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13. ABSTRACT (Maximum 200 Words) Research investigations involving the nonlinear wave propagation that arise in physically significant systems have been carried out. Applications include modeling and computational studies of wave phenomena in nonlinear optics, solutions of physically significant nonlinear equations, chaotic wave dynamics in physical systems and inverse scattering. There have been a number of important research contributions. During the past three years 19 papers were published or accepted for publication in refereed journals, 4 book chapters were published or accepted, and 14 invited lectures were given. New methods to find solutions to discrete equations in a nonlinear optical fiber array were discovered. Discrete diffraction managed systems and associated solitons were proposed. This work is relevant to recent experiments involving discrete optical waveguides. Experimental arrays occupy 5 microns in width and a total length of 2-5 millimeters. From first principles, the equations governing discrete systems in nonlinear optical arrays as well as discrete diffraction managed systems have been derived. The concept of dispersion management is being applied to the study of ultra-short laser pulse dynamics in Ti:sapphire lasers. In quadratic nonlinear optical media, a vector system of nonlinear Schrodinger (NLS) type with coupling to a mean field has been derived. It has been established that a universal type of chaotic wave dynamics can develop in physical and computational systems. Parameter regimes have been delineated where chaotic dynamics are predicted and observed. Such chaotic dynamics has been shown to arise in computational chaos, water waves and short pulses in nonlinear optical fibers. A class of free boundary problems has been investigated. New classes of localized solutions to multidimensional nonlinear wave problems have been obtained and analyzed.					
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Nonlinear Wave Propagation
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OBJECTIVES

To carry out fundamental and wide ranging research investigations involving the non-linear wave propagation that arise in physically significant systems. Applications include modelling and computational studies of wave phenomena in nonlinear optics, solutions of physically significant nonlinear equations, chaotic wave dynamics in physical systems and inverse scattering.

STATUS OF EFFORT

The research program of the PI in the field of nonlinear wave propagation is broadly based and very active. There have been a number of important research contributions during the past three years. During the period 1 December, 1999 – 30 November, 2002, 19 papers were published or accepted for publication in refereed journals, 4 book chapters were published or accepted, and 14 invited lectures were given. Some of the key results and research directions are described below in the section on accomplishments/new findings. Full details can be found in our research papers.

ACCOMPLISHMENTS/NEW FINDINGS

Nonlinear Optics in Waveguide Arrays .

In recent years there has been important experimental research involving optical waveguide arrays and the propagation of their nonlinear modes. Discrete optical waveguide arrays with 40-60 individual waveguides each occupying 5 microns in width and a total length of 2-5 millimeters can now be constructed. Such small scale structures can potentially be embedded in a large scale environment and can be used to guide light in a controllable manner. In our research the governing equations, which are semi-discrete nonlinear evolution equations, are derived and solutions to these equations are constructed and analyzed.

Motivated by these experiments we have undertaken wide ranging studies of discrete scalar and vector nonlinear Schrödinger (NLS) systems. In general, it is difficult to find solutions to discrete equations. New and effective methods have been developed in the Fourier domain to find localized and stationary and traveling wave pulse solutions to the governing discrete equations. The methods are robust and apply to a variety of equations; there is no need for the equations to be integrable. Traveling wave solutions pose certain difficulties. Unlike the continuous problem, in general one does not expect to be able to find discrete "uniformly" traveling soliton wave modes. But over the short scales involved we find that approximate traveling optical pulses can be obtained; they persist and are stable over the experimental distance.

We have applied the above analysis to a waveguide array which is a discrete diffraction managed system. The concept of diffraction management means that the waveguide array is alternately directed "positively" and "negatively" as a function of waveguide number over the propagation distance of the fiber. Using discrete Fourier methods we have obtained a nonlocal integral equation which governs the wave propagation in the discrete system and have found a class of discrete diffraction managed solitons as special solutions of this system. The theory has also been extended to include vector discrete systems and the interaction effects of traveling discrete solitons have been analyzed.

From first principles, employing asymptotic analysis the equations governing discrete systems in nonlinear optical arrays as well as discrete diffraction managed systems have been derived. The analysis can be applied to vector systems and new classes of vector discrete and discrete diffraction managed pulse solutions have been obtained. The collision process of interacting optical pulses has been studied in detail. Recent research has also indicated that the integrable discrete scalar NLS (IDNLS) equation can be derived in coupled optical arrays. This can have important implications in physics since the IDNLS system admits traveling wave solitons over wide parameter regimes.

