Computational Electromagnetics
AFOSR Contract# F49620-02-1-0052

Final Report

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CO-PIs/SUBCONTRACTORS:

None

OBJECTIVES

The objective of the effort sponsored by the present contract consists of the development of a suite of numerical algorithms, founded on a rigorous mathematical basis, for the accurate and efficient prediction of electromagnetic scattering returns. The techniques are based on integral equations, perturbation theory and high-frequency approximations; general numerical methods will be sought through suitable combinations of these.

The program will advance on several parallel and coordinated fronts with specific objectives, which include:

1) the development of a fast and high-order method for the solution of axisymmetric (BOR) scattering problems. In combination with high-order perturbation schemes, the resulting algorithm will enable efficient studies of a variety of practical configurations (e.g. certain types of ordnance, optical fibers, etc). In addition, the algorithm will be amenable to use as a

module to accelerate general multi-domain scattering codes (e.g. based on FEM, integral equations, etc) by allowing for an effective treatment of symmetric components of an arbitrary scatterer.

2) the development of a fast and high-order method for the solution of general, penetrable scattering problems. To this end, advantage will be taken of recently developed techniques that achieve high-order, fast solutions for configurations with globally smooth material properties. These algorithms, however, display only low (second) order convergence in the presence of material discontinuities. Thus, our objective is to introduce a suitable treatment of these that will result in a simulator with arbitrary, tunable convergence order without compromising speed. This newly designed treatment will be based on an accurate representation of fields scattered by "thin" penetrable bodies (such as those that arise when separating thin volumetric regions near a material discontinuity), and it will therefore also be directly applicable to other specific military interests (e.g. design of stealth coatings, identification of decoys, etc).

3) the development of stable boundary perturbation methods for the rapid evaluation of scattering returns. Here, the objective is to continue the search for high-order, stable, and efficient algorithms that exploit the simplicity of regular perturbation theory. Recent work on the conditioning of such methods will guide the design of new algorithms.

4) the development of general error-controllable high-frequency scattering solvers. The rather ambitious objective of this program is to derive a general purpose code that will allow for the prediction of electromagnetic signatures at arbitrarily large frequencies with prescribed accuracy and using discretizations that are *independent* of frequency, that is, without having to resolve on the scale of a wavelength. In other words, the objective here is to produce an algorithm that combines the virtues of rigorous methods (namely, error control) with those of high-frequency approximations (namely, frequency-independent discretizations). To achieve these goals, the program actually requires effort in both areas: on the one hand, efficient solvers for high-frequency approximate models must be developed to provide necessary high-frequency data and, on the other hand, suitable rigorous scattering solvers must be sought to process it. As explained below, these efforts will be undertaken concurrently.

STATUS OF EFFORT

Referring to the numbering above, accomplishments include:

1) the design and numerical implementation of a new fast integral-equation solver for the solution of (scalar) wave scattering problems in BOR

configurations, and its extension to a simulator that allows for the treatment of *general* (asymmetric) three-dimensional geometries. The implementations are based on the use of the relevant "addition theorems", to achieve a "separation of variables", and on Fast Legendre/Spherical-Harmonics Transforms ---FLT/FSHT--- (in the angular coordinates) and composite Chebyshev integration (in the radial direction). Rigorous error bounds have been established that confirm the observed correlation between the order of convergence of the algorithm and the global regularity of the solution to the (inhomogeneous) scattering problem.
[Work of former student Y. Han, currently a postdoc at Caltech]

2) the development and implementation of two- and three-dimensional (parallel) versions of a fast, high-order stand-alone "thin-volume" scattering solver. As projected, this thin-volume solver attains arbitrary convergence orders through especially designed quadratures in transverse and parallel directions, and it owes its efficiency to a suitable "equivalent-source" accelerator.
[Work of current student A. Anand]
In addition, a strategy for the integration of this approach into a general FFT-based methodology for the treatment of *general* scattering obstacles has been designed, and the two-dimensional version has been fully implemented, tested and parallelized.
[Work of former postdoc M. Hyde, currently an assistant professor at Rice Univ.]

