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The task is to collect, sunumarize, document, and put in useful form the basic cross sections, stopping powers, and other parameters characterizing the passage of energetic particles trough matter: photons, electrons, protons, alpha particles, neutrons, and heavy ions, penetrating all elements. A part of the task is to decide, of all available parameters, which ones are worth so documenting.

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THE PASSAGE OF ENERGETIC PARTICLES THROUGH MATTER

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FINAL REPORT

March 31, 2003

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PREFACE

The subject of the passage of neutral and charged particles through matter has been studied for nearly a century. As such, there is little new that can be said that hasn't been said before; as with many books that primarily collect information from a number of sources, there is only little original in the present one. Almost all the data collected here are available elsewhere, often on the Web, or in journal articles. However, workers in need of certain parameters (cross sections, stopping powers, etc.) may not be aware of the availability of the needed data, which can be well-known to other disciplines. Nuclear physicists, for example, may be very familiar with the cross section compilation in the Evaluated Nuclear Data Files (ENDF), but those studying the effects of cosmic rays on, say, satellites or micro-electronics, may be less so. It is not widely known that photo-atomic and electron-atomic cross sections are also tabulated in ENDF. Health physics workers, for instance, may not be aware of the full cross section compilations available. It is not widely known that models for electron multiple scattering are available on the Web. Knowing where to access those and other data, extracting a needed subset from all that is available, knowing how to interpret the format in which it is presented, can be time-consuming tasks. At least that has often been the experience of the author and, according to a short, informal survey, the author's colleagues as well. Graphs of parameters as a function of the relevant independent variable (cross sections vs. energy or vs. scattering angle, etc.) are sometimes what is wanted, and occasionally are sufficient for the purpose at hand.

The basic physics of the passage of photons, electrons, protons, and heavier charged particles through matter was worked out theoretically in the decades following the completion of quantum mechanics, largely in the 1930's, and neutron diffusion shortly thereafter. There is excellent summary documentation, from the 1950's and later. Today, one has available large data libraries with cross sections compiled from decades of experimental measurements and sophisticated calculations. There is, however, something of a gap between the early theoretical treatments and the modern data collections. The accuracy of data in current libraries far surpasses that of the early, relatively elementary, calculations. Below 100 keV, for example, the Bethe mean stopping power formula for electrons is less than 85% of the actual stopping power, and below 10 keV may be less than 60% of the correct value in some materials. In a solid this is seldom a problem, for the range of a 10 keV electron is so short (less than 10^{-3} cm) as to be of little interest except in specialized applications, but in a gas the difference can matter.

It was therefore deemed sensible to try to collect in one place as much of these often needed data as possible, together with enough background physics so the reader can feel comfortable applying them, having some understanding of where they come from and why they have the order of magnitude they have. We have also tried to digest the data in the form of useful graphs, showing dependencies over a wide range of the independent variable(s). And it was decided to include much of the data on a CDROM included with the book. Throughout, we include references to where the data came from, and where updates to them, and related information, can be found.

In addition, certain features of particles interacting with matter are not so well known. It is widely appreciated, for example, that when a photon Compton scatters from a free electron, its angular distribution peaks in the forward direction. But it is less widely recognized that when a photon Compton scatters from an *atom*, its angular distribution *vanishes* in the forward direction.

Similarly, for electrons, the continuous-slowing-down-approximation (in which each electron loses energy exactly at the mean rate) is widely used for energy loss calculations in electrons penetrating matter. But it is less widely recognized that when used for computing the slowing down electron energy spectrum it can be quite inadequate, and even violates detailed balance.

Likewise for electrons, it is common knowledge that the Coulomb cross section for scattering from an isolated charge (nucleus) diverges. Screening of atomic electrons makes the electron-atom elastic scattering cross section finite. But it is less widely appreciated that the elastic differential cross section in exactly the forward direction (θ=0) increases in proportion to the square of the incident electron energy, to a value of, say, 10^{13} barn/sr at 100 MeV on Fe(!). At an angle of only 1° it has fallen 9 orders of magnitude to 10^4 b/sr. 99% of the scattering occurs at angles less than 0.1°, in a solid angle of only 10^{-8} of a sphere.

It is well known that any cross section, say the photon-atom Compton cross section $\sigma(E)$, is a function of incident photon energy E for each target material. It is therefore obvious, yet not widely appreciated, that the cross section on elements, $\sigma(Z,E)$ is a function of atomic number Z and E, and can be represented as a surface in Z-E space, represented by, say, a contour plot in that space. One thereby displays an interaction for all materials (or at least for all elements) over essentially all energies of interest on a single graph, enabling global trends to be discerned and helping one to choose a material with desired characteristics (and, perhaps, enabling errors to be spotted). For a photon cross section, for example the total cross section $\sigma_{Tot}(Z,E)$, such a contour plot brings to light the difference between the atomic cross section (barn/atom) as a function of Z and E, and the bulk matter cross section (cm^2/gram) as a function of Z and E. In the conversion between the two the atomic weight $A(Z)$ of elements in their natural isotopic composition enters the conversion factor. Since $A(Z)$ has irregular behavior as a function of Z, contours of σ_{Tot}(cm^2/gm) exhibit an irregular pattern that does not occur in the smoother σ_{Tot}(barn/atom). Similar cross section graphs can be constructed for any other cross section or for stopping powers of charged particles.

Such contour plots are of more than academic interest. Not only does one see the full span of physics on a single page for, say, the total photon cross section as the dominant process passes from photo-electric absorption to Compton scattering to pair production, but in addition one can read the numerical value of the cross section often to better than 5%. Accuracies of that order are often sufficient for quick estimates. It seems worthwhile to bring these and other features to the attention of a wider audience.

Further, some published data are based on quite sophisticated calculations (for example self-consistent relativistic Dirac, Hartree, Fock models) that generally give a very believable result for cross sections, but may produce other unrealistic features. The self-consistent DHF model of elastic photon-atom scattering produces detailed form factors, but can be inaccurate for absorption edges. Such published data are most meaningful to other specialists in the field. The general user may wonder why a seemingly elementary quantity like the photo-ionization edge in Fe, which is given as 7.9024 eV in most tabulations, is given as 7.530 eV in LLNL's Evaluated Photon Data Library (EPDL) data base, and appears as 3.60 eV in calculations of elastic form factors [C.T. Chantler *J. Phys. Chem. Ref. Data* **24**:71 (1995)]. Likewise the separation of edges, such as L_{II} to L_{III}, may not be calculated accurately. The reason is that detailed models accurate for their intended purpose are not necessarily accurate for bound energies; they tend to break down at the 3-5 eV level. The resulting photo-ionization edge may be off by more than 1 eV. In particular, Livermore's EPDL photo-atomic library was constructed together with its EEDL electro-atomic counterpart for the purpose of having a consistent set of cross sections for electron-photon transport calculations. Here consistency between data sets is more important than absolute accuracy. The library documentation makes that purpose clear, and cautions against using the cross sections for other purposes without checking other sources. Merely bringing these and other related facts to the attention of a wider audience seemed sensible.

