ON THE ESTIMATION OF MUSCLE FIBER CONDUCTION VELOCITY USING A CO-LINEAR ELECTRODES ARRAY

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Abstract-**The problem of estimating the average Conduction Velocity (CV) in the muscle fibers is considered. The velocity is estimated from an array of noisy deterministic and unknown Motor Unit Action Potentials (MUAP) acquired by an array of *K* co-linear electrodes. In this array a vector of (*K*-1) independent time delays (TD) are to be estimated. For a given length of the array, the number *K* has to be determined in order to improve the estimation of the velocity. In this communication we show the influence of the value of *K* in the CV estimation in presence of noise and changes in the MUAP shape along the array.**
***Keywords* - Conduction Velocity, array of sensors, change in shape, Time Delay estimation**

I. INTRODUCTION

The problem of time delay (TD) estimation is of interest in many applications such as in Radar, Sonar and in biomedical instrumentation. Many authors tackled the problem of passive TD estimation between two spatially separated sensors in the presence of independent white Gaussian noise. The optimal method to estimate the TD τ is the Generalized Cross-Correlation method (GCC), but this requires a priori knowledge about signal and noise. In the biomedical applications, more precisely in Surface ElectroMyoGraphy (SEMG) cross-correlation method has been applied to estimate the TD and then the Conduction Velocity (CV) in the muscle fibers [1]: this method could give highly accurate estimations provided that the observed signals in the array are coherent, i.e. neither scaled in time nor different in shape, but only delayed with an amount of τ between each other. At low Signal to Noise Ratio (SNR) or in case of non ideal acquisition (e.g. the array is not strictly parallel to the muscle) this condition is not satisfied, leading to high estimation errors. In case of unknown signal, better TD estimates can be obtained if the signal is estimated before the correlation process. On the other hand, when multiple realizations *K* of a signal embedded in independent white noise sequences are available, synchronous averaging of these realizations decreases the standard deviation of the residual noise by an order of $\sqrt{K}$. Consequently, we look forward to enhance the TD estimation by applying synchronous averaging. Indeed, TD estimation by beamforming [2], which is itself a special case of the Maximum Likelihood Estimator (MLE) of TD, uses the synchronous averaging to estimate the TD between successive signals whose value is assumed to be unique in a linear array. If the TDs between successive signals are not equal, the synchronous averaging requires all the (*K*-1) independent TD estimates between the couples of signals in the array ; the problem is to estimate all these delays. Therefore, if the signal is unknown and the TDs are not equal, the performances of the TD estimators are decreased. By surveying the works done in the SEMG domain and especially the estimation of Conduction Velocity (CV) in the muscle fibers, one can conclude that array signal processing methods can be used to improve the CV estimates. The CV in the muscle fibers is an important parameter of the myoelectric signal which describes muscle fatigue manifestation during voluntary or elicited contractions. It may assess the contraction level of the muscle, the age, etc.. A review of this signal processing application, measurement techniques and clinical applications of the CV are discussed in [3]. In the literature, many studies [1] have been carried out using two electrodes to acquire the necessary signals for the CV estimation. However, the CV estimation using an array of EMG signals was tackled by Schneider et al. [4], where the estimation is carried out by the inclination of the line joining the maximums of the MUAPs. Schneider showed that there is a remarkable CV variability between different sites along the muscle fibers of a single Motor Unit (MU), that is, the interelectrode TD varies spatially. This fact is also proved from theoretical model of SEMG signals [5], therefore, an estimation of a spatial average of the CV contains all these variations. The estimated CV between two successive electrodes $\hat{v}$ can be found as $\hat{v} = l/\hat{\tau}$ where l is the spatial distance between the electrodes in meters and $\hat{\tau}$ the estimated TD between the two successive signals in seconds. The TD estimate $\hat{\tau}$ is a random variable (r.v.) with an expected value of τ. It is obvious that uncertainty on $\hat{v}$ is a decreasing function of the of τ value. Hence, it is better to estimate a velocity from large TDs rather than small ones. The question is, why use multiple electrodes with small interelectrode distances while two extreme electrodes on the muscle unit gives a lower variance? There are two reasons for this : firstly, the constant shape hypothesis is more pertinent between near signals, consequently better TD estimation using correlation ; secondly, (it's the aim of the present work) it can be shown that the average CV estimation along the whole muscle unit, can be improved by using intermediate CV estimates under some linear constraints on the TDs. We will show in simulation, the influence of *K* and changes in the MUAP shape onto this improvement.