Recent experimental research has shown that multidimensional discrete waveguide arrays can also be fabricated. Future research will investigate how to obtain localized multidimensional discrete optical pulses structures.

Discrete waveguide arrays allow us to carefully control and tune lightwaves. They provide us with the ability to navigate light in one and two dimensional networks. Discrete systems have superiority over bulk continuum media in the sense that it is easy to identify a path for light to propagate, and to readily modify optical pulse interactions in small and confined locations.

Potential applications involve beam management, pulse shaping, optical switching and the development of logic devices.

Nonlinear optics: dispersion management and the study of ultra-short laser dynamics in Ti:sapphire lasers

Research in nonlinear fiber optics has demonstrated that a new class of optical pulses, called dispersion managed solitons, and more recently, quasi-linear pulses can be obtained. These pulses arise from a transmission line composed of a periodic array of fibers, with different dispersion characteristics in each period. Such dispersion managed fibers have numerous advantages including sharply reduced inter-channel four wave mixing by-products as compared with classical soliton transmission.

Recent research investigations have shown that ultra short pulses in Ti:sapphire lasers are a manifestation of dispersion management. We are actively studying the dynamics of these ultra short dispersion managed lasers. The research we have done involving dispersion management is proving very useful in the understanding of the dynamics of Ti:sapphire lasers. In our research on Ti:sapphire lasers the goal is to understand the dynamics of the phase of the carrier wave relative to that of the envelope. Recent experiments have shown that the the phase difference between the envelope and carrier waves can be stabilized. This is an important advance in our ability to control and manipulate lightwaves. The measurement of the relative optical phase is important in applications. For a lightwave, the envelope function can provide a reference to measure carrier phase against. For example, the peak of the envelope pulse can be the reference with which to compare the carrier wave. This is called the carrier-envelope phase. We are currently developing analysis which can accurately predict the evolution of the carrier-envelope phase.

There is currently considerable interest in using ultra short optical pulses in order to control and measure both the amplitude and the phase of optical fields. Such control and detection can pave the way for significant advances in optical signal processing with ultra short optical pulses.

A unified theory that describes both dispersion managed solitons and quasi-linear pulses in nonlinear fiber communications has been developed. Quasi-linear pulses are of considerable interest in the communications community. Due to their relative simplicity and reception advantages at extremely high bit rates, quasi-linear pulses are being proposed as one of the main communications formats. It turns out that the same fundamental nonlinear equation governs both solitons and quasi-linear pulses. In the soliton format nonlinearity balances average dispersion. In the quasi-linear format, nonlinearity is reduced, or "managed" by properly using fiber design. The main additional parameter in dispersion management is the "map strength", s , which is a measure of the strength of the local dispersion. At large values of s , with fixed power, we show that the nonlinear terms are reduced by the factor : $O(\log s/s)$.

Dispersion management is effective in reducing unwanted inter-channel four wave mixing components generated by periodic amplification. However, with large values of map strength there is an additional serious penalty that occurs. Namely effects of intra-channel four wave mixing become very significant. This results in growth of zero bits (ghost modes), timing shifts and significant energy transfers between adjacent bits. We are studying how to alleviate such penalties.

This research is fundamental in the field of optical communications with many important applications. As mentioned above the concept of dispersion management applies to the dynamics of Ti:sapphire lasers. With the benefits of our research employing dispersion management in fiber optics we are in a strong position to make significant contributions to the understanding of the ultra short pulse dynamics in Ti:sapphire lasers. We have already made some important advances in this study.

Nonlinear optics: multi-dimensional pulse propagation in $\chi^{(2)}$ optical materials

In many applications the leading nonlinear polarization effect in an optical material is quadratic; they are referred to as " $\chi^{(2)}$ " materials. We have found that in multidimensional nonresonant $\chi^{(2)}$ materials, the nonlinear equation governing the slowly varying envelope of quasi-monochromatic wave trains is not the NLS equation but rather a coupled nonlinear system involving both the optical field and mean terms. We call these equations NLSM systems (M stands for the mean contribution). In water waves similar scalar systems were derived in 1968 by Benney and Roskes. A few years later, a special case of this system was found to be integrable. The latter system is frequently referred to as the Davey-Stewartson (DS) system.

