APPLYING THE RESONANCE EQUATION TO THE BLOOD PRESSURE WAVEFORM VARIATION IN AORTA BENDING AND RENAL LIGATION OF RATS

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Abstract - In the resonance theory, the radial dilatation is emphasized, and the blood pressure wave is transmitted in the form of “moving windkessel”. Based on this conjecture, we developed a semi-empirical procedure to describe the pressure distribution in a complex simulated model composed of a main tube and attached organs. Now we try to apply this fitting method to hemodynamics *in vivo*, and we tested our equations by two sets of experiments on the rats: ligating a renal artery and bending the aorta transversely. Abdominal aortic blood pressure of rat was measured through tubes inserted from the caudate artery and compared with curve-fitting deduced from semi-empirical resonance equations. The good fitting result illustrates that in spite of various complex structures of the arterial system, we can still provide good description for the blood pressure distribution by dividing the arterial system into sub-units and describing with few elastic parameters. It reinforces the conjecture of the resonance theory.

Keywords: blood pressure, resonance, hemodynamic, artery

I. INTRODUCTION

The blood pressure is generally thought to originate from resistance of blood flowing through vessels, which can be described conceptually by Poiseuille’s equation: P(pressure) = Q(flow)*R(resistance). Circulation theories deduced from the Navier-Stokes equation, what we call “flow theories”, is a combination of two forces: the inertial term and the viscous term [1, 2]. In electric analogue, a segment of vessel can be simulated by series of an inductance (L, inertial term) and a resistance (R, viscous term).

In the conjecture of the resonance theory, the arterial system is treated as a pressure-wave-transmitting system, the pulse pressure arriving at the terminal will drive the blood flux into the microcirculation [3-10]. Therefore the radial dilatation of the vessel wall was emphasized, and the pressure wave is transmitted in the form of “moving windkessel” [3-5]. We have derived an analytic pressure wave transmission equation in a long elastic tube [4, 5]. From this equation, we also developed a semi-empirical procedure to describe the pressure distribution in a complex simulated model composed of a main tube and attached organs with different sizes and at different sites [3]. We got good result in describing the pressure distribution in these simulating experiments.

Based on these previous works, now we try to apply these fitting methods to hemodynamics *in vivo*. Since the kidneys occupies the largest cardiac output in all organs, we treat the arterial system as a system composed of main artery and two kidneys, as shown in Fig. 1 (details see THEORY section). When we perform renal ligation, one kidney won’t participate in the coupled resonance with the main artery, thus lead to the alteration of blood pressure waveform. Similarly, when we bent the aorta transversely, the elastic property of the main artery will be changed, therefore the coupled resonance between the main artery and organs will also be altered, also lead to changes of blood pressure waveform.

Here we performed these two experiments to the rats. We observed the alteration of blood pressure waveform and applied semi-empirical fitting procedure deduced from the resonance theory to the experimental results. It can serve as a critical test for the resonance theory.

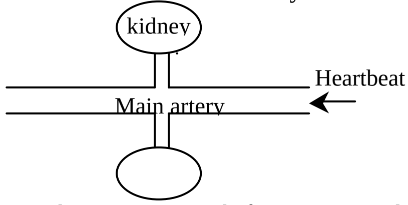

Figure 1: arterial system composed of main artery and two kidneys

II. THEORY

Part 1: a segment of aorta in the resonance theory

In the conjecture of the resonance theory, the blood pressure is generated by the momentum of the injecting blood from the heartbeat colliding on the vessel wall, it travels along the aorta in the form of transverse wave. The wave propagation is like a “moving windkessel”, an elastic chamber moving down the artery with finite speed. We can derive the wave propagation equation based on the radial movement for a small piece of wall [4, 5]:

$$\frac{\partial^2 P}{\partial t^2} + b\frac{\partial P}{\partial t} + n_0{}^2 P = n_\infty{}^2 \frac{\partial^2 P}{\partial Z^2} \qquad \text{(E1)}$$

$$n_0 = \sqrt{\frac{K}{m}} = \sqrt{\frac{E_n}{m r_0} - \frac{2p r_0}{C_A}},\ b = \frac{x}{m},\ n_\infty = \sqrt{\frac{t}{m}} = \sqrt{\frac{E_{rZ} h}{m}}$$

while $E_n = E_r + E_q$, $[\frac{E_n}{r_0} \equiv K]$

E_r: radial Young's modulus of the wall material

E_q: circumferential Young's modulus of the wall material

$m = r_w h + r_f h_f$, r_w: density of the tube material, r_f: density of the adherent fluid which moves together with the wall of thickness, h_f , h: tube thickness

