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VARIANCE REDUCTION OF PREDICTION ERROR USING FRACTIONAL DIGITAL DIFFERENTIATION : APPLICATION TO ECG SIGNAL PROCESSING

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Abstract—**This paper presents a prediction error variance reduction procedure based on fractional digital differentiation with negative order. This reduction is achieved by increasing correlation in the signals. Applications to ECG signals show that savings of more than one bit per residual signal sample can be attained.**

Keywords—Linear prediction, fractional differentiation, ECG signal.

I. INTRODUCTION

The fractional derivative generalizes the familiar derivative $D^n x(t) = \dfrac{d^n(t)}{dt^n}$ of a function x(t) with respect to the variable t, to noninteger values of n. Fractional differentiation is of use in Mathematics [1] as well as in Engineering [2]-[4].The formula used in this paper to compute the fractional digital derivative is, with the sampling period omitted, the same as the fractional differencing introduced by Hosking [5], and Granger and Joyeux [6] to difference time series to noninteger order. The fractional differencing (or fractional digital derivative with n < 0) of a zero-mean white noise gives a signal whose power spectral density is 1/f at low frequencies and an normalized autocorrelation function decaying hyperbolically with the lag instead of an exponential decaying as is the case of ARMA models. These features indicates a significant statistical dependence between distant signal samples. On the other hand, the original signal can be reconstructed from its derivative by the same algorithm with simple inversion of sign of the differentiation order. In this paper, fractional digital differentiation with negative order is used to increase the correlation in signals in order to reduce the error prediction variance. The paper is organized as follows. In Section II, fractional digital differentiation is presented. In Section III, linear prediction is reviewed. In Section IV, the discrete-time system realizing further variance reduction by fractional differentiation is described. In Section V, applications to ECG signal are intended to be illustrative. Finally, concluding remarks are presented in Section VI.

II. FRACTIONAL DIGITAL DIFFERENTIATION

There are several approaches to define the fractional derivative. We utilize the definition used by Oustaloup [2]. This definition is based on the generalization of the derivative of order n

$$D^n x(t) = \frac{(1-q^{-1})^n}{T_s^{\,n}} x(kT_s) \tag{1}$$

to noninteger values of n. T_s is the sampling period ($t = kT_s$) and q^{-1} is the backward shift operator ($q^{-1}x(kT_s) = x((k-1)T_s)$)). Using the series expansion of $(1-q^{-1})^n$ we get the following expression for $D^n x(t)$

$$D^n x(t) = \frac{1}{T_s^{\,n}} \sum_{i=0}^{\infty} a_i x(t - iT_s) \tag{2}$$

where the coefficients a_i are given by the following recurrent relation

$$a_i = \frac{i-n-1}{i} a_{i-1} \quad i = 1,2,3,\ldots \qquad a_0 = 1 \tag{3}$$

We note in the formula (2) the presence of all past signal samples. For a causal signal, i.e. x(k) = 0 for k < 0, and fixed T_s, the relation (2) gives a sampled signal

$$y(k) = \sum_{i=0}^{k} a_i x(k-i) \tag{4}$$

(the constant T_s is omitted) approximating the derivative of noninteger n. The smaller the sampling period T_s is, the better the approximation is. The relation (4) may be implemented taking advantage of recurrent property (3) which avoids explicit computation of coefficients a_i and their storage in memory.

In the relation (4), signal y(k) can be considered as the output of the fractional digital filter whose transfer function is

$$H(Z) = \frac{Y(Z)}{X(Z)} = (1 - Z^{-1})^n \tag{5}$$

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III. VARIANCE REDUCTION OBTAINED BY LINEAR PREDICTION

The linear prediction of order P uses the relation

$$\hat{x}(k) = \sum_{i=1}^{P} c_i x(k-i) \tag{6}$$

for predicting the sample x(k) by a linear combination of the P previous samples. The constants c_i, $1 \le i \le P$, are the coefficients of prediction. The difference

$$e(k) = x(k) - \hat{x}(k) \tag{7}$$

called prediction error or residual signal, is the signal to be quantized for the purpose of signal compression. If the original signal x(k) is correlated enough and the coefficients c_i are correctly chosen, the variance of error e(k) is smaller than that of the original signal. The coefficients c_i are chosen such that the mean squared error $\sigma_e^2 = E\left\{(x-\hat{x})^2\right\}$ is minimum. The minimal value of the variance residual signal σ_e^2 is given by the following relation [7]

$$\sigma_{e,\min}^2 = \sigma_x^2 \left[1 - \sum_{i=1}^{P} c_i \rho_x(i)\right] \tag{8}$$

where σ_x^2 is the variance of the signal x(k) and $\rho_x(i)$ its autocorrelation coefficients. The ratio of the variances

