The Adaptive Design of Experiments and Markovian Models

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Research was conducted in three broad areas: setting confidence intervals following an adaptive or sequential experiment; central limit theory for sums of stationary processes; the monotone change problem and related problems in isotonic inference. For the confidence intervals, existing techniques were extended to include grouped sequential methods and to allow for nuisance parameters. The work on central limit theory emphasized state space models (iterated random functions) and non-linear functionals of a linear process. In both cases, asymptotic distributions were obtained under very mild continuity conditions on the function or functional that is summed. The central limit theory was used to obtain the limiting distribution of a new test statistic in the context of a change point problem. For this the change point problem was reformulated to allow several gradual changes, as opposed to the single abrupt change implicit in the classical change point problem. The related problems in isotonic inference include a novel suggestion for the appropriate "degrees of freedom" in an isotonic regression problem.

Central limit theorem, change point problem, confidence intervals, isotonic inference, state space models, very weak expansions.
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Statement of the Problems Studied

Research conducted under the grant may be divided into four main categories, each of which is described below.

a) Corrected confidence sets for adaptively designed experiments.

b) Central limit theory for sums of a stationary process.

c) The monotone change problem and related topics from isotonic inference.

d) Miscellaneous other.

Parts a) and b) were explicitly part of the proposal for DAAG-55-0482, and the monotone change problem in c) is an important statistical application of b). The research described in d) was not envisioned when the proposal was written.

Summary of Important Results

a) Adaptively Designed Experiments

Corrected confidence intervals for sequentially designed experiments were studied in detail in [4], [5] and [12]. In [5], for example, it was shown that very weak expansions provide good approximations to actual coverage levels for confidence intervals set following a grouped triangular test when responses follow a normal distribution. This was encouraging, because there has been very little previous work on the accuracy of very weak approximations in a grouped sequential setting. The formulations in [4] and [12] are more general, allowing non-normal responses, nuisance parameters, and continuous inspection as well as grouped testing. The problem of setting confidence intervals for an unknown population size is studied in [1]. A main finding is that the problem is much harder in the presence of nuisance parameters, but possible for large populations. In [16] a new version of the non-linear renewal theorem is obtained which in the perturbations are required to
be approximately stationary, but not slowly changing. The limiting joint distribution of
the excess over the boundary and the last perturbation is obtained.

b) Central Limit Theory

One central theme of the research was to find limiting distributions for sums of station-
ary processes and to develop statistical applications. This work took several forms. In [8],
asymptotic normality was established for sums $S_n = g(X_1) + \cdots + g(X_n)$ of an iterated ran-
dom function $X_n = \psi(X_{n-1}, Y_n)$, where $X_0, Y_1, Y_2, \cdots$ are independent and $Y_1, Y_2, \cdots$ are
i.i.d.. The conditions in [8] are essentially growth conditions on the $L^2$ norms of $E(S_n | X_0)$.
They do not even require that $g$ be continuous and are applicable the indicators of sets
whose boundaries are not too messy. Iterated random functions were explored in more
detail in [13]. Central and non-central limit theorems were obtained for sums of non-linear
functionals of a linear process. After proper normalization such sums were shown to be
either asymptotically normal or to converge in distribution to a multiple Wiener integral,
depending on the non-linear function and the sizes of the coefficients in the linear process.
This work was reported in [7] and [19]. Applications to density estimation were developed
in [18].

c) The Monotone Change Problem

And Related Topics

A new approach to the change point problem was developed in [9]. The new approach
allows a gradual change as opposed to the abrupt change implicit in the change point
problem; it also allows dependence among the observations. The main contribution is an
asymptotic test for stationarity with good power against models with a change that is
monotone in an appropriate sense. This work uses the limit theory described above and
some related techniques from isotonic inference. The related techniques were developed
too. In [2] a novel suggestion for the appropriate "degrees of freedom" following an isotonic
regression is made and studied. In [10], [14], and [17] proposals for combining smoothness
and monotonicity in the estimation of a non-increasing density are made and studied.

d) Miscellaneous Other

Classical methods for setting confidence intervals often encounter difficulty when pa-
rameters are restricted—for example, when a parameter is know to exceed some threshold.
Such restrictions do not pose any special difficulty for Bayesian method, but these produce
credible sets, not confidence sets. In [11] and [21] Bayesian-frequentist compromises are
developed in which Bayesian credible intervals also have high frequentist coverage proba-
hand and in variance components models on the other. In [15], asymptotic distributions
are obtained for the number of clusters in a set of random points.
Publications

Published Research Articles

Peer Reviewed Journals


Other Publications

Theses


Submitted Research Articles


Technical Reports


Scientific Personnel Supported

The scientific personnel supported by the grant were Dong Yun Kim, Graciela Mentz, Ruby Weng, Michael Woodroofe, Wei-Biao Wu, Rong Zhang, and Tonglin Zhang. Except for Woodroofe, all were Ph. D. students in Statistics. Ruby Weng, Rong Zhang, and Wei-Biao Wu have completed their degrees; their thesis titles are listed above. Tonglin Zhang has completed the research for his thesis and will finish this summer. Dong Yun Kim and Graciela Mentz are expected to finish by the end of the next academic year.