Weak Duality and Iterative Beamforming Algorithms for Ad Hoc Networks

Ronald A. Iltis and Seung-Jun Kim
University of California
Department of Electrical and Computer Engineering
Santa Barbara, CA 93106-9560
email: iltis@ece.ucsb.edu
email: sjkim@ece.ucsb.edu

Abstract A unicasting ad hoc network is considered, where each node i is equipped with a transmit/receive beamformer pair ($\mathbf{w}_i$, $\mathbf{g}_i$). Each node's SNR Γ_i satisfies $\Gamma_i \geq \gamma_0$. It is first shown that the minimum sum power beamformers for the network satisfy a weak duality condition, in which the pairs ($\mathbf{g}_i^*$, $\mathbf{w}_i^*$) achieve the same sum power as the primal network. However, the optimum receive beamformer $\mathbf{w}_i$ is not in general equal to the optimum $\mathbf{g}_i^*$, in contrast to the case of certain networks with simpler topologies. A suboptimal iterative beamforming algorithm is then proposed in which $\mathbf{w}_i = \mathbf{g}_i^*$ is enforced. The algorithm is shown to be an instance of the Power Algorithm in which $\mathbf{g}_i$ is the maximizing eigenvector of an SNR-related objective matrix. The beamforming algorithm is also shown to have a Game Theory interpretation, in which the payoff is SNR, and the tax is related to interference caused to other nodes. The algorithm is also proven to have a Nash equilibrium(a).

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WEAK DUALITY AND ITERATIVE BEAMFORMING ALGORITHMS FOR AD HOC NETWORKS

Ronald A. Iltis and Seung-Jun Kim

Department of Electrical and Computer Engineering
University of California,
Santa Barbara, CA 93106-9560
{iltis,sjkim}@ece.ucsb.edu

ABSTRACT

A unicasting Ad hoc network is considered, where each node i is equipped with a receive/transmit beamformer pair $(\mathbf{w}_i, \mathbf{g}_i)$. Each node's SNR Γ_i satis£es $\Gamma_i \geq \gamma_0$. It is £rst shown that the minimum sum power beamformers for the network satisfy a weak duality condition, in which the pairs $(\mathbf{g}_i^*, \mathbf{w}_i^*)$ achieve the same sum power as the primal network. However, the optimum receive beamformer $\mathbf{w}_i$ is not in general equal to the optimum $\mathbf{g}_i^*$, in contrast to the case of cellular and time-division duplexing (TDD) networks. A suboptimal iterative beamforming algorithm is then proposed in which $\mathbf{w}_i = \mathbf{g}_i^*$ is enforced. The algorithm is shown to be an instance of the Power Algorithm in which $\mathbf{g}_i$ is the maximizing eigenvector of an SNR-related objective matrix. The beamforming algorithm is shown to have a Game Theory interpretation, in which the payoff is SNR, and the tax is related to interference caused to other nodes. The algorithm is also proven to have a quasi-Nash equilibrium(a). A sequential beamforming algorithm for rank-1 channels is also presented, which is proven to converge to an equilibrium and minimize a total interference function.

1. INTRODUCTION

Noncooperative game theory has recently been applied to problems in CDMA network power control [1],[2] and sequence design [3]. The application of game theory to distributed beamforming algorithms is an evolving area. Optimal solutions to transmit/receive beamforming have previously been developed for networks with well-de£ned topologies, e.g. cellular as in [4] or with TDD [5]. However, practical algorithms to obtain optimal beamforming, in terms of SNR constraints or capacity maximization, have not been obtained in the Ad hoc network case. The goal here is to use game theory to motivate "good" solutions to the distributed beamforming problem.

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An Ad hoc network is considered here with unicasting, where pairs of nodes $(i, l(i))$ such that $i \neq l(i)$ communicate only with each other, with $i, l(i) \in \{1, 2, \ldots, N\}$. Each node maintains a unit-norm receive/transmit beamformer pair $(\mathbf{w}_i, \mathbf{g}_i)$, and reciprocal channel matrices $\mathbf{H}_{i,j} = \mathbf{H}_{j,i}^T$ are assumed. An unnormalized beamformer $\tilde{\mathbf{g}}_i = \sqrt{P_i}\mathbf{g}_i$ is also de£ned where P_i is the transmitted power. It is £rst shown that this Ad hoc network exhibits a weak duality, in which the minimum sum power P_{sum} equals P'_{sum} for the dual network $(\mathbf{g}_i^*, \mathbf{w}_i^*)$. Next, a LEGO-type [5] iterative MMSE algorithm is developed for the Ad hoc network, which is shown to correspond to a noncooperative game. The action set comprises both the beamformers and powers, with a payoff function that increases with link SNR. However, the payoff includes a "tax" that increases with interference incurred by other links. A quasi-Nash equilibrium is proven to exist for the resulting Iterative Power Algorithm (IPA) beamforming game.

