1d1                       1.000000
em4w1d1                       1.000000
em5w1d1                       1.000000
```

```
em6w1d1                       1.000000
em14w1d1                      1.000000
em16w1d1                      1.000000
em18w1d1                      1.000000
em20w1d1                      1.000000
em1w1d2                       1.000000
em2w1d2                       1.000000
em4w1d2                       1.000000
em7w1d2                       1.000000
em8w1d2                       1.000000
em14w1d2                      1.000000
em17w1d2                      1.000000
em18w1d2                      1.000000
em20w1d2                      1.000000
em1w1d3                       1.000000
em2w1d3                       1.000000
em3w1d3                       1.000000
em4w1d3                       1.000000
em6w1d3                       1.000000
em7w1d3                       1.000000
em9w1d3                       1.000000
em15w1d3                      1.000000
em19w1d3                      1.000000
em3w1d4                       1.000000
em4w1d4                       1.000000
em7w1d4                       1.000000
em9w1d4                       1.000000
em10w1d4                      1.000000
em15w1d4                      1.000000
em16w1d4                      1.000000
em17w1d4                      1.000000
em19w1d4                      1.000000
em2w1d5                       1.000000
em5w1d5                       1.000000
em6w1d5                       1.000000
em8w1d5                       1.000000
em9w1d5                       1.000000
em16w1d5                      1.000000
em17w1d5                      1.000000
em18w1d5                      1.000000
em19w1d5                      1.000000
em2w1d6                       1.000000
em3w1d6                       1.000000
em5w1d6                       1.000000
em8w1d6                       1.000000
em9w1d6                       1.000000
em15w1d6                      1.000000
em17w1d6                      1.000000
em19w1d6                      1.000000
em20w1d6                      1.000000
em1w1d7                       1.000000
em5w1d7                       1.000000
em6w1d7                       1.000000
em7w1d7                       1.000000
em8w1d7                       1.000000
```

```
em15w1d7                      1.000000
em16w1d7                      1.000000
em18w1d7                      1.000000
em20w1d7                      1.000000
All other variables in the range 1-1034 are zero.
```