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The Ideal Constant Volume Limit of Pulsed Propulsion

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SOME ANALYTICAL SOLUTIONS FOR THE IDEAL CONSTANT VOLUME LIMIT OF PULSED PROPULSION

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ABSTRACT

A constant volume limit of pulsed propulsion is formulated which applies when blow down times are much longer than characteristic wave transit times in the combustion chamber. Under this limit, the combustion chamber is approximated as being time varying but spatially uniform, and the nozzle is approximated as being one dimensional but quasi-steady. Some analytical solutions for this limit with fixed expansion ratio nozzles are explored for the isentropic blow down of a constant γ ideal gas. The results are compared with a variable expansion ratio case where the expansion ratio is continuously varied to match the pressure ratio during blow down. The major conclusions are that constant volume devices should optimize at approximately the same mixture ratios as constant pressure devices, and that using a fixed expansion ratio results in only a modest impulse penalty, not exceeding 3% for the cases examined, compared to using a variable expansion ratio, as long as the fixed expansion ratio has been optimized to produce the maximum possible impulse for the blow down.

1. INTRODUCTION

Recent interest in pulsed detonation propulsion has spawned a number of attempts to model the system performance, the results from which have so far varied widely [1]. Constant volume (CV) combustion, as refered to here, is a limiting case for pulsed combustion cycles which is approached when blow down times become much longer than characteristic wave transit times in the combustion chamber (large combustion chambers and small nozzles). In this limit, the combustion chamber can be approximated as being time varying but spatially uniform, and the nozzle flow can be approximated as being one dimensional but quasi-steady. Compared with how a real device is likely to operate, the CV limit probably underpredicts the thrust (unrealistically small nozzles), but the specific impulse is probably close or equal to an upper bound. Thus the CV limit is a useful reference case. Some analytical solutions for the CV limit are explored here for the isentropic blow down of a constant γ ideal gas at fixed expansion ratio. The results are compared with the case where the expansion ratio is continuously varied to match the pressure ratio at all times during the blow down.

2. FORMULATION

2.1 General

The impulse produced by the unsteady blow down of the combustion gases in the CV limit is the integral of the thrust $F = \dot{m}v_e + (P_e - P_\infty)A_e$ over time, where $\dot{m}$ is the mass flow rate, P_∞ is the ambient pressure, and v_e, P_e, and A_e are the velocity, pressure, and area, respectively, at the exit plane. Under the transformation $dt = (dt/d\rho)d\rho = -(V/\dot{m})d\rho$ and the further transformation $r \equiv \rho / \rho_0$, where V is the combustion chamber volume and ρ_0 is the initial chamber density after combustion but before blow down at time $t = 0$, the total impulse I and blow down time t may be expressed as

$$I(t) = \int_0^t F(t)dt = \rho_0 V \int_r^1 (v_e + (P_e - P_\infty)A_e / \dot{m})dr\,, \qquad (1)$$

$$t = \int_0^t dt = \rho_0 V \int_r^1 dr / \dot{m} \qquad (2)$$

2.2 Fixed Nozzles

For isentropic flow, the instantaneous exit velocity is $v_e = \sqrt{2c_p(T - T_e)}$, where T and T_e are the instantaneous chamber and exit temperatures, respectively. Pressures and densities in both the chamber and the nozzle will be related in a simple fashion to temperatures according to $T/T_0 = (\rho/\rho_0)^{\gamma-1} = (P/P_0)^{(\gamma-1)/\gamma}$, where the subscript "0" denotes conditions in the chamber at time $t = 0$. With $c_0 = \sqrt{\gamma R T_0}$ and $c_p = \gamma R/(\gamma - 1)$, the exit velocity can be expressed as

$$v_e = c_0 r^{(\gamma-1)/2}\left(\frac{2}{\gamma-1}\right)^{1/2}\sqrt{1 - r_e^{\gamma-1}}\,, \qquad (3)$$

where γ is the ratio of specific heats and $r_e \equiv \rho_e / \rho$ is the ratio of the density at the exit plane to the density in the chamber. For isentropic nozzle flow, the exit density ratio is related to the expansion ratio $\varepsilon \equiv A_e / A^*$, where A^* is the throat area, by ([2], eq. 5.3)

$$\varepsilon = \frac{g(\gamma)}{r_e\sqrt{1 - r_e^{\gamma-1}}}; \qquad g(\gamma) \equiv \left(\frac{\gamma-1}{2}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{(1/2)(\gamma+1)/(\gamma-1)} \qquad (4)$$

Equation (4) has two solutions corresponding to subsonic and supersonic flow at the exit plane, and these solutions will depend only on ε and γ. For supersonic flow, the density ratio r_e will therefore remain constant during blow down as long as compression waves do not enter the nozzle. The exit velocity term in (1) will therefore be analytically integrable as long as this is true.

