NEW INSIGHTS INTO THE RELATIONSHIP BETWEEN POINCARÉ PLOT GEOMETRY AND LINEAR MEASURES OF HEART RATE VARIABILITY

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Abstract- **The Poincaré plot is an emerging Heart Rate Variability (HRV) analysis technique, the geometry of which has been shown to distinguish between healthy and unhealthy subjects in clinical settings. The Poincaré plot is able to display nonlinear aspects of the interval sequence and is therefore of interest in characterizing the nonlinear aspects of HRV. The problem is, how do we quantitatively characterize the geometry of the plot to capture useful descriptors that are independent of existing HRV measures? In this paper, we investigate a popular existing category of techniques and show that they measure linear aspects of the intervals which existing HRV indices already specify. The fact that these methods appear insensitive to the nonlinear characteristics of the intervals is an important finding because the Poincaré plot is primarily a nonlinear technique.**
***Keywords*: HRV, Poincaré plot, nonlinear analysis**

I. INTRODUCTION

The field of heart rate variability (HRV) studies the fluctuations in the intervals between heartbeats, known as RR intervals. The Poincaré plot, a technique taken from nonlinear dynamics, portrays the nature of these fluctuations graphically. It is a scatter-plot of each RR interval plotted against the next interval. Poincaré plot analysis is an emerging quantitative-visual technique whereby the shape of the plot is categorized into functional classes that indicate the degree of heart failure in a subject [1, 2]. The plot provides summary information as well as detailed beat-to-beat information on the behavior of the heart [3].

Support is increasing for nonlinear analysis techniques and quantitative descriptors as it has become evident that the cardiac systems are nonlinear in their function [4]. The Poincaré plot is becoming a popular technique due to its simple visual interpretation and its proven clinical ability as a predictor of disease and cardiac dysfunction [5]. The problem regarding Poincaré plot use has been the lack of obvious quantitative measures that characterize the salient features of the plot. Researchers have put forward a number of techniques that attempt to quantitatively summarize the plot's geometric appearance. The efforts can be summarized into 3 categories: geometrical descriptors, scanning parameters and image distribution parameters [6]. Of these, the geometrical descriptors are the most popular in the clinical and physiological HRV literature.

In this study, we consider the geometrical Poincaré plot descriptors. We provide expressions that connect each descriptor to existing linear measures of HRV. This accomplishes two things. Firstly, it provides insight into Poincaré plot geometry in terms of the well-understood existing indices of HRV. Secondly, it shows that these measures are not independent to the existing standard linear statistics. Therefore, the intrinsic ability of the Poincaré plot to identify nonlinear beat-to-beat structure is not being exploited by these techniques.

II. THE LINEAR HRV INDICES

This section describes the standard linear indices of HRV. In this paper, the time-course of the RR intervals is denoted by RR_n, with $n = 1..N$. We assume wide-sense stationarity so the following basic properties hold: $E[RR_n] = E[RR_{n+m}]$ and $E[RR_n^2] = E[RR_{n+m}^2]$.

A. Standard deviation of the RR intervals

The standard deviation of the RR intervals, denoted by $SDRR$, is often employed as a measure of overall HRV. It is defined as the square root of the variance of the RR intervals,

$$SDRR = \sqrt{E\left[RR_n^2\right] - \overline{RR}^2} \tag{1}$$

where the mean RR interval is denoted by $\overline{RR} = E\left[RR_n\right]$.

B. Standard deviation of the successive differences

The standard deviation of the successive differences of the RR intervals, denoted by $SDSD$, is an important measure of short-term HRV. It is defined as the square root of the variance of the sequence $\Delta RR_n = RR_n - RR_{n+1}$ (the delta-RR intervals),

$$SDSD = \sqrt{E\left[\Delta RR_n^2\right] - \overline{\Delta RR_n}^2}\ . \tag{2}$$

Note that $\overline{\Delta RR_n} = E[RR_n] - E[RR_{n+1}] = 0$ for stationary intervals. This means that $SDSD$ is equivalent to the root-mean-square of the successive differences, denoted $RMSSD$.

