

Scaling in Heartbeat Rate Variability

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- **Markus Feurstein, Wirtschaftsuniversität Wien**
- **Stefan Thurner, Allgemeines Krankenhaus Wien**

Report Documentation Page

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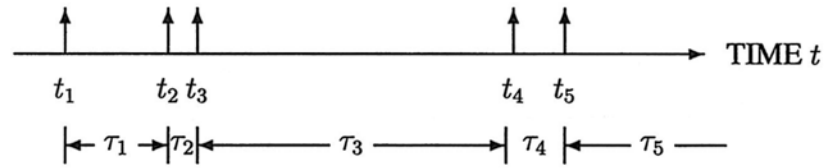
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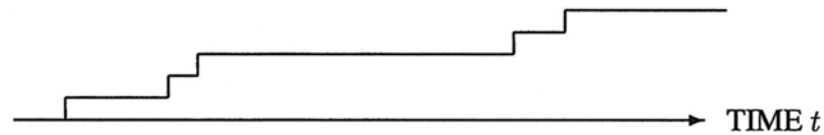


POINT PROCESSES

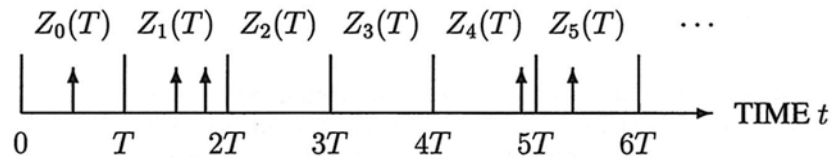
a) POINT PROCESS $dN(t)$



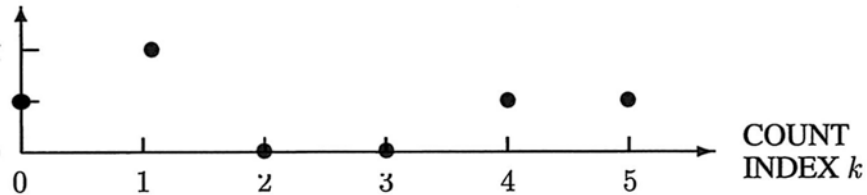
b) COUNTING PROCESS $N(t)$



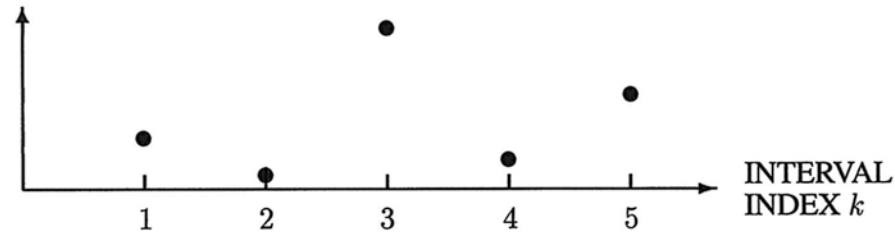
c) GENERATION OF COUNT SEQUENCE



d) COUNT SEQUENCE $\{Z_k(T)\}$



e) INTERVAL SEQUENCE $\{\tau_k\}$

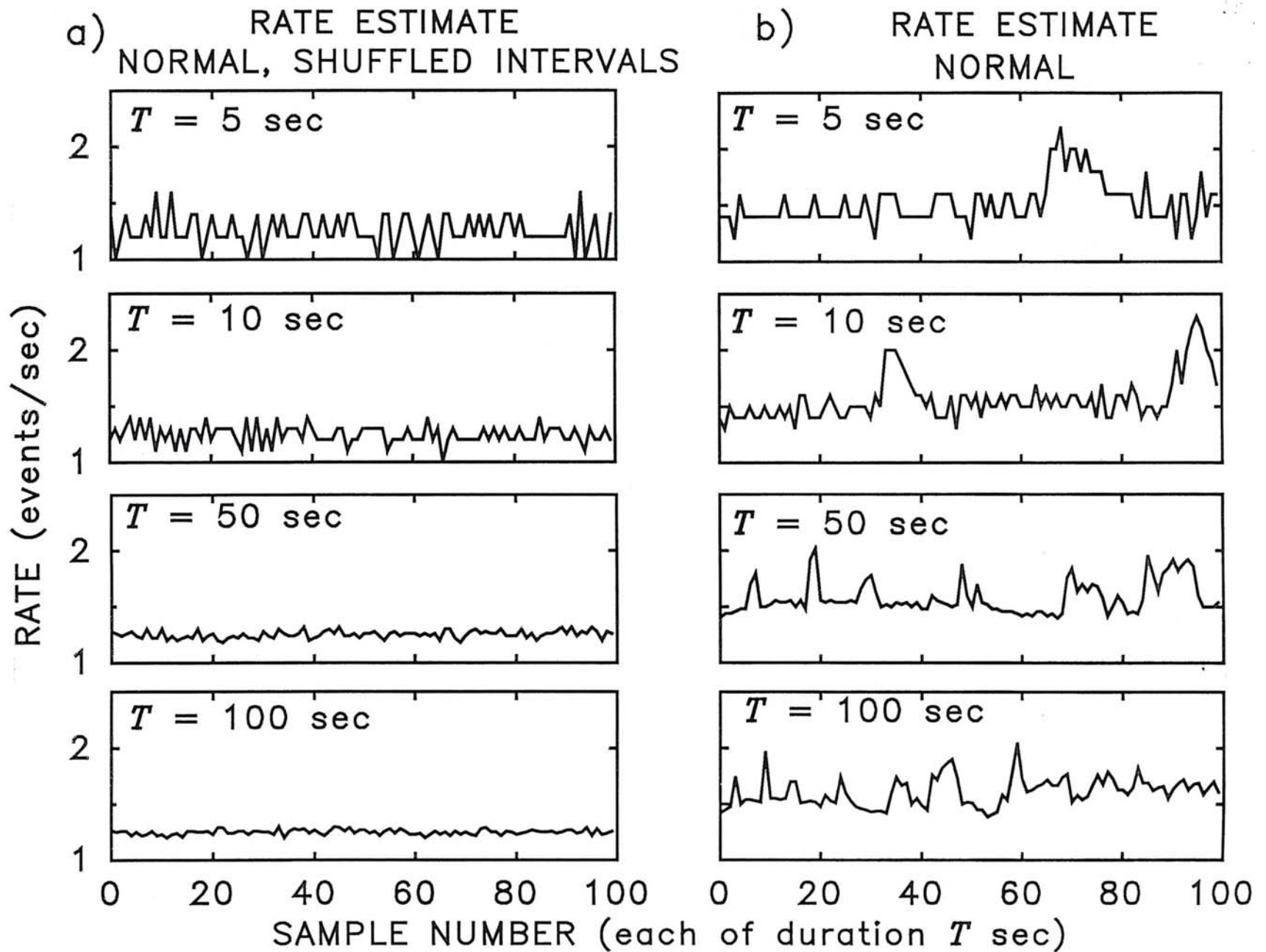




FRACTAL-BASED POINT PROCESSES

- Fractal point processes
- Fractal-rate point processes

S. B. Lowen and M. C. Teich, *Fractal-Based Point Processes*
(Wiley Series in Probability and Statistics, Hoboken, NJ, 2005)
ALMOST DONE!





INTERVAL-BASED MEASURES

CONGESTIVE HEART FAILURE

INABILITY OF HEART TO INCREASE CARDIAC OUTPUT IN PROPORTION TO METABOLIC DEMANDS

Symptom complex:

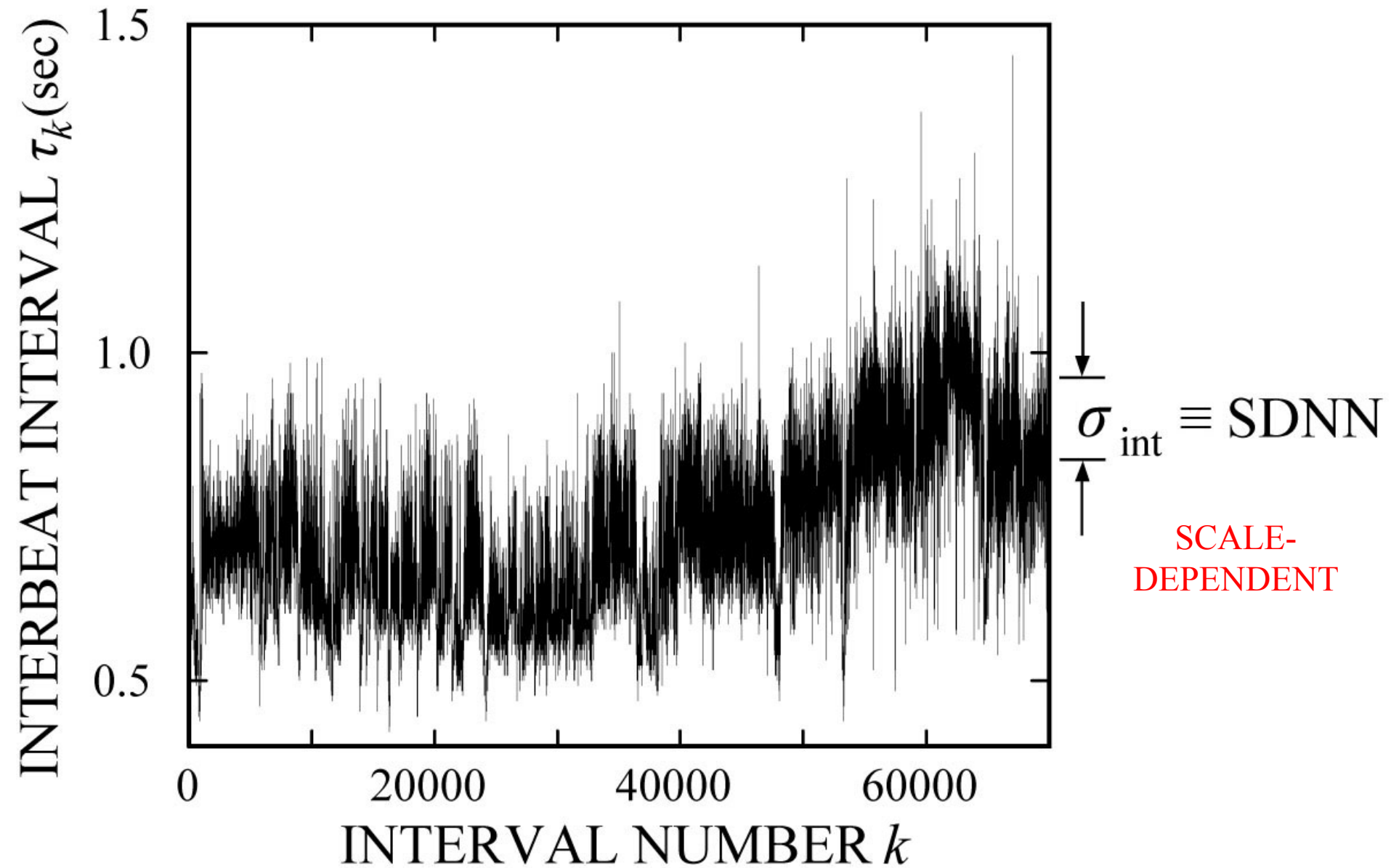
Many different presentations and etiologies

Typical symptoms:

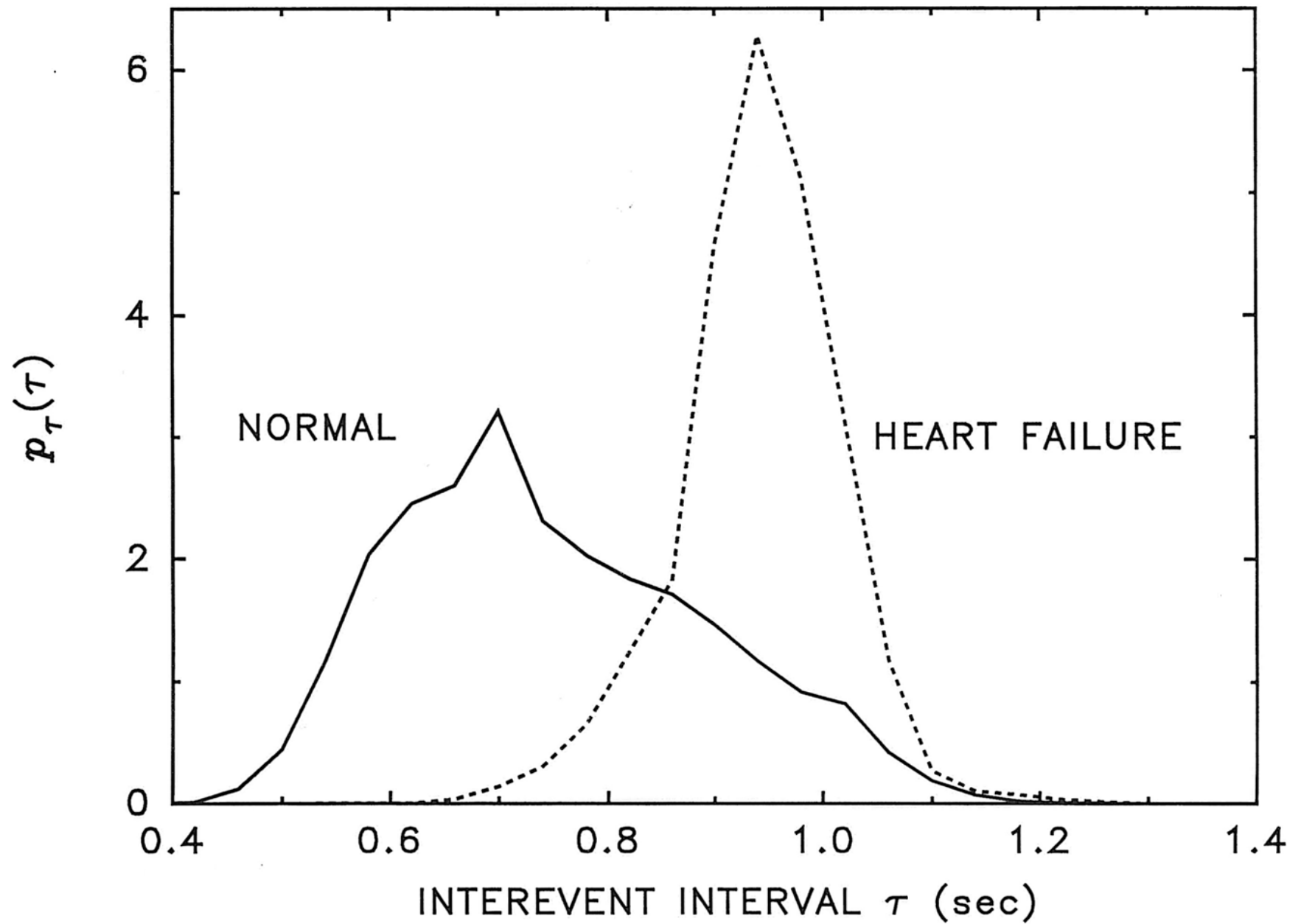
- Shortness of breath
- Swelling in legs
- General fatigue and weakness

Clinical diagnostics:

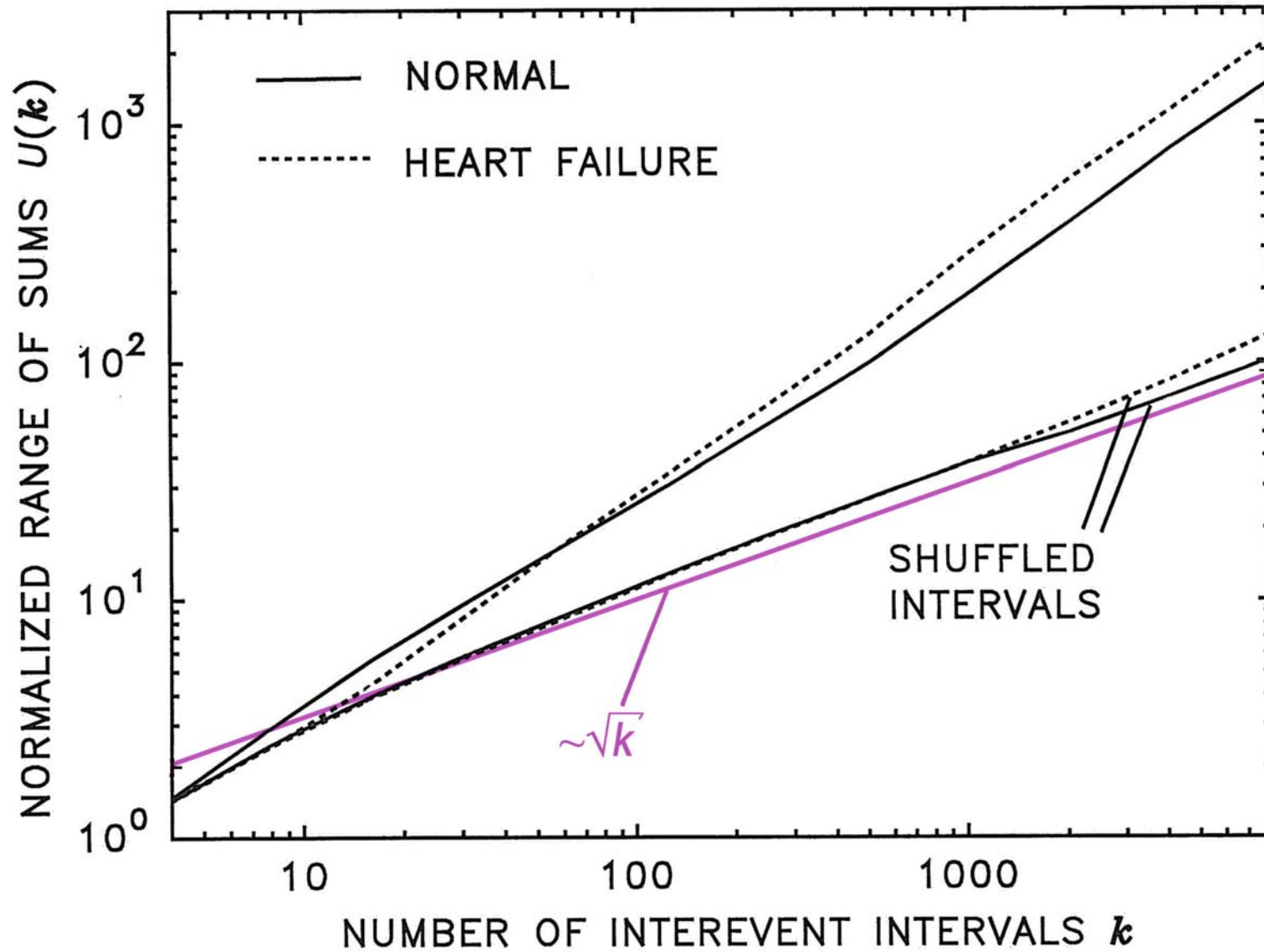
- Ascultate heart
- Carotid pulse
- Electrocardiogram
- Chest radiograph



INTEREVENT-INTERVAL HISTOGRAM

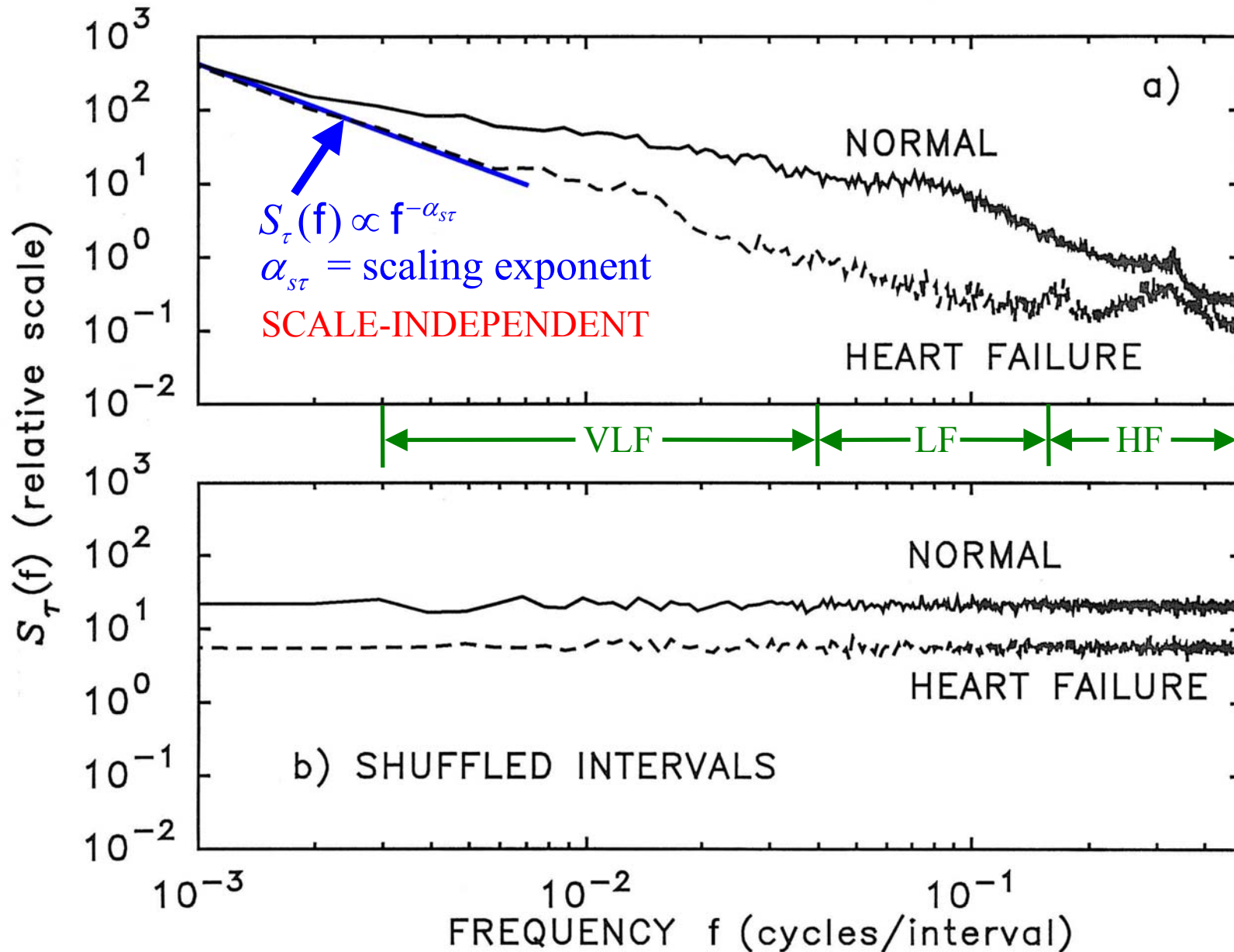


RESCALED RANGE ANALYSIS (R/S)