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r_0: the inner radius

$C_A = \dfrac{d(pr_0^2)}{dP}$: the arterial compliance

E_{rZ}: the shear modulus of the tube material

x: the viscosity constant for radial movement of the wall and the adherent fluid

We will have:

$$m\frac{d^2P}{dt^2} + bm\frac{dP}{dt} + mn_0^2 P = -mV_\infty{}^2 k^2 P$$

$$\left(k = \frac{2p}{l},\ l = \text{wavelength}\right) \qquad \text{(E2)}$$

at some particular position ($Z = Z_0$).

The term $mV_\infty{}^2 k^2 P$ is the external force F_{ext}. This term includes the heart pumping and all the other effects from the vascular bed.

Let

$L = m$, equivalent inertance from a segment of aorta

$R = x$, equivalent resistance from a segment of aorta

$1/C = mn_0^2$, equivalent compliance of a segment of aorta

It becomes:

$$L\frac{d^2P}{dt^2} + R\frac{dP}{dt} + \frac{P}{C} = \frac{F}{C} \qquad \text{(E3)}$$

In electric analogue, the left side of (E3) can be represented by series of an L (inductance), an R (resistance), and a C (capacitance).

Part 2: composite system of aorta and two symmetric kidneys in resonance theory

To take the kidneys into the system, we propose a pressure coupling mechanism as shown in Fig. 1. The renal arteries can be considered as a channel of pressure balance, and this pressure difference between the aorta and the kidneys is acting through the pressure coupling constants (e_{12}).

We may thus construct the coupled oscillation equations for the kidney system, which comprises a segment of aorta, two assumed symmetric kidneys, and two renal arteries serve to connect these three elements:

$$L_1\frac{d^2P_1}{dt^2} + R_1\frac{dP_1}{dt} + \frac{e_1}{C_1}P_1 + 2\frac{e_{12}}{C_1}(P_1 - P_2) = \frac{F_{ext}}{C_1} \qquad \text{(E4)}$$

$$L_2\frac{d^2P_2}{dt^2} + R_2\frac{dP_2}{dt} + \frac{e_2}{C_2}P_2 + \frac{e_{12}}{C_2}(P_2 - P_1) = 0 \qquad \text{(E5)}$$

F_{ext}: the applied force from the heart that is transmitted to this segment of aorta

P_1, P_2: the pressure in the segment of aorta and in the vascular bed of the kidneys

L_1, R_1, C_1: equivalent inertance, resistance, and compliance from the segment of aorta

L_2, R_2, C_2: equivalent inertance, resistance, and compliance from the vascular bed of the kidneys

e_i: the equivalent elastic coefficient of the whole vascular wall as one unit

$e_{ij}(P_i - P_j)$: the coupling force between the kidneys and the segment of aorta. e_{ij} is their coupling constant

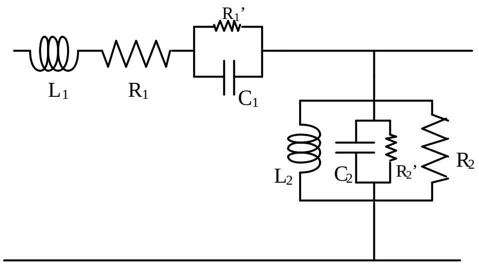


Figure 2: Electric analog for the circulatory system: the main artery, one organ, and the vein.

Similarly, in electric analogue as shown in Fig.2, the left side of (E4, the aorta) can be represented by a series of an RLC_1, while the left side of (E5, the kidney) represented by another series of RLC_2 parallel to the aorta. The return line is the vein.