II. ESTIMATION OF MULTIPLE TDs ALONG THE ARRAY

Several models of the recording of the propagating MUAPS along the muscle fiber has been proposed [6]. We will use the following:

$$x_k(n) = s(n - d_k) + w_k(n) \quad k = 1, \ldots, K; n = 1, \ldots, N \qquad (1)$$

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Abstract

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where $x_k(n)$ is the signal recorded by the electrode number k. In order to simplify we can assume that $d_1 = 0$. The $w_k(n)$'s are mutually uncorrelated, with zero mean and the same variance σ^2. This model is proposed assuming that the CV is a deterministic variable that varies spatially (i.e. along the muscle unit): therefore, the TDs between consecutive couples of electrodes will vary along the array.

As shown in [6], the Maximum Likelihood Estimation of the d_k's is given by:

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}}\left[\frac{1}{2\sigma^2}\sum_{i=1}^{K-1}\sum_{n=1}^{N}\left[-2x_i(n)\frac{1}{K}\sum_{k=i}^{K}x_k(n+d_k-d_i)\right]\right] \quad (2)$$

Defining $\tau_{ij} = d_i - d_j$, the solution of (2) is equivalent to

$$\hat{\boldsymbol{\tau}} = \arg\max_{\boldsymbol{\tau}}\Big(\frac{1}{\sigma^2 K}\Big(\sum_{k=2}^{K}\mathrm{fcorr}_{\mathbf{x_1},\mathbf{x_k}}(\tau_{1k}) + \sum_{k=3}^{K}\mathrm{fcorr}_{\mathbf{x_2},\mathbf{x_k}}(\tau_{2k}) + \cdots + \mathrm{fcorr}_{\mathbf{x_{K-1}},\mathbf{x_K}}(\tau_{(K-1)K})\Big)\Big) \quad (3)$$

where fcorr( ) stands for the cross-correlation function and defining vectors as : $\hat{\mathbf{d}} = [\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_K]^T$, $\mathbf{x_k} = [x_k(1), x_k(2), \ldots, x_k(N)]^T$ and $\hat{\boldsymbol{\tau}} = [\hat{\tau}_{12}, \hat{\tau}_{13}, \ldots, \hat{\tau}_{1K}, \hat{\tau}_{23}, \ldots, \hat{\tau}_{2K}, \ldots, \hat{\tau}_{(K-1)K}]^T$

Thus the optimal result is given by performing an exhaustive search over the d_i's, a (K-1) dimensional maximization, with substituting the τ_{ij} by $d_j - d_i$ in (3). Indeed the method is optimal but time consuming when high resolution is needed.

An alternative approach uses a suboptimal result given by a post correlation processing: maximizing each term of (3) with respect to all shift operations, we obtain the correlation estimates $\tilde{\tau}_{ij}$,we call coarse estimates. Note that the TDs are estimated continuously by fitting a parabola on the apex of the cross-correlation [1]. Then exploiting the K(K-1)/2 linear relations between the τ_{ij} as constraints (Chasles relations), leads to fine estimates. The advantage of this approach is the rapidity and high resolution result with respect to the optimal approach. This suboptimal approach can be presented as following.

First let us define the K(K-1)/2 linear relations between the τ_{ij}. These linear independent relations (Chasles relations) will supply additional information and can be exploited to improve the $\tilde{\boldsymbol{\tau}}$ estimate. These relations can be written as:

$$\begin{gathered}
\tau_{12} + \tau_{23} = \tau_{13} \\
\vdots \\
\tau_{12} + \tau_{2K} = \tau_{1K} \\
\tau_{23} + \tau_{34} = \tau_{24} \\
\vdots \\
\tau_{23} + \tau_{3K} = \tau_{2K} \\
\vdots \\
\tau_{(K-2)(K-1)} + \tau_{(K-1)K} = \tau_{(K-2)K}
\end{gathered}$$

