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13. ABSTRACT (Maximum 200 words)

Optical imaging of objects through atmospheric obscurants is an area of active research driven by national defense, commercial, and scientific interests. Multiple scattering of light by particles in the intervening medium (such as, cloud, aerosol, fog, smoke in the atmosphere) reduces signal intensity and signal-to-noise ratio, contributes to the blurring of image. Using pulsed lasers, time-gated detectors, and other techniques to sort out the ballistic and early snake photons from the diffusive photons it is possible to extract information about objects embedded in turbid media. The objective of the research is to carry out a theoretical investigation for photon starvation and image resolution, associated with imaging of a distant object through a scattering and obscuring medium. In this research, we estimate the feasibility of ballistic and snake light imaging at different distances computed using our analytical cumulant expansion method and compare it with results of the Monte Carlo simulation.

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The final report for contract DAAD19-01-1-0668

Feasibility Study of Optical Imaging through Atmospheric Obscurants

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I. Introduction

Imaging of objects through scattering media is an area of active research driven by national defense, commercial, and scientific interests, such as, space-based intelligence gathering, tactical reconnaissance, space-based terrestrial monitoring, free-space optical communication, and ground-based astronomical imaging. Multiple scattering of light by particles in the intervening medium (such as, cloud, aerosol, fog, smoke in the atmosphere) reduces signal intensity and signal-to-noise ratio, contributes to the blurring of image, and, in extreme cases, buries the image completely into background noise. Using pulsed lasers, time-gated detectors, and other techniques to sort out the ballistic and early snake photons from the diffusive photons it is possible to extract information about objects embedded in turbid media.

The objective of the research is to carry out a theoretical investigation for photon starvation and image resolution, associated with imaging of a distant object through a scattering and obscuring medium. The problem of "photon starvation" poses the question that given a target-detector separation in the intervening medium, will there be enough image-bearing photons to detect the target. The problem of "image resolution" asks what is the minimum separation between two points on the target, that can be resolved using reflected early photons. A theoretical estimation of these two parameters is crucial for design of experimental tests. These quantities require an accurate solution of the photon distribution function, $I(\mathbf{r}, \mathbf{s}, t)$, as function of position $\mathbf{r}$, direction $\mathbf{s}$, and time t. The mathematical equation governing photon propagation is the well-known radiative transfer equation (RTE):

$$\partial I(\mathbf{r},\mathbf{s},t)/\partial t + c\mathbf{s}\cdot\nabla_{\mathbf{r}} I(\mathbf{r},\mathbf{s},t) + \mu_a(\mathbf{r}) I(\mathbf{r},\mathbf{s},t) = \mu_s(\mathbf{r})\int P(\mathbf{s},\mathbf{s}',\mathbf{r})[I(\mathbf{r},\mathbf{s}'t) - I(\mathbf{r},\mathbf{s},t)]d\mathbf{s}' + \delta(\mathbf{r}-\mathbf{r}_0)\delta(\mathbf{s}-\mathbf{s}_0)\delta(t-0), \quad (1)$$

where the fundamental parameters of the background medium are the scattering rate $\mu_s(\mathbf{r})$, the absorption rate $\mu_a(\mathbf{r})$, and the differential angular scattering rate $\mu_s(\mathbf{r})P(\mathbf{s}, \mathbf{s} , \mathbf{r})$; c is the light speed in the medium.

Recently, we have developed a new approach to obtain an analytical solution of RTE, based on a cumulant expansion, in an infinite uniform medium with an arbitrary phase function $P(\mathbf{s}', \mathbf{s})$.[1,2] We have also derived the analytical cumulant solution for the vector radiative transfer equation.[3]

The salient features of this solution are:

(a) An explicit algebraic expression of spatial cumulants at any angle as functions of time is derived, exact up to an arbitrarily high order n (the first cumulant is the central position of the distribution, the second cumulant is the half-width of the distribution, the 3th cumulant describes the skewness or asymmetry of the distribution, 4th cumulant describes the "kurtosis" of the distribution, that is the extent to which it differs from the standard Gaussian shape, and so on). That means the distribution function $I(\mathbf{r}, \mathbf{s}, t)$ can be computed with very high accuracy.

(b) At the second order, $n = 2$, an analytic, hence, very useful explicit expression for distribution function $I(\mathbf{r}, \mathbf{s}, t)$ is obtained. This distribution is Gaussian in position. The central position and spread of the half-width of distribution are always exact.

