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TOTAL ARMY CAPITAL BUDGETING

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**CENTER FOR ARMY ANALYSIS
6001 GOETHALS ROAD
FORT BELVOIR, VA 22060-5230**

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**Director
Center for Army Analysis
ATTN: CSCA-RA
6001 Goethals Road
Fort Belvoir, VA 22060-5230**

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| 13. ABSTRACT (<i>Maximum 200 Words</i>) TACAB determined a method to better include the requirements for small-scale contingencies (SSCs) while making force structure decisions. TACAB developed a stochastic optimization for determining alternative Army force structures and considers the requirements for major contingencies as well as for small scale contingencies (SSCs). The stochastic element of the optimization represents the probabilities associated with SSCs occurring. | | | | |
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TOTAL ARMY CAPITAL BUDGETING

SUMMARY

THE PROJECT PURPOSE was to determine a method to better include small-scale contingency (SSC) requirements while making force structure decisions.

THE PROJECT SPONSOR was the Center for Army Analysis.

THE PROJECT OBJECTIVE was to develop a methodology that would use the information generated in Stochastic Analysis of Deployments and Excursions (SADE), Stochastic Analysis of Resources for Deployments and Excursions (SARDE), and Objective Force Planning, New and Extended (ONE) (and their follow-on projects) to aid senior Army decision makers in designing future forces.

THE LIMITATION OF THE PROJECT was that the data necessary to conduct this project is being developed; therefore, we used notional data.

THE MAIN ASSUMPTIONS are:

- (1) That historical data on deployments can be used to accurately forecast the probability of future actions.
- (2) That the units or missions are similar within each particular type of SSC. For example, each peacekeeping operation requires the same types of missions.
- (3) That the units used for SSCs will be diverted to major contingencies, if they should occur.

THE PRINCIPAL FINDING is that a stochastic optimization can be used to help develop force structure alternatives.

THE PRINCIPAL RECOMMENDATION is that, when data becomes available, the optimization formulated in this report be considered to aid in force structure decisions.

THE PROJECT EFFORT was conducted by Ms. Linda Coblentz.

COMMENTS AND QUESTIONS may be sent to the Director, Center for Army Analysis, ATTN: CSCA-RA, 6001 Goethals Road, Suite 102, Fort Belvoir, VA 22060-5230.

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1 INTRODUCTION

1.1 Total Army Capital Budgeting

The Total Army Capital Budgeting (TACAB) project was an internal research project for the Center for Army Analysis (CAA).

1.2 Problem Statement

To develop a method that considers the small-scale contingency requirements while making decisions on the units to include in future force structures.

1.3 Background

At the end of the Cold War, some expected that the United States Army would be lacking a well-defined mission. Time has not borne this out. Although we no longer face a peer opponent, the Army deployed more often in the past ten years than it had for the previous forty. Most of these deployments have been small-scale contingencies (SSCs); however, the current force is primarily designed for major contingency operations (MCOs). This is causing problems with the force as it stands now.

Historically, the Army planned its forces based on having to fight and win two major theaters of war. This was a reasonable approach, given the environment at the time. According to Army Vision 2010, from 1950 to 1990, the Army conducted ten notable deployments. However, the environment has changed significantly. During the years 1990 to 1996, the Army deployed twenty-five times; this is a 150% increase. The trend towards more deployments continues today. These recent deployments have mostly consisted of SSCs, such as humanitarian assistance for earthquakes and famine, show-of-force in Panama, and peace enforcing and peacekeeping in Bosnia. The Army is not currently structured to handle the pace of deployments it must accomplish. Certain types of units, such as military police, engineers, and psychological operations, are being overextended. Small-scale contingencies must be considered when planning the force in order to prevent readiness problems caused by the increased operational tempo of non-combat units.

It is difficult to plan for SSCs because we do not know when and where they are going to occur. We could not have planned five years ago for the Afghanistan action that is happening today. Humanitarian efforts due to natural disasters such as earthquakes provide no more than a day's warning. So the question is, how do we plan for force structures to meet SSC requirements when SSCs are so uncertain?

The Center for Army Analysis (CAA) has completed three studies that begin to deal with this complicated issue including Stochastic Analysis of Deployments and Excursions (SADE) (CAA-MR-99-9), Stochastic Analysis of Resources for Deployments and Excursions (SARDE) (CAA-MR-99-14), and Objective Force Planning, New and Extended (ONE) (CAA-SR-99-1).