Part 3: Solving the resonance equations for renal ligation experiment

set $k_i = \dfrac{R_i}{L_i}$, $n_{i0}{}^2 = \dfrac{e_i}{L_iC_i}$, $i = 1,2$

$a_{12}{}^2 = \dfrac{e_{12}}{L_1C_1}$, $a_2{}^2 = \dfrac{e_{12}}{L_2C_2}$

If the constants in equation (E4) and (E5) are not pressure-dependent, the equations become linear. We may set $F_{ext} = \sum_n F(jnw)e^{jnwt}$ $P_i = \sum_n P_i(jnw)e^{jnwt}$

$i = 1,2$, $j = \sqrt{-1}$

Let $w_n = nw$

$$\Delta_n = (-w_n{}^2 + ik_1w_n + n_{10}{}^2 + a_1{}^2)\times(-w_n{}^2 + ik_2w_n + n_{20}{}^2 + a_2{}^2) - a_1^2a_2^2$$

The solutions of the above equations are:

$$P_{n1} = \frac{F(-w_n{}^2 + ik_2w_n + n_{20}{}^2 + a_2{}^2)}{\Delta_n} \qquad \text{(E7.1)}$$

$$P_{n2} = P_{n3} = \frac{Fa_2{}^2}{\Delta_n} \qquad \text{(E8)}$$

with $F = F_{ext}/L_1C_1$

If one kidney is completely isolated from this system, the solution will become:

$$P_{n1}' = \frac{F(-w_n{}^2 + ik_2w_n + n_{20}{}^2 + a_2{}^2)}{(-w_n{}^2 + ik_1w_n + n_{10}{}^2 + a_{12}{}^2)(-w_n{}^2 + ik_2w_n + n_{20}{}^2 + a_2{}^2) - a_{12}{}^2a_2{}^2}$$

(E7.2)

We may take the ratio of P_{n1} and P_{n1}' to eliminate the F term and to predict the effects of renal ligation on the pressure pulses.

Part 4: Solving the resonance equations for aorta bending experiment

When we bent the aorta, the properties of the kidneys was assumed unchanged, while those of the aorta (k_1, n_{10}, a_{12}) were shifted to (k_1', n_{10}', a_{12}'). The solution for the blood pressure wave before aorta bending was (E7.1), while that after can be represented by:

$$P_{n1}'' = \frac{F(-w_n^2 + ik_2 w_n + n_{20}^2 + a_2^2)}{[-w_n^2 + ik_1' w_n + (n_{10}')^2 + (a_{12}')^2](-w_n^2 + ik_2 w_n + n_{20}^2 + a_2^2) - (a_{12}')^2 a_2^2}$$
(E7.3)

Similarly, we may take the ratio of P_{n1} and P_{n1}'' to eliminate the F term and to predict the effects of aorta bending on the pressure pulses.

III. MATERIALS AND METHODS

1. Animal preparation

7 male Wistar rats weighted 265.71±50.88 gw (mean ± SD) were anesthetized with urethane (1.2mg g^{-1} body weight). Abdominal aortic pressure was measured through a PE (polyethylene) tube (2F, outer diameter 0.6mm) filled with liquids (0.9 % NaCl and 0.3 % heparin) inserted from the caudate artery. The head of the tube was then moved to the level of the abdominal aorta. Pressure pulses were recorded by a catheter-tip pressure transducer (P10EZ transducer, RP-1500 Narco-Bio-System). The frequency response of this assembly system was uniform up to 60 Hz. The signal was connected to a simultaneous sample-and-hold panel AX753 and then to a 16-bit AD converter AX5621. These cards converted signals to digital data with sampling frequency 1500 Hz, and sent them to a personal computer for further analysis [8-10].

In aorta bending experiment, we opened the ventral side of the rat, separated the inferior mesenteric artery and tied prepared silk suture to it. The other end of silk suture was tied to a stereotaxic to provide a precise transverse displacement. When we bent the aorta, the renal artery was not moved at all. Blood pressure signal was recorded before and immediately after the bending. Right after the bending, the blood pressure was recorded again to check if it was the same as that before.