For the supersonic solution, the choked throat conditions will be functions only of r and γ. For a throat density and velocity given by ([2], eq.2.35) $\rho^* = r\rho_0[2/(\gamma+1)]^{1/(\gamma-1)}$ and $v^* = \sqrt{\gamma R T^*} = c_0(2/(\gamma+1))^{1/2} r^{\gamma-1}$, and for $P_e = P_0 r_e^\gamma r^\gamma$, the instantaneous mass flow and thrust become

$$\dot{m}(r) = \rho^* A^* v^* = \rho_0 c_0 A^* g(\gamma)\left(\frac{2}{\gamma-1}\right)^{1/2} r^{(\gamma+1)/2} \qquad (5)$$

$$F(r) = P_0 A^* g(\gamma)\left[r^\gamma \gamma \frac{2}{\gamma-1}\sqrt{1 - r_e^{\gamma-1}} + \frac{1}{r_e\sqrt{1 - r_e^{\gamma-1}}}(r_e^\gamma r^\gamma - \frac{P_\infty}{P_0})\right], \qquad (6)$$

where (4) has also been used. Thus all terms in (1) will integrable analytically.

Define $\phi_0 \equiv P_0 / P_\infty$ to be the ratio of the initial pressure at time $t = 0$ to the ambient pressure, and define $\iota \equiv I / \rho_0 c_0 V$ and $\tau \equiv t c_0 A^* / V$ to be the normalized impulse and the normalized time, respectively. Then integrating (1) and (2) gives

$$\iota(r) = \sqrt{1 - r_e^{\gamma-1}} \left(\frac{2}{\gamma-1}\right)^{1/2} \left(\frac{2}{\gamma+1}\right) a(r) + \\ + \frac{1}{\gamma r_e \sqrt{1 - r_e^{\gamma-1}}} \left(\frac{\gamma-1}{2}\right)^{1/2} \left[r_e^{\gamma} \frac{2}{\gamma+1} a(r) - \frac{1}{\phi_0} \left(\frac{2}{\gamma-1}\right) b(r) \right] \quad (7)$$

$$\tau(r) = \frac{1}{g(\gamma)} \left(\frac{2}{\gamma-1}\right)^{1/2} b(r), \quad (8)$$

where

$$a(r) \equiv 1 - r^{(\gamma+1)/2} \qquad b(r) \equiv (1/r)^{(\gamma-1)/2} - 1. \quad (9)$$

The quantity ι / τ will be related to the average thrust $\overline{F} = I / t$. Noting that $c_0^2 = \gamma R T_0$ and $P_0 = \rho_0 R T_0$, then combining constants after dividing leads to

$$\gamma \iota / \tau = \overline{F} / P_0 A^* \equiv \overline{c}_F, \quad (10)$$

where $\overline{c}_F$ is the average thrust coefficient for the blow down.

2.3 Variable Nozzles

Variable nozzles can be envisioned where the expansion ratio is continuously adjusted to match the pressure ratio at all times to produce the maximum possible impulse. With pressures always matched, only the exit velocity term $v_e = \sqrt{2 c_p (T - T_e)}$ in (1) will be of concern. For fixed nozzles, it was found that the term $T_e / T = r_e^{\gamma-1}$ was a constant, but here the exit temperature is fixed by the exit conditions $P_e = P_\infty$, $\rho_e = \rho_0 (1/\phi_0)^{1/\gamma}$, and $T_e = T_0 (1/\phi_0)^{(\gamma-1)/\gamma}$. Therefore T_e / T will not be constant in this case. If the exit area is assumed to be varied to match pressures at all times, as might approximately be the case with an automatically compensating nozzle such as a plug nozzle, the mass flow and blow down time based on a constant throat area are still given by (5) and (8), respectively. The normalized impulse becomes

$$\iota(r) = \left(\frac{2}{\gamma-1}\right)^{1/2} \int_r^1 \sqrt{r^{\gamma-1} - (1/\phi_0)^{(\gamma-1)/\gamma}}\, dr. \quad (11)$$

Eq. (11) has no convenient analytical solution, but may easily be integrated numerically.