SADE was developed to determine a methodology to forecast SSCs, given their uncertain nature. Essentially, SADE provides a way to determine the likelihood of an SSC occurring as well as the likelihood that another SSC would occur at the same time. SADE is updated regularly.

ONE developed mission task organized forces (MTOFs) for particular tasks that the Army must accomplish. In the ONE Report, an MTOF is defined as “a force list together with an associated set of objectives and tasks to be accomplished under specified condition and standard for a specific mission.” The generated force lists are illustrative of the types of units that would be necessary for a particular SSC. Generating MTOFs is an ongoing process with lists updated on a regular basis.

The SARDE study used the results of the SADE and ONE studies. SARDE determined the probability distributions of simultaneously required units. It used the probability distributions developed in SADE and the MTOFs developed in ONE as input. In essence, it gives the chance that a specific number of a type of unit will be needed at any point in time.

1.4 Purpose and Objectives

The purpose of TACAB was to develop and demonstrate a methodology that would use the information generated in SADE, SARDE, and ONE (and their follow-on projects) to aid senior Army decision makers in designing future force structures.

1.5 Limitations

Much of the data necessary to do TACAB is being developed in the Mission Task Organized Forces (MTOF) Generator project being performed at CAA. This project is currently on-going, so data is not available for TACAB; therefore, in order to demonstrate TACAB’s methodology, notional data was used.

1.6 Assumptions

TACAB’s assumptions for this project are:

- 1) That historical data on deployments can be used to accurately forecast the probability of future actions. In other words, the types and rates of deployments will remain stable. This allows us to be able to generate the probabilities associated with SSCs occurring.
- 2) That the units or tasks to be accomplished are similar within each particular type of SSC. For example, each peacekeeping operation requires the same types of tasks. If this is not the case and each occurrence of a type of SSC required different tasks, the size of any optimization model would be untenable.
- 3) That the units used for SSCs will be diverted to major contingencies, if they should occur. The force structure for SSCs is not competing with force structure for the major contingencies for portions of the personnel endstrength.

1.7 Approach

Figure 1 provides TACAB's approach. As mentioned in Section 1.5, some of the data is currently under development, in particular, the SSC task requirements. Because of this, the SSC task requirement, the MCO unit requirements, and force effectiveness measures used in this demonstration were randomly generated. When this methodology becomes fully functional, the MCO requirements can be obtained from the Total Army Analysis that CAA conducts. The force effectiveness measures are in the form of penalties when units for the MCOs are not available and for not being able to perform tasks in the SSCs. The determination of actual force effectiveness measures is not presented here. It is a follow-on research project. The data for the SSC probabilities was available from the SADE and SARDE projects and was used.

The optimization model creates a force structure based on the probabilities of SSCs, the SSC requirements, the MCO requirements and the Army endstrength. When the methodology becomes fully functional, the endstrength number would be determined by subtracting the number of soldiers in transit, training, hospitals, and schools from the congressionally mandated number of total soldiers. The model minimizes the penalties incurred for not satisfying MCO unit requirements and not being able to perform tasks in the SSCs. The penalties can be varied to produce alternative force structures.

Details will be discussed in following sections.