Fortunately for the author, the Air Force Office of Scientific Research agreed that this was a worthwhile undertaking, it hopefully saving future workers much time, and has funded its writing for some months. The author is grateful for having that support; there is no way the book could have been written without it.

Certain discussions may appear too elementary or unnecessarily detailed. For example the discussion of terminology and fluxes in Chapter 1 is indeed elementary, and it may appear that the angular distribution of δ-rays in Chapter 3 is an uncalled-for detail. But not including those discussions would be tantamount to pre-selecting and/or pre-judging the reader or the application. If a discussion is too detailed for some readers, no harm is done in including it for those who may find it helpful.

Data in the LLNL cross section data base is available in a certain format, commonly the so-called ENDF format which presents numerical values in fixed-width fields, six numbers to a line (three data points, each an energy and the cross section). To access those cross sections in useful form one needs to write a separate code to read that format. Here we present those cross sections on the included CDROM in the more useful columnar format, the energy in the first column and cross sections for individual processes in successive parallel columns.

Sources for these data and their updates are given as Web addresses. Unfortunately, internet URLs change over time; in a few years or months readers may not find the referenced address. We have found no simple way around this dilemma except to note that each address is usually associated with a particular organization which survives longer than the specific address for a subset of it that contains the data in question. Creative hunting for the new address within that organization may be necessary. While the address for photo-nuclear cross sections at the International Atomic Energy Agency, http://iaeand.iaea.or.at/photonuclear/reco mmended/, may change, the IAEA will still be there (currently at http://www.iaea.org/), and can be located by a search engine if needed.

This book is intended to be a working reference providing ready access to useful data, with enough discussion of the background physics to make understandable the order of magnitude of their numerical values. It is *not* intended to be a comprehensive treatise on the passage of particles through matter. References that together may be taken to constitute such a treatise are given in Chapter 1.

In addition to the cross sections, stopping powers, angular distributions, etc., presented here, there are numerous other quantities involving radiation interacting with matter that are of interest in various applications. Among them are:

- The photon build-up factor, the increase in photon and electron flux with depth due to cascade showering.

- Differences between electron and positron cross sections, stopping powers, and ranges. These are of interest, for example, in the medical applications of Positron Emission Tomography.

- Electron (and ion) restricted stopping power, the stopping power due to individual energy losses less than a specified amount. Or a related quantity, especially important in radiation effects in electronics, the radial profile of energy deposited along the trajectory of a primary electron or ion. This accounts in large measure for the principle difference between Linear Energy Transfer (LET) in micro-electronic applications and stopping power.

- The stopping power and ranges in compounds, as well as elements. Compound semi-conductors and particle detectors, plastics, metal alloys, dielectrics, biological tissue and other organic compounds, are some of the materials of interest.

- Stopping power and range of the mu meson, a component of sea-level cosmic rays. Only stable particles are discussed.

- Photo-electron and photo-photon yield; the number of backscattered electrons and photons created when a photon strikes a solid surface.

- The backscattered and secondary electron energy spectra, in addition to their number specified by their yield.

• Ionization caused by secondary nuclear particles following proton and heavier ion nuclear collisions.

Time limitations have prevented discussions of these and many other topics of interest; hopefully an opportunity will arise in the future that will permit their incorporation in a later edition.

Comments, noting of errors, and suggestions from readers are welcome. They may be addressed to the author at ncarron@alum.mit.edu. Being an e-mail forwarding service, this address will never change; it will expire only when the author does.

CHAPTER 1

INTRODUCTION

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

The study of the passage of photons, electrons, protons, α particles, neutrons, etc., through matter is essentially as old as the discovery of these radiations themselves. Indeed, it could hardly be otherwise. Original sources of energetic photons (X-rays), late in the nineteenth and early in the twentieth centuries, well before the name "photon" was invented, took place in vacuum chambers, and the radiations had to penetrate the walls of the chamber to propagate to detectors. Radiation detectors themselves were enclosed in a housing, and the radiation had also to penetrate its walls or entrance windows. Even radiation names were drawn from the notion of the passage of particles through matter. The names given to three forms of radiation, α, β, and γ, were selected in order of the "penetrating power" of the radiations, α particles being readily stopped by thin foils, β rays being more penetrating, and γ rays being capable of passing through relatively thick materials. The historically minded reader may follow the chronology in, for example, the beautiful account by Pais [Pa86].

Over the decades many fields of research have emerged in which energetic neutral and charged particles play a role. Their number seems to be increasing with time.

- Naturally trapped particles in Earth's inner and outer radiation belts routinely bathe satellites in beams of energetic electrons and protons, and internal electronics must be designed to withstand their effects.

- Energetic cosmic rays, and their secondary showers, unavoidably strike satellites and aircraft (and their passengers).

- Nuclear weapon bursts put out intense γ-rays, X-rays, β's, and neutrons, and an entire industry has grown to study their effects and shielding against them.

- Reactor laboratories need to understand the penetration of neutrons and γ's, and the secondary particles generated by them.

- Some laboratories routinely have pulsed or continuous X-ray sources, neutron guns, γ and β emitters, and other particle sources to study their effects on materials.

- The astronomy community is relying more and more on digital cameras, using CCDs in place of photographic emulsions. Performance of these electronic cameras is degraded over time as energetic particles, such as cosmic rays, impinge on them, and needs to be taken into account in the original design.

- Beams of charged particles, used for high energy research, or as microwave generators, or as tokamak feeds, or for plasma heating, may need vacuum-maintaining windows through which they pass. One needs to understand the response of the window to repeated passage of the charged particles, the secondary effects of the ensuing Bremsstrahlung, and the secondary effects of beam particles striking other surfaces.

- The health physics community is concerned with the effects of these radiations on biological tissue.

- Plasma physicists need to know the ionizing effects of charged particle beams passing through gases.

- Auroral and upper atmospheric studies require knowledge of the effects of energetic electrons and protons on atmospheric gases.

All of these endeavors require basic cross sections and stopping powers of particles in matter.

When a particle passes through material it changes the matter in a number of ways. A charged projectile will knock out atomic electrons, causing ionization (ion pairs in a gas, electron-hole pairs in a semiconductor); it can electronically excite atoms, which may de-excite by delayed photon emission (*fluorescence*); it may collide directly with a nucleus, causing it to recoil and displace in a solid; or it may produce Bremsstrahlung photons, which travel some relatively large distance to interact elsewhere in the material or which escape the material target. An energetic neutron or proton may cause a nuclear reaction ejecting other nuclear particles. The study of the effects of a particle or particles on the target material is known generally as *radiation effects*.

Likewise, the matter affects the particle, by scattering, slowing down, and/or absorbing it. The broad study of the effects of matter on the passage of particles is known generally as *radiation transport*. Both aspects of the process, the particle's effect on matter and the material's effect on the particle, are of interest. The former is of interest in radiation damage studies, as in cosmic ray effects in semiconductor electronics, and in particle detection, where the change in the target is used to register the passage of a particle. The latter, for example, is of interest in radiation shielding studies, or in the effect of a thin "attenuating" layer of solid purposely intended to slow down a proton beam.

