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# *The Dynamics of Growth in Worldwide Satellite Communications Capacity*

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Prepared for the United States Air Force

Project AIR FORCE

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## Preface

The Department of Defense (DoD) is considering increasing the amount of communications it owns and leases. Decisions on how much communications capacity to obtain, and how much will come from DoD–owned assets, will affect DoD investment in new communications satellites. A key factor in making investment decisions is the availability of commercial satellite communications capacity in different regions of the world. This report shows that the dynamics of growth in worldwide satellite communications capacity over the past two decades is closely related to general economic growth.

This research is a continuation of work undertaken at the request of Headquarters, United States Air Force (SC), the Assistant Secretary of the Air Force for Acquisition (SAF/AQS), and the Air Force Space Command (XP and SC). It is part of the Employing Commercial Communications task within Project AIR FORCE's Aerospace Force Development Program.

This research should be of interest to defense analysts concerned with obtaining satellite communications within the Air Force, the other military services, and the defense agencies.

## **Project AIR FORCE**

Project AIR FORCE, a division of RAND, is the Air Force federally funded research and development center (FFRDC) for studies and analysis. It provides the Air Force with independent analyses of policy alternatives affecting the development, employment, combat readiness, and support of current and future aerospace forces. Research is performed in four programs: Aerospace Force Development; Manpower, Personnel, and Training; Resource Management; and Strategy and Doctrine.

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## Summary

The Department of Defense (DoD) cannot afford to own all the satellite communications capacity it might possibly need in all areas of the world. As noted in a previous RAND report (Bonds et al., 2000), DoD planners estimate that they will need to provide about 16 Gigabits per second (Gbps) of bandwidth by 2010 to effectively support a joint-service operation. However, given current procurement plans, the DoD will only own only one-eighth of its projected desired capacity. Therefore, for the foreseeable future, the DoD will need to buy at least some of its communications capacity from commercial vendors. An ability to understand what drives growth in worldwide satellite capacity and to predict capacity would be useful to military communications planners in making decisions in advance to purchase and lease communications capacity in various parts of the world.

In the empirical analysis in this report, we show that there is a strong relationship between growth in total satellite communications capacity and economic growth, as measured by Gross Domestic Product (GDP). Adjustment to change is quite rapid; if there is an imbalance in the long-run equilibrium between supply and demand, we estimate that on average 25 percent of the adjustment is made within one year, although there is some regional variation. The analysis indicates that the market can adjust swiftly to a surge in demand, and thus there may be little need to buy satellite capacity in advance simply to ensure that capacity will be there if needed.

## Acknowledgments

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## Acronyms

- AR(1) Autoregressive of order one
- DoD Department of Defense
- ECM Error-Correction Model
- FGLS Feasible Generalized Least Squares
- Gbps Gigabits per second
- GDP Gross Domestic Product
- GEO Geosynchronous Earth Orbit
- I(j) Integrated of order j
- LEO Low Earth Orbit
- MEO Medium Earth Orbit
- MHz Megahertz
- OLS Ordinary Least Squares

## 1. Introduction

Satellite communications is a key part of Department of Defense (DoD) plans for information dominance in the future battlefield. Only satellites can provide the kind of worldwide coverage that the DoD needs. Fiber optic networks in combination with microwave relays may provide a feasible alternative in some parts of the world, but for many places where communications infrastructure is less well developed, satellites are the only feasible alternative.

However, important as they are, satellites cannot be allowed swallow the entire procurement budget; the DoD must be selective and prioritize essential satellite services—for the Single Integrated Operations Plan (SIOP), intelligence needs, and battlefield assessment. These needs require military-unique capabilities such as resistance to jamming or electromagnetic pulse (EMP). However, there is still a large part of DoD demand in both peacetime and wartime that can be met by minmally protected communications assets, which could be either military owned or commercially leased.

Although current DoD demand is under 4 Gigabits per second (Gbps), DoD planners estimate that they will need to provide approximately 16 Gbps of bandwidth by 2010 to effectively support a joint-service operation. About half of this potential demand could be provided by minimally protected assets.<sup>1</sup> However, given current procurement plans, the DoD may only own one-eighth of its projected desired capacity by 2010.

Thus, the DoD needs to turn to the commercial market, both for routine day-today needs and for a time of crisis.

This suggests a vital question: Do the dynamics of the market reflect any points the DoD needs to worry about? More specifically, can the market respond swiftly to increases in DoD demand?

Empirical analysis of the historical dynamics of the satellite telecommunications market can help to answer this question. It can help the DoD make better

<sup>&</sup>lt;sup>1</sup>DoD demand accounts for a small fraction of total worldwide capacity. Current global commercial capacity on C- and Ku-band is more than 175 Gbps, although the DoD prefers to deal with "international" systems in which U.S. companies hold an interest (e.g., INTELSAT), which accounts for only 70 Gbps of the total (Bonds et al., 2000).

decisions about whether and where to invest in owned capacity, and when to rely on the market to fill future needs.

The analysis below shows that there is a strong relationship between growth in total commercial satellite communications capacity and economic growth, as measured by Gross Domestic Product (GDP). Adjustment to change is quite rapid—if there is an imbalance in the long-run equilibrium between supply and demand, we estimate that on average 25 percent of the adjustment is made within one year, although there is some regional variation. The analysis indicates that in many regions the market can adjust rapidly to a surge in demand, and that there may be little need to buy satellite capacity in advance simply to ensure that capacity will be there when needed.<sup>2</sup>

### **Related Literature**

Satellite communications is part of telecommunications infrastructure, and there is a large literature on attempts to identify the relationship between infrastructure and economic growth. Gramlich (1994) provides a thorough review of the literature up to the mid-1990s. More recently, Fernald (1999) makes a careful study of the effect of the interstate highway system on the growth of the American economy. He provides persuasive evidence that the American economic slowdown in the early 1970s resulted, in part, from the construction of the interstate highway system—in other words, that the improvement in infrastructure had led to a one-time boost in productivity that accounted for part of the economic growth over the period of the interstate's construction.

