Energy Release Rate in a Constrained Polymeric Disk under Internal Pressure and Thermal Loads

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Abstract. In this study, the energy release rates in a centrally perforated star-shaped disk, which was made of a polymeric material, under internal pressure and thermal loads, were determined. The deformations of the disk were constrained by a circular steel ring enclosing the disk. Two constitutive models, namely, Hookean model and Ogden model, were used to model the constitutive behavior of the material. Three different loadings, internal pressure, isothermal load, and combined internal pressure and isothermal load, were considered. Numerical results showed that values of the energy release rate were very sensitive to Poisson's ratio for the pressure load. The decrease in the compressibility gave a higher value of energy release rate for the pressure load and a lower value of that for the isothermal load. The deformed crack profiles were also determined to address the compressibility effect on the energy release rate.

INTRODUCTION

Defects such as flaws and cracks may form in polymeric materials due to the manufacturing, handling or ageing. To ensure the integrity and reliability for such structural components, fracture toughness should be ascertained so that the onset of the crack growth can be determined based on the fracture resistance of the material. The energy release rate is a measure of the fracture toughness, and commonly used as a criterion to determine the maximum operating loads for a given pre-existing defect. Some polymeric materials exhibit mechanical behavior that remains nonlinearly elastic at large strains and has very little compressibility, and hence these materials are often referred to as fully or nearly incompressible. When these polymers are loaded in a highly confined state, even a small change in the compressibility can result in a dramatic difference in stress distributions. For example, Schapery [1] conducted an experiment for a circular polymeric disk under hydrostatic tension, and showed that a small change in Poisson's ratio can alter the stress distribution significantly.

Many investigations have been conducted to determine the relationship between the crack-tip stress and strain fields and the energy release rates for rubber-like materials. Thomas [2] was the first to study this relationship experimentally. He found that the average strain energy density in a sheet of rubber was uniquely related to the energy release rate regardless of the specimen type. Thomas's conclusion was validated later by Andrews [3] and Knauss [4]. Andrews used a microscopic, photoelastic technique to quantify the strain fields around the crack tip whereas Knauss used a printed-grid technique. Morman et al. [5] also gave an analytical solution relating the energy release rate to the crack tip radius.

Rice's development of the *J*-integral [6] gave a mathematical argument to characterize the local stress-strain field around a crack front. The *J*-integral was found to be equivalent to the energy release rate and independent of the path contours. The problem of a crack in an infinite, thin, and incompressible sheet subjected to a biaxial tension at infinity was studied, within the framework of nonlinear elasticity for a Neo-Hookean material, by Wong and Shield [7], and Chang [8] generalized the *J*-integral for nonlinear elastic materials with finite strains.

The presence of the crack surface tractions, which may be due to ignition pressures and pressurized

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14. ABSTRACT
In this study, the energy release rates in a centrally perforated star-shaped disk, which was made of a polymeric material, under internal pressure and thermal loads, were determined. The deformations of the disk were constrained by a circular steel ring enclosing the disk. Two constitutive models, namely, Hookean model and Ogden model, were used to model the constitutive behavior of the material. Three different loadings, internal pressure, isothermal load, and combined internal pressure and isothermal load, were considered. Numerical results showed that values of the energy release rate were very sensitive to Poisson's ratio for the pressure load. The decrease in the compressibility gave a higher value of energy release rate for the pressure load and a lower value of that for the isothermal load. The deformed crack profiles were also determined to address the compressibility effect on the energy release rate.

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fluids, is of practical interest in engineering applications. In this regard, the *J*-integral needs to be modified by including an additional line integral so that the path-independence of the *J*-integral can be preserved. Chang and Becker [9] scrutinized the effect of non-conservative surface traction applied to the crack faces on the energy release rate for a 2D rubber-like material. They pointed out that the usual expression for the energy release rate needed to be modified, and also demonstrated that the energy release rate depends upon the constitutive relation used to model the material response.

In this study, a finite element analysis was conducted to compute the energy release rate for a circular polymeric disk with a pre-existing crack. The disk had a star-shaped hole with six symmetrical notches emanating from the disc's center and was enclosed by a thin steel ring. We assumed that the disk was modeled either as a linearly elastic Hookean material or a nonlinear elastic Ogden material, and the steel ring was modeled as a Hookean material. Material constants obtained from uniaxial tensile test data were used to describe the Ogden strain energy potential. Three loading conditions, internal pressure, constant thermal load, and combined internal pressure and constant thermal load, were considered. To account for the material compressibility, Poisson's ratio of the Hookean material was assumed to vary from 0.48 to 0.4999.