SPECTRAL ANALYSIS

INTERVAL-BASED PERIODOGRAM



TIME-SCALE ANALYSIS

DISCRETE WAVELET TRANSFORM

EXAMINES ALL SCALES

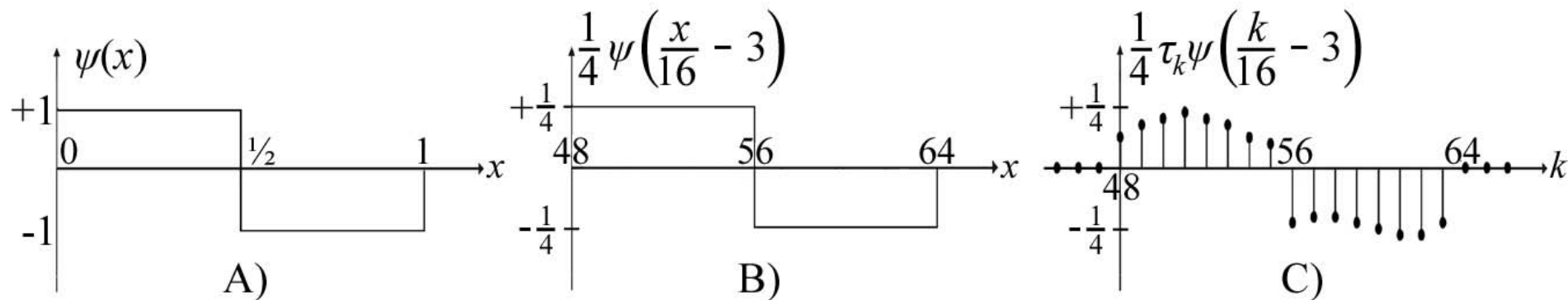
MITIGATES AGAINST NONSTATIONARITIES

m = scale index; 2^m = scale

$$W_{\psi, \tau}^{\text{wav}}(m, i) = \sum_k 2^{-m/2} \psi(2^{-m} k - i) \tau_k$$

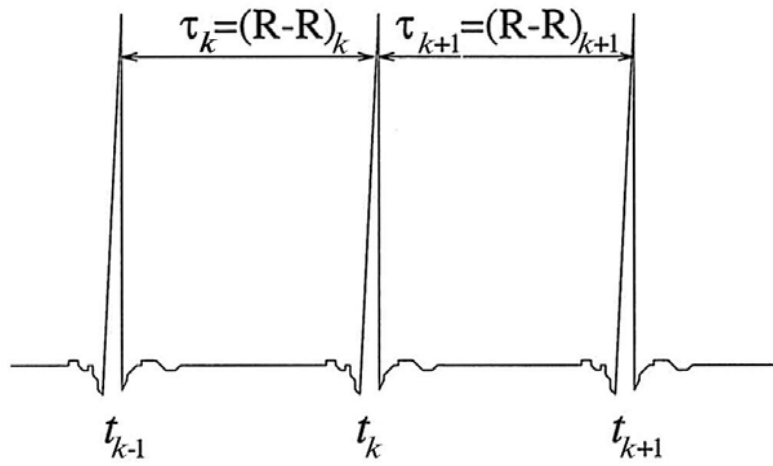
$$\sigma_{\text{wav}}^2 \equiv \text{Var} \left[W_{\psi, \tau}^{\text{wav}}(m, i) \right] = 2^{-m} \sum_k \sum_l \psi(2^{-m} k - i) \psi(2^{-m} l - i) R_{\tau}(l - k)$$

$$A_{\tau}(k) \equiv \text{Var} \left[W_{\psi, \tau}^{\text{wav}}(m, i) \right] / E^2[\tau]$$

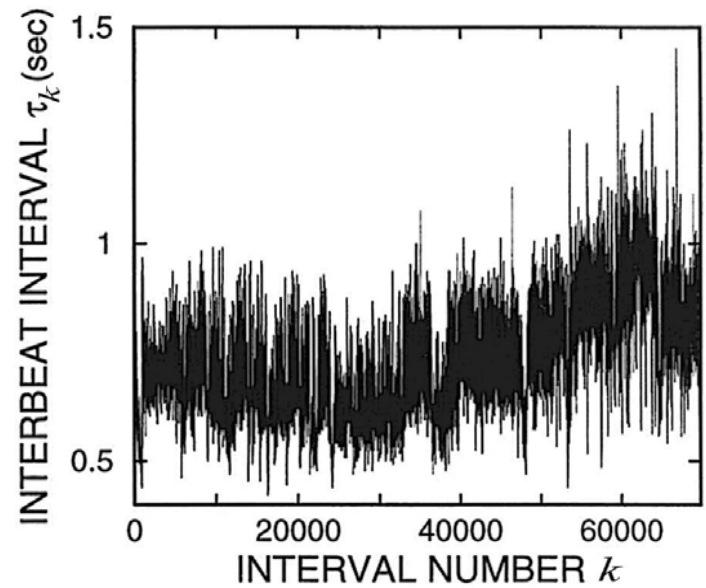


After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan,
in *Nonlinear Biomedical Signal Processing*, vol II, M. Akay, Ed.
(IEEE Press, NY, 2001), pp. 159-213.

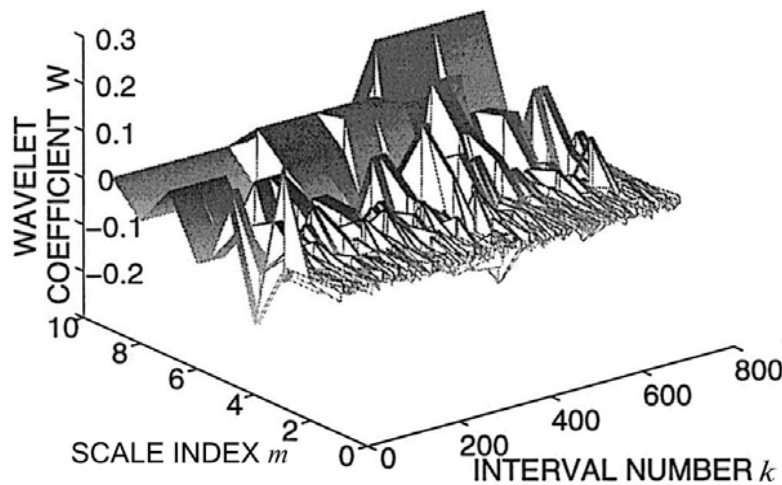
NORMAL



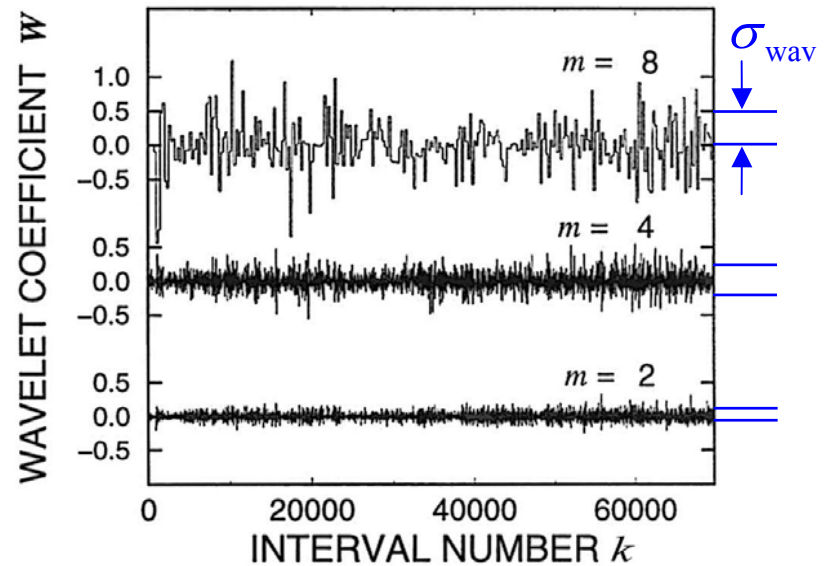
a)



b)

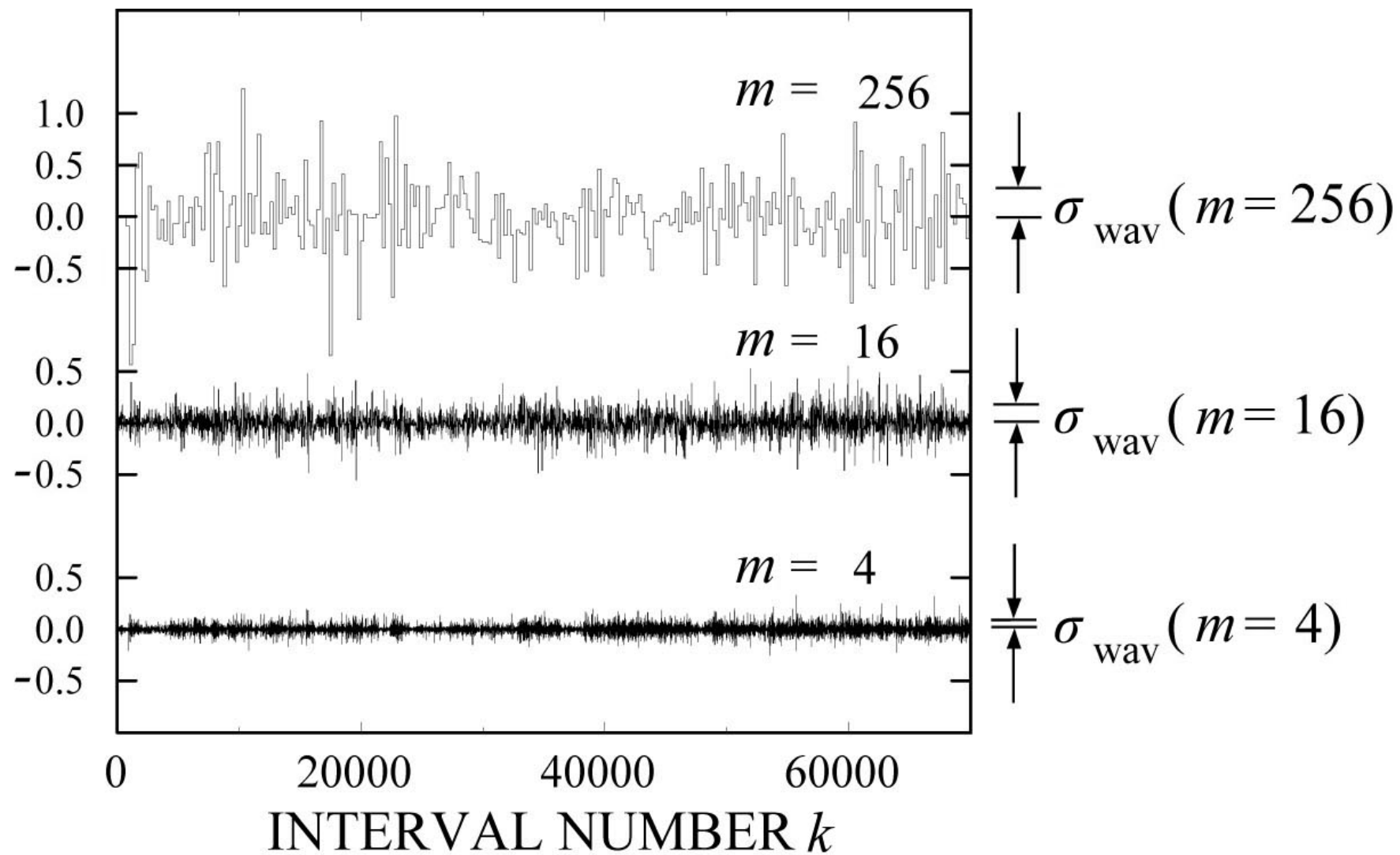


c)

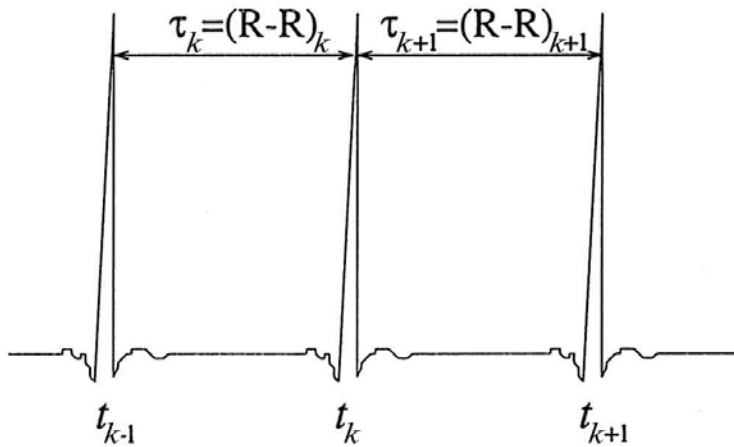


d)

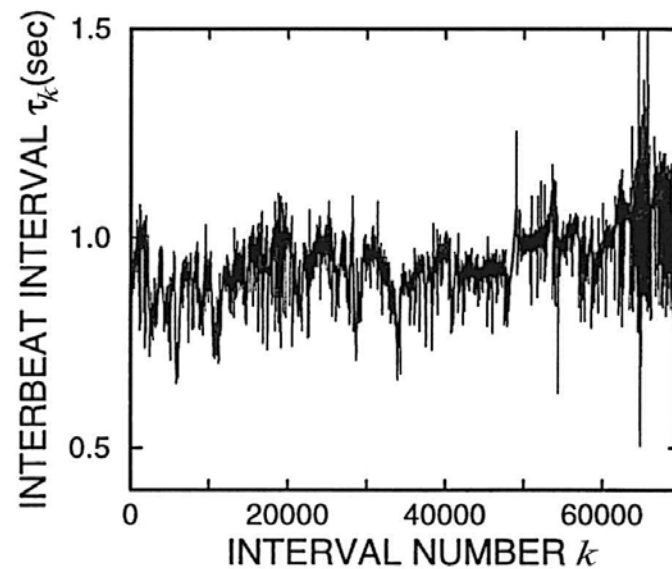
WAVELET COEFFICIENT W



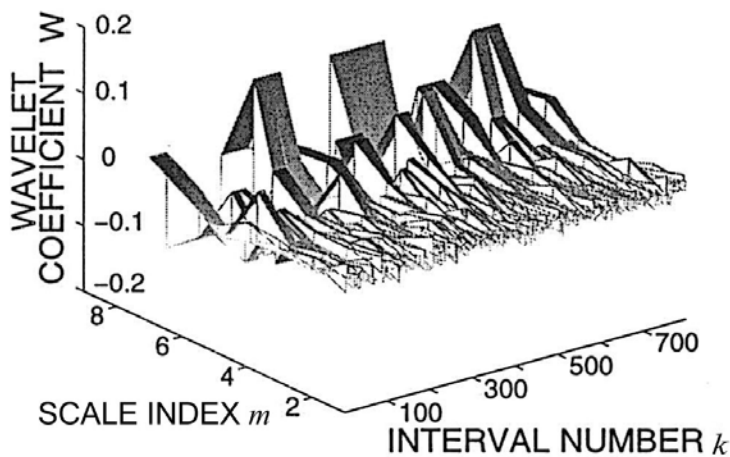
HEART-FAILURE



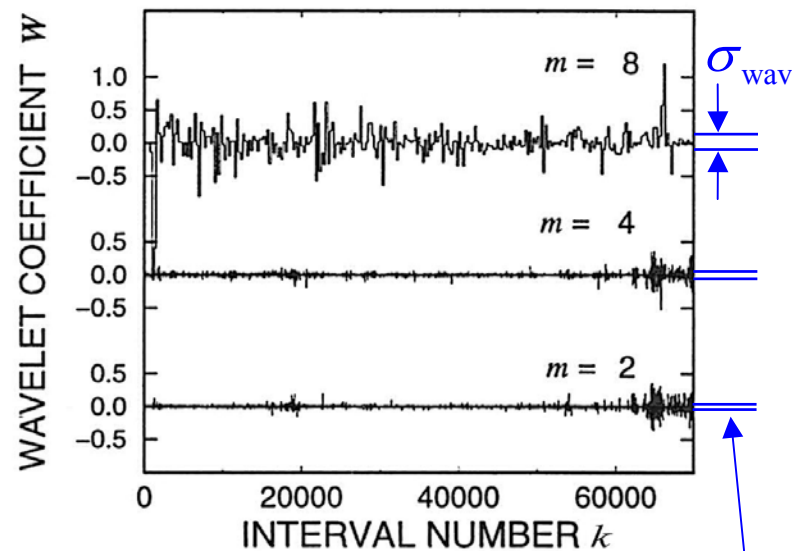
a)



b)



c)



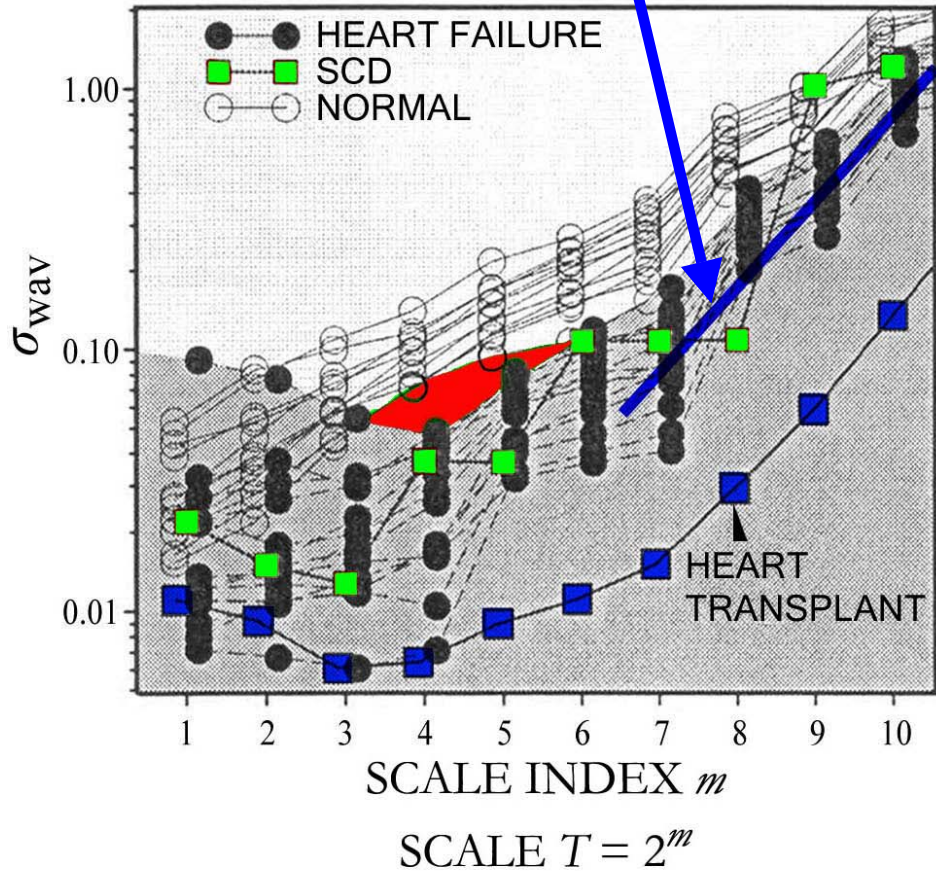
d)

SMALLER VALUES
OF σ_{wav} THAN FOR
NORMAL SUBJECTS

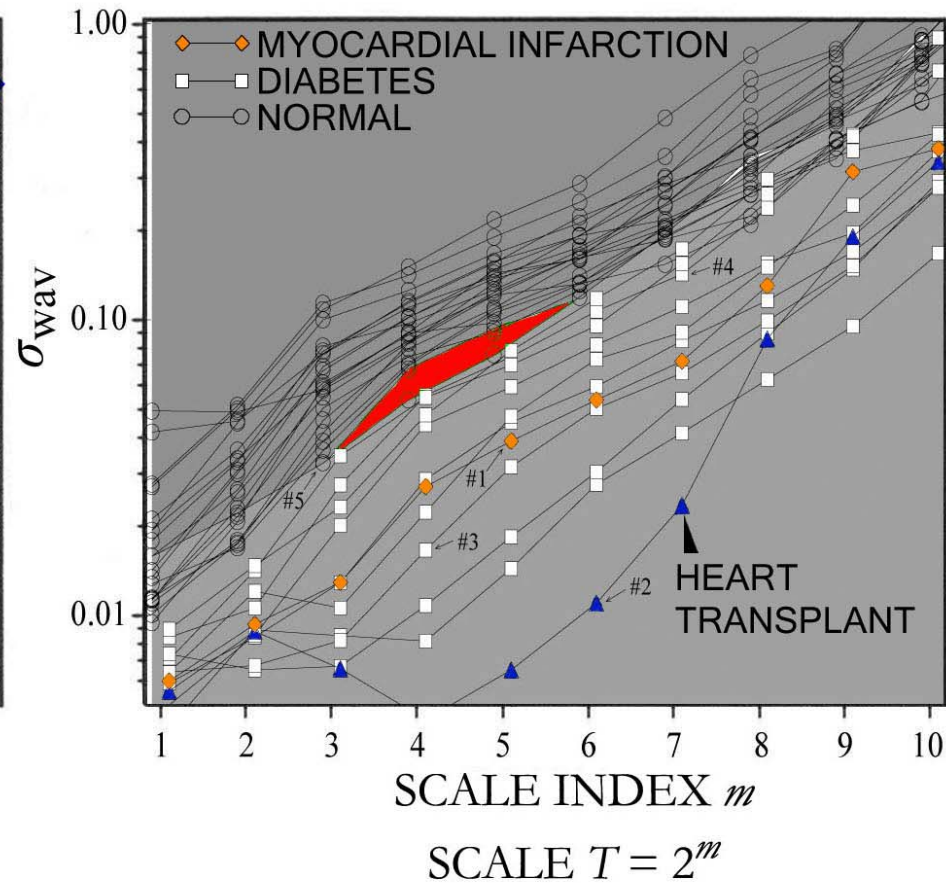
$$\sigma_{\text{wav}}^2(T) \propto T^{\alpha_{A\tau}}$$

$\alpha_{A\tau}$ = scaling exponent

SCALE-INDEPENDENT



After Teich, *Proc. Int. Conf. IEEE Eng. Med. Biol. Soc.* **20**, 1136-1141 (1998).