As needed for these studies, it would clearly be useful to have at the ready a collection of the basic data on relevant parameters concerning the passage of energetic particles through matter. A single place where one can find cross sections and mean free paths of photons and neutrons; stopping powers, ranges, straggling, and scattering of electrons, protons, α particles, and possibly other ions; kerma and dose; fluence-to-dose conversions, etc., in the elements and in compound materials of interest. It would not need to be a textbook, for most users would already be at least somewhat familiar with most concepts. But, in addition to data, it should have enough of a discussion of the physics to convince the user that he or she understands the data, what's behind them, and how to use them properly. The present book hopes to provide that source of information, at least partially. It is intended to complement the more fundamental references and provide a useful tool for researchers; to help workers *utilize* the mass of knowledge and data on the passage of energetic particles through matter, and to make it easier for the reader to find needed quantities. In this last regard, the most commonly used data (cross sections, stopping powers, fluence-to-kerma conversion factors, etc.) are provided on an accompanying CD-ROM, and many Web sites are cited.

In this well-studied subject, almost all of the just mentioned parameters can be found scattered in books, articles, or on the Internet in a wide variety of sources. In particular, the three oft-referenced works

H.A. Bethe and J. Ashkin, "Passage of Radiations Through Matter", in E. Segre, ed., *Experimental Nuclear Physics*, Vol. 1, J. Wiley (1953)

W. Heitler, *The Quantum Theory of Radiation*, 3rd edition, Oxford Univ. Press (1954)

R.D. Evans, *The Atomic Nucleus*, McGraw-Hill (1955)

are immortal classics, and will remain irreplaceable for their thorough, fundamental discussions. The series of reports by the Lawrence Livermore National Laboratory (LLNL), carrying basic number UCRL-50400, has provided a wide community with cross section data for years. Particularly useful as data sources are certain web sites of the National Institute of Standards and Technology (NIST), the Los Alamos National Laboratory (LANL), the National Nuclear Data Center at Brookhaven National Laboratory (BNL), and LLNL. But formulas and the discussion of the physics behind them are not so readily available on the web, and it can be no mean feat to get to the data from a laboratory's home page. And often enough some extension of the raw data is desired. Having most of this in one place would be a time saver.

Since the advent of powerful computers, and the growing need for particle transport calculations for shielding, a number of equally powerful codes has been developed to solve the transport through matter of photons, neutrons, and charged particles. One of the most widely used is MCNP (Monte Carlo n Particle, formerly Monte Carlo Neutron Photon). Others are COG, TIGER (now ITS), TART, EGS, GEANT, and so forth. These are all very successful, and in common use. They excel at handling complicated three dimensional geometries, and they are well developed. The December, 2000 MCNP Version 4C manual states that there has been some 450 man-years (person-years, actually) in its development up to that time, and its conceptual origins date back to von Neuman and Fermi.

Partially as a result of the availability of these codes and computers, there is a growing fad among scientists to run big, existing codes to solve almost every problem, rather than to think first with much effort about the physics involved. As a result, many studies in the disciplines listed above have been (and are being) carried out with the well-established codes. The work involved in solving a problem then becomes the work involved in setting up the code input, and deciding on useful forms of output from all those available. For complicated geometries this is not simple. This tendency toward numerical work is neither clearly bad nor clearly good. Calculations involving complicated geometries using well established codes are absolutely necessary. The geometries in most real problems are so involved that there is really no other way to get the right answer.

For the most part the individuals who make use of the large codes are not those who wrote them, and often are not intimately familiar with them. Not uncommonly, for the uninitiated, the documentation does not quite achieve the epitome of ultimate clarity. To check one's work, to make estimates, to scope the problem before setting up a large code computation, to convince oneself that numerical results are correct, to understand their meaning, one needs to see the raw data, its graphs, and perhaps scaling laws (with particle energy, or with atomic number) from basic equations and data.

Further, one often wishes to write one's own (smaller) code rather than use the standard large ones. It gives one better control over the entire problem, and an assurance of understanding.

The present book is intended primarily to present needed parameters, data, and concepts to satisfy these purposes.

One might expect most atomic cross sections and particle stopping powers to be quite well known and well documented by now. While that is true in a general sense, practical problems remain. Uncorrected errors in published journal articles; sizeable differences among different data compilations, with little guidance as to which is the more reliable; and errors in the documentation for the use of those compilations still exist. Some of those differences merely reflect the current state of knowledge of those parameters. While the situation seems to be improving with time, it can be a chore to get reliable numbers for actual application. Further, documentation of codes and data libraries is written, of course, by the code authors and data compilers themselves, i.e., those intimately familiar with their subject. But the reader is not so familiar, and it can be non-trivial for experts to write in words understandable to the non-expert, to try to anticipate questions by those not as familiar with the material as the authors. We provide here much of the data itself, together with sufficient discussion to provide the reader with a useful guide.

The standard data sources, some mentioned above, generally present neutral particle cross sections and charged particle stopping powers for the elements. For each element, these quantities are a function of projectile energy E. If we work with, say, 100 elements, there are then 100 tables and plots of, say, the total photon cross section as a function of incident photon energy E, and 100 tables and plots of the electron mean stopping power as a function of incident electron energy E, and so forth. Each quantity (a cross section, or a stopping power) is a function of Z and E, and therefore forms a surface in Z, E space. This surface can be represented by a contour plot in that space. In this way one can condense these usual 100 plots of cross sections and stopping powers in the elements into a single graph for a given process. The total photon cross section on all elements over the entire energy range of interest appears on one graph. Then, at a glance, one can grasp the entire behavior of any photon cross section over all elements for all energies. Trends are readily discerned. For example, when plotted in units of cm^2/gram, one can readily see that at 1 MeV the total photon cross section is very nearly 0.06 cm^2/gm in all elements; but when plotted in units of barn/atom, the cross section varies from 0.3 at Z=2 to more than 30 at $Z > 90$. Similarly, the electron or alpha particle stopping power at all energies in all elements can be seen in a single plot. These contour plots are presented in later chapters. For some processes the graphs can be read by the eye to better than 5% accuracy, which may itself be sufficient for rough work.

This book is in no way intended to be a comprehensive treatise or a textbook on the subject. Its primary purpose is to make modern data readily accessible, and to provide enough discussion to make those data understandable. There has been something of a disconnect between expositions of the fundamental theory and complete presentations of modern, best, compiled data. This book is intended to fill that gap, and to be a working reference.