In general, convergence to the equilibrium cannot be proven for the IPA. An alternative Sequential Distortionless-Response Beamforming (SDRB) algorithm is presented for the special case of rank-1 spatial channels. It is shown in a manner similar to [6] that the SDRB algorithm minimizes a Total Interference Function (TIF) that is the spatial channel analog to total squared correlation [6], and is hence convergent. Simulation examples for both the IPA and SDRB are presented.

2. WEAK NETWORK DUALITY

The duality results use the following de£nition of received SNR at node $l(i)$.

$$\Gamma_{l(i)} = \frac{P_i|\mathbf{w}_{l(i)}^H \mathbf{H}_{l(i),i}\mathbf{g}_i|^2}{\sum_{l \neq i, l(i)} P_l|\mathbf{w}_{l(i)}^H \mathbf{H}_{l(i),l}\mathbf{g}_l|^2 + \sigma_n^2}. \quad (1)$$

where $||\mathbf{w}_i||, ||\mathbf{g}_i|| = 1$. The SNR requirement is $\Gamma_i \geq \gamma_0$ for all i.

The following theorem describes duality for this network, and uses the approach of [7],[5].

Theorem 1 *Weak Network Duality: Consider a network with beamformer pairs* $(\mathbf{w}_i, \mathbf{g}_i)$*. If a feasible solution for the powers exists, then the solution for minimum sum power* $P_{sum} = \mathbf{1}^T\mathbf{p}$*, where* $\mathbf{1}$ *is the all-ones vector, exactly satis£es the constraints* $\Gamma_i = \gamma_0$*. De£ne a* dual network *by* $(\mathbf{g}_i^*, \mathbf{w}_i^*)$ *at all nodes. This network has the same sum power* $P'_{sum} = P_{sum}$ *as in the primal network. An optimal beamformer pair* $(\mathbf{w}_i^{opt}, \mathbf{g}_i^{opt})$ *exists for feasible* γ_0 *which minimizes* P_{sum}*, in which* $\mathbf{w}_i^{opt}$ *is always a Minimum Variance Distortionless Response (MVDR) beamformer. However,* $\mathbf{w}_i^{opt} \neq \mathbf{g}_i^{opt*}$ *in general for the Ad hoc network.*

Proof: An outline of the proof uses the gain de£nition $T_{i,j} = |\mathbf{w_i}^H \mathbf{H}_{i,j}\mathbf{g}_j|^2$, where $\mathbf{w}_i, \mathbf{g}_i$ are the optimum beamformers. Let $\mathbf{p} = [p_1, p_2, \ldots, p_N]^T$ be the power vector, which satis£es

$$\left[\mathbf{I} - \gamma_0 \mathbf{D}\tilde{\mathbf{T}}\right]\mathbf{p} = \gamma_0 \mathbf{D}\mathbf{1}, \qquad (2)$$

where $\mathbf{D}_{i,i} = 1/T_{l(i),i}$ is a diagonal matrix. Note that the i,j-th element of the matrix $\tilde{\mathbf{T}}$ satis£es $\tilde{\mathbf{T}}_{i,j} = T_{l(i),j}$. The solution for the minimum sum power in eq. (1) is

$$P_{sum} = \gamma_0 \mathbf{1}^T \left[\mathbf{D}^{-1} - \gamma_0 \tilde{\mathbf{T}}\right]^{-1}\mathbf{1}, \qquad (3)$$

Now de£ne the permuted vector $\mathbf{p}_i^\pi = \mathbf{p}_{l(i)}$, and $\tilde{\mathbf{T}}_{i,l}^\pi = T_{i,l(l)}$. Similarly, $\mathbf{D}_{i,i}^\pi = 1/T_{i,l(i)}$. The minimum sum power can be written in terms of these rearranged matrices/vectors as $P_{sum} = \gamma_0 \mathbf{1}^T \left[\mathbf{D}^{\pi-1} - \gamma_0 \tilde{\mathbf{T}}^\pi\right]^{-1}\mathbf{1}$. To complete the proof, the dual network $(\mathbf{g}_i^*, \mathbf{w}_i^*)$ has transfer functions $T'_{i,j} = T_{j,i}$ when $\mathbf{H}_{i,j} = \mathbf{H}_{j,i}^T$. Then the transfer function matrix in the dual network has entries $\tilde{\mathbf{T}}'_{i,j} = T_{j,l(i)}$. But note that $T_{j,l(i)} = \tilde{\mathbf{T}}_{j,i}^\pi$. Hence $\tilde{\mathbf{T}}'_{i,j} = \tilde{\mathbf{T}}_{j,i}^\pi$, and $\tilde{\mathbf{T}}'$ is just the transpose of $\tilde{\mathbf{T}}^\pi$. Furthermore, $\mathbf{D}'_{i,i} = 1/T_{i,l(i)} = \mathbf{D}_{i,i}^\pi$. The sum power for the dual network is then