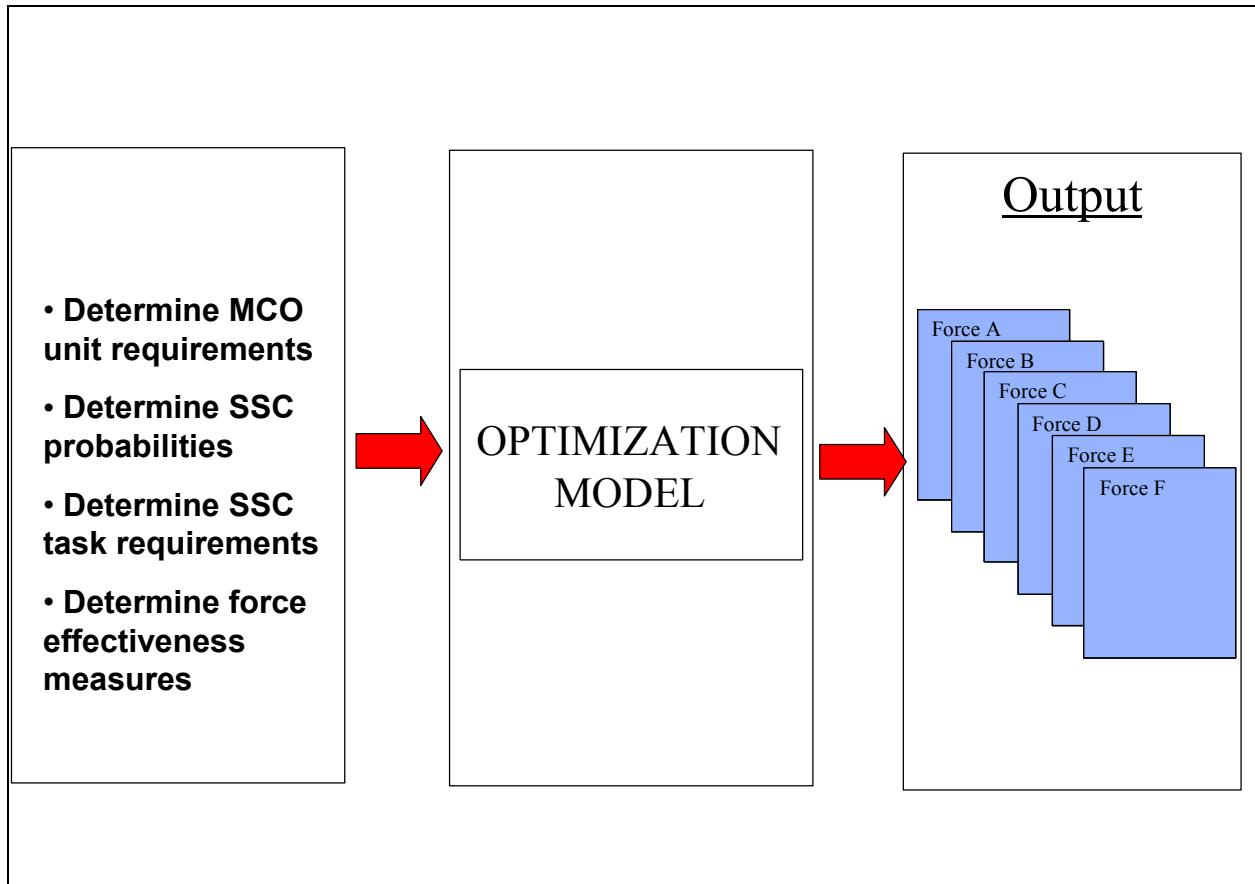


Figure 1. TACAB Approach

2 DATA DEVELOPMENT

This section discusses the development of the input data for the optimization model, which we divide into two categories: unit/task data and SSC probability data. Much of the unit/task data necessary to do this project is being developed in CAA's Mission Task Organized Forces (MTOF) Generator project. This project is currently on-going; therefore, in order to demonstrate the methodology, notional task/unit data was used. The details of each data category and generation of data is discussed below.

2.1 Unit/Task Data

The unit/task data consists of the following:

- the number of each unit needed for MCOs,
- the tasks that were required for each SSC,
- the units that could perform the tasks,
- the penalties for not having a unit in an MCO,
- the penalties for not being able to perform a task in an SSC,
- and the number of people in each unit.

We simulated multiple unit data sets using a random number generator in Excel.

Tables 1 through 3 provide examples of the unit/task data. In this example, there are three SSCs, ten units, and 15 tasks. The data is randomly generated. Table 1 gives the tasks that are necessary for each of the SSCs, the penalty for not being able to perform the task, and the number of times the task needs to be performed. Note that in this example, each task is needed only once.

| | Task | Penalty | Number of Times Task Performed |
|-------|------|---------|--------------------------------|
| SSC 1 | 1 | 5 | 1 |
| | 3 | 4 | 1 |
| | 5 | 3 | 1 |
| | 7 | 4 | 1 |
| | 8 | 2 | 1 |
| | 9 | 3 | 1 |
| | 11 | 2 | 1 |
| | 14 | 4 | 1 |
| SSC 2 | 4 | 1 | 1 |
| | 8 | 6 | 1 |
| | 12 | 4 | 1 |
| | 14 | 5 | 1 |
| SSC 3 | 1 | 2 | 1 |
| | 3 | 9 | 1 |
| | 8 | 1 | 1 |
| | 10 | 6 | 1 |
| | 13 | 1 | 1 |
| | 14 | 5 | 1 |