II. Results

1. Theoretical results of light distribution

We present the numerical results of the ballistic and the scattered components computed using our analytical cumulant expansion method and compare with Monte Carlo simulation. The Fig. 1 shows the photon distribution function $I(\mathbf{r}, \mathbf{s}, t)$ in an infinite uniform scattering medium as a function of time t for $g = 0.9$ (that represents the case of large scatterers, typically for cloud for laser wavelength $\lambda \sim 1$ μm). The detector is located at $R = 6\ l_t = 60\ l_s$ from the source front along direction of incident light, and the received direction is along the incident direction. The single Henyey-Greenstein phase function is used for calculation. There is no difficulty to use another form of the phase function in the cumulant expansion approach, but computing time in the Monte Carlo simulation will increase with the complexity of the phase function, for example, one obtained from

Mie formula. The absorption is negligible. The finite absorption can be easily included by multiplying a factor exp $(-c\,t\,/\,l_a)$ to the distribution function in an infinite uniform medium. We use l_t as the unit of length and l_t/c as the unit of time, so the unit of $I(\mathbf{r}, \mathbf{s}, t)$ is photons/[$l_t^3 \cdot$ rad] normalized to 1 photon injected from the source. Use of this scaling provides a normalized curve that can be applied for many cases of different value of the density of scatterers. In the figure, the solid curve is computed from approximation up to 10th order of cumulant; the dotted curves are computed from approximation up to second order of cumulant, and the discrete dots are from the Monte Carlo simulation. In order to reduce the statistical deviation to an acceptable level, 10^9 events are counted in the Monte Carlo simulation; and a box of $(0.2\ l_t)^3$ centered at $(0, 0, 6l_t)$ and a received solid angle up to $\cos\theta = 0.99$ (62.8 mrad) are taken for counting received events. The results are then expressed in the above mentioned units.

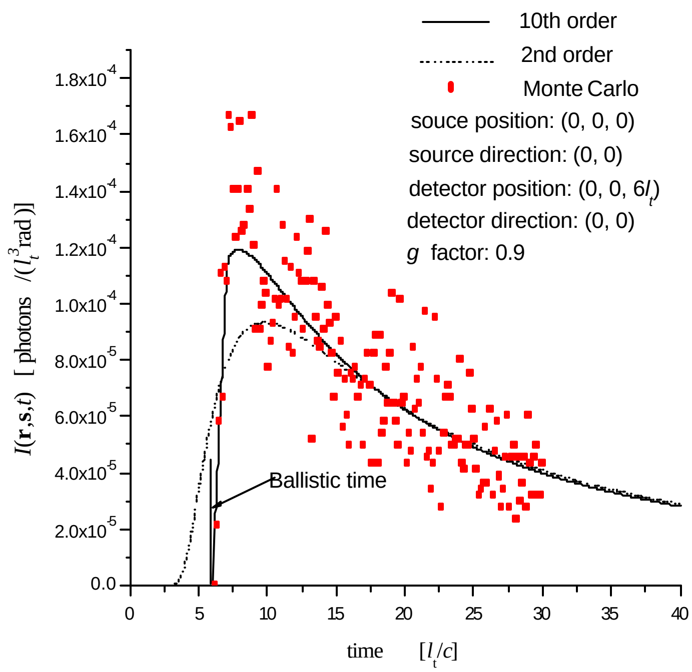

Fig.1 Transmitted photon distribution function $I(\mathbf{r}, \mathbf{s}, t)$ in an infinite uniform scattering medium as a function of time t for $g = 0.9$. The detector is located at $R = 6\ l_{tr} = 60\ l_s$ from the source front along direction of incident light, and the received direction is along the incident direction. The solid curve is computed from approximation up to 10th order of cumulant; the dotted curve is computed from approximation up to second order of cumulant, the discrete dark dots are from the Monte Carlo simulation.

The cumulant approximation terminated to second order provides a Gaussian distribution of $I(\mathbf{r}, \mathbf{s}, t)$ with exact spatial center position and exact spatial width, but it does not provide the correct detailed shape of the distribution. Note, especially, photons at front tail of the Gaussian distribution violate causality, as shown by dotted curve in Fig. 1. In order to overcome this fault of the second cumulant approximation, we have developed codes for computing the cumulant approximation up to an arbitrarily high order. The number of terms involved increases rapidly with increase of order of cumulant, resulting in a much longer CPU time than that in the second cumulant approximation.

If $g = 0$, $l_t = l_s$, so the ballistic component at $R = 6\ l_t$ decays as $e^{-(6\ lt\,/\,ls)} = e^{-6} = 2.5\text{x}10^{-3}$. On the other hand, in the case of $g = 0.9$, $l_t = 10\ l_s$, the ballistic component decays as $e^{-60} = 8.76\text{x}10^{-27}$, which is too weak to be shown in the figure 1.