Although it touches on the subjects of the effects of energetic particles on semiconductor electronic devices, it is decidedly not a textbook on that subject. Rather it presents the basic physical processes underlying all such effects. For a discussion of effects on semiconductor electronics, the reader is referred to standard works, such as

V.A.J. van Lint, T.M. Flanagan, R.E. Leadon, J.A. Naber, and V.C. Rogers, *Mechanisms of Radiation Effects in Electronic Materials*, Vol. 1, Wiley, New York (1980)

G.C. Messenger and M.S. Ash, *The Effects of Radiation on Electronic Systems*, 2nd Ed., Van Nostrand Reinhold (1992)

A. Holmes-Siedle and L. Adams, *Handbook of Radiation Effects*, Oxford University Press (1994)

Likewise, radiation shielding is a major subject in its own right, and good texts are available. Among them are

H. Goldstein, *Fundamental Aspects of Reactor Shielding*, Addison-Wesley (1959)

A.E. Profio, *Radiation Shielding and Dosimetry*, J. Wiley (1979)

J.K. Shultis and R.E. Faw, *Radiation Shielding*, American Nuclear Society (2000)

The high energy physics community, often requiring energies higher than those considered here, maintains its own summary of the passage of particles through matter, in the regularly updated review of particle properties, for example Groom and Klein in [PDG02] or http://pdg.lbl.gov. And the X-ray physics community keeps a site [http://xdb.lbl.gov] and a handy reference booklet [XRD01].

We first make note of some basic differences among the radiations.

1.2 CHARGED vs NEUTRAL PARTICLES

It is worth making a quick observation about the difference between a charged particle passing through matter and a neutral particle.

As will be seen, the cross section for a charged particle (electron, proton, or heavier ion) to interact with an atom is of order $\sigma \sim 10^{-16}$ cm^2, whereas that for a neutral particle (photon, neutron) is only of order 10^{-24} to 10^{-20} cm^2. Thus in matter of number density $N \sim 10^{23}$ cm^{-3}, the mean free path between collisions, $1/N\sigma$, is only a few Angstroms (1 Angstrom ≡ 1 Å = 10^{-8} cm) for a charged particle, but is microns to centimeters for a neutral one.

As a result, a neutral particle interacts relatively infrequently with target atoms, and at each collision is either absorbed or scattered out of the beam. The beam intensity is attenuated by a factor *e* in one mean free path (mfp), the quantity which then characterizes its passage, and the unscattered beam intensity decreases exponentially with distance traversed. Particles scattered out of the beam may or may not still be of interest. The mfp of a 1 MeV γ ray is approximately 16 gm/cm^2 in most elements (about 6.8 cm in Si, and about 125 m in STP air).

But a charged particle interacts with nearly every atom along its path, and, if its energy is $\gtrsim$ 10 eV, loses some energy each time to atomic excitation and/or ionization; but it continues on, perhaps scattered in direction. The relevant parameter for a beam of charged particles is its rate of energy loss. Eventually it loses all energy and stops after a fairly well defined track length. A 1 MeV electron has a track length of about 0.6 gm/cm^2 in most elements (about 0.25 cm in Si, and about 4.5 m in STP air).

Thus beams of photons or neutrons are characterized by exponential attenuation in one mfp, but beams of charged particles are characterized by a track length or *range.*

Charged particles (electrons, protons and heavier nuclei, charged mesons) are referred to as *directly ionizing* radiation. By their collisions with atoms they directly eject atomic electrons into the material. Many such ejected electrons, usually of low energy and small residual range, result from the passage of a single energetic charged particle.

Uncharged particles (photons, neutrons, neutral mesons) are referred to as *indirectly ionizing* radiation. The basic interaction of a neutral particle with matter may cause one or more energetic charged particles to emerge from the interaction (a Compton electron; a photo-electron; a recoil target nucleus following elastic neutron scattering; protons, alpha particles, and/or nuclear fragments following non-elastic neutron scattering; etc.). It is those energetic charged particles that then ionize the matter by direct collisions with atomic electrons. The original neutral particle does its ionizing only indirectly via the ionization by the energetic particle(s) kicked out by the interaction of the neutral projectile.

1.3 TERMINOLOGY

Over the decades each of the disciplines which has had occasion to study the passage of energetic particles through matter has historically developed terminology that may be inconsistent with, or at least confusing to, another. Even the simple terms *radiation, field,* and *flux*, have different meanings in different specialties. Here we try to explain them, or at least state unambiguously what we mean by the terminology we use.

Field, Radiation, Flux

The classical electromagnetic fields $\vec{E}$ and $\vec{B}$ are indeed classical *fields*, being continuously distributed quantities, continuous point functions of position and time. No particles are involved.

However, and somewhat surprisingly for electromagneticists, the same word, field, is used, for example, in the neutron measurement community to mean the collection of all neutrons that may be bouncing around in a room. In this community the neutron field is specified by the usual parameters describing a gas: either the number density of neutrons and their Boltzmann distribution function to specify their distribution in space and velocity; or, what is the same thing, their energy spectrum and angular distribution. A *neutron field* is the gas of neutrons in the vicinity of the observer, however one describes it, such as might occur near a reactor.

Once the particle nature of light and therefore of electromagnetic fields was realized in the 1920's, the term field in the context of electromagnetics has come to refer both to its original historical meaning of a classical, continuous point function, or, in the case of extremely high frequency fields whose particle nature is important, to the neutron-community-like specification of the distribution of numbers of photons. A *gamma field* is the gas of photons in the vicinity of the observer, also such as might occur near a reactor.

Even the elementary term "radiation" has evolved to have a dual meaning. Since Maxwell's era in the latter half of the 19th century, in electromagnetics the term *radiation* has meant classical electromagnetic fields, arising from a time-dependent current distribution, which at large distances r fall off as $1/r$. The energy flux in the radiation field then falls off as $1/r^2$. But the discoverers of α rays and β rays applied the term radiation to the emission of these particles from matter. Even the distance dependence of their intensity did not fall off as $1/r^2$, due to the additional exponential attenuation in passing through air or other materials, so the term as used had little in common with its electromagnetic origins. Today the term *Radiation* has the general meaning of the emission and transport of classical electromagnetic fields, or of photons, or α, or β particles, or neutrons, or anything else. A *radioactive* material is one that emits any of these particles.

In most areas of physics, the term *flux* means the rate at which some quantity passes through unit area. The number flux of particles is the number of particles passing per cm^2 per second.

In other disciplines the term *flux* is used to mean the rate at which some quantity passes an observer, generally through a specified area, but not per unit area. In radiometry, for example, the word is used to mean the rate at which photonic energy passes a specified area, and is measured in Watts

[Wo85]. Under this usage, a number flux would be the number of particles passing through a specified area per unit time, particles per sec. The number passing through unit area would be called *flux density*, the number of particles per cm^2 per second. Older neutron literature [e.g., ICRU Report 13, ref ICRU69] used just such terminology (although the term fluence, rather than fluence density, is used to mean particles per cm^2). Still other communities use the term "flux" to mean particles per cm^2 (which we call fluence).