C. Autocorrelation and autocovariance

The autocorrelation function is an important measure of HRV simply because its Fourier transform is the power spectrum of intervals. The autocorrelation function of the RR intervals is defined as,

$$\gamma_{RR}(m) = E\left[RR_n RR_{n+m}\right]. \tag{3}$$

Spectral analysis is normally performed on the mean-removed RR intervals, and therefore the mean-removed autocorrelation function, called the autocovariance function, is often preferred:

$$\phi_{RR}(m) = E[(RR_n - \overline{RR})(RR_{n+m} - \overline{RR})]\ . \tag{4}$$

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Abstract

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The autocovariance function is related to the autocorrelation function for stationary intervals by the relationship $\phi_{RR}(m) = \gamma_{RR}(m) - \overline{RR}^2$. The variance of the RR intervals is $SDRR^2 = \phi_{RR}(0)$, and the variance of the delta-RR intervals is $SDSD^2 = 2\left(\phi_{RR}(0) - \phi_{RR}(1)\right)$. Accordingly, these indices are linear measures of HRV.

III. The Geometrical Poincaré plot descriptors

The RR interval Poincaré plot typically appears as an elongated cloud of points oriented along the line-of-identity (see Fig. 1). The dispersion of points perpendicular to the line-of-identity reflects the level of short-term variability [5]. The dispersion of points along the line-of-identity is thought to indicate the level of long-term variability. The geometric descriptors use statistical moments, usually directed along the line of identity, to measure the shape of the plot. In this section, we relate several geometrical techniques that are popular in the literature to linear measures of HRV.

A. Ellipse fitting technique

By far the most popular technique used to characterize the shape of the plot numerically is the technique of fitting an ellipse to the plot [6-9], as Fig. 1 details. A set of axes oriented with the line-of-identity is defined [9]. The axes of the Poincaré plot are related to the new set of axes by a rotation of $\theta = \pi/4$ radians:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} RR_n \\ RR_{n+1} \end{bmatrix} \tag{5}$$

In the reference system of the new axes, the dispersion of the points around the x_1 axis is measured by the standard deviation denoted by $SD1$ [9]. This quantity measures the width of the Poincaré cloud, and therefore indicates the level of short-term HRV [2, 3, 5, 9]. The length of the cloud along the line-of-identity measures the long-term HRV and is measured by $SD2$ which is the standard deviation around the x_2 axis [2, 3, 5, 9]. These measures are related to the standard HRV measures in the following manner:

$$\begin{aligned} SD1^2 &= Var\left(x_1\right) = Var\left(\tfrac{1}{\sqrt{2}} RR_n - \tfrac{1}{\sqrt{2}} RR_{n+1}\right) \\ &= \tfrac{1}{2} Var\left(RR_n - RR_{n+1}\right) = \tfrac{1}{2} SDSD^2 \end{aligned} \tag{6}$$

Thus, the $SD1$ measure of Poincaré width is equivalent to the standard deviation of the successive intervals, except that it is scaled by $1/\sqrt{2}$. This means that we can relate $SD1$ to the autocovariance function:

$$SD1^2 = \phi_{RR}(0) - \phi_{RR}(1)\,. \tag{7}$$

It may also be shown that the length of the Poincaré cloud is related to the autocovariance function:

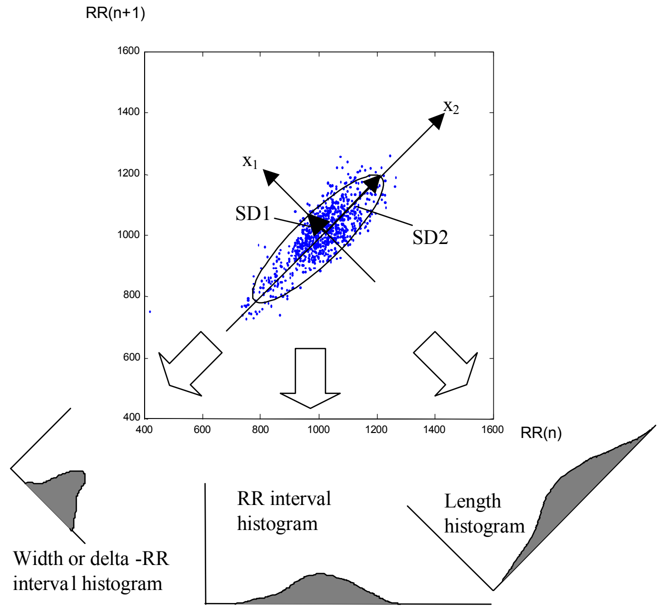

Fig. 1: An example Poincaré plot, also detailing the ellipse fitting process and the histograms derived from the plot.