FORMULATION OF THE PROBLEM

Figure.1 shows a cross-section of the specimen in which a polymeric disk is enclosed by a thin steel ring. A plane strain state of deformation was assumed to prevail in the body. The star-shaped hole at the center of the specimen had six symmetrically located notches. A crack with 1 *in.* length was assumed to exist directly ahead of the notch tip. The polymer and the steel casing were modeled as isotropic and homogeneous materials. Three loading conditions, 1000 *psi* internal pressure, -85^{o}F thermal load, and combined 1000 *psi* internal pressure and -85^{o}F thermal load, were considered. To account for the material compressibility, Poisson's ratio of the Hookean material was assumed to vary from 0.48 to 0.4999. Deformations of the steel ring were assumed to be infinitesimal and its isotropic material modeled by Hooke's law with Young's modulus = 29 M*psi*, Poisson's ratio = 0.3, and the coefficient of thermal expansion = $6.5\times10^{-6}/{}^oF$

We performed the finite element analysis using the commercial ABAQUS computer code to calculate the energy release rate for the problem studied. Due to the symmetry of the specimen geometry and loading conditions, only a 30^o sector of the specimen was investigated. Figure 2 exhibits the finite element mesh of the sector in which 4824 8-node quadrilateral elements were used with a dense mesh near the crack tip. Because the polymeric material was considered to be fully or nearly incompressible, hybrid elements were adopted in the analysis. Points on the bounding surfaces $\theta = 0^o$ and $\theta = 30^o$ were constrained to move radially, and tangential tractions on these surfaces were set equal to zero. The outer surface of the steel ring was taken to be traction free. The polymeric disk was assumed to be perfectly bonded to the steel ring so that displacements and surface tractions were continuous across their common interface.

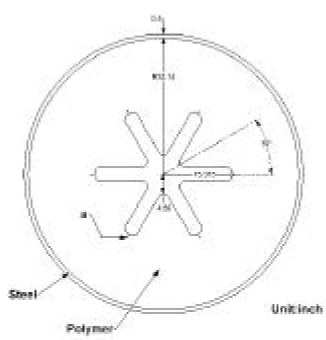

FIGURE 1. Specimen geometry

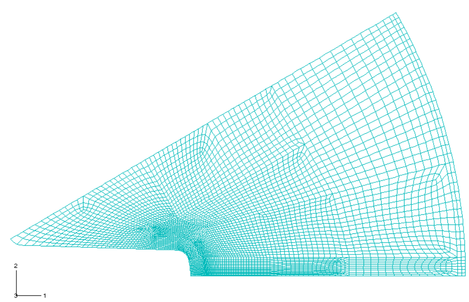

FIGURE 2. A finite element mesh with 4824 elements for the analysis of the problem

The polymer was modeled either as an Ogden or as a Hookean material. For the Ogden material with full incompressibility, the strain energy potential is

$$U(e) = \sum_{i=1}^{N} \frac{2m_i}{a_i^2}(\bar{l}_1^{a_i} + \bar{l}_2^{a_i} + \bar{l}_3^{a_i} - 3) \qquad (1)$$

where $\bar{l}_i$ are the deviatoric principal stretches; $\bar{l}_i = J^{-\frac{1}{3}} l_i$; l_i are the principal stretches; and m_i, a_i are temperature-dependent material parameters. J is the total volume ratio.

The shear modulus m_0 for the Ogden material can be given by

$$m_0 = \sum_{i=1}^{N} m_i \qquad (2)$$

The elastic volume ratio, J^{el} is related to J by

$$J^{el} = J\big/(1 + aT)^3 \qquad (3)$$

We set N equal to 2 in the Ogden strain energy, and ABAQUS computer code determined the material constants m_i, a_i from the uniaxial test data through a least-squares-fit procedure. Values of m_1 and m_2 determined by ABAQUS are

$$m_1 = -160.4\,psi,\; m_2 = 1643\,psi \qquad (4)$$

Computed values of the axial component of the first Piola-Kirchhoff stress vs. the nominal axial strain are compared with the experimental data in Fig. 3. It is clear that the two sets of data are very close to each other,implying that values given in Eq. (4) of m_1 and m_2 are very accurate.

In order to delineate the effect of the material compressibility, the polymer was also modeled as a Hookean material with Poisson's ratio n varying from 0.48 to 0.4999. The stress vs. strain curve for the Hookean material is also depicted in Fig. 3. The Young's modulus can be computed by

$$E = 2(1 + n)\,m_0 \qquad (5)$$

where m_0 is obtained by Eq. (2).