2. Estimation of photon starvation

Next, we estimate the number of photons backscattered from a target received by a detector. In order to count the photon flux through a detector during a time range, the following integration needed to be taken:

$$\int dt \int d^2\mathrm{r} \int d\mathrm{s} c I(\mathrm{r}, \mathrm{s}, t)\,, \qquad (2)$$

where c is speed of light. In Eq. (2), integration $d^2\mathbf{r}$ is taken over the area of detector, $d\mathbf{s}$ is taken over the received solid angle, and dt is taken over the time range being gated. In order to make a quantitative estimation of the effective scattered photons received by a detector, one must consider the size, received solid angle, and time-gating of the detector. Assuming a cloud medium with the scattering length $l_s = 50$ m (which means the visibility $V = 195$ m), so that $l_t = 500$ m for $g = 0.9$ (this g factor correspond to the size of water drop $r/\lambda \sim 1\text{-}2$, with r the radius of drop, and λ the wavelength of light), and absorption is negligible

First, we consider a detector located at $R = 3$ km from source, with the size A = 0.1 m^2 and receiving solid angle $\Omega = 10$ mrad, which leads to a reduction factor of $4\text{x}10^{-9}$. As shown in Fig. 1, the photon distribution rises to $\sim 0.8\text{x}10^{-4}$ in a time range of $1\ l_t\,/c$ (about 1.67 μsec). Hence, this detector received (according to Eq. 2, counting photons in an area of triangle in Fig. 1) $\sim 1.6\text{x}10^{-13}$ of photons incident from the source. If the time range is gated to $0.1\ l_t\,/c$ in order to improve the resolution of image, (the maximum path of light is $6.1\ l_t$, compared to $6\ l_t$ for ballistic light), then the received photons/incident photons will be $\sim 1.6\text{x}10^{-15}$. In any of the above cases the received early scattered photons are much stronger than ballistic photons, which are as $\sim 10^{-26}$ of the incident photons.

For estimation of a round trip of light reflecting from a target, detailed information of the target properties is needed. A rough estimation can be performed assuming that the target is a completely reflecting plane perpendicular to light direction with large enough area (a large mirror). If the mirror is located at $R_m = R/2$ from source, because of the "image mapping" effect of the mirror, the number of photons received by an "imaging detector" located at R is equal to number of reflected photons received by the detector located at the source position. Hence, the above estimation of photons received for photon transmission (one way) case by a detector at R = 3 km is equivalent to that of reflection case (two way) with a reflecting mirror located at R_m = 1.5 km.

Second, we estimate the pure ballistic photons reflected from a target located at distance R_m from the source and the detector (two way), assuming the target is a completely reflecting plane perpendicular to light direction with the large enough size. The source is uniformly distributed in a divergent angle ϕ and detector has size of A. For this geometry, the received reflected ballistic photons are $N = N_0 \exp(-2 R_m / l_s) F_g$, where N_0 is the number of photons from the source and the geometry factor F_g is given by

$$F_g = A/[\phi (2 R_m)^2] \quad \text{or} \quad F_g = 1 \quad \text{if } F_g > 1 \qquad (3)$$

If R_m = 500 m = 10 l_s = 1 l_t (l_s =50 m and g = 0.9), the laser divergent angle is $\phi \sim 20\text{x}10^{-6}$ rad, and the detector size is A = 0.1 m^2, the received ballistic photons are $\sim 10^{-11} N_0$. If the source power is 100 mJ, which provides ~ 5 x10^{17} photons with the wavelength λ = 1 μm, the received ballistic photons is ~5 x10^6. When R_m = 750 m, the ballistic photons received decrease to ~ 100. Considering that reflecting on the target is not perfect, therefore, R_m ~ 500 m (or 10 l_s) for $2R_m$ round trip as a standard for use of the pure ballistic photons in imaging seems to be proper.

Table 1 lists the received ballistic and snake photons through cloud at different distances between the target and the source-detector.

The photons reflected from a target through cloud,

received by a detector located at the source position

		Ballistic photons		Snake photons			
Medium	l_s	50 m		50 m			
	g factor	Not relevant		0.9			
Source	Power*	100 mJ		100 mJ			
		20 μrad		Not relevant			
Detector	Area***	0.1 m^2		0.1 m^2			
	Received angle	Not relevant		10 mrad			
	Time gating	~ 0 nsec		1670 nsec		167 nsec	
Target****	Distance R_m	500 m	750 m	1.5 km	2.5 km	1.5 km	2.5 km
		$5x10^6$	100	$8x10^4$	10^3	$8x10^2$	10

* the wavelength of laser is 1μm; the received photons are proportional to the power of source;
** the received ballistic photons are inversely proportional to the divergent angle of source;
*** the received photons are proportional to area, and the received solid angle of detector;
**** the target is assumed to be a large complete reflecting mirror; R_m is the distance from source; l_s is the scattering length, absorption is neglected.

3. Conclusion

(1) The number of received photons at certain distance of target-detector depends strongly on the weather.

(2) If only the ballistic photons are received, the detectable distance between target-detector is about 600 m.

(3) If the snake photons are received, the detectable distance between target-detector can be extended to 1.5-2 km. In this case, a recognizing process should be developed for improving the resolution.

Publications related to this research:

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