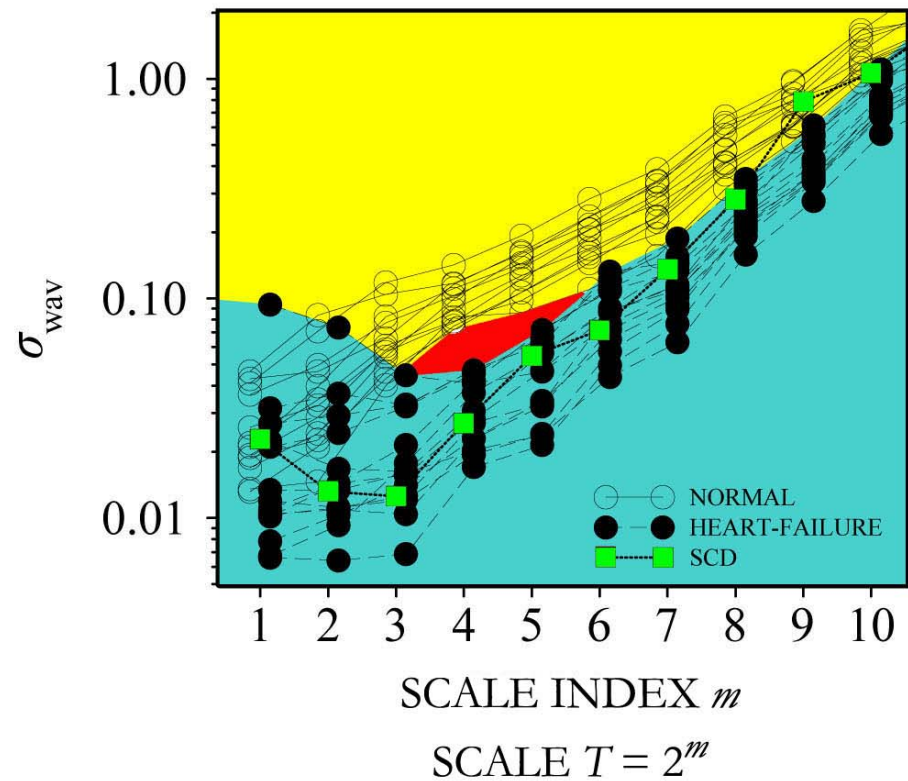
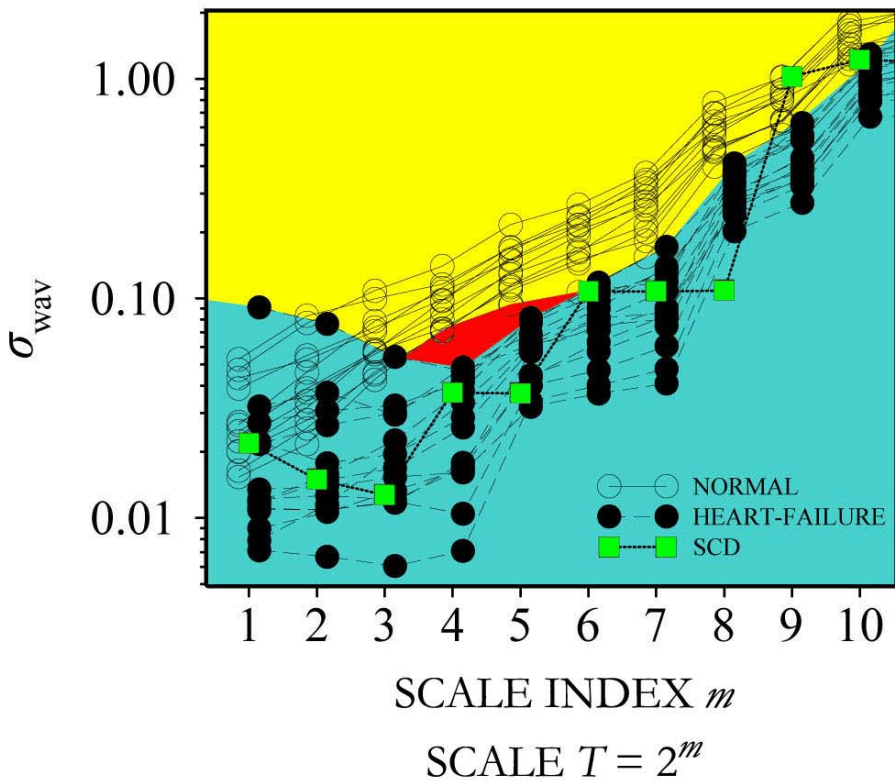


After Ashkenazy *et al.*, *Fractals* **6**, 197-203 (1998).

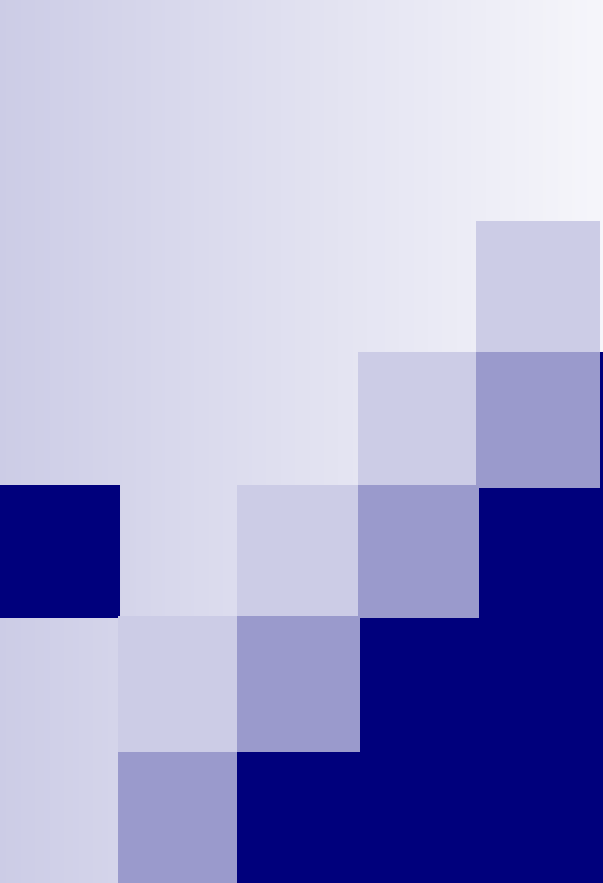
ROBUSTNESS WITH WAVELET FORM

Haar wavelet

Daubechies 10-tap wavelet



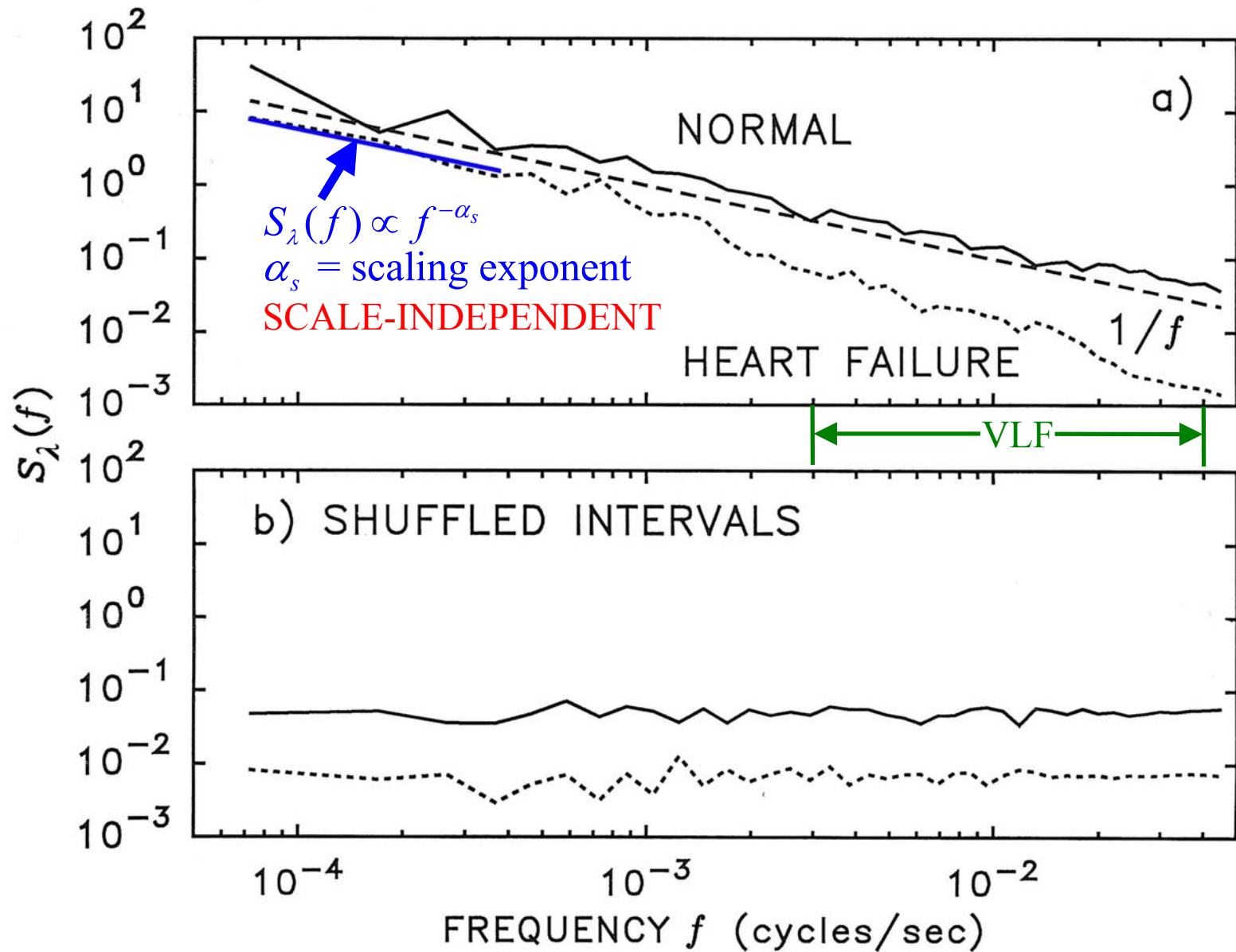
After Thurner, Feurstein & Teich, *Phys. Rev. Letters* **80**, 1544-1547 (1998).