3. RESULTS AND DISCUSSION

3.1 Effect of Thermochemistry

The dimensionless impulse and blow down time (7)-(9) depend explicitly on r and implicitly only on γ, r_ε (or ε), and ϕ_0. Of these, only γ is influenced by the thermochemistry, but only weakly. The major influence of thermochemistry comes through the initial speed of sound c_0 used to normalize the impulse. Thus the specific impulse $I_s = \iota c_0$ is maximized when c_0 is maximized. This in turn implies that the optimum thermochemistry is that which maximizes the initial combustion temperature T_0 and minimizes the molecular weight. The same general guidance is known to also apply to constant pressure devices. *Thus constant volume devices should optimize at approximately the same mixture ratios as constant pressure devices.* Also, like constant pressure devices, the thrust is maximized by maximizing the initial chamber pressure and the throat area, as implied by (10).

3.2 Fixed Nozzles

Much can be understood about the blow down of fixed nozzles by examining conventional steady state thrust coefficient curves such as can be found in standard texts [3]. The thrust coefficient is defined in (10), where P_0 in the steady state case is interpreted to be the steady chamber pressure of a constant pressure device. A set of these curves is given in Fig. 1 for $\gamma = 1.2$. Dimensionless isobars (curves of constant ϕ_0) initially increase with ε, reach a maximum, then decrease to a minimum where a shock enters the nozzle and the above formulation is no longer valid. The curve for $\phi_0 = \infty$ reaches a maximum only at $\varepsilon = \infty$. A curve connecting the maxima indicates the expansion ratio producing the maximum thrust for a given isobar.

The blow down of a fixed nozzle CV device proceeds along a vertical path of constant ε between two isobars. The thrust produced will be some average between the two isobars. Picking any two isobars in Fig. 1, say between $\phi_0 = 50$ and $\phi_0 = 20$, and following the vertical line between them for various ε, it can be envisioned that the average thrust will reach a maximum neither at large ε nor at $\varepsilon = 1$, but at some optimum ε in between. However, the average thrust cannot be calculated directly from Fig. 1, because the thrust coefficient is proportional to the thrust divided by the chamber pressure, not the thrust alone. A blow down from $\phi_0 = 50$ to $\phi_0 = 20$ is replotted in Fig. 2, where the curve for $\phi_0 = 20$ is normalized by the same pressure as for $\phi_0 = 50$, making all curves proportional to the thrust. This is accomplished by multiplying the thrust coefficient of Fig 1. by 0.4 for $\phi_0 = 20$. The curve labeled "b.d." gives the average thrust coefficient for the blow down and is seen to be an average of the curves for $\phi_0 = 50$ and $\phi_0 = 20$, as expected.

Average thrust coefficients are plotted as functions of ϕ_0 and ε for blow downs to $r = 0.75$, 0.5, and 0.25 in Figs. 3, 4, and 5, respectively. Dimensionless blow down times are plotted as a function of r for two values of γ in Fig. 6. As can be seen from (8) and Fig. 6, blow down times are independent of ϕ_0, and depend only weakly on γ. Because τ is fixed and independent of ϕ_0 for a fixed r, the curves in Figs. 3, 4, and 5 will also be proportional to the dimensionless impulse ι, as can be shown from (10). However, the constant of proportionality will be different for each r, increasing as r decreases, because τ increases as r decreases. The constant of proportionality is given in the figures. The overall trend in these figures is that the average thrust coefficient decreases as r decreases. This is due to τ increasing more rapidly than ι as r decreases, because ι also increases as r decreases.

The expansion ratio which produces the maximum impulse can in principle be found by taking the first derivative of (7) with respect to r_e and setting the result equal to zero. The resulting expression cannot be solved analytically, however, so the optimum expansion ratio is computed here by finding the maximum impulse using a numerical search. The optimum expansion ratio is plotted as a function of ϕ_0 and r in Fig. 7, and the dimensionless impulse produced at the optimum expansion ratio is plotted in Fig. 8. Because r is variable in these figures, τ is also, so the dimensionless impulse and the average thrust coefficient curves are no longer directly proportional as they were in earlier figures, where r was fixed. The optimum thrust coefficient is plotted in Fig. 9.

3.3 Variable Nozzles

Because the expansion ratio is always optimized for variable nozzles, the blow down of variable nozzles proceeds along the curve of maximum thrust in Fig. 1. The dimensionless impulse produced can be computed as a function of ϕ_0 and r by integrating (11), and compared with the performance of fixed nozzles operating at the optimum expansion ratio. The results are not easily viewed in the form of Fig. 8, however, so they are plotted instead here in Fig. 10 as the impulse penalty in using optimized fixed nozzles compared to the variable nozzle case, based on a percentage of the variable nozzle impulse. The penalty is seen to be very modest, not exceeding about 3% for all cases calculated. The reason the penalty is not more severe can be traced to the flatness of the curves in Fig. 1 near the points of maximum thrust. This is illustrated more clearly in Fig. 2, where the difference between the variable and optimized fixed nozzle blow down paths is shown. As can be seen, the maximum instantaneous thrust coefficient is not that much higher for the variable nozzle case than for optimized fixed nozzles, leading to calculated average thrust coefficients which are very close.