In matrix form this can be written as:

$$\mathbf{C}\boldsymbol{\tau} = \mathbf{0}$$

where $\mathbf{C}$ is the matrix of constraints with elements (-1,0,1) and size of (K-1)(K-1)/2xK(K-1)/2 and has a full rank. This matrix of constraints is involved in the final estimation step consisting in minimizing an error vector $\mathbf{e}$ in the set of linear equations that relates the vector to be estimated $\boldsymbol{\tau}$ to the continuous coarse TD already estimated $\tilde{\boldsymbol{\tau}}$ together with the constraints:

$$\begin{bmatrix}\mathbf{0}\\ \tilde{\boldsymbol{\tau}}\end{bmatrix} = \begin{bmatrix}\mathbf{C}\\ \mathbf{I}\end{bmatrix}\boldsymbol{\tau} + \begin{bmatrix}\mathbf{0}\\ \mathbf{e}\end{bmatrix} \quad (4)$$

where $\mathbf{0}$ is a (K-1)(K-2)/2x1 zero vector, $\mathbf{I}$ is a K(K-1)/2xK(K-1)/2 identity matrix, $\tilde{\boldsymbol{\tau}}$ is the K(K-1)/2x1 vector of coarse interelectrode TD estimates. The vector $\mathbf{e}$ contains K(K-1)/2 error elements with $e_{ij} = \tilde{\tau}_{ij} - \tau_{ij}$.

The least square estimator of the TDs $\hat{\boldsymbol{\tau}}$ from (4) and defining $\mathbf{Q} = \begin{bmatrix}\mathbf{C}\\ \mathbf{I}\end{bmatrix}$, would be given by:

$$\hat{\boldsymbol{\tau}} = (\mathbf{Q}\mathbf{R}^{-1}\mathbf{Q})^{-1}\mathbf{Q}^{\mathrm{T}}\mathbf{R}^{-1}\begin{bmatrix}\mathbf{0}\\ \tilde{\boldsymbol{\tau}}\end{bmatrix} \quad (5)$$

with $\mathbf{R}$, the covariance matrix of $\begin{bmatrix}\mathbf{0}\\ \mathbf{e}\end{bmatrix}$. Using some particular assumption one gets the final result:

$$\hat{\boldsymbol{\tau}} = \left[\mathbf{I} - \boldsymbol{\Phi}\mathbf{C}^{\mathrm{T}}(\mathbf{C}\boldsymbol{\Phi}\mathbf{C}^{\mathrm{T}})^{-1}\mathbf{C}\right]\tilde{\boldsymbol{\tau}} \quad (6)$$

with $\boldsymbol{\Phi} = \gamma\mathbf{I}$ and γ the variance of the error $\mathbf{e}$. Note that due to the form of (6) the value of γ is not needed.

Since $\hat{\boldsymbol{\tau}}$ is given by a suboptimal approach, the variance of the $\hat{\tau}_{ij}$ cannot reach the Cramer-Rao Lower Bound (CRLB) given by [6], for any value of K:

$$\mathbf{var}(\hat{\tau}_{ij}) = \mathbf{var}(\hat{d}_j - \hat{d}_i) = \mathbf{var}(\hat{d}_i) \geq \frac{2\sigma^2}{\mathbf{s'}^T\mathbf{s'}}$$

where $\mathbf{s'}$ is the first derivative of the signal with respect to time. One could notice that this theoretical bound doesn't depend on the number of sensors (K) meaning that increasing K doesn't lead to a performance improvement. In fact, the estimator variance will tend toward the CRLB as K tends to infinity meaning that the accuracy of the proposed estimator will be a function of K.

This improvement of the TDs estimation is crucial since the CV and TDs are linked by the inverse function. As we will see in the following, the calculation of the CV using the set of TDs can be achieved in various way.

III. VELOCITY ESTIMATION

The velocity can be estimated from the well known relation $v = l/d$ where l is the distance between two consecutive electrodes and d is the TD between these electrodes expressed in number of samples. An analytic form of the pdf of the velocity estimator can be obtained provided that the TD pdf is known. The choice of Gaussian pdf is a good one and can be satisfied, since the TD estimators are obtained from cross-correlations and this is a MLE. Moreover, the noise processes are considered to be Gaussian. The relation between the delay pdf $p_{\hat{\tau}}$ and the velocity pdf $p_{\hat{V}}$ is given by :

$$p_{\hat{V}}(v) = \frac{\rho}{v^2} p_{\hat{\tau}}(\frac{\rho}{v})$$

where $\rho = lf_s$, f_s being the sampling frequency in Hertz and l is the distance in meters. Taking a Gaussian pdf for the fine TD estimators $\hat{\tau}$, with mean $\mu_{\hat{\tau}}$ (this is equal to the exact TD for a non-biased delay estimator) and variance $\sigma_{\hat{\tau}}^2$, then the velocity pdf is given by :

$$p_{\hat{V}}(v) = \frac{\rho}{\sqrt{2\pi\sigma_{\hat{\tau}}^2 v^2}} \mathbf{exp}\left[-\frac{(\rho - v\mu_{\hat{\tau}})^2}{2\sigma_{\hat{\tau}}^2 v^2}\right] \qquad (7)$$