3) several significant developments in boundary perturbation schemes for wave scattering processes. Interestingly, these advances comprise both the popular low-order and the more widely applicable high-order versions. Indeed, guided by previous work on Dirichlet-to-Neumann maps and Calderon commutators, we have designed and implemented new low- and high-order schemes that exhibit a significant improvement in stability properties when compared to classical approaches (particularly in the near field). Moreover, in doing so, we have also provided a mathematical argument that sheds light on the mechanisms that result in superior performance for the specific boundary-variation scheme that we had designed (jointly with O. Bruno) under a previous AFOSR contract when compared with alternative shape-perturbation methods (e.g. the Operator Expansion Method of Milder et al.). In addition, we have also shown that our observations on the stability of these algorithms extend to the full electromagnetic case, general wavenumbers and multi-dimensional perturbation parameters.
[Work of former postdoc D. Nicholls, currently an assistant professor at the Univ. of Notre Dame]

4) significant advances in the design and implementation of new schemes for the simulation of (i) three-dimensional rough-surface scattering at high-

frequencies that greatly improves on the classical Kirchhoff approximation (KA), at a very modest additional cost; and (ii) high-frequency scattering from cylindrical, and general bounded three-dimensional geometries that rigorously accounts for every effect, including shadowing and multiple scattering. In connection with the solvers corresponding to (i) above, their design and implementation have been completed in their most general form, applicable to the vector electromagnetic problem and to general rough surfaces in three space dimensions. As we further explain below, these schemes are based on the observation that an integral equation formulation provides a favorable setting for the recursive evaluation of the terms in an asymptotic series for the surface current in powers of the wavelength, at least in the absence of shadowing effects (as in KA).
[Work of current graduate student C. Turc]
In fact, the more complex developments in (ii) were pursued as an alternative to consideration of fractional powers of the wavelength in an extension of (i) to shadowing configurations. As we have now shown, the newly developed algorithms of (ii) (as those in (i)) can deliver solutions without the need to numerically resolve the wavelength of radiation; in contrast with approximate methods (e.g. KA), however, these new schemes are *error-controllable*, that is, they can provide solutions with a prescribed accuracy at any fixed, finite frequency. The algorithms of (ii) are based on new developments in the treatment of integral equations (to ensure error-controllability) and also in high-frequency asymptotic solutions (to avoid the resolution of the wavelength). With regards to the former developments, a complete scheme has been designed, implemented and analyzed for arbitrary cylindrical configurations (including diffraction and occlusion effects) and the development of its three-dimensional counterpart has been initiated;
[Work of current graduate student F. Ecevit, and postdoc Y. Boubendir]
as for the latter, that is, concerning the solution of the asymptotic (eikonal) equations, we have also developed and implemented a novel spectral/discontinuous Galerkin (DG) method for the accurate evaluation of multi-valued solutions (as necessary for the a-priori determination of the phase of the currents implicit in the strategies for (ii) above).
[Work of former postdocs J. Wang, currently at Vital Images, and J. Qiang, currently a postdoc at UCLA]

ACCOMPLISHMENTS/NEW FINDINGS

As originally proposed and as anticipated above, our work in computational electromagnetics has advanced on several fronts, with an overall goal of developing a computational infrastructure capable of dealing with most configurations that arise in practice. To this end, our program has been closely coordinated with that of Dr. Bruno at Caltech, who will be reporting separately. As a general guideline, we have followed a path of designing the different components of a general computational suite, in a way so that the components are themselves of intrinsic and immediate interest and applicability.