Closer to the subject at hand, Röller and Waverman (2001) examine domestic telecommunications infrastructure, although not satellites specifically. Using data from 21 Organization for Economic Cooperation and Development (OECD) countries over 20 years, they find that telecommunications infrastructure has a significant impact on economic growth, particularly when telecommunications infrastructure reaches a "critical mass." They jointly estimate a micromodel of telecommunications investment and a macromodel relating telecommunications

<sup>&</sup>lt;sup>2</sup>One important caveat should be noted: this analysis relies on a macroscopic, time-series approach that offers no insight into potentially fundamental changes in the market that might lurk a few years off. Fundamental changes could include increasing penetration of close substitutes, such as fiber optic cable, into markets "traditionally" held by communications satellites (e.g., U.S. domestic distribution of cable-TV content to cable-heads), or the saturation of markets that have historically been subject to rapid growth. Such changes could result in a new long-run equilibrium relationship between GDP and satellite capacity or adjustments in the rapidity of response to market disequilibrium. Unfortunately, the data are not currently available for a microscopic analysis of the satellite communications market to enable us to identify the market fundamentals. However, such data may become available with the recent creation of open markets trading in satellite services, such as the London Satellite Exchange.

infrastructure to economic growth. Their setup recognizes that the arrow of causation can go both ways. In the micromodel, per-capita GDP along with price determines demand for telecommunications services, which in turn determines telecommunications investment, whereas in the macromodel, GDP is a function of the total stock of telecommunications infrastructure. In contrast, the macromodel we present below models causation as going from GDP to the stock of satellite transponders.

## Organization

We first discuss the empirical analysis, including the general characteristics of the data we are working with and the limitations on the analysis placed by the data. We then discuss the problems endemic to dealing with time-series data such as our satellite communications data. We briefly discuss the type of model we chose to estimate and the steps we took to ensure that we correctly specified and estimated the model. With estimates of the model, we discuss the implications of the estimates of the market dynamics of the supply of satellite communications capacity, and give some caveats. Finally, we make some concluding remarks on the dynamics of worldwide satellite capacity. An appendix discusses the goodness of fit of the model for each geographic region.

## 2. The Data

# The Data, and Limits of What We Can Hope to Model Based on the Data

The limits of an empirical analysis are often determined by the data available. While we would have liked to have constructed a traditional structural demandand-supply model of the communications satellite capacity market, the quantity, quality, and price data we would need are simply not available to estimate supply-and-demand curves with any confidence. Only limited price data are available. Although tariff schedules have been published in the past, and in this country vendors were for many years required to promptly inform the Federal Communications Commission of any changes in their tariff schedules, in fact most bilateral contracts between vendors and buyers of satellite capacity were negotiated as package deals, with implicit rates considerably lower than the published tariffs. In any case, the negotiated rates were, and still are in most cases, the private information of the vendors and purchasers of satellite capacity. For similar reasons, quantity data are also difficult to come by.

Instead, we have estimated a model based on the data that *are* available. We have estimated a model of how satellite communications capacity available relates to the total GDP in a particular region of the world.

Our data, spanning the years 1980 to 1999 and covering seven regions of the world (North America, Latin America, Western Europe, Central and Eastern Europe, Middle East and Africa, Southern Asia, and Asia Pacific), come from several sources. The satellite capacity data (measured in 36-MHz equivalent transponder units) is from the Euroconsult report, *World Satellite Communications and Broadcasting Markets Survey: Prospects to 2009.* The data include both regional and international systems in the C-, Ku-, and Ka-bands. Estimates of total GDP (1996 dollars) for each region of the world were derived from per-capita data from the Central Intelligence Agency and population data from the Bureau of the Census.

Satellite capacity and GDP both tend to be strongly trended; that is, they are both highly correlated with time (both have grown steadily over time). This brings some special problems in modeling and statistical estimation that we will discuss below.

### Some Details of the Empirical Analysis

A reader interested only in results, policy implications, and conclusions may wish to proceed to Section 3. What follows is a brief discussion of some of the pitfalls in analyzing time-series data such as our satellite data, as well as our strategy for avoiding those pitfalls.

#### Avoiding Possible Pitfalls

The supply of communications satellite capacity is strongly trended. Indeed, over 95 percent of the variance in worldwide communications capacity can be accounted for by a time trend alone. Many of the possible explanatory variables are also strongly trended, leading to a problem if we are trying to construct a model relating capacity to other variables. If we detect a strong relationship between capacity and some explanatory variable, is it because there is truly a relationship between the two variables, or is it merely because they are both strongly related to time? If two time series are growing, they may be correlated even though they are increasing for completely different reasons and may be increasing by increments that are uncorrelated.

Any series that tends to grow over time is known as a nonstationary series. (A stationary series would be one where the series of observations are all drawn from the same distribution; any growing series will have a changing mean, so is of necessity nonstationary.) Yule (1926), as cited in Banerjee (1993), demonstrated that spurious correlations might be detected between two nonstationary time series, although the underlying series themselves have no relationship. Granger and Newbold (1974) also demonstrated this using a Monte-Carlo analysis.

How can we avoid the potential pitfall of detecting a relationship when one in fact is not present? One approach is to take the first difference of the series in the hope that even though the series itself is nonstationary, the annual increments are stationary. Then, perhaps, we could run our regressions with more confidence, although we will have thrown away possibly valuable information encoded in the absolute levels of the variables.