One should note that from a theoretical point of view the mathematical expectation of $\hat{V}$ using (7) as pdf doesn't exist. Using truncation in numerical calculation avoid this problem. For a given velocity (i.e. a given constant ratio of $l / \mu_{\hat{\tau}}$ with arbitrary l and $\mu_{\hat{\tau}}$ values) the variance of the velocity estimator will be high when l and $\mu_{\hat{\tau}}$ are small, because noting the expression of $p_{\hat{V}}(v)$, we can see that the mathematical expectation of $\hat{V}$ explicitly depends on l and $\mu_{\hat{\tau}}$ and not only on the ratio $l / \mu_{\hat{\tau}}$. This remark is illustrated by giving mean ($\mu_{\hat{V}}$) and variance ($\sigma_{\hat{V}}^2$) of $\hat{V}$ for different values of on l and $\mu_{\hat{\tau}}$ with a constant ratio of the two variables, as in Tab.1.

TABLE I
MEAN AND VARIANCE OF CV

$\mu_{\hat{\tau}}$	l	$\mu_{\hat{V}}$	$\sigma_{\hat{V}}^2$
5	0.01	4.16	18.3
10	0.02	4.04	16.5
15	0.03	4.01	16.2

From the precedent theoretical analysis we conclude that the smaller the distance between electrodes (smaller TD), the higher will be the velocity estimator variance. This leads us to estimate TDs from widely spaced electrodes, but here again we are limited from the muscle unit length and spatial velocity resolution points of view. Another reason to take an array of signals with small interelectrode distance, is to get shape resemblance between signals in the array and consequently better TD estimation using correlation.

Keeping in mind that we have a vector of TD estimates obtained from different couple of electrodes in the electrode array, it is expected that an average estimate of the velocity from all the independent TDs will decrease the velocity variance. The average velocity is the harmonic average v_H , since we have equal electrode spacing and probably varying velocity. This is calculated as :

$$\frac{1}{\hat{v}_H} = \left(\frac{1}{K-1}\right)\left(\frac{1}{\hat{v}_2} + \frac{1}{\hat{v}_3} + \ldots + \frac{1}{\hat{v}_K}\right)$$

where $\hat{v}_i = lf_s / \hat{\tau}_{i-1,i}$, then

$$\hat{v}_H = \left(\frac{(K-1)lf_s}{\hat{\tau}_{12} + \hat{\tau}_{23} + \ldots + \hat{\tau}_{(K-1)K}}\right) \qquad (8)$$

When the total length of the sensors array is given equal to L, and adapted to the length of the muscle fibers, the distance between successive sensors is $l = L/(K-1)$. So, (9) becomes:

$$\hat{v}_H = \left(\frac{Lf_s}{\hat{\tau}_{12} + \hat{\tau}_{23} + \ldots + \hat{\tau}_{(K-1)K}}\right) \qquad (9)$$

The average CV can be also estimated by taking the TD ($\hat{\tau}_L$) (between the two most separated signals (using the most separated sensors) only:

$$\hat{v}_L = \frac{Lf_s}{\hat{\tau}_L} \qquad (10)$$

But as mentioned above there are some disadvantages when used with real signals because of the signal shape variation between larger spaced electrodes. From a theoretical point of view and taking the CRLB as variance for the estimated $\hat{\tau}_{ij}$, the variance of the sum $\hat{\tau}_{12} + \hat{\tau}_{23} + \ldots + \hat{\tau}_{(K-1)K}$ (appearing in (9)) is increasing as K becomes larger, leading to a loss of accuracy in the CV estimation. This fact seems to be in contradiction with the assumption that the accuracy of CV estimation will be higher as the number of sensors increases. We will see in the following that since practically the variance of $\hat{\tau}_{ij}$ is not the CRLB one, this conclusion is not valid.