$$P'_{sum} = \gamma_0 \mathbf{1}^T \left[\mathbf{D}^{\pi-1} - \gamma_0 (\tilde{\mathbf{T}}^\pi)^T\right]^{-1}\mathbf{1}. \qquad (4)$$

Again, the results of [7],[5] show that $P'_{sum} = P_{sum}$.

It should be emphasized that the result $P'_{sum} = P_{sum}$, where P_{sum} is the minimum power in the primal network, does *not* imply that the optimum $\mathbf{g}_i = \mathbf{w}_i^*$. That is, consider the receive/transmit beamformer pair in the primal network, $(\mathbf{w}_i, \mathbf{g}_i)$. The receive vector $\mathbf{w}_i$ must always be the normalized MVDR, or equivalently MMSE beamformer to maximize the SNR Γ_i (1). However, we cannot claim that $\mathbf{g}_i = \mathbf{w}_i^*$. It can only be proven that the dual network $(\mathbf{g}_i^*, \mathbf{w}_i^*)$ has the same minimum sum power as the primal.

For $n = 1, 2, \ldots$
  For each node i
    $m \leftarrow 1$
    While $||\mathbf{g}_i(m+1) - \mathbf{g}_i(m)|| > \epsilon$
      Update normalized MVDR beamformer at i
      $\mathbf{w}'_i(m+1) = \mathbf{R}_i^{-1}(\tilde{\mathbf{g}}_{-i}^n)\mathbf{H}_{i,l(i)}\mathbf{g}_{l(i)}(m)$
      $\mathbf{w}_i(m+1) \leftarrow \frac{\mathbf{w}'_i(m+1)}{||\mathbf{w}'_i(m+1)||}$
      $\mathbf{g}_i(m+1) \leftarrow \mathbf{w}_i^*(m+1)$
      Transmit packet to node $l(i)$ –
      Update node $l(i)$ beamformer.
      $\mathbf{w}'_{l(i)}(m+1) = \mathbf{R}_{l(i)}^{-1}(\tilde{\mathbf{g}}_{-i}^n)\mathbf{H}_{l(i),i}\mathbf{g}_i(m+1)$
      $\mathbf{w}'_{l(i)}(m+1) \leftarrow \frac{\mathbf{w}'_{l(i)}(m+1)}{||\mathbf{w}'_{l(i)}(m+1)||}$
      $\mathbf{g}_{l(i)}(m+1) \leftarrow \mathbf{w}_{l(i)}^*(m+1)$
    $m \leftarrow m+1$
    End while
    Update beamformer $\mathbf{g}_i^{n+1} \leftarrow \mathbf{g}_i(m)$
    Transmit estimated $\Gamma_{l(i)}$ from $l(i) \rightarrow i$
    Update power $P_i^{n+1} \leftarrow P_i^n \gamma_0 / \Gamma_{l(i)}$
  Next i
Next n

Table 1: Ad hoc beamforming algorithm

3. ITERATIVE POWER ALGORITHM BEAMFORMING GAME

The following iterative beamforming algorithm is proposed in Table 1 which is similar to LEGO [5], except that TDD is not required, and the SNR target γ_0 is met at every iteration. All nodes simultaneously update their unit-norm MVDR receiver beamformer $\mathbf{w}_i$, and then set their transmit beamformer to $\mathbf{g}_i^* = \mathbf{w}_i^*$ for subsequent transmission to $l(i)$. This procedure is repeated M times with beamformers $\mathbf{g}_{-i}$ (game theoretic notation) held £xed. The power P_i is then set to meet the SNR target γ_0. The interferer plus noise covariance matrix at node i is de£ned by

$$\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n) = \sum_{l \neq i, l(i)} \mathbf{H}_{i,l}\tilde{\mathbf{g}}_l^n \tilde{\mathbf{g}}_l^n \mathbf{H}_{i,l}^H + \mathbf{I}, \qquad (5)$$

where the thermal noise power is unity w.l.o.g. It should be emphasized that $\tilde{\mathbf{g}}_l = \sqrt{P_l}\mathbf{g}_l$, where $||\mathbf{g}_l|| = 1$, thus $\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n)$ incorporates the power of the interferers.