In this book we adopt the more common usage. The *flux* of a quantity is the amount passing through unit area per unit time. An electron beam of 10 Amperes and cross sectional area 5 cm^2 passes 2 Coulomb per cm^2 per second. Its number flux is $2/1.602\times10^{-19} = 1.248\times10^{19}$ electrons/cm^2/ sec. Its charge flux is 2 Coul/cm^2/sec = 2 Amp/cm^2, also called its *current density*.

The *fluence* of a flux of particles is simply the time integral of flux, the number of particles passing through unit area over a specified time, say particles/cm^2. *Energy fluence* is the energy of those particles that passes through unit area over a specified time, e.g., MeV/cm^2, or erg/cm^2. As discussed presently, it is further necessary to specify the orientation of the unit area relative to the incident particles to fully specify the flux. Various definitions of this orientation result in different kinds of flux, such as planar or omni-directional.

1.4 DISTRIBUTION FUNCTIONS AND FLUX

Although the focus of this book is on individual particles, one must eventually consider collections of particles. The dose in matter and the fluence-to-dose conversion factors make sense only for many particles. Here we discuss concepts and nomenclature of distribution functions and fluxes of particles.

From the previous discussion, once one defines flux as particles per unit area per unit time, it remains to specify the orientation of the unit area relative to the moving particles. Various orientations result in various definitions of flux, the *omni-directional flux*; the *planar, directed (current) flux*; or the *scalar directed flux*, each useful in certain applications. These all follow from the Boltzmann distribution function $f(\vec{r},\vec{\upsilon},t)$, which contains more detailed information. Different disciplines define different distribution functions and fluxes for convenience.

Distribution Function and Flux

The basic Boltzmann distribution function, $f(\vec{r},\vec{\upsilon},t)$ particles per cm^3 per $(cm/sec)^3$, contains the most detailed description of a field of particles. $f(\vec{r},\vec{\upsilon},t)d^3\upsilon$ is the number of particles per cm^3 with velocity in $d^3\upsilon$ about $\vec{\upsilon}$. The particle number density is $n(\vec{r},t) = \int f(\vec{r},\vec{\upsilon},t)d^3\upsilon$. The energy spectrum g is obtained by integrating over velocity angles Ω and using $E = \frac{1}{2}m\upsilon^2$ (non-relativistic) as the independent variable instead of υ: $g(\vec{r},E,t) = g(E) = \int d\Omega f(\vec{r},\vec{\upsilon},t)\upsilon/m$, where m is the particle mass.

f itself is fundamental in kinetic theory, but it is often not the most useful direct description in applications. Rather, one is often interested not in the instantaneous velocity distribution, but in the number of particles that pass an area in a specified time. The quantity

$$\upsilon f(\vec{r},\vec{\upsilon},t)d^3\upsilon$$

is the number of particles $cm^{-2}\ sec^{-1}$ with velocity in $d^3\upsilon$ about $\vec{\upsilon}$ that cross a square centimeter with normal parallel to $\vec{\upsilon}$. It contains as much information as f itself but addresses more directly the commonly needed quantity. Different disciplines define different differential fluxes. Seeing their connection to f and their relation among one another lends some unified understanding to the subject. Even within a single discipline (notably neutron physics), nomenclature can differ.

As mentioned, in f the three independent variables $\vec{\upsilon}$ can be replaced by $E=\frac{1}{2}m\upsilon^2$ and velocity direction Ω, so that $d^3\upsilon = \upsilon^2 d\upsilon d\Omega = (\upsilon/m)dEd\Omega$. Then $\upsilon f(\vec{r},\vec{\upsilon},t)d^3\upsilon = \upsilon f(\vec{r},E,\Omega,t)(\upsilon/m)\,dEd\Omega$ is the number of particles $cm^{-2}\ sec^{-1}$ with energy in dE about E with direction $d\Omega$ about Ω that pass through the unit area perpendicular to the direction of Ω. As indicated by their arguments, one takes $f(\vec{r},\vec{\upsilon},t)$ to have dimensions of, for example, particles/cm^3/(cm/sec)^3, but $f(\vec{r},E,\Omega t)$ to be particles/cm^3/MeV/sr. Thus the quantity

$$J(\vec{r},E,\Omega,t) = \upsilon f(\vec{r},E,\Omega,t) \qquad \text{(particles/cm}^2\text{/sec/MeV/sr)}$$

is a differential (in energy) flux with as much information as f itself. In neutron transport theory it is called the *angular flux* and is the fundamental quantity in that theory.

In nuclear reactor theory, Bell and Glasstone [Be70] use the notation $N(\vec{r},E,\Omega,t)$ for our $f(\vec{r},E,\Omega,t)$, and there N is called the *neutron angular density*, emphasizing its dependence on Ω. It is simply the neutron Boltzmann distribution function with independent variables E and Ω rather than υ_x, υ_y, υ_z. The angular flux $J = \upsilon f = \upsilon N$ is denoted $\Phi(\vec{r},E,\Omega,t)$[1]. When integrated over the neutron velocity angles Ω, the angular density becomes $\int N(\vec{r},E,\Omega,t)d\Omega = n(\vec{r},E,t)$, which is called the *neutron density*. Perhaps it should more properly be termed the *neutron spectral density*, for it is the number of neutrons per unit volume, per unit energy interval, for example neutrons/cm^3/MeV. The full number of neutrons per unit volume is $\int n(\vec{r},E,t)dE$. The angular flux Φ may be integrated over velocity directions Ω to obtain what is called the *total flux* $\phi(\vec{r},E,t) = \int \Phi(\vec{r},E,\Omega,t)d\Omega$ (particles/cm^2/sec/MeV). This too might more properly be called the *spectral flux*, since it is differential in energy, or even the *omni-directional spectral flux*, since it tabulates particles coming from all directions.

The same quantity $J(\vec{r},E,\Omega,t) = \upsilon f$ is commonly the quantity specified in cosmic ray data. In that context it is called the "unidirectional differential intensity" [Sm85].

In problems of radiative transfer (i.e., photon transport), the photons are characterized by their

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[1] Note that Weinberg and Wigner [We58] denote our $J = \upsilon f$ by f.

frequency ν and velocity direction $\vec{n}$, rather than their three-dimensional velocity vector (the magnitude of their velocity is, of course, always c, and is not a variable). The photon Boltzmann distribution function is then written $f_R(\vec{r},\vec{n},\nu,t)$ photons cm^{-3} Hz^{-1} sr^{-1}, and $f_R(\vec{r},\vec{n},\nu,t)d\Omega d\nu$ photons cm^{-3} is the number density of photons with frequency in $d\nu$ about ν, moving in direction $d\Omega$ about $\vec{n}$. The photon number flux is $cf_R(\vec{r},\vec{n},\nu,t)d\Omega d\nu$ photons cm^{-2} sec^{-1}. The energy flux is more useful than the number flux. As each photon has energy $h\nu$, the differential energy flux is

$$I(\vec{r},\vec{n},\nu,t) = h\nu c f_R(\vec{r},\vec{n},\nu,t) \qquad \text{erg cm}^{-2}\,\text{sec}^{-1}\,\text{Hz}^{-1}\,\text{sr}^{-1}$$

and is known as the "specific intensity" [Ch60, Mi78]. It too contains the same information as f, but is more useful for formulating the equation of radiative transfer.