$$G_P = \frac{\sigma_x^2}{\sigma_{e,\min}^2} = \left[1 - \sum_{i=1}^{P} c_i \rho_x(i)\right]^{-1} \tag{9}$$

or the prediction gain, expresses the achieved variance reduction by linear prediction of order P. By choosing the amplitude range of the quantizer some multiple factor of the standard deviation of the residual signal, the number of quantized levels is divided by $\sqrt{G_P}$ if the quantization step is maintained constant. This results in reduction of the number of bits per sample (1 bit reduction per 6 dB variance reduction). Linear prediction has been used by Ruttimann and Pipberger [8] for ECG compression. They showed that the variance reduction could not be substantially improved using P > 2 since the correlation between adjacent samples of the ECG signal is low.

IV. FURTHER VARIANCE REDUCTION BY FRACTIONAL DIGITAL DIFFERENTIATION

We used fractional digital differentiation with negative order which increases correlation in the signal to reduce the variance residual signal obtained by linear prediction. Fig.1 shows block diagram of the system used to enhance variance reduction. The output signal y(k) is divided by its standard deviation σ_y and then multiplied by the standard deviation σ_x of the input signal x(k) in order to have the same variance σ_x^2 for both signals $u(k)$ and x(k). In this case, the ratio of minimum values of the residual signals variances is given by

$$\frac{\sigma_{e,\min}^2}{\sigma_{r,\min}^2} = \frac{1 - \sum_{i=1}^{P} c_i \rho_x(i)}{1 - \sum_{i=1}^{P} d_i \rho_u(i)} \tag{10}$$

Autocorrelation coefficients $\rho_x(i)$ and $\rho_u(i)$ of x(k) and u(k) verify the inequality $\rho_x(i) < \rho_u(i)$ for $1 \le i \le P$.

V. ILLUSTRATIVE APPLICATIONS

Fig. 2 shows an ECG segment of 10 s duration taken from MIT/BIH arrhythmia database and the results obtained using a fractional digital differentiator with order n = -0.1, -0.3 and -0.45. Prediction order P = 2 and sampling frequency is 360 Hz. It can be seen the lower the order n is, the higher the dynamic range of residual signals decreases. In all cases, the error is more important at QRS complexes because of their rapid variations. The estimated prediction gain (9) is 17.41, 18.57, 21.40, et 24.45 dB respectively for n = 0, -0.1, -0.3 et -0.45. With n = -0.45, the savings in bits per sample is greater than one. Using the same ECG with sampling frequency of 500 Hz, derived by par interpolation and decimation of the original signal [9], the prediction gain estimated is 22.20, 23.53, 26.62, and 29.81 dB. The prediction gain estimated increases but the savings in bits per sample do not. Other experiments on ECG with various morphologies enable saving up to two bits. Although higher values of P can be considered since the signal correlation is increased, we have only used P = 2 ; the selection of this value being based on the pioneer work [8].

VI. DISCUSSION

In this paper we proposed a procedure based on fractional digital differentiation with negative order for the reduction of prediction error variance. After mean removal, the signal is applied to the input of a fractional digital differentiation in order to increase the correlation in the signal. The differentiated signal is reduced to the same variance as that of the original signal. From applications to ECG signals it is shown that savings of more than one bit per residual signal sample can obtained. Works presently in course aim to use this procedure for ECG compression by exploiting properties of strong correlation in fractionally differentiated signals to reduce the residual signal variance and invertibility of the fractional differentiation to reconstruct the original signal. Besides, this system only includes one parameter, the order of differentiation. And this is a solid argument in favor of the

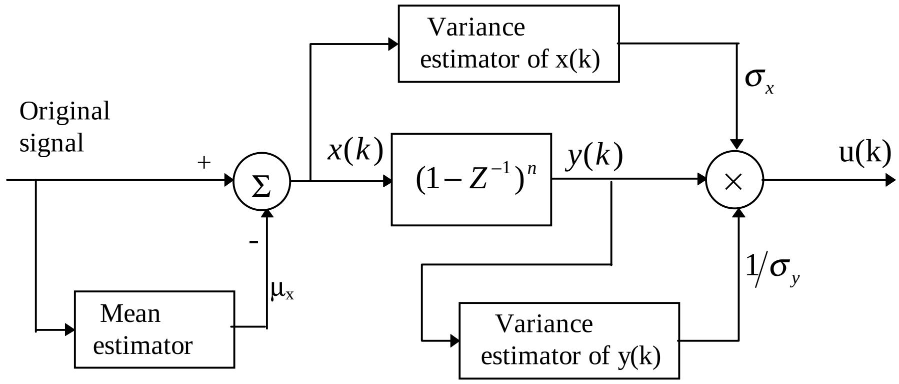

Fig. 1. Block diagram of the system used to reduce the variance of prediction error using fractional digital differentiation.