The power algorithm interpretation of the above algorithm is given in the following Proposition.

Proposition 1 *Consider the* n*-th overall update in the Ad hoc beamforming algorithm. The transmit vector* $\mathbf{g}_i^n$ *converges to the maximum eigenvector of the objective matrix*

$$\mathbf{G}_i = \mathbf{R}_i^*(\tilde{\mathbf{g}}_{-i}^n)^{-1}\mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i}^n)^{-1}\mathbf{H}_{l(i),i}. \qquad (6)$$

where $\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n)$ is the interference plus noise covariance matrix at node i in (5). The maximum eigenvector is also the maximizer of the ratio

$$\mathbf{g}_i^{n+1} = arg\max_{\mathbf{g}_i} \frac{\mathbf{g}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i}^n)^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i}{\mathbf{g}_i^H \mathbf{R}_i^*(\tilde{\mathbf{g}}_{-i}^n)\mathbf{g}_i}. \quad (7)$$

Proof: The MVDR beamformer $\mathbf{w}_i(m+1)$ is given by

$$\mathbf{w}_i(m+1) = \frac{1}{c}\mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\mathbf{g}_{l(i)}(m). \quad (8)$$

Recall that $\mathbf{g}_{l(i)}(m) = \mathbf{w}_{l(i)}^*(m)$. Thus,

$$\mathbf{g}_{l(i)}(m)^* = \frac{1}{c}\mathbf{R}_{l(i)}^{-1}(\tilde{\mathbf{g}}_{-i}^{n-1})\mathbf{H}_{l(i),i}\mathbf{g}_i(m), \quad (9)$$

where c is chosen for unit-norm $\mathbf{g}_{l(i)}$. Combining eqs. (8) and (9), and setting $\mathbf{g}_i(m+1) = \mathbf{w}_i(m+1)^*$ yields

$$\mathbf{g}_i(m+1) = \quad (10)$$
$$\frac{1}{c'}(\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n)^*)^{-1}\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i}^n)^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i(m),$$

which is a power algorithm iteration with c' normalizing $\mathbf{g}_i(m+1)$ to unity. Since $\mathbf{R}_i^*$ is always invertible from its de£nition (5), maximization of the ratio (7) is equivalent to £nding the maximizing eigenvector of the matrix in (6)

The algorithm in Table 1 is a noncooperative beamforming game, in which the i-th node utility function is

$$u_n(P_i, \mathbf{g}_i, \tilde{\mathbf{g}}_{-i}^n) = \eta\left(\gamma_0 - P_i\gamma_i(\mathbf{g}_i, \tilde{\mathbf{g}}_{-i}^n)\right) \quad (11)$$
$$+\ln \mathbf{g}_i{}^H\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}^{-1}(\tilde{\mathbf{g}}_{-i}^n)\mathbf{H}_{l(i),i}\mathbf{g}_i$$
$$-\ln \mathbf{g}_i{}^H\mathbf{R}_i^*(\tilde{\mathbf{g}}_{-i}^n)\mathbf{g}_i,$$

where $\eta(x)$ is any continuous, concave function with a global maximum at zero. The normalized SNR is

$$\gamma_i(\mathbf{g}_i, \tilde{\mathbf{g}}_{-i}) = \mathbf{g}_i^H\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i})^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i,$$

for unit norm $\mathbf{g}_i$. The second term in $u_n()$ is the log SNR, hence the payoff increases with increasing SNR and when the SNR constraint is met with equality. The last $\ln$ term however corresponds to an interference tax, de£ned by

$$\mathcal{T}(\mathbf{g}_i, \mathbf{g}_{-i}^n, P_{-i}^n) = \quad (12)$$
$$\mathbf{g}_i^H\mathbf{R}_i^*\mathbf{g}_i = \sum_{l\neq i,l(i)} \mathbf{g}_i^H\mathbf{H}_{i,l}^*P_l^n\mathbf{g}_l^{n*}\mathbf{g}_l^{nT}\mathbf{H}_{i,l}^T\mathbf{g}_i.$$

Since $\mathbf{g}_l^* = \mathbf{w}_l$, the tax $\mathcal{T}(\mathbf{g}_i)$ can be interpreted as the interference incurred by users $l \neq i, l(i)$ from transmitter i, weighted by the power of each user l. Finally, the noncooperative beamforming game is completely de£ned by

$$(\mathbf{g}_i^{n+1}, P_i^{n+1}) = \arg\max_{\mathbf{g}_i,P_i} u_n(P_i, \mathbf{g}_i, \tilde{\mathbf{g}}_{-i}^n). \quad (13)$$

The payoff function (11) lacks the properties of quasi-concavity and continuity required in most proofs of Nash equilibrium existence [1],[8]. Furthermore, since the action set involves beamformer vectors as well as powers, it is not possible to use supermodular theory which requires ordered action sets [1].