Flux and Three Dimensional Geometry

The distribution function $f(\vec{r},\vec{\upsilon},t)$ or the differential flux υf (or J or I) specify the most detailed information about a field of particles. Often such detail is not needed. Reduced fluxes, with less differential information, are defined for these purposes. They are usually obtained by integrating over velocity angles and/or magnitude.

In any application involving flux (particles per unit area per unit time) one must still specify the orientation of the unit area relative to the moving particles. Under various circumstances one has need for the *omni-directional flux*, or the *planar, directed (current) flux*, or the *scalar flux*, according to the following considerations. Unfortunately, some articles report a particle flux without specifying whether it is omni-directional or planar (they can differ by a factor of 2 or 4). Lack of specifying which flux is intended is often a source of confusion.

Fluxes of interest are broadly classified as (1) those particles crossing a fixed plane area, or (2) omni-directional.

The planar fluxes of type (1) may address particles crossing a fixed area in only one direction, say from $-z$ toward $+z$ across a unit area with normal parallel to the z axis, counting only those particles whose z component of velocity is positive. This is the flux of interest for studying, say, the flux of electrons striking a satellite surface and leading to space-craft charging.

Alternatively in a planar flux, one may wish the *net* number of particles crossing a fixed area from $-z$ toward $+z$, counting as negative those crossing from $+z$ toward $-z$. This is the *current* of particles in the $+z$ direction[2].

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[2] Documentation of the code MCNP uses the term *current* to mean the absolute sum of particles crossing a surface in either direction, what we call the scalar bi-directional flux. The user may, however, request a tally of particles crossing only in one direction.

Yet again, one may wish the total number of particles crossing the area in both directions, the *scalar bi-directional flux*, being the absolute sum of those crossing from $-z$ toward $+z$ and those crossing from $+z$ toward $-z$. This flux is needed for the reaction rate in a thin foil immersed in a particle field.

1. Planar Fluxes

Directed Planar Flux

In a gas of particles with Boltzmann distribution function $f(\vec{\upsilon})$ and number density $n = \int f(\vec{\upsilon}) d^3\upsilon$ particles/cm^3 (suppressing the $\vec{r},t$ variables), consider a plane circular disc of area $dA = 1$ cm^2 at the origin, with normal parallel to z. Gas particles stream through dA. Figure 1.1 shows those coming from angle ϑ.

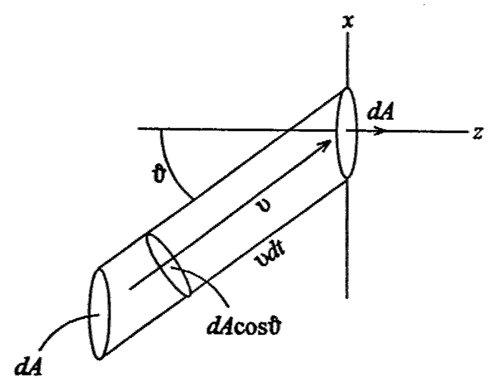

Figure 1.1. Particles streaming through fixed area element from angle ϑ.

The number of particles (of all velocities) per cm^2 per sec that pass through dA from one side, say from the left hemisphere ($z<0$), toward the other side, is the *directed, planar flux* (or *current density*) F_p through dA. Since the orientation of $d\vec{A}$ stays fixed during the integration over velocity directions, particles coming from angle ϑ see the smaller area $dA\cos\vartheta$ and contribute less to F_p than those at $\vartheta=0$. (The $\cos\vartheta$ factor can as well be considered as arising from $\upsilon_z = \upsilon\cos\vartheta$, where $\upsilon = |\vec{\upsilon}|$). As usual, F_p can be expressed as

$$F_p = \frac{1}{dA} d\vec{A}\cdot\int f(\vec{\upsilon})\vec{\upsilon}\, d^3\upsilon = \int f(\vec{\upsilon})\upsilon_z d^3\upsilon = \int f(\vec{\upsilon})\upsilon\cos\vartheta\, d^3\upsilon = n(>0)\overline{\upsilon_z(>0)} \tag{1.1}$$

with the integral taken over only $\upsilon_z>0$ ($\vartheta < \pi/2$). In Eq(1.1), $n(>0)$ is the density of particles with $\upsilon_z>0$ ($= n/2$ in an isotropic gas), and $\overline{\upsilon_z(>0)}$ is the average υ_z of all particles with $\upsilon_z>0$. For example, if dA is a fixed, small hole in the wall of a box containing a gas, $F_p dA$ is the rate at which molecules escape.

Bi-Directional Planar Flux and Current

F_p in (1.1) was computed for particles passing from left to right, $\upsilon_z>0$, $F_p = F_p(\upsilon_z>0)$. One can as well compute it for those particles passing through dA from right to left, $\upsilon_z<0$, obtaining $F_p(\upsilon_z<0)$. Their *scalar* sum, $F_{bi}^{scal} = F_p(\upsilon_z>0) + F_p(\upsilon_z<0)$ is the *scalar bi-directional flux*, the total number of particles traversing unit area per sec from both directions. In a uniform, isotropic gas $F_p(\upsilon_z>0) = F_p(\upsilon_z<0)$, and the scalar bi-directional flux is twice the planar flux in either direction.

The *algebraic* sum, $F_{bi}^{net} = F_p(\upsilon_z>0) - F_p(\upsilon_z<0)$, is the *net* number of particles crossing unit area in one direction (here from left to right). It is simply the (net) *current* of particles in the z direction.

F_{bi}^{net} is, for example, the flux determining the net charge transport across a surface in an electron gas. An example is the current passing through a surface at some fixed depth $z=d$ in a solid when a planar beam of electrons is externally incident on the surface of the solid (z=0). Here $F_p(z, \upsilon_z>0)$ is the flux of original electrons still moving forward at $z=d$, and $F_p(z, \upsilon_z<0)$ is the flux of electrons that have passed z and been back-scattered through the plane at $z=d$.

In neutron transport theory, for neutrons of a given energy E, the algebraic net F_{bi}^{net} is also called the *neutron current* and is commonly denoted [Be70] by $J(\vec{r},E,t)$. Using E and Ω instead of $\vec{\upsilon}$ for the independent variables,

$$J_z(\vec{r},E,t) = \int f(\vec{r},E,\Omega,t)\,\upsilon_z\,d\Omega = \int f(\vec{r},E,\Omega,t)\,\upsilon\cos\vartheta\,d\Omega = n(\vec{r},E,t)\,\overline{\upsilon_z} \tag{1.2}$$

where now the integral is taken over all ϑ ($0\leq\vartheta\leq\pi$). $n(\vec{r},E,t)$ is the number density per unit energy interval, and $\overline{\upsilon}_z$ is the average z component of velocity, over all directions, of neutrons with energy E. The terminology "neutron current" is entrenched by usage, but it is actually the neutron *spectral* current *density*, being differential in energy (spectral), and being per unit area (density)[3]. J has units of, for example, particles/MeV/cm^2/sec.