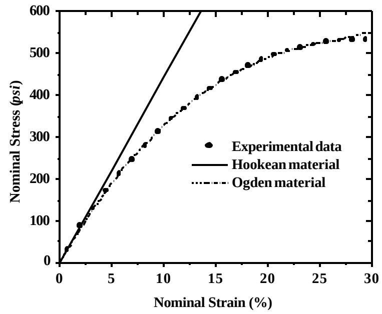

FIGURE 3. A comparison of the computed and the experimental axial nominal stress vs. nominal strain for the disk material

For both linear and nonlinear analyses, the coefficient of thermal expansion for the disk material equaled $5.6\times10^{-6}/^{o}$F.

When the inner surface of the hole and the surface of the starter crack, aligned with the $x_1 - \text{axis}$, are loaded by a uniform pressure p, the energy release rate G, for an elastic material is given by

$$G = \int_{\Gamma}\left(Wn_1 - T_{ij}n_j\frac{\partial u_i}{\partial x_1}ds\right) + \int_{\Gamma_c} p\frac{\partial u_2}{\partial x_1}dx_1 \quad (6)$$

where W is the strain energy density, **u** is the displacement of a point, Γ is a closed curve enclosing the crack tip, **n** is the outward unit normal to Γ, T_{ij} is the stress tensor, and Γ_c are the two crack faces. Note that the second term on the right-hand side of (6) represents the work done by the pressure on the crack surfaces. For the thermal load, there is no surface traction on the crack surface, and this term makes null contribution to the value of G. For a homogeneous hyperelastic material, T_{ij} equals the first Piola-Kirchhoff stress tensors.

RESULTS AND DISCUSSION

Table 1 lists values of the energy release rate for Hookean and Ogden materials for the three different loading conditions. In the linear analysis, the Poisson ratio n of the Hookean material is set equal to 0.4999 in order to compare results with those obtained for the Ogden material. The linear analyses give higher values of the energy release rate than those for the nonlinear analyses. Note that even for the linear analyses, values of the energy release rate for the combined load cannot be obtained by adding the energy release rates for the corresponding pressure and thermal loads.

TABLE 1. Energy Release Rate (*lb/in*)

Loading Type	Hookean	Ogden
Combined	57.99	26.67
Pressure	3.557	2.629
Thermal	32.82	31.32

The energy release rate as functions of normalized pressure and thermal loads for different Poisson's ratios are shown in Figures 4(a) and 4(b). It is seen that the value of the energy release rate depends on the magnitudes of the Poisson's ratio and the applied load.

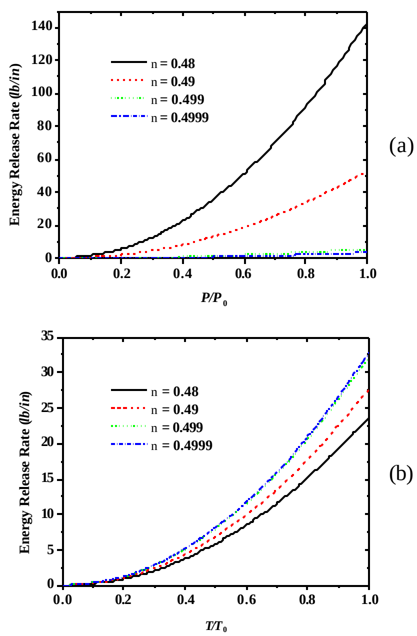

FIGURE 4. Variation of the value of the energy release rate with Poisson's ratio for **(a)** the pressure load **(b)** the thermal load

For the combined load, Fig. 5 displays a three-dimensional surface plot of the energy release rate vs. pressure and temperature. Due to the mutual coupling of both loads, the value of the energy release rate increases sharply as the pressure and the thermal loads are increased.

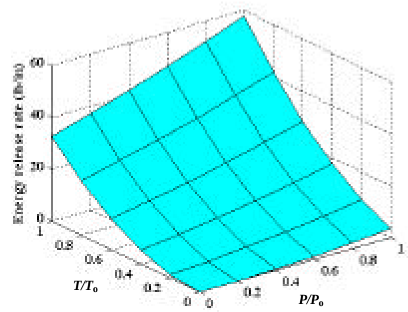

FIGURE 5. Variation of values of the energy release rate with the pressure load and thermal load for the Hookean material

Figure 6 shows the variation of the energy release rate with Poisson's ratio. It is clear that, for the pressure load and the combined load, the values of the energy release rate decrease sharply as the Poisson's ratio is increased from 0.48 to 0.5. However, for the thermal load, the Poisson's ratio has no significant effect on the energy release rate.