In the renal ligation experiment, we separated the left renal artery off the fat and then prepared the silk sutures to go around the artery. We then threaded both ends of the silk suture through a segment of PE tube. When we performed the ligation, we lifted the silk sutures and confined the PE tube at the same position to keep the renal artery from moving and thus restricted the aorta from bending. The transverse displacement of the renal artery was far less than 1 mm. Recording procedure was the same as that in aorta bending experiment.

After the surgical operation, we waited 30 minutes for the blood pressure wave to return to normal to start our experiments. At least 15 minutes were waited between each step to avoid interference from the last experiment. Between the experiments, the exposed tissues were kept moist with physiological saline and the environment was kept at 32^{o}C by a heat radiator.

2. Data analysis

In each recording, we have 10 pulses. Each pulse was picked by two relatively lowest points and transformed into frequency spectra by Fourier transform. To eliminate the effects of time-variant F, amplitude of each harmonic was normalized by its mean level (DC component) and named harmonic proportion. Because the amplitude decreased rapidly with the harmonic numbers, we focused only on the first six harmonics [8-10].

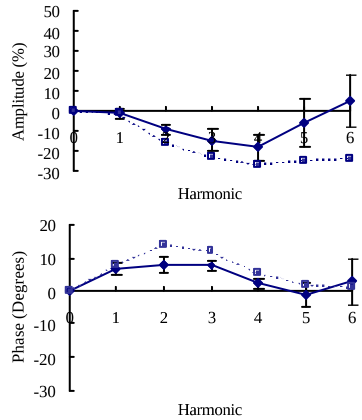


Figure 3: Harmonic variations of pressure wave by renal ligating. Upper: amplitude. Lower: phase angle. Solid line: measured data. Bars are SD. Dash line: fitting by the resonance theory.

The fitting procedure was similar with our previous work [3]. We first compare the measured and theoretic spectrum in renal ligation experiment and apply the least-mean-square-error criteria to both amplitude and phase angle of first 4 harmonics to get 6 parameters: k_1, $k_2(=k_3)$, n_{10}, $n_{20}(=n_{30})$, $a_{12}(=a_{13})$ and $a_2(=a_3)$. Similarly, we then turn to the result of aorta bending experiment and get another 9 parameters: k_1'(4mm), k_1'(5mm), k_1'(6mm), n_{10}'(4mm), n_{10}'(5mm), n_{10}'(6mm), a_{12}'(4mm), a_{12}'(5mm) and a_{12}'(6mm) (inside the parenthesis is the transverse bending of the aorta).

In all these equations, the F term from the heart is a problem, because it is an individual-dependent variable. Different rat will have different heartbeat, even for the same rat, this term can be time-dependent. However, 3-second bending is quick enough to avoid any reflex reaction and F can be treated as a constant. This is confirmed by carefully checking the pressure pulses before and after the bending to be the same. In the theoretical treatment, under the premise that the intervention does not change F, we may take the ratio of P_{01} and P_{01}' (or P_{01}'') to eliminate the F term, thus to predict the effects of renal ligation and aorta bending on the pressure pulses.

When we compare the first 6 harmonics, both with amplitude and phase angle, we have 12 equations. Considering 4 conditions of renal ligation and bending the aorta for 4, 5, 6 mm, we have totally 12×4=48 equations, while there are only 15 independent fitting parameters.

IV. RESULTS

Comparison between experimental data and curve fitting by the resonance theory in renal ligation experiment is shown in Fig. 3, and that for aorta bending is shown in Fig.