More formally, a series whose first difference is stationary is said to be integrated of order one, or I(1). The "integration" terminology comes from the idea that the series is a process that results from a cumulative sum over time. Similarly, if we have to take the differences of the differences to form a stationary series, that series is said to be integrated of order two, or I(2). A stationary series is integrated of order zero, or I(0). Differencing is not the only way to create a stationary series from nonstationary time series. Sometimes it is possible to create a stationary series by a weighted sum of nonstationary time series. That is, we might be able to take two I(1) series, {*x*} and {*y*}, and choose weights  $\alpha$  and  $\beta$  so that { $\alpha x + \beta y$ } is stationary. This is known as "cointegration." We could use variables that have a cointegrating relationship in our regressions without fear of being vulnerable to identifying spurious correlations due to nonstationarity. It also allows us to exploit the additional information we gain through having the variables enter the regression as levels as well as differences.

Another pitfall to avoid in regression analysis of time-series data is autocorrelated errors, which can lead to biased estimates and other problems. However, we can avoid this pitfall by the simple means of testing the residuals of our candidate regression specifications for serial correlation.

#### Technical Notes on Series Selection and Model Estimation Strategy

The basic strategy we followed in selecting the time series to use in estimating the model and in selecting the final structure of the model itself is described below.

First, test each series to see if it is I(1) against the null hypothesis that it is I(2), using the Augmented Dickey-Fuller test. If the null that the series is I(2) is refuted, we test to see if the series is I(0) against the null that the series is I(1) using Augmented Dickey-Fuller. If we fail to refute the null hypothesis, we accept the null and consider the series to be integrated of order one, or I(1). (Note that this is a relatively weak test; i.e., it is difficult to refute the null hypothesis.)

We next use the Johansen (1988) and Johansen and Juselius (1990) maximum likelihood procedure on the candidate set of I(1) variables to find maximum likelihood estimates of the cointegrating vectors and weights. Then, for each variable we use a likelihood ratio test to test the null that the variable does not enter into the cointegrating relationship. (This test is more useful in winnowing out alternative candidates because the null is to assume that *no* relationship exists.)

Finally, we use the surviving series in an Error-Correction Model (ECM) of the supply of communications capacity and test the residuals for serial correlation using the Durbin-Watson d statistic. We start with a generous lag structure for the ECM and test down to the final model. (Note that we choose to estimate the appropriate weights for the cointegrating variables within the ECM rather than use weights provided by the Johansen maximum likelihood procedure.)

## 3. The Model

The most striking feature of a graph of commercial communications satellite capacity over time is the sheer growth over the last thirty years. That growth has been driven by incremental capacity decisions made by private firms and governments (through monopolistic telecommunications operators) over the years. For the growth to be sustained every year, more new satellites go up and old satellites that have reached the end of their useful life are replaced by new satellites that often have larger capacity. The decision to buy, launch, and "run" a satellite or a constellation of satellites is nontrivial, generally requiring the expenditure of hundreds of millions of dollars.

So what drives the incremental capacity decisions that result in the increase in worldwide commercial satellite communications capacity over time? It seems reasonable to suppose that commercial telecommunications service providers will make incremental capacity decisions based on current and forecast demand for satellite communications services and the current capacity available.

This is the motivation for our dynamic model of the growth of commercial communications satellite capacity. We model the year-to-year growth in capacity as a function of the normal equilibrium relationship between supply and demand. If the normal equilibrium relationship is perturbed, market forces tend to bring the system back into equilibrium. If supply outpaces demand, market forces bring the growth rate of the supply down. Similarly, if demand outstrips supply, we would expect the growth rate to adjust upward.

In our model, demand is measured by GDP. Although GDP is not a direct measure of demand, because it is a measure of overall economic activity it seems plausible that it would be correlated with overall telecommunications demand.

## Some Technical Details

The type of model we are using to specify the equilibrium relationship and the dynamics of adjustment is a generalized ECM. The specification we use is

$$\Delta y_t = \alpha + \eta (y_{t-1} - x_{t-1}) + \beta \Delta x_t + \zeta x_{t-1} + \varepsilon_t$$

where  $\Delta y_t$  is the growth in communications capacity and  $(y_{t-1} - x_{t-1})$  is the error-correction term, giving the lagged difference between our measures of

supply and demand. The coefficient on the error-correction term,  $\eta$ , gives the rate at which the system adjusts back to long-run equilibrium.<sup>1</sup>

### **Estimates for the Regional Models**

The model of satellite communications capacity relates a measure of the growth rate of global transponder capacity to deviations from the equilibrium relationship of global transponder capacity to GDP. The equilibrium relationship is summarized by the error-correction term, and the coefficient on the errorcorrection term shows the strength of response to short-run deviations from equilibrium.

The measure we are using for demand for satellite communications capacity is less than ideal, and it should be viewed as an instrument for general demand for communications. It seems reasonable that the true underlying demand for satellite communications capacity would be correlated with a measure of general economic activity; however, one could imagine more apt measures of demand for satellite communications capacity (such as the amount actually traded in a given year.) The surprising result is that, even given this less-than-ideal measure of demand, the fit of the model is actually quite good for most regions of the world, as we will see below.

Variable definitions are shown in Table 1. The table defines each variable and gives the corresponding mathematical notation used in the technical description of the model above and the coefficient estimates in Table 2.