Table 1. SSC Task Data

Table 2 provides the units that can perform each task.

| | Unit |
|---------|------|
| Task 1 | 2 |
| Task 3 | 3 |
| Task 4 | 4,9 |
| Task 5 | 1 |
| Task 7 | 4 |
| Task 8 | 3 |
| Task 9 | 2 |
| Task 10 | 5 |
| Task 11 | 4 |
| Task 12 | 2,7 |
| Task 13 | 2 |
| Task 14 | 6 |

Table 2. Unit to Task Data

Table 3 provides the unit requirements for the MCOs, the penalties for not having the unit, and the number of people in each unit.

| | Major Conflict Requirements | Penalty | Strength |
|---------|-----------------------------------|---------|----------|
| Unit 1 | 53 | 5.6 | 400 |
| Unit 2 | 57 | 6.9 | 60 |
| Unit 3 | 28 | 9.1 | 1700 |
| Unit 4 | 30 | 8.3 | 700 |
| Unit 5 | 77 | 0.2 | 100 |
| Unit 6 | 1 | 5.4 | 2700 |
| Unit 7 | 76 | 9.1 | 3300 |
| Unit 8 | 81 | 4.3 | 2800 |
| Unit 9 | 70 | 6.7 | 4200 |
| Unit 10 | 4 | 5.0 | 500 |

Table 3. MCO Unit Data

2.2 SSC Probability Data

The SSC probability data is the probability that a specific number of a type of SSC occurs at the same time. For example, the probability of three peace keeping operations occurring at the same time is 0.15 (this is not the actual value, the actual value is classified).

Because of changes in the SADE/SARDE methodology, the probabilities of the number of SSCs occurring are no longer determined. In order to generate this data, we used the original SADE methodology and data from the SARDE 01 project. There are ten types of SSCs used in this project including peace keeping, noncombatant evacuation operations, domestic support, foreign humanitarian assistance, disaster relief, intervention, maritime operations, show of force, strike operations, and enforcing no-fly zones. The data associated with each SSC is close-hold; therefore the types of SSCs are not indicated in our example.

We developed a simulation model that mirrored the one used in SADE, which considers the probability distribution of the time between occurrences of SSCs, the distribution of the types of SSCs that occurred, and the distribution of the durations of each of the types of SSCs.

SSCs are considered to be entities in this model that occur based on the inter-arrival time distribution. They are assigned an SSC type based on the distribution of the types and remain in the system for an amount of time determined by the duration distributions and then the SSC exits the system, or ends.

At specified intervals, the number of SSCs in the system is output by each SSC type. This provides the number of each SSC type that occur at any one time. We repeat runs to determine probability distributions for the number of SSCs occurring at any point.

The probability data is used in the objective function in the optimization model. The optimization model is discussed in the next subsection.

2.3 Example of SSC Probability Data Generation

This section provides an example of SSC probability data generation. We used the arrival rate, SSC type distribution and duration data from SARDE 01.

In this example, the SSC arrival rate can be approximated using a lognormal probability distribution with a mean of 0.62 months and a standard deviation of 1 month.

Table 4 provides the probabilities that an incoming SSC will be of that type as well as the probability distribution of the duration, in months, for that type. For example, the probability that an SSC that occurs will be of type SSC 5 is 0.07. The duration of that SSC is distributed with a lognormal with mean of 12.57 months and standard deviation of 26.79.

| | Arrival Probability | Duration Probability (in months) |
|--------|---------------------|---|
| SSC 1 | 0.18 | Lognormal(8.77,19.62) |
| SSC 2 | 0.22 | P(1)=0.556,P(2)=0.1,P(3)=0.067,P(4)=0.044, P(>4)=Lognormal(16.86,14) |
| SSC 3 | 0.20 | 2 Months |
| SSC 4 | 0.02 | Triangular(1,5,28) |
| SSC 5 | 0.07 | Lognormal((12.57,26.79) |
| SSC 6 | 0.09 | P(1)=0.76, P(>1)=Lognormal(3.98,1.62) |
| SSC 7 | 0.06 | Gamma(0.62,8.97) |
| SSC 8 | 0.02 | 1 Month |
| SSC 9 | 0.12 | Lognormal(12,26) |
| SSC 10 | 0.02 | Lognormal(12,26) |