IV. SIMULATIONS AND RESULTS

In the following simulations, we will show that on the contrary to the ideal approach (TDs variance equalizes the CRLB) the performance of CV estimation (9) is improved when taking large value of K. We also show the influence on the estimates (9) and (10) of a change in shape of the MUAP along the fiber when K increases. In both cases the duration of the simulated signal is about 40 samples (corresponding to a 20ms duration MUAP sampled at a 2000Hz sampling rate) and the constant CV equal 1 m/sec. The criteria used in order to characterize the accuracy is the Mean Square Error (MSE) defined as $mse = \sigma_{\hat{v}}^2 + bias^2$ since it has been shown that uncertainty in the TDs estimation adds bias in the CV estimation.

A. Influence of K value on the CV estimation

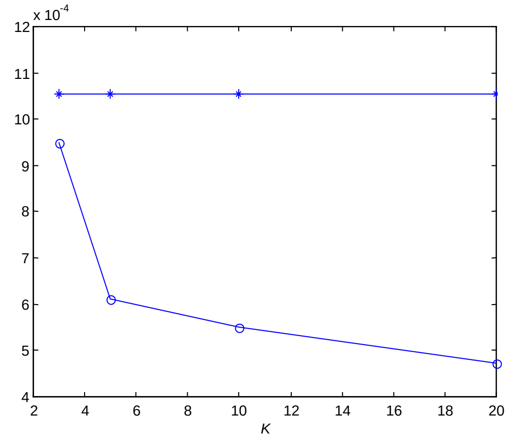

Fig. 1: MSE of CV estimation (9) and (10) for *K=3,5,10,20 (circle) and K=2 (only two sensors)(stars)*

In this simulation the SNR is about 18dB and the number of trials to calculate the *mse* is equal to 500. The result of this simulation is given in Fig.1 where the *mse*, characterizing (9) and (10), defined above is a function of $K = (3,5,10,20)$.

B. Influence of change in shape on the CV estimation

The aim of this simulation is to show the advantage in using more than two sensors when the signal is subject to change in shape along the muscle fiber. In Fig.2, signals used in this simulation (no noise) are shown for *K*=3, intermediate shape are easily deduced when *K* is higher than 3.
In Fig.3, *mse* for (9) and (10) are calculated adding a change in shape. Since signal are not noisy, *mse* criteria reveals only the bias.

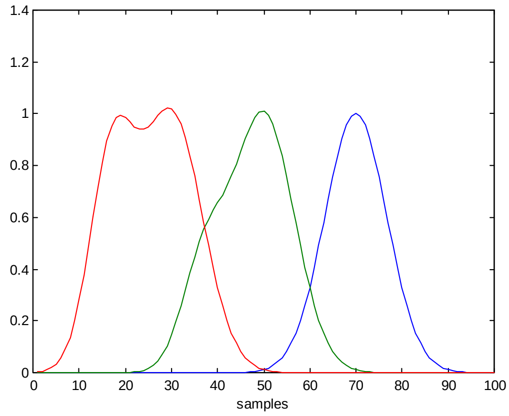

Fig. 2: three simulated signals when *K*=3

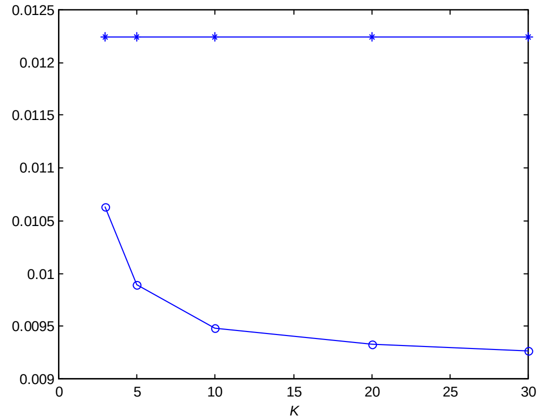

Fig. 3: MSE of CV estimation for *K=3,5,10,20,30 (circle) and K=2 (only two sensors)(stars)*

V. DISCUSSION

In this communication, in order to calculate the Conduction Velocity in the muscle fiber, Time Delays are estimated from an array of *K* sensors. After giving a theoretical study of the CV estimation performances, we have shown in simulations the obvious advantage in using more than 2 sensors (*K*>2) but due to the physical limitation (sensors size) and the limited gain (Fig.1and Fig.3) for *K*>15, a bound for the increasing of *K* can be proposed. When focusing on the bias reduction in Fig.3, the improvement is not very important probably due to the signal type used for the simulation.

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