In constructing the models, we followed the strategy for variable selection and model estimation given in the previous section. All the level variables were found to be I(1), and the Johansen likelihood ratio test showed that the variables entered into a cointegrating relationship. We then estimated the ECM using ordinary least squares, starting first with a generous lag structure and then testing down to the models shown below. (We chose to estimate the cointegrating weights within the ECM rather than use the weights produced by the Johansen maximum likelihood procedure. Once a cointegrating relationship

<sup>&</sup>lt;sup>1</sup>It may seem that the specification implies that in the equilibrium relationship  $x_{t-1}$  has a coefficient of one; however, the  $\zeta x_{t-1}$  term allows the error-correction term to "break homogeneity"; that is, to allow for the true error-correction term to be  $(y_{t-1} - \theta x_{t-1})$ , where  $\theta$  is not necessarily equal to one. The estimate of the rate at which the system converges back to equilibrium,  $\eta$ , is thus unaffected by our specifying the error-correction term as  $(y_{t-1} - x_{t-1})$  even though the true equilibrium relationship is described by  $(y_{t-1} - \theta x_{t-1})$  (Banerjee, 1993). We can, in fact, derive the  $\theta$  describing the long-run equilibrium relationship from the coefficients estimated in the specification given above using the formula  $\theta = -(\zeta - \eta) / \eta$ .

#### Table 1

#### **Variable Definitions**

Variable	Description
$y_t$	Satellite capacity (36-MHz transponder equivalents), logged
$x_t$	GDP (millions of 1996 dollars), logged
$\Delta y_t = y_t - y_{t-1}$	Growth in satellite capacity (36-MHz transponder equivalents)
$y_{t-1} - x_{t-1}$	Lagged error-correction term
$x_{t-1}$	Lagged GDP (millions of 1996 dollars), logged
$\Delta x_t = x_t - x_{t-1}$	Growth in GDP (millions of 1996 dollars)

#### Table 2

#### **Coefficient Estimates**

Variable	Region 1: North America	Region 2: Latin America	Region 3: Western Europe	Region 4: Central and Eastern Europe	Region 5: Middle East and Africa	Region 6: South Asia	Region 7: Asia Pacific
$y_{t-1} - x_{t-1}$	-0.26**	-0.46**	-0.32**	-0.24**	-0.15**	-0.30*	-1.01**
	(0.06)	(0.17)	(0.11)	(0.06)	(0.05)	(0.14)	(0.22)
$\Delta x_t$	0.23	3.44**	0.67	-0.36	1.91**	1.00	1.93**
	(0.96)	(1.23)	(0.85)	(0.35)	(0.81)	(1.17)	(0.55)
$x_{t-1}$	-0.02	1.18	1.93**	-0.49**	0.07	0.33	1.82**
	(0.16)	(0.63)	(0.71)	(0.18)	(0.52)	(0.37)	(0.40)
Constant	-1.96 (2.94)	–20.75* (10.29)	-33.32** (12.25)	5.05* (2.26)	-2.18 (7.48)	-6.56 (5.56)	-38.09** (8.30)
θ	0.94	3.59	6.97	-1.07	1.50	2.07	2.80
Durbin-Watsor	n 1.61	2.41	2.11	2.33	1.93	2.35	2.39
Observations	19	18	18	18	18	18	18
Adjusted R <sup>2</sup>	0.73	0.52	0.24	0.50	0.43	0.17	0.67

NOTE: Standard errors are given in parentheses; "\*" indicates coefficient is significant at the 0.05 level, "\*\*" at the 0.01 level.

among the variables is established, it is unnecessary to explicitly include it in the final regression.)

Broadly speaking, the model performs up to expectations. The Durbin-Watson statistics indicate that there is no significant serial correlation in any of the regional models. The R<sup>2</sup>s are fairly high in some cases, as is to be expected for a

model with three independent variables plus an intercept fit to only 18 or 19 observations.<sup>2</sup>

#### Interpreting the Regional Models

The coefficient on the change in GDP,  $\Delta x_t$ , is positive and significant at the 1percent level in three of the seven regions, indicating that GDP has a significant short-term effect on satellite capacity in those regions. The long-run effect of GDP on satellite capacity is captured by the error-correction term.

The coefficient on the error-correction term  $(y_{t-1} - x_{t-1})$ , indicating the speed of adjustment to long-run equilibrium, is negative and significant at the 1-percent level in all but one of the regions. The magnitude of the coefficient on the error-correction term can be interpreted as the fraction of the deviation from the long-run equilibrium in any given period that is made up within the next period. In many regions of the world, adjustment is quite rapid—25 percent or more in five of the seven regions. The size of the error-correction term for the Asia Pacific region is somewhat surprising, in that it implies that any one-period deviation from the long-run equilibrium is instantly made up within the next period.

The results show that capacity adjusts quite rapidly to changes in demand conditions, which is interesting given the long lead-time of most satellite procurement and launch contracts. Over the time period the model covers, the production cycle for major satellite manufacturers was well over 12 months, ranging from an average of 32 months in 1990 to 20 months in 1995, making "impulse purchases" difficult.<sup>3</sup> Counterbalancing this was the fact that several manufacturers during part of the period allowed customers to delay committing until as late as six months before the scheduled launch.<sup>4</sup> Launch delays were quite common over the time period studied.<sup>5</sup> It seems that that either investors were successfully forecasting fluctuations in demand or were able to exploit what flexibility there was in delivery and launch schedules to their advantage.

The model estimates also imply a long-run equilibrium relationship between capacity and GDP. This relationship is given by  $\theta$  in Table 2.<sup>6</sup> It can be

<sup>&</sup>lt;sup>2</sup>For the graphically inclined, the appendix gives charts, by region, showing how the predicted time series compares to the observed time series for these models.

<sup>&</sup>lt;sup>3</sup>Euroconsult, World Satellite Communications and Broadcasting Markets Survey: Prospects to 2007, The European Consulting Group, Paris, 1998, p. A-10.