Table 4. SSC Input Probabilities

Table 5 provides the probability generation results. The probabilities represent the probability that the number of that type of SSC will be occurring at any point in time. For example, the probability that 2 SSCs of type SSC 5 will occur at the same time is 0.34. A blank entry represents a probability of 0.

| | SSC 1 | SSC 2 | SSC 3 | SSC 4 | SSC 5 | SSC 6 | SSC 7 | SSC 8 | SSC 9 | SSC 10 |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| P(0) | 0.2690 | 0.5450 | 0.5892 | 0.7556 | 0.3064 | 0.5868 | 0.6179 | 0.9363 | 0.2269 | 0.6781 |
| P(1) | 0.2590 | 0.2853 | 0.2739 | 0.2045 | 0.3418 | 0.2866 | 0.2748 | 0.0621 | 0.2655 | 0.2592 |
| P(2) | 0.1870 | 0.1120 | 0.0955 | 0.0346 | 0.2097 | 0.0933 | 0.0815 | 0.0016 | 0.2071 | 0.0537 |
| P(3) | 0.1200 | 0.0391 | 0.0296 | 0.0047 | 0.0936 | 0.0253 | 0.0201 | | 0.1347 | 0.0080 |
| P(4) | 0.0722 | 0.0128 | 0.0086 | 0.0006 | 0.0339 | 0.0062 | 0.0045 | | 0.0788 | 0.0011 |
| P(5) | 0.0417 | 0.0040 | 0.0024 | | 0.0106 | 0.0014 | 0.0009 | | 0.0430 | |
| P(6) | 0.0234 | 0.0012 | 0.0007 | | 0.0030 | 0.0004 | 0.0002 | | 0.0224 | |
| P(7) | 0.0129 | 0.0004 | 0.0002 | | 0.0008 | | | | 0.0112 | |
| P(8) | 0.0070 | 0.0002 | | | 0.0002 | | | | 0.0055 | |
| P(9) | 0.0037 | | | | | | | | 0.0026 | |
| P(10) | 0.0020 | | | | | | | | 0.0012 | |
| P(11) | 0.0010 | | | | | | | | 0.0006 | |
| P(12) | 0.0005 | | | | | | | | 0.0003 | |
| P(13) | 0.0003 | | | | | | | | 0.0002 | |
| P(14) | 0.0003 | | | | | | | | | |

Table 5. Probability for Number of SSCs Occurring

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3 STOCHASTIC MODEL FORMULATION

3.1 Introduction

TACAB's stochastic optimization model needs to incorporate the probabilistic nature of the data. Unlike traditional optimization, stochastic optimization accounts for uncertain data, such as the unknown number of SSCs occurring at the same time. The impact of a solution is mitigated by the probability of all possible outcomes. For example, suppose a solution recommends including enough units in the force structure to accomplish a task in two SSCs when there is a chance that there can be up to fifteen occurrences at one time. The penalty for not being able to accomplish the task for the other thirteen occurrences is multiplied by the probability of the occurrences happening.

The objective function minimizes the ineffectiveness of the units chosen to be included in the force structure, while restricting the number of personnel to the mandated end strength. The necessity of being able to fight and win the MCOs was combined with the necessity of being able to perform a variety of SSCs. Planning for the MCOs was considered a given and the SSCs were treated as stochastic events.

The formulation is presented below.

3.2 Set Definitions

The primary sets, or indices, that are used in the formulation are:

$u \in U$ = set of unit types

$t \in T$ = set of task types

$s \in S$ = set of SSC types

$o \in O$ = an ordered set of the number of times each SSC type occurs at any point in time

O is a set of integers from zero to the largest number of occurrences that are possible. It is used as a counter to ensure the probabilities are calculated properly.

3.3 Data Definitions

Although the following are used as compound sets in the formulation, they are input data. These sets define the tasks that need to be performed and the units that can perform them.

$v \in V(u, t)$ = the set of units that can perform each task

$w \in W(t, s)$ = the set of tasks to be performed in each SSC

The following provides the notation for the remaining input data.