Examples of such components are those described in 1) and 2) above, dealing with BOR and thin-volumes, respectively. Over the course of the present contract we have designed and implemented a fast, high-order solver for the integral equations relevant to the BOR geometry and we have analyzed the results to gain insight into the precise correlation between the convergence properties of the scheme and the global regularity of the underlying solution. The solver is based on the observation that the pertinent integrals can be efficiently evaluated to high-order via FLTs and Chebyshev integration, provided certain "moments" of spherical Bessel functions are precomputed and stored. As for the error estimates, the observation that these estimates are not consistently sharp, has led us to uncover some interesting cancellation effects in the Legendre expansion of the fields, which should be the subject of further investigation. Concurrently with these studies we have further extended the scheme to apply to *general* three-dimensional geometries. A main ingredient to attain such an extension consists of the use of a FSHT that generalizes the FLT used in the BOR arrangement. A (scalar) version of the general three-dimensional solver, relying on an implementation of FSHT based on generalized three-term recurrences and fast interpolation via a one-dimensional FMM, has also been realized.

In connection with thin-volume integrators, as we said, two-and three-dimensional versions have been implemented, tested and parallelized. We recall that we designate a volume to be "thin" if its thickness in a specific (possibly non-uniform) direction is comparable to the wavelength of radiation. Our schemes to treat these rely upon new high-order quadratures that combine a number of elements, including parametric representations of the domains, a careful treatment of corner singularities of the volume potentials in the transverse direction and of "near-singularities" in the tangential direction, and various interpolation procedures; the evaluations are further expedited through an "equivalent-source" calculator that accelerates the computation of interactions between non-adjacent degrees of freedom. Further, as projected, we have also incorporated the resulting algorithm into a larger scheme that allows for the solution of *general* penetrable scattering problems; the two-dimensional version is fully operational and the three-dimensional implementation is currently underway. As we have shown, this solver is a first of its kind as it exhibits arbitrarily high orders of convergence with a computational cost that grows only linearly with problem size.

Work on stabilized versions of shape-deformation schemes, on the other hand, has resulted in the uncovering of additional shortcomings of classical approaches, as well as of promising new algorithms. In particular, we have extended our understanding of the conditioning of classical methods (Milder's

Operator Expansion --OE-- method, Rayleigh-Rice theory, etc) to encompass their typical numerical implementations in Fourier space. In doing so, we have provided a complete explanation for the observed stability of these schemes in the calculation of far- and near- field data, showing that the accuracy that can be achieved in the near-field is even more limited than that attainable in the far-field (and which, in turn, is already severely restricted by the conditioning of the underlying recurrences). Moreover, our analysis further yielded an elucidation for the mechanism behind the empiric superior performance of the specific boundary-variation scheme ("MVB") that we had previously designed (jointly with O. Bruno and under the auspices of a prior AFOSR contract) when compared with alternative shape-perturbation methods (e.g. OE). These observations coupled with our prior work on Dirichlet-to-Neumann operators (which, in fact, appear in the OE formulation), guided us in designing alternative implementations which, by avoiding cancellations, significantly enhance the applicability of these methods. These alternative implementations comprise both low- and high-order instances of perturbative methods, and should therefore appeal to a broad scientific community. In the low-order regime, our proposed approach entails a careful re-arrangement of the terms in the classical recurrences to *explicitly* account for cancellations. The challenges in identifying such cancellations are quite significant but, as we have shown, they *can* be overcome for low-order terms. Our approach to high-order calculations, on the other hand, is radically different as it relies on an *implicit* account of cancellations. As we have demonstrated, such an implicit account can be achieved in more than one way: a first possibility is to effect a change of variables *a-priori* of the derivation of the recursions to collapse the perturbed domains onto the unperturbed geometry. Alternatively (and, in fact, suggested by the first approach), the Dirichlet-to-Neumann operator entering the representation of the fields can be embedded into a family of "Dirichlet-to-Interior Neumann" operators for surfaces that continuously deform the unperturbed boundary to the desired obstacle, instead of embedding it in a family of plain Dirichlet-to-Neumann operators for these surfaces, as is classically done. As we have shown, effecting the differentiation *inside* the domain of definition has a pronounced stabilizing effect.