$$SD2^2 = \phi_{RR}(0) + \phi_{RR}(1)\,. \tag{8}$$

By adding equations (7) and (8) together, we obtain the result

$$SD1^2 + SD2^2 = 2\phi_{RR}(0) = 2SDRR^2\,. \tag{9}$$

Finally,

$$SD2^2 = 2SDRR^2 - \tfrac{1}{2} SDSD^2\,. \tag{10}$$

Equation (10) allows us to interpret $SD2$ in terms of existing indices of HRV and can also be used to argue that $SD2$ reflects the long-term HRV, however, we delay the discussion until later. Clearly, fitting an ellipse to the Poincaré plot does not generate indices that are independent of the standard time domain HRV indices. In fact, the width of the Poincaré plot is a linear scaling of the most common statistic used to measure short-term HRV, the $SDSD$ index. In other words, the width of the Poincaré plot should correlate extremely highly with other measures of short-term HRV, as indeed it does [3].

B. Histogram techniques

Another method to quantify the shape of the Poincaré plot is to measure the statistical properties of various projections of the plot via histogram distributions [2, 3, 5]. Fig. 1 shows the three main projections used. They are:

RR interval histogram. The histogram of the Poincaré plot points projected onto the x-axis (or the y-axis). This histogram is usually quantified by the mean and standard deviation, which correspond directly to the standard linear measures $\overline{RR}$ and $SDRR$. This view provides summary information on the overall HRV characteristics.

'Width', or delta-RR interval histogram. The Poincaré plot points are projected along the direction of the line-of-identity. It is not exactly equivalent to the delta-RR interval histogram as the abscissa has been scaled by the factor $1/\sqrt{2}$. Mathematically, it is the distribution of x_1. Therefore, the standard deviation of the width histogram is equal to $SD1$. This histogram provides summary information on the short-term characteristics.

'Length' histogram. The Poincaré plot points are projected onto the line-of-identity. The histogram is described mathematically by the distribution of x_2 and the standard deviation is therefore equivalent to $SD2$. Consequently, the length histogram portrays the long-term characteristics of HRV. The dispersion properties of these histograms are characterized by $SDRR$, $SD1$ and $SD2$. Hence, they are linked to the standard linear time-domain measures of HRV

C. Correlation coefficient

Some researchers have employed the correlation coefficient of the Poincaré plot to characterize its shape [10]. This measure is:

$$r_{RR} = \frac{E\left[(RR_n - \overline{RR})(RR_{n+1} - \overline{RR})\right]}{\sqrt{E\left[(RR_n - \overline{RR})^2\right]E\left[(RR_{n+1} - \overline{RR})^2\right]}}. \quad (11)$$

For the Poincaré plot, the correlation coefficient can be expressed in terms of the autocovariance function:

$$r_{RR} = \phi_{RR}(1)/\phi_{RR}(0). \quad (12)$$

Therefore, the correlation coefficient is a linear measure, even though it is based on the Poincaré plot which displays nonlinear features. None of the geometric descriptors that have been analyzed in this section are sensitive to the nonlinear features the Poincaré plot displays.

D. Short- and long-term variability

The relationships identified so far give clear mathematical insight into what the length and width of the Poincaré plot depict. The length and width of the Poincaré plot have been suggested as indicative of the levels of long- and short-term variability. It is reasonably clear that the standard deviation of the delta-RR intervals, as measured by $SDSD$, $RMSSD$ or $SD1$, is a measure of short-term HRV. In fact, this statement can be made even more precise: these indices are measures of the variability over a single beat. The standard deviation of the RR intervals, as measured by $SDRR$, is often employed as a measure of long-term HRV. However, there is a problem with this interpretation because this quantity measures *all* the variability, *long-term and short-term*.

For example, take a set of RR intervals that has variability only over a single beat. Such a sequence can be described as alternating between two values, e.g., a, b, a, b, etc. It is clear that the sequence contains variability only over a single beat: every second beat is equivalent. If $SDRR$ is taken to be an index of long-term variability, a contradiction arises because

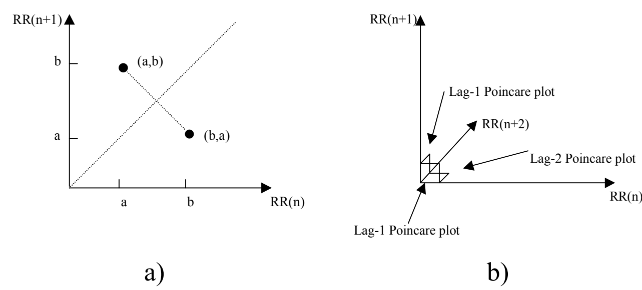

Fig. 2: a) Poincaré plot of alternating sequence. b) Diagram of a third-order Poincaré plot.