- r_u – the number of unit type u required for the major contingencies
- m_u – the number of people in unit type u
- q_u – the penalty for having a shortfall of unit type u in the major conflict
- $k_{t,s}$ – the penalty for not being able to accomplish task t in SSC s
- $p_{s,o}$ – the probability of SSC type s having o occurrences at any point in time
- $g_{t,s}$ – the number of times task t needs to be performed in each occurrence of SSC type s
- E – endstrength of the Army

3.4 Variable Definitions

Positive variables are defined below.

- x_u – the number of unit type u in the force structure
- $y_{u,t,s}$ – the number of unit type u assigned to perform task t in SSC s
- a_u – excess of unit type u for major conflicts
- b_u – shortage of unit type u for major conflicts
- $d_{t,s,o}$ – shortage of capacity to perform task t in SSC type s when s occurs o times
- $f_{s,t}$ – the number of occurrences for which task t can be performed for SSC type s
- $h_{s,t,o}$ – a binary variable that maintains the order of the occurrences for task t in SSC type s where 1 indicates that task t in SSC s has been performed for occurrence o

3.5 Objective Function

This objective function minimizes the penalties for not having units for major contingencies, $q_u b_u$, as well as for not being able to perform tasks in SSCs, indicated by $k_{t,s}(1 - h_{t,s,o})$. The penalties for not being able to perform a task are multiplied by the probability that a task will need to be accomplished (note: the probability that a task will need to be accomplished is the same as the probability that an SSC will occur).

The objective function was

$$\text{minimize } \sum_{u \in U} q_u b_u + \sum_{s \in S, t \in T} k_{t,s} \left(\sum_{o \in O} p_{s,o} (1 - h_{t,s,o}) \right)$$

3.6 Constraints

The objective function was subject to the following constraints:

(1) MCO requirements. This constraint introduces the MCO slack variables that allows the model to calculate excesses and shortages relative to the MCO requirement. This ensures that overages and shortfalls for the major contingencies are correctly accounted for.

$$x_u - a_u + b_u = r_u \quad \forall u \in U$$

(2) Endstrength. The endstrength of the Army may not be exceeded.

$$\sum_{u \in U} m_u x_u \leq E$$

(3) Assigning units to tasks. This constraint determines which type of unit will perform each task within an SSC and determines the task shortfalls.

$$\sum_{u \in U'(u,t)} y_{u,t,s} + d_{t,s,o} = g_{t,s} \quad \forall s \in S, t \in W(t,s), o \in O$$

(4) Unit accounting. The total of each type of unit assigned to tasks cannot be more than the number available in the force structure.

$$\sum_{s \in S, t \in T} y_{u,t,s} \leq x_u \quad \forall u \in U$$

(5) The following three equations ensure that the probabilities are counted in the proper order.

a. This constraint counts the total number of units assigned to perform a task for a type of SSC and divides it by the number of times a task needs to be performed in the SSC. This gives the number of occurrences for which this task can be accomplished.

$$\frac{\sum_{u \in U'(u,t)} y_{u,t,s}}{g_{t,s}} = f_{s,t} \quad \forall s \in S, t \in W(t,s)$$

b. This constraint ensures the binary variables indicating that a task can be accomplished sum to the number of occurrences determined above.

$$\sum_{o \in O} h_{s,t,o} = f_{s,t} \quad \forall s \in S, t \in W(t,s)$$

c. This constraint ensures that correct binary variables for each task are equal to one, i.e., that the binary variable for occurrence 12 is not equal to one when the binary variable for occurrence 3 is equal to zero.

$$h_{s,t,o} \geq h_{s,t,o+1} \quad \forall s \in S, t \in T, o \in O$$

4 EXAMPLE RESULTS

4.1 Introduction

This section provides the results of the model using the example data in section 2. Recall that in this example, there are three SSCs, ten units, and 15 tasks. The probability data that was used is in Table 5 for SSC 1, SSC 2, and SSC 3. An endstrength of 600,000 was used because it was approximately 70% of the number of soldiers required for the MCOs in the example.