4. The fitting parameters in Fig. 3 are $f_0 = 6.0Hz$, $w_0 = 2pf_0$, $k_1 = 70$, $k_2 = k_3 = 60$, $n_{10} = w_0$, $n_{20} = n_{30} = 2.6w_0$, $a_{12}{}^2 = a_{13}{}^2 = 3w_0{}^2$, and $a_2{}^2 = a_3{}^2 = 1.5w_0{}^2$ (w_0, n, k and a are in radians). The fitting parameters for Fig. 4 are the same as those in Fig. 3 except:

(A) $n_{10}' = 1.6n_{10}$, $a_{12}' = 0.83a_{12}$, $k_1' = 1.03k_1$;

(B) $n_{10}' = 2.1n_{10}$, $a_{12}' = 0.6a_{12}$, $k_1' = 1.05k_1$;

(C) $n_{10}' = 2.27n_{10}$, $a_{12}' = 0.45a_{12}$, $k_1' = 1.1k_1$.

V. DISCUSSION & CONCLUSION

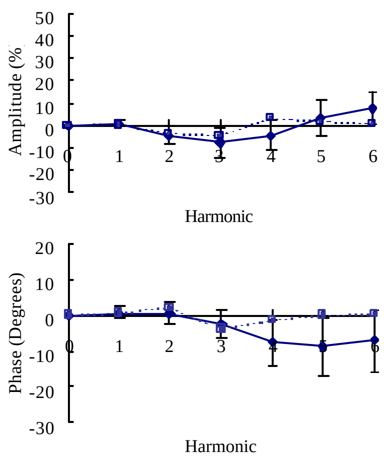
In the fitting results (Fig.4), the three changing parameters: k_1', n_{10}', a_{12}' at transverse displacement of 4, 5, 6 mm show rational trends. k_1' increased from $1.03k_1$, $1.05k_1$, to $1.1k_1$, n_{10}' from $1.6n_{10}$, $2.1n_{10}$, to $2.27n_{10}$, and a_{12}' changed from $0.83a_{12}$, $0.6a_{12}$ to $0.45a_{12}$. It is consistent with intuitive thought that when the aorta was bent more, these parameters deviated more from their unbent value accordingly. Besides, our fitting parameters are not chosen arbitrarily but have their physical meaning, therefore we call them "semi-empirical" ones [3-5].

Moreover, 48 linear equations can generally determine 48 parameters. However in our results, we can use only 15 independent parameters to fit totally 48 equations. We can see that the 48 equations are so highly dependent that these 48 parameters can be reduced into 15 independent ones as suggested in our description. It illustrates that even if there are so many complex structures in the arterial system, we can still get good description for the blood pressure distribution by dividing the arterial system into sub-units and describing with few elastic parameters as in the resonance theory.

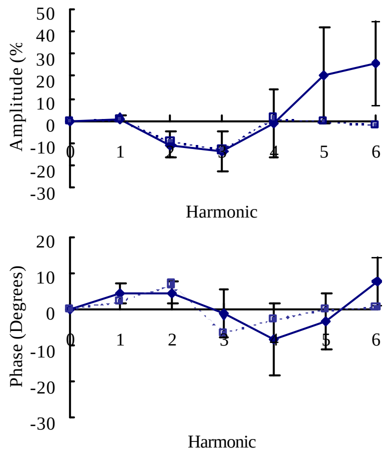
This study serves as the toughest test for our semi-empirical approach *in vivo*, and the result elucidates the conjecture of the resonance theory.

Figure 4: Harmonic variations of pressure wave when the aorta was bent for a transverse displacement of (A) 4 mm; (B) 5 mm; (C) 6 mm. Upper: amplitude. Lower: phase angle. Solid line: measured data. Bars are SD. Dash line: fitting by the resonance theory.

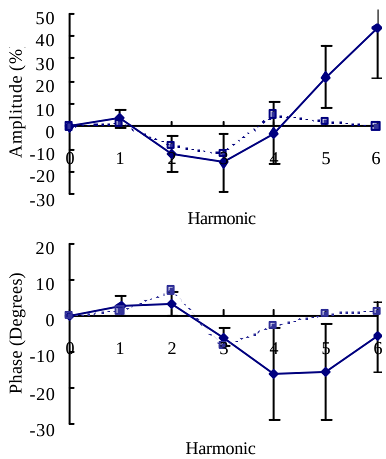
(A)



(B)



(C)



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