In $\chi^{(2)}$ optical materials, we derived both a scalar and more recently a vector NLSM system directly from Maxwell's equations. The vector NLSM systems generalize to multidimensions the well known 1+1 vector NLS equations. Such vector multidimensional systems

are new in mathematical physics; there is no analog in water waves. Investigations of the scalar NLSM system has shown that localized optical pulse solutions can be constructed. These localized pulses are induced by their interaction with mean terms that have nontrivial boundary values. Hence the localized pulses are boundary induced. This situation is similar to the situation that is known to occur for the Davey-Stewartson system; however in the optics problem the system is not integrable. These findings suggest that stable localized multidimensional pulses are a reproducible feature of these NLSM systems. In the future we will extend our analysis to spatial nonlinear optics where the boundary behavior can be more readily controlled.

Potential applications using such pulses include beam steering, pulse shaping, terahertz imaging, spatio-temporal light bullets and optical switching.

Chaotic wave dynamics—a universal phenomena in nonlinear wave systems

Our earlier research in computational chaos has motivated our studies of chaotic wave dynamics in physical problems. In both classes of problems the governing asymptotic equations are the NLS equation with suitable small perturbations representing the higher order terms. Such perturbed NLS equations are well known in the study of deep water waves and nonlinear optics. In the water wave problem we have collaborated with experimentalists in order to compare theory and experiment. The computational problems associated with the discrete NLS eq. (see also the above section on nonlinear optics in waveguide arrays) also yield a perturbed NLS eq. when one considers the effects of truncation error.

In the laboratory investigations carefully controlled modulated waves are excited at the entrance of the wave tank. Measurements are taken at downstream locations. The data received are then compared with identical experiments conducted at subsequent times. It was found that in the case of solitons, the results are reproducible. Namely different “identical” experiments (there is approx. 1% error between identical experiments) yield soliton waves with essentially the same amplitude and speed. On the other hand when the waves are excited in a periodically modulated manner, corresponding to three unstable modes of the underlying NLS equation, it is found that there are serious discrepancies between the data sets at downstream positions. The discrepancies are magnified as one proceeds downstream.

In our research we have developed an analytical/numerical framework which describes and explains this phenomena. The NLS equation with suitable small higher order corrections is the relevant equation. We call this the PNLSE equation (P stands for perturbed). With appropriate parameters, the simplest periodically generated waves of the NLS equation are unstable. The instability is associated with M unstable modes of the linearized version of the NLS equation. The value of M , an integer plays an important role in the theory. The PNLSE equation is used to compute the long time evolution. Our results based on

direct numerical simulations of the PNLSE equation with periodic boundary values and the numerical integration of the spectrum of the scattering problem associated with the IST solution of the NLS equation agree with the experimental observations.

Recently we have completed a study of modulational instability in nonlinear fiber optics. We find that for typical parameters, a similar scenario arises. The modulational instability produces wave growth which saturates in the nonlinear regime. The final state manifests itself as chaotic nonreproducible dynamics. In more detail it is found that traveling waves are excited which evolve chaotically between left to right running directions. We also considered another parameter regime where damping is important and amplifiers are introduced into the system. Once again the same type of chaotic dynamics is found.

Thus, we find that the chaotic dynamics which arises in physical systems governed by perturbed NLS equations with periodic boundary values have certain universal features. The scenario is described by left/right traveling waves which evolve chaotically across a fixed state. Since the NLS equation is a widely applicable equation, this research has many applications including nonlinear optics, fluid dynamics, plasma physics etc.

In our research on computational chaos a class of discrete equations which in the continuous limit are approximations to nonlinear Schrödinger equations (NLS), both scalar and vector and the sine-Gordon equation are investigated. These equations are physically interesting systems whose solutions and properties we have concrete analytical understanding. The discrete equations provide a vehicle by which: i) computational schemes can be compared and ii) errors in the schemes can be detected. We have found that in certain circumstances computationally irregular/chaotic temporal dynamics result. Since these are long time numerical integrations of nonlinear systems, there is no existing theory of error analysis which describes the phenomena.

For example we studied computational chaos associated with the discrete NLS equation with periodic boundary values. The discrete NLS eq. is a numerical scheme associated with the NLS eq. where the truncation error leads to a perturbed NLS eq. Thus, as with the physical problems described earlier, the study involves understanding the long time dynamics of the NLS equation under small perturbations,

As mentioned above, with appropriate parameters, the simplest periodically generated waves of the NLS equation are unstable. The instability is associated with M unstable modes of the linearized version of the NLS equation. The NLS eq. with small perturbations, due to the discretization, governs the long time evolution. Our computational results, shows that evolution near certain special parameter regimes, referred to as homoclinic manifolds, is highly unstable. Small perturbations due to numerical errors (or physical perturbations as discussed above) induce waves which evolve chaotically between left and right traveling

states across a fixed state. It is this same scenario which is also seen in physical phenomena such as discussed above for water waves and optics.

