REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)

2. REPORT DATE 03/20/04

3. REPORT TYPE AND DATES COVERED
Final Report
01 Jun 00 – 30 Nov 03

4. TITLE AND SUBTITLE
A Quantum Computer for Shor's Algorithm
2004 Final Report

5. FUNDING NUMBERS
DAAD19-00-1-0370
DAAD19-00-1-0373

6. AUTHOR(S)
Philip H. Bucksbaum

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Michigan
Ann Arbor, MI 48109-1120

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
~~ARO Proposal number~~
~~40315-PH-QC~~
40315.2-PH-QC

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

We employ learning algorithms, optical and terahertz pulse shaping and ultrafast laser techniques to control quantum coherence in Rydberg atom wave packet quantum data registers. Our goals are to discover efficient ways to limit decoherence in these systems, execute arbitrary unitary operations and projection operations, and produce and probe quantum entanglement. Rydberg atoms have many of the same quantum properties of other systems that have been proposed for processing and storing quantum information, and so they are models for techniques that could be employed in proposed quantum computers.

14. SUBJECT TERMS
Rydberg wave packets, Quantum Computing, Grover's Algorithm, Terahertz Half-Cycle Pulses

15. NUMBER OF PAGES
8

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

(1) Forward:

This is a report of a three year effort Our original goal was to build and operate a prototype quantum computer programmed to perform Shor's factoring algorithm, and we made considerable progress in our understanding of the difficulties that will be faced before this goal can be realized. In the end, we did accomplish the more modest goal of demonstrating a number of methods for efficient quantum searching of a quantum data base (Grover's algorithm), and came to a greater understanding of the issues involved in extending this to more complex computational tasks.

If these problems are to be pursued in the future (and we hope to be involved in those pursuits) then the insights gained in this research will have been well worth the effort.

This final report is divided along the lines of our research. First we will describe our Rydberg apparatus, which has been improved dramatically over the grant period as a result of this project. Next, we will describe our main experimental and theoretical results, which have been published.

(4) Statement of the problem studied:

We employ learning algorithms, optical and terahertz pulse shaping and ultrafast laser techniques to control quantum coherence in Rydberg atom wave packet quantum data registers. Our goals are to discover efficient ways to limit decoherence in these systems, execute arbitrary unitary operations and projection operations, and produce and probe quantum entanglement. Rydberg atoms have many of the same quantum properties of other systems that have been proposed for processing and storing quantum information, and so they are models for techniques that could be employed in proposed quantum computers.

(5) Summary of the most important results:

Demonstration of Grover's Search Algorithm in a Rydberg atom data base.
In earlier work prior to the ARO support grant, we had shown how simple superposition could be employed to search for data encoded as quantum phase in a multi-state Rydberg atom electronic wave packet. Our original technique relied on coupling between the Rydberg wave packet an a single non-Rydberg quantum level, such as the ground state. Furthermore, it required that most of the probability remain in the ground state. These two conditions hamper any attempt to extend this to large data registers, so we looked for a new technique without these restrictions. The answer was to achieve the same "inversion about the mean" operation with a transformation among the Rydberg states using a terahertz electric field pulse.

Our new Grover's algorithm proceeded as follows: A terahertz half-cycle pulse was used to retrieve information stored as quantum phase in an *N*-state Rydberg atom data register. The register was prepared as a wave packet with one state phase reversed from the others (the "marked bit"). A half-cycle pulse then drove a significant portion of the electron probability into the flipped state via multimode interference.

This interaction forced a constructive multimode interference only on a phase-flipped state. Direct integration of the time-dependent Schrödinger equation as well as an impulsive model calculation showed good agreements with the data. Beyond the problem of database searching, these results point toward the use of HCPs as general tools of analysis for unknown wave packets. This could be used in conjunction with optical techniques for wave packet tomography.

Analysis of Quantum-state information retrieval in a Rydberg-atom data register.
In our next paper, we analyzed the quantum search protocol that retrieved phase information from the Rydberg-atom data register using a subpicosecond half-cycle electric field pulse. Calculations show that the half-cycle pulse can perform the phase retrieval only within a range of peak field values. By varying the phases of the constituent orbitals of the Rydberg wave packet register, we can demonstrate coherent control of the phase retrieval process. By specially programming the phases of the orbitals comprising the initial wave packet, we showed that it is possible to use the search method as a way to synthesize single energy eigenstates.

The sensitivity of the phase retrieval to the peak HCP field is explained by multimode interference. We show how this implementation scales with increasing database size, increasing peak HCP field, and with increasing principal quantum number of the marked state. We demonstrate coherent control over the retrieval of phase information using an HCP. By programming the initial wave packet and the HCP, it is possible to create the correct interference conditions to produce any desired energy eigenstate.