$SDRR$ is not zero. We propose that Poincaré plot length is a more consistent and appealing measure of long-term variability as the length of the Poincaré plot is zero for an alternating sequence indicating zero long-term variability, as Fig. 2a shows.

With these ideas in hindsight, we can now explain the significance of equations (9) and (10). Equation (9) states that the sum of the short-term and the long-term variability is the total variability. Equation (10) affirms that the long-term variability is the total variability minus the contribution due to short-term variability.

IV. Generalizations of the Poincaré plot

Two different types of scatter-plots encountered in the literature can be considered simple generalizations of the Poincaré plot.

A. Lagged Poincaré plots

Instead of plotting RR_n against RR_{n+1}, some researchers have investigated plotting RR_n against RR_{n+m} where m is allowed to vary from 1 to some small positive value. In general, the plot is still clustered around the line-of-identity. However, the length and width of the plot are altered as the lag is increased. It is possible to show the width and length measures $SD1$ and $SD2$ can be generalized for lag m:

$$\begin{aligned} SD1(m)^2 &= \phi_{RR}(0) - \phi_{RR}(m) \\ SD2(m)^2 &= \phi_{RR}(0) + \phi_{RR}(m) \end{aligned} \quad (13)$$

The length and width of the lag-m Poincaré plot is related to the covariance function at lag m. Note also that:

$$\phi_{RR}(m) = \tfrac{1}{2}\left(SD2(m)^2 - SD1(m)^2\right) \quad (14)$$

This result is very interesting, as it states that the set of lagged Poincaré plot length and widths are a complete description of the autocovariance function, and hence, also the power spectrum of the intervals. Equation (14) also provides us with a geometrical relationship between the autocovariance function and the Poincaré plot's shape. If $\phi_{RR}(m) = 0$ then $SD1 = SD2$ and the length and width of the plot are equal. If $\phi_{RR}(m) > 0$ then $SD1 < SD2$ and the plot is longer than it is wide, i.e. dominated by short-term activity and vice versa for long-term variability. This is similar to the concept of a scatter plot's correlation coefficient sign. In fact the series of correlation coefficients of the lagged Poincaré plots are simply a scaled version of the autocovariance function:

$$r_{RR}(m) = \phi_{RR}(m)/\phi_{RR}(0) \tag{15}$$

B. Higher-order Poincaré plots

The standard Poincaré plot is a scatter-plot of the pairs (RR_n, RR_{n+1}), and is considered to be of first order. The second order Poincaré plot is a three-dimensional scatter-plot of the triples $(RR_n, RR_{n+1}, RR_{n+2})$. There are three orthogonal views of the three-dimensional shape of this plot, each being a view along one of the axis. These views result in two-dimensional projections of the three-dimensional cloud onto each of the coordinate planes (RR_n, RR_{n+1}), (RR_{n+1}, RR_{n+2}) and (RR_n, RR_{n+2}). The first two views are equivalent to the standard Poincaré plot and the third is the lag-2 Poincaré plot as Fig. 2b shows. This idea can be extended into higher dimensions, with the projections of the plot onto coordinate planes being lagged Poincaré plots. So, an order m Poincaré plot is geometrically described by the set of lagged Poincaré plots up to and including lag m. The results for lagged Poincaré plots carry over to higher-order Poincaré plots.

V. Conclusion

We have shown that the so-called geometric techniques that characterize the geometry of a Poincaré plot are related to linear indices of HRV. In addition, we provide arguments supporting the claim that the width of the Poincaré plot corresponds to the level of short-term HRV, while the length of the plot corresponds to the level of long-term variability.

The methods of quantifying the Poincaré plot that we have investigated herein are not capable of depicting the additional beat-to-beat variability information shown on a Poincaré plot. However, the additional information is likely to be of considerable value. The fact that that the length and width of a Poincaré plot corresponds so conveniently to standard time domain statistics is a very nice feature. However, simply treating the Poincaré plot as a tool for graphically representing these statistics is to ignore some of its potential capabilities. Therefore, we recommend nongeometric techniques, such as scanning parameters [7, 11] and image distribution measures [12], be investigated for a full appraisal of the clinical capabilities of the Poincaré plot.

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