Finally, a significant portion of our efforts has been devoted to the most ambitious part of our program relating to the design of rigorous solvers for *high-frequency* scattering applications. In this connection, over the course of this contract we have attained significant accomplishments in both (i) the implementation of a new, simple scheme for the simulation of three-dimensional rough-surface scattering at high-frequencies that greatly improves on the classical KA, at a very modest additional cost; and (ii) the design a more complex high-frequency scattering code for the treatment of general bounded geometries that rigorously accounts for every effect, including shadowing and multiple scattering. Specifically, in relation with the effort (i), its most general implementation for the solution of the Maxwell system in three space dimensions has been realized. The procedure is based on the solution of the relevant integral equations wherein the unknown surface current is written in the form of an envelope, expressed as a series in powers of the wavelength, modulated by the incident radiation; a suitable asymptotic evaluation of the resulting integrals (in a manner so as to resolve the singularity of the Green's function) leads to a recurrence for the terms of the series, which can be readily evaluated to arbitrary order. Our results show that high-order evaluations can deliver very significant improvements over KA (e.g. double precision accuracy) with minimal additional computational effort.

When shadowing occurs the power series alluded to above will generically entail fractional powers of the wavelength and they will also become singular expansions, due sharp light-to-shadow transitions at high frequencies. While the extension of these procedures to these cases has continued to be a subject of our work, the developments in (ii) mentioned above were designed as an effective alternative to consideration of singular power series. Indeed, in this approach, the asymptotic expansions for the slow current envelopes are replaced by direct *coarse* discretizations in the fully illuminated regions, and by appropriately refined ones in the penumbra. Our efforts have shown that, indeed, such a strategy can deliver solutions within arbitrarily prescribed accuracies in fixed computational times at all frequencies for single-scattering configurations (e.g. convex bodies). Moreover, we have also shown that multiple-scattering can be accounted for in an iterative manner wherein subsequent wave reflections are solved for in the order in which they (geometrically) occur. The multiple-scattering scheme is based on the interpretation of these reflections as terms in a Neumann series (a sort of high-order Born approximation) whose efficient computation is enabled by identifying the phase of the currents induced at each iteration through a geometrical optics solution. Our analysis of this procedure has verified the generic nature of its convergence at high frequencies, and it has further clarified the mechanism that allows for its acceleration via Pade approximation which, as we have shown, can have dramatic effects on the evaluation of the iterated series.

Our final project is closely related to our high-frequency integral equation solvers described above, as it concerns the accurate and efficient calculation of the phases as needed to produce an effective "ansatz" for the integral formulation (particularly when multiple scattering occurs). As we have previously described, this project concentrates on the design of a new implementation of an original formulation due to Osher et al. to solve "phase-space" versions of the eikonal equation to capture multi-valued solutions. The original simulation strategy for the resulting problems relied on spatial finite differences and Runge-Kutta time discretizations. More precisely, away from scattering boundaries, a fifth-order WENO-Godunov scheme was used for the space and phase variables while a third-order TVD-RK procedure was implemented to march forward in time; when dealing with (reflecting) boundaries, on the other hand, the order of the spatial discretization was reduced to first, due to the complications that arise in attempting to devise higher-order differencing schemes in such situations. On the other hand, we have now completed the design and implementation of an alternative solution technique that relies on entirely different discretizations and that is specifically designed (i) to take full advantage of the smoothness of solutions of the phase-space equations, and (ii) to facilitate the treatment of (possibly complex) scattering obstacles, all while retaining high-order convergence characteristics. The scheme is based on a spectral treatment of phase variables, and on arbitrary-order DG finite elements and SSP-RK schemes to resolve spatial and temporal variations respectively. As predicted, and as our numerical experiments confirm, the scheme can attain arbitrarily high convergence orders (spectral in phase variables and polynomial order) even in the presence of scattering boundaries.