The £nite power algorithm iterations in Table 1 are shown to yield a £xed point, and hence quasi-Nash equilibrium as follows. Using the unnormalized beamformer de£nition $\tilde{\mathbf{g}}_i = P_i\mathbf{g}_i$ with $||\mathbf{g}_i||^2 = 1$, the M power iterations (10) satisfying the SNR constraint are then equivalently written as

$$\tilde{\mathbf{g}}_i^{n+1} = \sqrt{\gamma_0}\frac{\mathbf{A}_i(\tilde{\mathbf{g}}_{-i}^n)^M\mathbf{v}_0}{\sqrt{\mathbf{v}_0{}^H\mathbf{A}_i(\tilde{\mathbf{g}}_{-i}^n)^{M\,H}\mathbf{B}_i(\tilde{\mathbf{g}}_{-i}^n)\mathbf{A}_i(\tilde{\mathbf{g}}_{-i}^n)^M\mathbf{v}_0}}, \quad (14)$$

where $\mathbf{B}_i = \mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}$, and $\mathbf{A}_i = (\mathbf{R}_i^*)^{-1}\mathbf{B}_i$. The quantity $\mathbf{v}_0$ is any vector not orthogonal to the maximum eigenvalue solution. It is readily veri£ed that (14) represents a vector transformation $\mathbf{g}^n = \mathbf{f}(\mathbf{g}_{-i}^n)$ which is differentiable, and hence continuous in $\mathbf{g}_{-i}$. Thus, the Brouwer £xed-point theorem guarantees that at least one equilibrium of (14) exists.

4. SDRB – RANK 1 CHANNELS

Efforts to prove convergence of the IPA in Table 1 have thus far failed, and simulations have demonstrated cases where the solutions for $(P_i, \mathbf{g}_i)$ oscillate between two points depending on initial conditions. The alternative SDRB algorithm is presented in Table 2 which is proven to converge for the special case of rank-1 channels. The rank-1 channel is de£ned by $\mathbf{H}_{i,j} = \mathbf{h}_{i,j}\mathbf{h}_{j,i}^T$. For example, in a environment free of multipath, $\mathbf{h}_{i,j}$ would represent the steering vector at node i corresponding to a plane wave received from node j. Each vector $\mathbf{g}_i$ is updated using the most recent values of $\mathbf{g}_{-i}$. Hence, following [6], when $\mathbf{g}_i$ is updated on the n-th pass of the algorithm,

$$\mathbf{g}_{-i}^n = \left\{\mathbf{g}_1^n, \mathbf{g}_2^n, \ldots, \mathbf{g}_{i-1}^n, \mathbf{g}_{i+1}^{n-1}, \ldots, \mathbf{g}_N^{n-1}\right\}$$

The interpretation of the SDRB is as follows: At each iteration, the MVDR beamformer $\mathbf{w}_i$, or equivalently $\mathbf{g}_i$ satis£es

$$\mathbf{g}_i \quad = \quad \arg\min_{\mathbf{g}} \mathbf{g}^H\mathbf{R}_i^*\mathbf{g} \quad (15)$$
$$\text{Subject to} \quad |\mathbf{g}_i^H\mathbf{h}_{i,l(i)}^*|^2 = \alpha.$$

However, for the rank-1 channel, this is equivalent to computing the unique solution to

$$\tilde{\mathbf{g}}_i = \arg\max_{\mathbf{g}} \frac{|\mathbf{g}^H\mathbf{h}_{i,l(i)}^*|^2\mathbf{h}_{l(i),i}^H\mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i})^{-1}\mathbf{h}_{l(i),i}}{\mathbf{g}^H\mathbf{R}_i^*(\mathbf{g}_{-i})\mathbf{g}} \quad (16)$$