4.0 CONCLUSIONS

Analytical solutions of the constant volume limit of pulsed propulsion for fixed expansion ratios have been explored and compared with the case where a variable nozzle is used to match the pressure ratio at all times during the blow down. The solutions apply for all supersonic flows where compression waves remain exterior to the nozzle. It is found that:

1. CV devices should optimize at the approximately the same mixture ratio as CP devices, namely the mixture ratio which maximizes the initial temperature and minimizes the molecular weight.

2. The thrust of CV devices is maximized in the same way as for CP devices, namely by maximizing the initial pressure and the throat area.
3. The blow down time depends on $r = \rho / \rho_0$, only weakly on γ, and is independent of the pressure ratio $\phi_0 = P_0 / P_\infty$.
4. In general an optimum expansion ratio exists which maximizes the impulse produced by the blow down of a CV device at a fixed expansion ratio.
5. Using an optimized fixed expansion ratio nozzle results in only a modest impulse penalty compared to using the more complicated variable expansion ratio nozzle. The penalty does not exceed 3% of the impulse of the variable nozzle for all cases considered.

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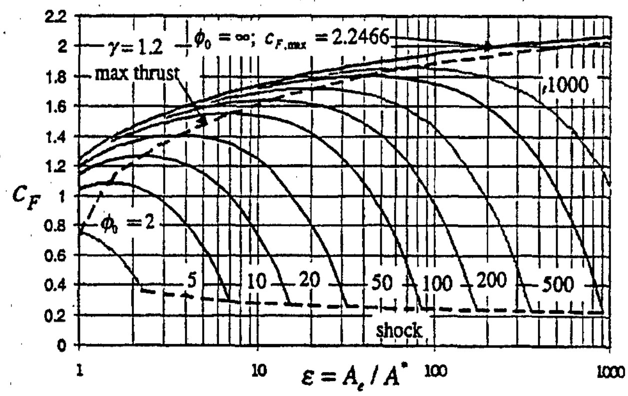

Fig. 1. Steady state thust coefficients

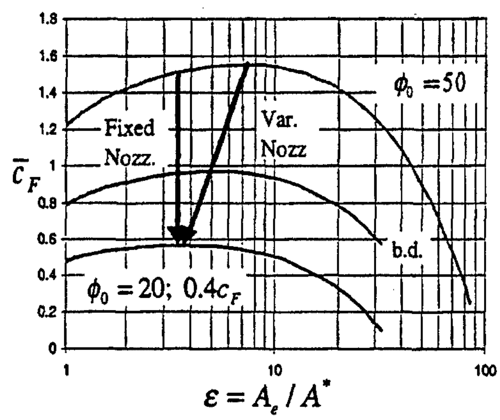

Fig. 2. Blow down from $\phi_0 = 50$ to $\phi_0 = 20$, where "b.d." curve is for the blow down. Arrows indicate paths for fixed and variable nozzles.