<sup>&</sup>lt;sup>4</sup>Euroconsult, p. A-10.

<sup>&</sup>lt;sup>5</sup>Euroconsult, p. A-11.

<sup>&</sup>lt;sup>6</sup>In Table 2,  $\theta = -(\zeta - \eta) / \eta$ , where  $\eta$  is the coefficient on the error-correction term  $(y_{t-1} - x_{t-1})$ , and  $\zeta$  is the coefficient on lagged GDP,  $x_{t-1}$ .

interpreted as an elasticity, giving the long-run percentage change in satellite capacity given a 1-percent change in GDP. The long-run equilibrium relationship between capacity and GDP was positive, as expected, in all but one of the regions (Central and Eastern Europe—probably because of unique historical circumstances, which we examine in more detail below). In North America, Latin America, the Middle East and Africa, and South Asia, the coefficient on  $x_{t-1}$  (which enters into the calculation of  $\theta$ ) is not statistically significant, which implies that  $\theta$  is not significantly different from one in these regions.

Setting aside the anomalous case of Eastern Europe for the moment, the Western Europe and the Asia Pacific regions both show fairly large  $\theta$  s that are statistically significantly different from one. While the value of 2.80 for the Asia Pacific region seems—just barely—to be in the range of the possible for a region that experienced unprecedented economic growth through 1997,<sup>7</sup> the value of 6.97 for Western Europe strains credulity. Perhaps recent rapid growth in the wake of government deregulation helps to explain this result. European satellites were long the province of the individual national telecommunications monopolies, although the introduction of the Astra system by Société Européenne des Satellites (SES) in 1988, the first private system in Europe, "transformed the European satellite market into one of the most dynamic in the world."<sup>8</sup> Since 1988 there has been rapid growth in satellites by 1997 and Eutelsat launching eight satellites over the same time period.<sup>9</sup>

### Estimates for Central and Eastern Europe

The results for Central and Eastern Europe are problematic. The estimated parameters for the model, if they are to be believed, indicate that there is, in equilibrium, a negative relationship between GDP and satellite capacity over the region. This is most likely a spurious relationship, however it is reflected in the raw data. Figure 1 shows GDP and satellite capacity over two decades. GDP declines precipitously over the eight years from 1989 to 1996, while satellite capacity shows fairly steady growth over the period. Therefore, it is perhaps unsurprising that the regression model indicates a negative relationship between GDP and satellite capacity.

<sup>&</sup>lt;sup>7</sup>Euroconsult, p. B-203.

<sup>&</sup>lt;sup>8</sup>Euroconsult, p. B-85.

<sup>&</sup>lt;sup>9</sup>Euroconsult, p. B-85.



Figure 1—GDP and Satellite Capacity in Central and Eastern Europe, 1980–1999

What are we to make of this result? Perhaps it arises from the particular history of the region. Central and Eastern Europe were deeply affected by the fall of the Soviet Union, which resulted in the decline in regional GDP. Satellites in the region have been largely a government enterprise, so it is perhaps unsurprising that the patterns of growth differ from those in regions where satellites are generally private. In addition, there may be data quality problems with the figures reported for the Soviet Union; however, that does little to explain the pattern observed from 1989 on. Given the puzzling behavior in this region, it would seem to make sense to view the model estimates with suspicion. Perhaps some other indicator of demand would make a better model for this region.

An alternative indicator of demand is the GDP for the neighboring region, Western Europe. Indeed, the Johansen likelihood ratio test shows that the cointegrating relationship between Western European GDP and Central and Eastern Europe transponders is much stronger than the relationship within Central and Eastern Europe. If Western European GDP is entered into the Johansen procedure along with Central and Eastern European satellite capacity and GDP, the Central and Eastern European GDP does not enter into the estimated cointegrating relationship in a statistically significant way.

Based on this result, we have reestimated the model of Central and Eastern European satellite capacity, substituting Western European GDP for Central and Eastern European GDP. The revised estimates are presented in Table 3, along with the original estimates for comparison.

#### Table 3

**Coefficient Estimates for Central and Eastern Europe** 

	Original	Revised
	( $x_t$ equals Central and	( $x_t$ equals Western
Variable	Eastern Europe GDP)	Europe GDP)
$y_{t-1} - x_{t-1}$	-0.24**	-0.50**
	(0.06)	(0.13)
$\Delta x_t$	-0.36	-0.08
-	(0.35)	(1.05)
$x_{t-1}$	-0.49**	1.91*
	(0.18)	(0.82)
Constant	5.05*	-35.04*
	(2.26)	(14.18)
heta	-1.07	4.85
Durbin-Watson	2.33	1.92
Observations	18	18
Adjusted R <sup>2</sup>	0.50	0.61

NOTE: Standard errors are given in parentheses; "\*" indicates coefficient is significant at the 0.05 level, "\*\*" at the 0.01 level.

The revised estimates are more intuitively appealing in that now there is a positive equilibrium relationship  $\theta$  between GDP and satellite capacity.  $\theta$  is large, indicating that for every 1-percent change in GDP, the supply will rise by nearly 5 percent, probably for the same reasons that  $\theta$  was large within Western Europe. (To quote the Euroconsult report, "the [Central European] satellite market has grown as an extension of the Western European market."<sup>10</sup>) The fit, as measured by adjusted R<sup>2</sup>, has also improved. The coefficient on the change in GDP,  $\Delta x_t$ , is not significantly different from zero, indicating small short-run changes attributed to GDP growth; however, the coefficient on the error-correction term  $y_{t-1} - x_{t-1}$  is very strong, second only to Asia's in magnitude, indicating a rapid adjustment to long-run equilibrium.