Control of Rydberg atoms to perform Grover's search algorithm We then proceeded beyond simple half-cycle pulses, to explore the limits of this technique. Using optimal control theory, a shaped terahertz pulse was designed that can perform the search algorithm better than an unshaped half-cycle pulse. Starting from an initial wave packet, we found that it is possible to use the search algorithm to synthesize single-energy eigenstates.

We derived three related, but different, results. Using an impulse model of the HCP, we showed that the phase retrieval performed by the HCP is closely related to the inversion-about-the-average operation central to the search algorithm. Using optimal control theory, shaped terahertz pulses were designed that can perform the search algorithm better than unshaped HCPs. Finally, we showed that a broadband terahertz pulse can be used to drive a Rydberg wave packet population into a single eigenstate.

Control of angular momentum evolution in Stark wave packets. Some quantum algorithms including Shor's factoring algorithm require at least two entangled degrees of freedom, with operators that can measure one

degree of freedom without destroying coherence in the other. In Rydberg states, these two degrees of freedom correspond to non-separable superpositions of at least two quantum numbers.
Furthermore, our work on data base searches has shown that in ultrafast implementations of quantum logic, there may be some advantage to quantum properties that are time-dependent. One idea we have investigated is using angular momentum quantum numbers in Stark Rydberg manifolds for quantum logic. Using techniques of ultrafast coherent control, we showed how to produce high-purity high-L angular momentum states in Rydberg Stark wave packets. The method works as follows: Two time-delayed phase-locked laser pulses excite the atom from a low-lying ‘‘launch state’’ into a low-L Rydberg wave packet, in the presence of a static electric field. By choosing the time delays between the pulses, and the static electric field, we find that a high-L state with high purity can be created at a specified target time. Using this technique, even high-L states can be created.

Control of Trapped-Ion Quantum States with Optical Pulses This is our first paper attempting to draw some connections between our work on quantum control, which has thusfar concentrated on Rydberg states, and more general implementations of quantum logic. We recently published a paper stating some general results on the quantum control of systems with infinitely large Hilbert spaces. A control-theoretic analysis of the control of trapped-ion quantum states via optical pulses was performed. We demonstrate how resonant bichromatic fields can be applied in two contrasting ways—one that makes the system completely uncontrollable and the other that makes the system controllable. In some interesting cases, the Hilbert space of the qubit-harmonic oscillator can be made finite, and the Schroedinger equation controllable via bichromatic resonant pulses. Extending this analysis to the quantum states of two ions, a new scheme for producing entangled qubits is discovered. Our results show that the specific temporal shapes of control fields are important in establishing controllability of quantum systems. A bichromatic control scheme that leads to the finite-dimensionality and controllability of the trapped ion system was presented. This can be extended to produce entangled states of two trapped-ion qubits.

(6) List of all publications:

(a) Manuscripts submitted, but not published

(b) Papers published in peer-reviewed journals

"Control of trapped-ion quantum states with optical pulses" by C. Rangan, A.M. Bloch, C.R. Monroe, and P.H. Bucksbaum, Physical Review Letters **92**, 11304 (2004).

"Control of angular momentum evolution in Stark wave packets" by H. Wen, C. Rangan, and P.H. Bucksbaum, Physical Review A **68**, 53405 (2003).

"Pseudospectral methods on a semi-infinite interval with application to the hydrogen atom: a comparison of the mapped Fourier-sine method with Laguerre series and Rational Chebyshev expansions" by John P. Boyd, C. Rangan and P.H. Bucksbaum, Journal of Computational Physics, **188**, 56 (2003).

"Control of Rydberg Atoms for Performing Grover's Search algorithm," C. Rangan, J. Ahn, D. Hutchinson, and P.H. Bucksbaum, Journal of Modern Optics **49**, 2239 (2003).

"Quantum-state information retrieval in a Rydberg-atom data register," J. Ahn, C. Rangan, D.N. Hutchinson, and P.H. Bucksbaum, Phys. Rev. A, **66,** art. no. 022312 (2002).

"Optimally shaped terahertz pulses for a quantum algorithm on a Rydberg atom data register", C. Rangan and P.H. Bucksbaum, Physical Review A, **64**, 033417 (2001). quant-ph/0103171

"Quantum phase retrieval of a Rydberg wave packet using a half-cycle pulse," J. Ahn, D. N. Hutchinson, C. Rangan, P. H. Bucksbaum, Phys. Rev. Lett. **86**, 1179-1182 (2001).