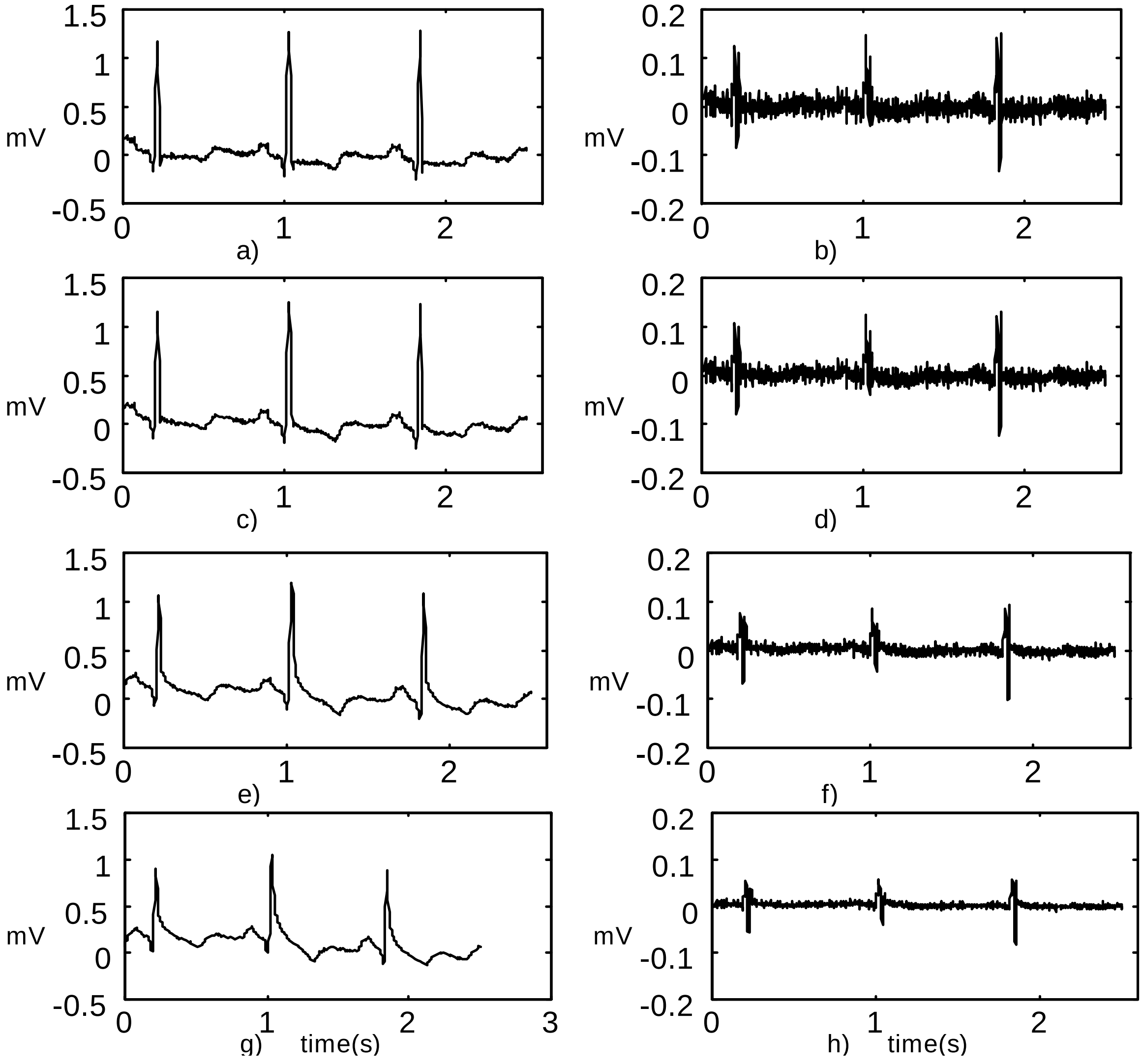

Fig. 2. a) original ECG and b) corresponding residual signal. c),e) and g) differentiated signals with n = -0.1, -0.3 and -0.45, d),f),h) corresponding residual signals.

use of this discrete-time system for data compression. Although values of n close to -0.5 assure a better reduction of the residual signal variance, they introduce, however, more distortions in the reconstructed signal that its values near zero since the amplification of the quantization noise by the noninteger derivative with a positive order is much more important when the order of differentiation is high. An important part of the amplified quantization noise may be reduced by a low pass filtering. Assimilating the output signal of fractional differentiator with positive order to a white noise, an ideal low pass filter of cut off frequency fc permits to divide its variance by $F_s/2fc$ where F_s is the sampling frequency. The hypothesis of whiteness of the amplified quantization noise is better verified for lower values of n.

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