PERSONNEL SUPPORTED

- *Graduate Students (SUPPORTED BY THIS GRANT)*
  - YoungAe Han (PhD, 2004, School of Mathematics, University of Minnesota; currently at Caltech).
  - Fatih Ecevit (School of Mathematics, University of Minnesota).
- *Faculty/Industrial Collaborators (NOT SUPPORTED BY THIS GRANT)*
  - Oscar Bruno (Applied Mathematics, California Institute of Technology).
  - Bernardo Cockburn (School of Mathematics, University of Minnesota).
  - Avner Friedman (Department of Mathematics, The Ohio State University).
  - David Nicholls (Department of Mathematics, University of Notre Dame).
  - K. K. Tamma (Department of Mechanical Engineering, University of Minnesota).
- *Post-Docs (NOT SUPPORTED BY THIS GRANT)*
  - Yassine Boubendir (School of Mathematics, University of Minnesota).
  - Mc Kay Hyde (NSF postdoctoral fellow, School of Mathematics, University of Minnesota; currently at Rice Univ.)
  - Jianliang Qian (Department of Mathematics, UCLA)
  - Jing Wang (Institute for Mathematics and its Applications, University of Minnesota; currently at Vital Images)
- *Graduate Students (NOT SUPPORTED BY THIS GRANT)*
  - Akash Anand (School of Mathematics, University of Minnesota).
  - M.-H. Chen (School of Mathematics, University of Minnesota).
  - Deepa Gupta (School of Mathematics, University of Minnesota).
  - Harun Kurkcu (School of Mathematics, University of Minnesota).

- Gerardo Ortigoza (PhD, 2003, School of Mathematics, University of Minnesota; currently at Universidad Veracruzana, Xalapa, Mexico).
- Catalin Turc (School of Mathematics, University of Minnesota).

PUBLICATIONS

- *SUBMITTED*
  - Journals

[1] B. Cockburn, J. Qian, F. Reitich and J. Wang, "An efficient spectral/discontinuous finite-element formulation of a phase-space-based level set approach to geometrical optics", *submitted.*

- ACCEPTED
  - Journals

[2] D. Nicholls and F. Reitich, "On analyticity of traveling water waves", *Proc. R. Soc. London A,* to appear.

[3] F. Reitich and C. Turc, "High-order solutions of three-dimensional rough surface scattering problems at high-frequencies. I: the scalar case", *Waves in Random Media*, to appear.

[4] M.-H. Chen, B. Cockburn and F. Reitich, "High-Order RKDG Methods for Computational Electromagnetics", *J. Sci. Comput.*, to appear.

[5] D. Nicholls and F. Reitich, "Shape deformations in rough surface scattering: cancellations, conditioning, and convergence", *J. Opt. Soc. Am. A* **21** (2004), 590-605.

[6] D. Nicholls and F. Reitich, "Shape deformations in rough surface scattering: improved algorithms", *J. Opt. Soc. Am. A* **21** (2004), 606-621.

[7] O. P. Bruno, C. A. Geuzaine, J. A. Monroe and F. Reitich, "Prescribed error tolerances within fixed computational times for scattering problems of arbitrarily high frequency: the convex case", *Phil. Trans. Roy. Soc. London* **362** (2004), 629-645.

[8] F. Reitich and K. K. Tamma, "State-of-the-art, trends and directions in Computational Electromagnetics", *CMES Comput. Model. Eng. Sci.* **5** (2004), 287-294.

[9] D. Nicholls and F. Reitich, "Analytic continuation of Dirichlet-Neumann operators", *Numer. Math.* **94** (2003), 107-146.

[10] A. Friedman and F. Reitich, "Quasi-static motion of a capillary drop, II: the three-dimensional case", *J. Diff. Equations* **186** (2002), 509-557.