For $n = 1, 2, \ldots$
  For each node i
    Update normalized MVDR beamformer at i
    $\mathbf{w}_i = \mathbf{R}_i(\tilde{\mathbf{g}}^n_{-i})^{-1}\mathbf{h}_{i,l(i)}$
    $\mathbf{g}^n_i = \leftarrow \frac{\mathbf{w}^*_i}{||\mathbf{w}_i||}$
    $P_i = \frac{\alpha}{|\mathbf{g}^H_i \mathbf{h}^*_{i,l(i)}|^2}$
  Next node i
Next n

Table 2: Sequential Distortionless-Response Beamforming Algorithm

which satis£es $|\tilde{\mathbf{g}}^H_i \mathbf{h}^*_{i,l(i)}|^2 = \alpha$. Hence, $\mathbf{g}_i$ still maximizes an SNR subject to an interference tax. The constraint $|\mathbf{g}^H_i \mathbf{h}^*_{i,l(i)}|^2 = \alpha$ for $\mathbf{w}_i = \mathbf{g}^*_i$ is the conventional distortionless-response requirement in MVDR beamforming.

To prove convergence of the algorithm in Table 2, £rst de£ne a Total Inteference Function as

$$TIF = \sum_i \sum_{l \neq i, l(i)} \tilde{\mathbf{g}}^H_i \mathbf{H}^*_{i,l} \tilde{\mathbf{g}}^*_l \tilde{\mathbf{g}}^T_l \mathbf{H}^T_{i,l} \tilde{\mathbf{g}}_i + 2\sum_i ||\tilde{\mathbf{g}}_i||^2, \quad (17)$$

where $\tilde{\mathbf{g}}_i = P_i \mathbf{g}_i$, with $||\mathbf{g}_i|| = 1$. The TIF is similar to total squared correlation in [6], except that it is de£ned in terms of £xed steering vectors $\mathbf{h}_{i,l}$ instead of adaptive signature sequences. The interference interpretation follows from the summands $\tilde{\mathbf{g}}^H_i \mathbf{H}^*_{i,l} \tilde{\mathbf{g}}_l$. When $\mathbf{g}_l = \mathbf{w}^*_l$, each such term represents the interference seen at node l due to transmitter i, multiplied by the product of the powers, $P_i P_l$. Furthermore, $||\tilde{\mathbf{g}}_i||^2$ represents the additive noise power incurred by receive beamformer i multiplied by P_i. Hence, TIF can indeed be viewed as the total mutual interference in the network.

The TIF after updating $\mathbf{g}^n_i$ can be decomposed as

$$\text{TIF}^n_i = 2(\tilde{\mathbf{g}}^n_i)^H \mathbf{R}^*_i(\tilde{\mathbf{g}}^n_{-i})\mathbf{g}^n_i + f(\mathbf{g}^n_{-i}), \quad (18)$$

for the symmetric channel case $\mathbf{H}_{i,j} = \mathbf{H}^T_{j,i}$, where $f(\mathbf{g}_{-i})$ is a function independent of $\mathbf{g}_i$. Convergence is claimed in the following.

Proposition 2 *The TIF is a non-increasing function of the sequential distortionless- response beamforming algorithm. Hence the vectors $\tilde{\mathbf{g}}_i$ converge.*

Proof: Follows directly from the statement of the SDRB. Let $\tilde{\mathbf{g}}^n_i$ be the most recently updated transmit vector. Recall that $\tilde{\mathbf{g}}^n_i$ minimizes $\tilde{\mathbf{g}}^H_i \mathbf{R}^*_i(\tilde{\mathbf{g}}^n_{-i})\tilde{\mathbf{g}}_i$ under the constraint $|(\tilde{\mathbf{g}}^n_i)^H \mathbf{h}^*_{i,l(i)}|^2 = \alpha$. Now the vector $\tilde{\mathbf{g}}^{n-1}_i$ satis£es the same constraint as $\tilde{\mathbf{g}}^n_i$, but minimizes $\tilde{\mathbf{g}}^H_i \mathbf{R}^*_i(\tilde{\mathbf{g}}^{n-1}_{-i})\tilde{\mathbf{g}}_i$ instead. Thus,

$$(\tilde{\mathbf{g}}^n_i)^H \mathbf{R}^*_i(\tilde{\mathbf{g}}^n_{-i})\tilde{\mathbf{g}}^n_i \leq (\tilde{\mathbf{g}}^{n-1}_i)^H \mathbf{R}^*_i(\tilde{\mathbf{g}}^n_{-i})\mathbf{g}^{n-1}_i, \quad (19)$$

which in turn implies that $TIF^n_i \leq TIF^n_{i-1}$, or that TIF is a nonincreasing sequence.