Planar Flux for an Isotropic Gas

In an isotropic gas $f(\vec{\upsilon}) = f(\upsilon)$, and with $\upsilon_z = \upsilon\cos\vartheta$, Eq(1.1) for F_p becomes

$$\begin{aligned} F_p &= \int_{\upsilon_z>0} f(\upsilon)\,\upsilon_z\,d^3\upsilon = \int f(\upsilon)\,\upsilon\cos\vartheta\,\upsilon^2\,d\upsilon\,d\Omega \\ &= \int_0^\infty d\upsilon f(\upsilon)\upsilon^3 \int_{2\pi} \cos\vartheta\,d\Omega \\ &= \frac{n}{2}\,\frac{\overline{\upsilon}}{2} \end{aligned} \tag{1.3}$$

where

$$\overline{\upsilon} = \frac{1}{n}\int f(\upsilon)\,\upsilon\,d^3\vec{\upsilon} \tag{1.4}$$

is the mean magnitude of the velocity over all particles (of density n), and $n = \int f(\upsilon)d^3\upsilon$ is the total number of particles per unit volume. In (1.3), one factor of ½ arises because the density of particles with $\upsilon_z>0$ (i.e., $\cos\vartheta>0$) is $n/2$. The other is because of the factor $\cos\vartheta$ from υ_z; the average υ_z over all particles with $\upsilon_z>0$ is $\overline{\upsilon}/2$:

$$\overline{\upsilon_z(>0)} = \frac{1}{n/2}\int d\upsilon\,\upsilon^3 f(\upsilon) \int_0^{2\pi} d\varphi \int_0^1 \mu\,d\mu = \frac{\overline{\upsilon}}{2} \tag{1}$$

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[3] We note that the widely used Monte Carlo transport code MCNP, having its distant origins in nuclear reactor computations, has adopted the flux and current definitions and terminology of Bell and Glasstone [Be70].

where $\mu \equiv \cos\vartheta$.

Of course, in an isotropic gas an equal number of particles pass the other way through *dA* (from right to left), so the net *directed* flux through *dA* vanishes. The total scalar, bi-directional flux is $F_{\mathrm{bi}}^{\mathrm{scal}} = 2F_p = n\bar{\upsilon}/2$, being the total number of particles that pass through a fixed *dA* in both directions.

Planar Flux for a Plane Beam

For a parallel, plane beam of particles of density n, velocity υ_o, the distribution function is

$$f(\vec{\upsilon}) = n\delta(\upsilon_x)\,\delta(\upsilon_y)\,\delta(\upsilon_z - \upsilon_o)\,. \tag{2}$$

When inserted in Eq(1.1), this should reproduce the usual flux $n\upsilon_o$ (/cm^2/sec); it is trivial to confirm that it does. If the beam has a spread in υ_z, $f(\vec{\upsilon}) = n\delta(\upsilon_x)\delta(\upsilon_y)g(\upsilon_z)$, then the planar flux is $n\bar{\upsilon}_z$.

In a gas of charged particles (usually electrons) the directed planar flux F_p is the relevant flux for the electromagnetic charge transported across a fixed surface with particles incident from only one side. Hence its alternate name *current density*. F_p is, for example, the rate at which a unit surface on the *exterior* of a spacecraft is struck by ambient electrons (causing spacecraft charging). The bidirectional planar flux $F_{\mathrm{bi}}^{\mathrm{scal}}$ would be needed if particles are incident from both sides, as for example would be the case for charging of a large, thin solar panel.

2. Omni-directional Flux

The *omni-directional flux* F_{om} is the number of particles passing through one cm^2 at the origin when the square cm is always directed toward the incoming particles. Thus $d\vec{A}$ changes direction during the integration always to be parallel to the instantaneous $\vec{\upsilon}$. It is integrated over all velocity directions (4π ster). Instead of Figure 1.1 we have

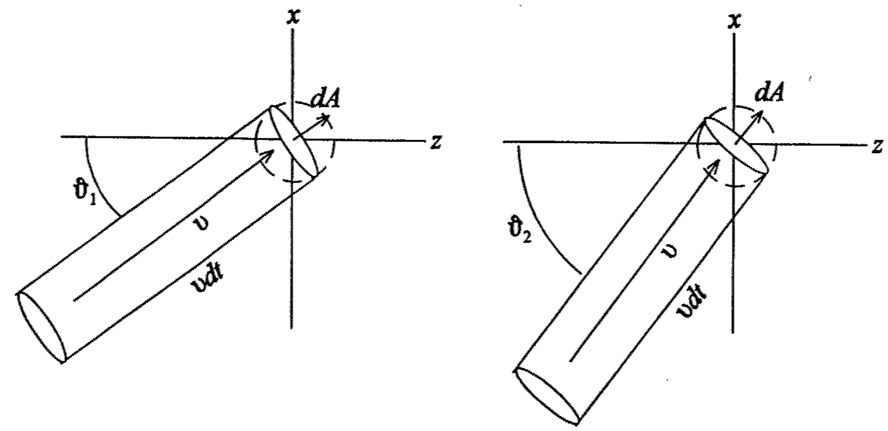

Figure 1.2. Particles streaming through area element always normal to velocity vector.

The number of particles passing through $d\vec{A}$ per sec is $\int d^3\upsilon f(\vec{\upsilon})\,\vec{\upsilon}\cdot d\vec{A}$. But since $d\vec{A}$ is always parallel to $\vec{\upsilon}$, $\vec{\upsilon}\cdot d\vec{A} = \upsilon\, dA$, and the number is $dA \int f(\vec{\upsilon})\,\upsilon\, d^3\vec{\upsilon}$. Thus, the expression for F_{om} is

$$F_{om} = \int f(\vec{\upsilon})\,\upsilon\, d^3\vec{\upsilon} \tag{3}$$

with the solid angle integral taken over 4π, and with no extra $\cos\vartheta$. The magnitude of velocity υ occurs in the integrand rather than $\upsilon_z = \upsilon\cos\theta$ as in Eq(1.1).

As indicated in Figure 1.2, the omni-directional flux is the number of particles passing into a sphere of unit cross sectional area ($\pi r^2 = 1$ cm^2).

The omni-directional flux is important, and the usually desired quantity in discussions of neutral particles (photons, neutrons), because it is the flux that addresses the following common question:

> Suppose you have a small volume of material containing N atoms/cm^3 buried in a gas of neutrons (or photons). How many collisions with N do the neutrons undergo? The answer is $N\sigma F_{om}$ collisions per cm^3/sec, where σ is the collision cross section.

The omni-directional flux F_{om} determines the reaction rate of the neutral particles with the material through which the particles traverse. By default, the flux computed by the Monte Carlo code MCNP, for example, is the omni-directional flux.