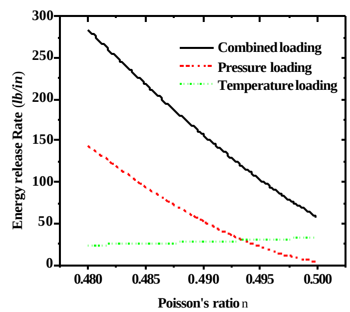

FIGURE 6. The dependence of the energy release rate upon Poisson's ratio for three loadings

The results of the analyses show that, under pressure load, the increase in ν will decrease the compressibility of the material which, in turn, will decrease the crack opening displacement between the two crack faces (Figure 7(a)). For the thermal load, the crack opening displacement, plotted in Fig 7(b), for different values of Poisson's ratios does not differ too much. It is shown that when Poisson's ratio equals to 0.4999, the deformed shapes between the Hookean material and Ogden material are almost indistinguishable.

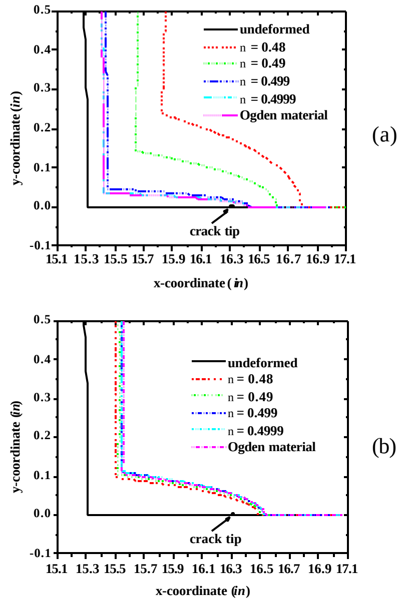

FIGURE 7. Deformed shapes of the crack face for the Ogden material and Hookean material with different values of Poisson's ratio for the **(a)** pressure load **(b)** thermal load

We further depict the strain fields near the crack tip. Figures 8(a) and 8(b) show the variation of the maximum principal strain with the distance from the crack tip for the pressure and the thermal loads, respectively. For the pressure load, it is clear that Poisson's ratio significantly influences the strain field in the vicinity of the crack tip.

For the thermal load, the maximum principal strains near the crack tip for the Hookean material with Poisson's ratio ranging from 0.48 to 0.4999 are essentially the same. At the crack tip, the maximum principal strain for the Ogden material is slightly larger than that for the Hookean material.

CONCLUSIONS

Finite element analyses were conducted to calculate the energy release rate for a polymeric disk with pre-existing cracks. It is shown that, for the loading conditions considered, the results obtained from the linear analyses give higher values of the energy release rate than those obtained from the nonlinear analyses. For the pressure load, the value of the energy release rate for a Hookean material strongly depends upon the values of the Poisson's ratio. This is due to the reason that the work done by the crack surface pressure increases noticeably as the value of the Poisson's ratio is decreased, resulting in a significant increase in energy release rate as the Poisson's ratio is decreased. For the thermal load, the value of the energy release rate does not differ much with the variation of the Poisson's ratio. It is interesting and important to point out that, even for the linear analysis, the value of the energy release rate for the combined load cannot be obtained by adding the energy release rates obtained from the pressure and the thermal loads separately.

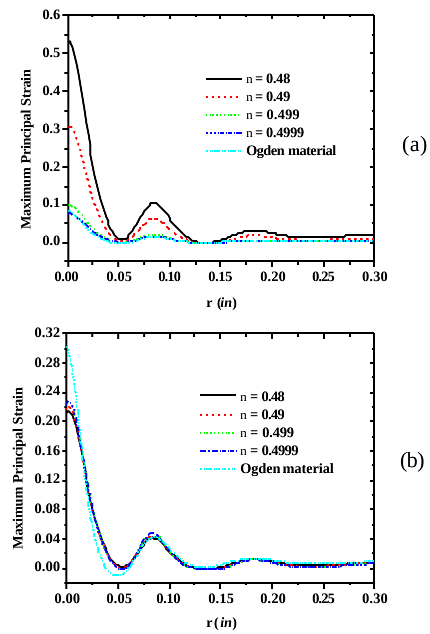

FIGURE 8. Variation of the maximum principal strain with the distance from the crack tip for the Hookean and Ogden materials;**(a)** pressure load **(b)** thermal load

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