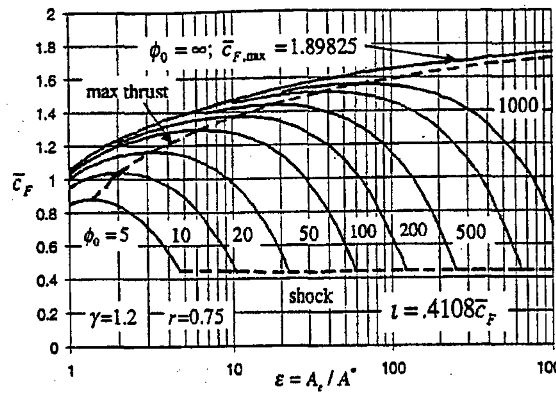

Fig. 3. Blow down to $r = 0.75$

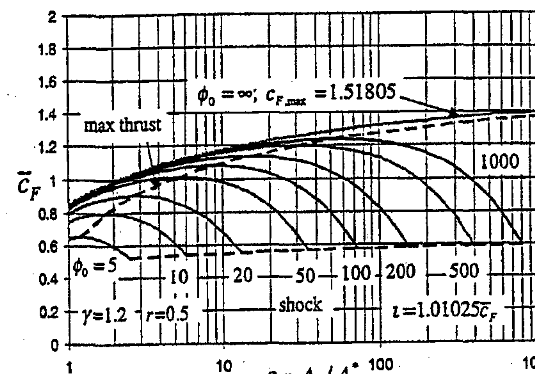

Fig.4. Blow down to $r = 0.5$

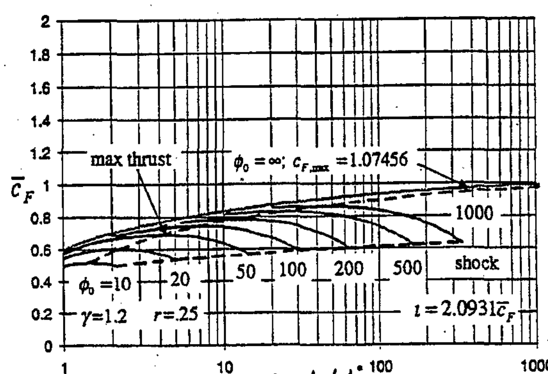

Fig. 5. Blow down to $r = 0.25$

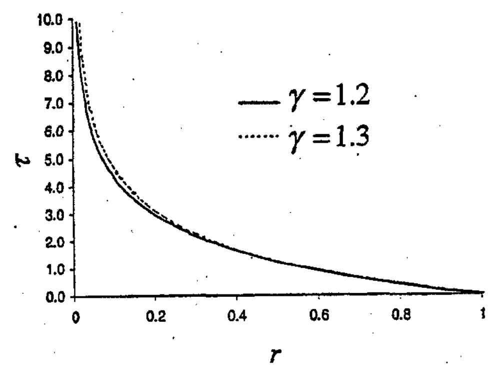

Fig. 6. Blow down times

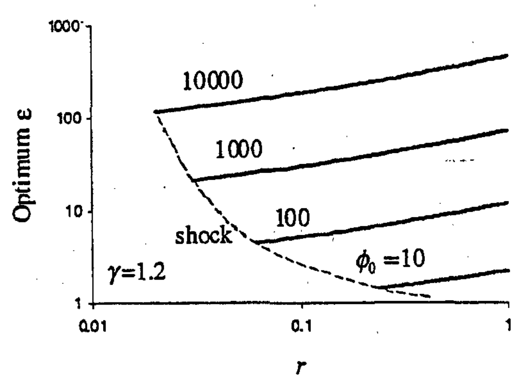

Fig. 7. Optimum expansion ratios

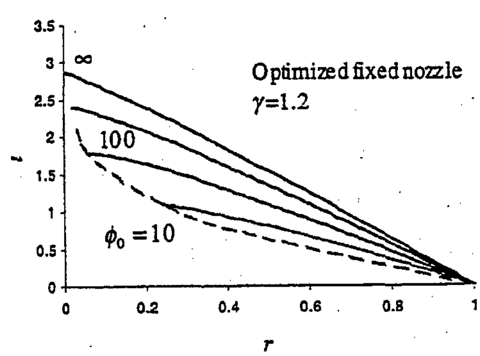

Fig. 8. Optimum impulse

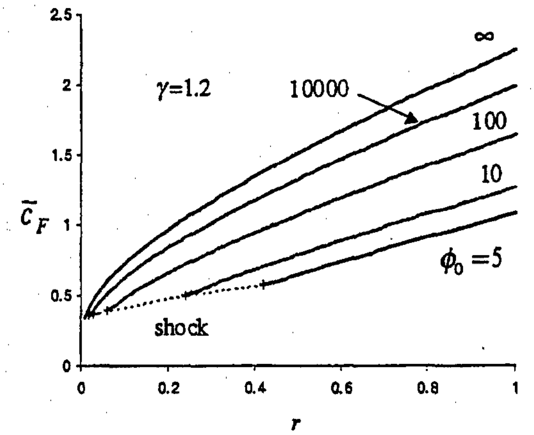

Fig. 9. Optimum thurst coefficient

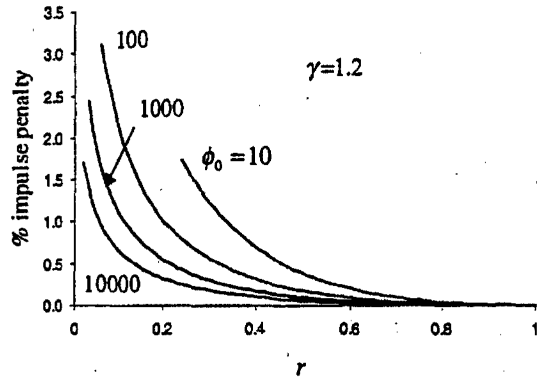

Fig. 10. Impulse penalty of an optimized fixed nozzle compared to a variable nozzle .