Depending on values of the parameters, in particular the number of unstable modes M , we have demonstrated that: computational chaos can result from truncation errors, or even from roundoff errors. We have also found that the spatial discretization plays a more important role than the temporal scheme for these nonlinear systems. Different spatial discretizations yield significantly different results. We find that "off the shelf" adaptive Runge-Kutta (RK) type algorithms (e.g. from standard libraries) perform about as well as symplectic integrators. The symplectic integrators performed better when they were higher order with fourth order symplectic algorithms performing about as well as RK algorithms. Further research is needed to decide whether symplectic integrators can be as useful for the integration of infinite dimensional Hamiltonian systems as they are for finite (low) dimensional dynamical systems.

Free boundary problems in nonlinear wave systems

Free boundary problems (FBP) arise widely in physical phenomena. They are important but usually are difficult problems to solve. From the mathematical point of view, the underlying complication of FBP involves not only solving the given partial differential equations, but also finding the unknown motion of the free boundary. Physically speaking, FBP arise in numerous contexts, e.g. surface dynamics of water waves, the internal evolution of the boundary between immiscible liquids, the motion of the free boundary between two energetically differing states such as those referred to as Stefan problems etc. The inherent difficulty in most FBP is that they require one to solve a nonlinear system. In some cases it is possible to prove existence theorems (at least for short time) but usually explicit solutions cannot be obtained. Recently in our research investigations a one-phase Stefan problem for the Burgers equation was considered. An exact traveling wave solution was obtained and the existence of the one-phase solution was proven for short intervals of time.

More recently, in the context of Burgers equation, we considered a more complicated two-phase FBP. The fundamental theory was constructed and exact free boundary solutions were obtained. The free boundary problem that we studied, corresponds to a one dimensional, nonstationary flow with two weakly nonlinear compressible fluids. We assume the two fluids to be immiscible, with different velocity fields and different viscosities, connected by continuity of velocity and a suitable energy balance condition at the free boundary. Due to the fact that Burgers equation can be linearized to the heat equation, we have a natural correspondence with the well known Stefan problem of the heat equation. For this reason we call this free boundary problem a Burgers-Stefan (B-S) problem.

Currently we are considering a class of discrete free boundary systems. In the discrete case we have considered a free boundary problem associated with the linear discrete diffusion equation and have obtained an equation governing the nonlinear free boundary. The study of discrete free boundary problems appears to be novel in mathematics and physics.

Solutions of Multidimensional Nonlinear Wave Equations

We have continued our studies of a class of multidimensional nonlinear wave equations. In our earlier work an important special class of solutions, namely two dimensional lumps which are solitons/coherent structures which decay in all directions was obtained.

We have extended the previous theory and have found a new class of lump type solutions to a $2+1$ generalization of the Korteweg-deVries equation, called the Kadomtsev-Petviashvili (KP) equation. Associated with the KP equation is a linear scattering problem which in this case is the nonstationary Schrödinger equation. Lump type solutions of the KP equations correspond to reflectionless potentials of the nonstationary Schrödinger problem. We have also found solutions of the nonstationary Schrödinger equation corresponding to these potentials. Given the importance of the nonstationary Schrödinger equation, this work has two equally important themes: solutions of the KP equation and solutions of the nonstationary Schrödinger equation.

Spectrally speaking, these new localized solutions correspond to multiple poles associated with certain eigenfunctions of the nonstationary Schrödinger problem. We have found that these solutions are characterized by an integer, called the charge. The charge is related to a winding number, or index.

The simplest new solution is explained as follows. In the usual spectral description of, say, a one lump solution, the scattering eigenfunction has one pair of poles symmetrically located in the upper/lower half planes. The charge associated with a simple lump is unity. In the case of a standard two lump solution, the eigenfunction has two pairs of poles symmetrically located in the upper/lower half planes. A two lump solution has an overall index of two obtained by simply adding the individual indices of each lump. We have shown, both by taking coalescing limits of (two) lump solutions and by direct analysis of the scattering problem, that the spectral configuration has a double pole in one of the half planes and a simple pole in the other. This new state has charge two, which is consistent with the fact that in the limit process one cannot lose "charge".