The correlation coef£cient α can be determined in terms of a target SNR. For unit-variance thermal noise, rank-1 channels, and in the absence of multiuser interference, the SNR at node $l(i)$ is given by

$$\Gamma_{l(i)} = |\tilde{\mathbf{g}}^H_i \mathbf{h}^*_{i,l(i)}|^2 |\mathbf{h}_{l(i),i}|^2. \quad (20)$$

The maximum value of α is then $P_i|\mathbf{h}_{i,l(i)}|^2$. For arrays with M unit-magnitude elements, we then have $\alpha_{max} = P_i M$ and $\Gamma_{l(i)} \leq P_i M^2 = \alpha_{max} M$. Thus, for target SNR Γ_0, α can be set to $\alpha = \Gamma_0/M$.

5. RESULTS AND CONCLUSIONS

The IPA and SRDB algorithms were simulated for $N = 10$ nodes in a 1km by 1km area, equipped with uniform linear arrays (ULAs). A rank-3 channel was chosen for the IPA algorithm with the direct path gain 3 dB above two paths at an angular spread of $\pi/4$ radians. For the SDRB, only a rank-1 channel was used de£ned by the steering vector for a ULA.

The beampatterns formed by the IPA algorithm are shown in Figure 1 for the case of $M = 8$ antenna elements, $\Gamma_0 = 10$ dB. A power ef£ciency is de£ned similarly to asymptotic ef£ciency in multiuser detection [9]. In the absence of multiuser interference, the power required to attain an SNR of Γ_0 is found using $\Gamma_0 = P^{su}_i \lambda_{max} \mathbf{H}^H_{l(i),i} \mathbf{H}_{l(i),i}$. Hence, the ef£ciency $\eta \in [0, 1]$ quanti£es the excess power required to maintain a constant error rate (assuming Gaussian MAI), and is de£ned by

$$\eta = \frac{P^{su}_i}{P_i} = \frac{\Gamma_0}{P_i \lambda_{max}\left(\mathbf{H}^H_{l(i),i}\mathbf{H}_{l(i),i}\right)}, \quad (21)$$

where P_i is the power resulting from the IPA. The power ef£ciency η is shown in Figure 2 for the scenario in Fig. 1. The algorithm clearly reaches the £xed point de£ned by the updates (8).

A typical result for the SDRB algorithm is shown in Fig. 3, for $M = 4$ antenna elements at each node. The corresponding TIF is plotted in Fig. 4, which is seen to be non-increasing as claimed. In this case, the target SNR was $\Gamma_0 = 10$ dB. This SNR is not reached in general, since satisfying the distortionless response constraint α only corresponds to $\Gamma_i = \Gamma_0$ in the ideal case of zero MAI.

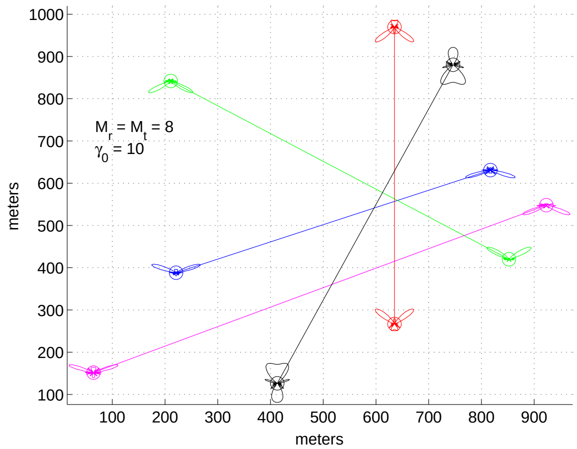

Figure 1: Beampattern for IPA algorithm, M = 8 elements/node.

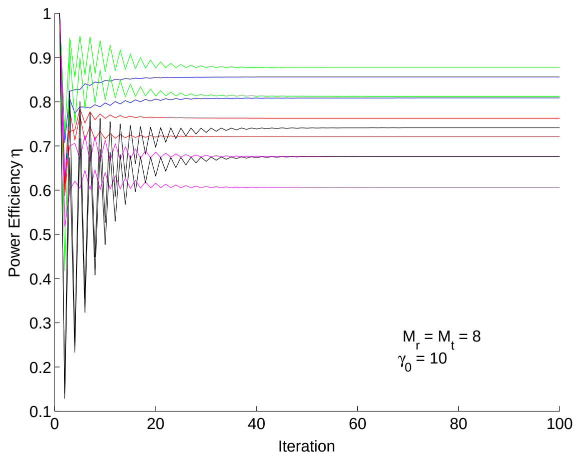

Figure 2: Power ef£ciency η for IPA algorithm, M = 8 elements/node.