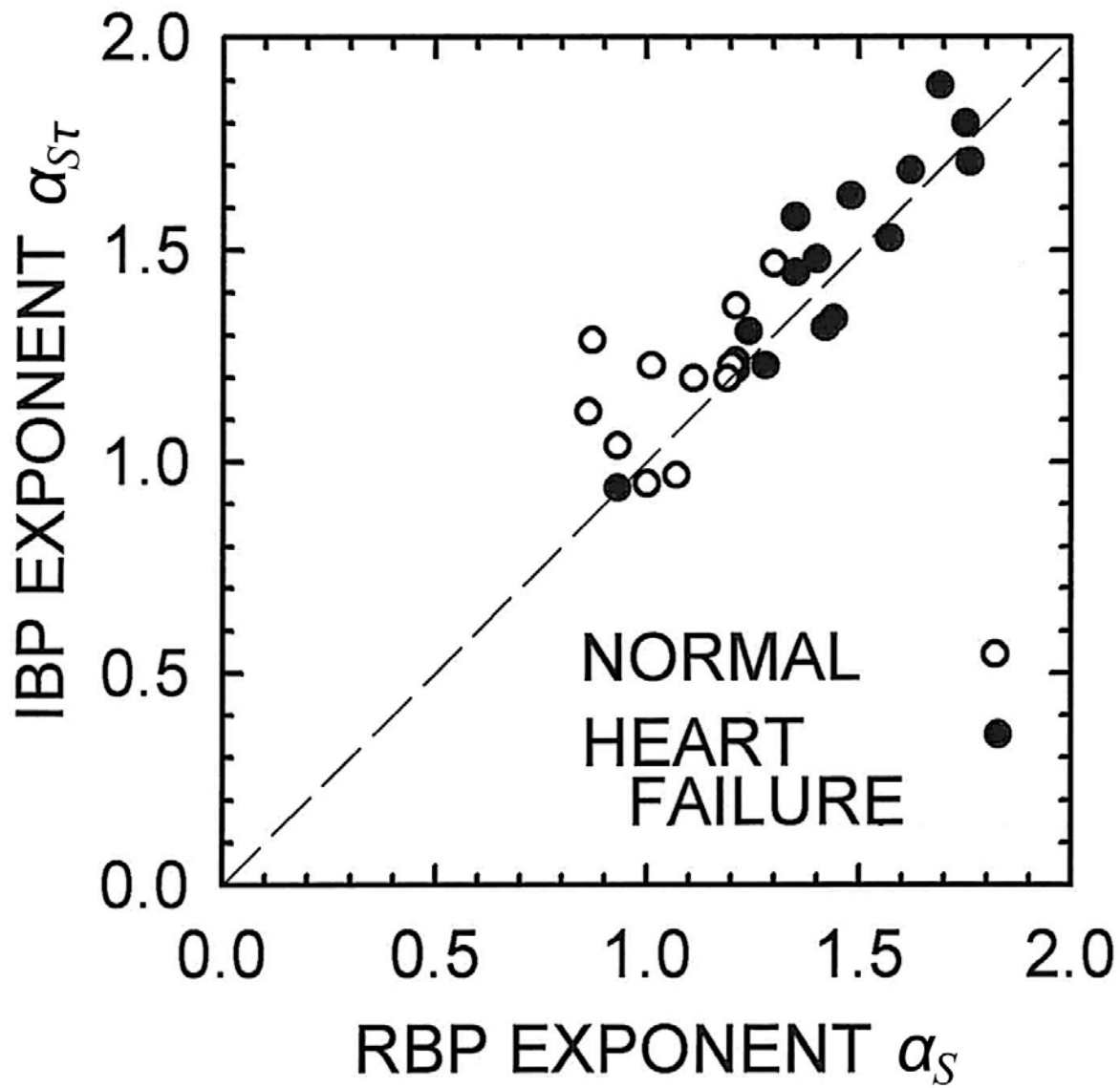


COUNT-BASED MEASURES

SPECTRAL ANALYSIS

GENERALIZED-RATE-BASED PERIODOGRAM

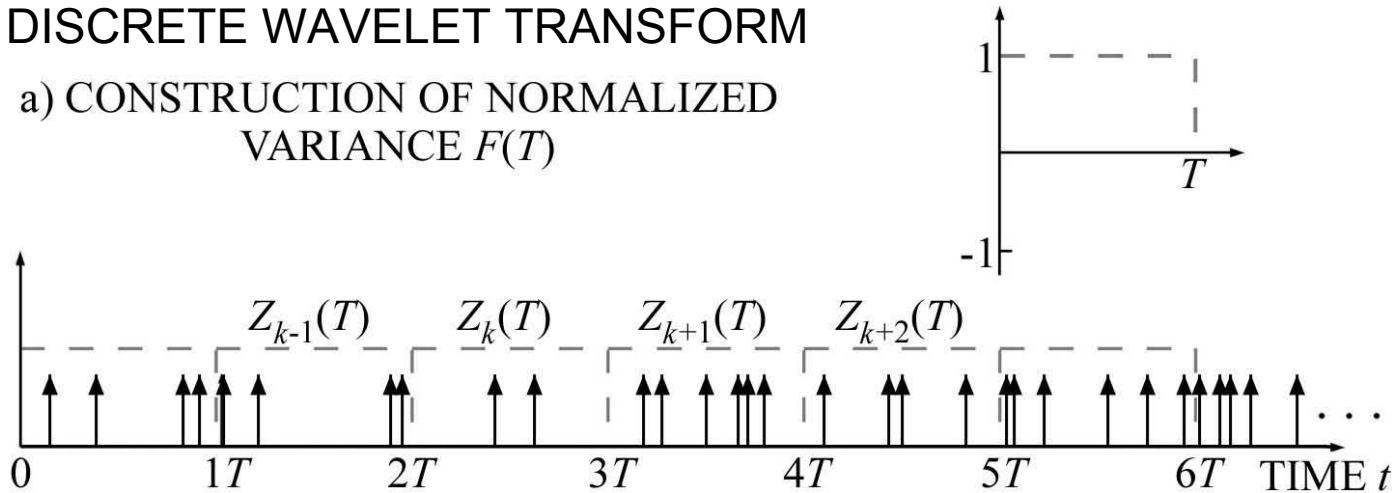




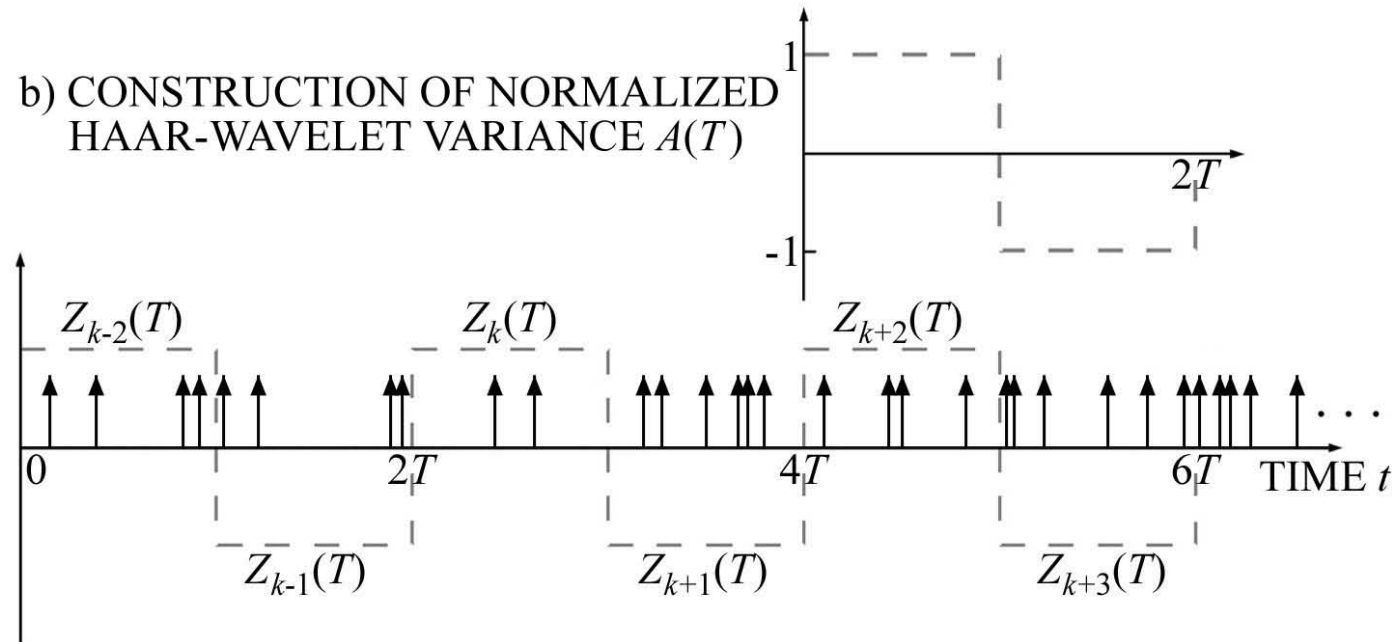
TIME-SCALE ANALYSIS

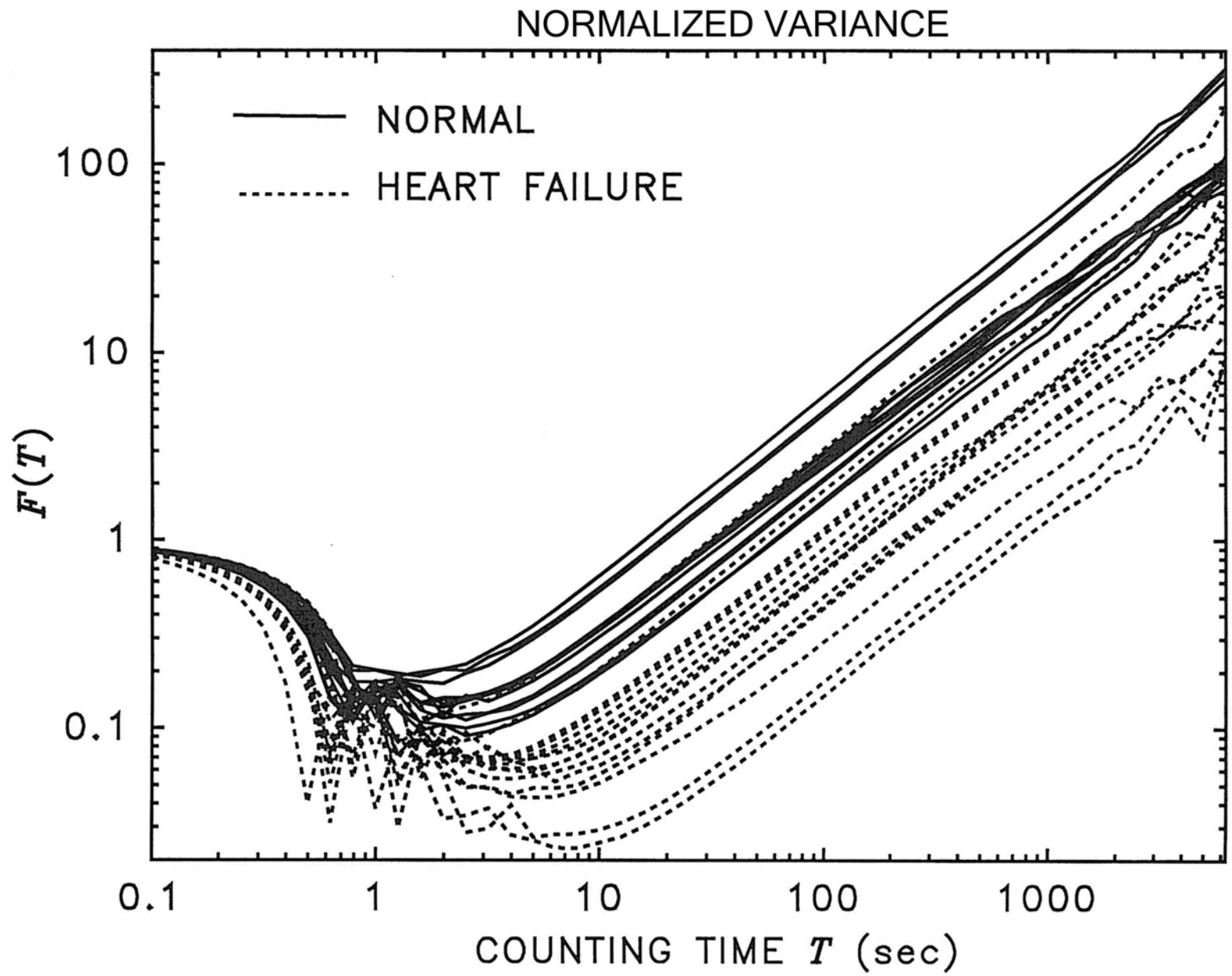
DISCRETE WAVELET TRANSFORM

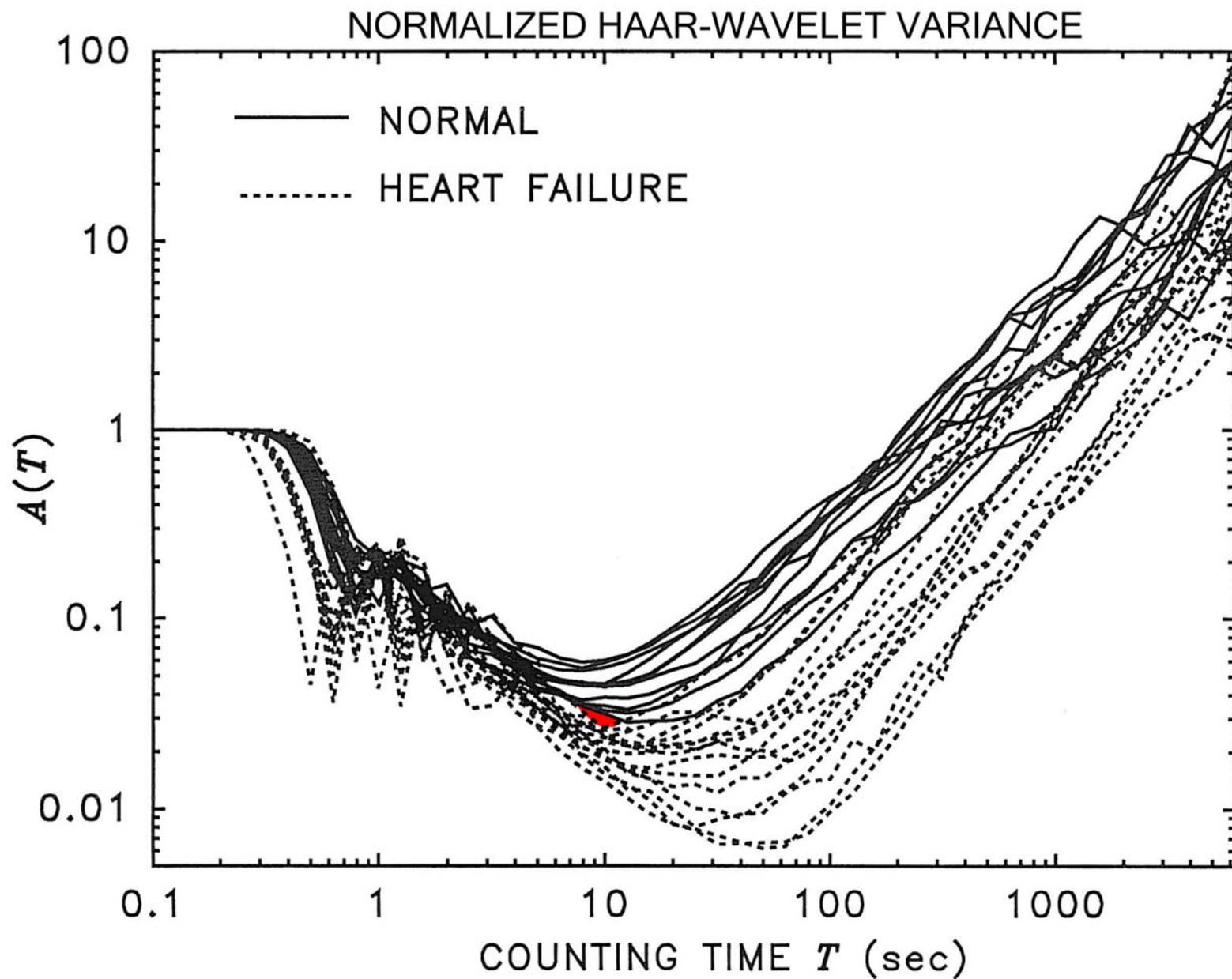
a) CONSTRUCTION OF NORMALIZED VARIANCE $F(T)$



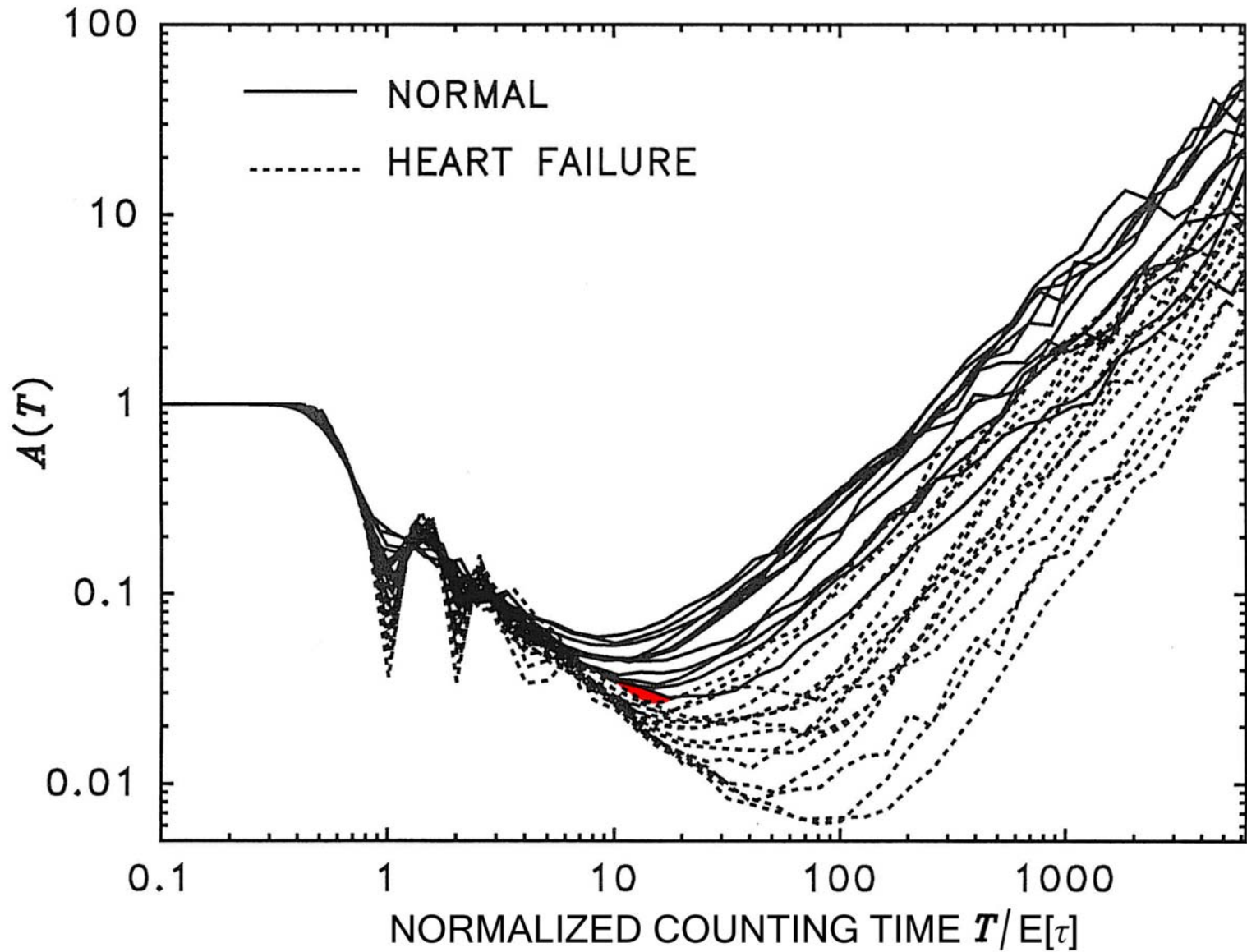
b) CONSTRUCTION OF NORMALIZED HAAR-WAVELET VARIANCE $A(T)$

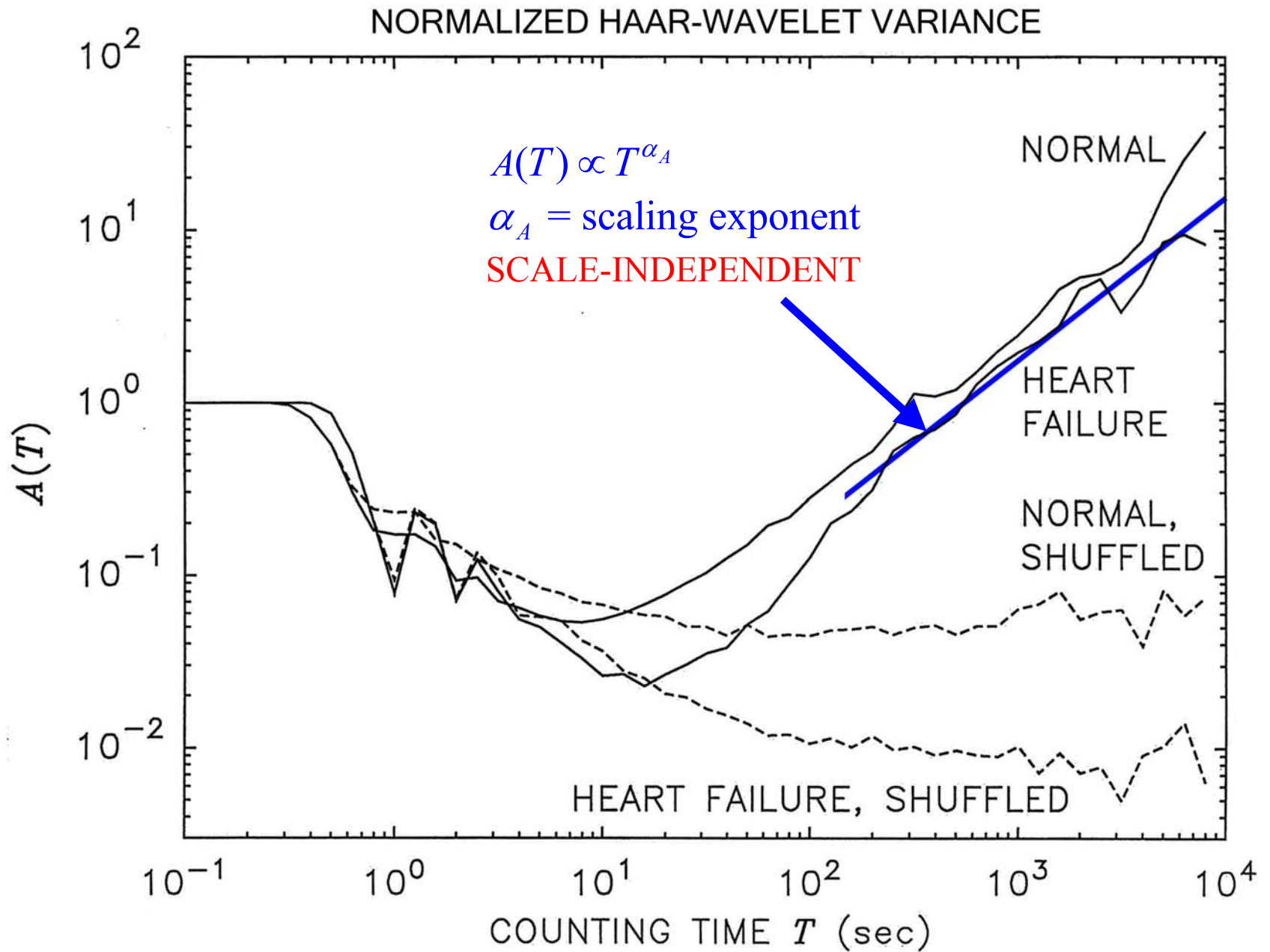




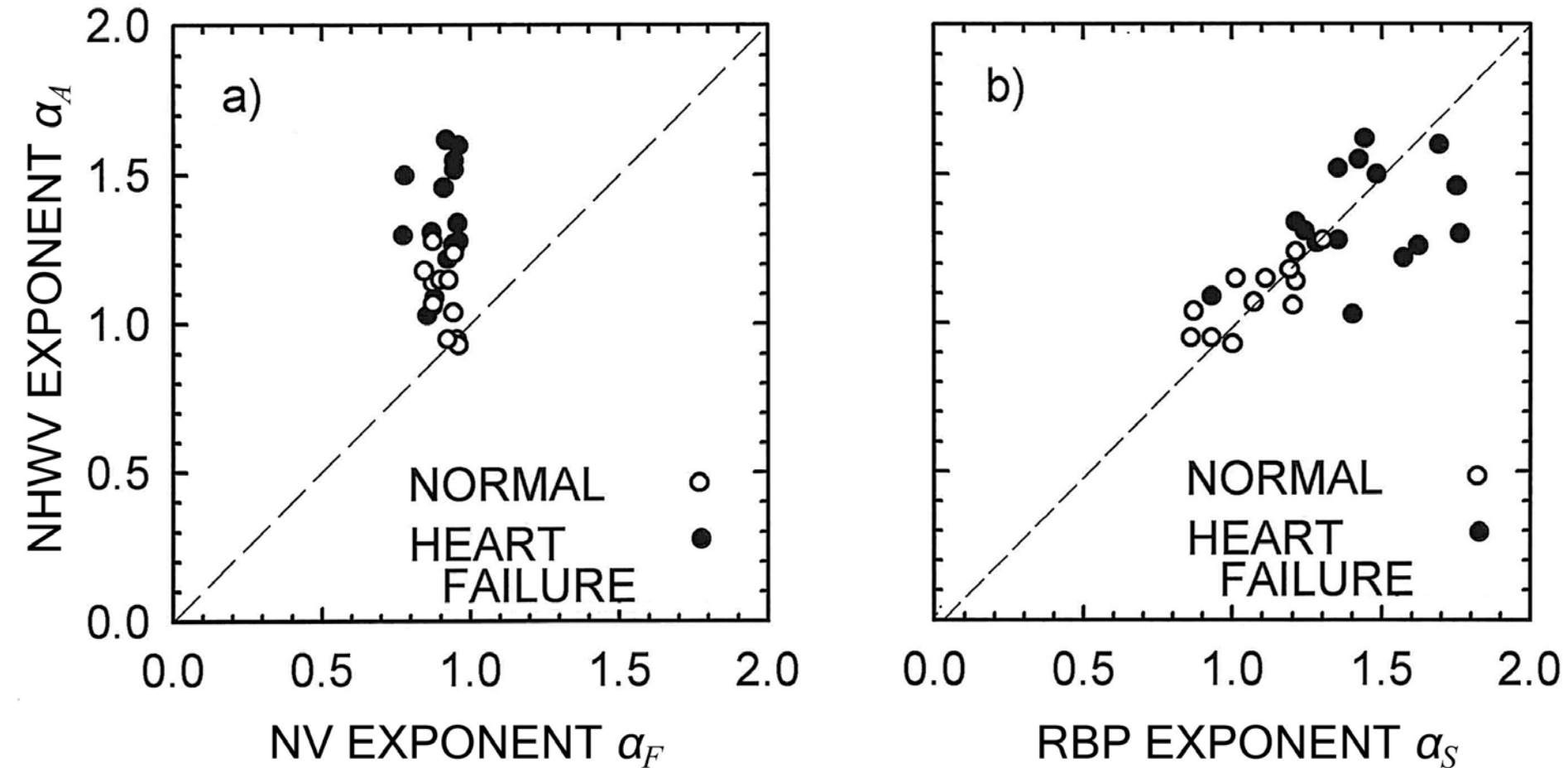


NHWV: NORMALIZED COUNTING TIME

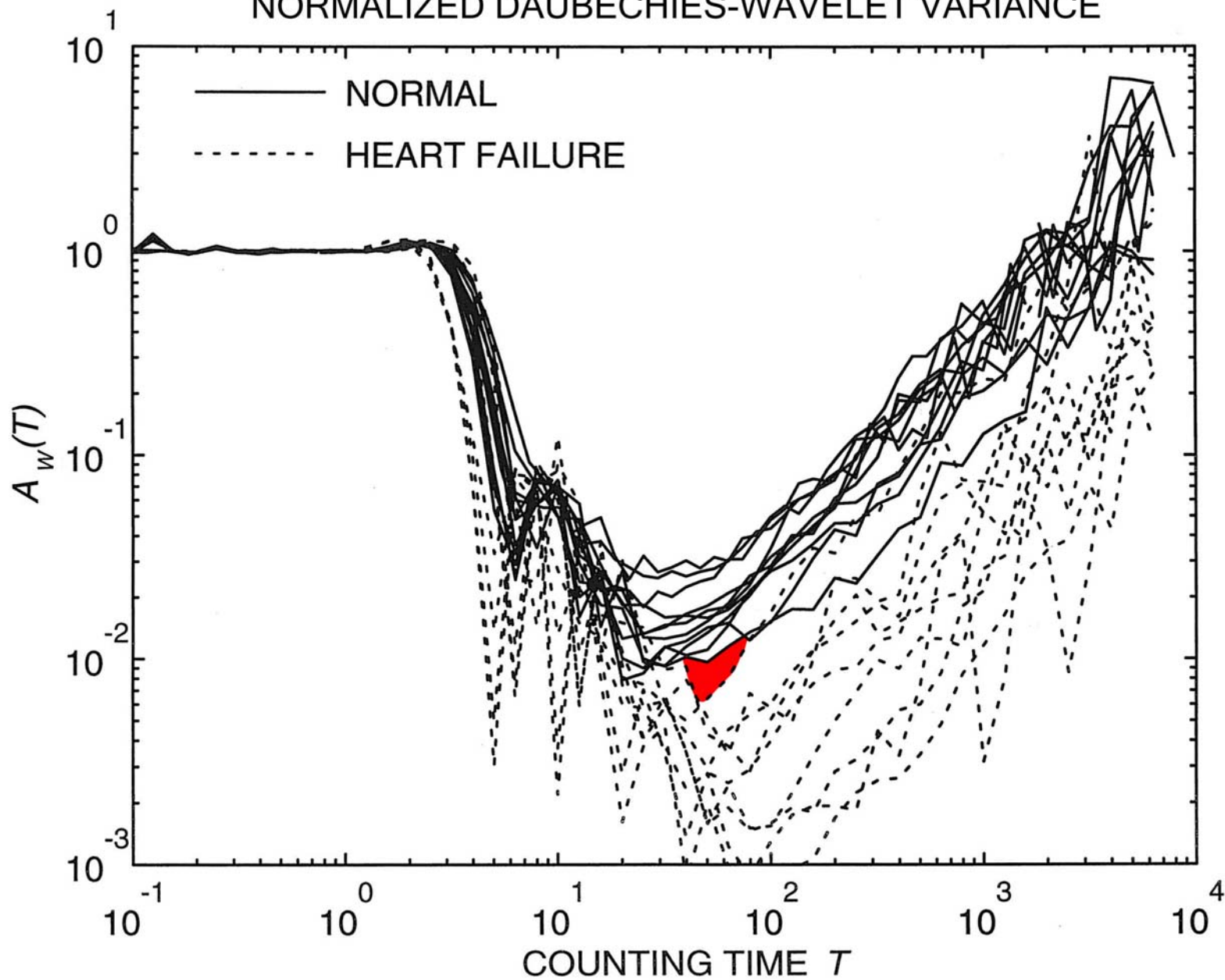




FRACTAL-EXPONENT ESTIMATORS



NORMALIZED DAUBECHIES-WAVELET VARIANCE

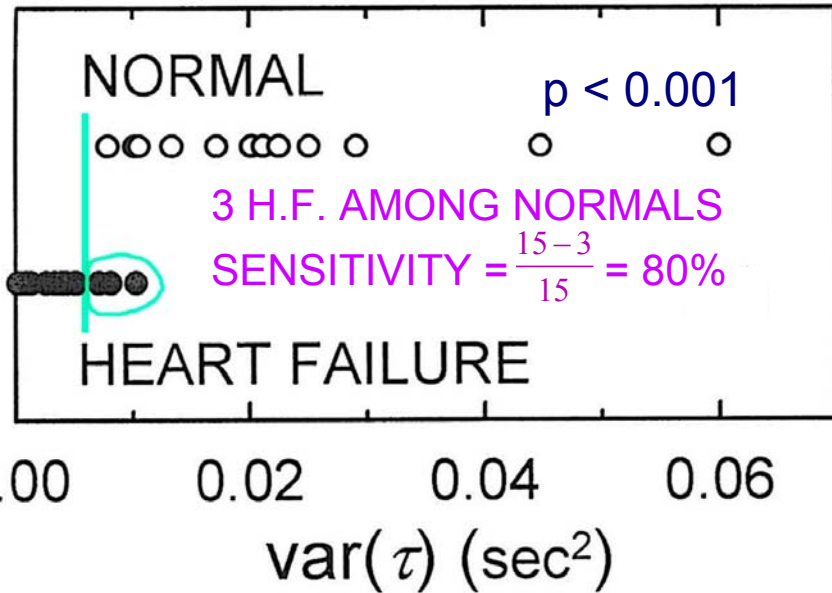


After Teich, *Proc. 18th Intern. Conf. IEEE Eng. Med. Biol. Soc.* **18**, 1128-1129 (1996).