[11] A. Friedman and F. Reitich, "Quasi-static motion of a capillary drop, I: the two-dimensional case", *J. Diff. Equations* **178** (2002), 212-263.

[12] D. Nicholls and F. Reitich, "A new approach to analyticity of Dirichlet-Neumann operators", *Proc. R. Soc. Edinburgh A* **131** (2001), 1411-1433.

[13] A. Friedman and F. Reitich, "Nonlinear stability of a quasi-static Stefan problem with surface tension: a continuation approach", *Ann. Scuola Norm. Sup. Pisa Cl. Sci.* (4) **30** (2001), 341-403.

- Conferences

[14] O. P. Bruno, C. Geuzaine and F. Reitich, "A new high-order high-frequency integral equation method for the solution of scattering problems. I: Single-scattering configurations", in *Proceedings of the 2004 ACES Conference*, ACES, 2004.

[15] O. P. Bruno, C. Geuzaine and F. Reitich, "A new high-order high-frequency integral equation method for the solution of scattering problems. II: Multiple-scattering configurations", in *Proceedings of the 2004 ACES Conference*, ACES, 2004.

[16] F. Reitich, "High-order domain variations in boundary value and free boundary problems", in *Partial Differential Equations and Inverse Problems,* Series on Contemporary Mathematics, vol. **362**, , C. Conca, R. Manasevich, G. Uhlmann and M. S. Vogelius, eds., American Mathematical Society (2004), 351-369.

[17] F. Reitich, "Shape deformations and analytic continuation in free boundary problems", in *Free Boundary Problems: Theory and Applications*, International Series of Numerical Mathematics, vol. **147**, P. Colli, C. Verdi and A. Visintin, eds., Birkhauser (2003), 265-280.

INTERACTIONS/TRANSITIONS

- *Participation/Presentations At Meetings, Conferences, Seminars, Etc*

    - Army Research Laboratories, Adelphi, November 1, 2004
    - AHPCRC Meeting, University of Minnesota, Minneapolis, August 17, 2004.
    - Mathematical Modeling in Industry - A Workshop for Graduate Students, Institute for Mathematics and its Applications, University of Minnesota, August 9-18, 2004 (*Organizer*).
    - IMA Workshop on Adaptive Sensing and Multimode Data Inversion, Minneapolis, June 28, 2004.
    - Minisymposium on High Order Discontinuous Galerkin Methods, ICOSAHOM 2004, Providence, June 22, 2004.
    - Session on Dynamics and Wave Propagation, IABEM 2004, Minneapolis, May 25, 2004.
    - AHPCRC Meeting, College Park, Maryland, May 19, 2004.
    - Seventh Riviere-Fabes Symposium on Analysis and Partial Differential Equations, Minneapolis, April 23-25, 2003 (*Organizer*).
    - Session on High Order Methods, 20th Annual Review of Progress in Applied Computational Electromagnetics, Syracuse, April 22, 2004.
    - Corning Inc., Modeling and Simulation Group, Corning, April 21, 2004.
    - Princeton University, Princeton, April 5, 2004.
    - Computational Methods in Multiscale Analysis and Applications, University of Florida, Gainesville, March 1, 2004.
    - AFOSR Contractors Meeting, San Antonio, January 8-10, 2004.
    - AFOSR High Frequency Workshop, Dayton, 28 August 2003.
    - Session on Numerical Methods in Electromagnetics and Acoustics, Conference on Partial Differential Equations and Applications, University of Notre Dame, South Bend, August 15, 2003.