4.2 Results

Table 6 provides the total number of each unit that was recommended, the number of units that are short, and the number that are over the MCO requirements. For example, we have an MCO requirement of 70 for Unit 9, but the model recommended having only 45 in the force structure, leaving a shortfall of 25. On the other hand, the MCO requirement for Unit 6 is one, but the model recommends having 10, making an overage of 9.

| | Number | MCO Shortfall | MCO Overage |
|---------|--------|------------------|----------------|
| Unit 1 | 53 | | |
| Unit 2 | 57 | | |
| Unit 3 | 28 | | |
| Unit 4 | 30 | | |
| Unit 5 | 77 | | |
| Unit 6 | 10 | | 9 |
| Unit 7 | 76 | | |
| Unit 8 | 0 | 81 | |
| Unit 9 | 45 | 25 | |
| Unit 10 | 4 | | |

Table 6. Total Number of Units in the Force Structure

Table 7 gives the unit that has been assigned to perform a task and the quantity that has been recommended. For example, eight units of Unit 3 are assigned to perform Task 3 in SSC 1, so Task 3 can be accomplished if eight SSCs of type 1 are occurring at the same time. Table 7 also provides the maximum number of occurrences for which the task will not be accomplished. For example, from the probabilities of SSC 1 occurring, at most 15 SSCs of Type 1 will occur at one

time, so in the worst case, the Army will not be able to accomplish Task 3 in seven SSCs of Type 1.

| | Task | Unit | Quantity | SSC Task Shortfall |
|-------|------|------|----------|--------------------|
| SSC 1 | 1 | 2 | 15 | |
| | 3 | 3 | 8 | 7 |
| | 5 | 1 | 15 | |
| | 7 | 4 | 15 | |
| | 8 | 3 | 7 | 8 |
| | 9 | 2 | 15 | |
| | 11 | 4 | 15 | |
| | 14 | 6 | 4 | 11 |
| SSC 2 | 4 | 9 | 9 | |
| | 8 | 3 | 5 | 4 |
| | 12 | 2 | 9 | |
| | 14 | 6 | 3 | 6 |
| SSC 3 | 1 | 2 | 8 | |
| | 3 | 3 | 5 | 3 |
| | 8 | 3 | 3 | 5 |
| | 10 | 5 | 8 | |
| | 13 | 2 | 8 | |
| | 14 | 6 | 3 | 5 |

Table 7. Task/Unit Assignments for SSCs

Note that, although only 1 unit of Unit 6 is required for the major conflicts, the optimization recommends resourcing 10 units because each SSC requires task 14, which can only be performed by Unit 6, and the inability to perform the task carries a relatively high penalty.

5 SUMMARY

5.1 Follow-on Actions

As mentioned in previous sections, this was a research project. Much of the data necessary to use this methodology is not yet available. When this data becomes available, research can begin to develop the penalties for the objective function in a more meaningful way. One way that can be used for this is to survey subject matter experts to develop a weighting scheme.

We recommend that, if this methodology is used, the optimization model be run several times, using variations on the penalties in the objective function. The penalties can be varied to give emphasis to whatever the decision makers deem important, showing the trade-offs for various policies. For example, one run could have a higher penalty for not being able to perform the tasks in an SSC of Type 1, while another could have a higher penalty for not having enough armor units for the MCOs. By varying the penalties, the project can provide the decision makers insights into force structure issues.

5.2 Summary

The information that can be gained from this type of model can provide valuable insights into force structure decision making. As seen in the example in Section 4, units that do not have a large requirement for major combat could be very beneficial in SSCs. In this case, it would make sense to have more of these units in the force structure, even though they would not be needed in a major conflict. This would help balance the needs of major conflicts with the needs of small-scale contingencies.

This methodology incorporates the probabilities associated with SSC occurrences into an optimization model. This has not been used in force structure decision making. When data becomes available, the model developed could provide valuable insights into the issues of balancing SSC requirements with major conflict requirements.

APPENDIX A PROJECT CONTRIBUTORS

1. PROJECT TEAM

a. Project Director:

Ms. Linda Coblentz

b. Team Members:

LTC Pat Dubois
Mr. Herman Orgeron

2. PRODUCT REVIEWERS

Dr. Ralph Johnson

3. EXTERNAL CONTRIBUTORS (If any)

Dr. Andrew Loerch, George Mason University
Dr. Karla Hoffman, George Mason University

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APPENDIX B STUDY DIRECTIVE