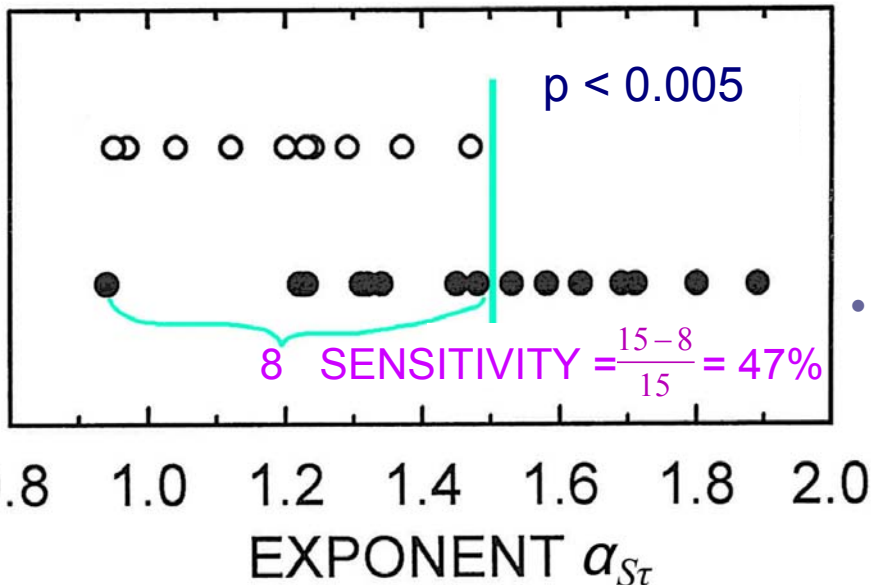


IDENTIFYING PATIENTS WITH CARDIAC DYSFUNCTION

MEASURES OF STATISTICAL SIGNIFICANCE



SCALE-DEPENDENT



SCALE-INDEPENDENT

- p VALUE, d', AND VARIANTS (rely on Gaussian assumption)

- SENSITIVITY/SPECIFICITY MEASURES OF CLINICAL SIGNIFICANCE (distribution free)

SENSITIVITY \equiv proportion of heart-failure patients that are properly identified

e.g., Hypothesis that all normal patients are so identified \equiv 100% SPECIFICITY

- ROC CURVES & AREA UNDER ROC

After Turcott & Teich,
Ann. Biomed. Eng. **24**, 269-293 (1996)

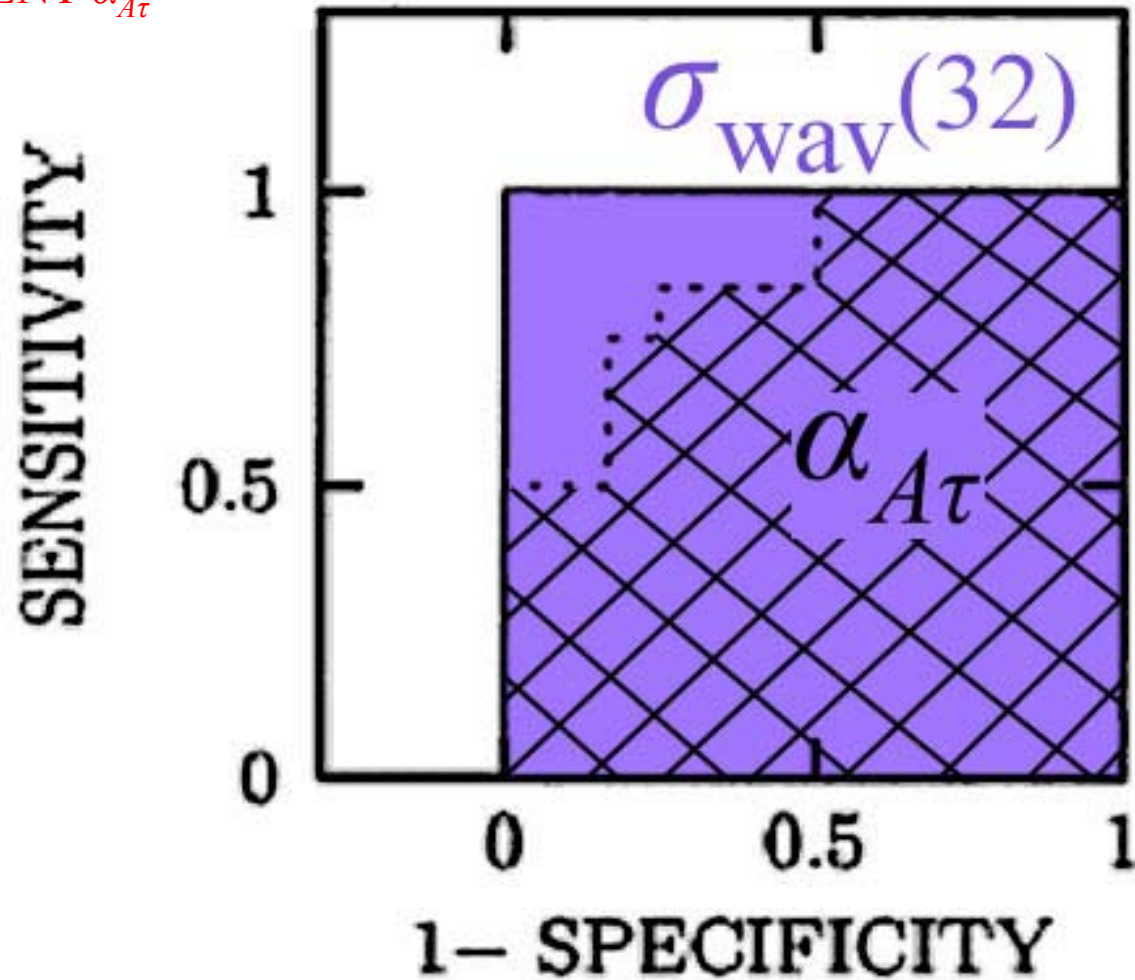
M. C. Teich 2004

ROC CURVES & AREA UNDER ROC

SCALE-DEPENDENT σ_{wav} (32)

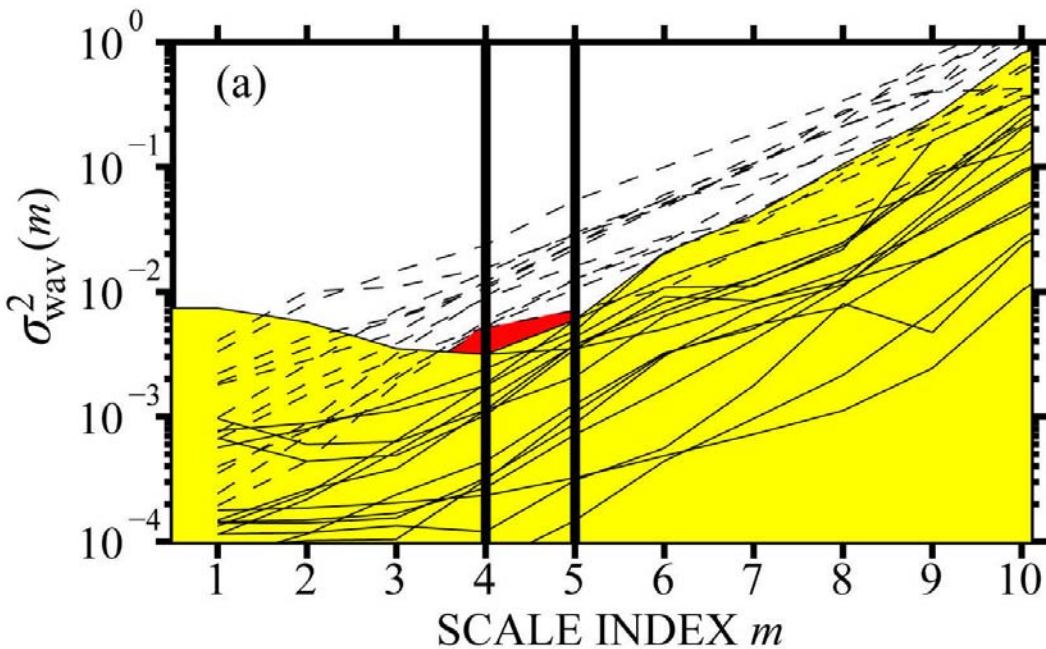
SCALE-INDEPENDENT $\alpha_{A\tau}$

HAAR WAVELET

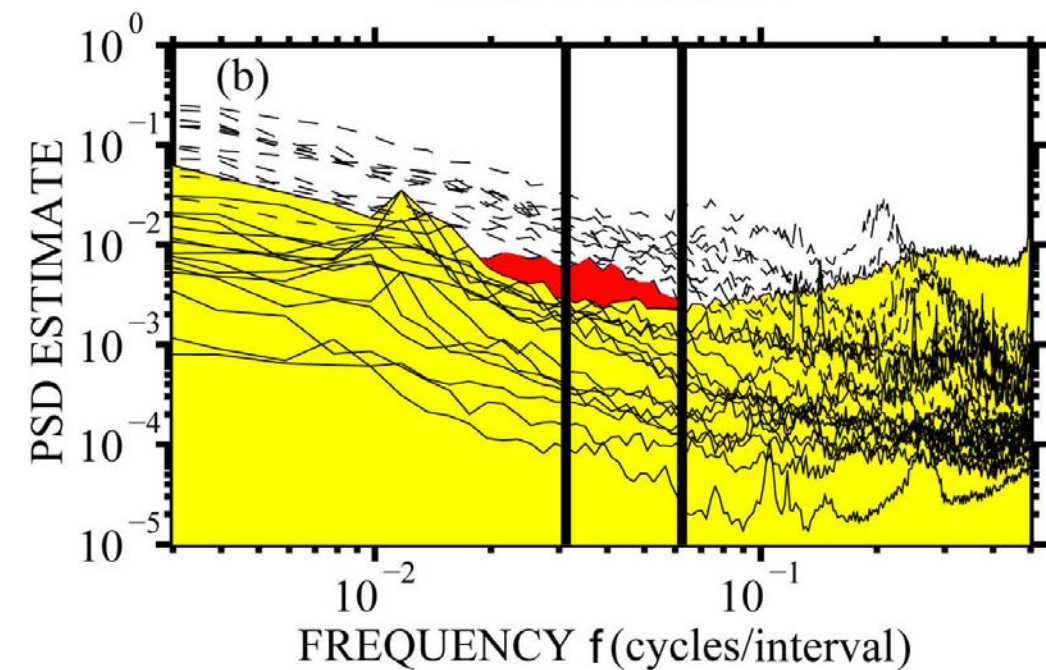


After Thurner, Feurstein, Lowen & Teich,
Phys. Rev. Letters **81**, 5688-5691 (1998).

M. C. Teich 2004



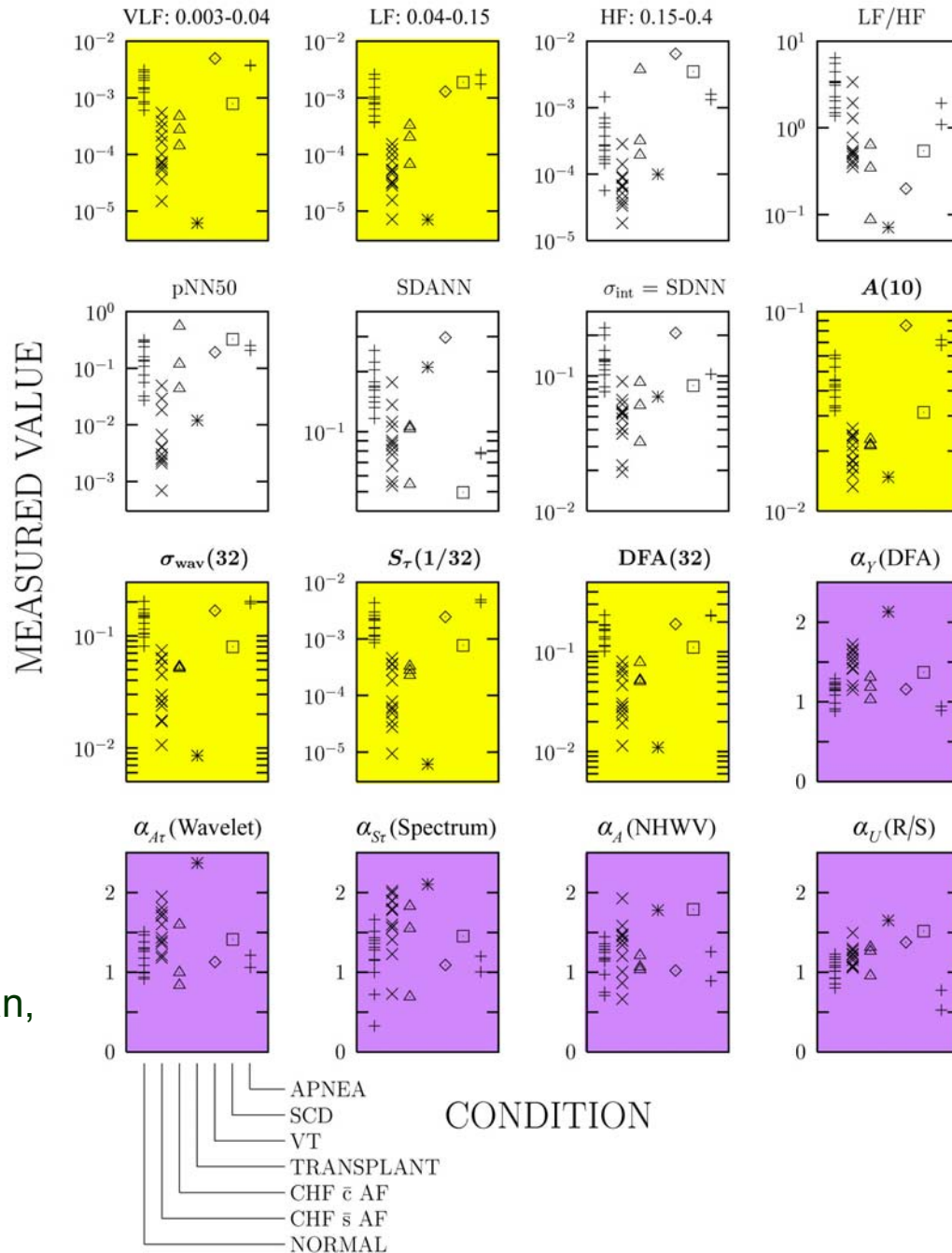
$$16 \leq 2^m \text{ (scale)} \leq 32$$



$$\frac{1}{32} < f \text{ (cycles/interval)} < \frac{1}{16}$$

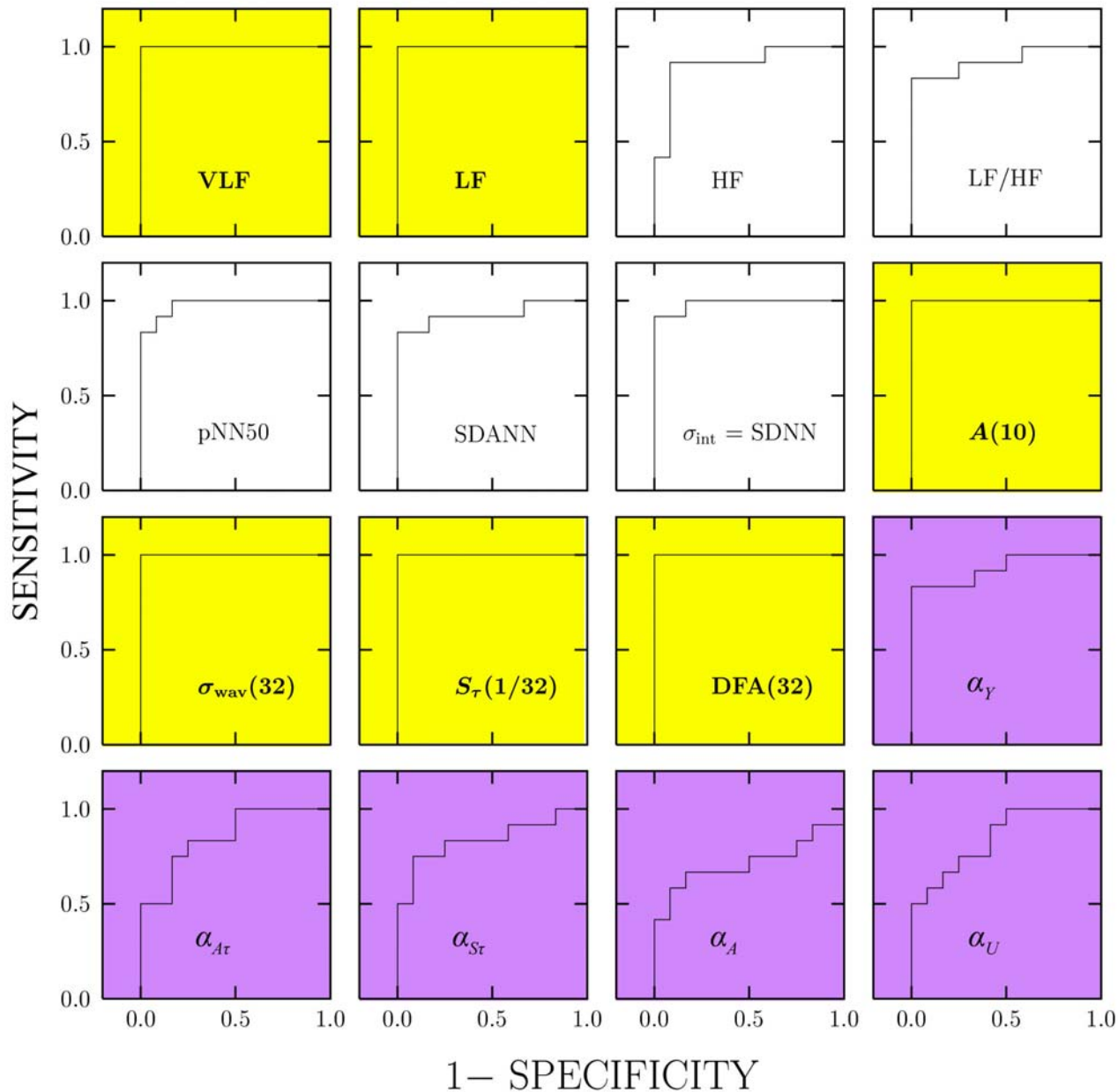
After Heneghan, Lowen, & Teich
Proc. 1999 ICASSP (Phoenix, AZ)
paper SPTM-8.2.