This process carries on to higher order multipole lumps. We have obtained a number of important results which are summarized below.

i) The multipole lump solutions are associated with an integer, referred to as the charge. Thus we have found a new underlying integer associated with the nonstationary Schrödinger

problem. Simple lumps have unit charge. Higher order lump type solutions can have any integer charge.

ii) The solutions of the nonstationary Schrödinger equation have multiple poles. The poles can have different orders in the upper/lower half planes. Our previously known solutions had only simple poles. The solution manifold is characterized by the order of the poles of the nonstationary Schrödinger equation and the charge.

iii) The solutions and dynamics of the multipole lump solutions associated with the Kadomtsev-Petviashvili equation have more complicated interaction properties than the simple lump solutions.

We have continued our investigations of the scattering theory associated with a class of multidimensional operators and are also considering the lump solutions of other physically significant 2+1 dimensional equations such as the Davey-Stewartson equation. We have made considerable progress.

This work is important for anyone studying scattering theory in multidimensions as well as nonlinear wave equations possessing multidimensional solitons. The underlying wave equations arise frequently in application as does the associated direct and inverse scattering problems.

PERSONNEL SUPPORTED

- Faculty: Mark J. Ablowitz
- Post-Doctoral Associate: Z. Musslimani
- Graduate Students: D. Trubatch, R. Horne
- Other (please list role) C. Schober, visiting research collaborator

PUBLICATIONS

• SUBMITTED

– Books/Book Chapters:

1. Discrete and continuous nonlinear Schrödinger systems, M. J. Ablowitz, B. Prinari and A. D. Trubatch. Submitted to: Cambridge University Press, 208pp

– Journals

1. Discrete spatial solitons in a diffraction managed nonlinear waveguide array: a unified approach, M. J. Ablowitz and Z. Musslimani, APPM[†] # 497 (2002) submitted to: *Physica D*

APPM[†]: Department of Applied Mathematics report

- Conference Proceedings: None

• ACCEPTED/PUBLISHED

– Books/Book Chapters

1. On the evolution and interaction of dispersion managed solitons, M. J. Ablowitz, G. Biondini and E. S. Olson, "Massive WDM and TDM Soliton Transmission Systems", Ed. A. Hasegawa, Kluwer Academic Publishers, 2000, 75-114.
2. On the scattering of multipole lumps for the Kadomtsev-Petviashvili equation, M. J. Ablowitz and J. Villarroel, *Scattering and Inverse Scattering in Pure and Applied Science*, Eds. P. Sabatier and E. R. Pike, Academic Press, (2001), 1792-1806.
3. Chaotic dynamics in nonlinear waves: computational and physical, M. J. Ablowitz and C. M. Schober, *Collected Lectures on the Preservation of Stability under Discretization* Eds: D. Estep and S. Tavener, (2002) SIAM, Phila.

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1. Modulated periodic waves in deep water, M. J. Ablowitz, J. Hammack, D. Henderson, and C. M. Schober, *Phys. Rev. Lett.*, **84** (2000) 887-890.
2. On integrability and chaos in discrete systems, M. J. Ablowitz, Y. Ohta, and A. D. Trubatch, *Chaos, Solitons and Fractals*, **11** (2000) 159-169.
3. A Novel Class of Solutions of the Non-stationary Schrödinger and the Kadomtsev-Petviashvili I Equations, M. J. Ablowitz, S. Chakravarty, A. D. Trubatch, and J. Villarroel, *Phys. Lett. A*, **267** (2000) 132-146.
4. On a Burgers-Stefan problem, M. J. Ablowitz and S. De Lillo, *Nonlinearity*, **13** (2000) 471-478.
5. Solutions of a Burgers-Stefan problem, M. J. Ablowitz and S. De Lillo, *Physics Letters* **271** (2000) 273-276.
6. Resonant nonlinear intrachannel interactions in strongly dispersion managed transmission systems, M. J. Ablowitz and T. Hirooka, *Optics Letters* **25** (2000) 175-1752.
7. Quasi-linear optical pulses in strongly dispersion managed transmission systems, M. J. Ablowitz, T. Hirooka, and G. Biondini, *Optics Letters* **26** (2001) 459-461.
8. Long time dynamics of the modulational instability of deep water waves, M. J. Ablowitz, J. Hammack, D. Henderson, and C. M. Schober, *Physica D* **152-153** (2001) 416-433.
9. Nonlinear Schrödinger equations with mean terms in non-resonant multi-dimensional quadratic materials, M. J. Ablowitz, G. Biondini and S. Blair, *Physical Review E* **63** (2001) 605-620.
10. Incomplete collisions of wavelength-division multiplexed dispersion-managed solitons, M. J. Ablowitz, G. Biondini and E. S. Olson *J. Opt. Soc. Am. B* **18** (2001) 577-583.
11. Nonlinear effects in quasi-linear dispersion managed pulse transmission, M. J. Ablowitz and T. Hirooka, *IEEE Photonics Technology Letters* **13** (2001) 1082-1084.