In neutron physics, the omni-directional flux is sometimes expressed as a function of particle energy,

$$F_{\mathrm{om}}(\vec{r},E,t) = \int f(\vec{r},E,\Omega,t)\, \upsilon\, d\Omega \tag{4}$$

and is the number of particles at $(\vec{r},t)$ per unit energy at energy E crossing unit area per second. In neutron physics it is called simply the *total* flux (Bell and Glasstone [Be70], p. 5), even though it is differential in energy; the "total" signifying all directions. Nomenclature is not universal, and the dimensions of some quantities of the same name differ according to displayed arguments; they may even use the same symbol. Weinberg and Wigner [We58], for example, use the term, "total flux", denoted by them Φ, to mean both $\Phi(\vec{r},E,t) = F_{\mathrm{om}}(\vec{r},E,t)$ and also the energy integral of $F_{\mathrm{om}}(\vec{r},E,t)$, $\Phi(\vec{r},t) = \int F_{\mathrm{om}}(\vec{r},E,t)dE = \int \Phi(\vec{r},E,t)dE$. In this case the meaning and units of Φ are determined by the displayed arguments.

Words mean different things to different people. In discussions involving particle flux, current, etc., a writer who wishes to be understood by those not already familiar with the field should specify at once his or her terminology for a quantity (e.g., "total flux"), its dimensions (e.g., particles per unit energy per unit area per unit time), and whether it is omni-directional or planar. If planar, whether it is in only one direction across the plane, or both directions. If both, whether it is the absolute sum of both directions (here called the bi-directional scalar flux) or the algebraic sum (here called current). Failure to spell out these different possibilities has been the source of much confusion.

Omni-Directional Flux for an Isotropic Gas

In the case of an isotropic gas, Eq(1.6) becomes

$$F_{\mathrm{om}} = \int f(\bar{\upsilon})\, \upsilon\, d^3\upsilon = n\bar{\upsilon} \tag{5}$$

with $\bar{\upsilon}$ given by (1.4). As stated, it is the total number of particles passing per sec through a unit area always normal to the particle velocity (i.e., through a sphere of radius r with $\pi r^2 = 1$ cm^2). It is 4 times the one-way *directed* planar flux F_p (Eq. 1.3). As mentioned following that equation, one factor of 2 comes from the area projection factor $\cos\theta$ being missing in the definition (1.7) of F_{om} relative to that of F_p. The other factor is because only half the particles have $\upsilon_z > 0$. And it is twice the bi-directional scalar planar flux $F_{\mathrm{bi}}^{\mathrm{scal}}$.

Omni-Directional Flux for a Plane Beam

With f from (1.6) inserted in (1.7), the integrals are trivial, and one finds $F_{\mathrm{om}} = n\upsilon_o$, the same as F_p and the common expression. All the particles/cm^2 in the beam enter a sphere of radius r with $\pi r^2 = 1$ cm^2. The omni-directional flux of a plane beam is the same as its ordinary planar flux.

1.5 ENERGY SPECTRUM AND SPECTRAL FLUENCE

As stated, one is often interested not in the instantaneous velocity distribution, but in the number of particles that pass an area in a specified time. Therefore a flux or fluence differential in energy raises a distinction between the energy spectrum *per se*, and the energy spectrum of those that pass through an area in a specified time. The two are different, and their difference can cause confusion, especially in neutron physics. Some clarifying remarks are in order.

A particle field can be thought of as a gas of particles, albeit perhaps highly non-Maxwellian. The energy spectrum, $g(E)$, of a gas, or of any collection of particles in space, is, strictly speaking, a point function of space and time, $g(E,\vec{r},t)$. At time t the number of particles in an infinitesimal volume d^3r with energy in interval $(E,E+dE)$ is $dn = g(E,\vec{r},t)d^3rdE$. With this definition, g has dimensions of, say, particles per cm^3 per MeV.

One often, however, is more interested in the energy spectrum of all those particles that pass through a small area dA in some specified time, say dt, or 1 sec.

The number of particles per unit volume in energy interval $(E,E+dE)$ is gdE. Of these, the number that pass through an area element dA, chosen always to be normal to the particle (non-relativistic) velocity $\upsilon = \sqrt{(2E/m)}$, in time dt is $gdE\,\upsilon dt\,dA$. Per unit area, this is

$$g\upsilon dEdt, \qquad \text{(particles/cm}^2\text{)}$$

a fluence. Further expressed as per second, it is

$$g\upsilon dE, \qquad \text{(particles/cm}^2\text{/sec)}$$

a flux. That is, whereas the simple number of particles at one instant of time t, at a point in space $\boldsymbol{r}$, in energy interval $(E,E+dE)$, is proportional to g, the flux or fluence of particles that pass through unit area is proportional to υg.

Now expressing this fluence or flux as per unit energy, as a function of energy, one has

$$g\upsilon dt, \qquad \text{(particles/MeV/cm}^2\text{)}$$

a *spectral fluence*, or a *differential* (in energy) *fluence*, and

$$g\upsilon, \qquad \text{(particles/MeV/cm}^2\text{/sec)}$$

a *spectral flux*, or a *differential* (in energy) *flux*. While the energy spectrum *per se* is $g(E,\vec{r},t)$, the energy spectrum of particles that pass through a given area, either per second or integrated over any time, is $\upsilon g(E,\vec{r},t)$.

As an illustration of this distinction, chosen to clarify a common point of confusion, consider the energy spectrum g of a gas of particles, with Boltzmann distribution function $f(\vec{r},\vec{\upsilon},t)$. The particle distribution in speed (velocity magnitude) is proportional to $\upsilon^2 f$. Then, using $E = \frac{1}{2}m\upsilon^2$, and $dE = m\upsilon d\upsilon$, the particle energy distribution g is proportional to $\upsilon f = \sqrt{E}\cdot f$.

Suppose the distribution is Maxwellian, $f \propto \exp(-E/T)$, with temperature T in energy units. For energies smaller than T, f is independent of E, and the low energy Maxwellian energy spectrum $g \sim \sqrt{E}\cdot f$

is proportional to $\sqrt{E}$. Accordingly, the low energy *spectral fluence* would be proportional to $\upsilon\sqrt{E}$, which is simply proportional to E.

Unfortunately, nomenclature in common use is not so precise. Once energetic neutrons have scattered many times through shielding material, air, and ground, they have been down-scattered enough to become largely thermal. Near and below thermal energies, they are Maxwellian. Figure 1.3 shows a neutron energy spectrum after moderation and thermalization, computed by MCNP. On this log-log plot one sees that below the thermal peak the spectrum is proportional to E. One may suspect an error, since the spectrum should be Maxwellian and proportional to $\sqrt{E}$. In fact the plotted quantity, referred to as the energy spectrum, is not the energy spectrum *per se*, but rather the spectral fluence. It is the spectral fluence that is the more useful quantity, and is the one commonly computed. Reports and articles often present what is called the energy spectrum $g(E)$, but what is in fact the spectral fluence $\upsilon g(E)$.

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