INDIVIDUAL VALUES: DATA

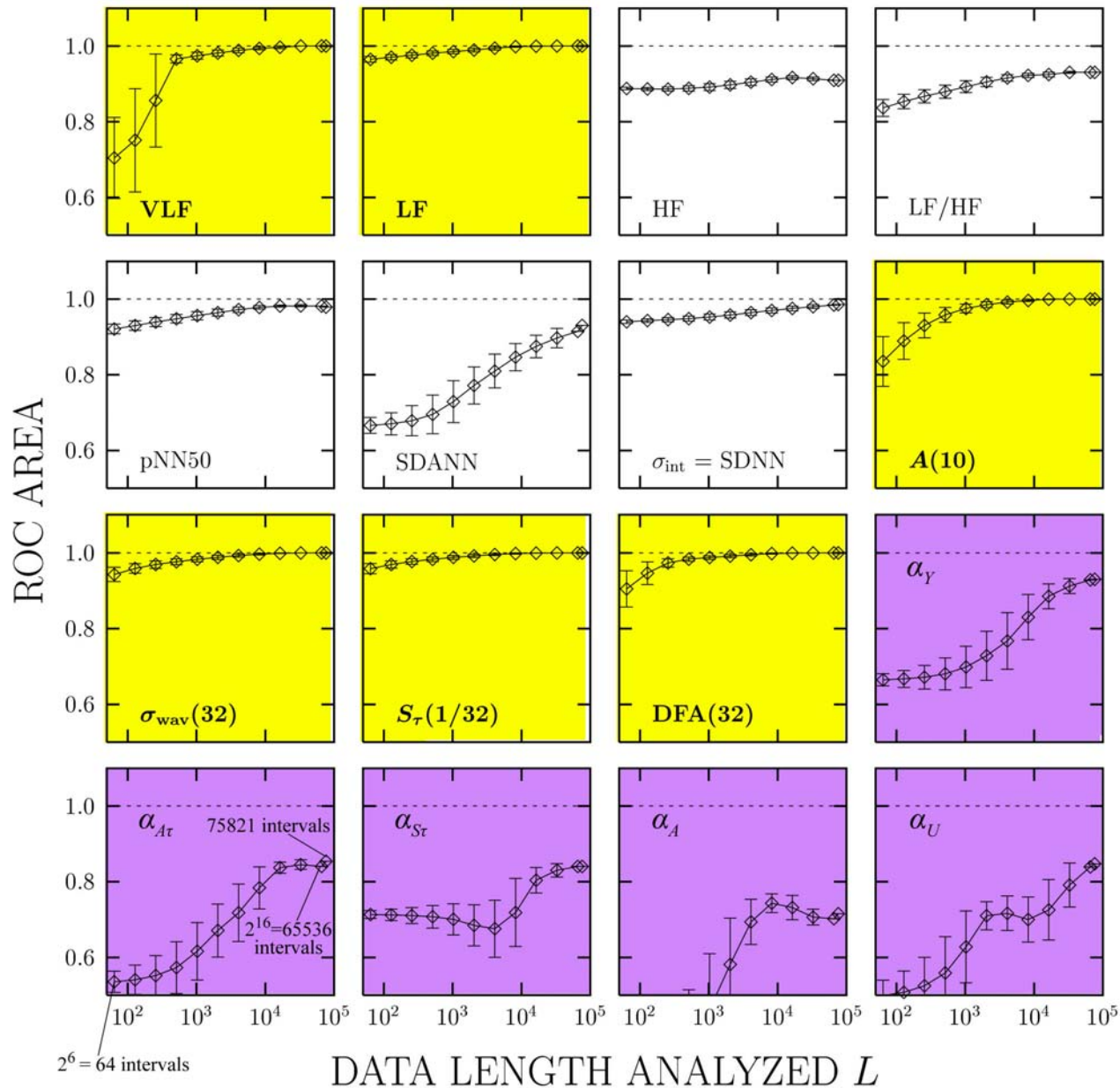


After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan, in *Nonlinear Biomedical Signal Processing*, vol II, M. Akay, Ed. (IEEE Press, NY, 2001), pp. 159-213.

ROC CURVES: NORMAL AND CHF DATA (\bar{S} AF; 75821 intervals)

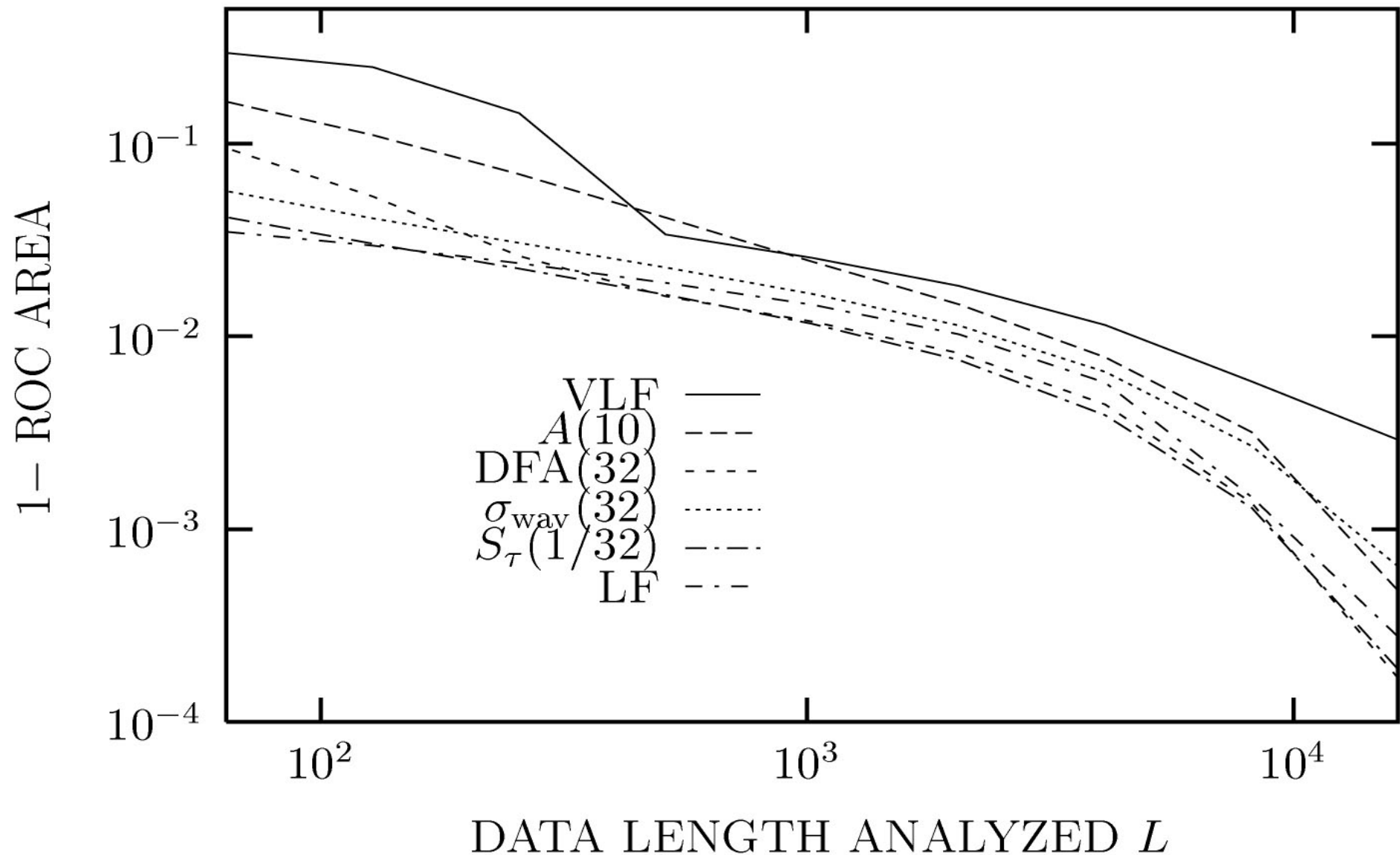


ROC-AREA CURVES: NORMAL AND CHF DATA



After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan, in *Nonlinear Biomedical Signal Processing*, vol II, M. Akay, Ed. (IEEE Press, NY, 2001), pp. 159-213.

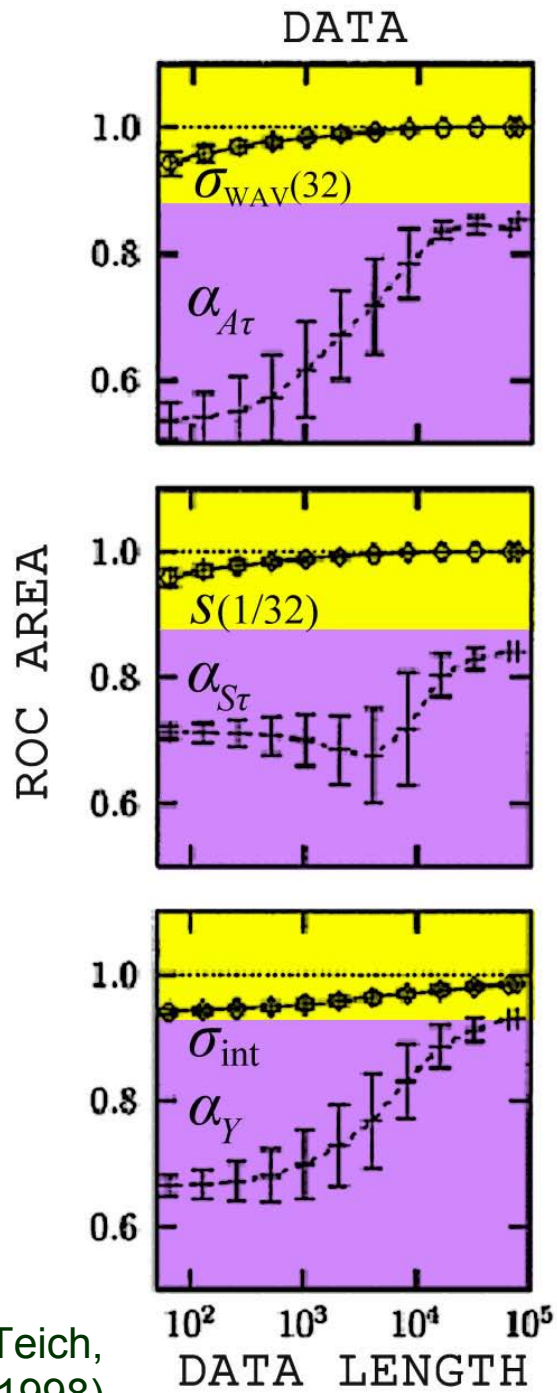
M. C. Teich 2004



After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan,
 in *Nonlinear Biomedical Signal Processing*, vol II, M. Akay, Ed.
 (IEEE Press, NY, 2001), pp. 159-213.

Measure	Execution Time (msec)
VLF, LF, HF, and LF/HF	330
pNN50	40
SDANN	160
σ_{int}	190
$A(10)$	160
$\sigma_{\text{wav}}(32)$	20
$S_{\tau}(1/32)$	60
DFA (32)	650,090
α_{γ}	650,110
$\alpha_{A\tau}$	220
$\alpha_{S\tau}$	920
α_A	610
α_U	570

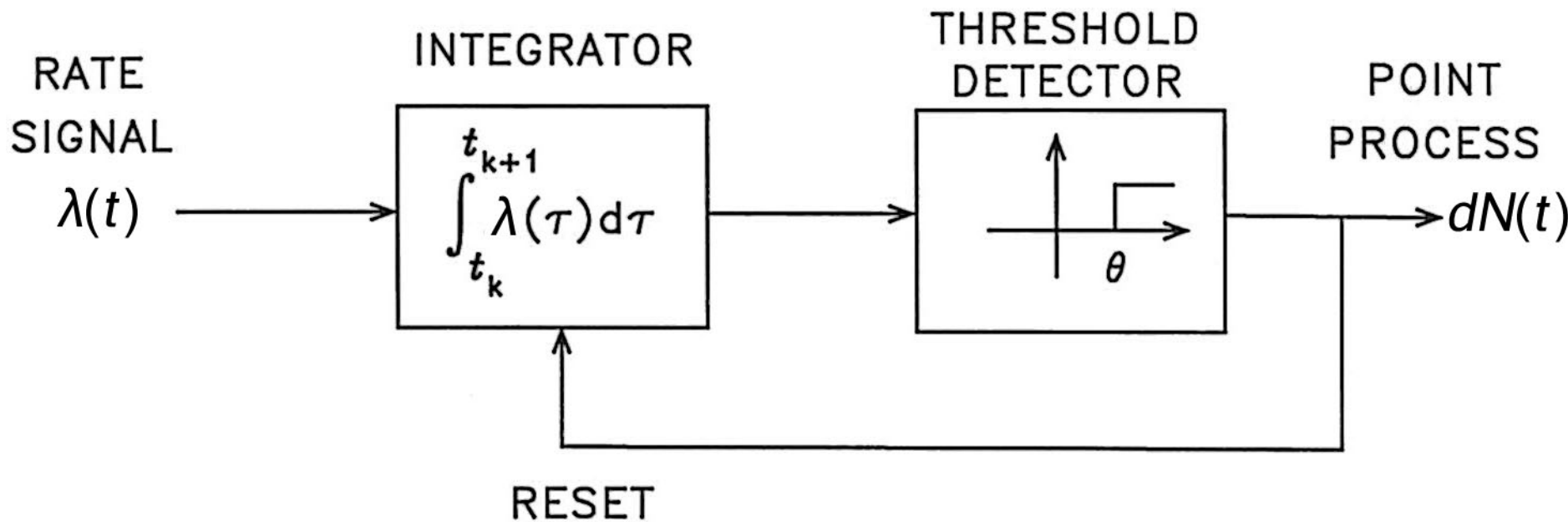
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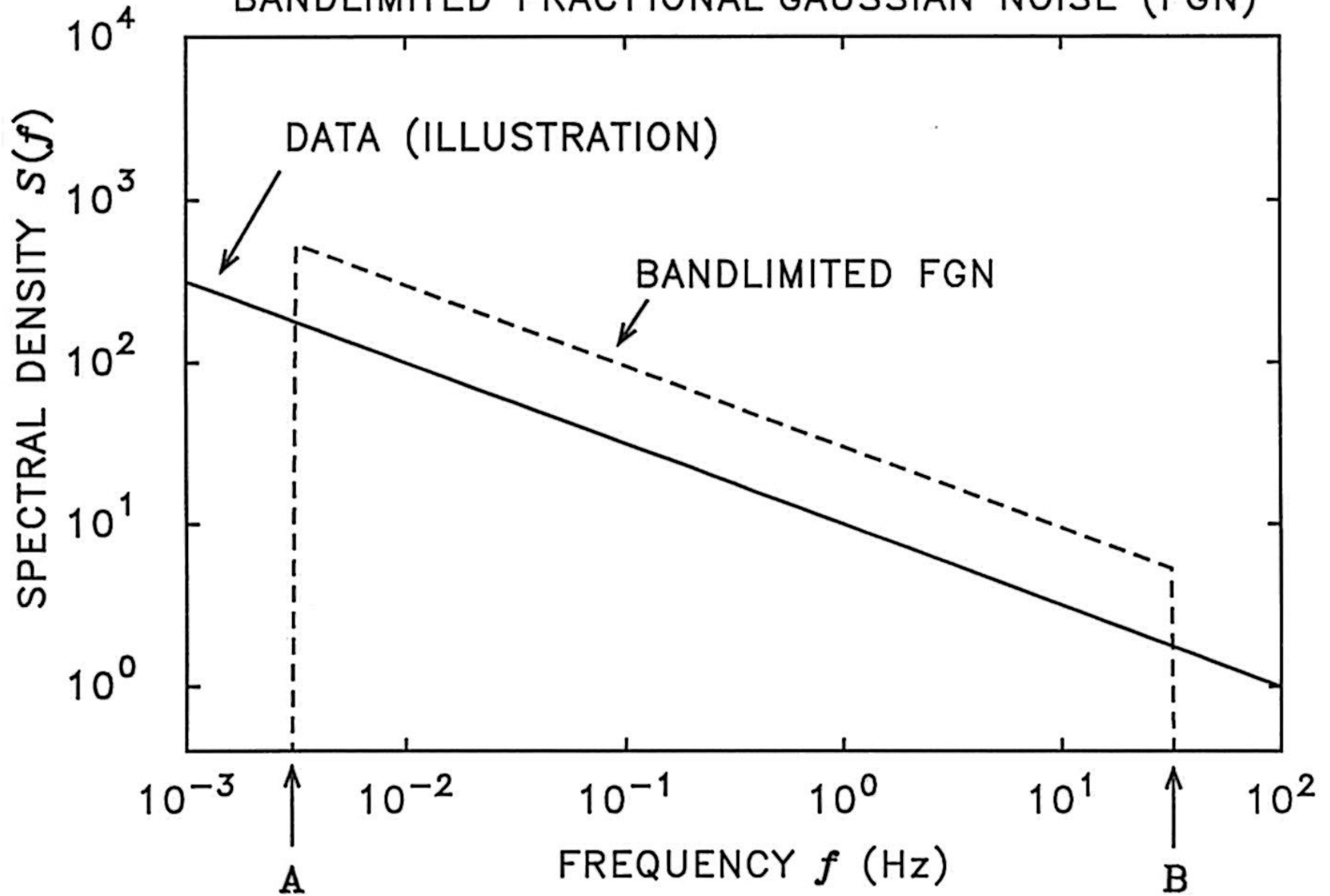
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Phys. Rev. Letters **81**, 5688-5691 (1998).



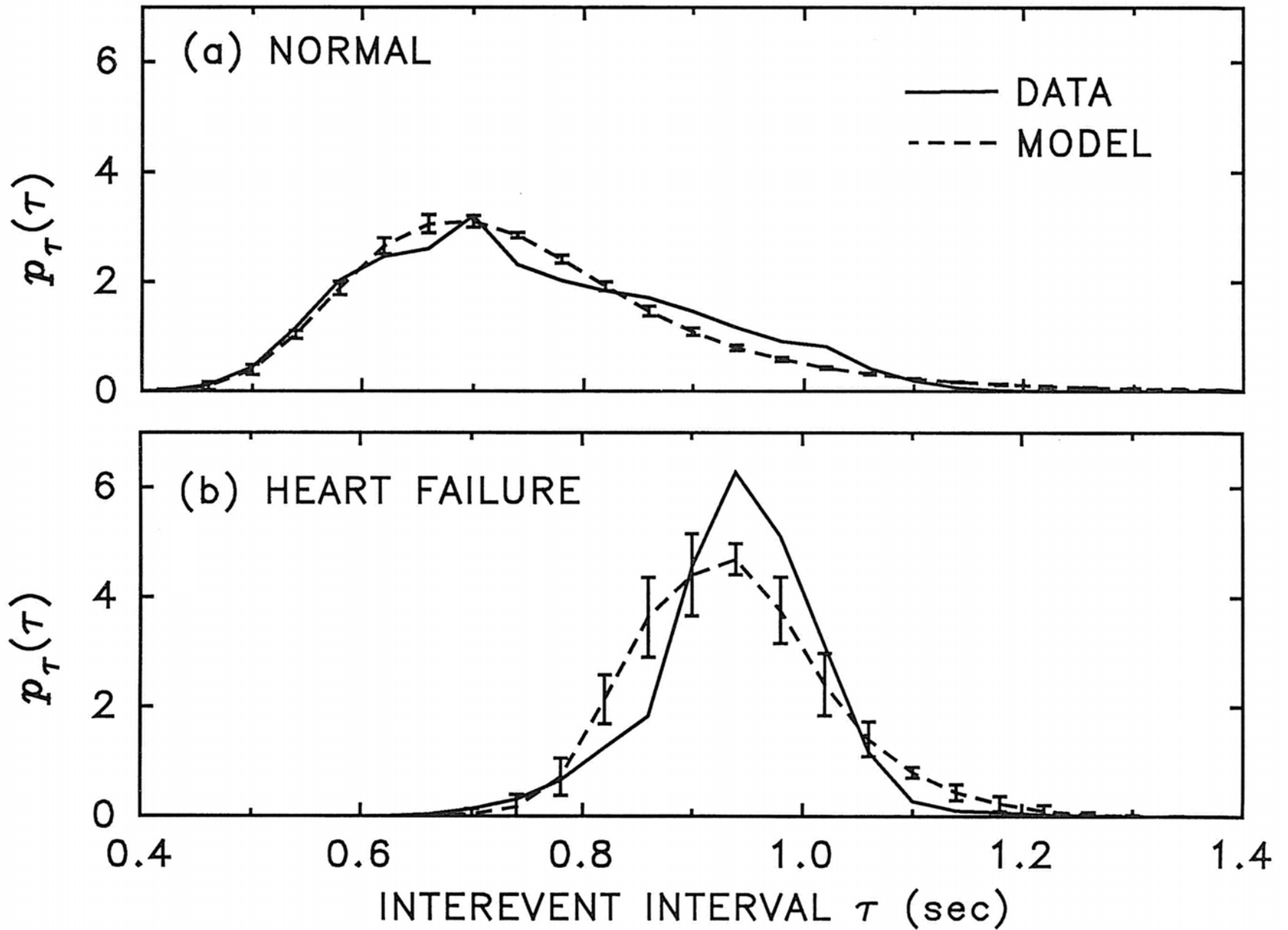
THEORY



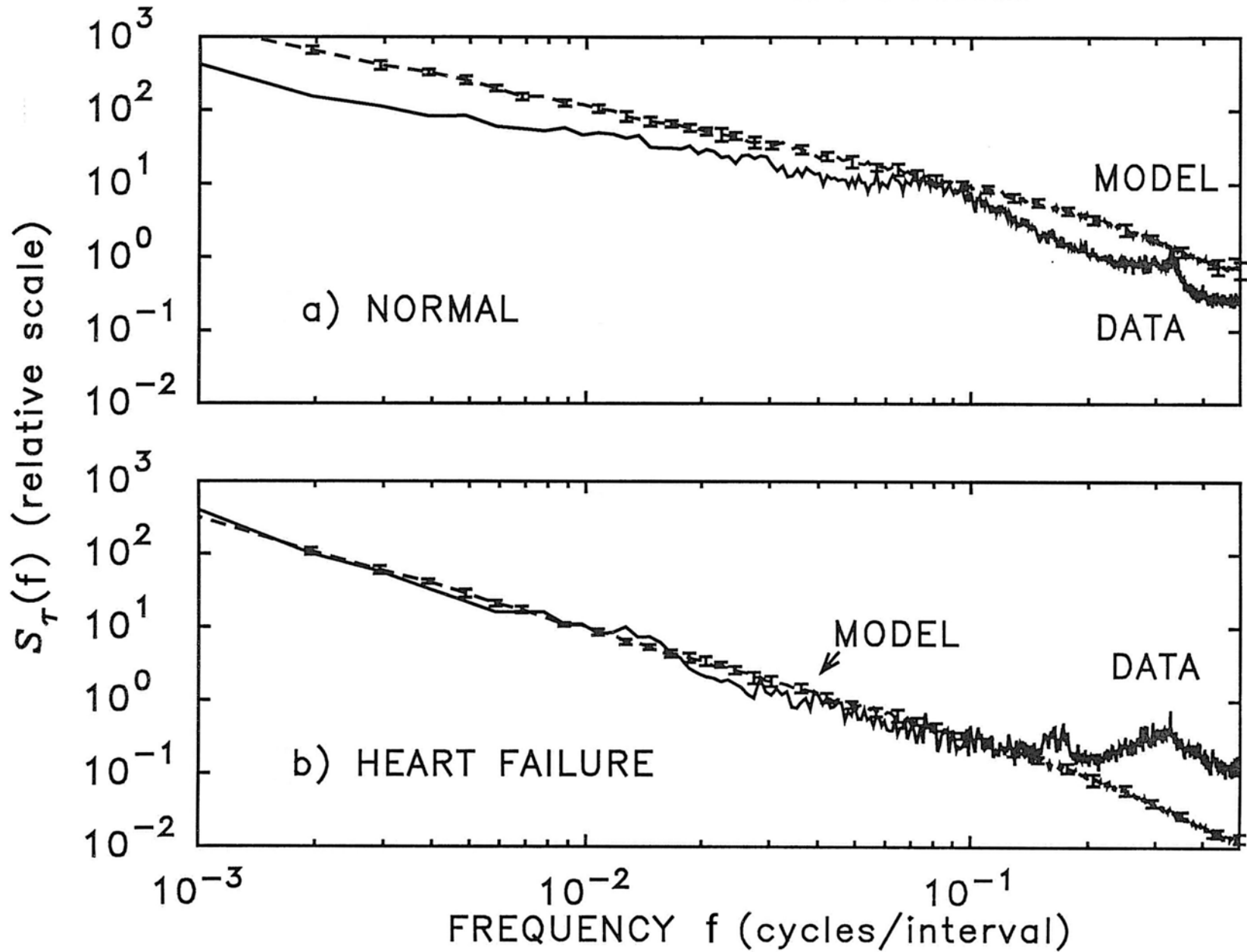
FREQUENCY-DOMAIN REPRESENTATION OF BANDLIMITED FRACTIONAL GAUSSIAN NOISE (FGN)