### **Pooling the Regional Data**

Given the model's performance for the individual regions, and some similarity in the results from region to region in signs and significance, it seems reasonable to ask if there are possible gains from pooling the data. Creating a model to take advantage of the pooled data brings its own problems, notably needing to allow

<sup>&</sup>lt;sup>10</sup>Euroconsult, p. B-123.

for the possibility of heteroskedasticity (differing error variances from region to region) and/or cross-sectional correlation across regions.

To estimate models allowing for cross-sectional correlation we needed to have balanced panels—i.e., no missing observations for any of the regions for any of the years in the sample. Thus, we dropped those years before 1980 and after 1997, leaving 17 observations per region, or 119 observations across seven regions. We also dropped the observations for Central and Eastern Europe (because of the issues identified above), leaving a total of 102 observations across six regions. Table 4 defines the variables.

In Table 5 we present results for three specifications using the pooled data. The first pair of columns gives the initial model, which is simply the original regional model, with the addition of indicator variables for each region interacted with

Variable	Description				
R <sub>i</sub>	Indicator variable for Region <i>i</i>				
$y_{j,t}$	Satellite capacity (36-MHz transponder equivalents), logged for Region <i>j</i>				
x <sub>j,t</sub>	GDP (millions of 1996 dollars), logged, for Region <i>j</i>				
$\Delta y_{j,t} = y_{j,t} - y_{j,t-1}$	Growth in satellite capacity (36-MHz transponder equivalents) for Region <i>j</i>				
$y_{j,t-1} - x_{j,t-1}$	Lagged error-correction term for Region <i>j</i>				
$R_i(y_{j,t-1} - x_{j,t-1})$	Lagged error-correction term for Region <i>j</i> interacted with Region <i>i</i> indicator				
$\Delta x_{j,t} = x_{j,t} - x_{j,t-1}$	Growth in GDP for Region <i>j</i>				
$R_i \Delta x_{j,t}$	Growth in GDP for Region $j$ interacted with Region $i$ indicator				
$x_{j,t-1}$	Lagged GDP (millions of 1996 dollars), logged, for Region $j$				
$R_i x_{j,t-1}$	Lagged GDP (millions of 1996 dollars), logged, for Region <i>j</i> interacted with Region <i>i</i> indicator				

Table 4
Variable Definitions for the Pooled Models

#### Table 5

#### **Coefficient Estimates for the Pooled Models**

	Specification 1		Specification 2		Specification 3	
$\Delta y_{j,t}$	Coefficient Std. Err.		Coefficient Std. Err.		Coefficient Std. Er	
$\overline{y_{j,t-1} - x_{j,t-1}}$	-0.26**	(0.05)	-0.25**	(0.02)	-0.25**	(0.02)
$R_2(y_{j,t-1}-x_{j,t-1})$	-0.55**	(0.15)	-0.56**	(0.14)	-0.51**	(0.15)
$R_3(y_{j,t-1} - x_{j,t-1})$	-0.11	(0.10)				
$R_5(y_{j,t-1}-x_{j,t-1})$	0.01	(0.06)				
$R_6(y_{j,t-1}-x_{j,t-1})$	-0.28*	(0.14)	-0.18	(0.10)		
$R_7(y_{j,t-1} - x_{j,t-1})$	-0.71**	(0.17)	-0.70**	(0.17)	-0.65**	(0.18)
$\Delta x_{j,t}$	-0.06	(0.74)	1.30**	(0.38)	1.39**	(0.24)
$R_2 \Delta x_{j,t}$	4.12**	(0.98)	2.74**	(0.84)	2.55**	(0.82)
$R_3 \Delta x_{j,t}$	1.71	(0.99)				
$R_5 \Delta x_{j,t}$	2.68**	(0.83)	1.33**	(0.51)	0.98*	(0.44)
$R_6 \Delta x_{j,t}$	2.05	(1.08)				
$R_7 \Delta x_{j,t}$	2.16*	(0.86)	0.66	(0.58)		
<i>x</i> <sub><i>j</i>,<i>t</i>-1</sub>	-0.07	(0.18)	-0.03**	(0.01)	-0.04**	(0.01)
$R_2 x_{j,t-1}$	2.35**	(0.49)	2.24**	(0.50)	2.05**	(0.51)
$R_3 x_{j,t-1}$	2.29**	(0.58)	1.48**	(0.20)	1.45**	(0.17)
$R_5 x_{j,t-1}$	0.37	(0.28)				
$R_6 x_{j,t-1}$	0.75*	(0.32)	0.45	(0.25)		
$R_7 x_{j,t-1}$	1.85**	(0.35)	1.77**	(0.31)	1.65**	(0.33)
Constant	-1.15	(3.13)	-1.65**	(0.28)	-1.54**	(0.26)
<i>R</i> <sub>2</sub>	-38.43**	(8.24)	-36.67**	(8.33)	-33.65**	(8.43)
<i>R</i> <sub>3</sub>	-37.23**	(9.94)	-23.43**	(3.08)	-22.93**	(2.63)
$R_5$	-5.04	(4.40)				
<i>R</i> <sub>6</sub>	-11.94*	(5.20)	-7.13	(3.89)		
<i>R</i> <sub>7</sub>	-36.00**	(7.03)	-34.51**	(6.50)	-32.15**	(6.79)
Observations	102		102		102	
Wald $X^2$	300.54		265.62		261.63	

NOTE: The dependent variable is  $\Delta y_{j,t}$ . Region 1, North America, is the omitted group. Region 5, Central and Eastern Europe, is excluded from the sample. Standard errors are given in parentheses; "\*" indicates coefficient is significant at the 0.05 level, "\*\*" at the 0.01 level. Coefficients that are not significant are grayed out. each explanatory variable and the constant term.<sup>11</sup> The remaining two pairs of columns of Table 5 show models derived from the first model, where we have dropped variables from the following pair of columns when the coefficients were not significant in the preceding pair at the 0.05 or 0.01 level.<sup>12</sup>

### **Interpreting the Pooled Model**

The principal findings from the pooled models bear a strong resemblance to the findings from the individual regional models. This is true in spite of the pooled models covering a slightly smaller time span and in spite of the different form of the model. However, in addition to the insights we gained from the individual models, we also gain new insights into the similarities and differences among regions. Although we presented three alternative specifications in Table 5, for brevity we will confine our attention to the third specification in our analysis and comparison with earlier results.