12. Intrachannel pulse interactions and timing shifts in strongly dispersion-managed transmission systems, M. J. Ablowitz and T. Hirooka, *Optics Letters* **26** (2001) 1846-1848.
13. Discrete diffraction managed spatial solitons, M. J. Ablowitz and Z. Musslimani, *Physical Review Letters* **87** (2001) 254102-1-254102-4.
14. Discretizations, integrable systems and computation, M. J. Ablowitz, B. M. Herbst and C. M. Schober, *J. Phys. A:Math. Gen.* **34** (2001) 10671-10693.
15. Nonlinear feedback shift registers via nonlinear evolution, M. J. Ablowitz, *Studies in Applied Math* 2000, **107** (2001) 127-136.
16. Methods for discrete solitons in nonlinear lattices, M. J. Ablowitz, Z. Muslimani and G. Biondini, *Physical Review E* **65** (2002) 026602-1-4.
17. Managing nonlinearity in strongly dispersion-managed optical pulse transmission, M. J. Ablowitz and T. Hirooka *JOSAB* **19** (2002) 425-439.
18. Discrete vector spatial solitons in a nonlinear waveguide array, M. J. Ablowitz and Z. Musslimani, *Physical Review E* **65** (2002) 056618-1-13.

– Conferences

INTERACTIONS/TRANSITIONS

- Participation/Presentations At Meetings, Conferences, Seminars, Etc
 1. Department of Electrical Engineering, University of New South Wales, Sydney Australia, "Dispersion managed solitons", January 10, 2000.
 2. Conference: *Geometric Integration* Durham, England, "Chaotic Dynamics of modulational Instability in water waves", July 18-28, 2000; July 23, 2000.
 3. Conference: *Nonlinear Optics Workshop*, Tucson, Arizona, Sept. 20-22, 2000, "Discrete vector nonlinear Schrödinger equations", September 20, 2000.
 4. Workshop: *Preservation of Stability under Discretization* Math Dept., Colorado State University, Fort Collins, CO. May 30-June 2, 2001; "Chaotic dynamics in nonlinear waves—computational and physical"; June 1, 2001.
 5. Euro Summer School "What is Integrability" Newton Institute, Cambridge UK August 13-24, 2001; "On discrete nonlinear Schrödinger systems"; Aug. 13, 2001.
 6. International Conference: "Nonlinear lattice structure and dynamics", Max-Planck Institute, Dresden Germany, Sept. 23-26, 2001, "Discrete solitons", Sept. 24, 2001.

7. International Conference "Arizona Applied Math Fest" Nov. 1-3, 2001, "Quasi-linear pulses in high bit rate optical communications", Nov. 1, 2001.
 8. Summer School on Theoretical Physics, Canberra Australia, Jan. 20-Feb. 1, 2002, Australian National University; Three lectures on "Nonlinear waves and (interesting) applications", Jan. 16-18, 2002.
 9. Department of Mathematics, Howard University, "Soliton Waves Everywhere", Mar. 1, 2002.
 10. Department of Mathematics, Ohio State University: "Integrability and nonlinear waves, 150+ years", April 18, 2002.
 11. Universidad Complutense University, Madrid Spain, International Conference on "Nonlinear Models in Physics: Perspectives for the XXI Century", June 7-8 2002, "Integrability and Nonlinear Waves — 150+ Years and Counting";, June 7, 2002.
 12. Advanced NATO Research Workshop, Cadiz, Spain June 9-19,2002; "Chaotic dynamics: computational and physical", June 14, 2002
 13. Department of Mathematics, MIT, "Nonlinear waves in high bit-rate communications", October 21, 2002.
 14. Department of Physics, University of Cuernavaca, "WWW: Waves, water and the web!", Dec. 3, 2002; "Discrete and continuous nonlinear Schrödinger equations", Dec. 4, 2002.
- **Consultative and Advisory Functions to Other Laboratories and Agencies:**
none
 - **Transitions:** none
 - **NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES:**
PATENT: US Patent 6,049,608: *Variable Length Nonlinear Feedback Shift Registers with Dynamically Allocated Taps*, Issued April 4, 2000

HONORS/AWARDS: Council on Research and Creative Work Faculty Fellowship, University of Colorado, Boulder, 2001-2002.