(c) Papers published in non-peer-reviewed journals or in conference proceedings

"Ultrafast Coherent Control in AMO Physics, Philip Bucksbaum, DAMOP 2004.

"Generation and detection of high angular momentum states in Rydberg Stark atoms," Haidan Wen, Santosh Pisharody, Joel Murray, Chitra Rangan, Philip Bucksbaum, DAMOP 2004.

"Theory of detection of angular momentum states in Rydberg atoms using half-cycle pulses," Chitra Rangan, Haidan Wen, Philip Bucksbaum, DAMOP 2004

"Controlling the population of angular momentum component in Rydberg states H.Wen and P.Bucksbaum, DAMOP 03.

"Rydberg Wave Packets in Quantum Information Science," Proceedings of the Fermi Summer School in Quantum Information, Springer (2002)

"Optimal control of trapped ion qubits using shaped optical pulses," C. Rangan et al., OSA Annual Meeting Bulletin, 2002.

"Control of trapped ion qubits using shaped optical pulses," C. Rangan, C. Monroe, P.H. Bucksbaum DAMOP APS Bulletin (2002).

"Partial Field Ionization of Entangled Stark States," Joel Murray, Catherine Herne, Jaewook Ahn, Ralph Conti, Phil Bucksbaum DAMOP APS Bulletin (2002).

"Quantum Control," P.H. Bucksbaum, Eugene Commins Festschrift Proceedings, ed. by D. Budker and S. Freedman, 2002.

"Performing Grover's search algorithm on a Rydberg atom data register " by C. Rangan, J. Ahn, D. Hutchinson and P.H. Bucksbaum, XXXII Winter Colloquium on the Physics of Quantum Electronics, Snowbird, UT, January 2002.

'Optimal control of trapped ion qubits using shaped optical pulses,' Chitra Rangan, Chris Monroe, Anthony Block, and P.H. Bucksbaum, OSA 2002, Long Beach, CA, October 2002.

(d) Papers presented at meetings, but not published in conference proceedings

"Learning about Quantum Dynamics from Learning Control Algorithms," Symposium on Molecular Reaction Dynamics in Condensed Matter, Laguna Beach, CA, March, 2004.

"What can we learn from learning feedback control algorithms?" Workshop on Coherent Control, Ringberg, Germany, December 2003.

"Quantum Information in Rydberg Atoms," QCPR, Nashville, TN, August 2002

"Quantum control in Rydberg wave packets: Applications to quantum information," Cargese Summer School on Coherent Control, October 2002.

'Quantum control research in atoms and molecules,' P.H. Bucksbaum, Texas Retreat symposium, Austin, TX, March 2002

'Ultrafast AMO Physics,' Invited talk at the fall CAMOS meeting, Irvine, CA, November 2002.

'Quantum control in Rydberg wave packets: Applications to quantum information," P.H. Bucksbaum, Invited lecture at the International Summer School on Quantum Control, Cargese, Corsica, October, 2002.

"Applications of Coherent Control: quantum computing to chemistry to x-rays," P.H. Bucksbaum, Frontiers in Spectroscopy Lecture III, Ohio State University 9 Feb 2001.

"Coherent Manipulation of a Rydberg Wave packet using a Terahertz Half-Cycle Pulse," Jae Ahn, ITAMP Workshop on Complex Phenomena Involving Rydberg Atoms and Molecules, Cambridge, April 26-28, 2001.
"Strong Field Coherent Control," Coherent Control Gordon Conference, Mt. Holyoke, August 2001

"Coherent manipulation of Rydberg wave packets toward quantum computing," Jaewook Ahn, Quantum Optics EuroConference 2001, San Feliu, Oct 9, 2001.

"Quantum wave packet sculpting," Philip H. Bucksbaum, ILS01, Long Beach, October, 2001.

"Performing Grover's search algorithm on a Rydberg atom data register " by C. Rangan, J. Ahn, D. Hutchinson and P.H. Bucksbaum, invited abstract, XXXII Winter Colloquium on the Physics of Quantum Electronics, Snowbird, UT, January 2002.

"Phase retrieval on a Rydberg atom data register through optimally shaped terahertz pulses" by C. Rangan and P.H. Bucksbaum, invited abstract, 3rd Annual Cross Border Workshop on Laser Science, Toronto, Canada, May 2001.

(7) List of all participating scientific personnel showing any advanced degrees earned by them while employed on the project

Jaewook Ahn, Ph.D. Received.
Joel Murray, advanced to candidacy
Haidan Wen, advanced to candidacy
Santosh Pisharody
Chitra Rangan, received junior faculty appointment