The short-term response to changes in GDP, given either directly by the coefficient on  $\Delta x_{j,t}$  or by the sum of the coefficient on  $\Delta x_{j,t}$  and the coefficient on  $R_i \Delta x_{j,t}$ , is positive and significant in all cases. This stands in contrast to the individual models, where the corresponding coefficients were positive and significant in only three of the seven regions. The coefficients for Region 2, Latin America, and Region 5, Middle East and Africa, are significantly higher than the rest.

The adjustment to long-term equilibrium, as given either directly by the coefficient on  $(y_{j,t-1} - x_{j,t-1})$  or by the sum of the coefficient on  $(y_{j,t-1} - x_{j,t-1})$  and the coefficient on  $R_i(y_{j,t-1} - x_{j,t-1})$ , is negative and significant in all regions, as it was in the individual models. The coefficients for Region 2, Latin America, and Region 7, Asia Pacific, are significantly more negative than the rest, with the magnitude of the Asia Pacific coefficient again being close to one, indicating that

$$\begin{split} \Delta y_{j,t} &= \alpha + \eta(y_{j,t-1} - x_{j,t-1}) + \beta \Delta x_{j,t} + \varsigma x_{j,t-1} \\ &+ \sum_{i \neq 1} \alpha_i R_i + \sum_{i \neq 1} \eta_i R_i(y_{j,t-1} - x_{j,t-1}) + \sum_{i \neq 1} \beta_i R_i \Delta x_{j,t} + \sum_{i \neq 1} \varsigma_i R_i x_{j,t-1} + \varepsilon_{j,t} \end{split}$$

where the variables are as defined in Table 4.

 $<sup>^{11}</sup>$ Specifically, the model is

<sup>&</sup>lt;sup>12</sup>The estimates were generated using feasible generalized least squares (FGLS), allowing for heteroskedasticity and contemporaneous correlation across regions. Several alternative specifications were tried, allowing for heteroskedasticity and/or cross-sectional correlation and/or AR(1) autocorrelation within and/or across the regions. None of the alternative specifications produced results qualitatively different from the results reported, and quantitatively the coefficient estimates typically differed by at most 10 percent.

any deviation from long-run equilibrium is almost completely made up within the next year.

The coefficient on lagged GDP, given either by the coefficient on  $x_{j,t-1}$  or the sum of the coefficients on  $x_{j,t-1}$  and  $R_i x_{j,t-1}$ , shows that  $\theta$  is significantly different from one in all cases, although the difference from one is estimated to be only 0.16 for Regions 1, 5, and 6, as indicated in Table 6. Once again, the value of  $\theta$  estimated for the Western European region seems extraordinarily high. Qualitatively, the pooled model seems to be largely in concordance with the regional models.<sup>13</sup>

	Specification 1	Specification 2	Specification 3
Region 1: North America	0.74	0.87**	0.84**
Region 2: Latin America	3.80**	3.73**	3.67**
Region 3: Western Europe	6.90**	6.74**	6.65**
Region 5: Middle East and Africa	2.21	0.87**	0.84**
Region 6: South Asia	2.24*	1.95**	0.84**
Region 7: Asia Pacific	2.83**	2.81**	2.78**

Table 6 Thetas Implied by the Pooled Models

NOTE: "\*" indicates the coefficient on  $x_{j,t-1}$  or  $(x_{j,t-1} + R_i x_{j,t-1})$  is significantly different from zero (and  $\theta$  is significantly different from one) at the 0.05 level, "\*\*" at the 0.01 level.

<sup>&</sup>lt;sup>13</sup>In addition to the pooling model presented here, we also tested two alternative specifications in an effort to see how sensitive our results were to certain assumptions. The first specification split the composite GDP variable into its component parts: per-capita GDP and population. We found that the significance of the error-correction term was unaffected, despite the fact that in many regions either the per-capita GDP variable or the population variable did not enter the cointegrating relationship (putting us at risk of identifying a spurious relationship as being significant). We also tested whether the relationship we were observing resulted from improved communications infrastructure causing economic growth, rather than the reverse relationship presupposed by our model specification. We tested this by running the model "backwards"; that is, by making GDP growth the dependent variable and the lagged difference in GDP and satellite capacity levels, the growth in satellite capacity, and the lagged level of satellite capacity the independent variables. Our hypothesis was that the fit of this model would be markedly inferior to that of the "forwards" model, which indeed it was.

## 4. Conclusion and Policy Implications

The above models show a strong link between regional GDP and the growth in supply of satellite communications capacity. Economic theory would predict that investment in capacity would be responsive to demand conditions; however, it does not explain how rapidly investors respond to demand conditions. We find that adjustment to change is quite rapid; if there is an imbalance between supply and demand, we estimate that on average 25 percent of the adjustment to the long-run equilibrium supply is made within one year, although there is some regional variation. The analysis indicates that in many regions the market can adjust rapidly to a surge in demand, and that there may be little need for the Department of Defense to buy satellite capacity in advance simply to ensure that capacity will be there when needed.