- Minisymposium on Novel Applied and Computational Mathematics Techniques for New Application Fields, ICIAM 03, Sydney, Australia, July 11, 2003.
- Minisymposium on Integral Technique for Harmonic Maxwell Equations: Iterative Techniques and Preconditioners, ICIAM 03, Sydney, Australia, July 10, 2003.
- Minisymposium on Truncation Techniques for Exterior Scattering Problems, ICIAM 03, Sydney, Australia, July 9, 2003.
- AHPCRC Meeting, Howard University, Washington DC, May 13, 2003.
- University of Notre Dame, South Bend, April 28, 2003.
- Sixth Riviere-Fabes Symposium on Analysis and Partial Differential Equations, Minneapolis, April 25-27, 2003 (*Organizer*).
- Special Session on Analytical and Computational Methods in Electromagnetics, AMS 2003 Spring Eastern Sectional Meeting, New York University, New York, April 12, 2003.
- McGill University, Montreal, Canada, April 7, 2003.
- Georgia Institute of Technology, Atlanta, February 24, 2003.
- Rutgers University, Newark, February 10, 2003.
- Pan-American Advanced Studies Institute (PASI) on Partial Differential Equations, Inverse Problems and Non-Linear Analysis, University of Chile, Santiago de Chile, January 15, 2003.
- AFOSR Contractors Meeting, San Antonio, January 9-11, 2003.
- Conference on Current Trends in Mathematics and its Applications in honor of Avner Friedman's 70th birthday, Minneapolis, November 8-10, 2002 (*Organizer*).
- Fall 2002 Midwest PDE Seminar, Northwestern University, Evanston, October 6, 2002.
- Georgia Institute of Technology, Atlanta, September 17, 2002.
- AHPCRC Meeting, University of Minnesota, July 23, 2002.

- Minisymposium on Recent Mathematical Advances in Solving Inverse Problems in Electromagnetics, PIERS 2002, Boston, July 2, 2002.
- Session on Advances in Computational Electromagnetics, PIERS 2002, Boston, July 2, 2002
- Workshop on recent advances, state-of-the-art and future directions in computational electromagnetics, June 27-28, 2002 (*Organizer*).
- FBP 2002, Free Boundary Problems: Theory and Applications, June 8, 2002.
- AHPCRC Meeting, Clark Atlanta University, May 15, 2002
- Fifth Riviere-Fabes Symposium on Analysis and Partial Differential Equations, April 5-7, 2002 (*Organizer*).
- AHPCRC Meeting, Army Research Labs, Adelphi, December 11, 2001.

- *Consultative And Advisory Functions To Other Laboratories And Agencies*
  - Member of the Review Panel on Numerical PDE, Applied and Computational Mathematics, National Science Foundation, March 3-5, 2004.
  - Portfolio Coordinator, Computational Electromagnetics Portfolio, Army High Performance Computing Research Center, ARL-University of Minnesota.

>the middle.
>
>In the second, an aircraft has to also learn to avoid hostile airspace.
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>In the third, two aircraft, taking off from different locations, have
>to
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>learn to meet in roughly the same area so that one tanker can service
>both aircraft.
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>The important feature of this technology is that we provided very
>little
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>information that is specific to the problem setting. We have to
>provide
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>the locations of the cargo aircraft and the tanker, and provide basic
>information that tell the cargo aircraft the direction they should
>roughly follow to get to their destination. Otherwise, there is no
>specific logic telling them they have to be refueled, or how they
>should
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>best be refueled.
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>The goal is to get to a system where someone can code basic physics
>(e.g. speed (may depend on location, direction and weather), rate of
>fuel consumption, probability of a failure (may depend on location))
>and
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>get an answer where the decisions look reasonable. We can include all
>kinds of random behavior (and misbehavior). The real test of the
>technology is when we can get good behavior for one dataset, switch to
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>new dataset and still get good behavior with a minimum of tuning.
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>A key complication of these new datasets is that they feature two
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>layers - both the cargo aircraft and the tanker can be independently
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>The current code only handles a few aircraft. The next version will
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>do,
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>but I hope to produce something that is much more transportable than my
>previous airlift simulator.
>
>Warren
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>

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13. ABSTRACT *(Maximum 200 words)*

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