To conclude, iterative beamforming algorithms were presented for Ad hoc networks. It was shown that the LEGO-type algorithms [5] correspond to power algorithm iterations and maximization of the ratio of SNR to an interference function. In terms of noncooperative game theory, the IPA corresponds to a payoff function for each node that is maximized when the target SNR is met subject to maximization of normalized link SNR. However, an interference tax is incurred by each node as a byproduct of the IPA. A sequential distortionless response beamforming algorithm was developed for rank-1 channels, and shown to converge by minimization of a Total Interference Function. The SDRB thus provides a connection between iterative beamforming and the iterative sequence construction algorithm of [6].

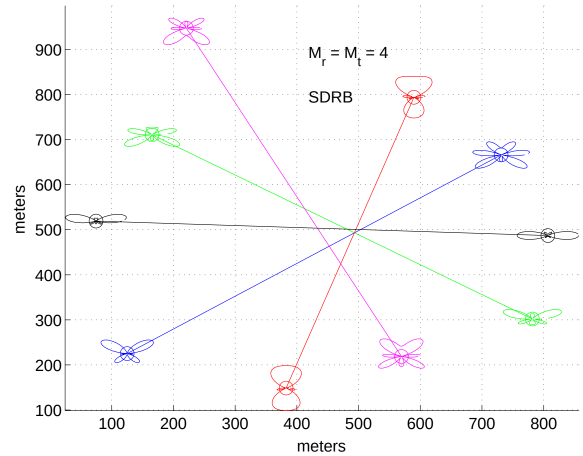

Figure 3: Beampattern for SDRB algorithm, M = 4 elements/node.

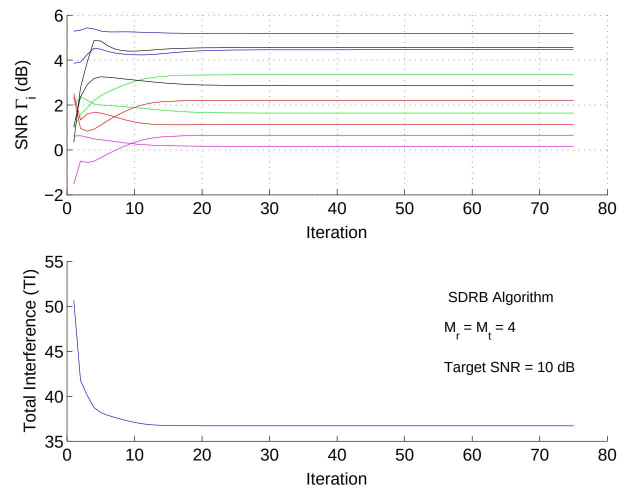

Figure 4: SNR and Total Interference Function trajectories for SDRB.

6. REFERENCES

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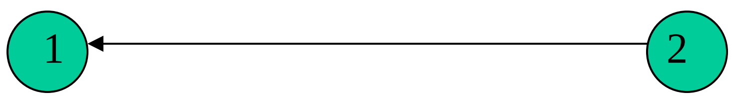
Weak Duality and Iterative Beamforming Algorithms for Ad hoc Networks -- Précis

Ronald A. Iltis and Seung-Jun Kim
Department of Electrical and Computer Engineering
University of California
Santa Barbara, CA 93106-9560
{iltis,sjkim}@ece.ucsb.edu

Iterative MMSE Beamforming – Accomplished via simple LMS/RLS algorithm and training sequences (as in 802.11b/g.)



$$\mathbf{w}_1(m+1) = \mathbf{R}_1(\mathbf{g}^n_{-i})^{-1}\mathbf{H}_{1,l(1)}\mathbf{g}_{l(1)}(m+1)$$

$$\mathbf{g}_1(m+1) = \mathbf{w}_1^*(m+1)/||\mathbf{w}_1(m+1)||$$

Equivalent to Power Algorithm.

Main Results

- Weak Duality – Replace receive transmit beamformer pair (w_i, g_i) at all nodes by (g_i^*,w_i^*), minimum sum power remains unchanged – Does not imply that optimum $g_i = w_i^*$.
- IMMSE Beamforming is a non-cooperative game.
- Utility function maximizes normalized SNR to meet target SNR at all nodes with minimum power. Includes tax proportional to interference to other nodes.
- Fixed point of the IMMSE algorithm exists. Convergence has been proven for a modified game with rank-1 channels based on Total Interference Function (TIF).

Abstract Presentation Paper Back to TOC

Example of Fixed-Point and Power Efficiency of IMMSE

Next Abstract