While the responsiveness of capacity to fluctuations in demand may make predicting future worldwide capacity difficult, it also means that the market can quickly adjust to changing demand. Even if worldwide capacity were to stay at current levels, the projected DoD demand circa 2010 would account for only a fraction of overall capacity; current global commercial capacity on C- and Kuband is over 175 Gbps compared to a projected total DoD demand of 16 Gbps by 2010 (Bonds et al., 2000). Thus, for most regions of the world, DoD surge demands for communications satellite capacity should be able to be largely met by a responsive commercial market.

This situation should only get better as time goes on. Several markets trading in satellite communications services have recently opened, and price data are increasingly becoming available. As the market matures, additional features of a fully developed commodity market will materialize, such as futures markets. With a new wealth of information becoming available on prices for current capacity and options on futures capacity, the Department of Defense may be able exploit techniques that have been used with other markets, such as real options analysis to evaluate when and where to invest in owned satellite capacity. The Department of Defense may also be able to take advantage of futures contracts to reduce risk. A maturing market in satellite communications can provide the Department of Defense with many new tools to manage price uncertainty. For the Department of Defense, a responsive market may be just as good as a crystal ball.

## Appendix Regional Fit

This appendix takes a graphical look at the fit of the individual regional models.

### **Region 1: North America**

Perhaps it is easiest to start by examining how the model performs in predicting the level of satellite capacity rather than the growth in satellite capacity. Figure A.1 shows the cumulative growth in satellite capacity since 1980. The solid line shows the observed growth in satellite capacity, whereas the dashed line shows the growth implied by the model estimates. The fit, while quite good, is not spectacular, in part because the model is in terms of logarithms, so the fit appears to be closer for the earlier years when there were fewer satellites. Also, since the model works in terms of year-to-year growth in satellite capacity, the individual predicted values have to be chained to give the cumulative growth over time.

Figure A.2 depicts the year-to-year growth in satellite capacity for North America. The raw model output, which is in terms of the difference of



Figure A.1—Observed Versus Fitted Cumulative Growth in Satellite Capacity in North America



Figure A.2—Observed Versus Fitted Growth in North America, 1980–1999

logarithms, has been translated into percentage growth for this figure. The solid line is the observed growth in satellite capacity, and the dashed line is the fitted line. The error bars indicate as plus and minus two times the standard error of prediction. While the fit is respectable, the precision of the fitted line may be a bit overstated, because while the upper and lower error bars nominally give a 95 percent confidence interval, the observed values are within the error bars for only 13 of the 18 observations, or 72 percent of the time. As it stands, though, the model replicates the major feature of the data; that is, the decline in the rate of growth in satellite capacity over the last two decades. Although satellite capacity is still growing, it is not growing at anywhere near the rate it was in the early 1980s.

### **Region 2: Latin America**

Growth in satellite capacity has varied considerably in Latin America over the past two decades, from a high of nearly 133 percent in 1985 to a low of -21 percent in 1990. See Figure A.3. Given how the growth rate bounces around, it is impressive that the model fits with an R<sup>2</sup> of over 0.52. The model succeeds in replicating some of the major features of the data, including the negative growth in 1990, although the model does not replicate the overall variance of the observations ("variance shrinkage").



Figure A.3—Observed Versus Fitted Growth in Latin America, 1980–1997

### **Region 3: Western Europe**

Western Europe has also shown considerable variance in growth from year to year over the past two decades, although nowhere near the variance of Latin America. See Figure A.4. As in the model of North America, the standard error of prediction as indicated by the error bars seems to overstate the precision of the estimates, as 5 of the 18 data points fall outside the error bars. Although the model replicates some of the variance of the data, it fails to replicate either of the two dips into negative growth.

### **Region 4: Central and Eastern Europe**

The case of Central and Eastern Europe was discussed in some detail in Section 3. Figure A.5 is a graphical depiction of the model using the Western European figures for GDP. The fit seems to be fairly good (as it should be, with an adjusted  $R^2$  of 0.61), but once again the error bars seem to overstate the precision of the estimate (the observed growth rates bounce around at the fringes of the error bars). Nevertheless, the overall performance of the model seems good, even when the observed growth rate dips into negative territory in 1997.





Figure A.4—Observed Versus Fitted Growth in Western Europe, 1980–1997



Figure A.5—Observed Versus Fitted Growth in Central and Eastern Europe, 1980–1997

### **Region 5: Middle East and Africa**

The fit of the model for this region seems reasonable (see Figure A.6), but, again, the precision of the estimates as indicated by the error bars seems a bit exaggerated (a number of observations fall slightly outside the error bars).



Figure A.6—Observed Versus Fitted Growth in the Middle East and Africa, 1980-1997

### **Region 6: South Asia**

The model for South Asia has the lowest R<sup>2</sup> of all the models (0.17), which is reflected in Figure A.7. The observed line bounces around the fitted line and the fitted line fails to mirror the dips into negative growth of the observed line. However, this lack of fit by no means invalidates conclusions based on examining the estimated coefficients; it merely means that the explanatory variables "explain" only a fraction of the variance in this region. As in several of the previous models, the standard error of prediction seems to overstate the precision of the estimates (a number of observed points fall outside of the error bars).



Figure A.7—Observed Versus Fitted Growth in South Asia, 1980–1997

## **Region 7: Asia Pacific**

In contrast to the model for South Asia, the fit of model for the Asia Pacific region is quite good. See Figure A.8. The model replicates the major features of the data, and nearly all the observed points fall inside the error bars.



Figure A.8—Observed Versus Fitted Growth in the Asia Pacific Region, 1980–1997

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