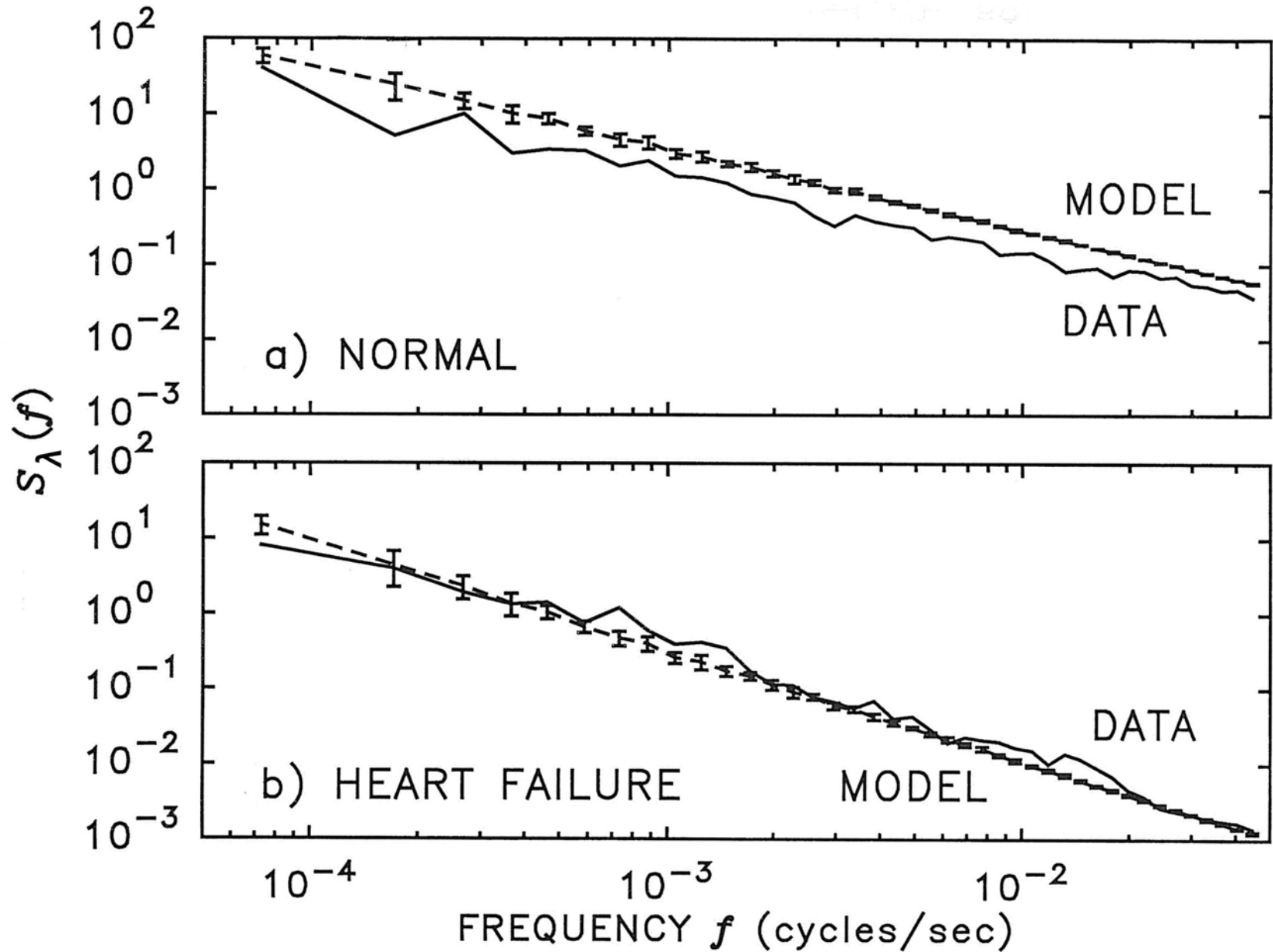
INTEREVENT-INTERVAL HISTOGRAM



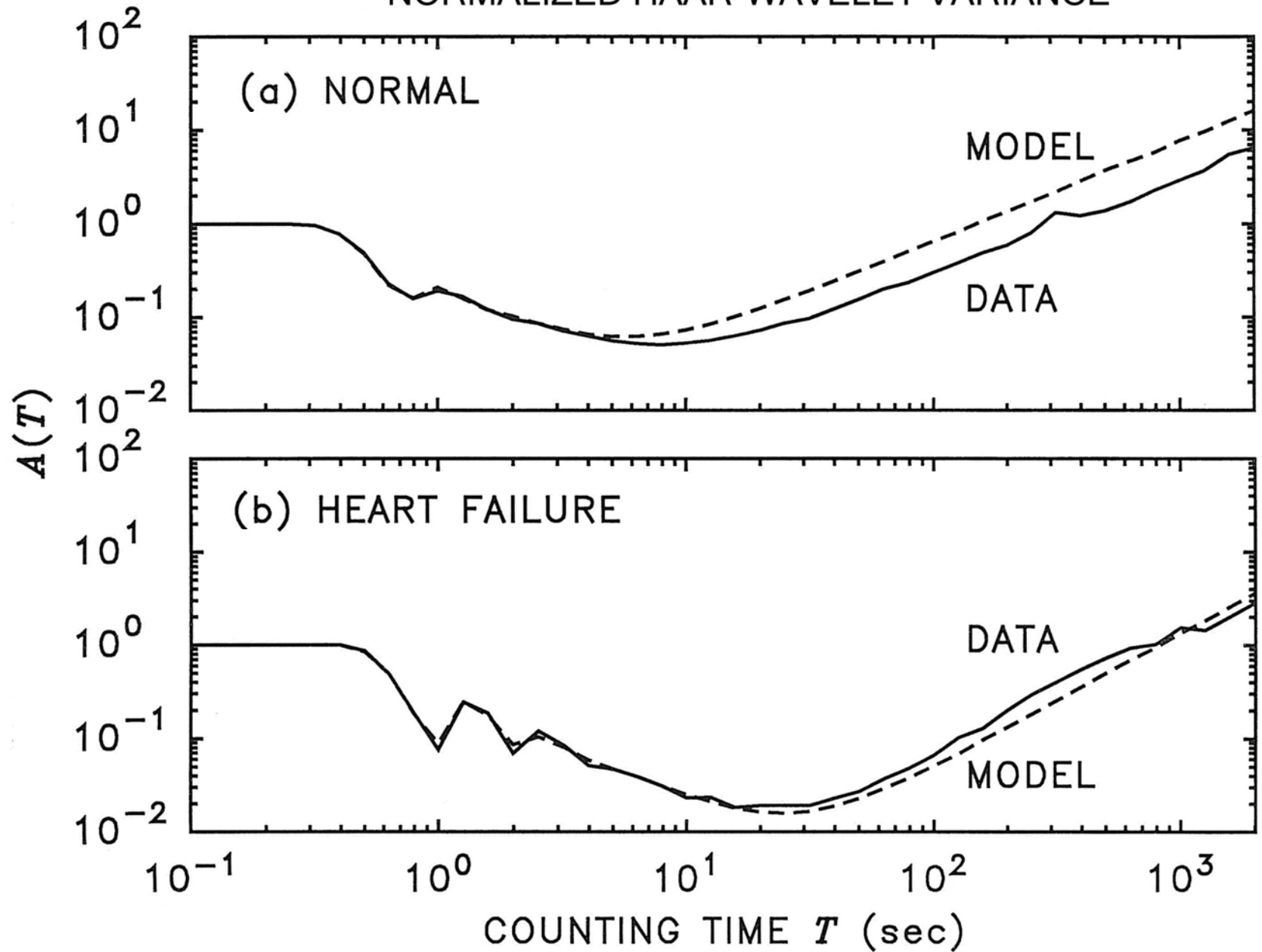
INTERVAL-BASED PERIODOGRAM



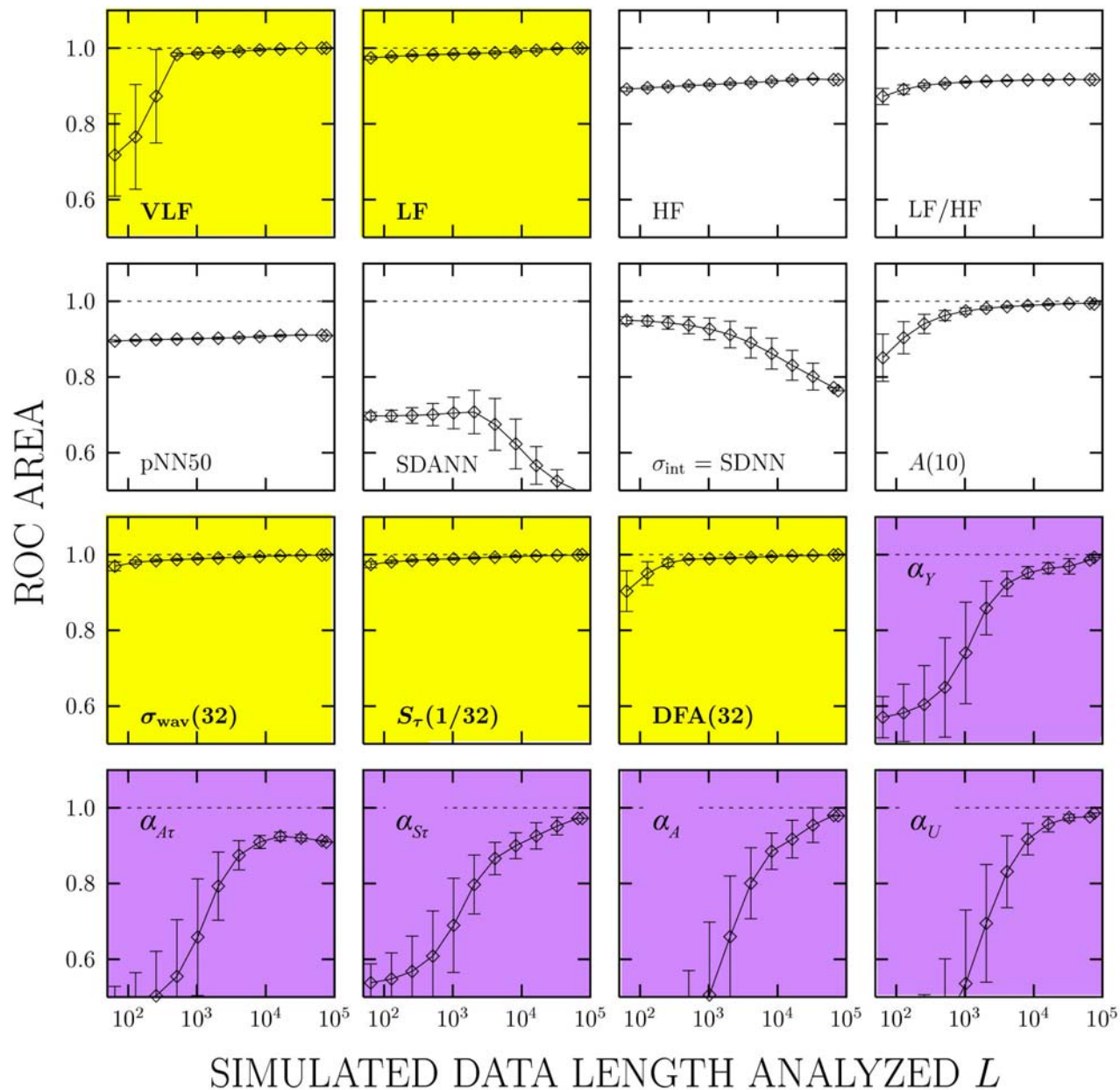
GENERALIZED-RATE-BASED PERIDOGRAM



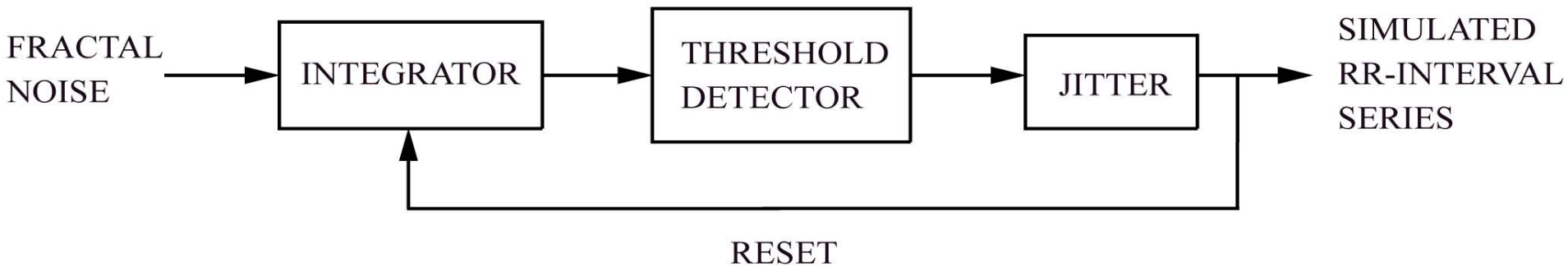
NORMALIZED HAAR-WAVELET VARIANCE



ROC-AREA CURVES: SIMULATION



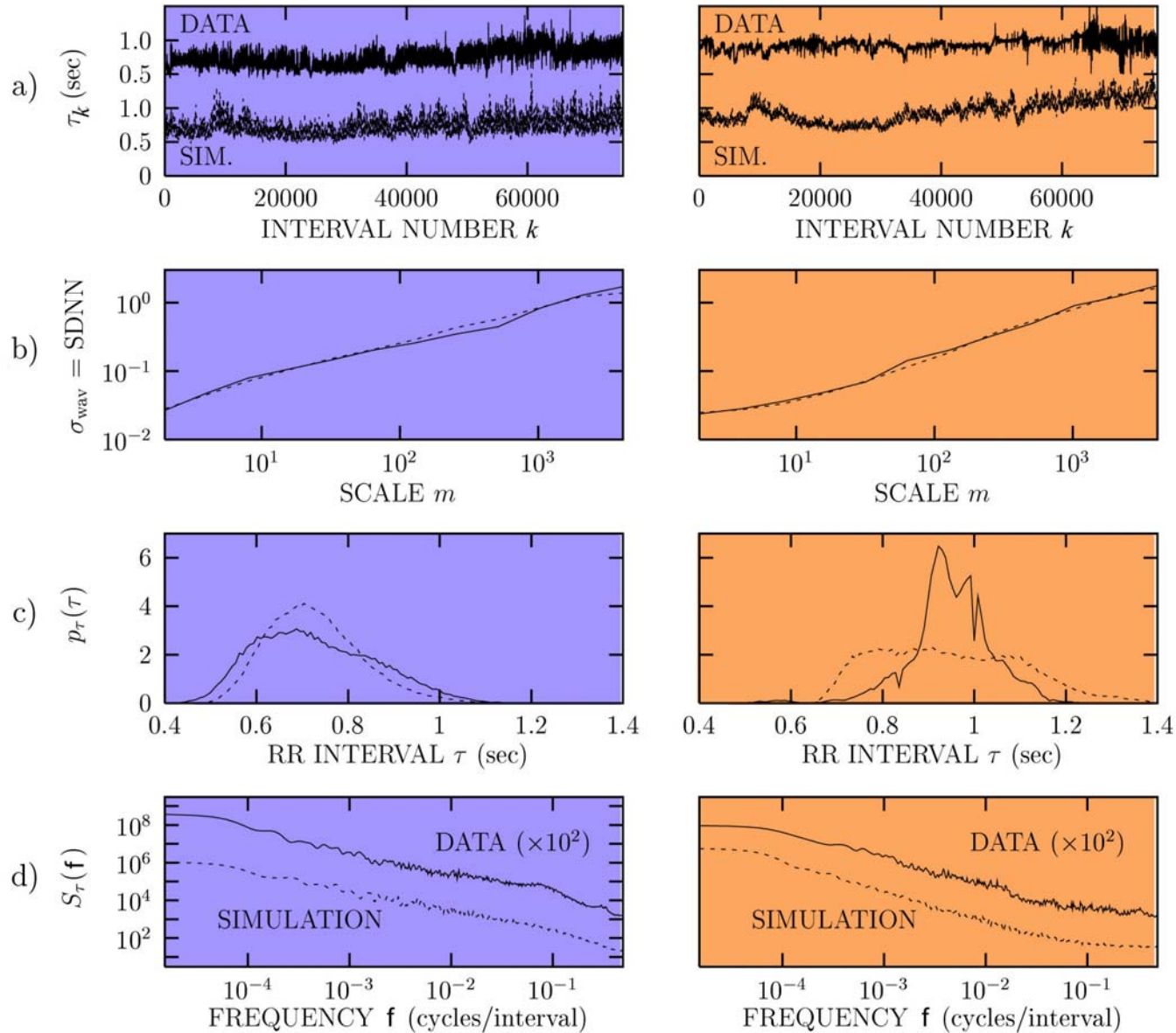
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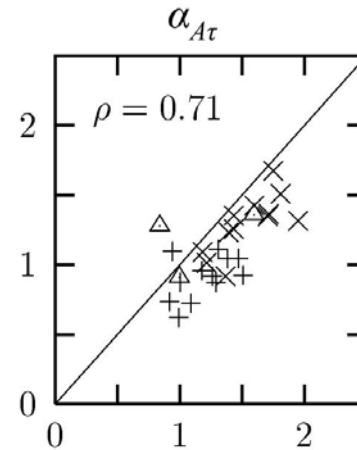
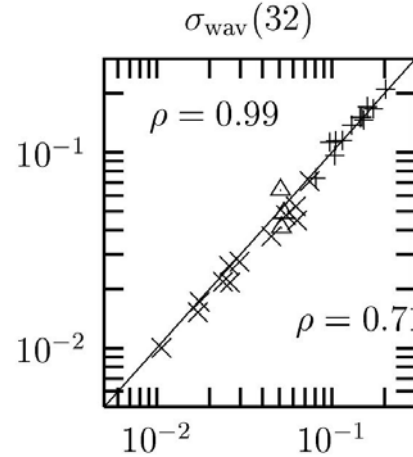
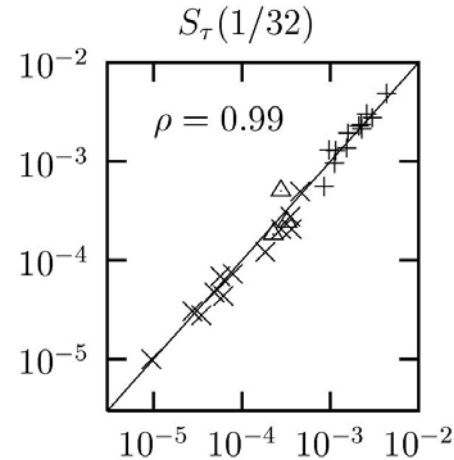
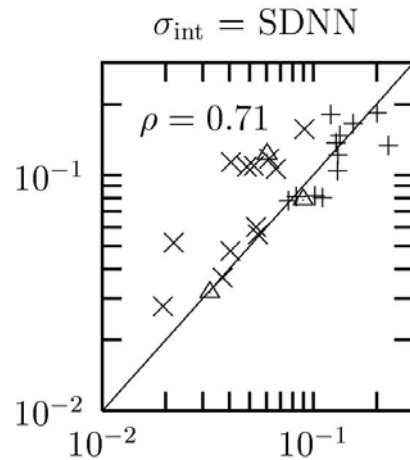
NORMAL

HEART FAILURE



SIMULATION ACCURACY

SIMULATION



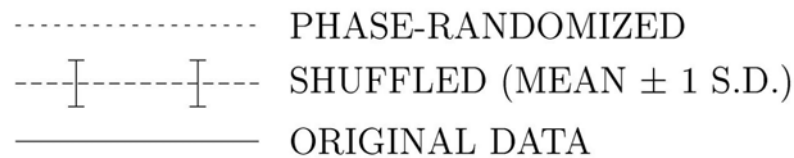
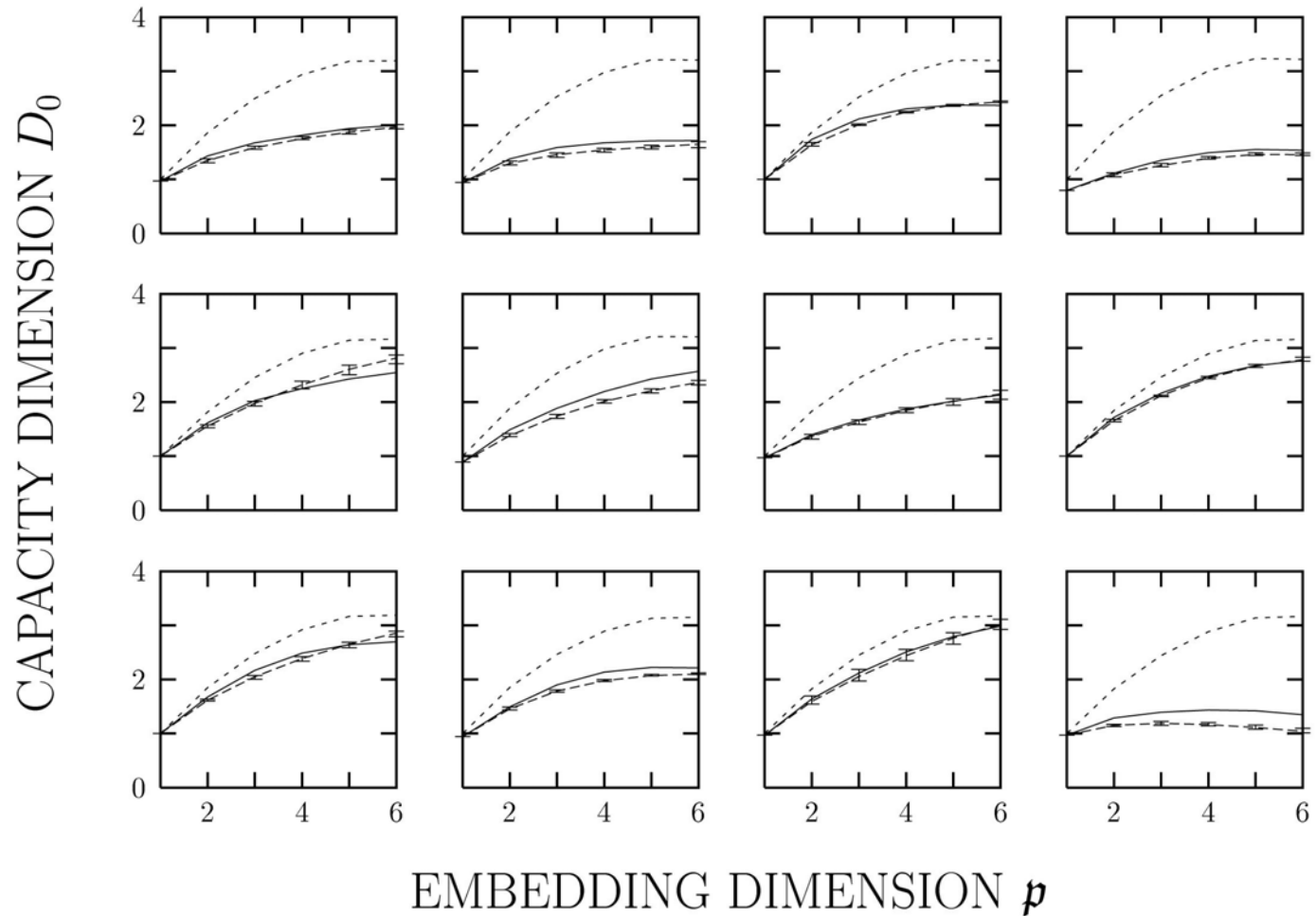
DATA

NORMAL +
 HEART FAILURE \bar{s} AF ×
 HEART FAILURE \bar{c} AF △

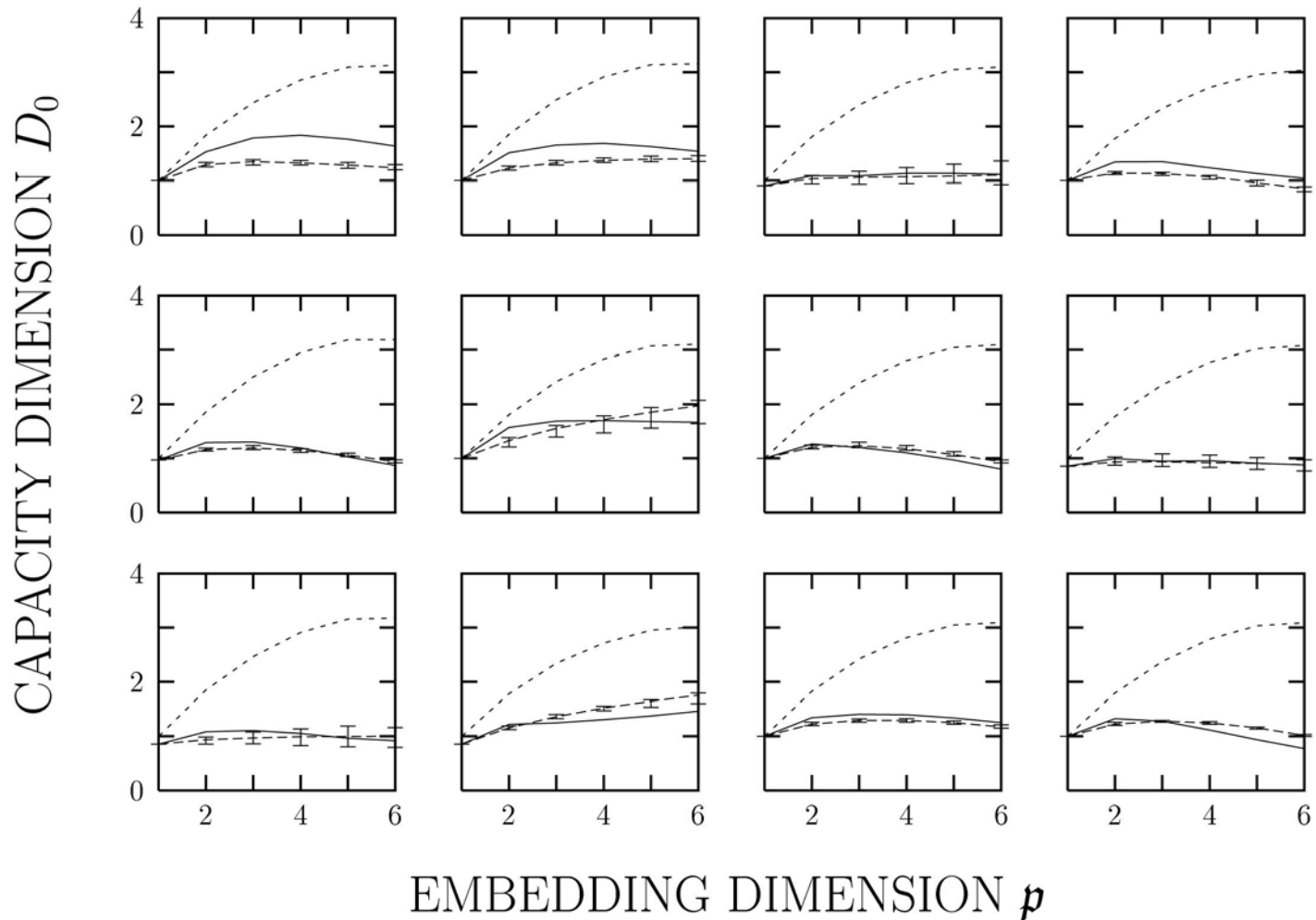


DOES THE HEARTBEAT REFLECT DETERMINISTIC CHAOS?

NORMAL



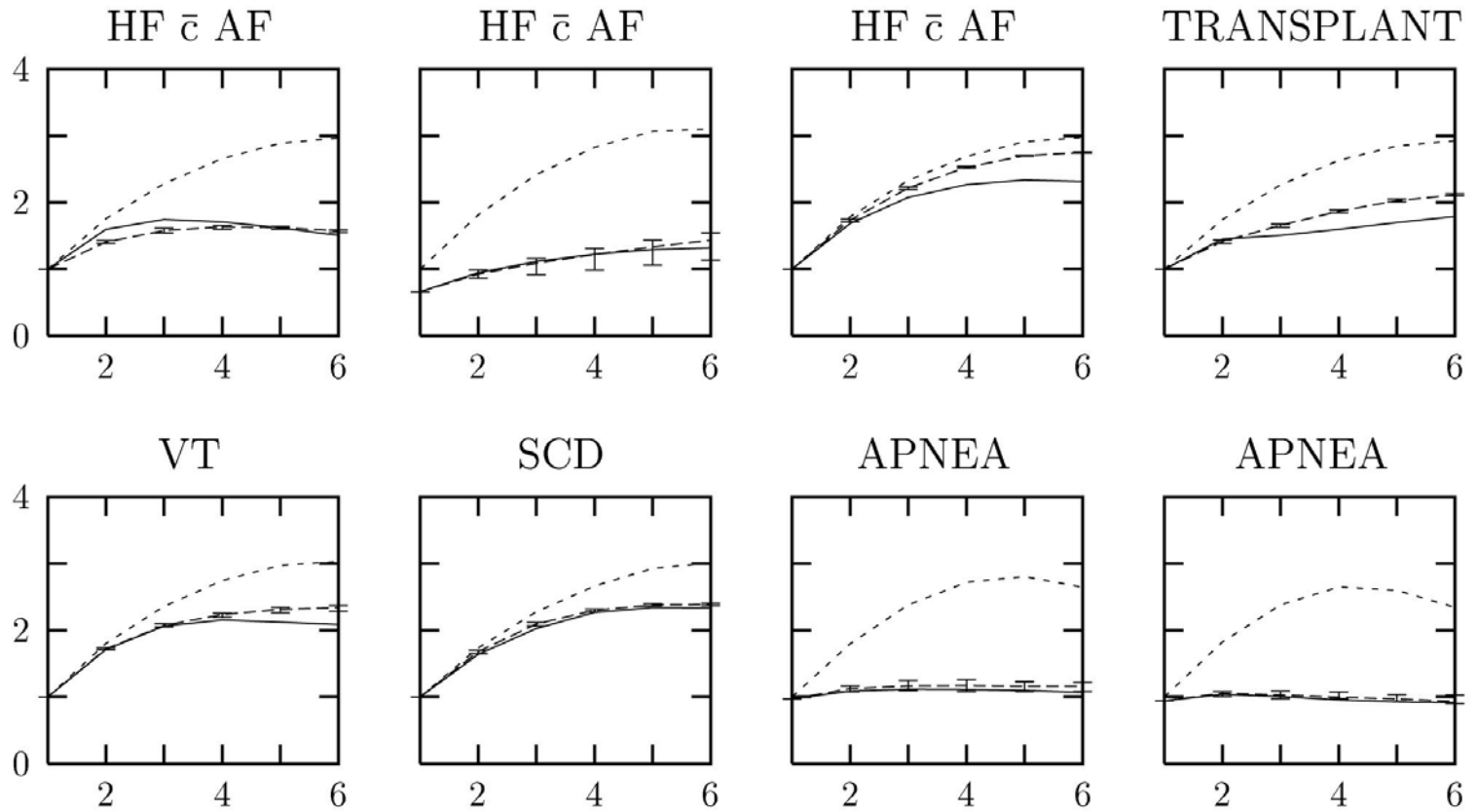
HEART FAILURE \bar{s} AF



- PHASE-RANDOMIZED
- |-|-|- SHUFFLED (MEAN \pm 1 S.D.)
- _____ ORIGINAL DATA

OTHER PATHOLOGIES

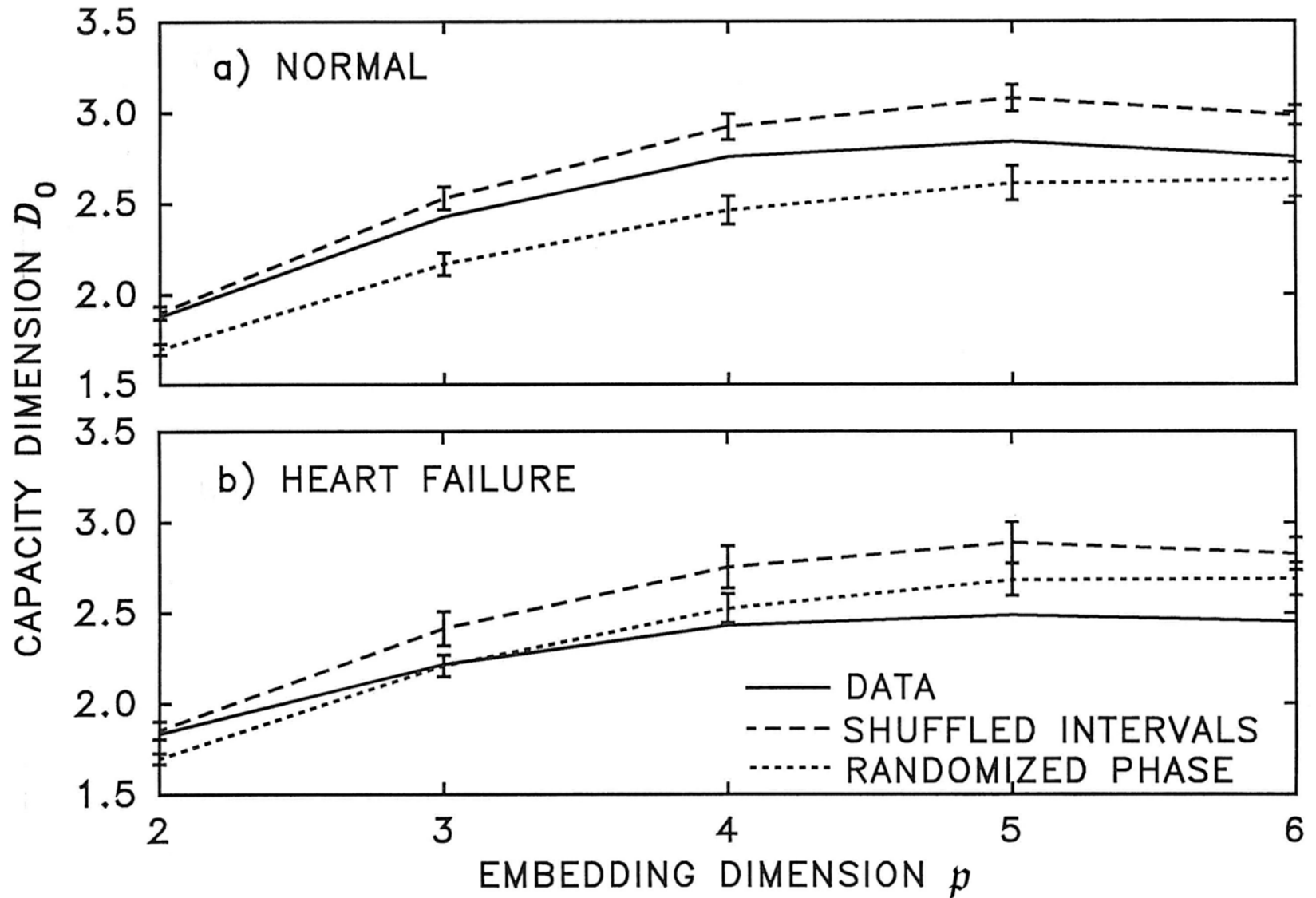
CAPACITY DIMENSION D_0



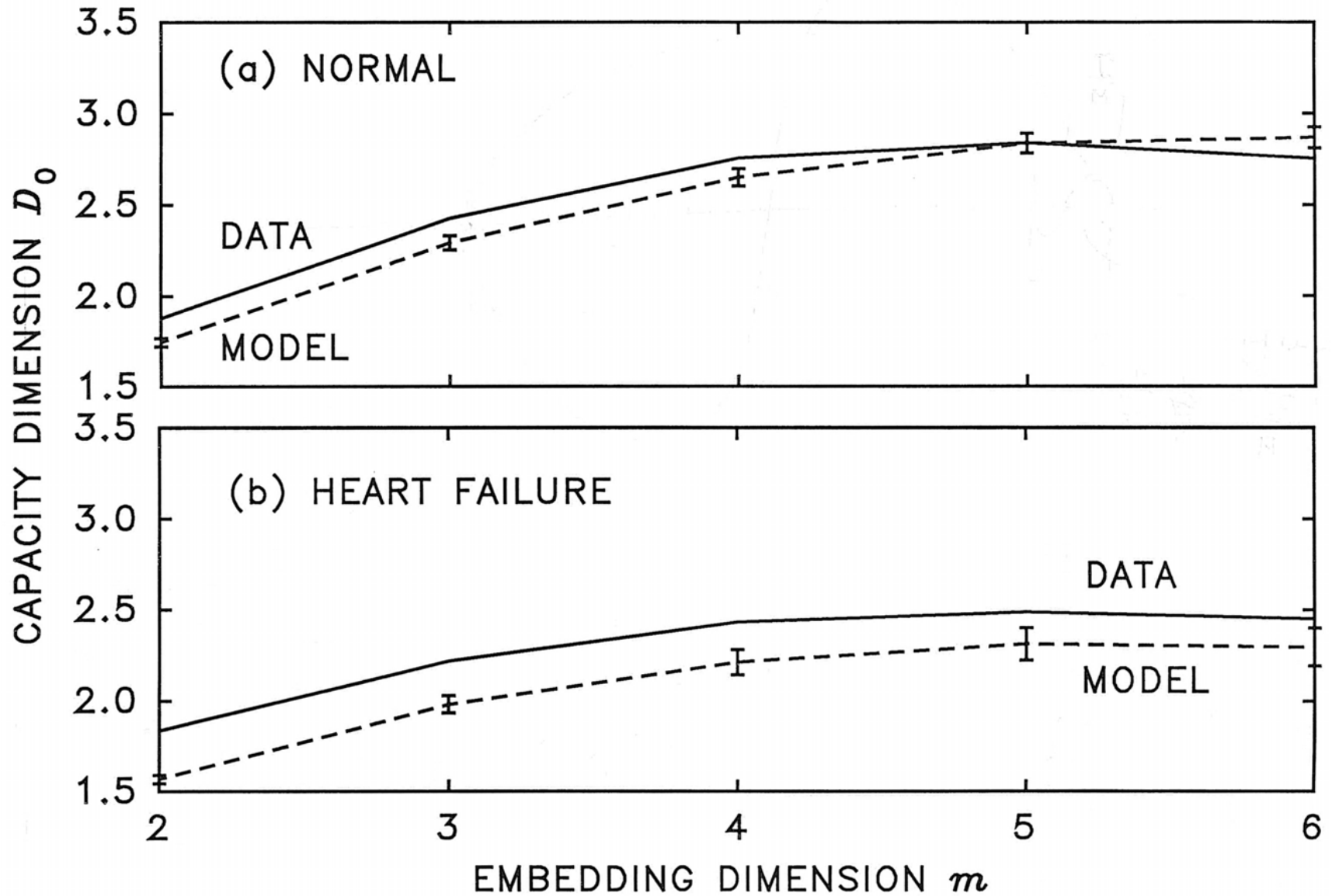
EMBEDDING DIMENSION p



GENERALIZED-RATE-BASED PHASE-SPACE RECONSTRUCTION



GENERALIZED-RATE-BASED PHASE-SPACE RECONSTRUCTION



References

- R. G. Turcott and M. C. Teich, "Long-Duration Correlation and Attractor Topology of the Heartbeat Rate Differ for Healthy Patients and Those with Heart Failure," *Proc. SPIE* **2036** (Chaos in Biology and Medicine), 22-39 (1993).
- R. G. Turcott and M. C. Teich, "Fractal Character of the Electrocardiogram: Distinguishing Heart-Failure and Normal Patients," *Ann. Biomed. Eng.* **24**, 269-293 (1996).
- M. C. Teich, "Fractal Behavior of the Electrocardiogram: Distinguishing Heart-Failure and Normal Patients Using Wavelet Analysis," *Proc. 18th Intern. Conf. IEEE Eng. Med. Biol. Soc.* **18**, 1128-1129 (1996).
- S. Thurner, S. B. Lowen, M. Feurstein, C. Heneghan, H. G. Feichtinger, and M. C. Teich, "Analysis, Synthesis, and Estimation of Fractal-Rate Stochastic Point Processes," *Fractals* **5**, 565-595 (1997).
- M. C. Teich, "Multiresolution Wavelet Analysis of Heart-Rate Variability for Heart-Failure and Heart-Transplant Patients," *Proc. Int. Conf. IEEE Eng. Med. Biol. Soc* **20**, 1136-1141 (1998).
- S. Thurner, M. C. Feurstein, and M. C. Teich, "Multiresolution Wavelet Analysis of Heartbeat Intervals Discriminates Healthy Patients from Those with Cardiac Pathology," *Phys. Rev. Lett.* **80**, 1544-1547 (1998).
- S. Thurner, M. C. Feurstein, S. B. Lowen, and M. C. Teich, "Receiver-Operating-Characteristic Analysis Reveals Superiority of Scale-Dependent Wavelet and Spectral Measures for Assessing Cardiac Dysfunction," *Phys. Rev. Lett.* **81**, 5688-5691 (1998).
- C. Heneghan, S. B. Lowen, and M. C. Teich "Analysis of Spectral and Wavelet-Based Measures Used to Assess Cardiac Pathology," in *Proc. 1999 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)* (Phoenix, AZ, 1999), paper SPTM-8.2.
- M. C. Teich, S. B. Lowen, B. M. Jost, K. Vibe-Rheymer, and C. Heneghan, "Heart Rate Variability: Measures and Models," in *Nonlinear Biomedical Signal Processing, Vol. II*, edited by M. Akay (IEEE Press, New York, 2001), pp. 159-213.